LBL-26020
Preprint c.2

Lawrence Berkeley Laboratory
UNIVERSITY OF CALIFORNIA

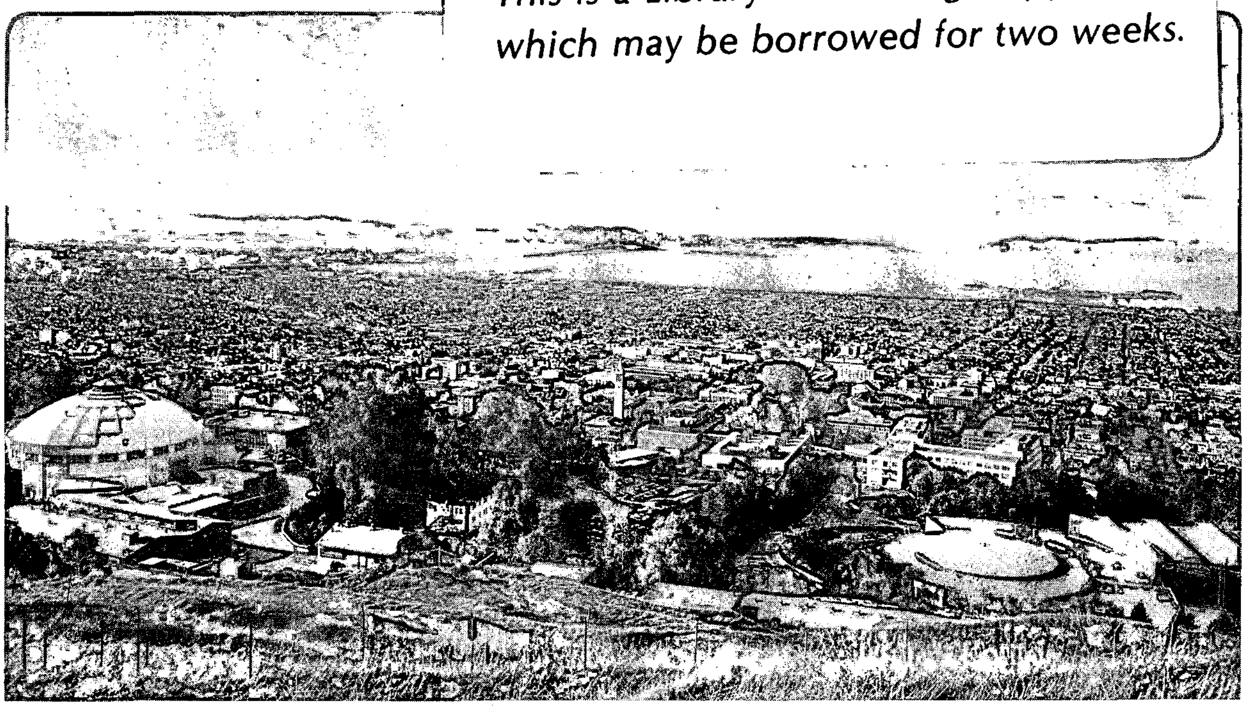
EARTH SCIENCES DIVISION

Submitted to Journal of Hydrology

Integral Method Solution for Diffusion into a Spherical Block

R.W. Zimmerman and G.S. Bodvarsson

September 1988



Prepared for the U.S. Department of Energy under Contract Number DE-AC03-76SF00098.

Integral Method Solution for Diffusion into a Spherical Block

Robert W. Zimmerman

Gudmundur S. Bodvarsson

Earth Sciences Division

Lawrence Berkeley Laboratory

University of California

Berkeley, CA 94720

Abstract

An approximate analytical solution is derived for the problem of a Newtonian fluid infiltrating into a porous spherical block. The fluid in the block is initially at a constant pressure p_i, and the pressure at the outer boundary is held at a constant value p_o. Using the simple assumption of linear pressure profiles, the instantaneous and cumulative fluxes into the sphere are predicted with surprisingly high accuracy. The solution applies to all other physical processes governed by the same equation, such as heat conduction and chemical diffusion. The solution should be very useful for incorporation into double-porosity models for fractured reservoirs and aquifers.

Submitted to the *Journal of Hydrology* (September 16, 1988)

1. Introduction

Fluid flow in fractured reservoirs has received considerable attention for the past two decades, since many petroleum and groundwater reservoirs, as well as nearly all geothermal reservoirs, reside in fractured rocks. Further interest in fractured reservoirs has arisen from research efforts in the subsurface disposal of nuclear waste. Early models of fractured reservoirs include the so-called "double porosity" model developed by Barenblatt et al. (1960) and Warren and Root (1963). In this model the high permeability fractures provide paths for the global flow, and the low permeability matrix blocks provide the fluid storage. Flow between the fractures and the matrix blocks was accounted for by assuming quasi-steady state conditions. Subsequent studies have extended the model to include transient flow between the matrix blocks and the fractures (Kazemi, 1969; Serra et al., 1983; Streltsova, 1983), and evaluated the effects of different matrix block geometries on the reservoir pressure response (deSwaan, 1976; Barker, 1985). These analytical solutions have been found valuable in the interpretation of pressure transient tests in naturally fractured reservoirs.

