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SANTA CRUZ

**ESSAYS ON MONETARY POLICY AT THE EFFECTIVE LOWER
BOUND AND WITH FINANCIAL DISRUPTION**

A dissertation submitted in partial satisfaction of the
requirements for the degree of

DOCTOR OF PHILOSOPHY

in

ECONOMICS

by

Akatsuki Sukeda

June 2019

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Abstract

Essays on Monetary Policy at the Effective Lower Bound and with Financial
Disruption

by

Akatsuki Sukeda

This dissertation consists of three self-contained chapters.

The first chapter analyzes the effectiveness of optimal sustainable forward guidance at the effective lower bound when the economy faces occasional financial disruptions. Financial disruptions make households and firms respond less to future monetary shocks because of precautionary saving and the value of current profit increases. As a result, the following three results emerge. First, forward guidance affects the current economy much less strongly without changing its effect when the economy exits the lower bound. Second, the effectiveness of forward guidance becomes weaker even if the central bank has perfect credibility and commit to stronger forward guidance. Third, since output gaps and inflation experience a larger deviation from the optimal level at exit periods, credibility concerns make the effectiveness of optimal sustainable forward guidance even weaker.

The second chapter analyzes the effectiveness of forward guidance in a low natural rate of interest environment. Since the strength of forward guidance depends on how lower and how many more periods the central bank will set the real interest below the natural rate, a low natural rate environment makes the central bank commit to

forward guidance with longer horizons to achieve the same effects on the current period. This increases the cost of forward guidance at the exit periods because the deviation from the optimal level becomes larger and lasts longer. As a result, the effectiveness of forward guidance becomes weaker even if the central bank has perfect credibility. Moreover, gaining credibility becomes harder and this even weakens the effectiveness of forward guidance.

The third chapter analyzes how the effects of monetary policy depend on the condition of the financial sector. With heterogeneous households in terms of the marginal cost of working, there exist essential financial needs. When it is more costly for the financial sector to extend lending than normal times, the effects of monetary policy becomes smaller. The larger the households heterogeneity is, the smaller the effects become. This is in sharp contrast with the case when there are no financial frictions in that the households heterogeneity does not affect the effects of monetary policy with the perfect financial market. The results also hold for forward guidance.

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Chapter 1

Credible Forward Guidance with Financial Disruption

1.1 Introduction

Forward guidance, which is announcements by central banks on the future policy interest rate, is an important monetary policy tool especially when the current policy interest rate hit the effective lower bound. Although the effectiveness of forward guidance in standard New Keynesian models is well understood, we do not know much about the effectiveness of forward guidance under financial disruption. This is the first paper which shows that the effectiveness of forward guidance might be smaller and the credibility on whether the central bank keeps the promise matters under financial disruption.

In standard New Keynesian models, forward guidance is powerful, and the

central bank almost never has the incentive to renege on the promise on the future policy rate. The key mechanism is that the output gap and the inflation rate in the current period respond to expectations on the future economic outcome. This effect is so strong that forward guidance has a larger effect when announced than when the actual interest rate cut occurs, which is coined as forward guidance puzzle. Then, in a severe recession which makes the effective lower bound binding, the central bank announces to keep the nominal interest rate low for several periods even when the economy gets out of the recession, and this has a huge expansionary effect without affecting the output gaps and the inflation when the economy exits the effective lower bound.

Then, once the economy gets out of the recession, the central bank has a strong incentive to keep the promise and set the interest rate low for several periods. The central bank compares the cost and the benefit of keeping the forward guidance announced previously to decide whether to renege on the promise or not. The cost comes from enduring slightly higher than the optimal level of the output gap and the inflation rate for several periods. The benefit is being able to use forward guidance again in the future recessions. Since the benefit is much larger than the cost, credibility concern is less relevant.

However, when there is financial distress, there are several channels through which the effects of forward guidance may change. First, when financial intermediaries are in trouble, borrowing credit becomes much more difficult. Then at the current period, those who want to borrow end up with low consumption. So the output gap responds less to the change in current interest rate. Also, since those who want to lend

today are afraid of having a hard time borrowing tomorrow, they consume less today than when there are no borrowing concerns. So the current output gap responds less to an increase in the future output gap. Second, firms also face problems in borrowing credit. This means that the current profit matters more to reduce financial frictions. Then, in response to an increase in the future inflation rate, firms are less inclined to increase prices because it decreases the current profit.

To capture the importance of occasional financial crises, I use a linearized New Keynesian model which behaves differently at the good state and the bad state. At the good state, the economy is characterized by a standard New Keynesian model. At the bad state, the output gap and the inflation rate respond less to future economic outcomes compared to the good state. The economy switches between the good state and the bad state following a Markov process, and agents in the economy correctly understand the probability of the state switches.

When I analyze forward guidance in this model, three results emerge. First, forward guidance affects the current economy much less strongly without changing its effect when the economy exits the lower bound. How forward guidance affects the current output gap and the inflation rate depends on how much these variables respond to future shocks, and in the bad state, the economy responds less to future shocks. However, at exit periods, when the central bank keeps the interest rate lower than the optimal level because of forward guidance, the economy is at the good state, and the economy responds future shocks as in a standard model.

Second, the effectiveness of forward guidance becomes weaker even if the cen-

tral bank has perfect credibility and commit to forward guidance with a longer horizon. Because the effect of extending forward guidance for one more period becomes smaller, the central bank wants to carry out forward guidance with longer horizons. However, this makes the output gap and the inflation rate much higher than the optimal levels at exit periods. As a result, even if the central bank is perfectly credible, the welfare gain is much smaller. In the bench mark case, the welfare gain becomes 48% of a standard case.

Third, credibility problems matters. When the effects of forward guidance are weakened, since the economy experiences the larger deviation from the optimal level at exit periods, the cost of committing to forward guidance becomes larger while the future benefit of mitigating deep crises will be smaller. As a result, the effectiveness of credible forward guidance becomes 33% of a standard case. The number becomes even smaller if there is a possibility of regaining credibility when the central bank reneges on the promise.

The paper proceeds as follows. In section 2, the relevant literature on the credibility of forward guidance and financial frictions are reviewed. Section 3 starts the model and section 4 explains the main driver of the results. Section 5 derives equilibrium outcomes and Section 6 defines optimal forward guidance and optimal sustainable forward guidance. Section 7 shows the results and section 8 concludes.

1.2 Literature Review

Monetary policy at the effective lower bound was first analyzed by Krugman (1998) in a two-period model and extended into infinite-horizon New Keynesian models by Eggertsson and Woodford (2003) and Jung et al. (2005). These papers point to the result that even if central banks cannot lower interest rate today, committing to keep interest rate lower in the future significantly expand the current economy without incurring much cost once the economy gets out of recessions. Adam and Billi (2006) and Nakov (2008) extend the models to incorporate anticipation of binding zero lower bound in the future.

The monetary policy at the effective lower bound got renewed attention after the Great Recession. McKay et al. (2016) analyzed a standard New Keynesian model and showed that forward guidance is very powerful and an announcement of an interest rate cut in distant future has a stronger effect on the current inflation. Del Negro et al. (2015) showed that this powerful effects of forward guidance also exist in a medium-scale New Keynesian model and coined it as forward guidance puzzle.

Since forward guidance is a commitment to future interest rate changes, the effectiveness crucially depends on whether the policy is credible or not. Bodenstein et al. (2012) relaxed the assumption of perfect credibility by assuming that the central bank reneged on the promise with fixed probability and showed that the effectiveness of forward guidance becomes smaller in this setup. One implication of the model is that the central bank should commit to stronger forward guidance to overcome imperfect

credibility. This implication also arises in the present paper. However, committing to stronger forward guidance incur a larger cost at exiting periods and this hurts credibility. This paper is the first which analyzes this channel in a monetary model where credibility is fragile.

Nakata (2018) and Walsh (2018) relaxed the assumption of an exogenous probability of reneging by introducing endogenous credibility. The central bank reneges on the promise whenever it leads to a smaller loss for the central bank. However, what they showed is that in standard New Keynesian models, the credibility of forward guidance is virtually perfect because forward guidance reduces the cost of severe recessions without incurring much cost when the economy is out of recessions.

The implication of endogenous credibility of forward guidance may depend on the models to be used because it changes the benefit and the cost of forward guidance. So in this paper, I show that how the implication of credibility of forward guidance change in a New Keynesian model which is intended to capture some aspects of the Great Recession.¹

But there are many aspects of the Great Recession. This paper singled out financial distress which affects two parts of the model. First, financial intermediaries are implicitly introduced in a similar way to Curdia and Woodford (2016) and Woodford (2010). In normal times, financial intermediaries have a perfectly elastic intermedia-

¹Very recently, Nakata et al. (forthcoming) analyze the optimal forward guidance under perfect credibility using the discounted Euler equation and the discounted Phillips curve. However, they do not analyze credibility. Nakata and Sunakawa (2019) analyze the optimal credible forward guidance using a New Keynesian model with the discounted Phillips curve. Neither of the papers assumes state dependent discounting.

tion supply curve, but in crises periods, financial intermediation supply curve becomes steeper. Second, firms who face financial frictions have to endure external finance premium as in Bernanke and Gertler (1989).

There are several papers which modify standard New Keynesian models in such a way that the effectiveness of forward guidance is muted. McKay et al. (2016, 2017) introduce heterogeneous agents with borrowing constraints, and it also showed that the model could be well approximated by a standard New Keynesian model with a discounted Euler equation. Angeletos and Lian (2018) relaxes the common knowledge assumption and introduce higher order uncertainty. Gabaix (2018) introduces bounded rationality and showed that the economy is characterized by a discounted Euler equation and a discounted New Keynesian Phillips Curve. Del Negro et al. (2015) introduced agents with a finite life.

However, there are two differences between their papers and my paper. First, only my paper attributes the discounted effects to financial disruption, which is the cause of hitting the effective lower bound. Second, my paper assumes that additional discounting occurs only at the crisis periods while their papers assume that additional discounting occurs all the time and the degree does not depend on the state. As a result, credibility plays a more significant role in my model. Also, in terms of calibration, to reduce the effectiveness of forward guidance, the discounting has to be somewhat large, so in their models, they have to claim that the degree of these modifications is large all the time while my model requires that discounting is large only at the crisis periods.

1.3 The Model

The basic structure of the model follows Eggertsson and Woodford (2003) in that there are two aggregate states of the world.² In addition, I assume that the economy is characterized by the same linearized New Keynesian model as Eggertsson and Woodford (2003) in the good state while the economy at the bad state is characterized by a linearized New Keynesian model with additional discounting on the expected output gap and the expected inflation. This is a model with financial disruption, and both households and firms change their response to future shocks under financial disruption.

In the good state, the economy is characterized by the standard dynamic IS curve and the New Keynesian Phillips curve,

$$x_t = E_t x_{t+1} - \frac{1}{\sigma}(i_t - E_t \pi_{t+1} - r_t^n) \quad (1.1)$$

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t, \quad (1.2)$$

where x_t is the output gap, i_t is the nominal interest rate, π_t is the inflation rate, and r_t^n is the natural rate of interest.

In the bad state, the economy is characterized by

$$x_t = M_1 E_t x_{t+1} - \frac{1}{\sigma}(i_t - E_t \pi_{t+1} - r_t^n) \quad (1.3)$$

$$\pi_t = \beta M_2 E_t \pi_{t+1} + \kappa x_t. \quad (1.4)$$

The two systems differ in three components. First, the natural rate of interest depends on the aggregate state as in Eggertsson and Woodford (2003). The natural rate of

²This structure is often used to analyze equilibrium at the effective lower bound on nominal interest rates. See, for example, Nakata (2018) and Walsh (2018).

interest rate is positive when the economy is in the good state and becomes negative when the economy is in the bad state,

$$r_t^n = \begin{cases} \rho > 0 & \text{if the state is good} \\ r_z < 0 & \text{if the state is bad.} \end{cases} \quad (1.5)$$

Second, there is a discounting term $M_1 < 1$ in the dynamic IS curve. This reduces how much the current output gap respond to the future output gaps. This captures a channel through which financial disruption affects households behavior. When households are more afraid of potential low consumption in the next period, they want to save more for the next period. As a result, when there is a positive shock to future income, households increase consumption relatively less. When $M_1 = 1$, the equation is identical to the standard dynamic IS curve.

Third, there is an additional discounting term $M_2 < 1$ in the New Keynesian Phillips curve. This reduces how much the current inflation rate responds to the future inflation rate. This captures a channel through which financial disruption affects firms behavior. When firms face financial frictions, the value of current profit relative to future profit becomes even more valuable because having higher net worth relaxes the financial friction. When $M_2 = 1$, the equation is identical to the standard New Keynesian Phillips curve.

The economy switches between the two systems following the Markov process,

$$P \equiv \begin{bmatrix} p_{gg} & p_{gb} \\ p_{bg} & p_{bb} \end{bmatrix} = \begin{bmatrix} s & 1-s \\ 1-q & q \end{bmatrix}, \quad (1.6)$$

where p_{ij} with $i \in \{g, b\}, j \in \{g, b\}$ is the probability of switching from the state i to state j , and g stands for the good state and b stands for the bad state.

To close the model, we need to specify monetary policy, which will be discussed in section 5.

1.4 Inspecting the Mechanism

Before considering the stochastic case governed by the transition probability, I examine a deterministic case and show why the effectiveness of forward guidance differs from standard models.

Suppose that the economy enters into the bad state in the current period and the bad state continues for 10 periods. Thus, the economy goes back to the good state at 10 periods ahead and stays at the good state thereafter.

Following McKay et al. (2016), consider an experiment in which the central bank announces that it will cut the real interest rate for one period starting in 10 periods so that the real interest rate will decrease by A ?

This effect can be seen by iterating forward the dynamic IS curve and the New

Keynesian Phillips curve. In standard models, the effect is

$$\Delta x_t = -\frac{1}{\sigma} \sum_{i=0}^{\infty} (r_{t+i} - r_{t+i}^n) \quad (1.7)$$

$$= \frac{1}{\sigma} A, \quad (1.8)$$

$$\Delta \pi_t = \kappa \sum_{i=0}^{\infty} \beta^i \Delta x_{t+i} \quad (1.9)$$

$$= \frac{\kappa}{\sigma} A \sum_{i=0}^{10} \beta^i, \quad (1.10)$$

where the variables Δx_t and $\Delta \pi_t$ denotes the difference from the case without the policy announcement. In the same way, we can calculate $\Delta x_{t+i}, \Delta \pi_{t+i}$ for any i . A numerical result is in figure 1 with label “ $M_1 = 1, M_2 = 1$ ”. As shown in McKay et al. (2016), the effects of announcing the rate cut are stronger if the rate cut is further in the future for inflation.

Moreover, when we compare the effects of these announcements on the current periods and the period when the central bank actually lowers the interest rate, the effects on the current periods are as large as on the later period for the output gap and much larger for the inflation rate.

$$\frac{\Delta x_t}{\Delta x_{t+10}} = 1, \quad (1.11)$$

$$\frac{\Delta \pi_t}{\Delta \pi_{t+10}} = \sum_{i=0}^{10} \beta^i = \frac{1 - \beta^{11}}{1 - \beta} > 1. \quad (1.12)$$

Also, what if the central bank announces a cut to the nominal interest rate for

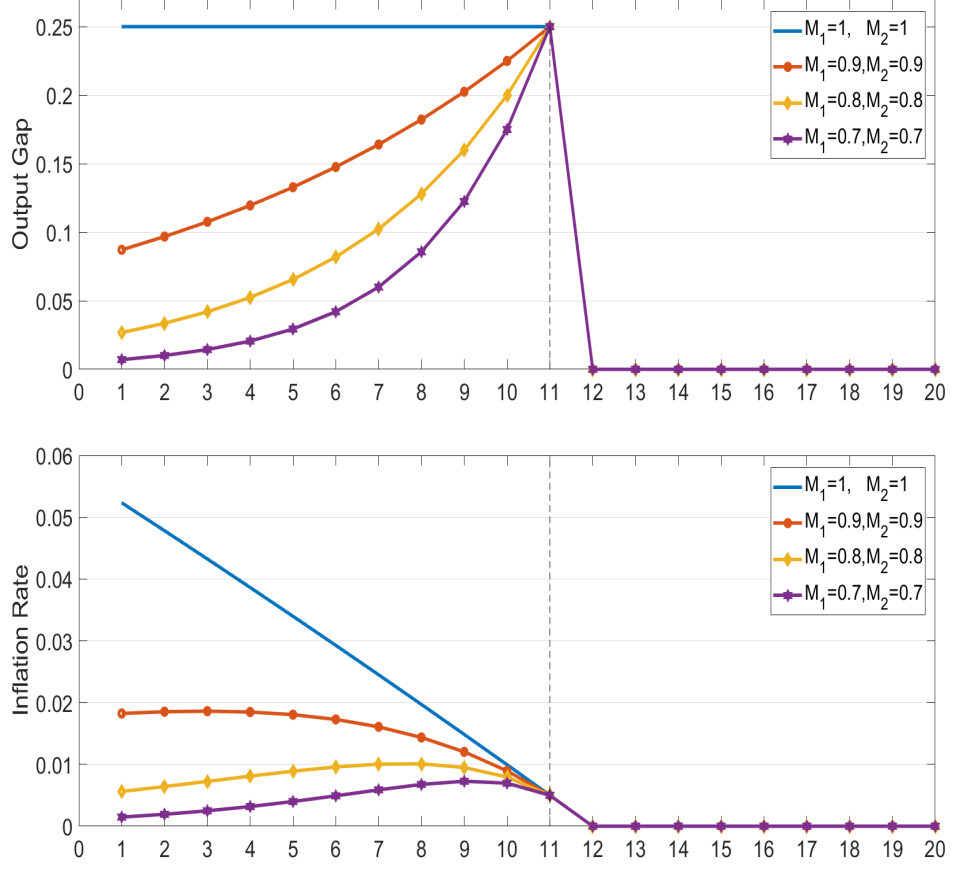


Figure 1.1: An Interest Rate Cut in 10 quarters ahead

multiple periods starting in 10 periods ahead? The effect is

$$\Delta x_t = -\frac{1}{\sigma} \sum_{i=0}^{\infty} (r_{t+i} - r_{t+i}^n) \quad (1.13)$$

$$= \frac{n}{\sigma} A, \quad (1.14)$$

$$\Delta \pi_t = \kappa \sum_{i=0}^{\infty} \beta^i \Delta x_{t+i} \quad (1.15)$$

$$= \frac{\kappa}{\sigma} A \left\{ n \sum_{i=0}^{10} \beta^i + \sum_{j=1}^{n-1} \beta^{10} \beta^j (n-j) \right\}. \quad (1.16)$$

where n is the periods of forward guidance. Thus, when the central bank announces a future interest rate cut for n periods, the effect becomes slightly more than n times larger than the one-period rate cut.

