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Enumeration by pattern recognition requires attention: Evidence against immediate holistic processing of canonical patterns

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Abstract

Enumeration of canonical patterns (e.g., faces of six-sided dice) has generally been characterized by researchers as a holistic process, in which all items are perceived collectively. In previous work, based on a holistic processing view of enumeration by pattern recognition, we predicted that enumeration of canonical forms would not be significantly affected by attentional load. In this paper, we present the results from two experiments designed to test this prediction using a divided-attention paradigm. In contrast to our predictions, enumeration of canonical patterns was disrupted by attentional load. Furthermore, enumeration of patterns under high attentional load showed evidence of conflation between patterns with similar contours, providing evidence against a holistic processing account of canonical pattern recognition.

Keywords: numerosity judgment; subitizing; enumeration; attention; pattern recognition; canonical patterns

Introduction

The processes underlying *subitizing*, the rapid and accurate enumeration of small numbers (≤ 4) of objects (Kaufman, Lord, Reese, & Volkmann, 1949), have long been a subject of contention. One longstanding question involves what role pattern recognition plays in subitizing. Ample evidence exists demonstrating that regular, common (often called *canonical*) arrangements of objects are enumerated more rapidly and accurately compared to arbitrary arrangements (Mandler & Shebo, 1982; Wender & Rothkegel, 2000). For example, patterns such as those found on six-sided dice are enumerated quickly (under 200–250 ms) and with little error (Wender & Rothkegel, 2000; Jansen et al., 2014). As canonical patterns hold processing advantages over non-canonical patterns, and enumeration of previously unfamiliar patterns can be improved with repeated exposure (Wolters, Van Kempen, & Wijlhuizen, 1987), the ability to leverage a pattern recognition process in numerosity judgment is uncontroversial. However, some researchers contend that not only does pattern recognition account for enumeration performance of canonical patterns, but that it may be the process underlying subitizing in general (Peterson & Simon, 2000; Logan & Zbrodoff, 2003; Krajcsi, Szabó, & Mórocz, 2013).

Because of the minimal reaction time increase per item and stable enumeration accuracy within the subitizing range, subitizing has long been thought of as involving a rapid process in which all items are perceived collectively. In other words, this view contends that enumeration in the subitizing range is a *holistic* process. To provide an explanatory account of subitizing as a holistic process, some have pointed to pre-attentive mechanisms of the object-tracking system (OTS) (Trick, 1992), while others have pointed to the approximate number system (ANS) and the accurate estimation of small quantities as an explanation (Dehaene & Changeux, 1993). However, researchers have recently demonstrated that increased attentional load is able to disrupt subitizing performance of non-canonical patterns, arguing against a holistic processing account of general subitizing (Railo, Koivisto, Revonsuo, & Hannula, 2008; Olivers & Watson, 2008; Egeth, Leonard, & Palomares, 2008; Vetter, Butterworth, & Bahrami, 2008; Burr, Turi, & Anobile, 2010).

Enumeration of canonical patterns has generally been conceptualized as a holistic process by both experimentalists (Mandler & Shebo, 1982; Krajcsi et al., 2013) and computational modelers (Peterson & Simon, 2000; Logan & Zbrodoff, 2003). Yet, enumeration of canonical patterns under conditions of divided-attention has not currently been investigated. If enumeration of canonical patterns was shown to be robust under conditions of attentional load, it would provide evidence against pattern recognition as a general mechanism underlying subitizing. In previous work (Briggs, Bridewell, & Bello, 2017), we presented a computational model of subitizing that accounted for enumeration accuracy of non-canonical arrangements of dots in the divided-attention task used by Railo and colleagues (2008). Additionally, we made the prediction, based on the holistic processing view, that enumeration of canonical forms in the same task would not be significantly affected by attentional load.

In this paper, we present the results from two experiments designed to test this prediction. However, in contrast to the predictions made in Briggs and colleagues (2017), enumeration of canonical patterns was disrupted by attentional load. Furthermore, analysis of the distribution of responses under higher attentional load showed evidence of conflation between patterns with similar contours, providing evidence against a holistic processing account of canonical pattern recognition.