The double porosity concept has also been utilized in numerical simulation of oil, gas, groundwater, and geothermal reservoirs (Duguid, 1973; Pruess and Narasimhan, 1975). However, for field-wide simulations the use of this approach is limited by the large number of gridblocks, and small time steps, that are often needed to accurately simulate the flow. It is therefore of great interest to investigate methods that can analytically estimate the fluid flow between the fractures and the matrix. If flow into or out of the matrix blocks could be represented analytically, the blocks could be treated merely as source/sink terms in numerical simulations, and only the fractures would need to be explicitly discretized. This would lead to considerable reductions in computational time, thereby allowing reservoirs to be simulated more economically. In the present paper, the "integral" or "boundary-layer" approach is used to derive an extremely simple and relatively accurate expression for the flux into or out of a ''spherical'' matrix block that is surrounded by fractures at a given pressure. Although sets of planar fractures define blocks that have the shape of parallelepipeds, as long as the blocks are not overly elongated, it has been shown (deSwaan, 1976; Pruess and Wu, 1988) that flow into such blocks can be approximated by flow into a sphere of equal volume.

The integral method of deriving approximate solutions of partial differential equations that arise in engineering was introduced by Pohlhausen in 1921 (see Schlichting, 1968) to treat the problem of laminar flow over a flat plate. The method seems to have been first brought to bear on diffusion problems by Landahl (1953), and has since been used for problems involving diffusion in half-spaces, slabs, and regions outside of cylindrical holes (see references given by Goodman, 1964). The method is often referred to as the ''boundary-layer'' approach, since in problems involving infinite domains attention is focused on a ''thin'' layer adjacent to the boundary of the problem. The salient feature of this method, however, is the choice of trial solutions that satisfy appropriate boundary and continuity conditions. These trial functions depend on some parameter, which varies with time, and which measures the extent of the diffusion. The governing partial differential equation is then integrated over the entire spatial domain, yielding an ordinary differential equation describing the time evolution of that parameter. The integration of this ODE, using the appropriate initial conditions, thus completes the (approximate) solution of the problem. Although its accuracy depends on the choice of the form of the trial solution, the usefulness of the method stems from the fact that the boundary flux, which is often the figure of merit, can be determined quite accurately from extremely simple trial functions. This feature makes the method attractive for applications such as to the double-porosity model in reservoir evaluations.

2. Formulation of Problem

The governing equation for spherically symmetric flow of a Newtonian fluid in an homogeneous porous medium, neglecting gravity, is (Muskat, 1937)

$$\frac{1}{r^2}\left[\frac{\partial}{\partial r}\left[r^2\frac{\partial p}{\partial r}\right]\right] = \frac{\phi\mu c}{k}\frac{\partial p}{\partial t}\,, \tag{1}$$

where p is the fluid pressure, r is the radial coordinate, t is time, ϕ is the porosity of the medium, k is the permeability of the medium, μ is the viscosity of the fluid, and c is a compressibility term that represents the storativity due to both the formation compressibility and the fluid compressibility. The left-hand side of eq. 1 represents the fluid flux (via Darcy's law), while the right-hand side represents the time rate of change of the fluid stored in a given region. Eq. 1 thus is essentially a statement of conservation of mass.

The basic problem to be solved is that of a spherical block of radius a which is initially at equilibrium at some pressure p_i. At some time (which can be arbitrarily denoted as $t=0$), the pressure at the outer boundary of the sphere is instantaneously changed to some new value p_o, after which it is held constant. The appropriate boundary conditions for this problem are therefore

$$p(r,t=0) = p_i\,, \tag{2}$$

$$p(r=a,t>0) = p_o\,. \tag{3}$$

It is convenient, and conventional, to recast this problem in dimensionless form before attempting a solution. This can be accomplished through the following changes of variables:

$$\hat{p} = \frac{p-p_i}{p_o-p_i} , \tag{4}$$

$$\tau = \frac{kt}{\phi\mu c a^2} , \tag{5}$$

$$\rho = r/a . \tag{6}$$

In terms of these new dimensionless variables, the governing equation and initial/boundary conditions are

$$\frac{1}{\rho^2}\left[\frac{\partial}{\partial\rho}\left[\rho^2\frac{\partial\hat{p}}{\partial\rho}\right]\right] = \frac{\partial\hat{p}}{\partial\tau} , \tag{7}$$

$$\hat{p}(\rho,\tau=0) = 0 , \tag{8}$$

$$\hat{p}(\rho=1,\tau>0) = 1 . \tag{9}$$

The problem stated in eqs. 7-9 has of course been solved using the separation of variables technique (Carslaw and Jaeger, 1959). The resulting expression for the instantaneous flux into the sphere is in the form of a very slowly (and non-uniformly) convergent infinite series, the numerical evaluation of which is computationally expensive. We will therefore use the boundary-layer technique to find an approximate solution that is quite accurate, but is much simpler in form.

3. Solution for "Small Times"

As mentioned above, the crux of the integral method is the assumption of a particular form of an approximate solution. The property of this method which makes it attractive for diffusion problems is that extremely accurate estimates of the flux can be found using very simple trial solutions. Consider some fixed value of dimensionless time τ, small enough such that the effects of the imposed pressure increment at the outer boundary of the sphere has not yet been felt at the sphere's center. Following the spirit of the boundary-layer approach, we assume that the pressure decreases to zero over a distance referred to as the penetration depth, which is denoted by $1-\zeta$ (Fig. 1),. and that $\hat{p}=0$ for $0<\rho<\zeta$. The simplest continuous pressure profile that satisfies this condition, along with boundary condition expressed by eq. 9, is one that varies linearly with radius over the range $\zeta<\rho<1$ (Fig. 1):

$$\hat{p} = \frac{\rho-\zeta}{1-\zeta} \qquad \text{for } \zeta<\rho<1 \,, \tag{10}$$

$$\hat{p} = 0 \qquad \text{for } 0<\rho<\zeta \,. \tag{11}$$

The parameter ζ, which measures the extent of diffusion, depends on τ in some manner which is *a priori* unknown.

