Title
MULTIPERIPHERAL THEORY OF DEEP INELASTIC ELECTRON-NUCLEON SCATTERING

Authors
Shei, Sun-Sheng
Tow, Don M.

Publication Date
1971-04-01

Submitted to
Physical Review

S. -S. Shei

UCRL-20677
Preprint

MULTIPERIPHERAL THEORY OF DEEP INELASTIC ELECTRON-NUCLEON SCATTERING

Sun-Sheng Shei and Don M. Tow*

April 22, 1971

AEC Contract No. W-7405-eng-48

*Filed as a Ph. D. thesis

DISCLAIMER

MULTIPERIPHERAL THEORY OF DEEP INELASTIC ELECTRON-NUCLEON SCATTERING*

Sun-Sheng Shei and Don M. Tow

Lawrence Radiation Laboratory
University of California
Berkeley, California 94720

April 22, 1971

ABSTRACT

The Amati-Bertocchi-Fubini-Stanghellini-Tonin multiperipheral model is modified to study deep inelastic electron-nucleon scattering. The behavior of the structure functions is derived in the limit of large energy and momentum transfer, ν and q^2, with $\omega \equiv \frac{2m\nu}{-q^2}$ fixed but large. In particular, scaling is derived. To accomplish this derivation it is necessary to introduce a cutoff for the momentum transfer of the exchanged nucleon, which directly couples to the photon in the multiperipheral chain. The multiperipheral derivation is compared with derivations that use other models. The generality of our derivation, in the sense of requiring only Regge behavior and a particular asymptotic off-shell dependence, is discussed.

I. INTRODUCTION

One method to probe the structure of hadrons is to do inelastic electron-nucleon scattering experiments. The electron interacts with the nucleon by means of exchanging photons. The part that is known about this reaction is the electron-photon vertex, given by ordinary quantum electrodynamics. The unknown part, due to the hadronic structure of the nucleon, is the photon-nucleon vertex. If only the final electron is observed in unpolarized inelastic e-N scattering, then in the one-photon-exchange approximation, all information is contained in $W_{\mu\nu}$ or the two structure functions W_1 and W_2. They are defined by

$$W_{\mu\nu} \equiv 4\pi^2 \frac{E_p}{m} \sum_n \langle p|J_\mu(0)|n\rangle\langle n|J_\nu(0)|p\rangle (2\pi)^4 \delta^4(q+p-p_n)$$

$$= -\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) W_1(q^2,\nu) + \frac{1}{m^2}\left(p_\mu - \frac{p\cdot q}{q^2} q_\mu\right)\left(p_\nu - \frac{p\cdot q}{q^2} q_\nu\right) \times W_2(q^2,\nu), \qquad (I.1)$$

where $|p\rangle$ is a one-nucleon state with four-momentum p_μ and mass m, $J_\mu(x)$ is the total hadronic electromagnetic current operator, q_μ, q^2, and $\nu \equiv \frac{p\cdot q}{m}$ are respectively the four-momentum of the virtual photon, its mass squared, and its energy in the laboratory system. The nucleon spin has been averaged in the definition of $W_{\mu\nu}$. The differential cross section in the laboratory system is given by

$$\frac{d^2\sigma}{dE'd\cos\theta} = \frac{8\pi\alpha^2 E'}{q^4}\left[\cos^2\left(\frac{\theta}{2}\right) W_2(q^2,\nu) + 2\sin^2\left(\frac{\theta}{2}\right) W_1(q^2,\nu)\right], \qquad (I.2)$$

where E' and θ are the final energy and the scattering angle of the electron. The kinematics are illustrated in Fig. 1.

A great deal of attention has recently been devoted to the study of deep inelastic e-N scattering from both experimental and theoretical points of view.[1] Many theoretical models have been proposed. They include parton model,[2] canonical field-theory model,[3] diffraction model,[4] vector-meson-dominance model,[5] and Veneziano-like model.[6] Particular emphasis is on the scaling property of W_1 and νW_2, as first suggested by Bjorken,[7] in the Bjorken limit of $\nu \to \infty$ with $\frac{2m\nu}{-q^2}$ fixed.

The purpose of this paper is to present another model[8] for deep inelastic e-N scattering. Our model is a simple modification of the Amati-Bertocchi-Fubini-Stanghellini-Tonin (ABFST) multiperipheral model (MPM),[9] which is known to explain many qualitative and quantitative features of high-energy hadronic interactions.[9-11] We derive several results, including the scaling property of W_1 and νW_2. All our results are consistent with present experimental data.[1,12] In Sec. II, we present our model and our results. The derivation of these results is given in Sec. III. A general discussion of our model and a comparison with previous models are contained in Sec. IV. The generality of our derivation, in the sense of requiring only Regge behavior and a particular asymptotic off-shell dependence, is also discussed in this section. Appendix A contains a generalization of the Matsuda-Suzuki argument[13] for a constant γNN vertex function. Appendix B shows our model satisfies current conservation in the asymptotic limit we are interested in.

II. OUR MODEL AND OUR RESULTS

To construct a model for inelastic e-N scattering, it is sufficient in the one-photon-exchange approximation to construct a model for Compton scattering of a virtual photon. Our model for virtual Compton scattering is a simple modification of the ABFST MPM.[9] We assume that the high-energy virtual-photon-nucleon scattering amplitude is dominated by t-channel pion poles, with the exception of the last link, that couples directly to the photon; for this last link we assume a t-channel nucleon pole. In other words, except for the last link our model is described by the ABFST MPM. Our model is shown in Fig. 2.

We now present our results, leaving the derivation to the next section. We find that a cutoff (or an extra damping factor) for the momentum transfer of the exchanged nucleon is needed. This cutoff is consistent with the experimental fact that for high-energy scattering the transverse momentum of secondaries is limited. When this cutoff is introduced, we find in the limit

$$\nu \to \infty, \quad -q^2 \to \infty, \text{ and } \quad \omega \equiv \frac{2m\nu}{-q^2} \text{ fixed but large} \qquad \text{(II.1)}$$

that

$$W_1 = c\omega^{\alpha}, \quad \nu W_2 = 2mc\omega^{\alpha-1}, \qquad \text{(II.2)}$$

which implies that

$$\frac{W_1}{\nu W_2} = \frac{\omega}{2m}, \qquad \text{(II.3)}$$

$$\frac{\sigma_S}{\sigma_T} = \frac{-q^2}{\nu^2} \to 0, \qquad \text{(II.4)}$$

where σ_S and σ_T are the "longitudinal" and "transverse" photo-absorption cross sections. They are defined in terms of W_1 and W_2 by

