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Authors
D'Mello, Sidney
Graesser, Art
Ventura, Matthew

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A Computational Approach to Mental Models

Matthew Ventura(mventura@memphis.edu)
Department of Psychology/ Institute for Intelligent Systems, 365 Innovation Drive
Memphis, TN 38152-3115 USA

Sidney D'Mello (sdmello@memphis.edu)
Department of Mathematical Science/ Institute for Intelligent Systems, 365 Innovation Drive
Memphis, TN 38152-3115 USA

Art Graesser(a-graesser@memphis.edu)
Department of Psychology/ Institute for Intelligent Systems, 365 Innovation Drive
Memphis, TN 38152-3115 USA

Abstract

This paper outlines the Mental Model Algorithm (MMA), a model of spatial reasoning that uses a coordinate based representation to specify the spatial relations between objects. Based on humans' performance on undetermined spatial reasoning problems, the MMA predicted preferences in the resolutions of undetermined positions of shapes. Additionally, the MMA is proposed to be a simpler and more scalable algorithm than propositional calculus models in that only a small set of rules are required to generate spatial inferences.

Introduction

Spatial comprehension in discourse can be claimed to require a construction process: *John is to the left of Mary, Mary is to the left of Steve,* that generates inferences such as *John is to the left of Steve* (Graesser, Singer, & Trabasso, 1994). In this construction process an important component consists of constructing mental models that arguably are generated from analog-based representations (Johnson-Laird, 1983; Johnson-Laird & Byrne, 1991). This theory has traditionally been contrasted with context-free propositional calculus (PC) models. The PC models stipulate that humans construct mental propositions by context-free formal rules of inference that apply to explicit assertions and that derive conclusions that are not sensitive to an argument's content (Hagert, 1985; Pylyshyn, 1981; Rips, 1986). PC models have formally been applied to the generation of spatial inferences but fail to make predictions of human spatial reasoning that are compatible with mental models (for a review, see McNAmara, Miller, & Bransford, 1996). Nevertheless a formal algorithm for mental models needs to be specified to advance theory development and experimentation on mental models. In creating such an algorithm, it is assumed that mental models are neither proposition lists nor detailed images, but a hybrid of symbolic representations that get instantiated in a formalized coordinate space (Johnson-Laird, 1983), much like Barsalou's (1999) perceptual symbol system. This distinction could be seen as a formalization of the "what" and "where" system of Landau and Jackendoff (1993). The "what" system handles object meaning and the "where" system handles the spatial orientations of objects in space. The purpose of this paper is to show how a simple coordinate system can generate inferences about where objects are located on a wide variety of spatial reasoning problems without extensive rules.

Mental models and Propositional calculus

Byrne and Johnson-Laird (1989) gave evidence that mental models predict difficulty for certain spatial reasoning problems. Consider problem 1.

    A is to the left of B.
    C is to the left of B.
    D is below B.
    E is below C.
    What is the relationship between D and E?

In this problem two mental models can be constructed:

    (a)   A   C   B
              E   D

    (b)   C   A   B
          E   D

Figure 1: Two mental models for problem 1.

The mental model theory predicted difficulty with this problem since there is indeterminacy with A and C. Since there then are two alternative models that can be constructed, humans have more difficulty with this type of problem. A PC model does not make this prediction since a conflict with A and C does not affect the inference question (Byrne & Johnson-Laird, 1989).

In the following sections we provide a computational framework for mental models that established further theoretical support for such models. In providing this theoretical support, mental models must be operationally defined under a representational system. Johnson-Laird & Byrne (1991) briefly describes how a coordinate based algorithm can be written that builds mental models for spatial reasoning. Additionally, Glenberg, Kruley, and Langston (1994), describe how mental models can be constructed in a three-dimensional spatial medium to account for increased activations of spatially close objects in sentence comprehension. The purpose of the present algorithm is to formally describe how a mental model can generate spatial inferences in discourse.

