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A New Formula for the Pattern Bandwidth of Fabry-Pérot Cavity Antennas Covered by Thin Frequency Selective Surfaces

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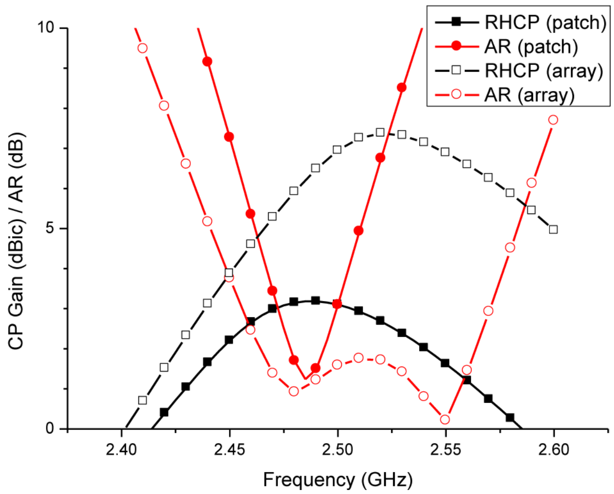

Fig. 9. Simulated AR and CP gain at zenith (+z-axis) of the antenna element and array shown in Fig. 7.

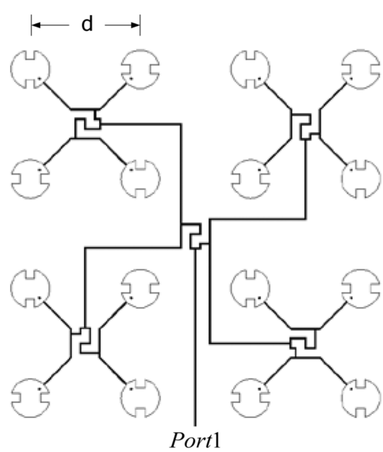

Fig. 10. Configuration of the 4 × 4 SR arrays using the presented SP feeds.

measurement. Using only a single transition of impedance transformation, the core layout of the proposed SP feed is uniform and neat. Two topologies of the presented SP feed are provided for the sizes of a $\lambda_g/4$ and a $3\lambda_g/8$. The SP feed of $\lambda_g/4$ can be employed for the case of critical demands on the small element spacing, while the design of $3\lambda_g/8$ can be used for general planar array applications. An example of $3\lambda_g/8$ SP feed with a patch as an array element is discussed in relation to imperfect performance. The presented SP feed can be extended to a $2^N \times 2^N$ feeding network, and is highly suitable for large-scale PCB arrays with less feed loss due to the shorter transmission line length in the total feeding network.

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A New Formula for the Pattern Bandwidth of Fabry-Pérot Cavity Antennas Covered by Thin Frequency Selective Surfaces

S. A. Hosseini, F. Capolino, and F. De Flaviis

***Abstract*—A new closed form expression is introduced to estimate the 3 dB pattern bandwidth of a Fabry-Pérot cavity antenna covered by a thin frequency selective surface (FSS) radiating at the broadside direction. The new formula has been obtained by using reciprocity, transmission line theory, and the susceptance model of the FSS. This formula estimates the 3 dB pattern bandwidth more accurately than previous expressions.**

***Index Terms*—Fabry-Pérot cavity (FPC) antenna, frequency selective surface (FSS), 3 dB pattern bandwidth.**

I. Introduction

A Fabry Pérot cavity (FPC) covered by a partially reflective surface was devised by Von Trentini [1] as a directive antenna. The FPC was later conceived as a grounded dielectric, covered by a denser layer as described in [2]. In [3], two layers of periodic rods made of alumina were placed above a patch antenna to increase its directivity. However, for fabrication reasons it is advantageous to cover the FPC by a frequency selective surface (FSS) made by an array

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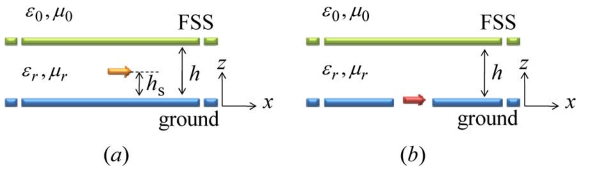

Fig. 1. Side view of the FPC antenna fed by either (a) an electric current in the middle of the cavity, or by (b) a magnetic current, modeling a slot, on the ground plane. The cavity is homogeneous in the x-y plane.