$$W_1 = \frac{K}{4\pi^2\alpha}\,\sigma_T ,$$

$$W_2 = \frac{K}{4\pi^2\alpha}\,\frac{q^2}{(q^2 - \nu^2)}\,(\sigma_T + \sigma_S) , \qquad \text{(II.5)}$$

where

$$K \equiv \nu + \frac{q^2}{2m} .$$

Result (II.4) corresponds to spin-$\frac{1}{2}$ partons[2] and to the Callan-Gross result[14] for a spin-$\frac{1}{2}$ quark current. Similarly, in the limit (II.1) we have

$$W_1^{(p)} = W_1^{(n)}, \qquad \nu W_2^{(p)} = \nu W_2^{(n)} , \qquad \text{(II.6)}$$

where superscripts p and n refer to proton and neutron, respectively. Result (II.6) is the same as that of the parton model[2] for the case of the nucleon consisting of three quarks and a quark-antiquark "sea." The average multiplicity of pions is given by

$$\langle n_\pi \rangle \sim \ell n\, \omega . \qquad \text{(II.7)}$$

Equation (II.7) implies that our model is expected to be correct only for large ω when the multiperipheral chain is long. Our results are consistent with present experimental data[1,12] and are very close to those of DLY.[3] In Sec. IV we compare the two models.

In our derivation we do not have to assume current conservation. But it can be shown that in the limit $\nu \to \infty$ with $\frac{2m\nu}{-q^2}$ fixed, our model is consistent with current conservation. In other words, in the limit $\nu \to \infty$ with $\frac{2m\nu}{-q^2}$ fixed, our model describes correctly the gauge-invariant aspect of deep inelastic e-N scattering.

III. DERIVATION

The absorptive part of the forward Compton scattering amplitude is given by the sum of diagrams of Fig. 2. This sum, together with the kinematics, is represented in Fig. 3. We first neglect the photon link on the left. Therefore, we start with the N-N amplitude. Before calculating this, we consider the π-N scattering amplitude when the spins of the external nucleons are averaged over.

From ABFST,[9] we know that the π-N amplitude is simply related to the π-π amplitude and that[9,15]

$$A_{\pi\pi}(\bar{s}',u''',u'') \underset{\bar{s}'\text{ large}}{\approx} (\bar{s}')^{\alpha}\, \phi_\alpha^{\pi\pi}(u''',u''), \qquad \text{(III.1)}$$

where

$$\phi_\alpha^{\pi\pi}(u''',u'') \underset{u''\text{ large}}{\sim} \frac{1}{(u'')^{\alpha+1}} . \qquad \text{(III.2)}$$

Figure 4 illustrates the kinematics. The absorptive part of the forward elastic π-N amplitude at the high-energy limit is given by

$$[-\mathcal{A}(s'',u'') + \not{q}''\mathcal{B}(s'',u'')]_{\pi N} = \frac{1}{8\pi^4}\int d\bar{s}_0[-\mathcal{A}^R(\bar{s}_0) + \not{q}\mathcal{B}^R(\bar{s}_0)]_{\pi N}$$

$$\times \iint \frac{d\bar{s}'\,du'''}{(u''' + \mu^2)^2}\; Q_{ABFST}(s'',-m^2;\, \bar{s}',u''';\, \bar{s}_0)\; A_{\pi\pi}(\bar{s}',u''',u''), \qquad \text{(III.3)}$$

where μ is the pion mass, the superscript R refers to low-energy π-N elastic scattering, and $Q_{ABFST}(s'',-m^2;\, \bar{s}',u''';\, \bar{s}_0)$ is the ABFST boundary function.

Since we are averaging over the spins of the nucleons, we don't have to know $\mathcal{A}$ and $\mathcal{B}$ separately. The combination we need is

$$\frac{1}{2m}\,\text{Trace}[(-\mathcal{A} + \not{p}''\mathcal{B})_{\pi N}(m + \not{p})] = 2\left(-\mathcal{A} + \frac{p\cdot p''\mathcal{B}}{m}\right)_{\pi N}$$

$$= \frac{1}{8\pi^4}\iiint \frac{d\bar{s}_0\, d\bar{s}'\, du'''}{(u''' + \mu^2)^2}\, Q_{ABFST}(s'', -m^2;\, \bar{s}', u''';\, \bar{s}_0)$$

$$\times\ [-2\,\mathcal{A}^R(\bar{s}_0) + \frac{2\,p\cdot Q}{m}\mathcal{B}^R(\bar{s}_0)]_{\pi N}\, A_{\pi\pi}(\bar{s}', u''', u''). \qquad \text{(III.4)}$$

Since

$$[-2\,\mathcal{A}^R(\bar{s}_0) + \frac{2\,p\cdot Q}{m}\mathcal{B}^R(\bar{s}_0)]_{\pi N}$$

$$= [-2\,\mathcal{A}^R(\bar{s}_0) + \frac{1}{m}(m^2 - \bar{s}_0 - u''')\,\mathcal{B}^R(\bar{s}_0)]_{\pi N}$$

does not have any u'' dependence, the u'' dependence of $A_{\pi N}$ (averaged over spin) is given by that of $A_{\pi\pi}$. And since $\bar{s}'^{\alpha}$ becomes s''^{α} after integration over $\bar{s}'$, we conclude that

$$A_{\pi N}(\text{averaged over spin}) \underset{s''\ \text{large}}{\approx} s''^{\alpha}\, \phi_\alpha^{\pi N}(u''), \qquad \text{(III.5)}$$

where

$$\phi_\alpha^{\pi N}(u'') \underset{u''\ \text{large}}{\sim} \frac{1}{(u'')^{\alpha+1}}\,. \qquad \text{(III.6)}$$

In other words $A_{\pi N}$(averaged over spin) has functional dependence similar to that of $A_{\pi\pi}$ in the limit of large s'' and u''. From now on we use $A_{\pi N}$ to designate $A_{\pi N}$(averaged over spin).