In contrast, in the model with discounting, the effect of announcing a one-period rate cut in 10 periods ahead is

$$\Delta x_t = -\frac{1}{\sigma} \sum_{i=0}^{\infty} M_1^i (r_{t+i} - r_{t+i}^n) \quad (1.17)$$

$$= \frac{1}{\sigma} A M_1^{10}, \quad (1.18)$$

$$\Delta \pi_t = \kappa \sum_{i=0}^{\infty} \beta^i \Delta x_{t+i} \quad (1.19)$$

$$= \frac{\kappa}{\sigma} A M_1^{10} \sum_{i=0}^{10} \beta^i M_2^i. \quad (1.20)$$

So the effects are muted by powers of M_1 and M_2 . A numerical simulation is in figure 1 for two different combinations of M_1 and M_2 less than unity. We can see that the effects on the current period are much weaker while the effects when the central bank actually lowers the policy rate are the same.

The comparison of the effects on the current period and later periods becomes

$$\frac{\Delta x_t}{\Delta x_{t+10}} = M_1^{10}, \quad (1.21)$$

$$\frac{\Delta \pi_t}{\Delta \pi_{t+10}} = M_1^{10} \sum_{i=0}^{10} \beta^i M_2^i \quad (1.22)$$

So now the feature that announcing rate cuts further in the future has stronger effects disappears for small values of M_1 and M_2 .

Also, the effect of announcing to cut the nominal interest rate for n periods

starting in 10 periods ahead is

$$\Delta x_t = -\frac{1}{\sigma} \sum_{i=0}^{\infty} M_1^i (r_{t+i} - r_{t+i}^n) \quad (1.23)$$

$$= \frac{n}{\sigma} A M_1^{10}, \quad (1.24)$$

$$\Delta \pi_t = \kappa \sum_{i=0}^{\infty} \beta^i M_2^i \Delta x_{t+i} \quad (1.25)$$

$$= \frac{\kappa}{\sigma} A M_1^{10} \left\{ n \sum_{i=0}^{10} \beta^i M_2^i + \sum_{j=1}^{n-1} (\beta M_2)^{10} \beta^j (n-j) \right\}. \quad (1.26)$$

So the fact that announcing an n -period rate cut in the future has effects that are roughly n times larger than announcing a one-period rate cut is still true.

As a result, in the present model with discounting because of financial disruption, the effects of announcing a future rate cut on the current period could be very weak, but the central bank can overcome this by announcing multiple-period rate cuts in the future. However, this could incur a much larger cost because even in the present model, the announcement has the same size of effects at later periods.

1.5 Equilibrium Outcomes

Now we discuss the stochastic case. To solve the model, we have to specify the monetary policy rule. I assume that the central bank chooses optimal discretionary policy unless the central bank commits to forward guidance.

The optimal discretionary policy is to set the real interest rate to achieve the optimal discretionary targeting rule in the good state and set the nominal interest rate at the effective lower bound in the bad state. The optimal discretionary targeting rule is

derived by minimizing the current period quadratic loss functions, which will be formally specified later, subject to the dynamic IS curve and the New Keynesian Phillips curve,

$$\kappa\pi_t + \lambda x_t = 0. \tag{1.27}$$

For simplicity, the effective lower bound is set at zero.

Forward guidance is defined as the additional periods when the central bank set the nominal interest rate at the effective lower bound beyond optimal discretionary policy. This choice follows Walsh (2018) and greatly simplifies the computation of the model. This forward guidance policy is more expansionary than the optimal discretionary policy because it lowers the real interest rate in the future and higher expected future output gap and inflation increases the current output gap and inflation.

For optimal discretionary policy and each length of forward guidance, we can calculate economic outcomes and the loss functions at each point in time.

1.5.1 The Optimal Discretionary Policy

When the central bank does not use forward guidance, I assume that it follows the optimal discretionary policy. For the optimal discretionary case, the central bank sets the nominal interest rate equal to zero in the bad state, which is the effective lower bound, and sets the nominal interest rate to achieve the targeting rule in the good state.

Then, the economy is characterized by five equations

$$\pi_n^d = \beta E_t \pi_{t+1}^d + \kappa x_n^d \quad (1.28)$$

$$x_n^d = E_t x_{t+1}^d - \frac{1}{\sigma} (i_n^d - E_t \pi_{t+1}^d - \rho) \quad (1.29)$$

$$\pi_z^d = \beta M_2 E_t \pi_{t+1}^d + \kappa x_z^d \quad (1.30)$$

$$x_z^d = M_1 E_t x_{t+1}^d + \frac{1}{\sigma} (E_t \pi_{t+1}^d + r_z) \quad (1.31)$$

$$0 = \kappa \pi_n^d + \lambda x_n^d, \quad (1.32)$$

where the last equation is the targeting rule. Superscript “ d ” stands for optimal discretionary policy. Subscript “ z ” stands for the periods when the zero lower bound is binding, and subscript “ n ” stands for the periods when the bound is not binding. Using the transition matrix, we can write out conditional expectations and obtain

$$\pi_n^d = \beta (s \pi_n^d + (1-s) \pi_z^d) + \kappa x_n^d \quad (1.33)$$

$$x_n^d = (s x_n^d + (1-s) x_z^d) - \frac{1}{\sigma} (i_n^d - (s \pi_n^d + (1-s) \pi_z^d) - \rho) \quad (1.34)$$

$$\pi_z^d = \beta M_2 (q \pi_z^d + (1-q) \pi_n^d) + \kappa x_z^d \quad (1.35)$$

$$x_z^d = M_1 (q x_z^d + (1-q) x_n^d) + \frac{1}{\sigma} ((q \pi_z^d + (1-q) \pi_n^d) + r_z) \quad (1.36)$$

$$0 = \kappa \pi_n^d + \lambda x_n^d. \quad (1.37)$$

This is a system of linear equations, and we can solve for these five economic variables.

When $M_1 = M_2 = 1$, this system is identical to standard New Keynesian models. What is distinct about these equilibrium conditions is that discounting terms, M_1, M_2 , show up only in equations at the bad state.

1.5.2 Forward Guidance Policy

First, we consider forward guidance policy for one period. The central bank set the nominal interest rate at zero in the bad state, and it also sets the nominal interest rate at zero in the first period when the economy moves from the bad state to the good state. The central bank then sets the nominal interest rate to achieve the targeting rule if the economy remains in the good state.

Then, the economy is characterized by the following seven equations

$$\pi_n^{fg1} = \beta(s\pi_n^{fg1} + (1-s)\pi_z^{fg1}) + \kappa x_n^{fg1} \quad (1.38)$$

$$x_n^{fg1} = (sx_n^{fg1} + (1-s)x_z^{fg1}) - \frac{1}{\sigma}(i_n^{fg1} - (s\pi_n^{fg1} + (1-s)\pi_z^{fg1}) - \rho) \quad (1.39)$$

$$\pi_e^{fg1} = \beta(s\pi_n^{fg1} + (1-s)\pi_z^{fg1}) + \kappa x_e^{fg1} \quad (1.40)$$

$$x_e^{fg1} = (sx_n^{fg1} + (1-s)x_z^{fg1}) + \frac{1}{\sigma}((s\pi_n^{fg1} + (1-s)\pi_z^{fg1}) + \rho) \quad (1.41)$$

$$\pi_z^{fg1} = \beta M_2(q\pi_z^{fg1} + (1-q)\pi_e^{fg1}) + \kappa x_z^{fg1} \quad (1.42)$$

$$x_z^{fg1} = M_1(qx_z^{fg1} + (1-q)x_e^{fg1}) + \frac{1}{\sigma}((q\pi_z^{fg1} + (1-q)\pi_e^{fg1}) + r_z) \quad (1.43)$$

$$0 = \kappa\pi_n^{fg1} + \lambda x_n^{fg1}, \quad (1.44)$$

where superscript “ $fg1$ ” stands for forward guidance with one period. Subscript “ e ” stands for the period when the central bank sets the nominal interest rate at zero even though the economy is at the good state because of forward guidance. This is a system of linear equations, and we can solve for these seven economic variables. Extending this analysis to forward guidance for k periods is straightforward. See the appendix for this.

To briefly illustrate how the discounting terms M_1 and M_2 affect equilibrium

outcomes, Table 1 compares the model outcome with the standard case using the calibration in section 7.

	$M_1 \approx 1, M_2 \approx 1$	$M_1 = 0.7, M_2 = 0.7$
(π_z^d, x_z^d)	(-0.01, -0.07)	(-0.01, -0.07)
(π_z^{fg1}, x_z^{fg1})	(-0.0011, -0.0585)	(-0.0097, -0.0698)

The Output gaps are quarterly and the inflation rates are annually.

Table 1.1: The Output Gaps and the Inflation Rate at the ELB

1.6 Credibility of Forward Guidance

Given this economic system, we define credible forward guidance. We just saw how forward guidance policy affects the economy with perfect credibility in (38)-(44). However, forward guidance is a commitment policy. This fact is obvious from the fact that this policy entails a deviation from the optimal discretionary policy by definition. Thus, there is an issue of credibility in that even if the central bank has an incentive to announce forward guidance at a point in time, it might want to renege on it in the later period and revert to the optimal discretionary policy. Then, when agents know this incentive structure of the central bank, they never form expectations believing the forward guidance, so announcements of the forward guidance do not have any effects. Since forward guidance does not have any actions until the future periods, it is crucial to understand the incentive structure of the central bank.

1.6.1 Loss Functions

I assume the central bank sticks to forward guidance only if it is desirable for the central bank to do so in terms of the bank's loss function. The central bank's loss function is the sum of discounted expected loss for each period

$$L_t = \sum_{i=0}^{\infty} \beta^i E_t(\pi_{t+i}^2 + \lambda x_{t+i}^2), \quad (1.45)$$

or written recursively,

$$L_t = (\pi_t^2 + \lambda x_t^2) + \beta E_t L_{t+1}. \quad (1.46)$$

For each monetary policy, we can calculate economic outcomes and the loss functions at each state. Then, we can compare these loss functions and see whether the central bank has the incentive to stick to forward guidance or not. We already saw how to calculate economic outcomes. Here it is assumed that there is no chance of regaining credibility once the central bank reneges on any promise. In section 7.3, I relax this assumption.

Under the optimal discretionary policy, the present discount values of periodic losses satisfy the following conditions

$$L_n^d = (\pi_n^d)^2 + \lambda (x_n^d)^2 + \beta E_t L_{t+1}^d \quad (1.47)$$

$$L_z^d = (\pi_z^d)^2 + \lambda (x_z^d)^2 + \beta E_t L_{t+1}^d. \quad (1.48)$$

Again, by writing out the conditional expectations using the transition matrix, we obtain

$$L_n^d = (\pi_n^d)^2 + \lambda (x_n^d)^2 + \beta (s L_n^d + (1-s) L_z^d) \quad (1.49)$$

$$L_z^d = (\pi_z^d)^2 + \lambda (x_z^d)^2 + \beta (q L_z^d + (1-q) L_n^d). \quad (1.50)$$

Given the four economic variables, $\pi_n^d, x_n^d, \pi_z^d, x_z^d$, this is another system of linear equations, and we can solve for the two loss functions.

Next, losses with one-period forward guidance satisfy the following conditions

$$L_n^{fg1} = (\pi_n^{fg1})^2 + \lambda(x_n^{fg1})^2 + \beta(sL_n^{fg1} + (1-s)L_z^{fg1}) \quad (1.51)$$

$$L_e^{fg1} = (\pi_e^{fg1})^2 + \lambda(x_e^{fg1})^2 + \beta(sL_n^{fg1} + (1-s)L_z^{fg1}) \quad (1.52)$$

$$L_z^{fg1} = (\pi_z^{fg1})^2 + \lambda(x_z^{fg1})^2 + \beta(qL_z^{fg1} + (1-q)L_e^{fg1}). \quad (1.53)$$

Given the output gaps and the inflation rate that satisfy (38)-(44), (51)-(53) are to solve for the present discount value of losses in each state.

Extending this analysis to forward guidance for k periods is straightforward.

See the appendix for this.

For ease of interpretation, the loss functions are transformed so that unit is steady-state consumption equivalence following Billi (2011),

$$\mu = (1 - \beta) \left(\frac{\omega\theta(1 + \eta\theta)}{(1 - \omega)(1 - \omega\beta)} \right) L.$$

When $\mu = 0.01$, this means that 0.01% of steady-state consumption is lost.

1.6.1.1 Definition of Optimal Forward Guidance

Given these losses at each state associated with each length of forward guidance, I define optimal forward guidance as the forward guidance which minimizes losses when the zero lower bound is binding. This is what the central bank wants to carry out if it has perfect credibility.

1.6.2 Credibility

However, in this model, credibility is endogenous. The definition of credible forward guidance depends on the assumption of the alternative policy which the central bank reverts to when it reneges on the forward guidance. I assume the alternative policy is forward guidance with shorter periods.

Here is the mechanism to check the credibility. Loss functions with the forward guidance of interest are compared with loss functions with alternative policies at all points in time, and if it is lower at all points in time, the forward guidance of interest is credible. Otherwise, the central bank has the incentive to deviate from the forward guidance at some point in time and agents do not trust this forward guidance from the very beginning.

1.6.2.1 Reverts-to-Forward Guidance with Shorter Periods

The definition of credible forward guidance comes from the idea that even if the central bank reneges on the promise, it does not necessarily lose all of the credibility because the loss functions are smaller in this case.

We check the credibility of forward guidance recursively. First, forward guidance with one period is credible if and only if

$$L_z^{fg1} < L_z^d, \tag{1.54}$$

$$L_e^{fg1} < L_n^d, \tag{1.55}$$

$$L_n^{fg1} < L_n^d. \tag{1.56}$$

Then, given that forward guidance with one period is credible, the alternative policy becomes forward guidance with one period. Thus, to check the credibility of forward guidance with two periods, we compare loss functions with forward guidance for two periods and those with forward guidance for one period. Forward guidance with two periods is credible if and only if forward guidance for one period is credible and

$$L_z^{fg2} < L_z^{fg1}, \quad (1.57)$$

$$L_e^{fg2} < L_e^{fg1}, \quad (1.58)$$

$$L_{e2}^{fg2} < L_n^{fg1}, \quad (1.59)$$

$$L_n^{fg2} < L_n^{fg1}. \quad (1.60)$$

In all cases considered, what is relevant is the first exit period. This result is because loss functions at the bad state are usually lower with forward guidance unless the length is not too long, and then loss functions at the normal state tend to be lower because the loss in the crises is lower. Finally, if the central bank wants to deviate from forward guidance at one of the exit periods, it often deviates from the very beginning of the exit periods because the cost of staying with forward guidance starts to accrue at the first exit period

As a result, we only have to check sign of gain and temptation. Gain is the difference between the two loss functions in (54) and (57). This measures whether at the bad state, the central bank has the incentive to extend the horizon of forward guidance for one more period or not. If the gain is negative, the forward guidance with one more period is not credible.

The temptation is the difference of the two loss functions in (55) and (58). This measure whether at the first exit period the central bank has the incentive to renege on the promise or not. If the sign of temptation is positive, the forward guidance in consideration is not credible.

In Nakata (2018) and Walsh (2018), the alternative policy is the optimal discretionary policy. When this alternative policy is used, sustainability is gained as long as losses with forward guidance for k periods at each state are smaller than losses with the optimal discretionary policy. As a result, it is possible that losses with forward guidance for specific periods are higher at one state than losses with forward guidance for shorter periods. On the contrary, when the alternative policy is forward guidance with shorter periods, forward guidance is sustainable if losses at each state are smaller than forward guidance for any shorter periods.

As a result, in some cases, optimal forward guidance can be achieved as the optimal sustainable forward guidance. In other cases, optimal forward guidance is not sustainable, and the best the central bank can do is to follow the optimal sustainable forward guidance, which minimizes the losses at the bad state among sustainable forward guidance.

1.7 Simulations and Results

Now we simulate an economy with occasional financial crises. The size of the negative shock is calibrated so that the inflation rate becomes -1% annually and

the output gap becomes -7% quarterly in the bad state with the optimal discretionary policy as in Nakata (2018). To achieve the target, a cost-push shock is added in this part on top of the negative shock to the natural rate of interest when the economy is at the bad state.³

1.7.1 Parameterization

Baseline parameters are summarized in Table 1. Most of these parameters are standard except parameters to govern the transition probability of the states, s and q , and discounting parameters on expected terms in dynamic IS curve, M_1 , and New Keynesian Phillips curve, M_2 . Results with several different values will be shown to see some specific parameter values do not drive important results.

The transition probability is chosen so that the expected duration of the good state is 50 years and the expected duration of the bad state is 2.5 years. This corresponds to $s = 0.995$ and $q = 0.9$ and implies that 4.8% of the time, the economy experiences financial disruptions. This value is lower than Walsh (2018), which reports that the U.S. economy was at the effective lower bound for 12% of the time. The difference can be somewhat justified because not all of the zero lower bound periods are associated with financial disruption, and financial disruption is at the center of the present model.

