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# RELATIONS BETWEEN THE HYPERON POLARIZATIONS

## IN ASSOCIATED PRODUCTION[*]

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In view of the recent discovery of a large up-down asymmetry in $\Lambda^0 \rightarrow p + \pi^-$,[1] we report in this note on some results on the polarizations of the produced hyperons, which will be useful in the interpretation of the experiments. We first show that in the decay $\Sigma^0 \rightarrow \Lambda^0 + \gamma$ from a polarized $\Sigma^0$ the $\Lambda^0$ is longitudinally polarized in the $\Sigma^0$ rest frame, and the value of its polarization is the same, except for having opposite sign, as the component of the $\Sigma^0$ polarization along the $\Lambda^0$ line of flight. It is assumed that both $\Lambda^0$ and $\Sigma^0$ have spin $\frac{1}{2}$, but the result is independent of their relative parity. Denoting by $\vec{u}$ the unit vector along the direction of emission of the $\Lambda^0$ in the $\Sigma^0$ rest frame, and by $\langle \vec{\sigma} \rangle_\Sigma$ and $\langle \vec{\sigma} \rangle_\Lambda$ the polarizations of the $\Sigma^0$ and of the $\Lambda^0$, one finds,

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[1] Crawford, Cresti, Good, Gottstein, Lyman, Solmitz, Stevenson, and Ticho, Detection of Parity Nonconservation in $\Lambda$ Decay, UCRL-8008, October 1957; and Phys. Rev. (to be published). Eisler et al., Phys. Rev. (to be published); L. Leipuner and R. Adair, Phys. Rev. (to be published).

after summing over the photon polarizations,

$$\langle \vec{\sigma} \rangle_\Lambda = -(\langle \vec{\sigma} \rangle_\Sigma \cdot \vec{u})\vec{u} \quad . \tag{1}$$

The angular distribution of the pion emitted in the subsequent $\Lambda^0$ decay is given by

$$W(\vec{v}) = 1 + \alpha(\langle \vec{\sigma} \rangle_\Lambda \cdot \vec{v}) = 1 - \alpha P_\Sigma(\vec{n} \cdot \vec{u})(\vec{u} \cdot \vec{v}) \quad , \tag{2}$$

where $\vec{v}$ is the unit vector along the direction of emission of the $\pi^-$ from $\Lambda^0$ decay in the $\Lambda^0$ rest frame, $\alpha$ is the asymmetry parameter for $\Lambda^0 \to p + \pi^-$, and we have written $\langle \vec{\sigma} \rangle_\Sigma = P_\Sigma \vec{n}$, where, in a two-body production process, such as $\pi^- + p \to \Sigma^0 + K^0$, $\vec{n}$ is a unit vector normal to the production plane. If $\alpha$ is known[2] from Eq. (2) one can measure $P_\Sigma$ at a given angle. One sees from Eq. (1) that the polarization for the larger sample including $\Lambda^0$'s emitted in all directions from the polarized $\Sigma^0$ is

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[2] Measurements of the up-down asymmetries in $\Lambda^0 \to p + \pi^-$ (for directly produced $\Lambda^0$) cannot determine the sign of $\alpha$, but only its magnitude--and that, presumably, not accurately. It has been pointed out that a measurement of the polarization of the nucleon emitted in $\Lambda^0$ decay would provide a direct determination of $\alpha$: R. Gatto, Possible Experiments on the Behavior of the Weak Hyperon Decay Interactions Under P, C, and T, UCRL-3795, June 1957; and T. D. Lee and C. N. Yang, Phys. Rev. (to be published).

$$\overline{\langle\vec{\sigma}\rangle}_\Lambda = -\frac{1}{3}\langle\vec{\sigma}\rangle_\Sigma \ ,$$

and for such a sample Eq. (2) reduces to

$$\overline{W(\vec{v})} = 1 - \frac{1}{3}\alpha P_\Sigma(\vec{n}\cdot\vec{v}) \ .$$

Therefore the only asymmetry expected for the larger sample is an up-down asymmetry with respect to the production plane (defined by the incoming $\pi^-$ and the outgoing $K^0$), and its measurement will permit a determination of $P_\Sigma$. In a smaller sample, for which the $\Lambda^0$ direction is observed, the asymmetry will not in general be up-down, but relative to the vector $\vec{u}$.[3]