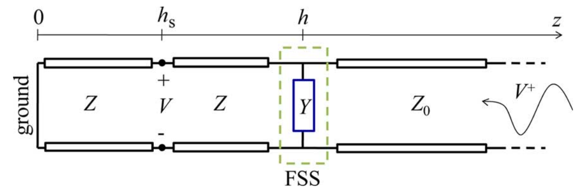

Fig. 2. Transmission Line (TL) model for a FPC antenna in the receiving mode illuminated by a plane-wave, fed by an electric dipole inside the cavity.

of either metallic patches or slots, as was used for the leaky wave (LW) antennas in [4]–[7]. The thin FSS made of periodic metallic patches or slots has been modeled in [5], [6] as a pure imaginary shunt admittance $Y = j\hat{b}Y_0$, where Y_0 is free space admittance and $\hat{b}$ is the normalized FSS susceptance, in a transmission line (TL). Based on the mentioned model, general formulas describing the pattern bandwidth (BW), half-power beamwidth, and the maximum radiated electric and magnetic fields as a function of $\hat{b}$ have been derived in [7], for this class of planar LW antennas. In [8], fundamental radiation properties of this class of LW antennas have been investigated in depth. Among other results, in [7], [8], a closed form formula was introduced to approximate the 3 dB pattern bandwidth of a FPC antenna covered by a thin FSS. The expression yields accurate results only for very large $\hat{b}$ values. The TL model of the FPC antenna was used in [9], to derive a general expression to calculate the theoretical gain of a FPC antenna fed by either electric or magnetic dipoles. Using the expressions in [9, Appendix A], one can determine that normalized susceptance values of $|\hat{b}| = 1, 5$, and 10 yield gains of 8.5 dB, 19.5 dB, and 25.5 dB for an FPC antenna filled with air, respectively. Therefore, large values of $\hat{b}$, e.g., $|\hat{b}| > 10$, would imply gains larger than 25.5 dB. Since FPC antennas constitute a useful design also for planar, low-profile antennas with moderate gains (between 10 and 20 dB for instance), it is believed that a more accurate BW formula is needed for this class of FPC antennas. Therefore, this work provides a better closed form approximation of the 3 dB pattern BW that is also applicable to moderate gain antennas using more precise approximations on the radiated power density. For large gain antennas (i.e., for large $\hat{b}$ values) this proposed BW expression tends to coincide with that in [7], [8]. Finally, analytical and numerical comparisons are made between the results calculated from the new formula and the numerically computed 3 dB pattern BW.

II. Improved Formula for the Pattern Bandwidth

Since the radiation properties of the FPC antenna (directivity, gain, gain BW, pattern BW) strongly depend on the cavity dimensions and the type of FSS used, the FPC antenna fed by an elementary dipole point-source is analyzed here. Ideally, a FPC antenna can be fed by either an electric elementary dipole inside the cavity at distance h_s from the ground plane, Fig. 1(a), or by an elementary magnetic dipole on its ground plane as shown in Fig. 1(b) which models a slot on the same plane. The arrows shown in Fig. 1 illustrate the polarization of the elementary electric (a) or magnetic (b) dipoles. The two above-mentioned designs are similar in their radiation characteristics. Although all calculations presented here are based on having the electric current (dipole) positioned in the middle of the cavity, the results achieved are also valid for a FPC antenna fed by a magnetic current.