The discounting parameters are harder to calibrate. Fuhrer and Rudebusch (2004) estimated the Euler equation and found the value of M_1 close to 0.6. Mavroeidis

³When M_1 and M_2 change, how strongly demand shocks affect the output gap relative to the inflation rate changes, and we cannot achieve the calibration target. The size of cost-push shocks changes for different values of M_1 and M_2 .

et al. (2014) estimated the New Keynesian Phillips curve and found the value of M_2 around 0.5 to 0.7. Therefore, I choose default values of the discounting as $M_1 = M_2 = 0.7$.

Parameters	Description	Values
β	Discount factor	0.99
σ	Relative risk aversion	2
ω	Probability of no resetting price opportunity	0.75
κ	Slope of the Phillips curve	0.02
θ	Elasticity of demand	7.66
λ	Relative weight on output gap volatility	0.003
s	Probability of staying at the good state	0.995
q	Probability of staying at the bad state	0.9
M	Discounting in Dynamic IS curve	0.7
N	Discounting in New Keynesian Phillips curve	0.7

Table 1.2: Parameter Values

1.7.2 Results

As a comparison, I first show the result when $M_1 = 0.99$, $M_2 = 0.99$, which is essentially a standard New Keynesian model.⁴ Figure 2 shows how the output gap and inflation rate behaves in the case where the bad state continues from period 1 to 6. Under the optimal discretionary policy, which corresponds to $k = 0$, the economy suffers from a negative output gap and especially large deflation. However, when the central bank commits to forward guidance with one period, $k = 1$, even at the bad state, inflation becomes very close to zero, which is the optimal level. At period 7, which is the first exit period, output gap and inflation rate are both positive, and this is not desirable

⁴The only reason M_1 and M_2 are not unity is because if so, forward guidance for one period is too strong and the output gap and the inflation rate at the ZLB becomes higher than those at the non-binding state.

$M_1 \approx 1, M_2 \approx 1$			
	L_z	Gain	Temptation
k=0	0.2013	—	—
<u>k=1</u>	0.0990	<u>0.1022</u>	<u>-0.0342</u>
k=2	0.1077	-0.0087	0.0035
k=3	0.2891	-0.1814	0.0623

Loss is measured in steady-state consumption level.

Table 1.3: Welfare Gain and Incentive to Deviate in Standard Case

in terms of optimal discretionary view. However, by accepting this undesirable boom at the exit period, the economy at the bad state performs much better. We can see this from table 3. Under the optimal discretionary policy, the value of loss function is 0.20 at the bad state. By committing to forward guidance with one period, the loss reduces to 0.10. The loss function associated with forward guidance with two periods is higher, and gain becomes negative, so forward guidance with one period is optimal forward guidance.

Next, we check whether the optimal forward guidance is sustainable or not. For $k = 1$, the temptation is negative. This means that the central bank does not have the incentive to renege on the promise. Therefore, in this benchmark case, the central bank only needs to commit to forward guidance with one period, and the policy is very powerful and credible.

Now, we see the result when $M_1 = 0.7$, $M_2 = 0.7$. Figure 3 shows how the output gap and the inflation rate behave. Under the optimal discretionary policy, the behavior is basically the same as the standard case. However, the behavior under optimal forward guidance, which is for 7 periods, $k = 7$, is very different. First, the

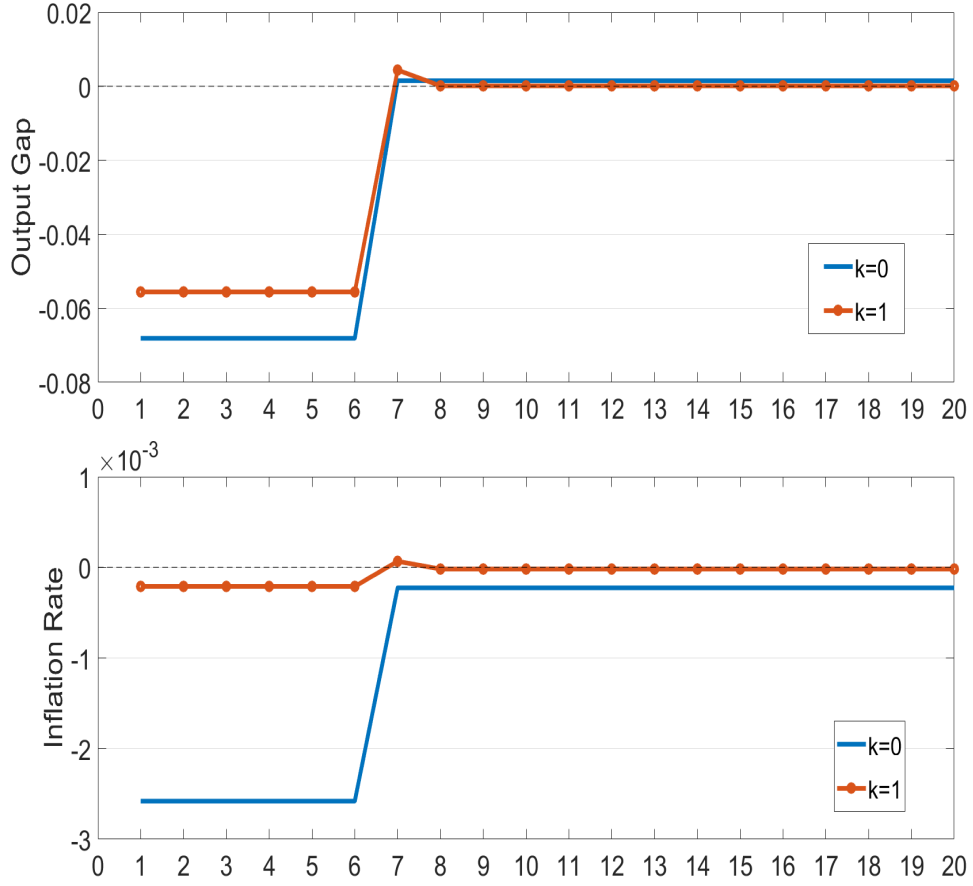


Figure 1.2: A Realized Time Series with a Standard Case ($M_1 \approx 1$, $M_2 \approx 1$)

central bank commits to forward guidance with longer horizons. This is because as we saw in section 4, when M_1 and M_2 are smaller, the effects of a future interest cut become smaller. Second, even with this longer forward guidance, the economy suffers from a large negative output gap and severe deflation. Third, at exit periods, the economy deviates significantly from the optimal level, which is zero. Table 4 shows loss functions. Gain is positive until forward guidance with 7 periods, $k = 7$, so this is the optimal forward guidance. The loss function at the bad state reduces significantly

from 0.204 to 0.160, but the welfare gain is much smaller compared to the benchmark case, which is from 0.20 to 0.10. So compared to the standard case, the welfare gain of optimal is only 48%.

Next, the optimal forward guidance is not sustainable anymore. In table 3, the value of temptation under forward guidance for 7 periods, $k = 7$, is negative. This means that if the central bank announces to keep the interest rate at zero for 7 periods once the economy goes back to the normal state, the central bank has the incentive to renege on the promise at the first exit period. Agents in the economy know this incentive structure of the central bank and the forward guidance with 7 periods is not sustainable. Forward guidance with 5 periods or above also has negative temptation, and this is not credible. As a result, optimal sustainable forward guidance is with 4 periods, $k = 4$. The reason why forward guidance with long periods is not sustainable in this case is that the economy has to suffer from large deviation from the optimal level at exit periods.

Under the optimal sustainable forward guidance, the loss function at the bad state reduces only to 0.173. As a result, compared to the standard case, the effectiveness of optimal sustainable forward guidance by 33%.

This result is driven by M_1 and M_2 being both lower than unity. For example when $M_1 = 1$ and $M_2 = 0.7$, optimal forward guidance, which is with 6 periods, $k = 6$, reduces loss functions from 0.20 to 0.04. Optimal sustainable forward guidance, which is with 5 periods, $k = 5$, still reduces loss functions to 0.05, so forward guidance is very effective. And the result is similar when $M_1 = 0.7$ and $M_2 = 1$, optimal forward

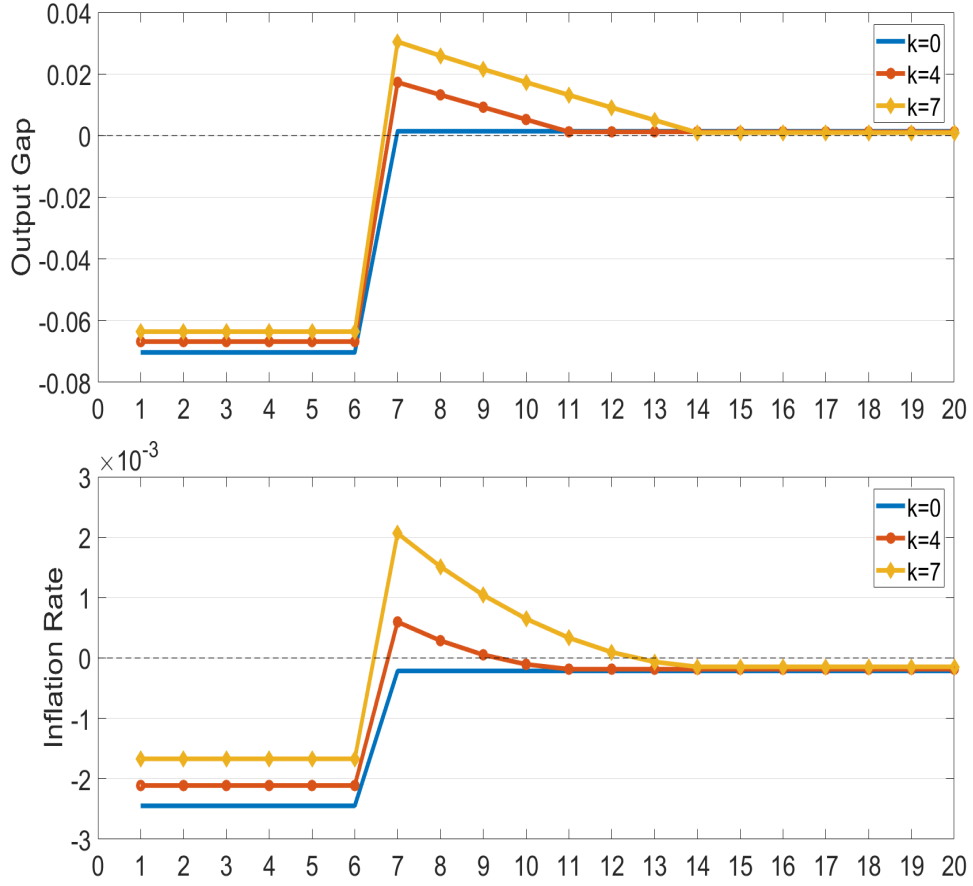


Figure 1.3: A Realized Time Series with a Discounted Case ($M_1 = 0.7, M_2 = 0.7$)

guidance, which is with 5 periods, $k = 5$, reduces loss functions from 0.203 to 0.117, which is 89% of the standard case, and optimal sustainable forward guidance, which is with 4 periods, $k = 4$, still reduces loss functions to 0.123, which is 83% compared to the standard case. This result is driven by the fact that there is complementarity between M_1 and M_2 about the effects of forward guidance on the current economy as we can see in equation (20).

Also, the result is driven by the fact that the discounting terms M_1 and M_2

$M_1 = 0.7, M_2 = 0.7$			
	L_z	Gain	Temptation
k=0	0.2037	—	—
k=1	0.1966	0.0071	-0.00231
k=2	0.1889	0.0077	-0.00235
k=3	0.1808	0.0081	-0.00207
<u>k=4</u>	0.1729	0.0079	<u>-0.00129</u>
k=5	0.1660	0.0070	0.00025
k=6	0.1610	0.0050	0.00287
<u>k=7</u>	0.1595	<u>0.0015</u>	0.00703
k=8	0.1635	-0.0040	0.01325
k=9	0.1757	-0.0122	0.02224

Loss is measured in steady-state consumption level.

Table 1.4: Welfare Gain and Incentive to Deviate in a Discounted Case

show up only in the bad state. For example, when I generalize the model in that the discounting terms in the good state are $M_{1,g}$ and $M_{2,g}$ and the discounting terms in the bad state are $M_{1,b}$ and $M_{2,b}$. The figure 4 shows the case when $M_{1,g} = M_{1,b} = 0.7$ and $M_{2,g} = M_{2,b} = 0.7$. The optimal forward guidance is for $k = 8$ and the optimal sustainable forward guidance is for $k = 4$.

Qualitatively, figure 3 and figure 4 are similar. However, we can see several differences. First, the negative output gap and the negative inflation rate at the bad state become severe in this case. This is because now the discounting occurs also at the good state, and the feature that doubling the length of forward guidance has roughly doubling effects on the current economy, which is shown in section 4, does not hold anymore. As a result, even though the central bank commits to more extended forward guidance, the effects at the bad state are smaller. Second, the size of deviation at exit periods is also smaller. This is again because now the discounting occurs even at the

good state.

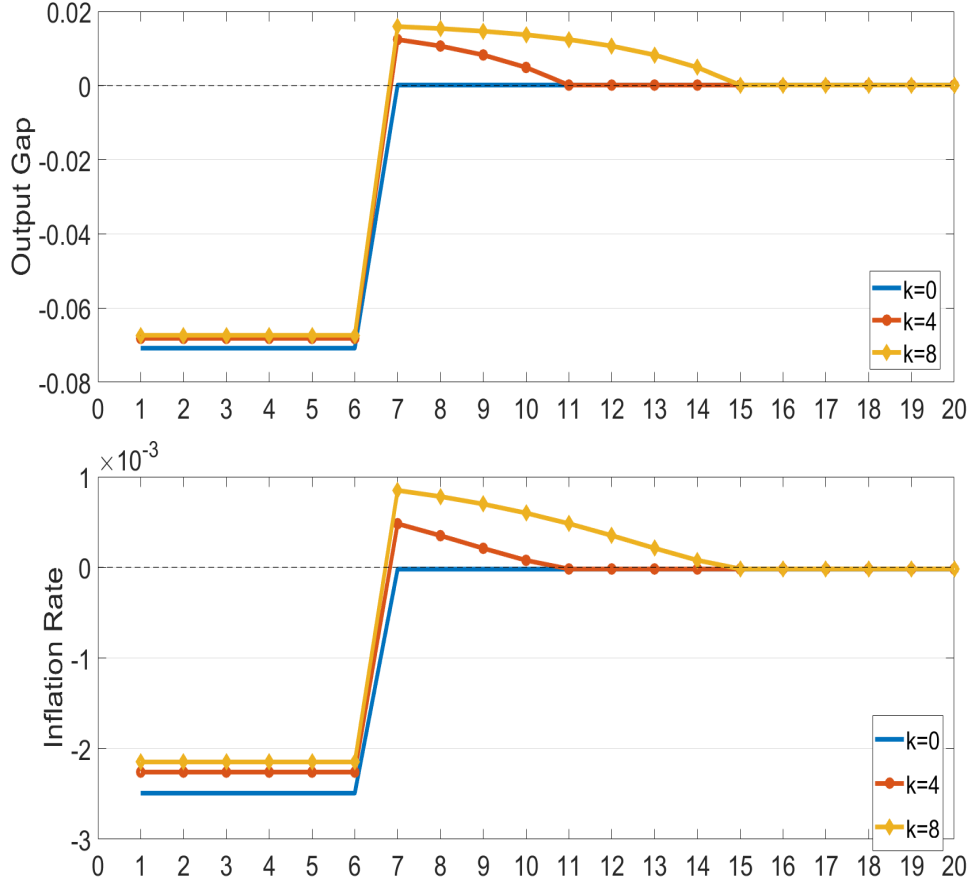


Figure 1.4: A Realized Time Series with an Always Discounted Case ($M_{1,g} = M_{1,b} = 0.7$, $M_{2,g} = M_{2,b} = 0.7$)

We can see the implications of these changes for the losses and the credibility in table 5. First, the effectiveness of forward guidance under perfect credibility is much smaller. In the state-dependent discounting case, optimal forward guidance reduces the losses from 0.20 to 0.16, but in the always discounting case, optimal forward guidance reduces the losses from 0.20 only to 0.18. Since the discounting is stronger, we can see that the gain from additional length of forward guidance decreases much faster in the

$M_{1,b} = M_{1,g} = 0.7, M_{2,b} = M_{2,g} = 0.7$			
	L_z	Gain	Temptation
k=0	0.2031	—	—
k=1	0.1959	0.0072	-0.00233
k=2	0.1904	0.0055	-0.00165
k=3	0.1863	0.0041	-0.00102
<u>k=4</u>	0.1834	0.0029	<u>-0.00046</u>
k=5	0.1815	0.0019	0.00002
k=6	0.1804	0.0011	0.00043
k=7	0.1799	0.0005	0.00076
<u>k=8</u>	0.1799	<u>-0.0000</u>	0.00103
k=9	0.1803	-0.0004	0.00125

Loss is measured in steady-state consumption level.

Table 1.5: Welfare in an Always Discounted Case

Gain column of table 5. The effectiveness of optimal forward guidance is 25% compared to the standard case.

Second, how much lack of credibility reduces the effectiveness of forward guidance becomes smaller. In the state-dependent discounting case, the difference between the effectiveness of optimal forward guidance and optimal sustainable forward guidance is 15% (=48%-33%) of the standard case. In the always discounting case, optimal sustainable forward guidance has 21% of welfare gain compared to the standard case. So the difference between the effectiveness of optimal forward guidance and optimal sustainable forward guidance is only 4% (=25%-21%). Credibility matters much more in the state-dependent discounting case.

1.7.3 Assumptions on Punishment

So far, we assumed that once the central bank reneges on the promise, there is no chance of regaining credibility. This might be a strong assumption, so the assumption is relaxed by introducing a fixed exogenous probability of regaining credibility.

Once the central bank reneges on forward guidance, credibility will be lost but there is a small probability, ϕ , in each period that the central bank will regain credibility. One advantage of this way of modeling is that when $\phi = 0$, the assumption goes back to everlasting punishment.

