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Publication Date

2004-11-01



Working Paper No. CPC05-051

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November 2004

JEL Classification: D23, D82

Keywords: Imperfect commitment, optimal contracting, delegation

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Abstract

Organizational theory suggests that authority should lie in the hands of those with information, yet the power to transfer authority is rarely absolute in practice. We investigate the validity and application of this advice in a model of optimal contracting between an uninformed principal and informed agent where the principal's commitment power is imperfect. We show that while full alignment of interests combined with delegation of authority is feasible, it is never optimal. The optimal contract is "bang-bang"—in one region of the state space, full alignment takes place, in the other, no alignment takes place. We then compare these contracts to those in which the principal has full commitment power as well as to several "informal" institutional arrangements. JEL Classification D23, D82.

1 Introduction

A key tenet of organizational theory is the "delegation principle" which says that the power to make decisions should reside in the hands of those with the relevant information (see, for instance, Milgrom and Roberts, 1992 or Saloner *et al.* 2001). Since information is likely to be dispersed within a firm, this principle implicitly advocates a largely horizontal structure, with decision making authority also dispersed throughout the firm. In practice, some corporations, such as Johnson & Johnson, have a generally decentralized structure, while others, such as General Electric, appear to be rather centralized.

The delegation principle implicitly assumes that the objectives of the person given authority do not differ from those of the firm. When such differences occur, it may be

^{*}This research was supported by the National Science Foundation (SES-0095639). We thank Ernesto Dal Bó as well as seminar participants at UC Berkeley and the Hoover Institution for helpful comments.

necessary to provide the right incentives to properly align the agent's objectives prior to delegation. Indeed, a second principle of organizational theory is the "alignment principle" which says that the alignment of incentives and the delegation of authority are complementary tools (see, for instance, Milgrom and Roberts, 1992, p. 17).

A second complication is that in practice, the authority given to subunits is rarely absolute. That is, senior management may and, on occasion, does find it beneficial to intervene on a case-by-case basis. The possibility of ex post intervention will obviously have an effect on the behavior of the subunits. In other words, there may be a commitment problem associated with the delegation of authority.

In view of these issues, how closely should the distribution of power match the distribution of information? How does the ability to commit, or the lack thereof, affect the delegation principle? With imperfect commitment, can a perfect alignment of objectives be attained? What is the optimal degree of alignment?

With the goal of shedding some light on these and related issues, we analyze the interaction between an uninformed principal and an agent who is informed about a payoff relevant state. Interest in the problem arises because the objectives of the agent may not coincide with those of the principal—a project that is optimal for one in a given state need not be optimal for the other.

In our model the principal may use (non-negative) monetary transfers as a tool to align the interests of the agent with her own. At the same time, the principal is assumed to have only a limited ability to commit. Specifically, the principal may commit to a compensation scheme but not to the project choice. So for instance, the composation scheme could commit the principal to pay the agent depending on the actual project chosen but the principal retains the freedom to make whatever choice is optimal for him. Contracts with imperfect commitment of this form are prevalent in many settings. Investment banking contracts specify a fee schedule depending on the "project" undertaken by the CEO but the CEO is free to make whatever choices he wants in light of the advice offered by the investment bank. Managers at retail stores (like Wal-Mart) may offer advice on the optimal square footage for the store at a particular location to the corportate headquarters and their actual compensation may be tied to the resulting sales via a compensation contract. In the case of Wal-Mart, where such decisions are notably centrally controlled, the ultimate decision on retail square footage at a location is retained by headquarters in Arkansas. Bajari and Tadelis (2001) note that construction contracts between commercial developers and general contractors consist of compensation schedules based on the project ultimately undertaken (with the advice of the cntractor) but with authority ultimately retained by the developer.

Our goal, therefore, is to characterize optimal contracts when the principal's power to commit is imperfect. The structure of the optimal contract then sheds light on the delegation and alignment principles in environments where commitment is imperfect.

Investigating optimal contracts in such environments is complicated by the fact that the standard "revelation principle," which allows one to restrict attention to direct contracts with truth-telling, cannot be invoked. Indeed, the standard revelation principle is known to fail when commitment is imperfect (Bester and Strausz, 2001). Without the revelation principle, there is no systematic way to determine the optimal contract—the class of contracts one may consider is necessarily *ad hoc*.

A methodological contribution of this paper is to find a contract under this form of imperfect commitment that is optimal in the class of *all* feasible contracts. We do this by first establishing that a limited form of the revelation principle—sufficient for our needs—continues to hold even though commitment is imperfect.¹ This allows us to restrict attention to direct contracts and to study the structure of the optimal contracts. In a direct contract, the agent provides (possibly noisy) information about the true state, the principal makes appropriate inferences and chooses a project.

Our findings regarding contracts with imperfect commitment are as follows:

- 1. There exists a contract that *fully* aligns the agent's interests with her own. As a result, the principal is able to delegate authority to the agent, confident in the knowledge that in every state, the project chosen will be the same as if she herself were in possession of the information. While feasible, *a full alignment contract is never optimal*.
- 2. In a leading case of the model—the so-called uniform-quadratic case—optimal contracts can be explicitly characterized and involve no payment for imprecise information.
- 3. It is never the case that the interests of the agent are only partly aligned. The optimal contract is of the bang-bang variety—in one region of the state space, a full alignment contract is implemented, in the other region, the principal makes no attempt to align the agent's interests with his own. This is consistent with the conventional wisdom that incentives and delegation are complements.

For purposes of comparison, we also derive the structure of contracts with perfect commitment—that is, situations in which the contract specifies both how the agent will be compensated and how project choices will be made. Here we find:

- 1. The ability to perfectly commit does not alter the conclusion that full alignment contracts are never optimal.
- 2. The optimal contract again involves no payment for imprecise information.
- 3. With perfect commitment, it is *always* the case that the interests of the agent are only partly aligned. The power of perfect commitment removes the need for the principal to fully align interests in any state.

¹The positive result of Bester and Strausz (2001) cannot be applied to our model.

Related Literature

Our analysis builds on the classic "cheap talk" model of Crawford and Sobel (1982) which studies the interaction between an informed agent and an uninformed principal. In their setup, the principal effectively has no commitment power whatsoever. In contrast, we allow the principal to commit to transfer payments and, in the perfect commitment section, to projects as well.

Much of the related literature focuses on a particular specification with negative quadratic preferences and a uniform distribution of states. This so called "uniform-quadratic" case is notable for its tractability and has been extensively used in subsequent applications. For instance, the political science literature on the efficacy of legislative rules (see Gilligan and Krehbiel, 1987 & 1989 and Krishna and Morgan, 2001), largely concerns itself with this case.

Baron (2000) studies the effect that "contracting" arrangements have on the interaction between an uninformed legislature and an informed committee for the uniform-quadratic case. His model differs from ours in many respects. First, he restricts the set of contracts to those that either involve full revelation over some interval and no revelation over another (in that case, the committee is said to be "discharged"). Second, the limited liability constraint is replaced by an endogenous participation constraint. This means that transfers between the legislature and the committee could be in either direction. Indeed the contract that is optimal in his class involves transfers from the agent to the principal. Finally, the principal is assumed to be able to commit to a transfer only when information is revealed. This is a critical assumption as it can be shown that the principal can improve his payoff by means of a contract that also involves transfers even when the agent is "discharged."

Ottaviani (2000) also examines how the use of transfers can enhance the amount of information that the agent shares with the principal. Again, for the uniform-quadratic case, he shows the possibility of full alignment contracts (this is a special case of our Proposition 2) and that this contract is dominated by one that delegates authority to the agent directly but involves no transfers. He does not study optimal contracts under either imperfect or perfect commitment.

Dessein (2001) also examines the benefits of delegation in a similar model, again in the uniform-quadratic case.² Unlike our setting, Dessein does not allow for the possibility of transfer payments by the principal; hence the "alignment principle" is inoperative in his setting. Further, the principal is assumed to be able to commit not to intervene in the project chosen by the agent; thus, issues associated with imperfect commitment are also absent. In Section 5, we compare optimal contracts in our setting with delegation contracts along the lines of Dessein. In a similar model with transfers, Krähmer (2004) allows the principal to commit whether or not she wants to delegate authority to the agent depending on the message sent by the agent. He shows that such "message contingent delegation" may be superior both to ex ante

²Dessein also looks at cases where the preferences are concave functions of the quadratic loss specification.

or "unconditional" delegation as in Dessein (2002) and to unconditional retention of authority.

A separate strand of the literature is concerned with solving the moral hazard problem of information gathering on the part of the agent or agents (see, for example Aghion and Tirole, 1997 and Dewatripont and Tirole, 1999). In contrast, our primary interest is in the role of contracts to elicit information from already informed agents. In these papers, incentive alignment for efficient information transmission, once the agent has gathered information, is a secondary consideration.

Finally, our paper is somewhat related to questions addressed in Bester and Strausz (2001). That paper seeks to extend the revelation principle to settings where the principal is unable to commit to one or more dimensions of the contracting space—as in the question we consider. They show that when the set of states is *finite*, any incentive efficient outcome of that mechanism—that is, an equilibrium outcome not Pareto dominated by another equilibrium outcome—can be replicated by an equilibrium of a direct mechanism. Their result, however, does not apply to our model since it has a continuum of states. We derive a revelation principle in Proposition 1 that in the context of our model, applies to all incentive feasible outcomes, not only those that are also incentive efficient.