To find an expression for the dependence of ζ on τ, the assumed profile given by eqs. 10 and 11 is substituted into the governing eq. 7, which is then integrated over the entire volume of the sphere:

$$\iiint_{Vol} \frac{1}{\rho^2} \frac{\partial}{\partial \rho} \left[\rho^2 \frac{\partial \hat{p}}{\partial \rho} \right] dV = \iiint_{Vol} \frac{\partial \hat{p}}{\partial \tau} dV \ . \tag{12}$$

By Green's theorem (or by explicit integration), the left hand side integrates out to

$$\iiint_{Vol} \frac{1}{\rho^2} \frac{\partial}{\partial \rho} \left[\rho^2 \frac{\partial \hat{p}}{\partial \rho} \right] dV = 4\pi \, \rho^2 \left. \frac{\partial \hat{p}}{\partial \rho} \right|_{\rho=1} = \frac{4\pi}{1-\zeta} , \tag{13}$$

where the assumed pressure profile from eqs. 10 and 11 has been used in the last step. Note that eq. 13 represents the instantaneous flux into the sphere, integrated over the entire outer surface.

Since the region of integration in eq. 12 does not vary with time, the time derivative on the right hand side can be taken outside of the integral. After integrating out the contribution of the angular coordinates, we have

$$\frac{4\pi}{1-\zeta} = \frac{d}{d\tau} \left[4\pi \int_0^1 \hat{p}(\rho) \rho^2 \, d\rho \right] . \tag{14}$$

Substituting the pressure profile given by eqs. 10 and 11 into eq. 14, and cancelling out the factors of 4π, leads to:

$$\frac{1}{1-\zeta} = \frac{d}{d\tau} \int_{\zeta}^{1} \frac{\rho-\zeta}{1-\zeta} \rho^2 \, d\rho \ . \tag{15}$$

The integral extends only from ζ to 1, since the integrand equals zero for $0<\rho<\zeta$. Carrying out the integration in eq. 15, we arrive at the differential equation that governs the change of ζ with τ:

$$\frac{1}{1-\zeta} = \frac{d}{d\tau}\left[\frac{\zeta^4-4\zeta+3}{12(1-\zeta)}\right] . \tag{16}$$

To solve eq. 16, we first factor the term $(1-\zeta)$ out of the numerator on the right hand side, leading to

$$\frac{1}{\zeta-1} = \frac{d}{d\tau}\left[\frac{(\zeta^3+\zeta^2+\zeta-3)}{12}\right] . \tag{17}$$

Expanding out the right-hand side leads to

$$\frac{1}{\zeta-1} = \frac{(3\zeta^2+2\zeta+1)}{12}\frac{d\zeta}{d\tau} . \tag{18}$$

By separating the variables ζ and τ, the indefinite integral of eq. 18 is readily found to be

$$12\tau = \frac{3}{4}\zeta^4 - \frac{1}{3}\zeta^3 - \frac{1}{2}\zeta^2 - \zeta + C , \tag{19}$$

where C is a constant of integration. The initial condition for eq. 19 is that when $\tau=0$, ζ must equal 1 (see Fig. 1, and eqs. 8 and 9). This implies that $C=13/12$, and the relationship between ζ and τ is therefore

$$12\tau = \frac{3}{4}\zeta^4 - \frac{1}{3}\zeta^3 - \frac{1}{2}\zeta^2 - \zeta + \frac{13}{12} . \tag{20}$$

If we define $\varepsilon = 1-\zeta$ to be the penetration distance into the sphere, eq. 20 can also be written as

$$\tau = \frac{1}{4}\varepsilon^2 - \frac{2}{9}\varepsilon^3 + \frac{9}{144}\varepsilon^4 . \tag{21}$$

It should be noted that the solution derived above is valid only for $\zeta > 0$, or $\varepsilon < 1$, which corresponds to $\tau < 13/144$. For larger values of τ, the pressure profile given by eq. 10 and 11 is not physically meaningful, and the solution will have a different form (see next section).

For many purposes, particularly application to the double-porosity model, the important quantity is the total flux into the sphere. The instantaneous flux into the sphere can be determined from Darcy's law and the pressure profile given by eq. 10 as

$$\dot{Q} = A \left.\frac{\partial \hat{p}}{\partial \rho}\right]_{\rho=1} = \frac{4\pi}{1-\zeta} = \frac{4\pi}{\varepsilon} . \tag{22}$$

(Note that the normalization process has the effect of setting the permeability, the viscosity, and the outer radius of the sphere all equal to 1). The total flux from time 0 to time τ is found by integrating the instantaneous flux:

$$Q[0 \rightarrow \tau] = \int_0^\tau \dot{Q}\, d\tau = \int_0^\tau \frac{4\pi}{\varepsilon}\, d\tau$$

$$= \int_0^\varepsilon \frac{4\pi}{\varepsilon} \frac{(3\varepsilon^3 - 8\varepsilon^2 + 6\varepsilon)}{12}\, d\varepsilon$$

$$= \frac{\pi}{3}\,(\varepsilon^3 - 4\varepsilon^2 + 6\varepsilon)\ , \tag{23}$$

where eq. 21 has been used to convert $d\tau$ to $d\varepsilon$. Eqs. 21 and 23 thus describe (implicitly) the total flux as a function of time, for $\tau < 13/144 \approx 0.09$. Note that when $\tau = 13/144$ (i.e. $\varepsilon = 1$), the total flux $Q = \pi$.