There are several details on this assumption. The first one is that when the economy is at “bad” state or “exit” periods, there is no possibility of regaining credibility. The second one is that there is no possibility of regaining credibility on the period when the central bank breaks the promise. These two assumptions are just operational, and we can easily modify these.

Since now agents know that there is a positive probability that the central bank regains credibility, the economic variables behave differently from previous cases. At “normal” state, the economy can stay at the same state or go to “bad” state as before. On top of this, if the central bank has not regained credibility yet, there is probability ϕ that the central bank regains credibility conditioning on the fact that the state in the next period is good. Since the economy at exit periods and crisis periods depends on the situation in the good state, all of the economic variables change with the possibility of regaining credibility. As a result, loss functions also change, and the implications for

credibility is different. The details of all the equations are in the appendix.

Now we go back to the table 4, which is for $M_1 = 0.7$, $M_2 = 0.7$. Only changes are at the temptation column and the length of optimal sustainable forward guidance. When $\phi = 0.0125$, which is 5% probability of regaining credibility a year, the optimal sustainable forward guidance is with 3 periods, $k = 3$, and it reduces loss functions from 0.204 only to 0.181. Compared to the standard case, the effectiveness of optimal sustainable forward guidance is 25%. Thus, lack of credibility reduces 23% ($=48\%-25\%$) of the welfare gain in the standard case. When $\phi = 0.04$, the effectiveness is 16%, and 32% ($=48\%-16\%$) of welfare in the standard case is lost due to lack of credibility.

1.8 Conclusions

The paper analyzed how the introduction of financial disruption changes implications of forward guidance. In crises periods, both households and firms respond less to future monetary shocks because of precautionary saving and the value of current profit increases more than in normal times. As a result, three results emerge. First, forward guidance affects the current economy much less strongly without changing its effect when the economy exits the lower bound. Second, the effectiveness of forward guidance becomes weaker even if the central bank has perfect credibility and commit to forward guidance with a longer horizon. Third, since output gaps and inflation experience larger deviation from the optimal level at exit periods, credibility concerns make the effectiveness of optimal sustainable forward guidance even weaker. The results sug-

gest that even if the economy responds to future shocks in normal times, if the economy responds less to future shocks in crisis periods, the effectiveness of forward guidance becomes weaker.

Future research is required to analyze the connection between financial disruption and forward-looking degree of the economy more generally. Then, we can analyze the complementarity between forward guidance and other instruments to help financial intermediaries. Also, investigating other implications from state-dependent forward-looking degree is of interest.

Chapter 2

Credible Forward Guidance in a Low Natural Rate Environment

2.1 Introduction

When the current interest rate is already at the effective lower bound, forward guidance, which is announcements on future interest rates, becomes an important monetary policy rule for central banks. Actually, many central banks around the world conducted some forms of forward guidance after the global financial crisis originated from the Great Recession.

However, many central banks could not achieve their inflation target despite their very aggressive forward guidance. For example, in Japan, the Bank of Japan has not achieved its inflation target for the last 20 years.

This is hard to be consistent with standard New Keynesian models because

forward guidance is very powerful in these models. Also, since the failure of achieving the inflation target lasted a long time, sharp changes due to financial crises cannot be the sole reason for the inability of forward guidance.

One of the significant long-term changes occurred in interest rates. As Eggetson et al. (2019) pointed out, in the last 25 years, interest rates has been steadily declining in many developed countries. This might have a huge implication on monetary policy through the current interest rate. Whether the current interest rate is expansionary or contractionary depends on the underlying neutral rate. If the underlying neutral rate has decreased and continues to stay low, the room for expansionary monetary policy might become smaller because interests cannot go below the effective lower bound.

However, implications on forward guidance is less clear. For each period of forward guidance, the room for expansionary policy becomes smaller in the same logic, however, central banks can extend the horizons of forward guidance. By doing so, central bank do not necessarily lose their ability to control business cycles.

Therefore, in this paper, I analyze the effectiveness of forward guidance in a low natural rate environment. The economy switches between the good state and the bad state, which differs in the natural rate of interest. At the bad state, the natural rate of interest is so low that the central bank has incentive to rely on forward guidance on top of setting the current interest rate at the effective lower bound.

The most important result is that, when the natural rate of interest rate is lower even after recovering from recessions, the central bank has to implement longer

forward guidance to achieve the same size of effects on the current period. However, it is shown that this prolonged forward guidance is more costly for the society because the economy has to deviate more from the optimal level. This cost becomes larger when the economy respond less to future shocks.

Then, this leads to the fact that optimal forward guidance becomes less effective even when the central bank has perfect credibility. On top of that, it becomes harder to maintain credibility because the cost of keeping the promise becomes larger. As a result, the effectiveness of forward guidance becomes even weaker.

The paper proceeds as follows. In section 2, the relevant literature are reviewed. Section 3 starts the model and section 4 explains the main driver of the results. Section 5 delivers equilibrium outcomes and defines optimal forward guidance and optimal sustainable forward guidance. Section 6 shows the results and section 7 concludes.

2.2 Literature Review

There are some papers which analyze the effectiveness of forward guidance with endogenous credibility. Walsh (2018) and Nakata (2018) use standard New Keynesian models and found that credibility is not an issue because forward guidance in standard New Keynesian models affects the current economy more than later periods.¹

There are several papers which analyze the effectiveness of forward guidance with endogenous credibility using modified New Keynesian models. Nakata and

¹Nakata et al. (forthcoming) analyze how the optimal forward guidance differs using a New Keynesian model with the discounted Euler equation and the discounted Phillips curve under perfect credibility.

Sunakawa (2019) analyze the optimal credible forward guidance using the discounted New Keynesian Phillips curve. Sukeda (2019) showed that when the economy respond less to future shocks in recessions, the effectiveness of forward guidance becomes smaller and credibility limits the effectiveness of forward guidance. Neither of the papers analyze how the optimal forward guidance depends on a low natural rate of interest environment.

Summers (2014) pointed out the possibility of secular stagnation in the U.S., which is represented by persistently low natural rate of interest. Eggertsson et al. (2019) showed that interest rates have been steadily declining in the last 30 years in several developed countries. How this affects the effectiveness of forward guidance is not answered yet.

2.3 The Model

The basic structure of the model follows Eggertsson and Woodford (2003) in that there are two aggregate states of the world.²

The economy is characterized by the standard dynamic IS curve and the New Keynesian Phillips curve,

$$x_t = M_1 E_t x_{t+1} - \frac{1}{\sigma} (i_t - E_t \pi_{t+1} - r_t^n) \quad (2.1)$$

$$\pi_t = \beta M_2 E_t \pi_{t+1} + \kappa x_t, \quad (2.2)$$

where x_t is the output gap, i_t is the nominal interest rate, π_t is the inflation rate, and r_t^n is the natural rate of interest. M_1 and M_2 are parameters which determine

²This structure is often used to analyze equilibrium at the effective lower bound on nominal interest rates. See, for example, Nakata (2018) and Walsh (2018).

how the current output gap and the inflation rate responds to future variables. Since standard New Keynesian models, which have $M_1 = 1$ and $M_2 = 1$, are known to have unrealistically large response from forward guidance, I introduce these parameters.

The natural rate of interest depends on the aggregate state as in Eggertsson and Woodford (2003). The natural rate of interest rate is positive when the economy is in the good state and becomes negative when the economy is in the bad state,

$$r_t^n = \begin{cases} r_n > 0 & \text{if the state is good} \\ r_z < 0 & \text{if the state is bad.} \end{cases} \quad (2.3)$$

Note that unlike Sueda (2019), M_1 and M_2 are not state dependent, and only the natural rate of interest is state dependent.

The economy switches between the two systems following the Markov process as in Nakata (2018) and Walsh (2018),

$$P \equiv \begin{bmatrix} p_{gg} & p_{gb} \\ p_{bg} & p_{bb} \end{bmatrix} = \begin{bmatrix} s & 1-s \\ 1-q & q \end{bmatrix}, \quad (2.4)$$

where p_{ij} with $i \in \{g, b\}, j \in \{g, b\}$ is the probability of switching from the state i to state j , and g stands for the good state and b stands for the bad state.

To close the model, we need to specify monetary policy, which will be discussed in section 5.

2.4 Inspecting the Mechanism

Before considering the stochastic case governed by the transition probability, I examine a deterministic case and show why the effectiveness of forward guidance differs from standard models.

Suppose that the economy enters into the bad state in the current period and the bad state continues for 10 periods. Thus, the economy goes back to the good state at 10 periods ahead and stays at the good state thereafter.

Following McKay et al. (2016), consider an experiment in which the central bank announces that it will cut the real interest rate for one period starting in 10 periods so that the real interest rate will decrease by A ?

This effect can be seen by iterating forward the dynamic IS curve and the New Keynesian Phillips curve. The effect is

$$\Delta x_t = -\frac{1}{\sigma} \sum_{i=0}^{\infty} M_1^i (r_{t+i} - r_{t+i}^n) \quad (2.5)$$

$$= \frac{1}{\sigma} A M_1^{10}, \quad (2.6)$$

$$\Delta \pi_t = \kappa \sum_{i=0}^{\infty} \beta^i M_2^i \Delta x_{t+i} \quad (2.7)$$

$$= \frac{\kappa}{\sigma} A M_1^{10} \sum_{i=0}^{10} \beta^i \left(\frac{M_2}{M_1} \right)^i, \quad (2.8)$$

where the variables Δx_t and $\Delta \pi_t$ denotes the difference from the case without the policy announcement. In the same way, we can calculate Δx_{t+i} , $\Delta \pi_{t+i}$ for any i . As shown in McKay et al. (2016), when the values of M_1 and M_2 are unity, the effects of announcing the rate cut are stronger if the rate cut is further in the future for inflation, and here

we can see that the claim holds as long as the values are close enough to unity.

Moreover, when we compare the effects of these announcements on the current periods and the period when the central bank actually lowers the interest rate, the effects on the current periods are as large as on the later period for the output gap and much larger for the inflation rate as long as values of M_1 and M_2 are close to unity.

$$\frac{\Delta x_t}{\Delta x_{t+10}} = M_1^{10}, \quad (2.9)$$

$$\frac{\Delta \pi_t}{\Delta \pi_{t+10}} = M_1^{10} \sum_{i=0}^{10} \beta^i \left(\frac{M_2}{M_1} \right)^i. \quad (2.10)$$

Both of these points can be seen from a simulation result in Figure 1. The result is identical to Sueda (2019) even though this model is different in that M_1 and M_2 are not state dependent.

Now, consider a policy announcement that the central bank will cut the real interest rate by half of A , but on the other hand, the rate will be lower for two periods. The effect of this announcement on the current period is

$$\Delta x_t = -\frac{1}{\sigma} \sum_{i=0}^{\infty} M_1^i (r_{t+i} - r_{t+i}^n) \quad (2.11)$$

$$= \frac{1}{\sigma} \left(\frac{A}{2} + \frac{A}{2} M_1 \right) M_1^{10} = \frac{1}{\sigma} A M_1^{10} \left(\frac{1 + M_1}{2} \right), \quad (2.12)$$

$$\Delta \pi_t = \kappa \sum_{i=0}^{\infty} \beta^i M_2^i \Delta x_{t+i} \quad (2.13)$$

$$= \frac{\kappa}{\sigma} A \left(M_1^{10} \sum_{i=0}^{10} \beta^i \left(\frac{M_2}{M_1} \right)^i + \frac{1}{2} \beta^{11} M_2^{11} \right). \quad (2.14)$$

Thus the effect of the two policy experiments on the current periods are very similar (0% change for the output gap and 4.2% change for the inflation, given $\beta = 0.99$, $M_1 = 1$,

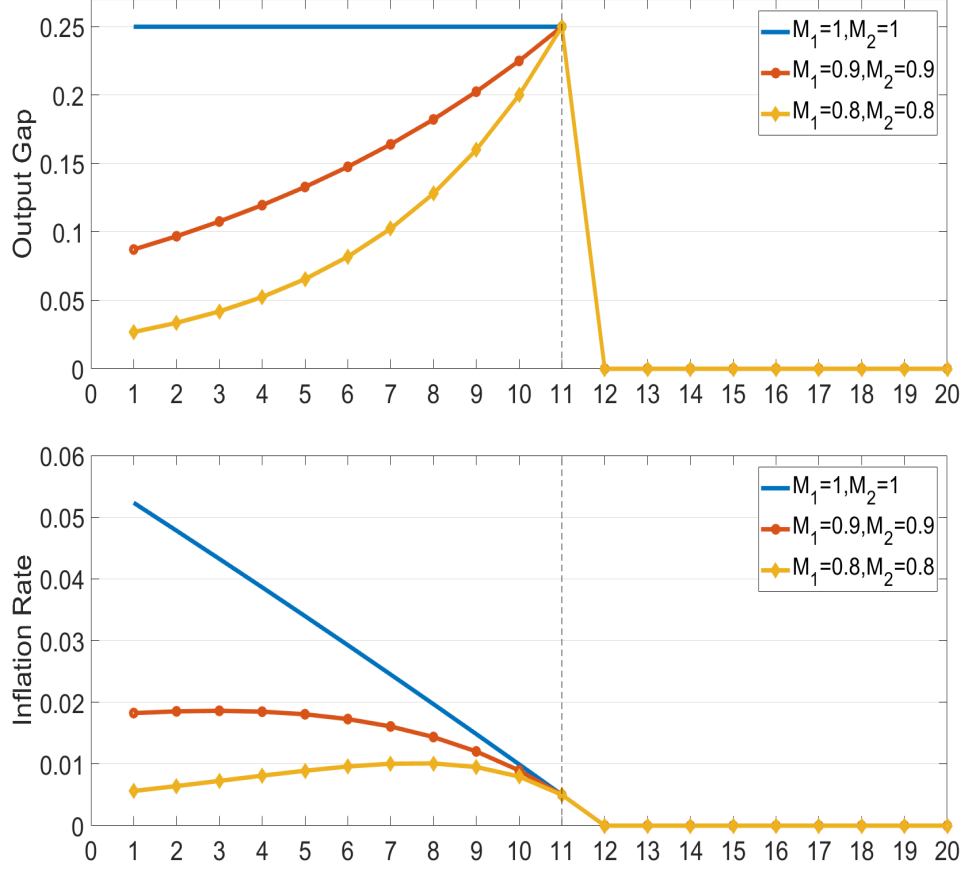


Figure 2.1: An Interest Rate Cut in 10 quarters ahead

$M_2 = 1$). However, the cumulative effects after the rate cut are

$$\Delta x_{t+10}^c = \Delta x_{t+10} + \Delta x_{t+11} \quad (2.15)$$

$$= \frac{1}{\sigma} A \left(\frac{1+M_1}{2} \right) + \frac{1}{\sigma} \left(\frac{A}{2} \right) = \left(1 + \frac{M_1}{2} \right) \left(\frac{1}{\sigma} A \right), \quad (2.16)$$

$$\Delta \pi_{t+10}^c = \Delta \pi_{t+10} + \Delta \pi_{t+11} \quad (2.17)$$

$$= \frac{\kappa}{\sigma} A \left(\frac{1+M_1+\beta M_2}{2} \right) + \frac{\kappa}{\sigma} A \left(\frac{1}{2} \right) \quad (2.18)$$

$$= \frac{\kappa}{\sigma} A \left(1 + \frac{M_1+\beta M_2}{2} \right). \quad (2.19)$$

Thus, the effect of the two policy experiments on the cumulative effects after the rate cut is very different (50% increase for the output gap and 100% increase for inflation given $\beta = 0.99$, $M_1 = 1$, $M_2 = 1$). For the three-period case, the numbers become 100% for output gap and 232% for inflation given the same parameters, and 150% and 396% for the four-period case. The details of the numbers are shown in the appendix.

A simulation result is shown in Figure 2. The simulation compares the effect of an interest rate cut for one period starting in 10 quarters ahead and the effect of an interest cut for three periods starting also in 10 quarters ahead with similar effects on the initial period. The value of M_1 and M_2 are set to be $M_1 = 0.8$, $M_2 = 0.8$. First, as shown theoretically in the case for $M_1 = 1$ and $M_2 = 1$, we can see that an interest rate cut for multiple periods affect more at and after the 10 quarters ahead.

Moreover, when M_1 and M_2 are less than unity, doubling the length of interest cuts is not enough to double the effect on the current period. When the central bank tries to achieve the same effect on the current period, it has to rely on more aggressive forward guidance than just doubling the length of the rate cuts. The simulation result shows that the cumulative effects after the rate cut is 110% for output gap and 324% for inflation rate. As a result, there is interaction between how much the economy is forward looking and how costly the lower natural rate of interest rate is.

Therefore, in a low natural rate environment where the central bank might have to commit to forward guidance with longer horizons, the economy might have to incur much larger cost from forward guidance at the exit periods.