Utilizing the reciprocity theorem [10], [11], as an alternative to calculate directly the radiated far-field of antenna fed by an electric dipole, the far-field radiation pattern is found by determining the induced electric field at the feeding point in receiving mode, when the antenna is illuminated by a plane-wave. The received field is calculated by using a TL model, as already done in [12]. Therefore, it is assumed without loss of generality that the feeding electric dipole in the middle of the cavity, at $z = h_s$ in Fig. 1(a), is oriented along the x direction. According to reciprocity property detailed in [12, p. 1330], the far-zone x-polarized electric field E_x^{ff} radiated by an antenna at broadside ($\theta = 0^\circ$) is related to the electric field induced at the feeding point $z = h_s$ of the antenna in the receiving mode E_x^{pw} due to an incident x-polarized plane wave $E_x^{\mathrm{inc}}(z) = E_0\, e^{jk_0 z}$ propagating from the z direction (Fig. 2), as $E_x^{\mathrm{ff}} = E_x^{\mathrm{pw}}$. The magnitude of the plane-wave is $E_0 = -j\omega\mu_0/(4\pi)$ as calculated using the reciprocity theorem as discussed in [12, p. 1330], $k_0 = \omega\sqrt{\mu_0\varepsilon_0}$ is the free space wavenumber and ω the radian frequency. The 3 dB pattern BW of a planar FPC antenna, placed in xy-plane, is evaluated only for the *broadside* radiation (or reception) direction, i.e., where the antenna radiates (receives) its maximum power at its operational frequency. That is why calculations, here, are restricted to the receiving mode with plane-wave illumination from the z-direction (Fig. 2). Using the TL model of the receiving antenna [12, p. 1331], the received electric field E_x^{pw} at the point $z = h_s$ is modeled by a voltage as $E_x^{\mathrm{pw}} = V(z = h_s)$, produced by an incident traveling wave $V^+ e^{jk_0 z}$ with $V^+ = E_0$, as in Fig. 2. Therefore, the far-field radiation by the antenna at broadside is found as

$$E_x^{\mathrm{ff}} = V(z = h_s). \tag{1}$$

In the TL model $Z = Z_0\sqrt{\mu_r/\varepsilon_r}$ is the characteristic impedance of the material filling the cavity, and $Z_0 = Y_0^{-1} = \sqrt{\mu_0/\varepsilon_0}$ is the free space impedance. A very thin FSS is modeled as a lumped imaginary admittance Y as shown in Fig. 2, [5]–[8], whose normalized FSS-susceptance is $\hat{b} = Y/(jY_0)$. Capacitive and inductive FSSs result in positive and negative $\hat{b}$ values respectively.

The TL voltage induced at the feeding point $z = h_s$ of a FPC antenna is calculated as

$$V = \frac{2V^+}{\sin(kh)}\frac{\sin(kh_s)}{1 + jZ_0 B_{\mathrm{tot}}} \tag{2}$$

where $k = \omega/v$ is the wavenumber inside the cavity, with $v = 1/\sqrt{\mu_r\mu_0\varepsilon_r\varepsilon_0}$, and B_{tot} is the total imaginary admittance of the FPC antenna, looking leftward at $z = h + 0^+$ (Fig. 2)

$$B_{\mathrm{tot}} = Y_0[\hat{b} - \xi_r \cot(kh)] \tag{3}$$

where $\xi_r = \sqrt{\varepsilon_r/\mu_r}$. The broadside radiation power density of a FPC antenna is therefore determined as (detailed in [8])

$$P = \frac{|V|^2}{2Z_0} = \frac{2Y_0|V^+|^2}{\sin^2(kh)}\left(\frac{\sin^2(kh_s)}{1 + Z_0^2 B_{\mathrm{tot}}^2}\right). \tag{4}$$

The total imaginary admittance B_{tot} is zero at the operational frequency ω_{op} of the FPC antenna (which is defined as the resonance frequency of the FPC), which implies that the resonance height h is given by

$$h = \frac{1}{k_{\mathrm{op}}}\tan^{-1}\left(\frac{\xi_r}{\hat{b}}\right) \cong \frac{\lambda_{\mathrm{op}}}{2}\left(1 + \frac{\xi_r}{\pi\hat{b}}\right) \tag{5}$$

where the most-right hand side is an approximation for large values of $\hat{b}$. Here, it is assumed that the dipole feed is placed in the center of the cavity ($h_s = h/2$, where the cavity electric field is maximum) which, for moderate or high-gain FPC antennas implies that $\sin(kh_s) \cong 1$. Therefore, using (4), the maximum power density at broadside is approximated as