The absorptive part of the forward N-N amplitude is related to that of π-N by[9]

$$-A(p',p) + \not{p}B_1(p',p) + \not{p}'B_2(p',p)$$

$$= \frac{1}{8\pi^4}\int ds_0 \int d^4p''\, \frac{[-A^R(s_0) + \not{p}''B_1{}^R(s_0) + \not{p}'B_2{}^R(s_0)]}{(p''^2 - \mu^2)^2}$$

$$\times\ \delta[(p' - p'')^2 - s_0]\, A_{\pi N}(p'',p)\,. \qquad \text{(III.7)}$$

The reason for having two B terms is that the nucleon of momentum p' is off mass-shell. By taking trace, it follows from Eq. (III.7) that

$$A(p',p) = \frac{1}{8\pi^4}\iint ds_0 d^4p''\, \frac{A^R(s_0)\, \delta[(p' - p'')^2 - s_0]}{(p''^2 - \mu^2)^2}\, A_{\pi N}(p'',p),$$

$$\not{p}B_1(p',p) + \not{p}'B_2(p',p)$$

$$= \frac{1}{8\pi^4}\iint ds_0 d^4p''\, \frac{[\not{p}''B_1{}^R(s_0) + \not{p}'B_2{}^R(s_0)]}{(p''^2 - \mu^2)^2}\, A_{\pi N}(p'',p)\delta[(p'-p'')^2-s_0]. \qquad \text{(III.8)}$$

Let us define

$$\not{p}b_1(p',p) + \not{p}'b_2(p',p)$$

$$= \frac{1}{8\pi^4}\iint ds_0 d^4p''\, \frac{\not{p}''B_1{}^R(s_0)\, \delta[(p' - p'')^2 - s_0]}{(p''^2 - \mu^2)^2}\, A_{\pi N}(p'',p). \qquad \text{(III.9)}$$

In terms of b_1, b_2, we can express B_1 and B_2 as

$$B_1(p',p) = b_1(p',p),$$

$$B_2(p',p) = b_2(p',p) + \frac{1}{8\pi^4}\iint ds_0 d^4p''\, \frac{B_2{}^R(s_0)\delta[(p'-p'')^2-s_0]A_{\pi N}(p'',p)}{(p''^2 - \mu^2)^2}. \qquad \text{(III.10)}$$

From Eq. (III.9), we can determine b_1 and b_2 by multiplying by $\not{p}$ and $\not{p}'$ and then taking traces. The results are

$$b_1(p',p) = \frac{1}{4m^2p'^2 - (s' - p'^2 - m^2)^2} \iint \frac{ds_0 d^4p''}{8\pi^4(p''^2 - \mu^2)^2}$$

$$\times B_1^R(s_0)\ \delta[(p' - p'')^2 - s_0]$$

$$\times A_{\pi N}(p'',p)[2p'^2(s'' - p''^2 - m^2) + (s' - p'^2 - m^2)(s_0 - p''^2 - p'^2)],$$

$$b_2(p',p) = \frac{1}{4m^2p'^2 - (s' - p'^2 - m^2)^2} \iint \frac{ds_0 d^4p''}{8\pi^4(p''^2 - \mu^2)^2}$$

$$\times B_1^R(s_0)\ \delta[(p' - p'')^2 - s_0]$$

$$\times A_{\pi N}(p'',p)[-(s' - p'^2 - m^2)(s'' - p''^2 - m^2) - 2m^2(s_0 - p''^2 - p'^2)]. \qquad \text{(III.11)}$$

So far, we have concentrated on the N-N amplitude. Now we add the photon link by using ordinary Feynman rules. The tensor $W_{\mu\nu}$ is then given by

$$W_{\mu\nu}(q,p) = \frac{f^2}{16m\pi^5} \int d^4p'\ \frac{\delta[(p' - q)^2 - m^2]}{(p'^2 - m^2)^2}$$

$$\times \text{Trace}\{(-A + \not{p}B_1 + \not{p}'B_2)(\not{p}' + m)\ \gamma_\mu(\not{p}' - \not{q} + m)\ \gamma_\nu(\not{p}' + m)\}, \qquad \text{(III.12)}$$

where f is the γNN vertex function, which has been assumed to be a constant. In Appendix A, we give a generalization of the Matsuda-Suzuki plausibility argument[13] to justify this assumption in the asymptotic limit in which we are interested. Let us denote the trace in (III.12) by $T_{\mu\nu}(q,p',p)$. Then

$$T_{\mu\nu}(q,p',p) \equiv \text{Trace}\{(-A + \not{p}B_1 + \not{p}'B_2)(p' + m)\gamma_\mu(\not{p}' - \not{q} + m)$$

$$\times \gamma_\nu(\not{p}' + m)\}$$

$$= 4g_{\mu\nu}[q^2(p'\cdot pB_1 + p'^2B_2 - mA) + (m^2 - p'^2)(q\cdot pB_1 + q\cdot p'B_2)]$$

$$+ 8(p'\cdot pB_1 + p'^2B_2 - mA)[p'_\mu(p' - q)_\nu + p'_\nu(p' - q)_\mu]$$

$$+ 4(m^2 - p'^2)[(p' - q)_\mu(p_\nu B_1 + p'_\nu B_2) + (p_\mu B_1 + p'_\mu B_2)(p' - q)_\nu]. \qquad \text{(III.13)}$$

We show the explicit calculation only for the B_1 term. Similar calculations can be done for the A and B_2 terms. We first change to invariant variables by defining $W_{\mu\nu}^{B_1}(s,u)$ and $B_1(s',u')$ by

$$W_{\mu\nu}^{B_1}(q,p) = \iint dsdu\ W_{\mu\nu}^{B_1}(s,u)\ \delta[(p + q)^2 - s]\ \delta(u + q^2), \qquad \text{(III.14a)}$$

$$B_1(p',p) = \iint ds'du'\ B_1(s',u')\ \delta[(p + p')^2 - s']\ \delta(u' + p'^2), \qquad \text{(III.14b)}$$

where the superscript B_1 means that we are considering just the B_1 term. Substituting this into (III.12) gives

$$W_{\mu\nu}^{B_1}(s,u) = \frac{f^2}{16m\pi^5} \iint ds'du'\ \frac{B_1(s',u')}{(u' + m^2)^2}\ Q_{\mu\nu}^{B_1}(s,u;\ s',u';\ m^2), \qquad \text{(III.15)}$$

where

$$Q_{\mu\nu}^{B_1}(s,u;\ s',u';\ m^2) = \int d^4p'\ T_{\mu\nu}^{B_1}(q,p',p)\ \delta[(p' - q)^2 - m^2]$$

$$\times\ \delta[(p + p')^2 - s']\ \delta(u' + p'^2). \qquad \text{(III.16)}$$

Since $T^{B_1}_{\mu\nu}$ is symmetric as can easily be seen from (III.13), the most general form for $Q^{B_1}_{\mu\nu}$ is the sum $c_1 g_{\mu\nu} + c_2 q_\mu q_\nu + c_3 p_\mu p_\nu + c_4(p_\mu q_\nu + q_\mu p_\nu)$.