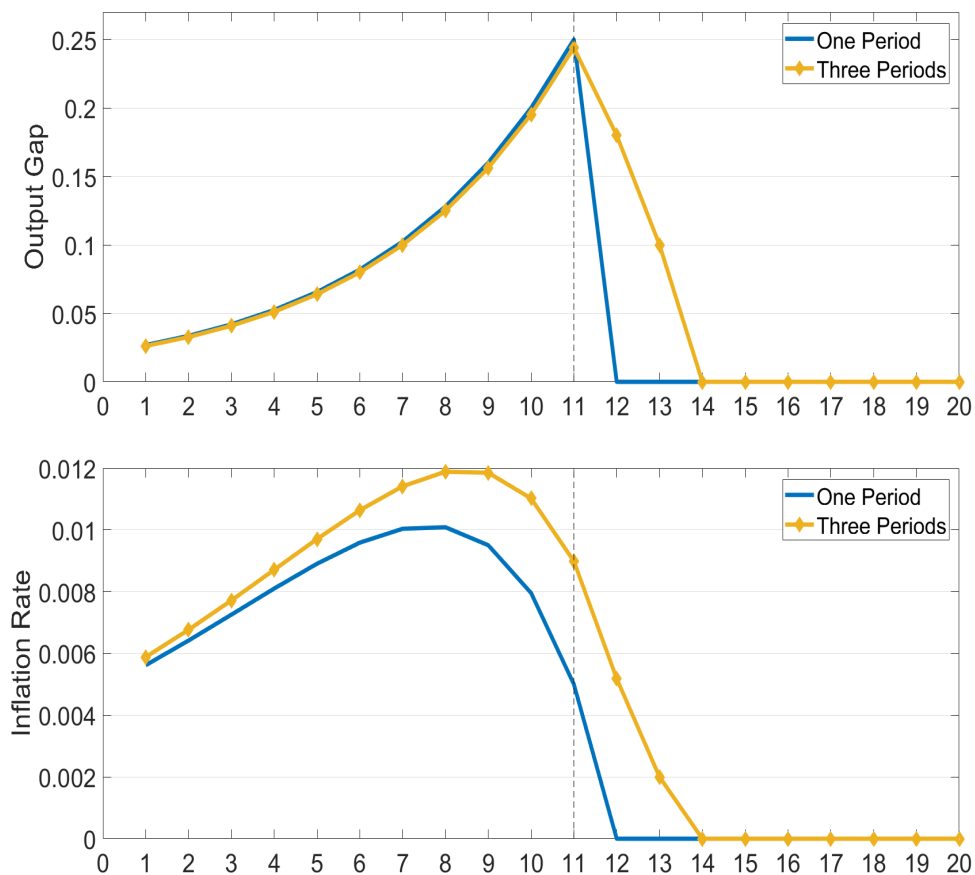


Figure 2.2: Comparison between an Rate Cut for One period and Three periods

2.5 Equilibrium Outcomes and Credibility of Forward Guidance

Now we discuss the stochastic case. Since the assumptions of monetary policy and the definition of credibility are the same as Sueda (2019), this section follows the paper closely. To solve the model, we have to specify the monetary policy rule. As in Walsh (2018), I assume that the central bank chooses optimal discretionary policy

unless the central bank commits to forward guidance. So the monetary policy rule for the optimal discretionary case is conducted to satisfy

$$\kappa\pi_t + \lambda x_t = 0. \quad (2.20)$$

For simplicity, the effective lower bound is set at zero.

Forward guidance is defined as the additional periods when the central bank set the nominal interest rate at the nominal interest rate at the effective lower bound beyond optimal discretionary policy. This choice follows Walsh (2018) and greatly simplifies the computation of the model.

For optimal discretionary policy and each length of forward guidance, we can calculate economic outcomes and the loss functions at each point in time. The only difference is that the discounting parameters are not state dependent. See appendix for the details.

The central bank's loss function is the sum of discounted expected loss for each period

$$L_t = \sum_{i=0}^{\infty} \beta^i E_t(\pi_{t+i}^2 + \lambda x_{t+i}^2), \quad (2.21)$$

or written recursively,

$$L_t = (\pi_t^2 + \lambda x_t^2) + \beta E_t L_{t+1}. \quad (2.22)$$

For each monetary policy, we can calculate economic outcomes and the loss function at each state. Then, we can compare these loss functions and see whether the central bank has the incentive to stick to forward guidance or not. Here it is assumed

that there is no chance of regaining credibility once the central bank reneges on any promises. We relax this assumption in a later part.

For ease of interpretation, the loss functions are transformed so that unit is steady-state consumption equivalence following Billi (2011),

$$\mu = (1 - \beta) \left(\frac{\omega\theta(1 + \eta\theta)}{(1 - \omega)(1 - \omega\beta)} \right) L. \quad (2.23)$$

When $\mu = 0.01$, it means that 0.01% of steady-state consumption is lost.

2.6 Simulations and Results

Now we simulate an economy with stochastic shocks. As in Nakata (2018), parameters are calibrated to match -7% for output gap and -1% for the annualized inflation rate when the economy is at the bad state. Parameters about the frequency of the good and the bad state follow Sueda (2019).

The key parameter in this paper is the natural rate of interest at the good state. Without any wedges, the natural rate of interest is determined by

$$\beta(1 + r_n) = 1. \quad (2.24)$$

Since the discount factor affects the value of future gain and cost, I introduce a wedge such that the natural rate of interest can change without changing the value of the discount factor. The wedge, α , is given as

$$\beta \left(1 + \frac{r_n}{\alpha} \right) = 1 \quad (2.25)$$

When α is larger, the natural rate of interest becomes larger. To see how does low natural rate of interest environments affect the effectiveness, I compare the case in which $\alpha = 1$ as the benchmark and $\alpha = 1/3$ as a case in which the natural rate of interest is low. The calibrated parameters are summarized in Table 1.

Parameters	Description	Values
β	Discount factor	0.99
σ	Relative risk aversion	2
ω	Probability of no resetting price opportunity	0.75
κ	Slope of the Phillips curve	0.02
θ	Elasticity of demand	7.66
λ	Relative weight on output gap volatility	0.003
s	Probability of staying at the good state	0.995
q	Probability of staying at the bad state	0.9
M_1	Discounting in Dynamic IS curve	0.9
M_2	Discounting in New Keynesian Phillips curve	0.9
α	Adjustment of the Natural Rate of Interest	1/3

Table 2.1: Parameter Values

First, I show the result for the benchmark case. Table 2 summarizes the loss for different duration of forward guidance. The optimal forward guidance, which minimizes the loss under perfect credibility is for $k = 7$. This reduces the loss from 0.20 to 0.09 compared to the optimal discretionary case, $k = 0$. However, forward guidance with seven periods, $k = 7$, is not sustainable. This is because temptation is positive at $k = 7$. Since forward guidance with six periods, $k = 6$, also has marginally positive temptation, the optimal sustainable forward guidance is for five periods. With this policy, the loss is reduced only to 0.10. Compared to the perfect credibility case, the effectiveness is 91%. Thus, credibility lowers the effectiveness of forward guidance, but the quantitative

$M_1 = 0.9, M_2 = 0.9, \alpha = 1/3$			
	L_z	Gain	Temptation
k=0	0.2036	—	—
k=1	0.1764	0.0272	-0.0090
k=2	0.1519	0.0245	-0.0079
k=3	0.1308	0.0212	-0.0065
k=4	0.1135	0.0173	-0.0047
<u>k=5</u>	0.1006	0.0129	<u>-0.0025</u>
k=6	0.0927	0.0079	0.0000
<u>k=7</u>	0.0902	<u>0.0025</u>	0.0031
k=8	0.0936	-0.0034	0.0066

Loss is measured in steady-state consumption level.

Table 2.2: Welfare Gain and Incentive to Deviate in a Benchmark Case

impact is not large in this case.

Figure 3 shows how the output gap and the inflation rate behaves. The duration of the bad state was assumed to be ten periods ex post. Forward guidance significantly brings the output gap and the inflation rate at the bad state close to zero. However, it also brings the output gap and the inflation rate once the economy goes back to the good state away from zero. This deviation is larger for the forward guidance with longer periods, and this is the reason why forward guidance with seven periods, $k = 7$, is not credible.

Now we see how a low natural rate of interest environment affect the effectiveness of forward guidance. Table 3 summarizes the results. The optimal forward guidance under perfect credibility is for seventeen periods, $k = 17$, which is much longer than the benchmark case. Also, the loss reduces from 0.20 only to 0.12. Compared to the perfect credibility case in the benchmark, the effectiveness is 74%. So the effec-

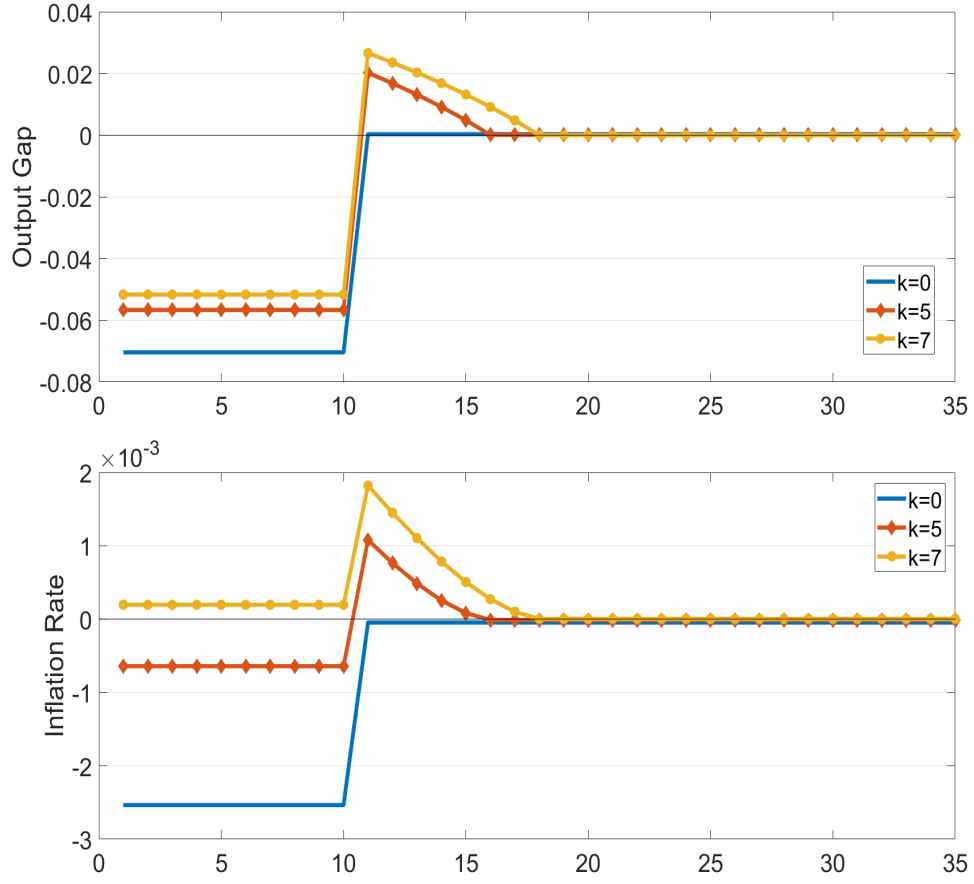


Figure 2.3: Output Gap and Inflation Rate in the Benchmark Case

tiveness of forward guidance under perfect credibility decreases in a low natural rate environment. Moreover, the optimal sustainable forward guidance is with twelve periods, $k = 12$, and the loss associated with it is 0.13. The effectiveness of forward guidance is 66% compared to the perfect credibility case in the benchmark case.

Figure 4 shows how the output gap and the inflation rate behaves. Compared to the benchmark case, the output gap and the inflation rate at the bad state is more away from zero.

$M_1 = 0.9, M_2 = 0.9, \alpha = 1/3$			
	L_z	Gain	Temptation
k=0	0.2036	—	—
k=1	0.1957	0.0079	-0.0026
k=2	0.1879	0.0078	-0.0026
k=3	0.1803	0.0076	-0.0025
k=4	0.1729	0.0074	-0.0024
k=5	0.1658	0.0071	-0.0022
k=6	0.1591	0.0067	-0.0020
k=7	0.1528	0.0064	-0.0018
k=8	0.1468	0.0059	-0.0016
k=9	0.1414	0.0054	-0.0013
k=10	0.1365	0.0049	-0.0009
k=11	0.1321	0.0044	-0.0005
<u>k=12</u>	0.1284	0.0038	<u>-0.0001</u>
k=13	0.1253	0.0031	0.0003
k=14	0.1228	0.0024	0.0008
k=15	0.1211	0.0017	0.0014
k=16	0.1201	0.0010	0.0020
<u>k=17</u>	0.1200	<u>0.0002</u>	0.0026
k=18	0.1206	-0.0006	0.0032

Loss is measured in steady-state consumption level.

Table 2.3: Welfare Gain and Incentive to Deviate in a Low Natural Rate Case

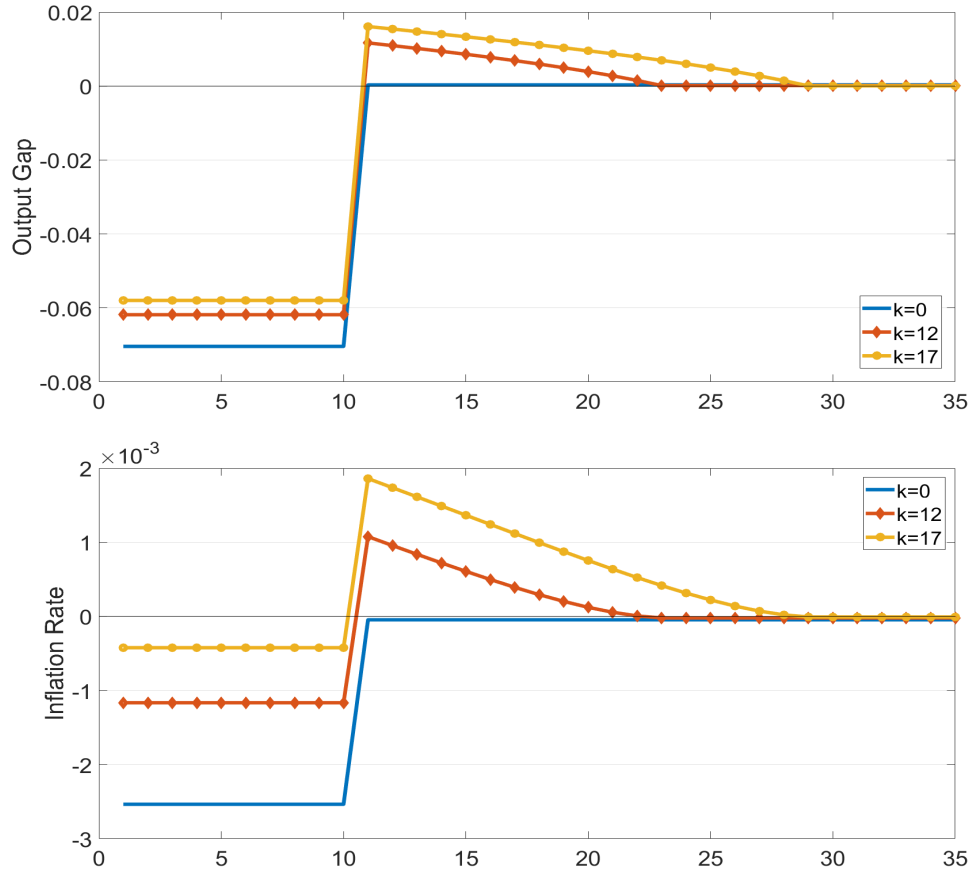


Figure 2.4: Output Gap and Inflation Rate in a Low Natural Rate Case

2.7 Conclusions

The paper analyzes how the low natural rate of interest environment affects the effectiveness of forward guidance. Forward guidance relies on the relatively high natural rate of interest rate when the economy gets out of recession. However, when this natural rate of interest rate becomes lower, the central bank has to implement longer forward guidance to achieve the same size of the effects on the current period,

and this is more costly because the economy has to deviate from the optimal level more and longer once recessions are over. As a result, the effectiveness of forward guidance becomes smaller even with perfect credibility. Moreover, it becomes harder to maintain credibility and this limits the effectiveness of forward guidance even more.

In the model, the low natural rate was exogenously assumed. To analyze the complementarity between monetary policy and other instruments to tackle the low natural rate of interest, we have model factors which determine the level of the natural interest rate.

Chapter 3

The Effects of Monetary Policy with Financial Disruption

3.1 Introduction

Monetary policy is given a dominant role to counteract excessive business cycles and there is a large consensus that monetary policy can affect business cycles in normal times. However, recent events such as hitting the effective lower bound of the policy interest rate and underachieving inflation targets cast doubts about our understanding of the effects of monetary policy under financial crises.

The role of financial distress is not captured well in standard New Keynesian models. What is assumed in those models is the perfect financial market. Financial intermediaries do not charge any markup over funding cost. As a result, we do not need to explicitly model the financial sector with the perfect financial market. Also,

since there is no heterogeneity among households in the models, there is zero amount of borrowing in equilibrium among households.

However, one of the characteristics of financial crises is very sharp rises in credit spreads. When the financial sector charges high spreads over funding rate, in response to an increase in credit demand, the equilibrium amount of credit becomes smaller relative to the case with the perfect financial market. As a result, monetary policy might affect the economy less in financial crises.

To generate a nontrivial finance needs in the model, I introduce two types of households, who differ in the marginal cost of working. Each household stochastically switches between the two types over time, and they are homogeneous ex ante. However, once the type for the current period is realized, each household is classified by the marginal cost of working. As a result, when a household has higher marginal cost of working in the current period, they want to borrow from those with lower marginal cost because their permanent income is almost identical.

Then, the financial market plays a key role in the model, and the degree of financial intermediation is captured by the marginal cost of extending loans. When the marginal cost is zero, the economy becomes very close to the standard New Keynesian models.

However, when the marginal cost is higher, the amount of lending in response to an interest rate cut becomes smaller, and this affects the amount of labor for each type and thus outputs become smaller. Also, this dampening effects becomes larger when the households heterogeneity is larger. This is in sharp contrast with the case when there

are no financial frictions in that the households heterogeneity does not affect the effects of monetary policy with the perfect financial market. These results hold not only for current interest cuts but also for forward guidance.

The paper proceeds as follows. In section 2, the relevant literature are reviewed. Section 3 lays out the model. In section 4, quantitative experiments are conducted, and the results are shown. Section 5 concludes.

3.2 Literature Review

There are several papers which analyzed the relationship between monetary policy and financial frictions. Many of them, however, focused on how monetary policy effects are amplified with financial frictions. Bernanke et al. (1999) introduced financial frictions between entrepreneurs and lenders and showed that the effects of monetary policy on economic variables such as output and investment becomes larger with financial accelerator channel. Gertler and Karadi (2011) finds the same result when they introduce financial frictions between financial intermediaries and lenders.