The remainder of the paper proceeds as follows: In Section 2 we sketch the model. Section 3 presents results on full alignment contracts and shows that these contracts are never optimal. We then derive various structural properties of optimal contracts and then uses these to characterize in closed form the optimal contract under imperfect commitment. Section 4 does the parallel exercise for perfect commitment. Section 5 compares the value of contracting with several alternative schemes. Finally, Section 6 concludes.

2 Preliminaries

In this section we sketch a simple model of decision making between an informed agent and an uninformed principal. The model is virtually the same as that of Crawford and Sobel (hereafter 'CS') but is amended to allow for transfers from the principal to the agent. In addition, we allow the principal to contract on certain aspects of her choices. Which aspects are contractible and which are not is specified in more detail below.

Consider a principal who has authority to choose a project $y \in \mathbf{R}$, the payoff from which depends on some underlying state of nature $\theta \in \Theta \equiv [0,1]$. The state of nature θ is distributed according to the density function $f(\cdot)$. The principal has no information about θ , but this information is available to an *agent* who observes θ .

The payoff functions of the agents, not including any transfers, are of the form $U(y, \theta, b_i)$ where b_i is a bias parameter which differs between the two parties. The bias of the principal, b_0 , is normalized to be 0. The bias of the agent, $b_1 = b > 0$. In what follows we write $U(y, \theta) \equiv U(y, \theta, 0)$ as the principal's payoff function. We

suppose that U is twice continuously differentiable and satisfies $U_{11} < 0$, $U_{12} > 0$, $U_{13} > 0$. Since $U_{13} > 0$ the parameter b measures how closely the agent's interests are aligned with those of the principal and it is useful to think of b as a measure of how biased the agent is, relative to the principal. We also assume that for each i, $U(y, \theta, b_i)$ attains a maximum at some y. Since $U_{11} < 0$, the maximizing project is unique. The biases are commonly known.

These assumptions are satisfied by "quadratic loss functions." In this case, the principal's payoff function is

$$U(y,\theta) = -(y-\theta)^{2} \tag{1}$$

and the agent's payoff function is

$$U(y, \theta, b_i) = -(y - (\theta + b))^2$$
(2)

where b > 0.

Define $y^*(\theta) = \arg\max_y U(y, \theta)$ to be the *ideal* project for the principal when the state is θ . Similarly, define $y^*(\theta, b) = \arg\max_y U(y, \theta, b)$ be the ideal project for the agent. Since $U_{13} > 0$, b > 0 implies that $y^*(\theta, b) > y^*(\theta)$.

Notice that with quadratic loss functions, the ideal project for the principal is to choose a project that matches the true state exactly: for all θ , $y^*(\theta) = \theta$. The ideal project for an agent with bias b is $y^*(\theta, b) = \theta + b$.

In the basic CS model, upon learning the state θ , the agent is assumed to offer some "advice" to the decision maker. This advice is in the form of a costless message m chosen from some fixed set M. Upon hearing the advice offered by the agent, the principal chooses the project y.

We augment the CS model and allow the principal to contract with the agent and perhaps make transfer payments. We suppose that preferences of the two parties are quasi-linear. Thus, if a payment $t \geq 0$ is made to the agent, then the payoff of the principal from project y in state θ is

$$U(y,\theta) - t$$

while the payoff of the agent is

$$U(y,\theta,b)+t$$

We assume that only nonnegative transfers $(t \ge 0)$ from the principal to the agent are feasible. In effect, the agent is protected by a "limited liability" clause and cannot be punished too severely.³

Two contracting environments are studied.

³While the precise characterizations of the optimal contract make use of this assumption, many of the qualitative features of the model are unaffected if we replace this with the usual ex ante participation constraint. For instance, Proposition 3 below continues to hold.

- 1. Imperfect commitment. In this case, the principal commits to transfer payments but retains ultimate authority over the choice of the project. That is, the principal cannot commit not to *intervene* in the choice of projects.
- 2. Perfect commitment. In this case, the principal contracts in advance on both the project choice and the transfer.

While the precise form of these contracts is specified below, throughout this paper we suppose that the two parties cannot contract on the state of nature θ . Contracts are not allowed to depend directly on the realized state of nature since even after the fact, it may be difficult to verify for a third party.⁴ With this one exception we allow the contract to depend on any other variable that is mutually observable and can be verified by a third party. For instance, the contract could specify how the compensation will depend on the project actually undertaken by the principal. This, of course, allows an indirect dependence of the contract on the state of nature but is based on something that is verifiable.

3 Optimal Contracts with Imperfect Commitment

We first study a situation in which the ability of the principal to commit is imperfect. By this we mean that the principal is unable or unwilling to bind herself to choose a particular project as a result of the advice offered by the agent—that is, she retains executive authority.

Without loss of generality, any mechanism in this setting can then be represented as a pair (M,T), where M is an arbitrary set of messages and $T(\cdot)$ is a transfer scheme that determines the compensation $T(m) \geq 0$ that the agent will receive if he sends the message m. The idea of imperfect commitment is captured by assuming that of the two instruments available to the principal, project choices y and transfers t, he can contract on, and commit to, only one. The purpose of the contract is, of course, to align the interests of the agent more closely with those of the principal.

While the specification that compensation is based on the message ("advice") alone seems unnatural; so it is useful to examine how, exactly, such contracts are, indeed, without loss of generality. The key is to notice that more "realistic" contracts, such as those described in the introduction, are all accommodated within this framework. For instance, the compensation contract for an investment bank may be of the form T(y). The investment bank's advice, m, ultimately leads the CEO to undertake a project, y(m), and hence to compensation T(y(m)). Thus we may suppose that T depends on the message m itself.

When the principal can perfectly commit—that is, to both a project Y(m) and a transfer T(m)—then the revelation principle may be invoked. Specifically, for any

⁴See Prendergast (1993) for a variety of other reasons why contracting on the realized state (or equivalently on realized payoffs) may be problematic.

(full-commitment) mechanism (M, Y, T) and any equilibrium of this mechanism, (i) there exists a direct mechanism—in which the agent reports his private information, so that $M = \Theta$ —such that (ii) truth-telling is an equilibrium that is outcome equivalent to the given equilibrium of the original mechanism. The underlying argument is very simple. The original mechanism can be composed with the strategies that constitute an equilibrium of the said mechanism to obtain a direct mechanism. It is then easily verified that truth-telling is an equilibrium of the direct mechanism—it is incentive compatible—and that the resulting outcomes are the same as in the given equilibrium of the original mechanism. Put another way, incentive compatible outcomes of direct mechanisms span the set of equilibrium outcomes resulting from all mechanisms.

The statement above highlights the fact that the standard revelation principle has two components. First, it allows one to restrict attention to direct mechanisms. Second, only truth-telling equilibria of direct mechanisms need be considered. The revelation principle is a powerful tool because, rather than searching over the space of all possible indirect mechanisms, an impossible task, it allows the analyst to restrict attention to direct mechanisms—that is, it is the first component that renders the task of finding an optimal contract feasible.

When the principal's ability to commit is imperfect, the second component of the revelation principal clearly fails in general. This is because if the principal is not committed to act in prespecified ways, when the agent truthfully reveals, the principal will use this information to his own advantage. Knowing this, the agent will, in general, be better off not fully revealing what he knows—that is, some loss of information is likely. To see this in the context of the model of the previous section, suppose that the principal can commit to neither decisions not transfers. In that case, the model is identical to CS, who showed that full revelation cannot be an equilibrium.

It remains to see if the first component continues to hold. For the reasons described above, knowing even this component would be extremely helpful. But, as we demonstrate below (our example is similar to one by Bester and Strausz, 2001), with imperfect commitment, even the first component of the revelation principle may fail—there may be equilibrium outcomes of an indirect mechanism that cannot be replicated by a direct mechanism.

Suppose that the principal and agent have quadratic loss functions where the agent's bias is $b = \frac{1}{6}$, but, in a departure from our model, the state space is *binary*, that is, $\theta \in \Theta = \{\theta_1, \theta_2\}$ where $\theta_1 = \frac{1}{4}$ and $\theta_2 = \frac{3}{4}$. Each state is equally likely. Consider a contract in which the set of messages has three elements so that $M = \{m_1, m_2, m_3\}$ and the associated transfer scheme:

$$T(m_1) = \frac{1}{6}$$
, $T(m_2) = \frac{7}{48}$ and $T(m_3) = 0$

Suppose the agent follows the reporting strategy:

in state θ_1 , send either m_1 or m_2 with probability $\frac{1}{2}$ in state θ_2 , send either m_2 or m_3 with probability $\frac{1}{2}$

The message m_1 is sent only in state θ_1 and thus reveals to the principal that the state is θ_1 . Similarly, m_3 reveals that the state is θ_2 . The message m_2 , however, is sent in both states and so the principal is still unsure as to which state has occurred. Thus the posterior beliefs of the principal after hearing message m_i are

$$p(\theta_1|m_1) = 1$$
, $p(\theta_1|m_2) = \frac{1}{2}$, and $p(\theta_1|m_3) = 0$

Given these beliefs, the optimal project choices of the principal following m_i are

$$y(m_1) = \theta_1, y(m_2) = \frac{1}{2}\theta_1 + \frac{1}{2}\theta_2, \text{ and } y(m_3) = \theta_2$$

It is routine to verify that, given the $y(m_i)$ as above, it is a best response for the agent to behave in the manner specified. Thus when the set of messages is $M = \{m_1, m_2, m_3\}$, there is a contract T and a perfect Bayesian equilibrium (PBE) of the resulting game, in which the principal chooses three possible project with positive probability.