We show in Appendix B that current is conserved in the limit of $s \to \infty$ with s/u fixed, i.e., we can write

$$Q^{B_1}_{\mu\nu}(s,u;\, s',u';\, m^2) = -\eta_1^{B_1}(s,u;\, s',u';\, m^2)\left(g_{\mu\nu} + \frac{q_\mu q_\nu}{u}\right) + \frac{\eta_2^{B_1}(s,u;\, s',u';\, m^2)}{m^2}\left[p_\mu + \frac{(p\cdot q)q_\mu}{u}\right]\left[p_\nu + \frac{(p\cdot q)q_\nu}{u}\right] \quad . \qquad \text{(III.17)}$$

Therefore, our model describes correctly the gauge invariant aspect of deep inelastic electron-nucleon scattering. The structure functions are therefore given by

$$W_i^{B_1}(s,u) = \frac{f^2}{16m\pi^5}\iint ds'du'\, \frac{B_1(s',u')}{(u'+m^2)^2}\,\eta_i^{B_1}(s,u;\, s',u';\, m^2). \qquad \text{(III.18)}$$

Appendix B also shows that in the limit of $s \to \infty$ with s/u fixed,

$$\frac{\eta_2^{B_1}}{m^2}\left[m^2 + \frac{(p\cdot q)^2}{u}\right] \approx \eta_1^{B_1} \approx -4[(m^2+u')(p\cdot q) - u(p\cdot p')], \qquad \text{(III.19)}$$

which reduces to

$$\frac{s^2}{4m^2u}\,\eta_2^{B_1} \approx \eta_1^{B_1} \approx 2(us' - su' - m^2 s)\, Q_{ABFST}(s,u;\, s',u';\, m^2) \qquad \text{(III.20)}$$

when s/u is large.

Before we can calculate $W_i^{B_1}$, we must first calculate $B_1(s',u')$. Changing variables as in (III.14) and using (III.10) and (III.11), we have

$$B_1(s',u') = \int ds_0\, \frac{B_1^R(s_0)}{8\pi^4}\iint \frac{ds''du''}{(u''+\mu^2)^2}\, Q_{ABFST}(s',u';\, s'',u'';\, s_0) \times A_{\pi N}(s'',u'')\left[\frac{2u'(s''+u''-m^2) - (s'+u'-m^2)(s_0+u''+u')}{4m^2u' + (s'+u'-m^2)^2}\right]. \qquad \text{(III.21)}$$

In the limit of large s' and s'/u', this becomes

$$B_1(s',u') \approx \frac{1}{16\pi^3}\int ds_0\, B_1^R(s_0)\int_0^{s'}\frac{ds''}{s'}\int_{\frac{s''}{s'}\left(u' + \frac{s_0}{1-\frac{s''}{s'}}\right)}^{\infty} du''\, \frac{\phi_{\pi N}(u'')\, s''^{\alpha}}{(u''+\mu^2)^2} \times \left[\frac{2u'(s''+u''-m^2) - (s'+u'-m^2)(s_0+u''+u')}{4m^2u' + (s'+u'-m^2)^2}\right]$$

$$\approx -\frac{1}{16\pi^3 s'}\int ds_0\, B_1^R(s_0)\int_0^{s'}\frac{ds''}{s'}\int_{\frac{s''}{s'}\left(u' + \frac{s_0}{1-\frac{s''}{s'}}\right)}^{\infty} du''\, (s_0+u''+u')\, \frac{\phi_{\pi N}(u'')\, s''^{\alpha}}{(u''+\mu^2)^2}. \qquad \text{(III.22)}$$

When we change variable from s'' to $x \equiv \frac{s''}{s'}$, we immediately find that we have Regge behavior:

$$B_1(s',u') \underset{s'\ \text{large}}{\approx} (s')^{\alpha-1}\, \phi_\alpha^{B_1}(u'). \qquad \text{(III.23)}$$

The expression $\phi_\alpha^{B_1}(u')$ is given after integrating over x by

$$\phi_\alpha^{B_1}(u') \approx -\frac{1}{16\pi^3(\alpha+1)(2u')^{\alpha+1}}\int ds_0\, B_1^R(s_0)\, J(\alpha,s_0,u'), \qquad \text{(III.24)}$$

where

$$J(\alpha,s_0,u') \equiv \int_0^\infty du'' \, \phi_\alpha^{\pi N}(u'') \, (s_0 + u'' + u')$$

$$\times \frac{\{s_0 + u' + u'' - [(s_0 + u' + u'')^2 - 4u'u'']^{\frac{1}{2}}\}^{\alpha+1}}{(u'' + \mu^2)^2} \quad . \qquad \text{(III.25)}$$

If u' is also large, we can approximate $J(\alpha,s_0,u')$ by

$$J(\alpha,s_0,u') \approx u' \int_0^{u'} du'' \frac{(2u'')^{\alpha+1}}{(u'' + \mu^2)^2} \phi_\alpha^{\pi N}(u'')$$

$$+ \; u' \int_{u'}^\infty du'' \frac{(2u')^{\alpha+1}}{(u'')^2} \; \phi_\alpha^{\pi N}(u'')$$

$$\approx u' \int_0^b du'' \frac{(2u'')^{\alpha+1}}{(u'' + \mu^2)^2} \phi_\alpha^{\pi N}(u'') + 2^{\alpha+1} u' \int_b^{u'} du''(u'')^{\alpha-1} \phi_\alpha^{\pi N}(u'')$$

$$+ (2u')^{\alpha+1} u' \int_{u'}^\infty du'' \frac{\phi_\alpha^{\pi N}(u'')}{(u'')^2} \quad , \qquad \text{(III.26)}$$

where b is a constant such that for $u'' > b$, $\phi_\alpha^{\pi N}(u'') \sim \frac{1}{(u'')^{\alpha+1}}$.

Evaluating the integrals in (III.26), we find that, for large u', $J(\alpha,s_0,u') \sim u'$. This, together with Eq. (III.24), allows us to conclude

$$\phi_\alpha^{B_1}(u') \underset{u' \text{ large}}{\sim} \frac{1}{(u')^\alpha} \quad . \qquad \text{(III.27)}$$

Substituting Eqs. (III.20), (III.23), and (III.27) into Eq. (III.18) and following the method we just used to evaluate the asymptotic expression for B_1, we find in the limit $s \to \infty$, $u \to \infty$, and s/u fixed but large

$$W_1^{B_1}(s,u) \sim (\tfrac{s}{u})^\alpha \, \ell n \, u$$

and

$$W_2^{B_1}(s,u) \sim \tfrac{1}{u}(\tfrac{s}{u})^{\alpha-2} \, \ell n \, u .$$

As in DLY's derivation,[3] if a cutoff or an extra damping factor is introduced in the u' integration, we obtain

$$W_1^{B_1}(s,u) \approx c'(\tfrac{s}{u})^\alpha ,$$

$$W_2^{B_1}(s,u) \approx \frac{4m^2c'}{u} (\tfrac{s}{u})^{\alpha-2} , \qquad \text{(III.28)}$$

where c' is a proportionality constant. Similar results are obtained from the A and B_2 terms, except that no cutoff is needed for the A term. Remembering that $u = -q^2$ and $s \approx 2m\nu$ for large ω, we obtain the results quoted in (II.1) - (II.4). Equation (II.6) follows from the fact that the vacuum Regge pole couples identically to $p\bar{p}$ and $n\bar{n}$ channels. If the vacuum Regge pole has intercept 1, $\nu W_2(\omega)$ should approach a constant.