Enumeration and Attention

In order to investigate enumeration of canonical patterns in the context of attentional constraints, we adapted the dual-task paradigm presented by Railo and colleagues (2008). In their study, subjects had two potential tasks: (A) report which of the vertical or horizontal axes of a centrally located cross

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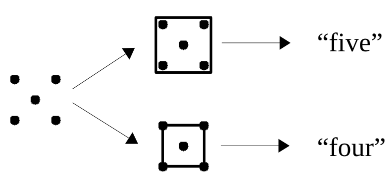
was longer and (B) report the number of dots randomly clustered in a quadrant outside the central cross (see Figure 1). Initially, subjects viewed and responded to a few trials in which only the cross appeared (task A only). Then in a critical trial, in which a peripheral dot cluster appeared for the first time (and subjects were unaware of the enumeration task), they were asked whether or not they noticed anything other than the cross. If so, subjects were then asked how many other objects they saw. After this critical trial, subjects then performed trials in two counterbalanced blocks on videos containing both a cross and a peripheral cluster of dots. In one block, subjects were asked to perform both task A and B, and in the other, subjects were asked only to enumerate (task B only). Thus, trials could be grouped into three attentional conditions vis-á-vis enumeration: (1) the *inattention* condition, consisting of the critical trial; (2) the *divided attention* condition; and (3) the *full attention* condition.

Railo and colleagues (2008) found that in the inattention condition, in the absence of awareness of the enumeration task, subjects were only able to accurately enumerate 1-2 dots. They also found that enumeration accuracy in the divided attention condition was significantly lower than in the full attention condition, and significant accuracy decreases began within the subitizing range. That subitizing appears interruptable is evidence against what we have deemed a holistic account, in which information from multiple items are integrated effectively simultaneously. However, as mentioned previously, the clusters of dots used by Railo and colleagues (2008) were explicitly non-canonical. How canonical forms would be enumerated in similar conditions would depend on the process underlying pattern recognition.

Enumerating Canonical Forms

Two main accounts of enumeration by pattern recognition have been put forth: (1) the *holistic processing* model, and (2) the *outline detection* model (Krajcsi et al., 2013). Below we illustrate the numerosity judgment predicted by each account for the canonical, dice-pattern arrangement of five dots.



In the holistic processing account (top), all the items contribute to a numerosity judgment. The outline detection model (bottom), however, does not consider dots within the contour defined by the arrangement. As such, the outline detection model would predict that people conflate dice-pattern arrangements of five and six dots with four, and dice-pattern arrangements of three dots with two. Intuitively, there is a clear sense in which the holistic processing model is right and the outline detection model is wrong. Anyone who has played a board game can attest that reading the results of his or her dice rolls does not appear to be an error-prone, slow, or otherwise effortful process. Empirical results from enumeration studies also corroborate this intuition (Mandler & Shebo, 1982; Wender & Rothkegel, 2000; Krajcsi et al., 2013; Jansen et al., 2014).

However, rejecting the outline detection model and accepting the holistic processing model as defined above does not necessarily help us predict enumeration performance of canonical patterns under conditions of attentional load. These models are purely high-level, functional accounts that make no claims about the time-course or interruptability of processing. The question remains: does enumeration of patterns involve integrating information in a rapid and uninterruptable manner (akin to a special, more accurate, case of ANS estimation), or is it a rapid, but interruptable process of integrating information (like general subitizing)? The data in this regard are less clear, which we discuss below.

Pattern Recognition Under Attentional Load

Computational implementations of the holistic processing model of enumeration by pattern recognition have generally assumed simultaneous integration of information from all objects to be enumerated (Peterson & Simon, 2000; Logan & Zbrodoff, 2003). However, the data on the time course of enumeration of canonical patterns are mixed. Mandler and Shebo (1982) report negligible reaction time differences for canonical patterns up to five, whereas other studies have shown statistically significant (if slight) reaction time increases for enumerating dice-pattern arrangements of more than four dots (Wender & Rothkegel, 2000). One study shows that when subjects are given very brief (< 30 ms) presentations of structured patterns, either arranged linearly or as vertices of regular polygons, enumeration accuracy suffers more when items are arranged linearly (Allen & McGeorge, 2008), indicating that some outline-based conflation may be occurring. Allen and McGeorge (2011) also showed that expert subjects (in their case, air traffic controllers) enumerate structured patterns more accurately than novices, but do so at a time cost. Thus, while recognition of canonical patterns may be holistic in the sense of "counting" every item, the integration of the information from all items may not be instantaneous. Enumeration by pattern recognition may be interruptable. In the following section, we present the results of an experiment designed to interrupt it, if possible.