In a direct mechanism, that is, if $M = \Theta$, then following any report $\theta \in \Theta$, at most one project would be optimal for the principal. Thus, the principal chooses two possible projects. This means that a direct mechanism cannot replicate the workings of the indirect mechanism specified above; nor can it replicate the resulting payoffs. Without imperfect commitment, both components of the revelation principle may fail. Thus, it is not clear how one should proceed to find an optimal contract—one that is best for the principal. The set of outcomes depends on the number of messages that the agent may use to convey information and that the number required messages may be more than the number of states. What is the "right" number of messages?

In the example above, there were only two states. In our model, there is a *continuum* of states and, as we show below, this restores the first component of the revelation principle: even with imperfect commitment, *any* equilibrium outcome of an indirect mechanism can be replicated by an equilibrium of a direct mechanism.⁵

3.1 A "Revelation Principle" with Imperfect Commitment

Consider a contract (M, T) in which the agent sends messages m in some set M. Given a message m, the principal transfers T(m) to the agent. After that he is free to choose any project y that he wishes. This defines a game between the principal and agent.

⁵Following Bester and Strausz (2001), in what follows we refer to this conclusion—the first component—as a "revelation principle without commitment."

A perfect Bayesian equilibrium (μ, Y, G) of this game consists of (i) a strategy for the agent $\mu: \Theta \to \Delta(M)$ which assigns for every state θ , a probability distribution over M; (ii) a strategy for the principal $Y: M \to \mathbf{R}$; and (iii) a belief function $G: M \to \Delta(\Theta)$ which assigns for every m a probability distribution over the states θ . It is required that following any message m, the principal maximizes her expected utility given her beliefs; G is derived from μ using Bayes' rule wherever possible; μ is optimal given Y.

In what follows, an "equilibrium" is always understood to mean "perfect Bayesian equilibrium."

Proposition 1 Consider a contract (M,T) with imperfect commitment and any equilibrium under this contract. Then there exists an equilibrium under a direct contract (Θ,t) which is outcome equivalent.

Proof. Suppose that (μ, Y, G) is a perfect Bayesian equilibrium under the contract (M, T). Consider two states $\theta_1 < \theta_2$. Let $y_1 = \max\{Y(m) : m \in \text{supp } \mu(\cdot \mid \theta_1)\}$ and $y_2 = \min\{Y(m) : m \in \text{supp } \mu(\cdot \mid \theta_2)\}$. Then we claim that $y_2 \geq y_1$. Suppose to the contrary that $y_1 > y_2$. If T_1 and T_2 are the transfers associated with y_1 and y_2 , respectively, then by revealed preference of y_1 in state θ_1 we have that $U(y_1, \theta_1, b) - U(y_2, \theta_1, b) \geq T_2 - T_1$. Since $U_{12} > 0$, we have that $U(y_1, \theta_2, b) - U(y_2, \theta_2, b) > T_2 - T_1$ which is a contradiction since this means that it is better to induce action y_1 and transfer T_1 in state θ_2 . Thus, $y_1 \leq y_2$ and so in equilibrium, any two states have at most one project in common, and projects are strongly monotonic in the state.

Next, we show that for almost every state, at most one project is induced in equilibrium. Suppose to the contrary that there is some open interval of states I such that for all $\theta \in I$, there are two projects $\underline{Y}(\theta) < \overline{Y}(\theta)$ that are induced. If for all $\theta \in I, \underline{Y}(\theta)$ is a constant or $\overline{Y}(\theta)$ is a constant then this would violate the monotonicity derived above. This means that over every open interval $I, \underline{Y}(\cdot)$ and $\overline{Y}(\cdot)$ are not constant. Then for a small enough open subset of I, we have that both $\underline{Y}(\cdot)$ and $\overline{Y}(\cdot)$ are strictly increasing. But now again, the strong monotonicity property derived above is violated. Thus for almost every state θ there can be at most one project $y(\theta)$. To summarize, we have so far shown that, in any equilibrium of any indirect mechanism, the agent induces a unique project $y(\theta)$, and hence a unique transfer $t(\theta)$, in (almost) every state.

Suppose that under the contract (M,T), the project $y(\theta)$ and transfer $t(\theta)$ are equilibrium outcomes in state θ . Now consider the direct contract (Θ,t) and the following strategy for the agent: Suppose that in state θ , the equilibrium calls for project $y(\theta)$ to be induced. Define

$$Z(\theta) = \{\sigma : y(\sigma) = y(\theta)\}\$$

⁶Because of the assumption that the principal's utility $U(\cdot, \theta)$ is strictly concave, it is unnecessary to allow for strategies in which the principal randomizes.

to be the set of states in which the project induced is the same as that induced in state θ . By the monotonicity property, $Z(\theta)$ is an interval, possibly degenerate.

To complete the proof, let the equilibrium strategy of the agent in the direct contract, $\mu^*(\cdot \mid \theta)$ be the uniform distribution over the elements of $Z(\theta)$. This strategy leads the principal to hold posterior beliefs G^* identical to those in the equilibrium of the indirect contract, and so the project chosen by the principal in state θ will be the same in the two equilibria. Thus, the direct contract (Θ, t) is outcome equivalent to the contract (M, T) and this completes the proof.

Like the standard revelation principle, Proposition 1 allows us to restrict attention to direct mechanisms—bypassing the plethora of possible indirect contracts. It should be contrasted with a similar result of Bester and Strausz (2001) because (a) it concerns situations with a continuum of states and, as a result, (b) it applies to all incentive feasible contracts, not only those that are incentive efficient.

3.2 Full Alignment Contracts

Organization and management texts suggest that the provision of incentives and delegation of authority are complementary. It is argued that the principal should delegate decision making authority to the agent only after providing incentives such that the agent's objectives are aligned with those of the principal. In this subsection, we examine the extent to which this principle applies in our model. In particular, we examine two related questions: First, with imperfect commitment, is it even possible for the principal to design a contract that fully aligns the agent's interests with her own? Second, and more importantly, if it is possible, under what circumstances is this the best contract for the principal? Proposition 1 allows us to restrict attention to direct contracts. A full alignment direct contract will, of course, induce the agent to reveal perfectly his private information in every state.

In the absence of any contracting ability whatsoever, CS have shown that it is impossible for the principal to induce the agent to fully reveal his private information. However, we show below that when the principal can contract, albeit imperfectly, this is no longer he case—full revelation is always implementable. We then show that, despite the fact that such a contract always leads to the principal obtaining her most desired project in every state, it is never cost effective. That is, full alignment contracts are *never* optimal.

To see that full alignment contracts are always feasible, first notice that under such a contract the agent offers truthful advice; that is, $\mu\left(\theta\right)=\theta$. Further, the principal anticipates that this will be the case; hence $y\left(\theta\right)=y^{*}\left(\theta\right)$. For truth-telling to be a best response requires that in every state θ

$$U(y^*(\theta), \theta, b) + t(\theta) \ge U(y^*(\theta'), \theta, b) + t(\theta')$$

for all $\theta' \neq \theta$. The first-order condition for the agent's maximization problem results

in the differential equation

$$t^{*\prime}(\theta) = -U_1(y^*(\theta), \theta, b) y^{*\prime}(\theta)$$

Since $U_1(y^*(\theta), \theta, b) > 0$ and $y^{*'}(\theta) > 0$, a contract that induces full revelation is downward sloping. Thus among all contracts that induce full revelation and satisfy limited liability, the least-cost one is:

$$t^*(\theta) = \int_{\theta}^{1} U_1(y^*(\alpha), \alpha, b) y^{*\prime}(\alpha) d\alpha$$
 (3)

It is routine to verify that the contract in (3) indeed induces full revelation—that is, no nonlocal deviations are profitable either. To summarize:

Proposition 2 Under imperfect commitment, full alignment contracts are always feasible.

Proposition 2 can also be derived as follows. A standard result in contract theory (see Salanié, 1997, p. 31) is that with full commitment every monotonic project choice can be implemented via a truthful direct mechanism with an appropriate transfer scheme. This implies that y^* can be so implemented. But since y^* is ex post optimal for the principal under truth-telling, no commitment is needed to ensure that the principal will in fact, choose $y^*(\theta)$ in state θ . Thus y^* can be implemented even without commitment.