$$P(\omega_{\rm op}) \cong 2Y_0|V^+|^2/\sin^2(\omega_{\rm op}h/v). \tag{6}$$

The relative 3 dB pattern BW is defined as

$$\mathrm{BW}_{3\ \mathrm{dB}} = \frac{\omega_{3\ \mathrm{dB}}^{+} - \omega_{3\ \mathrm{dB}}^{-}}{\omega_{\rm op}} \tag{7}$$

where $\omega_{3\ \mathrm{dB}}^{\pm}$ are the frequencies that half the power radiated at broadside

$$P(\omega_{3\ \mathrm{dB}}^{\pm}) = \frac{1}{2}P(\omega_{\rm op}). \tag{8}$$

Equation (8) can be solved numerically to find the 3 dB pattern BW of the antenna, but without providing for the capability of further analytical investigations. Since for a FPC antenna covered by a thin FSS several radiation parameters can be defined as a function of $\hat{b}$ [5]–[8], a closed-form formula is useful in determining the 3 dB pattern BW of the antenna, which can also be employed for further studies to increase the 3 dB pattern BW, as in [13]. Furthermore, general trends can be inferred by inspecting the closed form formula. In order to find the $\omega_{3\ \mathrm{dB}}^{\pm}$ values analytically a few considerations are necessary. Note that the resonance condition $B_{\rm tot}(\omega_{\rm opt}) = 0$ for large values of $\hat{b}$ also implies that $\cos(\omega_{\rm op}h/v) \cong -1$ whereas $\sin(\omega_{\rm op}h/v)$ is approximated as

$$\sin\left(\omega_{\rm op}\frac{h}{v}\right) \cong \frac{-\xi_r}{\hat{b}}. \tag{9}$$

Also, defining $\Delta = \omega - \omega_{\rm op}$ and assuming that $\Delta/\omega_{\rm op}$ is a small number, i.e., much less than unity, and using the Taylor approximations around the operational frequency of the antenna (i.e., for $\Delta \cong 0$) one has

$$\sin\left(\omega_{3\ \mathrm{dB}}^{\pm}\frac{h}{v}\right) \cong \frac{-\xi_r}{\hat{b}} - \left(\frac{h}{v}\right)\Delta^{\pm} \tag{10-1}$$
$$\cos\left(\omega_{3\ \mathrm{dB}}^{\pm}\frac{h}{v}\right) \cong -1 + \frac{\xi_r}{\hat{b}}\left(\frac{h}{v}\right)\Delta^{\pm}. \tag{10-2}$$

For thin FSSs modeled as inductive, $\hat{b} = -Z_0/(\omega_{\rm op}L)$, or capacitive, $\hat{b} = Z_0\omega_{\rm op}C$, shunt loads [5]–[7], we can assume that changes of $\hat{b}$ are negligible within the 3 dB BW of the antenna, which is usually narrow. Thus, using (9) and (10), (8) is approximated as $A\Delta^2 + B\Delta + C = 0$ where

$$A = \left(\frac{h}{v}\right)^2\left(\hat{b}^4 + \hat{b}^2\left(2\xi_r^2+1\right) + \xi_r^4\right)$$
$$B = 2\xi_r\hat{b}\left(\frac{h}{v}\right), \quad C = -\xi_r^2. \tag{11}$$

Solving for Δ, the upper and lower limits of the 3 dB pattern BW are found as

$$\Delta^{\pm} = \xi_r\left(\frac{v}{h}\right)\frac{-1 \pm \sqrt{\hat{b}^4 + \hat{b}^2\left(2\xi_r^2+2\right) + \xi_r^4}}{\hat{b}^4 + \hat{b}^2\left(2\xi_r^2+1\right) + \xi_r^4}. \tag{12}$$

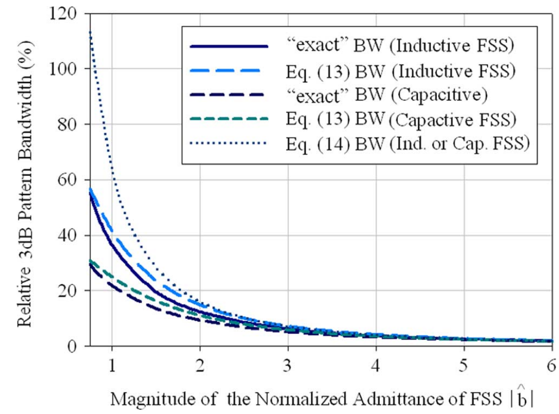

Fig. 3. Comparison between "exact" values of the 3 dB pattern BW of a FPC antenna covered by inductive or capacitive FSS and the results calculated from the approximated formulas: our result (13), and previous result (14).