Experiment 1: Randomized vs. Dice Patterns

The aim of Experiment 1 was two-fold. First, we sought to replicate the results from Railo and colleagues (2008) for randomized, non-canonical arrangements using subjects recruited on the Amazon Mechanical Turk platform (Paolacci, Chandler, & Ipeirotis, 2010). Because viewing conditions for subjects recruited online could not be as tightly controlled, we needed to assess the viability of conducting such experiments. The second aim of the study was to investigate the effects of attentional load on enumeration of canonical patterns.

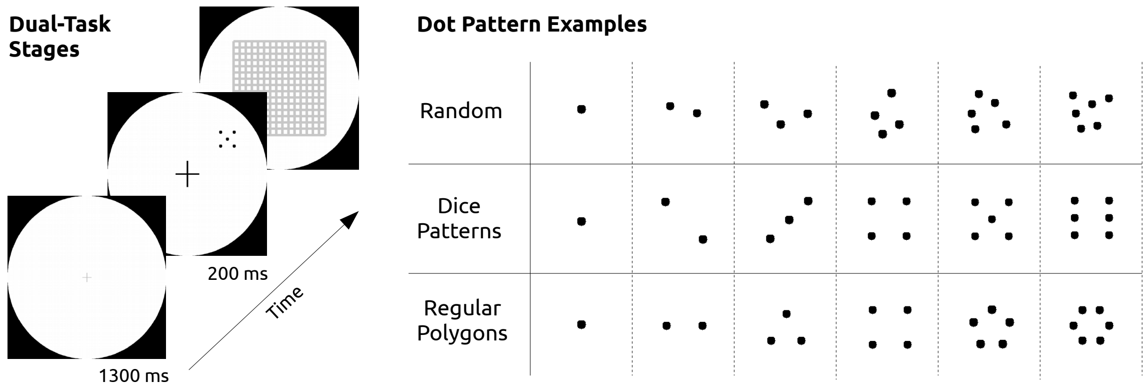


Figure 1: Time course of dual-task videos (left). Examples of dot arrangements used in each pattern category (right).

To achieve these goals, we replicated the task from Railo and colleagues (2008) with two dot arrangement conditions. One was a *randomized arrangement* condition, which served as a control and basis for replication. The second was a *dice pattern* condition, in which all peripheral clusters of dots were arranged canonically. The right side of Figure 1 illustrates examples of randomized and dice patterns for each numerosity probed. For the purposes of this study, we were concerned with the divided and full attention trials for each subject, yielding a 2x2 mixed factorial design. Comparisons of arrangement type would be between subjects, whereas comparisons between attentional conditions were within-subjects.

Under the simultaneous processing interpretation of holistic pattern recognition, we predicted that neither attentional load nor numerosity should significantly affect enumeration accuracy for subjects given dice pattern arrangements (Briggs et al., 2017). This stands in contrast with the effects found for randomized arrangements by Railo and colleagues (2008), where both enumeration accuracy generally decreased as the number of peripheral dots increased, and decreased significantly for larger numbers in divided attention trials relative to full attention trials.

Method

Participants: Seventy-two subjects volunteered through the Amazon Mechanical Turk platform. Participants were evenly distributed between the randomized arrangement and dice pattern conditions. The task was configured to be unavailable to users on mobile platforms to ensure appropriate viewing size of task videos.

Stimuli: The left side of Figure 1 illustrates the time course of the dual-task. First, a small fixation cross appears in the center of a circular viewing area for 1.3 seconds. The task stimulus, consisting of the centrally-located cross and the peripherally-located cluster of dots, then appears for 200ms. A masker then appears and remains on the screen while the subject responds to the task questions. In each video, the dimensions of the viewing area and task objects were designed to replicate the stimuli from Experiment 2 from Railo and colleagues (2008) as closely as possible, given the inability to control the viewing hardware of each subject. All pixel dimensions were calculated based on an assumption of 100 pixels per inch (ppi) using the metrical information specified by Railo and colleagues (2008). For dice patterns with vertical or horizontal asymmetry (i.e., two and three), both mirrored variants were generated.