Now we show that a full alignment contract is never cost-effective. To see this, consider an alternative contract $t(\cdot)$ that induces the following: the agent reveals any state $\theta \in [0, z]$ where z < 1 and pools thereafter. No payment is made if the reported state m > z. At $\theta = z$, the agent must be indifferent between reporting that the state is z and reporting that it is above z. If we denote by t_z the payment in state z, then we must have

$$U(y^*(z), z, b) + t_z = U(y([z, 1]), z, b)$$
 (4)

where $y\left([z,1]\right) = \arg\max E\left[U\left(y,\theta\right) \mid \theta \in [z,1]\right]$ is the optimal project conditional on knowing that $\theta \in [z,1]$. Since for z close to 1, $U\left(y^*\left(z\right),z,b\right) < U\left(y\left([z,1]\right),z,b\right)$, it follows that $t_z > 0$.

It is routine to verify that

$$\frac{dt_z}{dz}\Big|_{z=1} = U_1(y^*(1), 1, b) \left(\frac{d}{dz}y[z, 1]\Big|_{z=1} - y^{*'}(1)\right)$$

Incentive compatibility over the interval [0, z] requires that

$$t(\theta) = t_z + \int_{\theta}^{z} U_1(y^*(\alpha), \alpha, b) y^{*\prime}(\alpha) d\alpha$$

which is again always greater than zero, so this alternative contract is also feasible. It is useful to note that:

$$\frac{dt\left(\theta\right)}{dz} = \frac{dt_z}{dz} + U_1\left(y^*\left(z\right), z, b\right) y^{*\prime}\left(z\right)$$

That is, on the interval [0, z], the new contract t is parallel to the full alignment contract t^* . Indeed, for all $\theta \leq z$ we have,

$$t(\theta) - t^*(\theta) = t_z - t^*(z)$$

The expected utility resulting from the new contract is

$$V = \int_{0}^{z} \left(U\left(y^{*}\left(\theta \right), \theta \right) - t\left(\theta \right) \right) f\left(\theta \right) d\theta + \int_{z}^{1} U\left(y\left[z, 1 \right], \theta \right) f\left(\theta \right) d\theta$$

Differentiating with respect to z, we obtain

$$\frac{dV}{dz} = (U(y^*(z), z) - t_z) f(z) - U(y[z, 1], z) f(z)
- \int_0^z \left(\frac{dt(\theta)}{dz}\right) f(\theta) d\theta
= (U(y^*(z), z) - t_z) f(z) - U(y[z, 1], z) f(z)
- \int_0^z \left(\frac{dt_z}{dz} + U_1(y^*(1), 1, b) y^{*'}(1)\right) f(\theta) d\theta$$

When z = 1, we have

$$\frac{dV}{dz}\Big|_{z=1} = -\frac{dt_z}{dz}\Big|_{z=1} - U_1(y^*(1), 1, b) y^{*\prime}(1)
= -\left(U_1(y^*(1), 1, b) \left(\frac{d}{dz}y[z, 1]\Big|_{z=1} - y^{*\prime}(1)\right)\right) - U_1(y^*(1), 1, b) y^{*\prime}(1)
= -U_1(y^*(1), 1, b) \frac{d}{dz}y[z, 1]\Big|_{z=1}
< 0$$

where the inequality follows from the fact that $U_1(y^*(1), 1, b) > 0$ and $\frac{d}{dz}y[z, 1] > 0$. Thus we have shown that for z close enough to 1, the alternative contract $t(\cdot)$ yields a higher expected utility for the principal than the full alignment contract $t^*(\cdot)$.

The argument above establishes the main result for this section:

Proposition 3 Under imperfect commitment, full alignment contracts are never optimal.

The economic trade-off captured in this result is the following: For states near the highest possible state, the direct contracting costs of inducing truth-telling are relatively inexpensive ($t^*(\theta)$) is close to zero when θ is close to 1); however the indirect effect of obtaining this revelation is to increase the information extraction costs for all of the lower states. The alternative contract shows that the informational benefits of additional revelation in the high states never justifies these increased costs. The principal can locally give up a small amount of information by inducing pooling for the highest states, but more than recovers this in the global reduction in the costs of information extraction for lower states.

Delegation and Incentives Consider a situation in which all decision making authority is transferred to the subordinate (agent), and an appropriate contract is in place so that he has the incentive to choose the right decision from the perspective of the principal. The following (indirect) contract, mimicking the one in (3) achieves the desired result:

$$T^{*}(y) = \int_{\phi(y)}^{1} U_{1}(\gamma, \phi(\gamma), b) d\gamma$$
 (5)

where $\phi(\gamma) \equiv y^{*-1}(\gamma)$ denotes the state in which project γ is optimal for the principal. Indeed, the contract in (5) is obtained from that in (3) merely by a change of variable from θ to γ .

It is easy to see that such a contract fully aligns the objectives of the agent with those of the principal. So if the former is given authority to choose the project (even if the principal might override the agent's decision), the agent can do no better than to choose $y^*(\theta)$ in state θ . Since this is the principal's most preferred project, she will not override the agent. But as we have shown above, such an arrangement is suboptimal—it is too expensive to align the agent's incentives completely and then to transfer authority. Thus, the optimal contract necessarily entails an incomplete alignment of interests.

3.3 Optimal Contracts in the Uniform-Quadratic Case

What is the structure of optimal contracts under imperfect commitment? To obtain an exact characterization requires placing more structure on the distribution of states and the payoff functions of the actors. In this section, we offer an explicit characterization for the uniform-quadratic case.

We begin by establishing some structural properties of optimal contracts. Notice that, because equilibrium projects are nondecreasing in the state, the state space may be delineated into intervals of *separation*—where the agent fully reveals his private information—and intervals of *pooling*—where the agent discloses only that the state lies in some interval.

No separation to the right of pooling We first establish that inducing separation by fully aligning the interests of the agent with those of the principal is only cost-effective in low states. That is, once a contract calls for a pooling interval over a set of states, it never pays to induce separation for higher states. Specifically,

Proposition 4 The optimal contract under imperfect commitment involves separation in low states and pooling in high states.

Proof. Suppose there is pooling in the interval [w, x] and revelation in the interval [x, z]. In the interval [x, z] the contract must satisfy

$$t(\theta) = 2b(z - \theta) + t(z) \tag{6}$$

Then the indifference condition at x is

$$-\left(\frac{w+x}{2} - (x+b)\right)^2 + t_{wx} = -b^2 + t(x) \tag{7}$$

Notice that $t_{wx} > 0$. Otherwise, at x, both the projects $\frac{w+x}{2}$ and x are too low for the agent.

At w, the agent must be indifferent between some equilibrium project y together with some transfer t_y , and the project $\frac{w+x}{2}$ together with the transfer t_{wx} . Hence, we have

$$t_y = (y - (w+b))^2 - \left(\frac{w+x}{2} - (w+b)\right)^2 + t_{wx}$$
$$= w^2 + 2zb + y^2 - 2yw - 2yb + t(z)$$

using (7) to substitute for t_{wx} . It is important to note that the transfer t_y does not depend on x.

Hence, the principal's utility in this interval

$$EV = \int_{w}^{x} \left(-\left(\frac{w+x}{2} - \theta\right)^{2} - t_{wx} \right) d\theta - \int_{x}^{z} \left(2b\left(z - \theta\right) + t\left(z\right) \right) d\theta$$
$$= wx^{2} - xw^{2} + t\left(z\right)w - w^{2}b - \frac{1}{3}x^{3} + \frac{1}{3}w^{3} + 2bzw - bz^{2} - t\left(z\right)z$$

Now consider a small change in x, keeping fixed all projects and transfers not in the interval [w, x]. As noted above, this does not affect the transfer t_y associated with the project y to the left of w. Moreover, since $t_{wx} > 0$, a small change in x is feasible. The change in expected utility from an increase in x is:

$$\frac{dEV}{dx} = -\left(w - x\right)^2$$

and this is negative provided x > w. This means that no contract in which there is pooling over some nondegenerate interval [w, x] followed by separation over some interval [x, z] can be optimal.

The property derived above implies that an optimal contract consists of separation for some set of low states, say for θ below some threshold a, followed by a number of pooling intervals that subdivide [a, 1].

No payment for pooling The next proposition establishes an important property: it is never optimal for the principal to make positive transfers for *partial* revelation. Put differently, for states where the principal does not obtain her most preferred project, she should offer no compensation whatsoever. Formally,

Proposition 5 The optimal contract under imperfect commitment involves no payment in any pooling interval.

Proof. See Appendix A.

Propositions 4 and 5 together imply that, under the optimal contract, the agent is induced to reveal up to some state a and not compensated thereafter. Further, for any value of a, it can be shown that the number of pooling intervals, K, is uniquely determined—it is the no contracting outcome that maximizes the principal's expected payoffs. (For a formal statement, see Lemma 1 in Appendix A.)

Thus, the optimal contract can be completely characterized as the solution to the problem of choosing a to maximize

$$EV = -\int_{0}^{a} (2b(a - \theta) + t(a)) d\theta - \sum_{k=1}^{K} \int_{x_{k-1}}^{x_{k}} (y([x_{k-1}, x_{k}]) - \theta)^{2} d\theta$$

where K is determined as in Lemma 1 in Appendix A.

Finally, we show that the interval over which separation takes place and contractual payments are made is "relatively small." In particular, the optimal contract never involves paying for information more than one-fourth of the time.