From (7), one has $\mathrm{BW}_{3\ \mathrm{dB}} = (\Delta^+ - \Delta^-)/\omega_{\rm op}$, which leads to

$$\mathrm{BW}_{3\ \mathrm{dB}} = \xi_r\left(\frac{\lambda_{\rm op}}{\pi h}\right)\frac{\sqrt{\hat{b}^4 + \hat{b}^2\left(2\xi_r^2+2\right) + \xi_r^4}}{\hat{b}^4 + \hat{b}^2\left(2\xi_r^2+1\right) + \xi_r^4}. \tag{13}$$

Note that the cavity height h is determined by (5) and its value also depends on the inductive or capacitive choice of the FSS; in other words, besides the magnitude of $\hat{b}$, also its sign affects the resonance value of h. In (13), the ratio v/h has been equivalently rewritten as $v/h = \omega_{\rm op}\lambda_{\rm op}/(2\pi h)$. Note that for very large values of $\hat{b}$, which imply high gain FPC antennas, using (5), $h = \lambda_{\rm op}/2$, and retaining only the $\hat{b}^4$ terms, (13) can be further simplified as

$$\mathrm{BW}_{3\ \mathrm{dB}} = \left(\frac{2}{\pi\hat{b}^2}\right)\xi_r \tag{14}$$

which is the approximated BW formula for high gain antennas covered by a thin FSS obtained in [7], [8].

III. Illustrative Examples

Comparisons are made between the 3 dB pattern BW values calculated analytically using (13) and (14) and the numerical results carried out by full-wave simulations for the same FPC antennas to demonstrate that (13) can be used as design BW estimation with high accuracy. These comparisons also verify the better accuracy of (13) over previously found (14). The comparison is shown in Fig. 3 for negative and positive FSS susceptance $\hat{b}$ and for $\varepsilon_r = 1$ and $\mu_r = 1$, against an "exact" result obtained by solving (8) numerically.

As expected, both (13) and (14) are accurate for high values of $\hat{b}$, however for lower susceptance values $\hat{b}$, i.e., in case of low-gain FPC antennas, (13) yields more accurate results with respect to (14). Furthermore, one can notice that (13) provides more accurate results also in distinguishing between capacitive and inductive FSSs, while this is not possible with (14). In Table I, the relative 3 dB pattern BW values calculated using (13) and (14) are compared with the numerical results calculated for the same structures simulated by a full-wave (FW) simulation (Ansys HFSS). The comparison is carried out for FPCs with inductive and capacitive low values of the FSS susceptance $\hat{b}$, designed at 10 GHz. The simulated FPC antenna is fed by an ideal magnetic dipole (slot) on its ground plane, and covered by an FSS (made of Copper with thickness of 10 μm) with infinite extent along x and y. The inductive and capacitive FSSs, with a square FSS-unit-cell of $12 \times 12\ \mathrm{mm}^2$, are

TABLE I
RELATIVE 3 dB PATTERN BW OF A FPC ANTENNA FED BY IDEAL MAGNETIC DIPOLE (SLOT) COVERED WITH INDUCTIVE/CAPACITIVE FSSS, COMPARISON BETWEEN FW RESULTS AND APPROXIMATED EXPRESSIONS (13) AND (14)