To derive Eq. (1) we observe that in the decay $\Sigma^0 \to \Lambda^0 + \gamma$, from the three available vectors-- $\vec{\sigma}$, Pauli spin operator, e , polarization vector of the $\gamma$, and $\vec{u}$, the last two satisfying $(\vec{e}\cdot\vec{u}) = 0$ imposed by gauge invariance--we can form only one pseudoscalar, $(\vec{\sigma}\cdot\vec{e})$, and only one scalar, $(\vec{\sigma}\cdot\vec{e} \times \vec{u})$, which contain $\vec{e}$ linearly. Therefore the transition matrix $T$ is, apart from constants, $(\vec{\sigma}\cdot\vec{e})$, if $\Sigma^0$ and $\Lambda^0$ have opposite (relative) parity (E1 transition), and $(\vec{\sigma}\cdot\vec{e} \times \vec{u})$ if they have same parity (M1 transition). The polarization of the emitted $\Lambda^0$ is given by

$$\langle\vec{\sigma}\rangle_\Lambda = \mathrm{Tr}\left[T(1 + \langle\vec{\sigma}\rangle_\Sigma\cdot\vec{\sigma})T\ \vec{\sigma}\right]\Big/\mathrm{Tr}\left[T(1 + \langle\vec{\sigma}\rangle_\Sigma\cdot\vec{\sigma})T^\dagger\right] \ ,$$ which

reduces in the present case to $\mathrm{Tr}\left[T\langle\vec{\sigma}\rangle_\Sigma\cdot\vec{\sigma}\ T^\dagger\ \vec{\sigma}\right]\Big/\mathrm{Tr}\left[TT^\dagger\right]$ ,

giving $\langle\vec{\sigma}\rangle_\Lambda = -\langle\vec{\sigma}\rangle_\Sigma + 2(\langle\vec{\sigma}\rangle_\Sigma\cdot\vec{e})\vec{e}$ in the case of opposite parity

and $\langle\vec{\sigma}\rangle_\Lambda = -\langle\vec{\sigma}\rangle_\Sigma + 2(\langle\vec{\sigma}\rangle_\Sigma\cdot\vec{e} \times \vec{u})(\vec{e} \times \vec{u})$ in the case of equal

[3] In particular these $\Lambda^0$'s produced through intermediate $\Sigma^0$ may exhibit a forward-backward asymmetry, simulating parity doublets.

parity of $\Sigma^0$ relative to $\Lambda^0$. It is evident from these last expressions that, after averaging over the $\gamma$ polarization, one has $\langle \vec{\sigma} \rangle_\Lambda$ the same for both cases of relative parity, and one immediately derives Eq. (1).

We now discuss the restrictions imposed by charge independence on the hyperon polarizations in the reactions (a): $\pi^- + p \to \Sigma^- + K^+$; (b): $\pi^- + p \to \Sigma^0 + K^0$; (c): $\pi^+ + p \to \Sigma^+ + K^+$; and examine what information on the production matrix can be obtained from measurement of the intensities and of the polarizations. We express the angular distribution and the $\Sigma$ polarization in each of such reactions in the forms $I = \frac{1}{2} \text{Tr} \left[ MM^\dagger \right]$ and $\langle \vec{\sigma} \rangle_\Sigma = P_\Sigma \vec{n} = I^{-1} \text{Tr} \left[ MM^\dagger \vec{\sigma} \right]$, in which M is the transition matrix for the particular reaction. If the relative parity of K with respect to $\Sigma N$ in $-1$ the matrices M are of the form $E + F(\vec{\sigma} \cdot \vec{n})$; if such relative parity is $+1$ they are of the form $G(\vec{\sigma} \cdot \vec{m}) + H(\vec{\sigma} \cdot \vec{m}')$, where $\vec{m}$ and $\vec{m}'$ are unit vectors in the direction of the in- and of the outgoing momentum respectively, and E, F, G, H are, for each reaction, functions of the energy and of $\cos \theta = \vec{m} \cdot \vec{m}'$. We introduce, for each reaction, the quantities $f^\pm$, defined by $f^\pm = \frac{1}{\sqrt{2}} (E \pm F)$, or by $f^\pm = \frac{1}{\sqrt{2}} (G e^{\pm i\theta} + H)$, according to the two cases of relative parity. One verifies that $|f^+|^2$ and $|f^-|^2$ are the probabilities for production at a given angle of $\Sigma$ with spin up or down respectively: $|f^\pm|^2 = \frac{1}{2} \text{Tr} \left[ MM^\dagger \Lambda(\pm \vec{n}) \right] = \frac{1}{2} I(1 \pm P_\Sigma)$, where $\Lambda(\pm \vec{n})$ are the projection operators $\frac{1}{2} (1 \pm \vec{\sigma} \cdot \vec{n})$. The quantities $f^\pm$ are not properly quantum-mechanical amplitudes, since an average on the proton-spin orientation is involved in their definition. However,