Design and Procedure: After completing six cross-only trials, subjects then completed two blocks corresponding to each attentional condition. Each block consisted of nine trials and was preceded by an updated set of instructions. The order of these blocks was counterbalanced; half of the subjects within each arrangement condition received the full attention first, while the other half received the divided attention block first. Each peripheral cluster numerosity was presented within each block at least once (three numerosities presented once, three presented twice). The order and frequency of presentation for each numerosity were randomized. The quadrant of the viewing area that the peripheral cluster would appear in was also randomized (appearing with equal probability in each quadrant). When subjects were asked to report perceived numerosity of the peripheral dots, they were given the option to select values ranging from one to six in a drop-down menu. Subjects were also asked how confident they were in their responses (from 1 = very unsure to 5 = very sure).

Results

Randomized Patterns Analysis of the data from subjects in the randomized arrangement condition showed successful replication of the key effects found by Railo and colleagues (2008). Enumeration accuracy in each condition is graphed in Figure 2. Friedman tests of enumeration accuracy showed significant effects of numerosity in both the divided attention

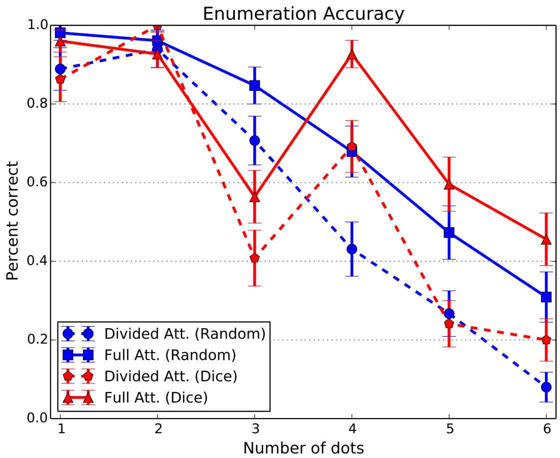


Figure 2: Accuracy of numerosity judgment in divided and full attention conditions for randomized arrangements (blue) and dice pattern dot arrangements (red). Error bars indicate standard error.

trials ($\chi^2 = 88.66$, $df = 5$, $p < .001$), and the full attention trials ($\chi^2 = 81.51$, $df = 5$, $p < .001$).

Wilcoxon tests between adjacent numerosities indicated significant differences in enumeration accuracy during the divided attention trials between three and four ($p = .001$) and five and six ($p = .008$).[†] Within the full attention conditions, enumeration accuracy also decreased as numerosities increased, though there was only a marginally significant difference between four and five ($p = .030$). Two-tailed Z-proportion tests were used to compare enumeration accuracies for numerosities between attentional or arrangement conditions. Enumeration accuracy in the divided attention trials were significantly lower than in full attention trials for numerosities four ($Z = -2.54$, $p = .011$), five ($Z = -2.24$, $p = .025$), and six ($Z = -2.85$, $p = .004$).

As with the original study (Railo et al., 2008), performance on the cross task in the divided attention trials was above chance (69% correct) and was not correlated with enumeration accuracy (Spearman's rho, $\rho = -.06$, $p = .739$), indicating that subjects were dividing attention between the cross and enumeration tasks.

Dice Patterns Contrary to our original predictions, we found that enumeration accuracy of dice patterns was affected by both numerosity and attentional condition. Friedman tests showed significant effects of numerosity in both the divided attention trials ($\chi^2 = 86.38$, $df = 5$, $p < .001$) and full attention trials ($\chi^2 = 58.24$, $df = 5$, $p < .001$).

Wilcoxon tests between adjacent numerosities indicated significant differences in enumeration accuracy during the divided attention trials between one and two ($p = .004$), two and three ($p < .001$), three and four ($p = .009$), and four and five ($p < .001$). Within the full attention condition, significant differences were found between two and three, three and four ($p < .001$), and four and five ($p = .002$). Enumeration accuracy was also lower in the divided attention trials relative to the full attention trials for numerosities four ($Z = -3.12$, $p = .002$), five ($Z = -3.71$, $p < .001$), and six ($Z = -2.88$, $p = .004$).