$\hat{b}$	Max Gain (dB)	FW Broadside Gain BW_{3dB}(%)	FW Broadside Pattern BW_{3dB}(%)	Apprx. BW_{3dB} Eq.13 (%)	Apprx. BW_{3dB} Eq.14 (%)
-1.5	10.58	17.9	19.2	23.72	28.29
-2	12.12	11.87	11.38	14.79	15.92
-3	14.86	6.26	5.71	7.07	7.07
1.5	9.22	15.08	13.24	16.24	28.29
2	11.15	9.85	8.68	10.99	15.92
3	14.13	5.13	4.63	5.76	7.07

respectively made of periodic rectangular slots and strips with width of 5 mm and length L. Different values of FSS susceptance $\hat{b}$ are obtained by varying L (see [14]), i.e., for the inductive FSSs (slots), $L = 10.93$ mm, 10.40 mm, and 9.48 mm resulting in $\hat{b} = -1.5, -2$, and -3, respectively. While for the capacitive FSSs (strips), $L = 9.58$ mm, 10.25 mm, and 10.95 mm resulting in $\hat{b} = 1.5, 2$, and 3 respectively. Then, the antenna broadside radiation power, gain, and pattern and gain BWs are calculated using the formulas described in [9, Appendix A]. As shown in Table I, the pattern BW values calculated from (13) and those obtained from the full-wave simulation results are in better agreement than the previous BW formula (14) as expected.

The accuracy of (13) is also investigated by considering the example in [15, Table I], in which a FPC antenna fed by a patch (6×6 mm^2) and covered by a finite-size (7×28 elements) FSS made of periodic patches (0.95×0.1 cm^2), and FSS-unit-cell size of 1.15×0.3 cm^2, was designed at 12.4 GHz for a cavity partially filled with air (12.792 mm) and partially with a dielectric substrate ($\varepsilon_r = 3.38$ and thickness of 0.508 mm (20 mil)). As reported in [15], the FPC cavity fed by a patch antenna provided the maximum gain of approximately 18 dB at 12.4 GHz, whereas, the patch antenna (without FPC) provided a gain of 5–6.5 dB in the entire operational frequency band of 10 to 14 GHz. The reported 3 dB BW of the directivity of the FPC antenna, as shown in [15, Fig. 7], was approximately 875 MHz (7.05%). The procedure discussed in [14], when applied to the FSS in [15], yields $\hat{b} = 2.35$. For simplicity, in our calculation the FPC antenna is filled with air (which is what the FPC in [15] is mostly filled with), and the FPC resonates at 12.4 GHz for a FPC height, using (5), $h = 13.65$ mm (the total height in [15] was 13.3 mm). Using (8), the 3 dB pattern BW of the FPC antenna, with FSS susceptance $\hat{b} = 2.35$ (and thus we assume it constant in the narrow 3 dB BW frequency range), $h = 13.65$ mm, and fed by an ideal magnetic dipole, is 7.92%. Instead, using the analytic formulas (13) and (14), the estimated 3 dB pattern BW is calculated as 8.6% and 11.5%, respectively. Alternatively, by using a more accurate frequency-dependent model of the FSS based on the method discussed in [14] and the formulas in [9, Appendix A], the theoretical gain and broadside power density of the FPC fed by an ideal magnetic dipole on the ground plane are calculated, with a maximum gain of 12 dB (which can be interpreted as the difference between the gain of the FPC antenna and that of the patch at 12.4 GHz), and its 3 dB pattern BW is numerically calculated to be equal to 7.13%. In summary, the estimated 3 dB pattern BW value of 8.6%, by (13), is very close to the pattern BW obtained from (8), (7.92%), and the pattern BW of the FPC fed by an ideal magnetic dipole calculated numerically based on the values calculated using [9], (7.13%), and to the directivity BW of the FPC fed by the patch (7.05%, obtained from [15, Fig. 7]), demonstrating a better accuracy with respect to the pattern BW calculated by (14), (11.5%). This result also shows that the pattern BW of a FPC antenna (usually narrow) is mainly determined by the resonant cavity and not by the feed.

IV. CONCLUSION

A new closed form expression (13) was proposed for the estimation of the 3 dB pattern bandwidth of a FPC antenna covered by a thin FSS that, for very high-gain antennas, converges to a previous formula (14) proposed in [7], [8]. Both analytical and numerical comparisons prove the new formula (13) to be a significant improvement over the previous one (14) for medium-gain antennas. The proposed formula is well suited for FPCs covered by either capacitive (patches) or inductive (slots) FSSs, and it is an effective tool for engineers for providing an estimation of the resulting pattern bandwidth before starting a computationally expensive simulation campaign for designing the FPC and FSS.

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