they can be expressed in terms of corresponding quantities for definite

isotopic spin, $f_3^\pm$ for $I = \frac{3}{2}$, and $f_1^\pm$ for $I = \frac{1}{2}$, in exactly

the same way as the amplitudes themselves are expressed in terms of

the amplitudes for definite isotopic spin. Namely:

$$|f_a^\pm|^2 = |f_3^\pm|^2 + 4 \, \text{Re}(f_3^{\pm *} f_1^\pm) + 4 \, |f_1^\pm|^2,$$

$$|f_b^\pm|^2 = 2|f_3^\pm|^2 - 4 \, \text{Re}(f_3^{\pm *} f_1^\pm) + 2 \, |f_1^\pm|^2, \quad \text{and}$$

$$|f_c^\pm|^2 = 9 \, |f_3^\pm|^2 \quad , \text{ where, according to the two cases of parity,}$$

$f_j^\pm = \frac{1}{\sqrt{2}}(E_j \pm F_j)$ or $f_j^\pm = \frac{1}{\sqrt{2}}(G_j \, e^{\pm i\theta} + H_j)$, in which $j = 1, 3$

and in which $E_j$, $F_j$, $G_j$, and $H_j$ refer to definite isotopic spin.

Calling $|f_a^\pm|^2 = x_1^\pm$, $2|f_b^\pm|^2 = x_2^\pm$, $|f_c^\pm|^2 = x_3^\pm$, these

equations imply, as well known, the existence of a triangle with

sides $\sqrt{x_1^+}$, $\sqrt{x_2^+}$, $\sqrt{x_3^+}$ and of a triangle with sides

$\sqrt{x_1^-}$, $\sqrt{x_2^-}$, $\sqrt{x_3^-}$.[4] Such conditions are all the conditions

imposed on the process by charge independence. The triangle condition

for the intensities I follows as a consequence. Provided the two

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[4] Necessary and sufficient condition for the existence of a triangle

with sides $\sqrt{x_1}$, $\sqrt{x_2}$, $\sqrt{x_3}$ is the inequality

$x_1^2 + x_2^2 + x_3^2 - 2(x_1 x_2 + x_1 x_3 + x_2 x_3) \leq 0$, which is equivalent

to any of the three conditions $|\sqrt{x_r} - \sqrt{x_s}| \leq \sqrt{x_t} \leq \sqrt{x_r} + \sqrt{x_s}$,

rst being a permutation of 123.

triangle conditions are satisfied, the above equations can be solved, giving the six real quantities $|f_3^{\pm}|^2$, $|f_1^{\pm}|^2$, $Re(f_3^{+*} f_1^{+})$ and $Re(f_3^{-*} f_1^{-})$ in terms of the six observed $|f_a^{\pm}|^2$, $|f_b^{\pm}|^2$, $|f_c^{\pm}|^2$. Since the total transition matrix at a given angle is defined in terms of four complex numbers $(E_3, F_3, E_1, F_1,$ or $G_3, H_3, G_1, H_1)$, one relative phase in the transition matrix still remains undetermined from a measurement of the intensitites and of the $\Sigma$ polarizations at that angle. Such an undetermined phase (it can be taken as that between $f_3^{+}$ and $f_3^{-}$, or that between $f_1^{+}$ and $f_1^{-}$) is a relative phase between an amplitude for spin up and an amplitude for spin down--it is measurable, in principle, by more complicated experiments such as a measurement of the $\Sigma$ polarizations when the initial protons are polarized.