4. Solution for "Large Times"

For $\tau > 13/144$, the diffusion front has reached the center of the sphere, and eqs. 10 and 11 can no longer be used to approximate the pressure profile. The simplest profile that can be used for $\tau > 13/144$ is (see Fig. 2):

$$\hat{p} = \hat{p}(0) + [1 - \hat{p}(0)]\,\rho\,, \tag{24}$$

where $\hat{p}(0)$ represents the pressure (*a priori* unknown) at the center of the sphere. Although $\hat{p}(0)$ does not in any sense represent a "penetration distance", it plays a role analogous to that of ζ. Following the same procedure as for the "small time" solution, the pressure profile from eq. 24 is substituted into eq. 12 and integrated, leading to:

$$\left.\frac{\partial \hat{p}}{\partial \rho}\right|_{\rho=1} = \frac{d}{d\tau}\int_0^1 \left[\hat{p}(0) + [1 - \hat{p}(0)]\rho\right]\rho^2\,d\rho\ . \tag{25}$$

The result of carrying out the integration on the right-hand side of eq. 25 is the following differential equation for $\hat{p}(0)$ as a function of τ:

$$\frac{d\hat{p}(0)}{d\tau} = 12[1 - \hat{p}(0)] \ . \tag{26}$$

Since the initial condition is that $\hat{p}(0)$ must be 0 when $\tau = 13/144$ ($= \tau^*$), the solution to eq. 26 is

$$\hat{p}(0) = 1 - \exp[-12(\tau - \tau^*)] \ . \tag{27}$$

From Darcy's law and eq. 24, the instantaneous flux per unit area is simply $1 - \hat{p}(0)$, so that the total flux for the entire sphere up until any arbitrary time $\tau > 13/144$ is

$$Q[0 \to \tau] = Q[0 \to \tau^*] + 4\pi \int_{\tau^*}^{\tau} \exp[-12(\tau - \tau^*)] d\tau$$

$$= \pi + \frac{\pi}{3}\left[1 - \exp[-12(\tau - \tau^*)]\right] \ . \tag{28}$$

5. Discussion of Solution

For very small values of time, eq. 21 shows that $\varepsilon \approx 2\sqrt{\tau}$. The instantaneous flux $\dot{Q}$ is therefore (from eq. 22) approximately equal to $2\pi/\sqrt{\tau}$, and the cumulative flux up to time τ is (from eq. 23) approximately equal to $4\pi\sqrt{\tau}$. This $\sqrt{\tau}$ dependence is characteristic of linear diffusion problems, since for very small times the curvature of the sphere is not felt. In the limit of very small times, the exact expression for the instantaneous flux can be shown (Crank, 1965, p. 91) to be asymptotically equal to $4\sqrt{\pi/\tau}$, so that the approximate result is initially about 11% too low. (The approximate solution fortuitously "compensates" for this error as time increases, so that the cumulative flux will have a relative accuracy of better than 11% for all subsequent times).

As the diffusion front moves into the sphere, the area available for flow decreases, and thus the instantaneous flux decreases at a rate faster than $1/\sqrt{\tau}$ (Fig. 3). In general, for $\tau < 13/144$, eq. 21 gives ε as an implicit function of τ, while eqs. 22 and 23 express the instantaneous and cumulative fluxes as functions of ε. Although an explicit expression for the flux as a function of time would be more convenient, eq. 21 can be inverted numerically with little effort. For example, if $\varepsilon_o = 2\sqrt{\tau}$ is used as the initial guess, ε can be found to six digit accuracy with at most three iterations using the Newton-Raphson method.

Since a finite sphere with a finite pressure at its outer boundary will only absorb a finite amount of fluid, the instantaneous flux will eventually go to zero. For $\tau > 13/144$, the approximate solution gives the instantaneous flux into the sphere explicitly as $\dot{Q} = 4\pi\exp[-12(\tau-\tau^*)]$. Note that although the *form* of the solution is "discontinuous" at $\tau = 13/144$, the predicted instantaneous flux is nevertheless a continuous function of time for all times. In this sense the approximate solution developed here is superior to the "small time" and "large time" approximations that can be found using Laplace transform techniques, since they would not correctly match up for intermediate values of τ. In Fig. 3, the instantaneous flux predicted by the approximate solution is compared to the exact solution, which can be found in Crank (1965). While the approximate flux is lower than the actual flux for some values of τ, and higher for others, in general the two results agree very closely.

If the cumulative flux is divided by the total volume of the sphere, $4\pi/3$, the resulting normalized cumulative flux varies from 0 to 1, and can be referred to as the "fractional uptake", f. For $\tau < \tau^* = 13/144$, the fractional uptake (Fig. 4) is found by numerically solving eqs. 21 and 23. Note that when $\tau = \tau^*$, $\varepsilon = 1$ and $Q = \pi$, so $f = 3/4$. For $\tau > \tau^*$, the fractional uptake is given explicitly from eq. 28 as

$$f = 1 - \frac{1}{4}\exp[-12(\tau - \tau^*)] \ . \tag{29}$$

The exact expression for the fractional uptake is (Crank, 1965)

$$f = 1 - \frac{6}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{n^2}\exp(-n^2\pi^2\tau) \ . \tag{30}$$

The cumulative fluxes predicted by the approximate solution agree extremely well with the exact values, over the entire range of times (Fig. 4). Although eq. 30 is valid for all values of τ, it is computationally inconvenient for very small times (due to very slow convergence), whereas the approximate value of f can be found to six-digit accuracy from eqs. 21 and 23 with very minimal computation.

The integral method is not intended to provide accurate details of the exact pressure distribution in the sphere, and indeed such details are generally not needed for the double-porosity model. However, it is interesting to note that despite the extreme simplicity of the assumed pressure profiles, the approximate integral method is surprisingly accurate, at least in a qualitative sense, in its depiction of the infiltration process (Fig. 5). Note that when the pressure profiles are plotted against the cube of the normalized radius, the cumulative fluxes are directly proportional to the areas under the curves. Somewhat more accurate approximations can be obtained by using nonlinear pressure profiles that behave smoothly at $\rho = 0$ and $\rho = \zeta$, but the additional complexity of the resulting $Q - \tau$ relationships renders such approximate solutions less useful than the one developed here.