The expression (II.7) for the average pion multiplicity can be derived following the usual approach of ABFST.[9,10] The only change for the case of deep inelastic e-N scattering is that we have ω^α, instead of s^α. The average multiplicity is therefore given by $\langle n_\pi \rangle \sim \ell n \, \omega$. The fact that our derivation requires ω to be large is consistent with this logarithmic growth of the multiplicity, because large ω means a long miltiperipheral chain and so justifies the use of the integral equation approach.

IV. DISCUSSION AND COMPARISON WITH PREVIOUS MODELS

1. Our multiperipheral derivation is concerned with unitarity diagrams, and not Feynman diagrams, so it is not necessary to consider renormalizations. In DLY's field-theory derivation,[3] only the leading term is kept in each order; they had to assume that the sum of these leading terms is the dominant contribution. For this reason, they cannot determine the constant in Eq. (II.2). On the other hand, the multiperipheral integral approach keeps all terms; only at the end does one approximate by keeping the leading Regge poles. In this respect, the multiperipheral derivation is more rigorous.[16] Furthermore, in principle we can determine the eigenfunction of the inhomogeneous integral equation and therefore the constant in (II.2).

2. Our model is consistent with the ABFST multiperipheral assumption of meson-exchanged dominance. Note that our model has only one exchanged nucleon, as compared with DLY's, in which all exchanged particles are nucleons (see Fig. 5). We have shown that deep inelastic e-N scattering can be adequately described by a minimal modification of the ABFST MPM.

3. The cutoff or extra damping factor can be justified because the nucleon propagator does not sufficiently damp out large transverse momenta (corresponding to large momentum transfers). This cutoff also allows all other momentum transfers to be small, consistent with the notion of peripheralism. Note that no cutoff is needed for momentum transfers corresponding to exchanged pions. In DLY's model, a cutoff is needed for every exchanged particle.

4. As already mentioned in Sec. II, our results correspond to the parton model where the partons have spin-$\frac{1}{2}$ and the nucleon consists of three quarks and a quark-antiquark "sea."

5. In our model, the contribution of any Regge trajectory scales, in contrast to Harari's diffraction model[4] in which contributions of "ordinary" trajectories do not scale.

6. We have also calculated the diagram with no nucleon exchange, i.e., the photon couples to two pions. In this case we get $\nu W_2 = 2m\bar{c}\omega^{\alpha-1}$ (same functional dependence as before) and $W_1 = \bar{c}\,\omega^{\alpha}\,\frac{\ell n(-q^2)}{(-q^2)}$, i.e., $W_1 \to 0$ in the limit (II.1).[17] Therefore, at least for W_1, in the limit of large $-q^2$ the diagram with γ coupling to spin-$\frac{1}{2}$ particle dominates over the diagram with γ coupling to spin-0 particle. When both diagrams are included, we obtain a finite but nonzero value for $\frac{\sigma_S}{\sigma_T}$, i.e., $\frac{\sigma_S}{\sigma_T} = \frac{\bar{c}}{c}$.

7. Our model predicts that an energetic $N\bar{N}$ pair is produced in deep inelastic e-N scattering. We can conclude from the previous paragraph that the likelihood of producing an energetic $N\bar{N}$ pair increases as $(-q^2)$ increases. If such energetic $N\bar{N}$ pair is not observed experimentally, it means that the meson-exchange diagram is not yet negligible in the present kinematic region. Since our derivation of scaling (as in almost all other models) is an asymptotic derivation, it still leaves unanswered the question of why scaling sets in so early.

8. Our derivation is actually more general than the ABFST MPM, since once any extra damping factor for u' is introduced, all that is needed in our derivation is Regge behavior as in Eq. (III.23) and asymptotic off-shell dependence as in Eq. (III.27). It is not necessary to know how these are derived. For example, scaling is true in the model where all exchanged particles are nucleons as long as a

cutoff is introduced for each momentum transfer (this is just the result of DLY). Again, it is true in a model where any combination of pions and nucleons is exchanged as long as the photon couples to the nucleon and a cutoff is introduced for each exchanged nucleon.

If only the final electron is observed, then inelastic e-N scattering represents the inclusive reaction $e + N \to e + X$, where X means any hadron state. It is interesting to speculate whether one can derive our results by assuming some J-plane analyticity structure of the six-line connected part as in the work of Mueller[18] for pure hadronic scattering.

9. The method of this paper can of course be used to study ν-N and $\bar{\nu}$-N scatterings. By crossing, it can also be used to study $e^- + e^+ \to p$ + anything.

ACKNOWLEDGMENT

We thank Professor Geoffrey F. Chew and Professor Mahiko Suzuki for valuable comments.

APPENDIX A. CONSTANT γNN VERTEX FUNCTION

Matsuda and Suzuki[13] presented a plausibility argument to show that for three spinless particles the vertex function is a constant when two of the particles are far from their mass-shells. Here we present a simple generalization of this plausibility argument to our case of a photon coupling to two spin-$\frac{1}{2}$ nucleons. The essential argument in this generalization is that electromagnetic field is minimially coupled to spin-$\frac{1}{2}$ particles and there is no derivative coupling among hadrons.

Our starting point is the matrix element of the time-ordered product

$$I^{\mu} = \int d^4x \, e^{ip\cdot x} \langle 0 | (A^{\mu}(0) \, \psi(x))_+ | a \rangle, \tag{A.1}$$

where $\psi(x)$ is the nucleon field, $|a\rangle$ is the one-nucleon state with momentum p_a, and A^{μ} is the electromagnetic potential. We can write Eq. (A.1) as

$$I^{\mu} = \frac{1}{q^2(\not{p} - m)} \Gamma^{\mu} \, u_a(p_a), \tag{A.2}$$

where q is the momentum of the photon, p is the momentum of nucleon, and Γ^{μ} is the vertex function.