Proposition 6 The optimal contract under imperfect commitment involves: (i) positive payments and separation over an interval $[0, a^*]$ where $a^* \leq \frac{1}{4}$; (ii) no payments and a division of $[a^*, 1]$ into a number of pooling intervals.

Proof. We claim that the optimal value of a is

$$a^* = \frac{3}{4} - \frac{1}{4}\sqrt{4 + \frac{1}{3}(3 - 8bK(K - 1))(8bK(K + 1) - 3)}$$
 (8)

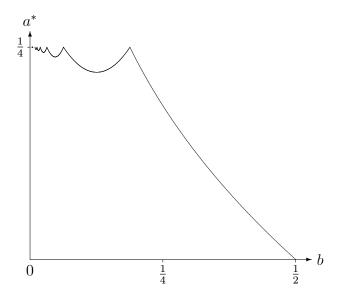


Figure 1: Optimal a

where K is the unique integer such that

$$\frac{3}{8K(K+1)} \le b < \frac{3}{8K(K-1)} \tag{9}$$

It is routine to verify that $a^* \leq \frac{1}{4}$. How a^* varies with b is depicted in Figure 1.

First, we show that for all b, the payoff to the principal from choosing $a > a^*$ is worse than her payoff from choosing $a = a^*$. At $a = \frac{1}{4}$, the most informative partition has K elements where K is the unique integer satisfying (9). For any $a > a^*$,

$$\frac{\partial EV}{\partial a} = \frac{1}{6} \frac{8b^2 K^4 - 8b^2 K^2 - 6bK^2 + 3(1 - 2a)(1 - a)}{K^2} < 0$$

using (8). This shows that all $a > a^*$ are suboptimal since for any such a the most informative partition of [a,1] can have at most K elements. In particular, $\frac{dU}{da} < 0$ at $a = \frac{1}{4}$.

Next, we show that for all b, the payoff to the principal from choosing $a < a^*$ is worse than her payoff from choosing $a = a^*$. For $a < a^*$ and fixed K, one may readily verify that

$$\frac{\partial EV}{\partial a} > 0$$

The only thing left to verify is that for $a < a^*$, the utility is lower than at a^* even if the number of elements in the most informative partition of [a, 1] is greater than K.

Suppose that when a = 0, the maximal size of the partition of [a, 1] is N (as in CS).

For L = N - 1, N - 2, ...K + 1, K define a_L to be the smallest a for which it is not possible to make a size L + 1 partition. That is,

$$-\frac{1}{2} + \frac{1}{2}\sqrt{1 + \frac{2(1 - a_L)}{b}} = L$$

The principal's expected payoff function is not differentiable at the points a_L since there is a "regime change" from L+1 to L element partitions. We can however, find the right and left derivatives of EV at a_L and a_{L-1} , respectively.

The right derivative of EV at $a = a_L = 1 - 2bL(L+1)$ is,

$$\left. \frac{\partial EV}{\partial a} \right|_{a=a_L}^{+} = \frac{1}{3} 8b \left(2L + 1 \right) \left(L + 1 \right) \left(b - \frac{3}{8L \left(L + 1 \right)} \right) \tag{10}$$

But since for all $a \in [a_L, a_{L-1})$, there does not exist a partition of [a, 1] with L + 1 elements and $a < \frac{1}{4}$, we have

$$b \ge \frac{(1-a)}{2L(L+1)} > \frac{3}{8L(L+1)}$$

and so (10) is positive.

Similarly, the left derivative of U at $a = a_{L-1} = 1 - 2bL(L-1)$

$$\frac{\partial EV}{\partial a}\Big|_{a=a_{L-1}}^{-} = \frac{1}{3}8b(2L-1)(L-1)\left(b - \frac{3}{8L(L-1)}\right)$$
 (11)

But since at a_{L-1} , there does not exist a partition of $[a_{L-1},1]$ with L elements and $a_{L-1}<\frac{1}{4}$

$$b \ge \frac{(1 - a_L)}{2L(L - 1)} > \frac{3}{8L(L - 1)}$$

and so we have that (11) is also positive.

The proof is completed by noting that when L = K, we have

$$\left. \frac{\partial EV}{\partial a} \right|_{a=a_K}^+ > 0 \text{ and } \left. \frac{\partial EV}{\partial a} \right|_{a=a_{K-1}}^- < 0$$

Discussion The structure of the optimal contract has the somewhat unusual property that the interval in which the principal finds it optimal to compensate the agent to fully align interests is *nonmonotonic* in the bias of that agent. (See Figure 1.)

Why is this? The key trade-off is that, by reducing the size of the separating interval, the principal can induce more information transmission for higher states.

Obtaining this information for higher states is cost-effective for the agent in two respects. First, the more precise the information immediately to the right of a^* , the less expensive is the compensation contract to induce aligned interests since this creates a parallel shift downward in the transfer schedule. At the same time, there is a direct benefit of obtaining more information in higher states at no cost whatsoever. As this trade-off becomes more or less favorable with changes in the bias, the optimal contract adjusts the length of the separating interval. Interestingly, the net benefits from increased information in higher states always outweigh the upside from increasing the separating interval for states above $\theta = \frac{1}{4}$. That is, it is never in the principal's interest to compensate the agent more than one-fourth of the time.

4 Optimal Contracts with Perfect Commitment

In the face of the commitment problems implied by intervention in the choice of projects by upper management, strategic management guides often counsel that managers invest in a reputation for a consistent style of handling intervention. This investment can involve setting well-established routines before upper management can exercise authority in intervening in project choice. Alternatively, the firm can seek to develop a culture for non-intervention or highlight prescribed circumstances for intervention in its mission statement. What is the value of this commitment? How does the ability to commit not to intervene affect the structure of optimal contracts? Does it now benefit the firm to employ full alignment contracts more extensively?

We study these issues in the context of our model. Adding the power of perfect commitment means that the principal can now commit to both instruments—transfers t and project choices y. When perfect commitment is possible, the standard revelation principle applies, and it is sufficient to consider direct contracts—that is, those in which M = [0,1]—which satisfy incentive compatibility. A direct contract (y,t) specifies for each message $\theta \in [0,1]$, a project $y(\theta)$ and a transfer $t(\theta)$. A direct contract (y,t) is incentive compatible if for all θ , it is best for the agent to report the state truthfully, that is, if $\sigma = \theta$ maximizes $U(y(\sigma), \theta, b) + t(\sigma)$. Standard arguments show that, under perfect commitment, necessary and sufficient conditions for incentive compatibility requires that: (i) $y(\cdot)$ is nondecreasing; and (ii) $t'(\theta) = -U_1(y(\theta), \theta, b)y'(\theta)$ at all points θ where $y(\cdot)$ is differentiable (see, for instance, Salanié, 1997).

One might be tempted to apply standard techniques for analyzing this class of problems; however, there are several features of the CS model that prevent the application of standard techniques. Specifically, a usual assumption about the agent's utility is that $U_2 > 0$; that is, a given project yields higher utility in higher states (see, for instance, Sappington, 1983). This guarantees that the agent's payoff in any incentive compatible contract is non-decreasing in the state the limited liability constraint (or a participation constraint) is indeed met for all θ if it is met for the lowest type. In the CS model, however, the agent's payoff is nonmonotonic— $U(y, \theta, b)$ is

maximized at $y = y^*(\theta, b)$. Hence, it is not enough to ensure the limited liability constraint only for extreme types and the analysis becomes non-standard.

Full Alignment Contracts First, we revisit the optimality of full alignment contracts. When commitment was imperfect, we saw that these contracts were feasible but not optimal (Propositions 2 and 3). Clearly any contract that is feasible under imperfect commitment is feasible under perfect commitment. And since the full alignment contract was never optimal in the former circumstances, it is never optimal under the latter either. Thus it immediately follows that:

Corollary 1 Under perfect commitment, full alignment contracts are always feasible but never optimal.

Thus, even when the principal can perfectly commit not to intervene, the best policy is still not to align the incentives of the agent and delegate decision making responsibility fully to the informed party.

Optimal Contracts The optimal contract is the solution to the following control problem

$$\max \int_{0}^{1} (U(y, \theta) - t) f(\theta) d\theta$$

subject to the law of motion

$$t' = -U_1(y, \theta, b) u \tag{12}$$

and the constraints

$$y' = u$$
$$t > 0$$

where y and t are the state variables and u is the control variable. Notice that local incentive compatibility constraints are captured in the law of motion, which says that either: (i) y is locally strictly increasing, and in that case y and t are related according to (12); or (ii) y and t are both locally constant. That local incentive compatibility implies global incentive compatibility follows from standard arguments.

Necessary conditions that the optimal contract must satisfy can be obtained using standard methods of control theory and some salient features of the optimal contract under perfect commitment can be inferred from these. Appendix B contains the detailed analysis and shows that the optimal contract under perfect commitment involves:

1. Non-alignment of objectives. The principal (almost) never fully aligns the agent's objectives with her own. Instead, in states in which the principal compensates the agent, he does so in such a way that the chosen project lies between her most preferred project and that of the agent. (Lemmas 2 and 4 in Appendix B.)