Mental Model Algorithm

The Mental Model Algorithm (MMA) is proposed to formally model spatial reasoning by using a simple analogical representation in 3-D space to construct coordinate boundaries of linguistically described spatial relations. After spatial relations are plotted, an inference process translates the spatial array back into linguistic descriptions.

To assigned coordinate boundaries the bounding box model was used (Regier, & Carlson, 2001) to specify 6 spatial relationships (i.e., above, below, left, right, behind, front right). In the bounding box model (see Figure 2), each preposition states a coordinate rule which specifies a three dimensional box relative to a *landmark*. A *trajector* is any object that falls in the two dimensional coordinates of the landmark.
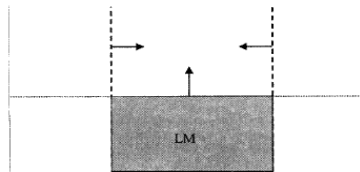

Figure 2: Bounding box model for above relative to a landmark (LM).

The MMA constructs simple spatial relationships by linguistic propositions stating the relationship between two objects (e.g., (1a) *A is on the left of B* (1b) *B is on the left of C)*. The MMA displays the spatial relations between the three shapes by using the box model to configure each pair of shapes in each proposition. On a *n* x *n* x *n* coordinate plane, the MMA constructs (1a) by representing *B* as a point at the origin *(0,0)* and *A* as a point at *(-1,0);* MMA always places the first landmark of a spatial problem at the origin. Where the landmark thereafter in each pair is always the object already set in the coordinate plane. Then in (1b) the MMA adds *C* to the plane at point *(1,0)* to display *A B C.*

Spatial inferences are made between the points in 3-D space with a simple interpreter. This inference process takes nonlinguistic information and recodes it back into linguistic

descriptions. In this space the MMA takes each point *P* and checks for other points that fall anywhere on the x, y, or z axis. Points on higher/lower y coordinates than *P* are considered above/below, points higher/lower than *P* on the x coordinate is considered right/left, and points higher/lower than *P* on the z coordinate is considered in front/behind (see figure 3). This process will yield all the linguistic spatial relations for each point. With simple process the model will report the inference that the *A is in left of C* without explicit rules.
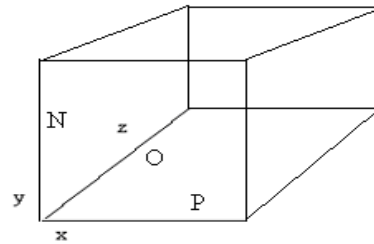

Figure 3: Three dimensional plane for inferring spatial relations relative to *P*. In this figure, P is in front, and to the right of O. P is also to the right, and below, N.

Model Complexity for 2-D problems

One question that arises in the MMA and PC models are the number of rules required in solving spatial inference problems. Increasing rules suggest limitations in scalability towards varying applications. In the following example we will show how increasing complexity in spatial configurations poses increasing rules for PC models. Conversely, the MMA only has a small set of rules that need no modification for increasingly complex spatial configurations. Consider problem 2.

A is to the left of B.
B is to the left of C.
D is below A.
E is below C.
What is the relationship between D and E?

The depiction is shown below where object-pair relations are assumed to be adjacent (Byrne & Johnson-Laird, 1989).
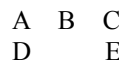
A   B   C
D   E

Figure 4: Mental model for problem 2.

In this case 6 rules (from Hagert, 1983) in the logic program Prolog are required to answer that D is to the left of E for any length between A and C, A and D, or C and E. Rules are written in non-variable form to better understand the rules for the problem.

(1) Left (A, C) **If** Left (A, B) & Left (B, C).
(2) Left (D, B) **If** Below (D, A) & Left (A, B).
(3) Left (B, E) **If** Below (E, C) & Left (B, C).
(4) Below (A, C) **If** Below (A, B) & Below (B, C).
(5) Left (A, B) = Right (B, A).
(6) Above (A, D) = Below (D, A).