We consider the Bjorken limit $p_0 \to \infty$, in which (A.1) can be expanded in powers of $\frac{1}{p_0}$ to give[19]

$$I^{\mu} = \int d^3x \, e^{-i\underset{\sim}{p}\cdot\underset{\sim}{x}} \langle 0 | \{ \frac{1}{ip_0} [A^{\mu}(0), \psi(\underset{\sim}{x},0)] + \frac{1}{(ip_0)^2} [A^{\mu}(0), \dot{\psi}(\underset{\sim}{x},0)] + \frac{1}{(ip_0)^3} [A^{\mu}(0), \ddot{\psi}(\underset{\sim}{x},0)] + \cdots \} | a \rangle . \tag{A.3}$$

If we use the Gupta-Bleuler formalism for the electromagnetic field, the A^{μ}'s are independent dynamical variables. Their equal-time commutation relations are

$$[A^{\mu}(x), A^{\nu}(x')]_{x_0=x'_0} = 0,$$

$$[A^{\mu,0}(x), A^{\nu}(x')]_{x_0=x'_0} = ig^{\mu\nu}\, \delta(\underset{\sim}{x} - \underset{\sim}{x}'), \qquad (A.4)$$

where

$$A^{\mu,0}(x) = \frac{\partial}{\partial t} A^{\mu}(x).$$

If the electromagnetic field is minimally coupled to the ψ field, then we can write

$$(i\not\partial - e\not A)\psi - m\psi = h, \qquad (A.5)$$

where h is a function of ψ field, but not its derivatives. The function h describes the interactions among hadrons. From Eq. (A.5), we obtain

$$\partial_0\psi = \frac{1}{i}\,(eA_0\psi + i\gamma^0\underset{\sim}{\gamma}\cdot\nabla\psi - e\gamma^0\underset{\sim}{\gamma}\cdot\underset{\sim}{A}\psi + m\gamma^0\psi + \gamma^0 h). \qquad (A.6)$$

We also know

$$[A^{\mu}(0), \psi(\underset{\sim}{x},0)] = 0. \qquad (A.7)$$

This, together with Eqs. (A.4) and (A.6), implies

$$[A^{\mu}(0), \dot\psi(\underset{\sim}{x},0)] = 0, \qquad [A^{\mu}(0), \ddot\psi(\underset{\sim}{x},0)] \neq 0 . \qquad (A.8)$$

Therefore, we find that in the Bjorken limit,

$$I^{\mu} \to \frac{1}{(p_0)^3}\,\text{constant}. \qquad (A.9)$$

On the other hand, the right-hand side of Eq. (A.2) gives us

$$I^{\mu} \xrightarrow[\substack{p_0\to\infty \\ p_a \text{ fixed}}]{} \frac{1}{(p_0)^3}\,\Gamma^{\mu}\, u_a(p_a). \qquad (A.10)$$

Comparing Eqs. (A.9) and (A.10), we conclude that $\Gamma^{\mu} u_a(p_a) \to$ constant, or $\Gamma^{\mu} \to$ constant. Thus we have extended the plausibility argument of Matsuda-Suzuki to the case of a photon coupling to two spin-$\frac{1}{2}$ nucleons.

Before concluding this Appendix, we want to remark that the constancy of γNN vertex in the asymptotic limit can also be easily proved if we use the radiation gauge formulation of the electromagnetic field.

APPENDIX B. CURRENT CONSERVATION

In Sec. III, we made the statement that it can be proved that in the limit of $s \to \infty$ with s/u fixed, $Q_{\mu\nu}^{B_1}$ can be written as Eq. (3.17), i.e., $Q_{\mu\nu}^{B_1}$ satisfies current conservation. We now present the proof of this statement.

From Eqs. (III.13) and (III.16), $Q_{\mu\nu}^{B_1}$ is given by

$$Q_{\mu\nu}^{B_1} = \int d^4p' \{4g_{\mu\nu}[q^2(p'\cdot p) + (m^2 - p'^2)q\cdot p] + 8p'\cdot p[p'_\mu(p' - q)_\nu + p'_\nu(p' - q)_\mu] + 4(m^2 - p'^2)[(p' - q)_\mu p_\nu + p_\mu(p' - q)_\nu]\} \times \delta[(p' - q)^2 - m^2]\,\delta[(p + p')^2 - s']\,\delta(u' + p'^2). \quad \text{(B.1)}$$

Since $Q_{\mu\nu}^{B_1}$ is symmetric, its most general form can be written as

$$Q_{\mu\nu}^{B_1} = -\eta_1^{B_1}\left(g_{\mu\nu} + \frac{q_\mu q_\nu}{u}\right) + \frac{\eta_2^{B_1}}{m^2}\left[p_\mu + \frac{(p\cdot q)q_\mu}{u}\right]\left[p_\nu + \frac{(p\cdot q)q_\nu}{u}\right] + \gamma q_\mu q_\nu + \delta p_\mu p_\nu. \quad \text{(B.2)}$$

Dotting (B.2) with $g^{\nu\mu}$, $p^\nu p^\mu$, $q^\nu p^\mu$, and $q^\nu q^\mu$, we can determine the four coefficients in (B.2). The four equations for $\eta_1^{B_1}$, $\eta_2^{B_1}$, γ, and δ are

$$-3\eta_1^{B_1} + \frac{\eta_2^{B_1}}{m^2}\left(m^2 + \frac{(p\cdot q)^2}{u}\right) - \gamma u + \delta m^2 = g^{\nu\mu} Q_{\mu\nu}^{B_1},$$

$$-\eta_1^{B_1}\left(m^2 + \frac{(p\cdot q)^2}{u}\right) + \frac{\eta_2^{B_1}}{m^2}\left(m^2 + \frac{(p\cdot q)^2}{u}\right)^2 + \gamma(p\cdot q)^2 + \delta m^4 = p^\nu p^\mu Q_{\mu\nu}^{B_1},$$

$$-\gamma u(p\cdot q) + \delta m^2(p\cdot q) = q^\nu p^\mu Q_{\mu\nu}^{B_1},$$

$$\gamma u^2 + \delta(p\cdot q)^2 = q^\nu q^\mu Q_{\mu\nu}^{B_1}. \quad \text{(B.3)}$$