- 2. "Caps" on project choice. The principal places a "cap" on the highest project that the agent can induce and does not compensate the agent if he does so. As a consequence, for high states, projects are unresponsive to the state. Put differently, the optimal contract always involves pooling in high states. (Lemma 5 in Appendix B.)
- 3. No strategic "overshooting". The principal can set incentives in such a way that in some states the agent induces a project that is higher than his most preferred action. While this is clearly undesirable for the principal in a local sense, such "strategic overshooting" could, in theory, reduce the principal's costs of obtaining more preferred projects in other states. While this kind of contract is feasible, it is never optimal.⁷ (Lemma 2.)

Recall that under imperfect commitment, it was optimal for the principal to set the compensation scheme such that, in low states, the interests of the agent were fully aligned with the principal. The key advantage of perfect commitment is that the principal now finds it in her interest *never* to align the interests of the agent with her own in any state. Put differently, the power of commitment saves the principal the expense of full alignment.

"Capped" Delegation and Optimal Contracts While our characterization concerns the direct contract, an outcome equivalent indirect contract is as follows: The principal delegates the decision about project choice to the agent, but restricts the agent to select from a menu of projects which is "capped"—the highest project available to the agent, \bar{y} (say) is less than $y^*(1)$. Further, the principal specifies a compensation schedule as a function of the project chosen by the agent. This schedule entails higher levels of compensation for low projects and no compensation when \bar{y} is selected. Since the principal is committed not to intervene, the agent simply chooses her most preferred project (taking into account the compensation scheme and the cap) for the given state.

Notice that the optimal delegation scheme does not entail a full alignment of objectives; nor does it entail giving the agent complete freedom in project choice.

Uniform-Quadratic Case We conclude this section with an explicit characterization of the optimal contract for the uniform-quadratic case under perfect commitment. The qualitative features of the contract when the bias is low differ somewhat from those when the bias is high.

When the bias is low, that is, if $b \leq \frac{1}{3}$, the optimal contract has three separate pieces (see Figure 2). In low states, that is when $\theta \leq b$, the project $y(\theta) = \frac{3}{2}\theta + \frac{1}{2}b$ lies between that optimal for the principal $(y^*(\theta) = \theta)$ and that optimal for the agent $(y^*(\theta, b) = \theta + b)$. As θ increases, the project chosen tilts increasingly in favor of the

⁷In contrast, strategic overshooting is a feature of contracts in the class studied by Baron (2000).

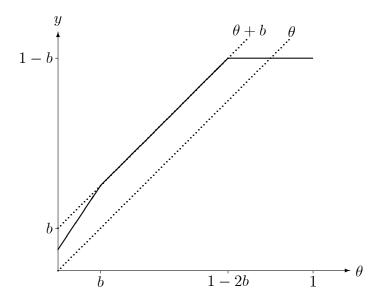


Figure 2: Optimal Contract with Perfect Commitment, $b \leq \frac{1}{3}$

agent, with a commensurate decrease in the transfer payments. For states between b and 1-2b, the project that is best for the agent $(y^*(\theta,b)=\theta+b)$ is played and no transfers are made. It is as if the project choice were delegated to the agent. The set of feasible projects is "capped" at $\bar{y}=1-b$. For states above 1-2b, the project is unresponsive to the state—that is, the agent always chooses project \bar{y} and there is, effectively, pooling over this interval.

When the bias is high, that is, $\frac{1}{3} < b \le 1$, the optimal contract consists of only two pieces (see Figure 3). In low states, the project again lies between the project ideal for the principal and that ideal for the agent. As in the case when the bias is low, the choice tilts in favor of the agent as the state increases with a corresponding decrease in the transfer payments. The set of feasible projects is again capped, but at a lower level. Indeed, as the agent becomes more biased, the cap decreases; that is, the agent becomes more constrained in her choice of projects. For high states, the agent always chooses the highest feasible project and there is, effectively, pooling over this interval. Unlike the case of low bias, there is no region in which the principal effectively delegates authority to the agent.

For very high biases, that is when b > 1, contracting is of no use—the optimal contract is no contract at all.

5 The Value of Contracting

It is argued (e.g., Coase, 1937, Williamson, 1975) that creating formal contracting arrangements between principals and agents is inherently costly. Further, as contracts become more complex, as in the case of perfect commitment, these contracting costs

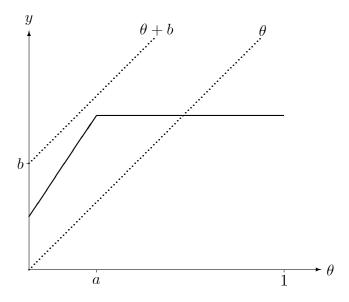


Figure 3: Optimal Contract with Perfect Commitment, $b > \frac{1}{3}$

might increase. In this section, we compare the value of contracting under full and imperfect commitment with two, arguably less costly regimes: no contracting and full delegation. When contracting is costly, when is it worthwhile to write contracts? How does the value of contracting differ depending on the degree of commitment? When is full delegation worthwhile? In the uniform-quadratic case explicit answers to these questions may be obtained.

Gains from Contracting Figure 4 depicts the expected payoffs from three alternative arrangements: the optimal contract under imperfect commitment, the optimal contract under perfect commitment, and finally, no contract at all.⁸ The key thing to notice about the figure is that the gains from contracting—with or without perfect commitment—are nonmonotonic in the degree of bias. Clearly, when the preferences of the agent and the principal are closely aligned the latter's payoff is close to her first-best level. In this case, the potential upside from contracting is quite limited. As the bias increases, the informational losses to the principal become more severe and there is more scope for contracting to "fix" the incentive problem. For cases of severe bias, $b \ge \frac{1}{4}$, absent contracts, the agent can credibly reveal no information. Resorting to contracts improves the situation, but the cost of aligning the agent's preferences increases until, at $b \ge \frac{1}{2}$, it becomes prohibitively costly for the principal. Thus, when the agent's preferences are extreme, the gains from contracting are also limited. This suggests that if there were some costs associated with "formalizing" the exchange of information between principals and agents by writing contracts, one

⁸To compute the payoffs under no contracting, we select the equilibrium maximizing the decision maker's payoffs in the CS game.

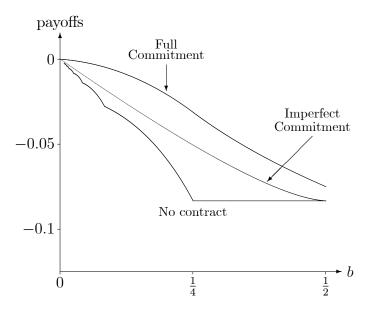


Figure 4: Value of Contracting

would expect to see contracts in cases of intermediate bias, but not when incentives are relatively closely aligned nor when the agent being consulted is an extremist.

Contracting versus Full Delegation Strategic management texts often suggest that, for businesses faced with decentralized information, delegation (or a flat organizational structure) is the appropriate response. For example, Saloner, Shepard, and Podolny (2001, pp. 79-80) write: "One basic principle of organization design is to assign authority to those who have information."

In the context of our model, the validity of the delegation principle may be examined by comparing full delegation—the unconditional assignment of authority to the person with information—to the optimal contracting arrangement with imperfect commitment. By "full" delegation we mean that the principal commits not to exercise any discretionary authority and so no longer has the freedom to intervene ex post. Specifically, there are no "caps" on what project the agent may choose. In that case, the agent will, of course, choose his favorite project $y^*(\theta, b) = \theta + b$ in each state, and the payoff of the principal is simply $-b^2$.

Figure 5 compares the principal's expected payoffs from the optimal contract under imperfect commitment with those from full delegation. As the figure shows, contracting under imperfect commitment is superior to full delegation only when the bias of the agent is high, b > 0.244. Recall that the optimal contract lies between the principal's favorite project and that of the agent. This arises because it is more cost-effective for the principal to economize on transfers by compromising on projects.

⁹In an important paper, Dessein (2002) has shown, again for the uniform-quadratic case, that delegation is superior to no contracting when the bias of the agent is not too extreme.

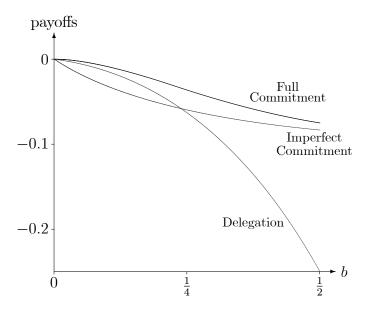


Figure 5: Contracts vs. Delegation

Full delegation is an extreme version of this idea—the principal pays no transfers but instead of a compromise, in effect concedes to the agent, giving him the freedom to choose his preferred project. When the preferences of the two parties are relatively closely aligned, the complete transfer of authority is more cost-effective for the principal than aligning incentives via transfers and retaining authority. As the bias increases, the transfer of authority becomes increasingly costly for the principal and transfers start to become more cost-effective.

The same figure also depicts the value derived from the optimal perfect commitment contract of the previous section. As is obvious, the perfect commitment contract dominates full delegation and the gains from the optimal contract increase as the bias increases. Notice that full delegation already implies that the firm is able to commit not to intervene in the project choice of the agent. As the figure shows, when the principal has such commitment power, she is better advised to impose caps on the set of feasible projects and to partially align incentives. Indeed, as the preference divergence between herself and the agent grows large, the upside from a creating a more nuanced delegation relationship (rather than the blunt instrument of full delegation) becomes considerable.