For the MMA 3 algorithms are needed to answer any 3-D spatial relation. Since the rules in the MMA are constrained by space, they pose a greater advantage for solving any spatial configuration. Algorithm 1 and 2 state the steps involved in plotting trajectors relative to landmarks in key-value hash tables.

Algorithm 1: subdirection
Input: Two objects, the *landmark,* the *trajector,* and a spatial relationship *relation* between the *landmark* and *trajector.*
Preconditions: The *landmark* should already have its co-ordinates mapped in the hashtables.
Postconditions: The hashtables, **x_map and y_map**, maintaining the co-ordinates of each object in the X and Y directions respectively will be updated.

subdirection(*trajector, landmark, relation*)
{
  if(*relation* == "LEFT")
  {
    x_map(*trajector*) = *x_map*(*landmark*) - 1;
    y_map(*trajector*) = *y_map*(*landmark*);
  }
  else if(*relation* == "RIGHT")
  {
    x_map(*trajector*) = *x_map*(*landmark*) + 1;
    y_map(trajector) = *y_map*(*landmark*);
  }
  else if(*relation* == "BELOW")
  {
    x_map(*trajector*) = *x_map*(*landmark*);
    y_map(*trajector*) = *y_map*(*landmark*) - 1;
  }
  else if(*relation* == "ABOVE")
  {
    x_map(*trajector*) = *x_map*(*landmark*);
    y_map(*trajector*) = *y_map*(*landmark*) + 1;
  }
}

Algorithm 2: construction
Input: A set of propositions in an array *P*. For clarity but not necessity, the l*andmark* is always assumed to be the third position, the landmark is assumed to be in the first position and the relation is in the middle position.
Preconditions: None
Postconditions: Two global Hashtables, **x_map and y_map**, maintaining the co-ordinates of each object in the X and Y directions respectively will be created.

construction(*P)*
{
  foreach proposition *p* in *P*
  {
    landmark = **parse**(*p, 0*);
    relation = **parse**(*p, 1*);
    trajector = **parse**(*p, 2*);

    if *(***empty**(*x_map*) && **empty**(*y_map*)
    {
      x_map(*landmark*) = 0;
      y_map(*landmark*) = 0;
    }
    if(**exists**(*x_map,landmark*)&& **exists**(*y_map,landmark*))
      subdirection(*trajector, landmark, relation*);
    else
      print_error("undefined landmark");
  }

}
Figure 5: Algorithm 1 and 2 for the x and y axis.

Algorithm 2 takes each proposition and parses the pair into the landmark and the trajector. The landmark is always the object that already has been plotted in the plane. Then algorithm 1 assigns coordinates to each trajector relative to the landmark.

After all objects have been plotted in the plane, Algorithm 3 generates spatial relations between all objects on the x and y axis.

Algorithm 3: spatial relations
Input: None
Preconditions: Coordinates for each object should be mapped to the hashtables *x_map* and *y_map.*
Postconditions: Spatial relations between all objects are listed.

relations()
{
  foreach *landmark* in *x_map*
  {
    foreach *trajector* in *y_map*
    {
      if(*landmark* != *trajector*)
      {
        if(*x_map(landmark)* < x_map(*trajector)*)
          print(*landmark* "LEFT" *trajector)*
        else if(*x_map(landmark)* > x_map(*trajector)*)
          print(*landmark* "RIGHT" *trajector)*
        else if(*y_map(landmark)* < y_map(*trajector)*)
          print(*landmark* "BELOW" *trajector)*
        else if(*x_map(landmark)* > x_map(*trajector)*)
          print(*landmark* "ABOVE" *trajector)*
      }
    }
  }
}
Figure 6: Algorithm3 for the x and y axis. This assigns spatial orientations between every object on the x and y axis.