Woodford (2010) analyzes the effects of monetary policy in a static IS-MP model with the explicit financial sector. The paper makes a case that monetary policy affects less when the financial sector is unwilling to supply credit. Curdia and Woodford (2016) introduced a financial sector into otherwise a standard New Keynesian model and analyzes the optimal monetary policy.

This paper analyzes the effects of monetary policy when the financial sector

is disrupted using a model similar to Curdia and Woodford (2016). Doing this will generalize the claim in Woodford (2010) that monetary policy affects under financial disruption into standard dynamic monetary models which are widely used for monetary policy analyses. Also, this paper analyzes the effects of forward guidance.

One important difference in this paper is that households heterogeneity comes from the marginal cost of working instead of the degree of elasticity of intertemporal substitution as in Curdia and Woodford (2016). When the heterogeneity comes from the degree of elasticity of intertemporal substitution, borrowers are more willing to substitute consumption across time compared to lenders in response to an interest rate change. This might not square with data when borrowers are associated with being poorer. In that case, borrowers care more about the current period and does not have incentive to save a lot in response to an interest rate increase. Also, in their model, the steady-state consumption is higher for borrowers, but this also does not have empirical support.

On the other hand, when the heterogeneity comes from the marginal cost of labor, steady-state consumption is smaller for borrowers, and borrowers do not have higher elasticity of intertemporal substitution. In this economy, there is always some households or agents who are poorer at earning or generating internal funds. When the financial market is functioning, this is not a problem because these agents simply have to borrow and lenders are willing to lend. However, when the financial sector is disrupted, this channel becomes broken and these agents with poorer ability to earn cannot spend much. This might be a situation similar to financial crises.

3.3 The Model

The model is a New Keynesian model with the financial sector based on Curdia and Woodford (2016). The model is appealing because it generates finance needs and the financial sector plays a role while it still nests the standard New Keynesian model.

One crucial aspect of the model is that there are two types of households and each household switch their types stochastically over time. Type is denoted by $\tau \in \{b, s\}$. In equilibrium, type b households borrow and type s households saves. Each agent draws a lottery with probability $1 - \delta$ each period. When they draw a lottery, each agent becomes type b with probability π_b and type s with probability π_s , and $\pi_b + \pi_s = 1$. The probability of becoming each type is independent of the type before the lottery.

The difference of two types comes only from different marginal disutility of labor. The functional form is assumed to be

$$v^\tau(h_t^\tau(j)) = \frac{\psi_\tau}{1 + \nu} [h_t^\tau(j)]^{1+\nu}, \quad (3.1)$$

where $h_t^\tau(j)$ is the amount of labor supplied by type τ for firm j . I assume that marginal disutility of labor is higher for type b given the same level of labor, and thus the steady-state level of labor amount becomes smaller for type b . As a result, the steady-state level of consumption is lower for type b .¹ The relationship between the marginal cost and labor hours at the steady state is shown in the appendix.

¹Curdia and Woodford (2016) also assume that there is heterogeneity in the marginal cost of working. However, on top of that, they assume that the elasticity of intertemporal substitution is higher for type b . As a result, the steady-state level of consumption is lower for type s .

Following Curdia and Woodford (2016), I assume that households have access to state-contingent insurance through which they can insure away idiosyncratic risk. However, they can receive transfer from the insurance agency only when they draw a lottery and before their type will be revealed. By this assumption, we can avoid the situation that households' respective marginal utilities do not diverge, and we can rely on local methods. Also, this implies that in any period, all households of a given type have the same marginal utility.

These assumption captures a following situation. For some households who became type b today, it is more costly to earn for some duration and the opposite holds for type s . However, since the probability of each type is independent of the current type and because of the insurance, their expected marginal disutility and the starting wealth at the transfer are the same across agents. As a result, type b household want to borrow because their marginal disutility of labor is relatively lower than expected marginal disutility for some duration and the opposite holds for type s .

Another crucial aspect of the model is that it is costly for the financial sector to extend loans. To make real loan b_t , the financial intermediaries have to use real resources $\Xi_t(b_t)$. As a result, the interest rate borrowers pay, i_t^b becomes higher than the deposit rate, i_t^d . We implicitly define credit spread ω_t as follows,

$$1 + i_t^b = (1 + \omega_t)(1 + i_t^d). \quad (3.2)$$

The perfectly competitive loan market and the deposit market imply that the equilib-

rium credit spread is given by

$$\omega_t(b_t) = \Xi'_t(b_t). \quad (3.3)$$

In standard New Keynesian model, $\Xi'(b) = 0$ and thus $\omega = 0$. One of the feature of financial distress can be characterized by high marginal cost of extending loans. The functional form is assumed to be $\Xi_t(b_t) = \tilde{\Xi}_t b_t^\eta$.

3.3.1 Equilibrium Conditions

Each household maximizes utility subject to consumption, labor, and private bond holding. The Euler equation for type b is

$$\lambda_t^b = (1 + i_t^d)(1 + \omega_t)\beta E_t \left[(\delta + (1 - \delta)\pi_b) \frac{\lambda_{t+1}^b}{\Pi_{t+1}} + (1 - \delta)(1 - \pi_b) \frac{\lambda_{t+1}^s}{\Pi_{t+1}} \right], \quad (3.4)$$

where λ_t^b, λ_t^s is the marginal utility of consumption for type b and s respectively, and Π_{t+1} is the gross inflation rate. Because of the insurance, all households of a given type have the same marginal utility. The Euler equation for type s is

$$\lambda_t^s = (1 + i_t^d)\beta E_t \left[(1 - \delta)\pi_b \frac{\lambda_{t+1}^b}{\Pi_{t+1}} + (\delta + (1 - \delta)(1 - \pi_b)) \frac{\lambda_{t+1}^s}{\Pi_{t+1}} \right]. \quad (3.5)$$

By aggregating debt of borrowers, the law of motion of aggregate real private debt is

$$\begin{aligned} (1 + \pi_b \omega_t)b_t &= \pi_b(1 - \pi_b)B(\lambda_t^b, \lambda_t^s, Y_t, \Delta_t) - \pi_b b_t^g \\ &\quad + \delta(b_{t-1}(1 + \omega_{t-1}) + \pi_b b_{t-1}^g) \frac{1 + i_{t-1}^d}{\Pi_t}, \end{aligned} \quad (3.6)$$

where b_t^g is the government debt, and $B_t(\lambda_t^b, \lambda_t^s, Y_t, \Delta_t)$ is an auxiliary variable, which will be defined later.

Firms have isoelastic production function for their differentiated goods, $Y_t(j) = Z_t h_t(j)^{1/\phi}$. Under the assumption of infrequent price adjustment opportunity a la Calvo (1983), the optimal pricing implies that the law of motion of price is

$$\frac{1 - \alpha \Pi_t^{\theta-1}}{1 - \alpha} = \left(\frac{F_t}{K_t} \right)^{\frac{\theta-1}{1+\omega_y\theta}}, \quad (3.7)$$

where α is the probability of no price resetting opportunity, F_t and K_t are auxiliary variables, $\omega_y \equiv \phi(1 + \nu) - 1$, and θ is the elasticity of substitution across differentiated goods. The law of motion of price dispersion, Δ_t , under the infrequent pricing opportunity is

$$\Delta_t = \alpha \Delta_{t-1} \Pi_t^{\theta(1+\omega_y)} + (1 - \alpha) \left(\frac{1 - \alpha \Pi_t^{\theta-1}}{1 - \alpha} \right)^{\frac{\theta(1+\omega_y)}{\theta-1}}. \quad (3.8)$$

As already discussed, the equilibrium credit spread, ω_t , is given by

$$\omega_t = \eta \tilde{\Xi}_t b_t^{\eta-1}. \quad (3.9)$$

The good market clearing condition is

$$Y_t = \pi_b \bar{C}_t^b (\lambda_t^b)^{-\sigma} + (1 - \pi_b) \bar{C}_t^s (\lambda_t^s)^{-\sigma} + \tilde{\Xi}_t b_t^\eta + G_t, \quad (3.10)$$

where σ is the inverse of the elasticity of intertemporal substitution, and G_t is the government spending.

Lastly, the central bank is assumed to follow a Taylor rule,

$$\frac{1 + i_t^d}{1 + \bar{i}} = \left(\frac{\Pi_t}{\bar{\Pi}} \right)^{\phi_\pi} \left(\frac{Y_t}{\bar{Y}} \right)^{\phi_y} e^{\varepsilon_m}. \quad (3.11)$$

The definition of auxiliary variables, $\{B_t, F_t, K_t\}$ are defined in the Appendix.

3.4 Simulations and Results

3.4.1 Calibration

Now we simulate an economy to analyze how the effects of monetary policy depends on the conditions of the financial sector. The calibration of parameters are summarized in Table 1.

Parameters	Description	Values
ϕ	Inverse of concavity of production function	1/0.75
δ	Probability of not drawing a new type	0.5
π_b	Probability of type b	0.5
ν	Inverse of Frisch elasticity	1
θ	Elasticity of demand	7.66
α	Probability of no resetting price opportunity	0.66
β	Discount factor	0.99
σ	Average Relative risk aversion	6.25
σ_{bs}	Ratio of Relative risk aversion of borrowers to lenders	1
η	Marginal cost of extending loans	{1, 51}
h_{bs}	Ratio of borrowers' steady-state labor to lenders'	{0.01, 0.9}

Table 3.1: Parameter Values

There are two key parameters in the exercises. The first parameter, η , governs how costly it is for the financial sector to extend loans. When $\eta = 1$, the spread will be zero as in the standard New Keynesian models.

The other key parameter, h_{bs} , governs the degree of households heterogeneity. When the value is unity, there is no heterogeneity in households. When this value becomes smaller, borrowers become poorer to generate internal funds.

Two other parameters we have to pay attention is the probability of not drawing a new type, δ , and the ratio of the inverse of elasticity of intertemporal substitution

between borrowers and lenders, σ_{bs} . When δ is close to 0, the model becomes closer to the standard New Keynesian model because the current type does not persist in the long periods.² The other parameter is σ_{bs} . In this paper, we do not allow heterogeneity in the elasticity of intertemporal substitution.

The exercises of interest is to compare the case with and without financial disruption. This can be captured by different values of η . For all of the cases, we will see the effects of an 1% (annualized) interest rate cut.

3.4.2 Results

The results are shown in Figure 1. When the financial sector is less constrained ($\eta = 1$), an 1% interest rate cut increases the output by 1.55% and the inflation rate by 0.15% annually. However, when the financial sector is more constrained ($\eta = 51$), it increases the output only by 1% and the inflation rate by 0.095%.

For both of these cases, the amount of borrowing increases in response to an interest rate cut. However, when the value of δ becomes very close to one, the amount of borrowing decreases as in Curdia and Woodford (2016). But even in their paper, when the value of δ becomes lower, the amount of borrowing increases. Since there is no strong reasons to believe that the value δ is very close to unity, it is important to analyze the case in which δ is away from unity and thus the amount of borrowing increases.³

²Curdia and Woodford (2016) use a very high value of $\delta (= 0.95)$, but for some parameterization, the sign of borrowing in response to monetary policy shock switches when δ is close to one. As explained in the text, when δ is close to zero, the model becomes closer to the standard New Keynesian models, so I conservatively choose a low value of δ .

³Curdia and Woodford (2016) do not allow huge heterogeneity in the marginal cost of working. When

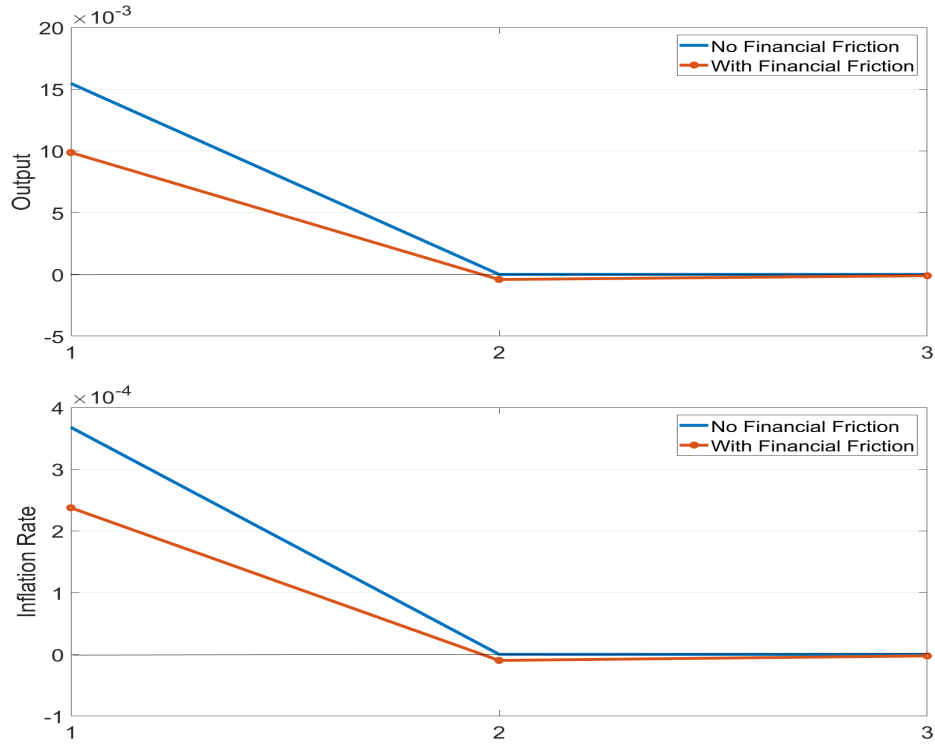


Figure 3.1: Impulse Response to an Interest Rate Cut

To look closely how the degree of households heterogeneity matters, I repeat the exercise with different parameters of the marginal cost of working. The result is in Figure 2. As we can see, the dampening effect becomes larger when the degree of households heterogeneity increases. When the economy relies more on borrowing, financial distress reduces the effects of monetary policy

In contrast, when the financial sector is not financially constrained, the degree of households heterogeneity does not affect how monetary policy affects the economy

δ is low, the amount of borrowing in the economy becomes very small, and there is little implications for financial disruption. This is not the case in this paper because the steady-state hours of working can be different across types.

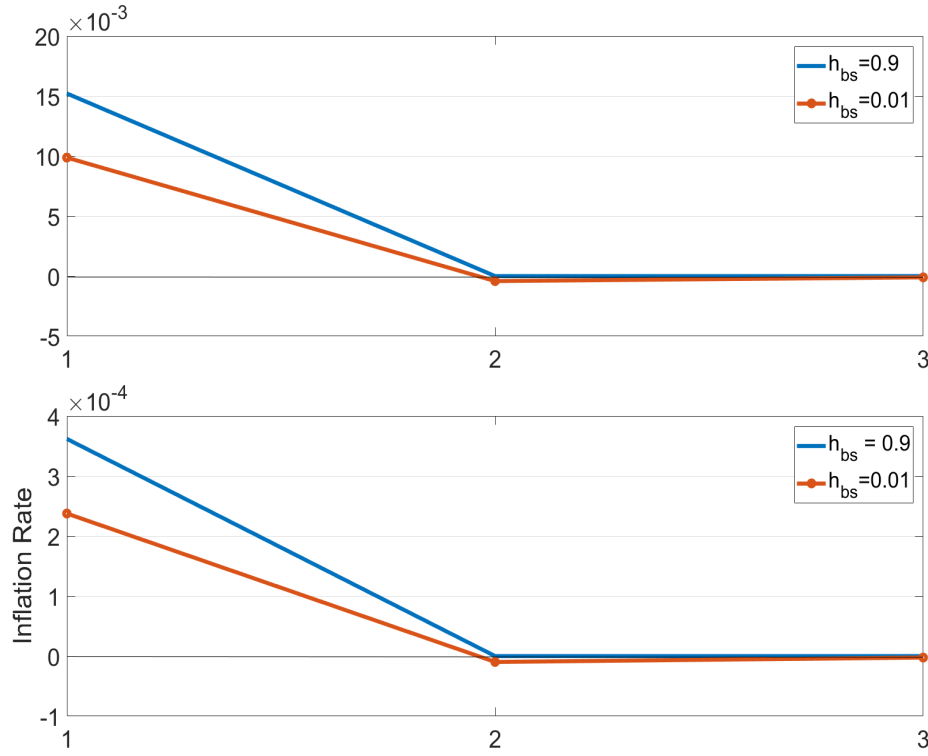


Figure 3.2: Impulse Response to an Interest Rate Cut: Different Degree of Households Heterogeneity

(the figure is not shown). When $\eta = 1$, regardless of the value of h_{bs} , the output increases by 1.55% and the inflation by 0.15%.