Unlike in the randomized arrangement conditions, mean enumeration accuracy did not monotonically decrease. Rather, enumeration accuracy for three was lower for dice patterns relative to randomized patterns for both divided attention ($Z = 3.11$, $p = .002$) and full attention trials ($Z = 3.34$, $p < .001$). Conversely, enumeration accuracy for four was higher for dice patterns for both divided attention ($Z = -2.67$, $p = .008$) and full attention trials ($Z = -3.26$, $p = .001$). Performance on the cross task in the divided attention trials was above chance (72% correct) and was not correlated with enumeration accuracy (Spearman's rho, $\rho = -.23$, $p = .181$).

Response Distribution Not only did the data show attentional effects on enumeration accuracy for dice pattern arrangements, but the pattern of responses was consistent with subjects relying on shape/outline information to guide numerosity judgments, especially in the case of high attentional load. The fact that dice patterns of three were enumerated less accurately than random arrangements of three dots is suggestive of this, as randomly arranged patterns of three dots are much more likely to form triangles rather than a linear arrangement. Likewise, the square shape of the dice arrangement of four dots may generate a stronger shape/outline cue than a random arrangement of four dots. To illustrate this pattern of response further, we present a chart of the most common response for each numerosity in each arrangement and attentional condition below (response frequency in parentheses).

Actual Number:	**1**	**2**	**3**	**4**	**5**	**6**
Mode Response Random-DivAtt.	1 (.89)	2 (.94)	3 (.71)	3 (.47)	3 (.36)	4 (.46)
Mode Response Random-FullAtt.	1 (.98)	2 (.96)	3 (.85)	4 (.68)	5 (.47)	5 (.45)
Mode Response Dice-DivAtt.	1 (.86)	2 (1.0)	2 (.53)	4 (.69)	4 (.33)	4 (.53)
Mode Response Dice-FullAtt.	1 (.96)	2 (.93)	3 (.56)	4 (.93)	5 (.60)	6 (.46)

In the divided attention trials for dice patterns, not only is three confused with two, but five and six appear to be frequently confused for four. This is consistent with predictions made by the outline detection model of enumeration by pattern recognition. One possible interpretation of these results is that subjects are first focusing on shape/outline cues, then attempting to disambiguate similar patterns by subsequently focusing on uniquely identifying sub-regions. However, with the limited amount of time to enumerate (especially in the divided attention trials), this second step is not achieved. These results are consistent with the findings from Allen and McGeorge (2008), who found evidence for conflation of linearly arranged patterns. It is worth noting that Allen and McGeorge (2008) used patterns located in the center of a subject's field

[†]Acceptance level set at $p = .025$.

of view, which would argue against this effect being simply an artifact of eccentricity.

Experiment 2: Regular Polygon Arrangements

The aim of Experiment 2 was to further investigate the relationship between outline detection and enumeration of canonical patterns. We presented subjects with patterns with no internal or linearly arranged dots. If people are relying on shape/outline cues to provide information about numerosity, then conflation between patterns should not be as frequent if the elements of the pattern all resided on the pattern's contour. For example, enumeration accuracy for patterns of three dots in this experiment should not suffer in the same manner as three dots arranged in the traditional dice pattern.

Method

Participants: Thirty-four subjects volunteered through the Amazon Mechanical Turk platform. As in Experiment 1, the task was unavailable to users on mobile platforms.

Stimuli, Design, and Procedure: The stimuli and procedure were the same as in Experiment 1 with the exception of the arrangement of the peripheral dot clusters. Instead of randomized or dice pattern arrangements, dots were arranged to appear as the vertices of regular polygons (see Figure 1).

Results

Like the previous experiment, performance on the cross task in the divided attention trials was above chance (66% correct) and was not correlated with enumeration accuracy (Spearman's rho, $\rho = -.05$, $p = .784$). Mean enumeration accuracy for polygon arrangements is found in Figure 3. Friedman tests indicate significant effects of numerosity in both the divided attention ($\chi^2 = 44.91$, $df = 5$, $p < .001$) and full attention trials ($\chi^2 = 59.53$, $df = 5$, $p < .001$). Wilcoxon tests between numerosities indicate only a marginal decrease in accuracy between four and five in the divided attention trials ($p = .046$), and a significant decrease in accuracy between four and five in the full attention trials ($p < .001$). Enumeration accuracy in the divided attention trials was only significantly lower than in the full attention trials for numerosities three ($Z = -2.34$, $p = .020$), four ($Z = -3.78$, $p < .001$), and six ($Z = -2.11$, $p = .035$).