Currently it may appear that the MMA is more complex than the PC models. But as we will see the MMA does not change for any 2-D problems. While PC models can solve any symmetrical variations of problem 2, difficulties arise under simple spatial augmentations. Consider problem 3 with E placed directly left of D:

A is to the left of B.
B is the left of C.
D is below A.
D is to the left of E.
What is the relationship between E and B?

A depiction is shown in Figure 7.

A B C
D E

Figure 7: Mental model for problem 3.

The inference that E is below B can not be solved by the current rule set using Prolog. By adding the following rule:

(7) Below (E,B) **If** Left (D,E) & Left (A,B) & Below (D,A).

Now consider problem 4 with E placed on the left of D. In this situation the PC model would not be able to infer that E is to the left of A. The following rule would be needed:

(8) Left (E,A) **If** Left (E,D) & Below (D,A).

A final variation worth mentioning is to consider problem 4 with F added below E. In this case F is also below D. The PC model would not be able to infer this relationship with the current rule set.

Thus it appears that PC models are not scalable to variations of spatial reasoning problems. There are other spatial relations not mentioned in problem 2 that would need additional rules (e.g., D is below B and C, E is below A and B, etc…). This is contrasted with 4 inference rules needed in the MMA to calculate all possible 2-D spatial relations. The following section will illustrate further complexity in PC models for 3-D spatial relations.

Model Complexity for 3-D problems

Adding a third dimension in spatial reasoning creates three spatial relations for every object. Consider problem 5.

A is behind B.
B is behind C.
D is below A.
D is to the left of E.
What is the relationship between E and B?

Where a depiction would look like the following:

A/B/C
D E

Figure 8: Mental model for problem 5. Where x/y indicates x is behind y.

The MMA will have to incorporate 2 plotting rules and 2 more inference rules (front/behind) to detect the relations between E and B (i.e., behind, below, and to the right). PC models need rules to account for relations between the z to y and z to x axis. Here is a sample of a few.

(9)   Behind (A,B) = Front (B,A).
(10) Behind (D,B) **If** Below (D,A) & Behind (A,B).
(11) Behind (D,B) **If** Left (D,A) & Behind (A,B).
(12) Behind (A,C) **If** Behind (A,B) & Behind (B,C).
(13) Behind (E,B) **If** Left (E,D) & Behind (D,B).
(14) Behind (E,B) **If** Left (D,E) & Behind (D,B).
(15) Left (B,E) **If** Behind (D,A) & Left (D,E).
(16) Left (E,B) **If** Behind (D,A) & Left (E,D).
(17) Below (E,B) **If** Left (E,D) & Below (D,A).
(18) Below (E,B) **If** Left (D,E) & Below (D,A).

Thus for any 3-D spatial problem the MMA has 6 inference rules while a PC model has at least 18. In addition to the MMA advantages computationally, the following experiment will investigate further theoretical predictions of the MMA and PC models.

Experiment: spatial inferences in undetermined spatial reasoning

This construction of a mental model assumes an incremental updating of the representation on the basis of the present and past input. So the resultant representation in any given moment guides the interpretation of subsequent input. A question arises as to the integration of previous information with the interpretation of each new sentence (see Sanford & Garrod, 1989). One issue in this integration process is how to resolve conflict in indeterminate spatial problems. In this process the MMA assumes comprehension proceeds in a linear fashion and can not have objects share the same space in the coordinate plane. One key prediction of the MMA is to prefer certain resolutions over others for simple indeterminate problems such as *A left of B, B right of C*. Where resolutions are assumed to be depicted on one dimension as either A C B or C A B (see Hayward and Tarr, 1995; Regier, & Carlson, 2001). Givon's iconicity assumption (1992), would predict C A B, which states humans assume an event *t* stated in discourse gets constructed and remains unchanged when integrated with new information *q*. To build this depiction, the MMA constructs the first premise and displays A B. Since C can not occupy the same space as A, C is placed on the left of the A to make C A B (see Figure 9 for the added subroutine for this process). PC models would not predict any

preference for a resolution since there are no constraints of space for PC, thus it does not matter that A and C occupy the same space.