Also, to see more about the mechanism, Figure 3 shows how the effects of monetary policy depends on the marginal cost of working for both type. When there is no financial friction, the impulse response looks almost the same. However, when there is a financial friction, the interest rate cut affects less when the marginal cost of working is high for both types. This is likely to be because borrowing and working is a substitute for funding. When the marginal cost of working is higher, agents rely more on borrowing. However, when the financial friction is high, borrowing becomes also

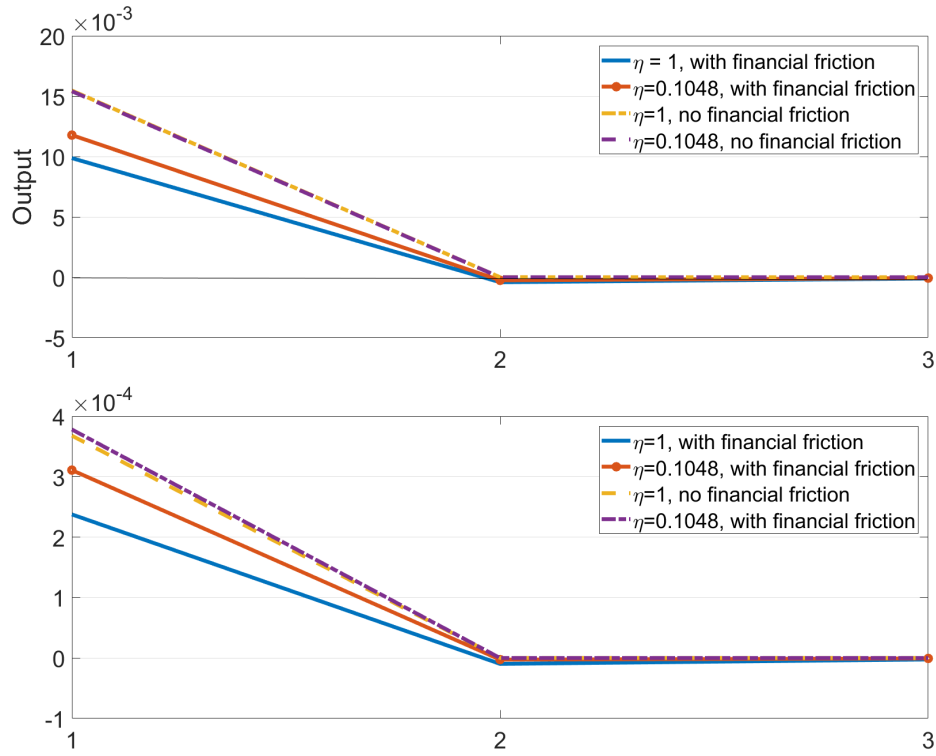


Figure 3.3: Impulse Response to an Interest Rate Cut: Different Degree of Marginal Cost of Working

costly and spending responds less to the interest rate cut.

3.4.3 Forward Guidance Case

Since one of the advantage of this model is the dynamic feature, I will also analyze how the effects of forward guidance depends on the condition of the financial sector. I repeat a similar exercise, but now the interest rate cut will occur in four quarters ahead.⁴

⁴Since we assume that the central bank follows a Taylor rule, giving a monetary policy shock in four quarters ahead will lead to an endogenous response of the interest rate in the first four quarters. To isolate the endogenous response of the interest rate, I give monetary policy shocks in such a way that the interest rate in the first four quarters do not move. See Figure 4 for the behavior of the interest rate.

The result is shown in Figure 4. In response to an announcement to cut the annualized interest by 1%, both the output and inflation increase in the current period. When the financial sector is less constrained, the forward guidance increases the output by 4.30% and the annualized inflation rate by 1.21%. On the contrary, when the financial sector is constrained, it increases the output by 0.73% and the inflation rate by 0.34%. As a result, we can also see the dampening effect for forward guidance too.

One of the biggest differences comes from the response of the private bonds. Of course, the magnitude is much smaller in the case with financial friction. Moreover, without financial friction, the peak of the borrowing occurs before the actual interest rate cut, but this is not the case with financial friction. The differences of the output and the inflation rate across different degrees of financial friction somewhat resemble those across the different degrees of how forward-looking the economy are as in Sukeda (2019).

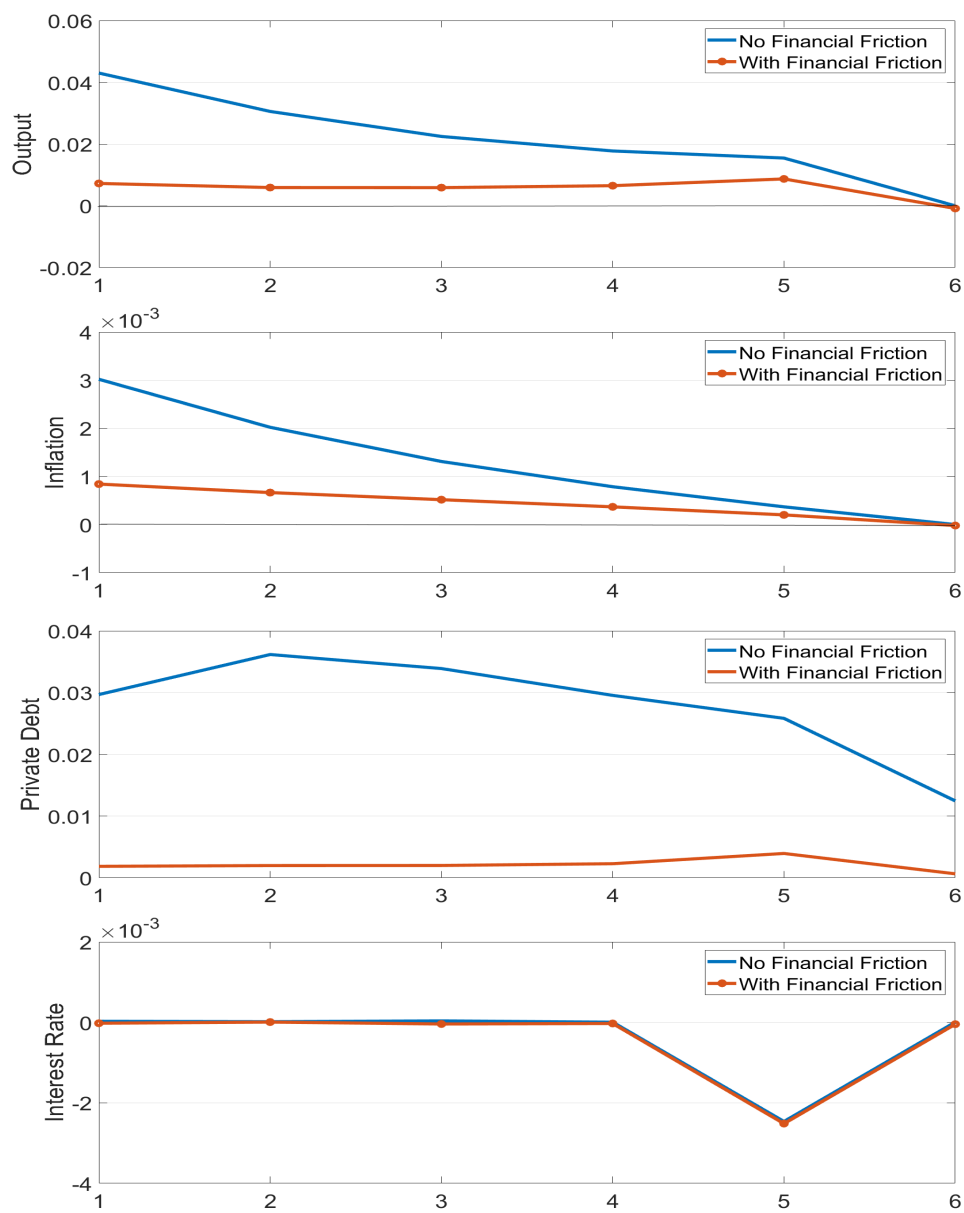


Figure 3.4: Impulse Response to Forward Guidance

3.5 Conclusions

The paper analyzed how the condition of financial sector affects the effects of monetary policy. In the model, households are heterogeneous in their marginal cost of working. This generates a nontrivial role of lending. When the financial market is perfect, the households heterogeneity does not affect the effects of monetary policy. However, once it becomes costly for the financial sector to extend loans, the effects of monetary policy becomes smaller. This dampening becomes larger when the households heterogeneity becomes larger. These results also hold for forward guidance.

In this paper, the cost of extending loans is given exogenously. To analyze the complementarity between monetary policy and other instruments to help the financial sector, we need to derive the cost of extending loans from an optimization problem of the financial sector. Doing this also sheds light on quantitative severity of financial sector in financial crises.

Also, as for forward guidance puzzle, how heterogeneous agents models can amplify or dampen the effects of forward guidance are well-studied in models with hand-to-mouth agents. However, very little is known when the tightness of borrowing is endogenous. This model might be good to start with.

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Appendix A

Appendix for Chapter 1

A.1 Appendix 1: Derivations

A.1.1 Details of Solving the Model

Equilibrium conditions can be summarized in a matrix form. Under forward guidance for one period, they become $Ax = b$, where

$$A = \begin{bmatrix} 1 - \beta s & -\kappa & 0 & 0 & -\beta(1 - s) & 0 & 0 \\ -s & \sigma(1 - s) & 0 & 0 & -(1 - s) & -(1 - s)\sigma & 1 \\ -\beta s & 0 & 1 & -\kappa & -\beta(1 - s) & 0 & 0 \\ -s & -\sigma s & 0 & \sigma & -(1 - s) & -\sigma(1 - s) & 0 \\ 0 & 0 & -\beta M_2 & 0 & 1 - \beta M_2 q & -\kappa & 0 \\ 0 & 0 & -(1 - q) & -\sigma M_1(1 - q)/\xi & -q & \sigma(1 - M_1 q)/\xi & 0 \\ \kappa & \lambda & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$x = \begin{bmatrix} \pi_n & x_n & \pi_{e1} & x_{e1} & \pi_z & x_z & i_n \end{bmatrix}',$$

$$b = \begin{bmatrix} 0 & \rho & 0 & \rho & 0 & r_z & 0 \end{bmatrix}'.$$

This can be solved for x by $x = A^{-1}b$.

This can be generalized for any period k of forward guidance. Under forward

guidance for two periods, the equilibrium conditions become $Ax = b$, where

$$A = \begin{bmatrix} 1 - \beta s & -\kappa & 0 & 0 & 0 & 0 & -\beta(1 - s) & 0 & 0 \\ -s & \sigma(1 - s) & 0 & 0 & 0 & 0 & -(1 - s) & -(1 - s)\sigma & 1 \\ -\beta s & 0 & 1 & -\kappa & 0 & 0 & -\beta(1 - s) & 0 & 0 \\ -s & -\sigma s & 0 & \sigma & 0 & 0 & -(1 - s) & -\sigma(1 - s) & 0 \\ 0 & 0 & \beta s & 0 & 1 & -\kappa & -\beta(1 - s) & 0 & 0 \\ 0 & 0 & -s & -\sigma s & 0 & \sigma & -(1 - s) & -\sigma(1 - s) & 0 \\ 0 & 0 & 0 & 0 & -\beta M_2 & 0 & 1 - \beta M_2 q & -\kappa & 0 \\ 0 & 0 & 0 & 0 & -(1 - q) & -\sigma M_1 \frac{(1-q)}{\xi} & -q & \sigma \frac{(1-M_1 q)}{\xi} & 0 \\ \kappa & \lambda & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$x = \begin{bmatrix} \pi_n & x_n & \pi_{e2} & x_{e2} & \pi_{e1} & x_{e1} & \pi_z & x_z & i_n \end{bmatrix}',$$

$$b = \begin{bmatrix} 0 & \rho & 0 & \rho & 0 & \rho & 0 & r_z & 0 \end{bmatrix}'.$$

By comparing these two cases, we can see the rule for generalization. For the first two rows, the first two columns and the last three columns are identical across these two cases. Other columns are all zeros. The last row is identical across these two cases. For the last two rows after removing the very last row, the last five columns are the same across these two cases. Other columns are all zeros.

In the middle rows, the last three columns are the same across these two cases.

For other columns, we can see that two matrices show up in different columns. These matrices are

$$\begin{bmatrix} -\beta s & 0 \\ -s & -\sigma s \end{bmatrix}, \begin{bmatrix} 1 & -\kappa \\ 0 & \sigma \end{bmatrix}.$$

The other columns are all zeros. With this rule, we can solve for forward guidance for any period.

We can solve for loss functions in a similar way. Under forward guidance for one period, conditions for loss functions are $By = d$, where

$$B = \begin{bmatrix} 1 - \beta s & 0 & -\beta(1 - s) \\ -\beta s & 1 & -\beta(1 - s) \\ 0 & -\beta(1 - q) & 1 - \beta q \end{bmatrix},$$

$$y = \begin{bmatrix} L_n & L_{e1} & L_z \end{bmatrix}',$$

$$d = \begin{bmatrix} \frac{1}{2}(\pi_n^2 + x_n^2) & \frac{1}{2}(\pi_{e1}^2 + x_{e1}^2) & \frac{1}{2}(\pi_z^2 + x_z^2) \end{bmatrix}'.$$

Under forward guidance for two periods, conditions for loss functions are $By = d$, where

$$B = \begin{bmatrix} 1 - \beta s & 0 & 0 & -\beta(1 - s) \\ -\beta s & 1 & 0 & -\beta(1 - s) \\ 0 & -\beta s & 1 & -\beta(1 - s) \\ 0 & 0 & -\beta(1 - q) & 1 - \beta q \end{bmatrix},$$

$$y = \begin{bmatrix} L_n & L_{e2} & L_{e1} & L_z \end{bmatrix}',$$

$$d = \begin{bmatrix} \frac{1}{2}(\pi_n^2 + x_n^2) & \frac{1}{2}(\pi_{e2}^2 + x_{e2}^2) & \frac{1}{2}(\pi_{e1}^2 + x_{e1}^2) & \frac{1}{2}(\pi_z^2 + x_z^2) \end{bmatrix}'.$$

Again we can see the rule for generalization. For the first row, the first, and the last column are the same across these two cases and other columns are all zeros. For the last row, the last two columns are the same across these two cases and other columns are all zeros.

For the middle rows, the last columns are identical. For other columns, we can see a matrix showing up in different columns. The matrix is

$$\begin{bmatrix} -\beta s & 1 \end{bmatrix}.$$

The rest of the columns are all zeros. In this way, we can calculate the loss functions under forward guidance for any period.

A.2 Appendix 2: Details of Regaining Case

When we change an assumption on punishment, we have to solve the model again because economy depends on expected future variables and future variables depend on monetary policy in the future. The model is already solved for optimal discretionary case and forward guidance for period k , so output gap, inflation rate, and loss functions for these cases are available.

A.2.1 Discretionary Case with Regaining Credibility

A.2.1.1 Economic Variables

First, we will solve for the economic outcome with discretionary policy when regaining probability is positive, $\phi > 0$. Superscript “d” stands for discretionary policy

as in Walsh (2018). Superscript r stands for “regain”. The equations for “bad” state look almost identical except that all of the variables have superscripts r (when $M_1 = M_2 = 1$). Even though these equations look almost identical to Walsh (2018), they are not identical because there is a positive probability of regaining credibility once the economy goes back to “good” state.

$$\begin{aligned} x_z^{d,r} &= M_1 E_t x_{t+1} + \frac{1}{\sigma} (E_t \pi_{t+1} + r_z) \\ &= M_1 q x_z^{d,r} + M_1 (1 - q) x_n^{d,r} + \frac{1}{\sigma} (q \pi_z^{d,r} + (1 - q) \pi_n^{d,r} + r_z) \end{aligned} \quad (\text{A.1})$$

$$\begin{aligned} \pi_z^{d,r} &= \beta M_2 E_t x_{t+1} + \kappa x_z^{d,r} \\ &= \beta M_2 q \pi_z^{d,r} + \beta M_2 (1 - q) \pi_n^{d,r} + \kappa x_z^{d,r} \end{aligned} \quad (\text{A.2})$$

The equations for “good” state looks more different because there is a probability ϕ that the central bank regains credibility.

$$\begin{aligned} x_n^{d,r} &= M_1 E_t x_{t+1} - \frac{1}{\sigma} (i_n - E_t \pi_{t+1} - \rho) \\ &= M_1 s \phi x_n^{FG(1)} + M_1 s (1 - \phi) x_n^{d,r} + M_1 (1 - s) x_z^{d,r} \\ &\quad - \frac{1}{\sigma} (i_n - (s \phi \pi_n^{FG(1)} + s (1 - \phi) \pi_n^{d,r} + (1 - s) \pi_z^{d,r} - \rho)) \end{aligned} \quad (\text{A.3})$$

$$\begin{aligned} \pi_n^{d,r} &= \beta M_2 E_t \pi_{t+1} + \kappa x_n^{d,r} \\ &= \beta M_2 (s \phi \pi_n^{FG(1)} + s (1 - \phi) \pi_n^{d,r} + (1 - s) \pi_z^{d,r}) + \kappa x_n^{d,r} \end{aligned} \quad (\text{A.4})$$

The economy starts with these variables when this was the period when the central bank reneged on the promise or when the central bank has not been regaining credibility yet

since they broke the promise.

Some comments on these equations are that I used two operational assumptions mentioned in the summary part. There is no chance of regaining credibility in the period when the central bank reneged on forward guidance, and there is no chance of regaining credibility if the central bank has not regained credibility while the economy is at “good” state. The first assumption simply enables us to treat the first period of breaking promise equally. The second assumption reduces the cases to consider a lot, and these cases carry a very small probability of occurring.

When the economy is at “good” state, there are three possibility in the next state. (i) With probability $s\phi$, the economy is still in “good” state, and the central bank regains credibility. (ii) With probability $s(1 - \phi)$, the economy is still in “good” state, but the central bank does not regain credibility yet. (iii) With probability $1 - s$, the economy goes into “bad” state and in this case, there is no chance to regain credibility.

With probability $s\phi$, the economy is still “good” and the central bank regained credibility. In the above example, output gap and inflation rate have superscript “FG(1)”, but this depends on another assumption on credibility that credibility must be checked recursively. In the case of Walsh (2018), we should calculate these for “FG(k)” because in Walsh (2018) discretionary case serves as an alternative policy for forward guidance for any period k. In the case of recursive credibility, discretionary policy serves as an alternative policy only for forward guidance with period 1.

Given $x^{FG(1)}$ and $\pi_n^{FG(1)}$ calculated already, we have 4 unknowns and 4 equations and we can solve for $x_z^{d,r}, \pi_z^{d,r}, x_n^{d,r}, \pi_n^{d,r}$.

When $\phi = 0$, these equations are identical to Walsh (2018).