Below we present the most frequent responses for each numerosity.

Actual Number:	1	2	3	4	5	6
Mode Response Polygon-DivAtt.	1 (.91)	2 (.81)	3 (.69)	4 (.59)	5 (.40)	5 (.33)
Mode Response Polygon-FullAtt.	1 (.98)	2 (.91)	3 (.88)	4 (.91)	5 (.52)	6 (.48)

Unlike with the dice patterns in the first experiment, there does not appear to be any systematic confusion between patterns of different numerosities. The enumeration accuracy of three dots in the polygon arrangement condition does not suffer like the dice pattern condition, and is in fact more consistent with the enumeration accuracies of randomized arrangements. Enumeration accuracy for numerosities one to four

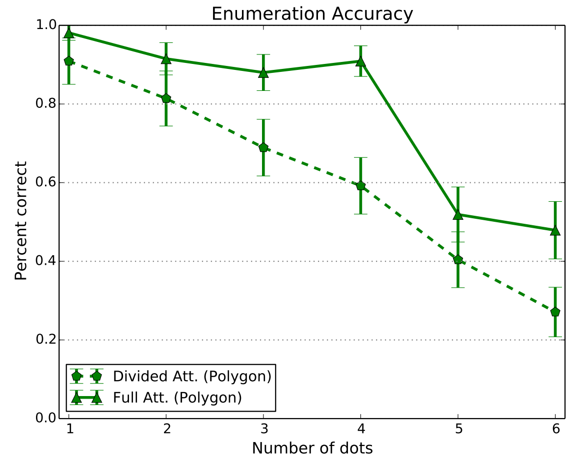


Figure 3: Accuracy of numerosity judgment in divided and full attention conditions for regular polygon arrangements. Error bars indicate standard error.

in the full attention trials exhibits the behavior we originally predicted for pattern recognition (i.e., no significant performance decrease over different canonical forms). Overall, this is consistent with our predictions based on initial focus on shape/outline cues. Interestingly, though, there still appears to be a significant processing limit of four items in the full attention case. Whether this is due to relative unfamiliarity of pentagon and hexagon shapes in an enumeration context (e.g., Resnick, Verdine, Golinkoff, & Hirsh-Pasek, 2016), or some deeper capacity-limitation, requires further investigation (e.g. Trick, 1992).

General Discussion

The results from our two experiments demonstrate that enumeration of canonical patterns requires attention. More specifically, enumeration of canonical patterns requires sufficient time to integrate information from all items within a patterned arrangement. This may be indicative of the need to deploy attention sequentially: first to establish a set of possible responses consistent with the gist of the arrangement, then to focus on subregions that would disambiguate between the multiple possibilities. Regardless, these findings stand in stark contrast with the predictions made by holistic processing accounts of enumeration by pattern recognition and warrant further work in a variety of directions.

One direction would be establishing whether or not the initial gist of a pattern is solely based on the shape/outline. Despite the evidence that items along the outlines of clusters strongly affect initial senses of numerosity, we cannot completely eliminate the possibility that information from interior items is also being integrated. Some holistic models of pattern recognition are based on calculating similarity between arrangements (e.g., Logan & Zbrodoff, 2003), and patterns that share contours are likely to be rated as having high similiarity. Additionally, visual items in the interior or along the contours of patterns may be weighted less due to effects such as crowding (Valsecchi, Toscani, & Gegenfurtner, 2013), further contributing to ambiguity even in the case of holistic processing.

One intriguing implication of our findings concerns how knowledge of the numerosity of novel patterns is acquired. If enumeration by pattern recognition is not immediate and holistic, then learning novel patterns may not be simply a matter of learning associations between static arrangements of dots and numbers. Rather, learning to enumerate novel patterns is a matter of learning how to strategically deploy attention to uniquely identify particular arrangements. For instance, in the case of learning complex patterns over a series of repeated sessions (e.g., Wolters et al., 1987), subjects may be learning to attend to particularly informative subregions of patterns. Based on this view, we would expect that perturbing items in these informative subregions would affect enumeration performance more than in other subregions.