As mentioned earlier, the solution derived above can be incorporated into a numerical model that discretizes the fractures and treats the matrix blocks as source/sink terms. Obviously such a model can handle spheres of different sizes in different parts of the field, which would be useful if data were available regarding fracture patterns. We have incorporated this simple solution into a numerical model (Pruess, 1987), and found that computational speed can typically be increased by factors of about 5-10; details of this implementation will be presented in a future paper. Finally, note that this method as described is useful for problems in which the (fracture) pressure at each point in the reservoir changes either monotonically, or oscillates with a characteristic time that is larger than about $\Delta\tau \approx 1$ (see Fig. 4). Problems with more rapidly oscillating fracture pressures require a somewhat different formulation for the fracture-matrix flow problem (cf. Vinsome and Westerveld, 1980; Pruess and Wu, 1988).

6. Conclusions

The integral method has been used to derive a very simple, but highly accurate, solution to the problem of flow into a porous sphere that is initially at a constant pressure, and whose outer boundary is held at some other fixed pressure. The approximate solution and the known exact solution agree particularly well with regard to the predicted cumulative flux into the sphere. This will make the solution convenient for use in conjunction with double-porosity models of petroleum, groundwater or geothermal reservoirs.

While the analysis in this paper has explicitly dealt with the flow of a single-phase Newtonian fluid in a porous medium, the results of course apply to any geometrically similar problem that is governed by the ''diffusion equation''. For example, in order to interpret eq. 1 as governing heat conduction, it is merely necessary to identify p as the temperature, k as the thermal conductivity, and $\phi\mu c$ as the product of the density and the specific heat. The term $k/\phi\mu c$ then represents the thermal diffusivity. Eq. 1 can represent diffusion of a substance that is governed by Fick's law if p is thought of as the concentration, and $k/\phi\mu c$ as the diffusion coefficient (Crank, 1965). Other physical processes governed by the same equation are discussed by Carslaw and Jaeger (1959, pp. 28-29).

Acknowledgements

This work was carried out under U.S. Department of Energy Contract No. DE-AC03-76SF00098, administered by the DOE Nevada Office, in cooperation with the United States Geological Survey, Denver. The authors thank Kenzi Karasaki and Yu-Shu Wu of Lawrence Berkeley Laboratory for reviewing this paper.

References

Barenblatt, G.E., Zheltov, Y.P. and Kochina, I.N., 1960. Basic concepts in the theory of seepage of homogeneous liquids in fissured rocks. J. Appl. Math. Mech., 24: 1286-1303.

Barker, J.A., 1985. Block-geometry functions characterizing transport in densely fissured media. J. Hydrology, 77: 263-279.

Carslaw, H.S. and Jaeger, J.C., 1959. Conduction of Heat in Solids. Clarendon Press, Oxford.

Crank, J., 1965. The Mathematics of Diffusion. Clarendon Press, Oxford.

deSwaan, A., 1976. Analytic solutions for determining naturally fractured reservoir properties by well testing. Soc. Pet. Eng. J., 16: 117-122.

Duguid, J.O., 1973. Flow in Fractured Porous Media. Ph.D. thesis, Princeton University, Princeton, N.J.

Goodman, T.R., 1964. Application of integral methods to transient nonlinear heat transfer. In: Advances in Heat Transfer, 1: 51-122.

Kazemi, H., 1969. The behavior of naturally fractured reservoirs with uniform fracture distribution. Soc. Pet. Eng. J., 9: 317-326.

Landahl, H.D., 1953. An approximate method for the solution of diffusion and related problems. Bull. Math. Biophys., 15: 49-61.

Muskat, M., 1937. The Flow of Homogeneous Fluids Through Porous Media. McGraw-Hill, New York.

Pruess, K., 1987. TOUGH User's Guide, Report LBL-20700, Lawrence Berkeley Laboratory, Berkeley, Ca.

Pruess, K., and Narasimhan, T.N., 1985. A practical method for modeling fluid and heat flow in fractured porous media. Soc. Pet. Eng. J., 25: 14-26.

Pruess, K. and Wu, Y.S., 1988. A semi-analytical method for heat sweep calculations in fractured reservoirs. 13th Workshop on Geothermal Reservoir Engineering, Stanford University.

Schlichting, H., 1968. Boundary-Layer Theory, 6th Ed. McGraw-Hill, New York.

Serra, K.V., Reynolds, A.C. and Raghavan, R., 1983. New pressure transient analysis methods for naturally fractured reservoirs, J. Pet. Tech., 35: 1903-1914.

Streltsova, T.D., 1983. Well pressure behavior of a naturally fractured reservoir. Soc. Pet. Eng. J., 23: 769-780.

Vinsome, P.K.W. and Westerveld, J., 1980. A simple method for predicting cap and base rock heat losses in thermal reservoir simulators. J. Can. Pet. Tech., July-Sept: 87-90.

Warren, J.E. and Root, P.J., 1963. The behavior of naturally fractured reservoirs. Soc. Pet. Eng. J., 3: 245-255.

Nomenclature

a	outer radius of sphere [m]
A	dimensionless outer surface area of sphere, 4π
c	compressibility [Pa^{-1}]
f	fractional fluid uptake of sphere
k	permeability [m^2]
p	pressure [Pa]
$\hat{p}$	normalized pressure, $(p - p_i)/(p_o - p_i)$
Q	cumulative dimensionless flux into sphere
$\dot{Q}$	instantaneous dimensionless flux into sphere
r	radial coordinate [m]
t	time [s]

Greek symbols

ε	normalized front penetration distance
ϕ	porosity
μ	viscosity [Pa s]
ρ	normalized radial coordinate, r/a
τ	dimensionless time, $kt/\phi\mu c a^2$
τ^*	value of τ when front reaches center of sphere
ζ	normalized radial position of front, $1-\varepsilon$

Subscripts

i	initial condition $(t=0)$
o	outer surface of sphere $(r=a)$

Figure Captions

Fig. 1. Assumed form of the pressure profile, in terms of normalized variables $\hat{p}$ and ρ. This form is used until the pressure front reaches the center of the sphere, i.e. until $\zeta \rightarrow 0$.