A.2.1.2 Loss Function

Once equations for economic variables are understood, the equations for loss functions are straightforward. Loss functions with possibility of regaining credibility are

$$L_z^{d,r} = ((\pi_z^{d,r})^2 + \lambda(x_z^{d,r})^2) + \beta q L_z^{d,r} + \beta(1 - q)L_n^{d,r}, \quad (\text{A.5})$$

$$\begin{aligned} L_n^{d,r} &= ((\pi_n^{d,r})^2 + \lambda(x_n^{d,r})^2) \\ &+ \beta s \phi L_n^{FG(1)} + \beta s(1 - \phi)L_n^{d,r} + \beta(1 - s)L_z^{d,r}. \end{aligned} \quad (\text{A.6})$$

Given $L_n^{FG(1)}$ calculated already, we have 2 unknowns and 2 equations and we can solve for $L_z^{d,r}$ and $L_n^{d,r}$. As we expect, since $L_n^{FG(1)} < L_n^{d,r}$, loss functions will be closer to zero when ϕ is larger. When $\phi = 0$, these equations are identical to Walsh (2018).

A.2.2 Forward Guidance for One Period with Regaining Credibility

A.2.2.1 Economic Variables

Equations at “bad” state looks the same as discretionary case.

$$\begin{aligned} x_z^{FG(1),r} &= M_1 E_t x_{t+1} + \frac{1}{\sigma} (E_t \pi_{t+1} + r_z) \\ &= M_1 q x_z^{FG(1),r} + M_1 (1 - q) x_n^{FG(1),r} + \frac{1}{\sigma} (q \pi_z^{FG(1),r} + (1 - q) \pi_n^{FG(1),r} + r_z) \end{aligned} \quad (\text{A.7})$$

$$\begin{aligned} \pi_z^{FG(1),r} &= \beta M_2 E_t x_{t+1} + \kappa x_z^{FG(1),r} \\ &= \beta M_2 q \pi_z^{FG(1),r} + \beta M_2 (1 - q) \pi_n^{FG(1),r} + \kappa x_z^{FG(1),r} \end{aligned} \quad (\text{A.8})$$

We have now additional equations for exiting periods. However, because of the assumption that there is no chance of regaining credibility at “exit” periods, the equations look almost identical with standard case.

$$\begin{aligned}
x_e^{FG(1),r} &= M_1 E_t x_{t+1} - \frac{1}{\sigma} (0 - E_t \pi_{t+1} - \rho), \\
&= M_1 S x_n^{FG(1),r} + M_1 (1 - s) x_z^{FG(1),r} + \frac{1}{\sigma} (s \pi_n^{FG(1),r} + (1 - s) \pi_z^{FG(1),r} + \rho)
\end{aligned} \tag{A.9}$$

$$\begin{aligned}
\pi_e^{FG(1),r} &= \beta M_2 E_t \pi_{t+1} + \kappa x_e^{FG(1),r}, \\
&= \beta M_2 s \pi_n^{FG(1),r} + \beta (1 - s) \pi_z^{FG(1),r} + \kappa x_e^{FG(1),r}.
\end{aligned} \tag{A.10}$$

Equations at “good” state looks the same as discretionary case.

$$\begin{aligned}
x_n^{FG(1),r} &= M_1 E_t x_{t+1} - \frac{1}{\sigma} (i_n - E_t \pi_{t+1} - \rho) \\
&= M_1 s \phi x_n^{FG(2)} + M_1 s (1 - \phi) x_n^{FG(1),r} + M_1 (1 - s) x_z^{FG(1),r} \\
&\quad - \frac{1}{\sigma} (i_n - (s \phi \pi_n^{FG(2)} + s (1 - \phi) \pi_n^{FG(1),r} + (1 - s) \pi_z^{FG(1),r} - \rho)),
\end{aligned} \tag{A.11}$$

$$\begin{aligned}
\pi_n^{FG(1),r} &= \beta M_2 E_t \pi_{t+1} + \kappa x_n^{FG(1),r} \\
&= \beta M_2 (s \phi \pi_n^{FG(2)} + s (1 - \phi) \pi_n^{FG(1),r} + (1 - s) \pi_z^{FG(1),r}) + \kappa x_n^{FG(1),r}.
\end{aligned} \tag{A.12}$$

A.2.2.2 Loss Functions

Once equations for economic variables are understood, the equations for loss functions are straightforward. Loss functions with possibility of regaining credibility are

$$L_z^{FG(1),r} = ((\pi_z^{FG(1),r})^2 + \lambda(x_z^{FG(1),r})^2) + \beta q L_z^{FG(1),r} + \beta(1-q)L_e^{FG(1),r}, \quad (\text{A.13})$$

$$L_e^{FG(1),r} = ((\pi_e^{FG(1),r})^2 + \lambda(x_e^{FG(1),r})^2) + \beta s L_n^{FG(1),r} + \beta(1-s)L_z^{FG(1),r}, \quad (\text{A.14})$$

$$\begin{aligned} L_n^{FG(1),r} = & ((\pi_n^{FG(1),r})^2 + \lambda(x_n^{FG(1),r})^2) \\ & + \beta s \phi L_n^{FG(2)} + \beta s(1-\phi)L_n^{FG(1),r} + \beta(1-s)L_z^{FG(1),r}. \end{aligned} \quad (\text{A.15})$$

Given $L_n^{FG(2)}$ calculated already, we have 3 unknowns and 3 equations and we can solve for $L_z^{FG(1),r}$, $L_e^{FG(1),r}$, and $L_n^{FG(1),r}$.

A.2.3 Forward Guidance for Multiple Periods with Regaining Credibility

Because of the simple structure, the cases with multiple periods follows immediately given that code for multiple-period forward guidance without regaining possibility is automated. We only have to introduce new variables, and we do not need to modify equations for “bad” state and “exit” periods. We only have to modify equations for “n (non-binding)” state.

In a matrix form, equations for economic variables become (changes are high-

lighted by underline)

$$\begin{aligned}
& \begin{bmatrix} 1 - \beta M_2 s \underline{(1 - \phi)} & -\kappa & \dots \\ -s \underline{(1 - \phi)} & -\sigma(M_1 s \underline{(1 - \phi)} - 1) & \dots \\ \dots & \dots & \dots \end{bmatrix} \begin{bmatrix} \pi_n^{FG(k),r} \\ x_n^{FG(k),r} \\ \dots \end{bmatrix} \\
&= \begin{bmatrix} \underline{\beta M_2 s \phi \pi_n^{FG(k+1)}} \\ \rho + \underline{\sigma M_1 s \phi x_n^{FG(k+1)}} + \underline{s \phi \pi_n^{FG(k+1)}} \\ \dots \end{bmatrix}. \tag{A.16}
\end{aligned}$$

In general, $FG(k)$ could be d and $FG(k+1)$ could be any periods of forward guidance of interest to check credibility. (In Walsh (2018) with regaining possibility, $(FG(k), r)$ should be (d, r) and $FG(k+1)$ should be $FG(k)$.)

Equations for loss functions become (changes are highlighted by underline)

$$\begin{aligned}
& \begin{bmatrix} 1 - \beta s \underline{(1 - \phi)} & \dots \\ \dots & \dots \end{bmatrix} \begin{bmatrix} L_n^{FG(k),r} \\ \dots \end{bmatrix} \\
&= \begin{bmatrix} ((\pi_n^{FG(1),r})^2 + \lambda(x_n^{FG(k),r})^2) + \underline{\beta s \phi L_n^{FG(k+1)}} \\ \dots \end{bmatrix}. \tag{A.17}
\end{aligned}$$

A.3 Appendix 3: More Results

A.3.1 Different Values of M_1 and M_2

The effect of reducing the discounting values M_1 and M_2 is not monotone. The welfare loss arising from a lack of credibility is zero when M_1 and M_2 are close to unity.

When $M_1 = M_2 = 0.8$, the optimal forward guidance reduces the losses from 0.200 to 0.122, which is 84% of the standard case while the optimal sustainable forward

guidance reduces them to 0.135, which 70% of the standard case. The welfare loss arising from a lack of credibility corresponds to 14% of the welfare gain in the standard case. When the regaining probability is $\phi = 0.0125$, the value becomes 27%. So, in this case, the welfare loss arising from lack of credibility is larger than the loss arising from the additional discounting.

As in the main text, when $M_1 = M_2 = 0.7$, the optimal forward guidance reduces the losses from 0.200 to 0.122 while the optimal sustainable forward guidance reduces them to 0.135. The welfare loss arising from a lack of credibility corresponds to 15% of the welfare gain by the optimal forward guidance. When the regaining probability is $\phi = 0.0125$, the value becomes 23%.

When $M_1 = M_2 = 0.6$, the optimal forward guidance reduces the losses from 0.200 to 0.176, which is 25% of the standard case, while the optimal sustainable forward guidance reduces them to 0.181, which is 20%. The welfare loss arising from a lack of credibility corresponds to 5% of the welfare gain in the standard case. When the regaining probability is $\phi = 0.0125$, the value becomes 10%.

Appendix B

Appendix for Chapter 2

B.1 Appendix 1: Details of Numerical Results in Section 4

For the three-period case, the cumulative effects for the output gap after the rate cut are

$$\Delta x_{t+10}^c = \Delta x_{t+10} + \Delta x_{t+11} + \Delta x_{t+12} \quad (\text{B.1})$$

$$= \frac{1}{\sigma} A \left(\frac{1 + M_1 + M_1^2}{3} \right) + \frac{1}{\sigma} A \left(\frac{1 + M_1}{3} \right) + \frac{1}{\sigma} A \left(\frac{1}{3} \right). \quad (\text{B.2})$$

When $M_1 = 1$, it becomes

$$\Delta x_{t+10} = \frac{1}{\sigma} A \left\{ \frac{3}{3} + \frac{2}{3} + \frac{1}{3} \right\} \quad (\text{B.3})$$

$$= 2 \left(\frac{1}{\sigma} A \right), \quad (\text{B.4})$$

so the cumulative effect is 100% increase for the output gap. For the inflation rate, the cumulative effect is

$$\Delta\pi_{t+10}^c = \Delta\pi_{t+10} + \Delta\pi_{t+11} + \Delta\pi_{t+12} \quad (\text{B.5})$$

$$= \frac{\kappa}{\sigma} A \left\{ \frac{(1 + M_1 + M_2^2) + \beta M_2(1 + M_1) + (\beta M_2)^2}{3} + \frac{(1 + M_1) + \beta M_2}{3} + \frac{1}{3} \right\}. \quad (\text{B.6})$$

Thus, when $M_1 = 1$ and $M_2 = 1$, it becomes

$$\Delta\pi_{t+10}^c = \frac{\kappa}{\sigma} A \left\{ \frac{3 + 2\beta + \beta^2}{3} + \frac{2 + \beta}{3} + \frac{1}{3} \right\}, \quad (\text{B.7})$$

so the cumulative effect is 232% increase for the inflation rate. The four-period case when $M_1 = 1$ and $M_2 = 1$ follows the above case, and

$$\Delta x_{t+10}^c = \frac{1}{\sigma} A \left\{ \frac{4 + 3 + 2 + 1}{4} \right\}, \quad (\text{B.8})$$

$$\Delta\pi_{t+10}^c = \frac{\kappa}{\sigma} A \left\{ \frac{4 + 3\beta + 2\beta^2 + \beta^3}{4} + \frac{3 + 2\beta + \beta^2}{4} + \frac{2 + \beta}{4} + \frac{1}{4} \right\}, \quad (\text{B.9})$$

so the increase is 150% for the output gap and 396% for the inflation rate.

B.2 Appendix 2: Equilibrium conditions

The equilibrium conditions under optimal discretionary case are

$$\pi_n^d = \beta M_2(s\pi_n^d + (1-s)\pi_z^d) + \kappa x_n^d \quad (\text{B.10})$$

$$x_n^d = M_1(sx_n^d + (1-s)x_z^d) - \frac{1}{\sigma}(i_n^d - (s\pi_n^d + (1-s)\pi_z^d) - \rho) \quad (\text{B.11})$$

$$\pi_z^d = \beta M_2(q\pi_z^d + (1-q)\pi_n^d) + \kappa x_z^d \quad (\text{B.12})$$

$$x_z^d = M_1(qx_z^d + (1-q)x_n^d) + \frac{1}{\sigma}((q\pi_z^d + (1-q)\pi_n^d) + r_z) \quad (\text{B.13})$$

$$0 = \kappa\pi_n^d + \lambda x_n^d. \quad (\text{B.14})$$

The equilibrium conditions under forward guidance for one period are

$$\pi_n^{fg1} = \beta M_2(s\pi_n^{fg1} + (1-s)\pi_z^{fg1}) + \kappa x_n^{fg1} \quad (\text{B.15})$$

$$x_n^{fg1} = M_1(sx_n^{fg1} + (1-s)x_z^{fg1}) - \frac{1}{\sigma}(i_n^{fg1} - (s\pi_n^{fg1} + (1-s)\pi_z^{fg1}) - \rho) \quad (\text{B.16})$$

$$\pi_e^{fg1} = \beta M_2(s\pi_n^{fg1} + (1-s)\pi_z^{fg1}) + \kappa x_e^{fg1} \quad (\text{B.17})$$

$$x_e^{fg1} = M_1(sx_n^{fg1} + (1-s)x_z^{fg1}) + \frac{1}{\sigma}((s\pi_n^{fg1} + (1-s)\pi_z^{fg1}) + \rho) \quad (\text{B.18})$$

$$\pi_z^{fg1} = \beta M_2(q\pi_z^{fg1} + (1-q)\pi_e^{fg1}) + \kappa x_z^{fg1} \quad (\text{B.19})$$

$$x_z^{fg1} = M_1(qx_z^{fg1} + (1-q)x_e^{fg1}) + \frac{1}{\sigma}((q\pi_z^{fg1} + (1-q)\pi_e^{fg1}) + r_z) \quad (\text{B.20})$$

$$0 = \kappa\pi_n^{fg1} + \lambda x_n^{fg1}. \quad (\text{B.21})$$

After calculating the economic variables, we can calculate the loss at each state using the following conditions

$$L_n^d = (\pi_n^d)^2 + \lambda(x_n^d)^2 + \beta E_t L_{t+1}^d \quad (\text{B.22})$$

$$L_z^d = (\pi_z^d)^2 + \lambda(x_z^d)^2 + \beta E_t L_{t+1}^d. \quad (\text{B.23})$$

By writing out the conditional expectations using the transition matrix, we obtain

$$L_n^d = (\pi_n^d)^2 + \lambda(x_n^d)^2 + \beta(sL_n^d + (1-s)L_z^d) \quad (\text{B.24})$$

$$L_z^d = (\pi_z^d)^2 + \lambda(x_z^d)^2 + \beta(qL_z^d + (1-q)L_n^d). \quad (\text{B.25})$$

Conditions of loss functions for forward guidance with one period are

$$L_n^{fg1} = (\pi_n^{fg1})^2 + \lambda(x_n^{fg1})^2 + \beta(sL_n^{fg1} + (1-s)L_z^{fg1}) \quad (\text{B.26})$$

$$L_e^{fg1} = (\pi_e^{fg1})^2 + \lambda(x_e^{fg1})^2 + \beta(sL_n^{fg1} + (1-s)L_z^{fg1}) \quad (\text{B.27})$$

$$L_z^{fg1} = (\pi_z^{fg1})^2 + \lambda(x_z^{fg1})^2 + \beta(qL_z^{fg1} + (1-q)L_e^{fg1}). \quad (\text{B.28})$$

Appendix C

Appendix for Chapter 3

C.1 The Relationship Between the Marginal Cost of Labor and Steady-State Level of Hours Worked

The optimality conditions from the households' optimization problem yields the intratemporal substitution between consumption and labor,

$$\psi_\tau [h_t^\tau(j)]^\nu = \lambda_t^\tau w_t(j), \quad (\text{C.1})$$

where λ_t^τ is the marginal utility of consumption for type $\tau \in \{b, s\}$ and $w_t(j)$ is the wage for differentiated labor type $j \in [0, 1]$. By taking the ratio of the two types,

$$\left(\frac{h_t^s(j)}{h_t^b(j)} \right)^\nu = \frac{\lambda_t^s \psi_b}{\lambda_t^b \psi_s}. \quad (\text{C.2})$$

As a result, in the steady state, the higher marginal cost is associate with lower labor hours.

C.2 Equilibrium Conditions and Auxiliary Variables

The definition of auxiliary variables used in the equilibrium conditions are as follows.

$$K_t = \Lambda(\lambda_t^b, \lambda_t^s) \mu^p \phi \tilde{\lambda}(\lambda_t^b, \lambda_t^s)^{-1} \left(\frac{Y_t}{Z_t} \right)^{1+\omega_y} + \alpha \beta E_t \left[\Pi_{t+1}^{\theta(1+\omega_y)} K_{t+1} \right], \quad (\text{C.3})$$

$$F_t = \Lambda(\lambda_t^b, \lambda_t^s) (1 - \tau_t) Y_t + \alpha \beta E_t \left[\Pi_{t+1}^{\theta-1} F_{t+1} \right], \quad (\text{C.4})$$

where $\mu_t^p \equiv \theta/(\theta - 1)$,

$$\Lambda(\lambda_t^b, \lambda_t^s) \equiv \pi_b \lambda_t^b + (1 - \pi_b) \lambda_t^s, \quad (\text{C.5})$$

and

$$\tilde{\lambda}(\lambda_t^b, \lambda_t^s) \equiv \left[\pi_b \left(\frac{\lambda_t^b}{\psi_b} \right)^{\frac{1}{\nu}} + (1 - \pi_b) \left(\frac{\lambda_t^s}{\psi_s} \right)^{\frac{1}{\nu}} \right]^{\nu}. \quad (\text{C.6})$$

$$\begin{aligned} B(\lambda_t^b, \lambda_t^s, Y_t, \Delta_t) &\equiv \bar{C}_t^b [\lambda_t^b]^{-\sigma} - \bar{C}_t^s [\lambda_t^s]^{-\sigma} \\ &\quad - \left[\left(\frac{\lambda_t^b}{\psi_b} \right)^{\frac{1}{\nu}} - \left(\frac{\lambda_t^s}{\psi_s} \right)^{\frac{1}{\nu}} \right] \left(\tilde{\lambda}_t \right)^{-\frac{1+\nu}{\nu}} \left(\frac{Y_t}{Z_t} \right)^{1+\omega_y} \Delta_t. \end{aligned} \quad (\text{C.7})$$