Our results suggest that appropriate organizational design needs to account for the degree of preference misalignment between a subunit with relevant information and the overall objectives of the business. But if the power to perfectly commit is available, it is *never* optimal to simply "assign authority to those who have information."

6 Conclusions

We have studied optimal contracting in environments where an agent possesses information that is important to project choice by the principal and where the objectives of principal and agent do not coincide. The usual advice in this situation is for the principal to delegate decision making authority to the agent while using contract to align the agent's interests. A key difficulty, however, is that the principal may be unable to commit not to intervene in the project choice of the agent—that is, the principal may be unable to perfectly commit to the scheme. Our main findings concern the structure of optimal contracts when commitment is imperfect as well as when perfect commitment is possible.

First, we show that, while it is always feasible for the principal to design a contract aligning the agent's incentives and transferring (however imperfectly) authority to choose the project, such contracts are *never* optimal for the principal. The key trade-off here is that by *not* incentivizing the agent to fully reveal, the principal is able to gain a direct benefit by receiving more precise information from the agent in circumstances where no payments are made as well as an indirect benefit—more precise information from agents decreases the cost of creating incentives to fully reveal. Indeed, in the uniform-quadratic case, full alignment contracts are always worse for the principal than offering no contract at all.

Under imperfect commitment, the best strategy for the principal is to use compensation contracts to align incentives only a fraction of the time and to pay no compensation otherwise. When no compensation is paid, one can think of the optimal arrangement as one where the principal informally solicits advice from the agent (in the form of cheap talk) and then makes the project choice herself. Interestingly, the proportion of the time the principal chooses to align interests is not directly proportional to the bias of the agent. In the case of perfect commitment, delegation plays a larger role, but it never pays for the principal to use contracts to fully align incentives. Instead, the optimal contract partially aligns incentives and delegates the authority for project choice to the agent, but with "caps" on the set of projects the agent can choose. In short, the optimal contract entails a more nuanced approach than the simple advice of using contracts to align interests and then delegating.

Finally, we study the gains from contracting relative to the case where no contracts are possible and relative to the case where the principal fully delegates but does nothing to align interests. Compared to no contracting, the principal gains most from contracting when the bias of the agent is moderate. Contracting is less valuable when incentives are either closely aligned or far apart. Compared to full delegation, contracting under imperfect commitment is more valuable when incentives are poorly aligned. Compared to perfect commitment, full delegation is, of course, never optimal. Again, the gains from contracting tend to be higher when interests are less well aligned.

In studying optimal contracts, we have focused on the problem of information

sharing within the firm, abstracting away from the role of contracts in providing the right incentives for information acquisition on the part of the agent. Of course, both problems are important in organizational design. In some instances, the two problems—information gathering and information acquisition—can be effectively decomposed and our analysis is directly relevant. In other cases the problems cannot be considered separately. It remains for future research to study how our conclusions about the nature of optimal contracts change in cases where effort incentives are also important.

7 Appendix A

This appendix contains proofs of some results pertaining to the structure of optimal contracts under imperfect commitment in the uniform-quadratic case, specifically, Proposition 5. The following lemma is a first step.

Lemma 1 Suppose that a contract calls for revelation on [0, a] and pooling with no payment thereafter. Such a contract is feasible if and only if the no-contract equilibrium that subdivides [a, 1] into the maximum number of pooling intervals is played.

Proof. First, suppose that with no contracts, a size K partition of [a, 1] is possible, then the "break-points" of the partition are

$$a_{j} = \frac{j}{K} + \frac{K - j}{K}a - 2bj(K - j)$$

for j = 1, 2, ..., K.

For a size K partition to be feasible $(a_1 > a)$ and a size K + 1 partition to be infeasible $(a_1 \le a)$ together requires that:

$$\frac{1-a}{2K(K+1)} \le b < \frac{1-a}{2K(K-1)} \tag{13}$$

In state a, incentive compatibility implies that the agent is indifferent between the project a and the project $\frac{1}{2}(a + a_1)$,

$$-b^{2} + t_{0} = -\left(\frac{a+a_{1}}{2} - (a+b)\right)^{2}$$

where t_0 is the transfer associated with a report $\theta = a$. Substituting for a_1 yields

$$t_0 = \frac{1}{4} \frac{(1 - a - 2K(K - 1)b)(2bK(K + 1) - (1 - a))}{K^2}$$

The condition that $t_0 \ge 0$ in any feasible contract is the same as (13), the condition that there be at most K partition elements in the interval [a, 1].

Proof of Proposition 5. Proposition 4 implies that an optimal contract must have separation over some interval $[0, z_0]$ (possibly degenerate) and then a number of pooling intervals (say n^*). Suppose that the total expected transfer in this contract is B^* . Since the contract is optimal it must also maximize the principal's expected payoffs among all contracts in which the expected expenditure is B^* , which one may think of as the "budget" of the principal. We will argue that every solution to a budget constrained problem—and the optimal contract must be a solution to such a problem—has the "no payment for pooling" property.

Choose $n \ge \max(n^*, N(b))$ where N(b) is the maximum number of partition elements of [0,1] with no transfers. Further, let the budget B be arbitrary. Given a budget B, we want to construct the equilibrium maximizing the principal's expected utility among those that consist of revealing over the interval $[0, z_0]$ followed by at most n intervals of pooling in a way that the expected transfers add up to exactly B. Let the revealing interval be $[0, z_0]$ and let the cut points be denoted by $z_1, z_2, ..., z_{n-1}$ with payments t_i over the interval $[z_{i-1}, z_i]$. Payments for any θ in the revealing interval $[0, z_0]$ are $t_0 + 2b(z_0 - \theta)$. For notational convenience, we adopt the convention that $z_n = 1$.

For i = 1, 2, ..., n-1, incentive compatibility on the part of the agent implies that, in state z_i ,

$$-\left(\frac{z_i+z_{i-1}}{2}-(z_i+b)\right)^2+t_i=-\left(\frac{z_i+z_{i+1}}{2}-(z_i+b)\right)^2+t_{i+1}$$

and solving this recursively, we obtain

$$t_{i} = \frac{1}{4} (z_{i} - z_{i-1})^{2} - (z_{i} + z_{i-1}) b - \frac{1}{4} (1 - z_{n-1})^{2} + (1 + z_{n-1}) b + t_{n}$$
 (14)

Incentive compatibility also implies that, in state z_0 ,

$$-b^{2} + t_{0} = -\left(\frac{z_{0} + z_{1}}{2} - (z_{0} + b)\right)^{2} + t_{1}$$

and, using the solution for t_1 obtained in (14) we get

$$t_0 = -2z_0b - \frac{1}{4}(1 - z_{n-1})^2 + (1 + z_{n-1})b + t_n$$
(15)

Given a budget B, the optimal contract under imperfect commitment is the solution to the following:

Problem 1 Choose $z_0, z_1, ..., z_{n-1}$ and t_n to maximize

$$EU = -\frac{1}{12} \sum_{i=1}^{n} (z_i - z_{i-1})^3$$

subject to the constraints that (i) the total expected transfers

$$z_0 (bz_0 + t_0) + \sum_{i=1}^n t_i (z_i - z_{i-1}) \le B$$

and (ii) for i = 0, 1, ..., n - 1,

$$t_i \geq 0$$

where t_i are given by (14) and (15).

The Lagrangian associated with Problem 1 is

$$L = U + \lambda \left(B - z_0 \left(b z_0 + t_0 \right) - \sum_{i=1}^{n} t_i \left(z_i - z_{i-1} \right) \right) + \sum_{i=0}^{n-1} \mu_i t_i$$

where λ and μ_i are multipliers. The first-order necessary conditions require that the following expressions equal zero:

$$\frac{\partial L}{\partial z_0} = \frac{1+3\lambda}{4} (z_1 - z_0)^2 - 2\mu_0 b - \frac{1}{2}\mu_1 (z_1 - z_0 + 2b)$$
(16)

for i = 1, 2, ..., n - 2

$$\frac{\partial L}{\partial z_i} = \frac{1+3\lambda}{4} \left((z_{i+1} - z_i)^2 - (z_i - z_{i-1})^2 \right) + \frac{1}{2} \mu_i \left(z_i - z_{i-1} - 2b \right) - \frac{1}{2} \mu_{i+1} \left(z_{i+1} - z_i + 2b \right)$$
(17)

$$\frac{\partial L}{\partial z_{n-1}} = \frac{1+3\lambda}{4} \left((1-z_{n-1})^2 - (z_{n-1}-z_{n-2})^2 \right) - \frac{1}{2}\lambda \left(1-z_{n-1}+2b \right)
+ \frac{1}{2} \left(1-z_{n-1}+2b \right) \left(\sum_{i=0}^{n-2} \mu_i \right) + \frac{1}{2}\mu_{n-1} \left(1-z_{n-2} \right)$$
(18)

$$\frac{\partial L}{\partial t_n} = -\lambda \left(z_0 \frac{\partial t_0}{\partial t_n} + \sum_{i=1}^n \frac{\partial t_i}{\partial t_n} \left(z_i - z_{i-1} \right) \right) + \sum_{i=0}^{n-1} \mu_i \frac{\partial t_i}{\partial t_n}$$

$$= -\lambda + \sum_{i=0}^{n-1} \mu_i \tag{19}$$

Notice that the expected cost of full alignment is b. Thus, when the budget is large enough, that is, $B \geq b$, then full alignment is feasible and clearly solves the budget constrained problem.