Or

$$-3\eta_1^{B_1} + \frac{\eta_2^{B_1}}{m^2}\left(m^2 + \frac{(p\cdot q)^2}{u}\right) - \gamma u + \delta m^2 = g^{\nu\mu} Q_{\mu\nu}^{B_1},$$

$$-\eta_1^{B_1} + \frac{\eta_2^{B_1}}{m^2}\left(m^2 + \frac{(p\cdot q)^2}{u}\right) + \gamma\frac{(p\cdot q)^2}{\left(m^2 + \frac{(p\cdot q)^2}{u}\right)} + \delta\frac{m^4}{\left(m^2 + \frac{(p\cdot q)^2}{m}\right)} = \frac{p^\nu p^\mu Q_{\mu\nu}^{B_1}}{\left(m^2 + \frac{(p\cdot q)^2}{u}\right)},$$

$$\gamma u = \frac{1}{\left((p\cdot q)^2 + m^2 u\right)}\left[m^2(q^\nu q^\mu Q_{\mu\nu}^{B_1}) - (p\cdot q)(q^\nu p^\mu Q_{\mu\nu}^{B_1})\right],$$

$$\delta(p\cdot q) = \frac{1}{\left((p\cdot q)^2 + m^2 u\right)}\left[u(q^\nu p^\mu Q_{\mu\nu}^{B_1}) + (p\cdot q)(q^\nu p^\mu Q_{\mu\nu}^{B_1})\right]. \quad \text{(B.4)}$$

By explicit calculation from (B.1), we find

$$g^{\nu\mu}Q_{\mu\nu}^{B_1} = 4[2p\cdot p'(2m^2 - u) + 2p\cdot q(m^2 + u')]Q_{ABFST},$$

$$p^{\nu}p^{\mu}Q_{\mu\nu}^{B_1} = 4\{m^2[-u(p\cdot p') + (m^2 + u')p\cdot q]$$
$$+ (p\cdot p' - p\cdot q)[4(p\cdot p')^2 + 2m^2(m^2 + u')]\}Q_{ABFST},$$

$$q^{\nu}p^{\mu}Q_{\mu\nu}^{B_1} = 4\{(m^2 + u')[2p\cdot p'(p\cdot q - p\cdot p') + m^2(p'\cdot q + u)]\}Q_{ABFST},$$

$$q^{\nu}q^{\mu}Q_{\mu\nu}^{B_1} = 4[(m^2 + u')^2 (p\cdot p' - p\cdot q)]Q_{ABFST}. \quad (B.5)$$

By a straightforward calculation one can see that in the limit of $s \to \infty$ with s/u fixed,

$$g^{\nu\mu}Q_{\mu\nu}^{B_1} \approx O(su')Q_{ABFST},$$
$$p^{\nu}p^{\mu}Q_{\mu\nu}^{B_1} \approx O(su'^2)Q_{ABFST},$$
$$q^{\nu}p^{\mu}Q_{\mu\nu}^{B_1} \approx O(su'^2)Q_{ABFST},$$
$$q^{\nu}q^{\mu}Q_{\mu\nu}^{B_1} \approx O(su'^2)Q_{ABFST}. \quad (B.6)$$

Using the last two equations in (B.4) and (B.6), we can conclude that

$$\gamma \approx O\left(\frac{u'^2}{s}\right)Q_{ABFST},$$
$$\delta \approx O\left(\frac{u'^2}{s}\right)Q_{ABFST}. \quad (B.7)$$

When we substitute γ and δ into the first two equations of (B.4), we see immediately that we can neglect the γ and δ terms, i.e., current is conserved in the limit of $s \to \infty$ with s/u fixed. This method can also be used for the A and B_2 terms.

Solving for $\eta_1^{B_1}$ and $\eta_2^{B_1}$ then gives

$$\frac{\eta_2^{B_1}}{m^2}\left[m^2 + \frac{(p\cdot q)^2}{u}\right] \approx \eta_1^{B_1} \approx -4[(m^2 + u')(p\cdot q) - u(p\cdot p')], \quad (B.8)$$

in the limit of $s \to \infty$ with s/u fixed. This is just Eq. (III.19).

FOOTNOTES AND REFERENCES

* This work was done under the auspices of the U.S. Atomic Energy Commission.

1. For a review, see articles by F. Gilman and R. E. Taylor, in <u>International Symposium on Electron and Photon Interactions at High Energies, Liverpool, England, 1969</u>, edited by D. W. Braben and R. E. Rand (Daresbury Nuclear Physics Laboratory, Daresbury, Lancashire, England, 1970).
2. J. D. Bjorken and E. A. Paschos, Phys. Rev. <u>185</u>, 1975 (1969).
3. S. D. Drell, D. J. Levy, and T.-M. Yan, Phys. Rev. Letters <u>22</u>, 744 (1969), Phys. Rev. <u>187</u>, 2159 (1969), and Phys. Rev. <u>D1</u>, 1035 (1970). In this paper we designate their model as DLY.
4. H. D. I. Abarbanel, M. L. Goldberger, and S. B. Treiman, Phys. Rev. Letters <u>22</u>, 500 (1969); H. Harari, Phys. Rev. Letters <u>22</u>, 1078 (1969), and <u>24</u>, 286 (1970).
5. J. J. Sakurai, Phys. Rev. Letters <u>22</u>, 981 (1969); C. F. Cho and J. J. Sakurai, Phys. Letters <u>31B</u>, 22 (1970); W. K. Tung, Phys. Rev. Letters <u>23</u>, 1531 (1969); J. D. Sullivan, Nucl. Phys. <u>B22</u>, 358 (1970).
6. P. V. Landshoff and J. C. Polkinghorne, Nucl. Phys. <u>B19</u>, 432 (1970).
7. J. D. Bjorken, Phys. Rev. <u>179</u>, 1547 (1969).
8. A condensed version of this paper has already been published. See S.-S. Shei and D. M. Tow, Phys. Rev. Letters <u>26</u>, 470 (1971).
9. L. Bertocchi, S. Fubini, and M. Tonin, Nuovo Cimento <u>25</u>, 626 (1962); D. Amati, A. Stanghellini, and S. Fubini, Nuovo Cimento <u>26</u>, 896 (1962). This model is called the ABFST multiperipheral model.

10. D. M. Tow, Phys. Rev. <u>D2</u>, 154 (1970).
11. G. F. Chew, T. Rogers, and D. R. Snider, Phys. Rev. <u>D2</u>, 765 (1970).
12. E. D. Bloom et al., Recent Results in Inelastic Electron Scattering, SLAC preprint, Sept. 1970.
13. S. Matsuda and M. Suzuki, Phys. Rev. <u>D1</u>, 1778 (1970).
14. C. G. Callan and D. J. Gross, Phys. Rev. Letters <u>22</u>, 156 (1969).
15. C. Ceolin, F. Duimio, R. Stroffolini, and S. Fubini, Nuovo Cimento <u>26</u>, 247 (1962).
16. Our derivation is similar to that of Abarbanel, Goldberger, and Treiman (Ref. 4), who used a Bethe-Salpter integral-equation approach to study a ϕ^3 model with a spinless photon.
17. G. Altarelli and H. R. Rubinstein, Phys. Rev. <u>187</u>, 2111 (1969), also obtained this result in a ϕ^3 model.
18. A. H. Mueller, Phys. Rev. <u>D2</u>, 2963 (1970).
19. J. D. Bjorken, Phys. Rev. <u>148</u>, 1467 (1966).