Fig. 2. Assumed form of the pressure profile, in terms of normalized variables $\hat{p}$ and ρ. This form is used after the pressure front has reached the center of the sphere.

Fig. 3. Instantaneous flux as a function of dimensionless time, according to both the exact solution and the approximate solution obtained from the integral method using a linear pressure profile.

Fig. 4. Normalized cumulative flux as a function of dimensionless time, according to both the exact solution and the approximate solution obtained from the integral method using a linear pressure profile.

Fig. 5. Pressure profiles at various values of dimensionless time, according to both the exact and approximate solutions.

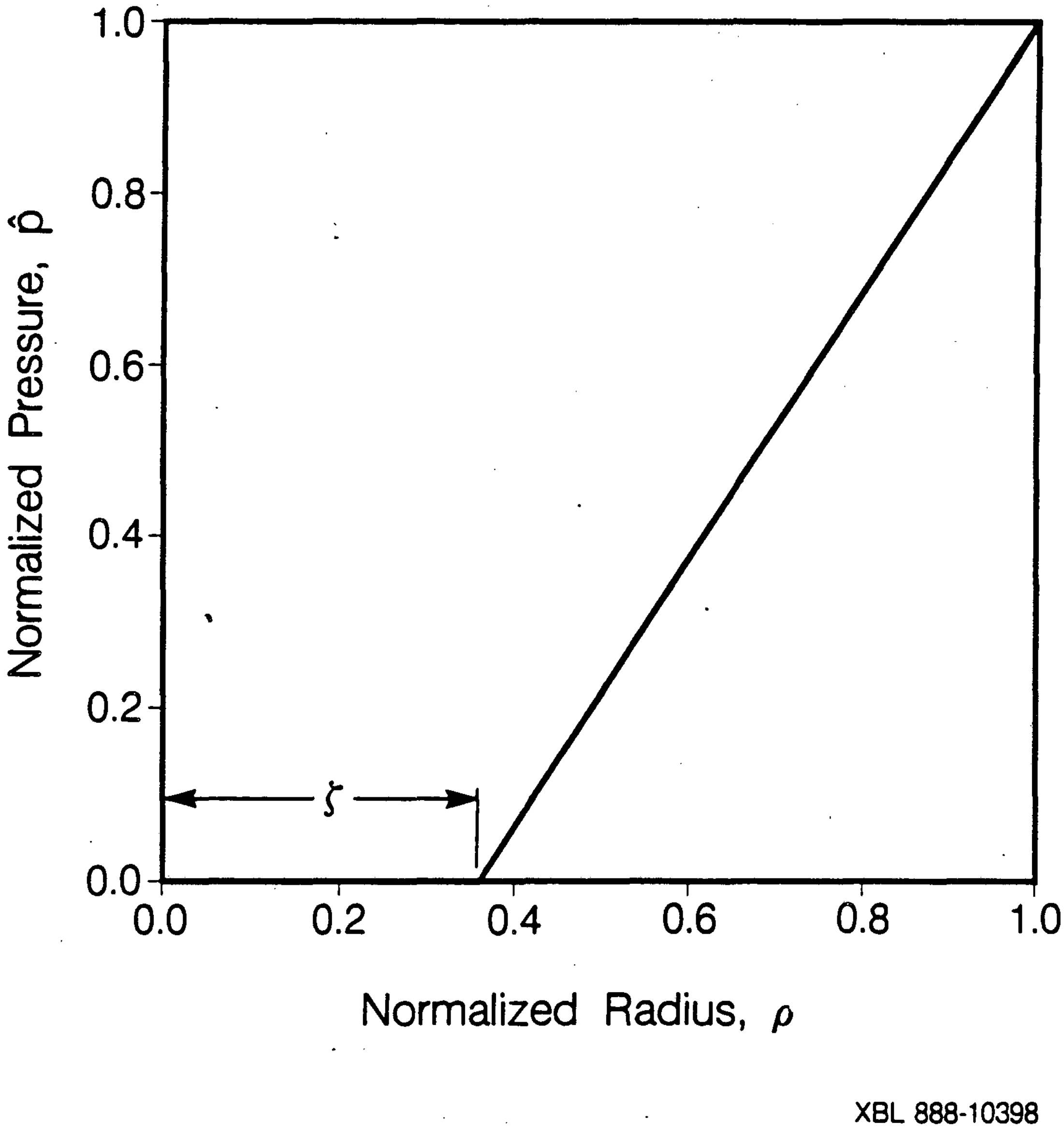


Fig. 1. Assumed form of the pressure profile, in terms of normalized variables $\hat{p}$ and ρ. This form is used until the pressure front reaches the center of the sphere, i.e. until $\zeta \rightarrow 0$.