Finally, these findings complicate the larger question of pattern recognition's role in subitizing. Instead of cleanly dissociating pattern recognition from general subitizing by showing that enumeration of canonical forms is uninterruptable, we found that it was interruptable. Moreover, not only is it interruptable, but it exhibits similar behavioral patterns as models of subitizing that involve rapid serial deployment of attention (Briggs et al., 2017). This blurs the line between object-tracking system and pattern recognition based accounts of subitizing. For example, it is difficult to tease out whether people are rapidly focusing on four individual items, or whether they are noticing a triangle arrangement, plus an extra item.

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References

Allen, R., & McGeorge, P. (2008). Enumeration: Shape information and expertise. *Acta Psychologica*, *129*, 26–31.

Allen, R., & McGeorge, P. (2011). Enumeration: Experts take their time. *Applied Cognitive Psychology*, *25*, 588–592.

Briggs, G., Bridewell, W., & Bello, P. F. (2017). A computational model of the role of attention in subitizing and enumeration. In *Proceedings of the 39th Annual Meeting of the Cognitive Science Society* (pp. 1672–1677). London, UK.

Burr, D. C., Turi, M., & Anobile, G. (2010). Subitizing but not estimation of numerosity requires attentional resources. *Journal of Vision*, *10*, 1–10.

Dehaene, S., & Changeux, J.-P. (1993). Development of elementary numerical abilities: A neuronal model. *Journal of Cognitive Neuroscience*, *5*, 390–407.

Egeth, H. E., Leonard, C. J., & Palomares, M. (2008). The role of attention in subitizing: Is the magical number 1? *Visual Cognition*, *16*, 463–473.

Jansen, B. R., Hofman, A. D., Straatemeier, M., Bers, B. M., Raijmakers, M. E., & Maas, H. L. (2014). The role of pattern recognition in children's exact enumeration of small numbers. *British Journal of Developmental Psychology*, *32*, 178–194.

Kaufman, E. L., Lord, M. W., Reese, T. W., & Volkmann, J. (1949). The discrimination of visual number. *The American Journal of Psychology*, *62*, 498–525.

Krajcsi, A., Szabó, E., & Mórocz, I. Á. (2013). Subitizing is sensitive to the arrangement of objects. *Experimental Psychology*, *60*, 227–234.

Logan, G. D., & Zbrodoff, N. J. (2003). Subitizing and similarity: Toward a pattern-matching theory of enumeration. *Psychonomic Bulletin & Review*, *10*, 676–682.

Mandler, G., & Shebo, B. J. (1982). Subitizing: An analysis of its component processes. *Journal of Experimental Psychology: General*, *111*, 1–22.

Olivers, C. N., & Watson, D. G. (2008). Subitizing requires attention. *Visual Cognition*, *16*, 439–462.

Paolacci, G., Chandler, J., & Ipeirotis, P. G. (2010). Running Experiments on Amazon Mechanical Turk. *Judgment and Decision Making*, *5*, 411–419.

Peterson, S. A., & Simon, T. J. (2000). Computational evidence for the subitizing phenomenon as an emergent property of the human cognitive architecture. *Cognitive Science*, *24*, 93–122.

Railo, H., Koivisto, M., Revonsuo, A., & Hannula, M. M. (2008). The role of attention in subitizing. *Cognition*, *107*, 82–104.

Resnick, I., Verdine, B. N., Golinkoff, R., & Hirsh-Pasek, K. (2016). Geometric toys in the attic? a corpus analysis of early exposure to geometric shapes. *Early Childhood Research Quarterly*, *36*, 358–365.

Trick, L. M. (1992). A theory of enumeration that grows out of a general theory of vision: Subitizing, counting, and finsts. *Advances in Psychology*, *91*, 257–299.

Valsecchi, M., Toscani, M., & Gegenfurtner, K. R. (2013). Perceived numerosity is reduced in peripheral vision. *Journal of Vision*, *13*, 1–16.

Vetter, P., Butterworth, B., & Bahrami, B. (2008). Modulating attentional load affects numerosity estimation: Evidence against a pre-attentive subitizing mechanism. *PLoS One*, *3*, 1–6.

Wender, K. F., & Rothkegel, R. (2000). Subitizing and its subprocesses. *Psychological Research*, *64*, 81–92.

Wolters, G., Van Kempen, H., & Wijlhuizen, G.-J. (1987). Quantification of small numbers of dots: Subitizing or pattern recognition? *The American Journal of Psychology*, *100*, 225–237.