For any B < b, we will show that a solution to the budget constrained problem is characterized as follows:

First, for any point $\theta = a$ define K to be the integer satisfying

$$\frac{1-a}{2K\left(K+1\right)} \le b < \frac{1-a}{2K\left(K-1\right)}$$

We know from CS that there is a partition equilibrium of [a, 1] into K intervals with cut points

$$a_{j} = \frac{j}{K} + \frac{K - j}{K}a - 2bj(K - j)$$

for j = 0, 1, 2, ..., K and no transfers. Clearly since $a \le 1$, it follows immediately that $K \le N(b)$ and from Lemma 1, $t_0 \ge 0$..

Second, let a be the solution to:

$$a\left(ba - \left(\frac{a+a_1}{2} - (a+b)\right)^2 + b^2\right) = B$$

that is, a is such that the entire budget is exhausted in getting the agent to reveal all states $\theta \in [0, a]$.

Case 1: n = K. It is useful to begin with the case in which n = K. The solution in this case is: for n = 0, 1, 2, ..., n - 1,

$$z_j = a_j \tag{20}$$

where $a_0 \equiv a$. In addition,

$$t_n = 0 (21)$$

We also need to specify the values for the various multipliers. These are:

$$\lambda = -\frac{\frac{4}{3}K^2(K^2 - 1)b^2 + (1 - a)^2}{(2K(K + 1)b - 1)(2K(K - 1)b - 1) - 4a + 3a^2}$$
(22)

which is positive.

$$\mu_0 = 0 \text{ and } \mu_1 = \frac{1+3\lambda}{2} \frac{r_1^2}{f(0)}$$
 (23)

and for i = 2, ..., n - 1

$$\mu_{i} = \frac{(1+3\lambda)}{g(i-2)g(i-1)} \left(4b \sum_{j=0}^{i-2} g(j)^{2} + \frac{1}{2}r_{1}^{2}g(-1)\right)$$
(24)

where $r_1 = \frac{1-a}{K} - 2b(K-1)$ and $g(j) = r_1 + 4jb + 2b$.

It may be verified that the values for z_i , t_n together with the multipliers λ and μ_i solve the necessary first-order conditions for Problem 1.

Case 2: n > K. When n > K, a solution to the first-order conditions can be obtained by setting $z_0 = z_1 = ... = z_{n-K} = a$ and for i = 1, 2, ..., K, $z_{n-K+i} = a_i$. The indices of the remaining variables are also displaced by n - K.

This completes the argument that the solution specified in (20) to (24) satisfies the necessary first-order conditions (16) to (19) associated with Problem 1. We now show that in fact this is an optimal solution. We do this by showing that it satisfies both the necessary and sufficient conditions for an equivalent problem.

Consider the following alternative specification of the budget constrained problem in which the choice variables are the lengths of the intervals $r_i = z_i - z_{i-1}$ rather than their end points z_i .

Problem 2 Choose $z_0, r_1, ..., r_n$ and t_n to maximize

$$EU = -\frac{1}{12} \sum_{i=1}^{n} r_i^3$$

subject to the constraints that: (i) the total expected transfers

$$z_0 (bz_0 + t_0) + \sum_{i=1}^n t_i r_i \le B$$

(ii) for
$$i = 0, 1, ..., n - 1$$
,

$$t_i > 0$$

and (iii)

$$z_0 + \sum_{i=1}^{n} r_i = 1$$

where t_i are given by (14) and (15).

Problem 2 is the same as Problem 1 except for a change of variables. Since they share all local extrema, for every solution to the first-order conditions for Problem 1 there exists a corresponding solution to the first-order conditions for Problem 2. But in Problem 2, the objective function is concave in the choice variables and the constraints are all convex functions, the first-order conditions for Problem 2 are also sufficient. Thus any solution to the first-order conditions for Problem 1 constitutes a global optimum.

We have thus shown that the optimal solution to the budget constrained problem entails that except for t_0 , all other $t_i = 0$. In other words, in the optimal contract, the principal never pays for pooling. This completes the proof of Proposition 5.

8 Appendix B

This appendix derives properties of the optimal contract under perfect commitment. The optimal contract is the solution to the following control problem

$$\max \int_{0}^{1} (U(y,\theta) - t) f(\theta) d\theta$$

subject to the law of motion

$$t' = -U_1(y, \theta, b) u \tag{25}$$

and the constraints

$$y' = u$$
$$t > 0$$

where y and t are the state variables and u is the control variable.

If we write the generalized Hamiltonian

$$L = (U(y, \theta) - t) f(\theta) - \lambda_1 U_1(y, \theta, b) u + \lambda_2 u + \mu t$$

the resulting Pontryagin conditions are: there exist non-negative costate variables λ_1, λ_2 and a nonnegative multiplier μ that satisfy:

$$\lambda_{1}' = -\frac{\partial L}{\partial t} = f(\theta) - \mu \tag{26}$$

$$\lambda_{2}' = -\frac{\partial L}{\partial y} = -U_{1}(y,\theta) f(\theta) + \lambda_{1} U_{11}(y,\theta,b) u$$
(27)

$$0 = \frac{\partial L}{\partial u} = -\lambda_1 U_1(y, \theta, b) + \lambda_2 \tag{28}$$

$$0 = \mu t \tag{29}$$

and the transversality conditions are:

$$\lambda_1(1) = 0 \text{ and } \lambda_2(1) = 0 \tag{30}$$

Lemma 2 For all $\theta \in (0,1)$, $y(\theta) \leq y^*(\theta,b)$.

Proof. Suppose that the contrary is true, that is, there exists a θ such that $y(\theta) > y^*(\theta, b)$. Recall that in any optimal contract

$$-\lambda_1 U_1(y, \theta, b) + \lambda_2 = 0$$

and since $\lambda_1(\theta) \geq 0$ and $\lambda_2(\theta) \geq 0$. If $\lambda_1(\theta) > 0$, then the contradiction is immediate since $U_1(y, \theta, b) < 0$. Suppose that $\lambda_1(\theta) = 0$ then $\lambda_2'(\theta) = -U_1(y, \theta) f(\theta) > 0$ and hence $\lambda_2(\theta) > 0$ and again there is a contradiction.

An immediate implication of the previous lemma is that the transfers are nonincreasing in the state.

Lemma 3 $t(\cdot)$ is nonincreasing.

Proof. The law of motion (25), is

$$t' = -U_1(y, \theta, b) u$$

and from the fact that any incentive compatible $y(\cdot)$ is nondecreasing, we know that $u = y' \ge 0$. Now Lemma 2 implies that $U_1(y, \theta, b) \ge 0$ and so $t' \le 0$.

Lemma 4 If $t(\theta) > 0$, then $y^*(\theta) < y(\theta)$.

Proof. If $t(\theta) > 0$, then from Lemma 3, for all $\sigma < \theta$, $t(\sigma) > 0$. This means that $\mu(\sigma) = 0$ for all $\sigma \in [0, \theta]$. Now (26) implies that

$$\lambda_1(\theta) = F(\theta) + \lambda_1(0)$$

where F is the cumulative distribution function associated with f and from (28)

$$\lambda_2(\theta) = (F(\theta) + \lambda_1(0)) U_1(y, \theta, b)$$

and differentiating this results in

$$\lambda_{2}'(\theta) = f(\theta) U_{1}(y, \theta, b) + (F(\theta) + \lambda_{1}(0)) (U_{11}(y, \theta, b) u + U_{12}(y, \theta, b))$$

Equating this with the expression in (27), we get

$$U_{1}(y, \theta, b) + U_{1}(y, \theta) = -\frac{F(\theta) + \lambda_{1}(0)}{f(\theta)}U_{12}(y, \theta, b) < 0$$

since $U_{12} > 0$. But since $y \leq y^*(\theta, b)$ this implies that $y > y^*(\theta)$.

Finally, the optimal contract must involve some pooling in high states. Thus, even though the principal has the option of full revelation, this is too expensive and never optimal.

Lemma 5 There exists a z < 1, such that y is constant over [z, 1].

Proof. We claim that there exists a z < 1, such that t(z) = 0. If $t(\theta) > 0$ for all $\theta \in (0,1)$, then we have that for all $\theta \in (0,1)$, $\mu(\theta) = 0$. Now (26) together with the transversality condition implies that $\lambda_1(\theta) = F(\theta) - 1$, which is impossible since $\lambda_1(\theta) \geq 0$.

The uniform-quadratic case. In the uniform-quadratic case, the Pontryagin conditions (26) to (29) are also sufficient since the relevant convexity conditions are satisfied (see for instance, Seierstad and Sydsæter, 1987). Some qualitative features of the solution differ depending on whether the bias b is less than or exceeds $\frac{1}{3}$. These are depicted in Figures 2 and 3, respectively.¹⁰

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¹⁰The exact solutions in the uniform-quadratic case may be obtained from the authors.

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