subundetermined(*trajector*)
```
{
  if(exists(x_map, trajector)
  {
    if(relation == "LEFT")
    {
      x_map(trajector) = min(x_map) – 1;
      y_map(trajector) = y_map(landmark);
    }
    if(relation == "RIGHT")
    {
      x_map(trajector) = max(x_map) +1;
      y_map(trajector) = y_map(landmark);
    }
  }
    if(exists(y_map, trajector)
    {
      if(relation == "BELOW")
      {
        y_map(trajector) = min(y_map) -1;
        x_map(trajector) = x_map(landmark);
      }
      if(relation == "ABOVE")
      {
        y_map(trajector) = max(y_map) +1;
        x_map(trajector) = x_map(landmark);
      }

    }

  }
}
```

Figure 9: Undetermined coordinate assignment for the x and y axis. This rule says that if the assigned coordinate already exists in the hash table, replace its x or y axis with the (the min) -1 or (the max) + 1 of the values in the hash table.

Data Collection

Twenty college students were presented with 6 undetermined spatial problems consisting of left and right, (e.g., (1a) *circle is on the right of the square*, (1b) *square is on the left of the triangle)*. Instructions were to list all possible shape spatial relations that were not already stated in the 2 propositions. Subjects were not explicitly instructed to draw, but were told they could use any means necessary to generate the spatial relations between the shapes. The dependent measure was the subjects' first spatial inference given for each problem.

Results and Discussion

For undetermined spatial inference problems, 54% of the observations preferred the MMA prediction, 38% supported other correct alternatives, and 8% were incorrect. A comparison of the correct alternatives yielded a significant one-tailed difference $x^2 = 3.25$, p < .05. Additionally, all subjects drew the spatial depictions to generate inferences. In these drawings 94% were drawn on one dimension, supporting theories of spatial prototypically (Hayward & Tarr, 1995; Regier, & Carlson, 2001).

Discussion

Our data suggest that a simple coordinate based construction system is sufficient to model many characteristics of spatial reasoning in discourse. This is consistent with intuitions of proponents of Mental Models (Johnson-Laird & Byrne, 1991). By using simple Euclidian based rules for spatial relations between object pairs, inferences can be made as to spatial relationships between complex spatial arrays. Additionally, the MMA requires a small set of rules that can be generalized to spatial reasoning problems that pose complexity for PC. By testing formally PC models and the MMA it is apparent that mental models of MMA have advantages theoretically as well as computationally.

Arguments have been made that humans may not process mental models in a Euclidian like space (Langston, Kramer, & Glenberg, 1998), these experiments looked at implicit spatial relations in text comprehension. Spatial reasoning mainly involves explicit instructions to look for spatial relations between objects. This process can be argued to be Euclidian based since it is a simple strategy to use for generating spatial inferences. Further research should be conducted to determine the validity of Euclidian based representations.

Additionally, the MMA can be easily applied to 3 dimensional problems since the subroutine for assigning coordinates and scanning directions is not qualitatively different for a third dimension. Including a third dimension also would allow for an "in/out" relation. This relation would have to incorporate notions of space for objects beyond points, since the notion of containment has assumptions of size for landmarks and trajectors.

MMA in Language Comprehension

In the domain of language comprehension, spatial reasoning plays an important role. Landau and Jackendoff (1993) describe language comprehension as a combination of "what and "where" process. This "where" process can be seen as a formalization in the MMA to establish situation models of objects or agents in discourse. A major theoretical question arises as to how humans combine the "what" with the "where" in situation models. One option possibility is that semantic "what" information is bound to "where" information through pointers in long-term memory (Sanford & Garrod, 1981). The details of this possibility pose a challenge for further research.

Conclusion

This paper represents a scalable computational approach to Mental Models using simple coordinates in 3-D space.

While PC models can also generate spatial relationships, the nature of its representation causes challenges both theoretically and computationally.

Acknowledgments

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