FIGURE CAPTIONS

Fig. 1. Kinematics for inelastic electron-nucleon scattering.

Fig. 2. Diagram representing our model; solid, dashed, and wavy lines are nucleons, pions, and photons, respectively.

Fig. 3. Diagram representing unitarity sum and defining kinematic variables; solid, dashed, and wavy lines are nucleons, pions, and photons, respectively.

Fig. 4. Kinematics for the π-N amplitude and the π-π amplitude.

Fig. 5. Diagram for DLY's field-theory model; solid, dashed, and wavy lines are nucleons, pions, and photons, respectively.

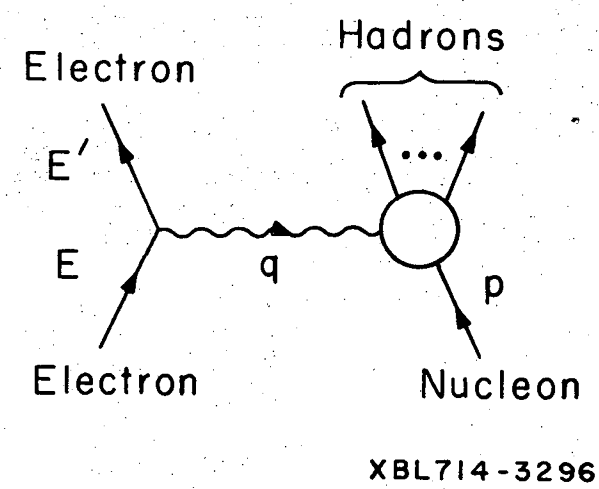
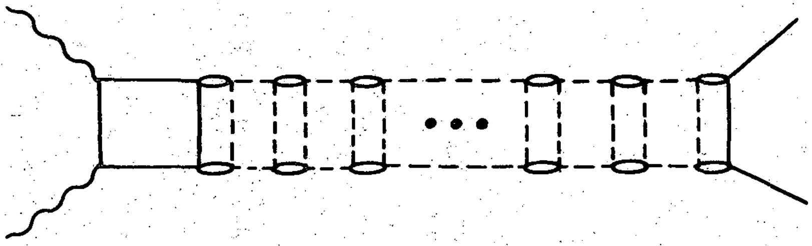
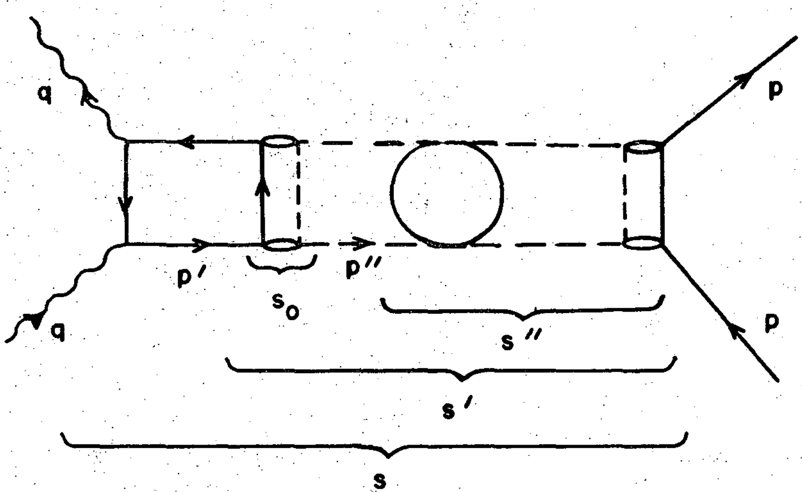


Fig. 1



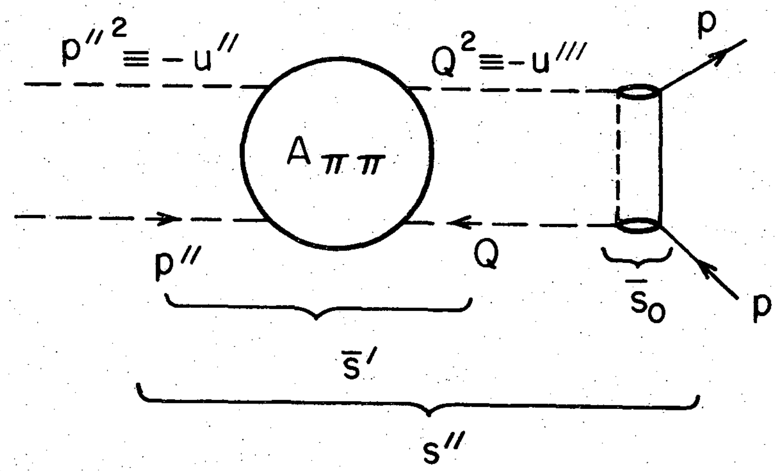
XBL 7012 - 4212

Fig. 2



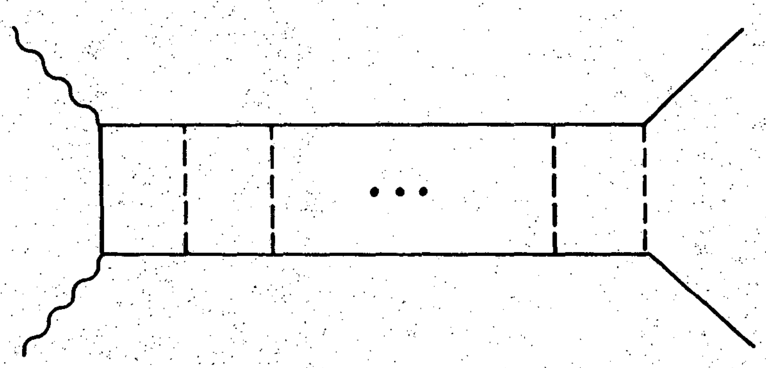
XBL 7012- 4211

Fig. 3



XBL 714-3298

Fig. 4



XBL714-3297

Fig. 5

TECHNICAL INFORMATION DIVISION
LAWRENCE RADIATION LABORATORY
UNIVERSITY OF CALIFORNIA
BERKELEY, CALIFORNIA 94720