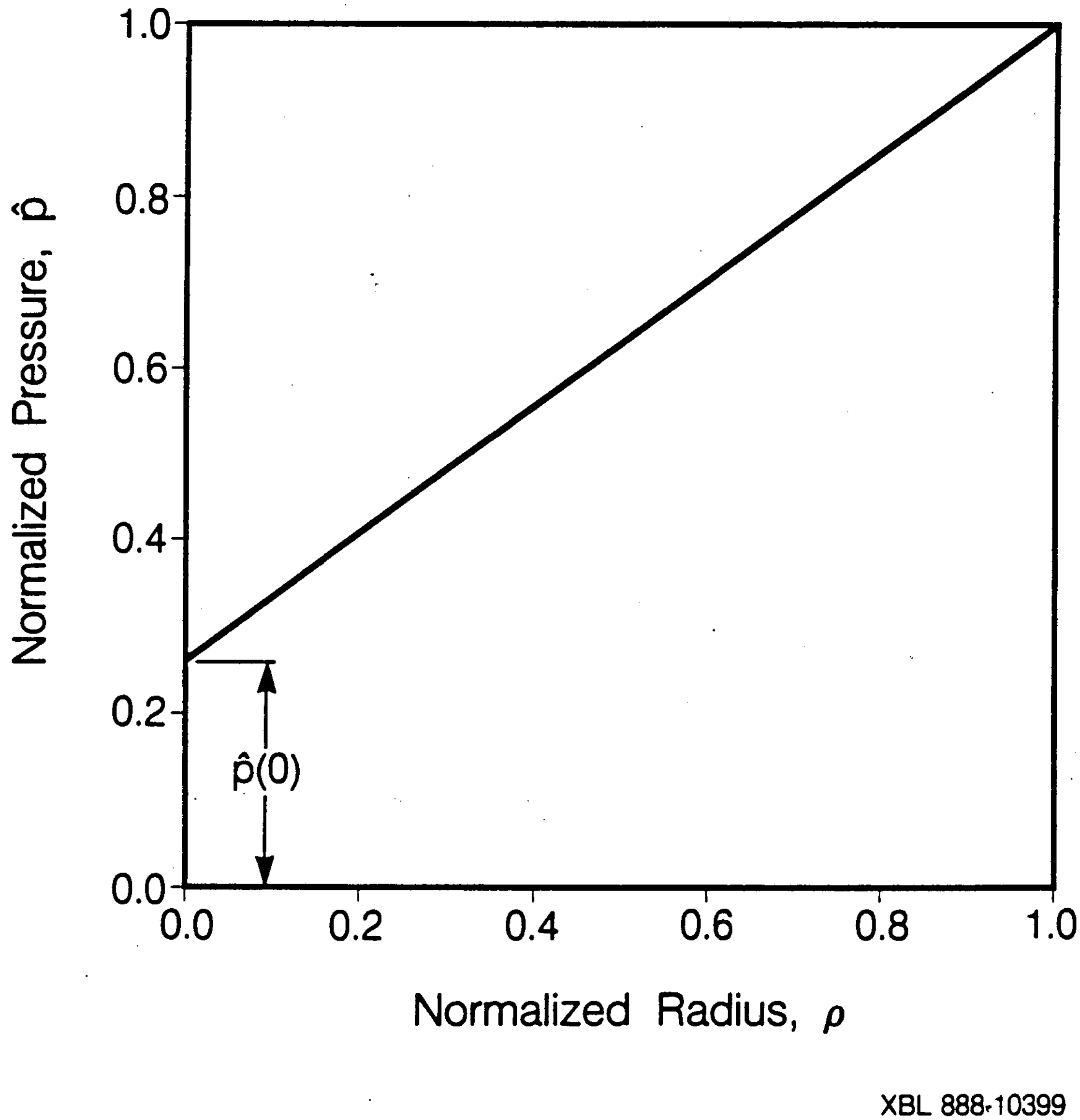
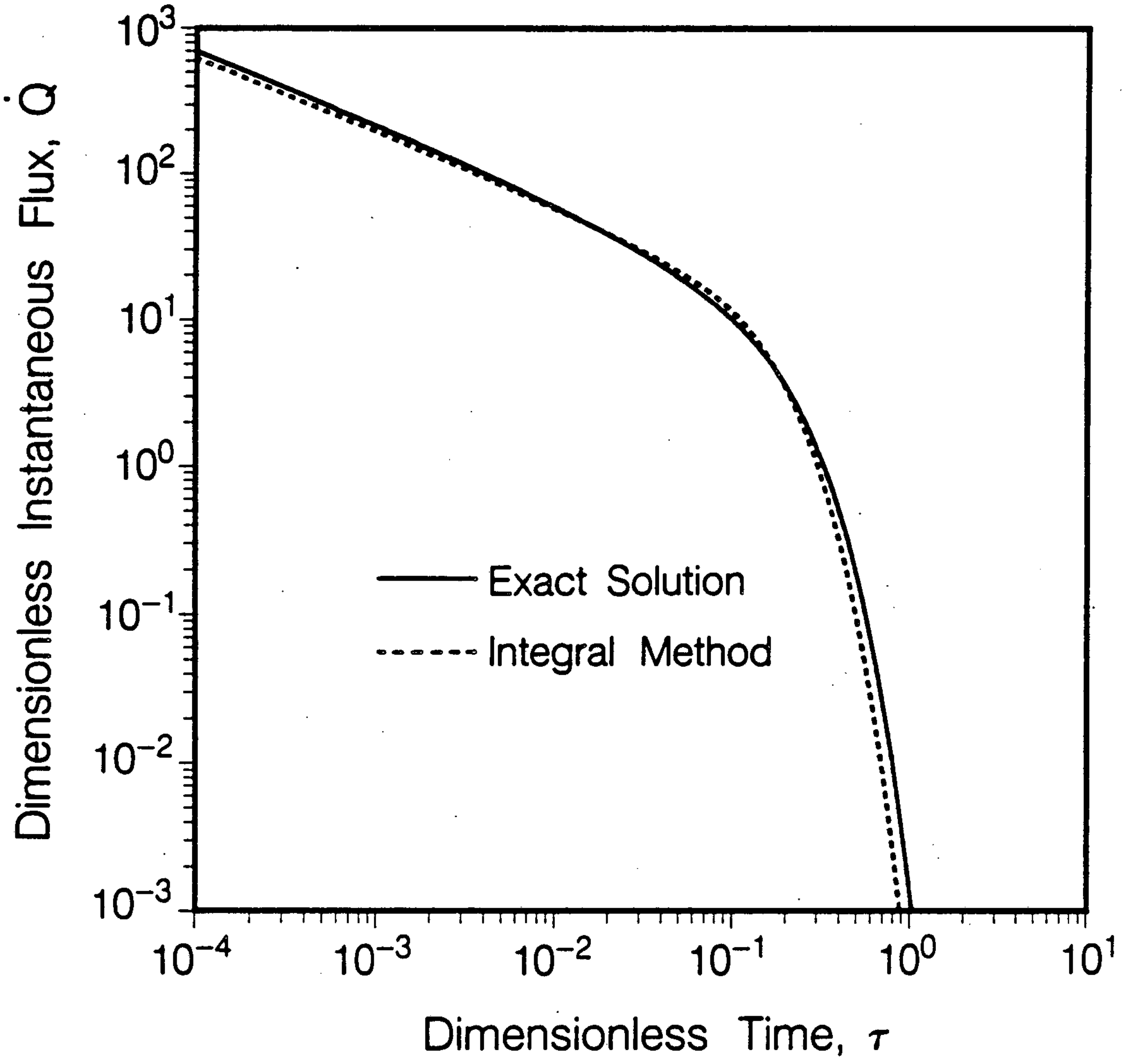
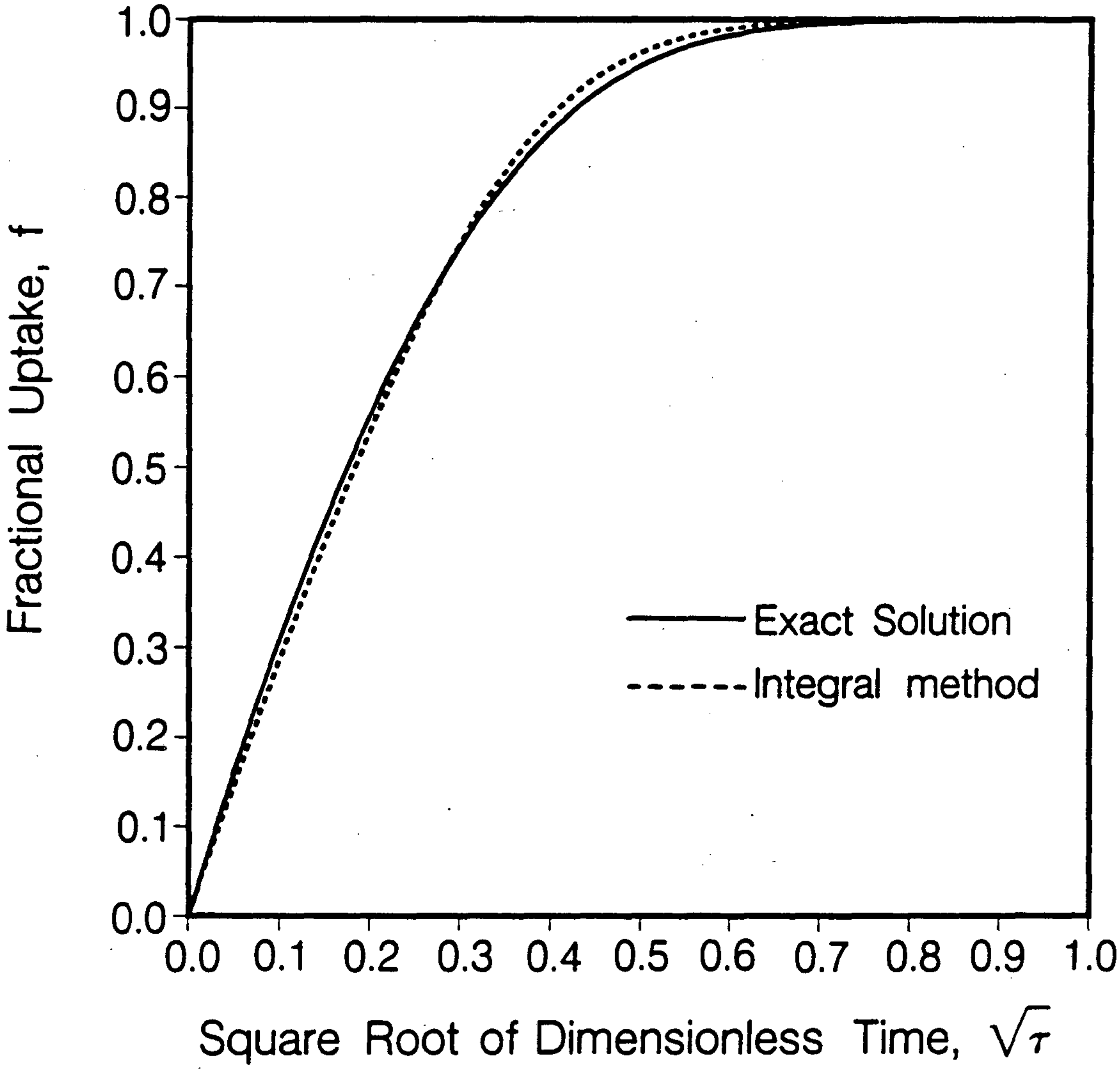


Fig. 2. Assumed form of the pressure profile, in terms of normalized variables $\hat{p}$ and ρ. This form is used after the pressure front has reached the center of the sphere.



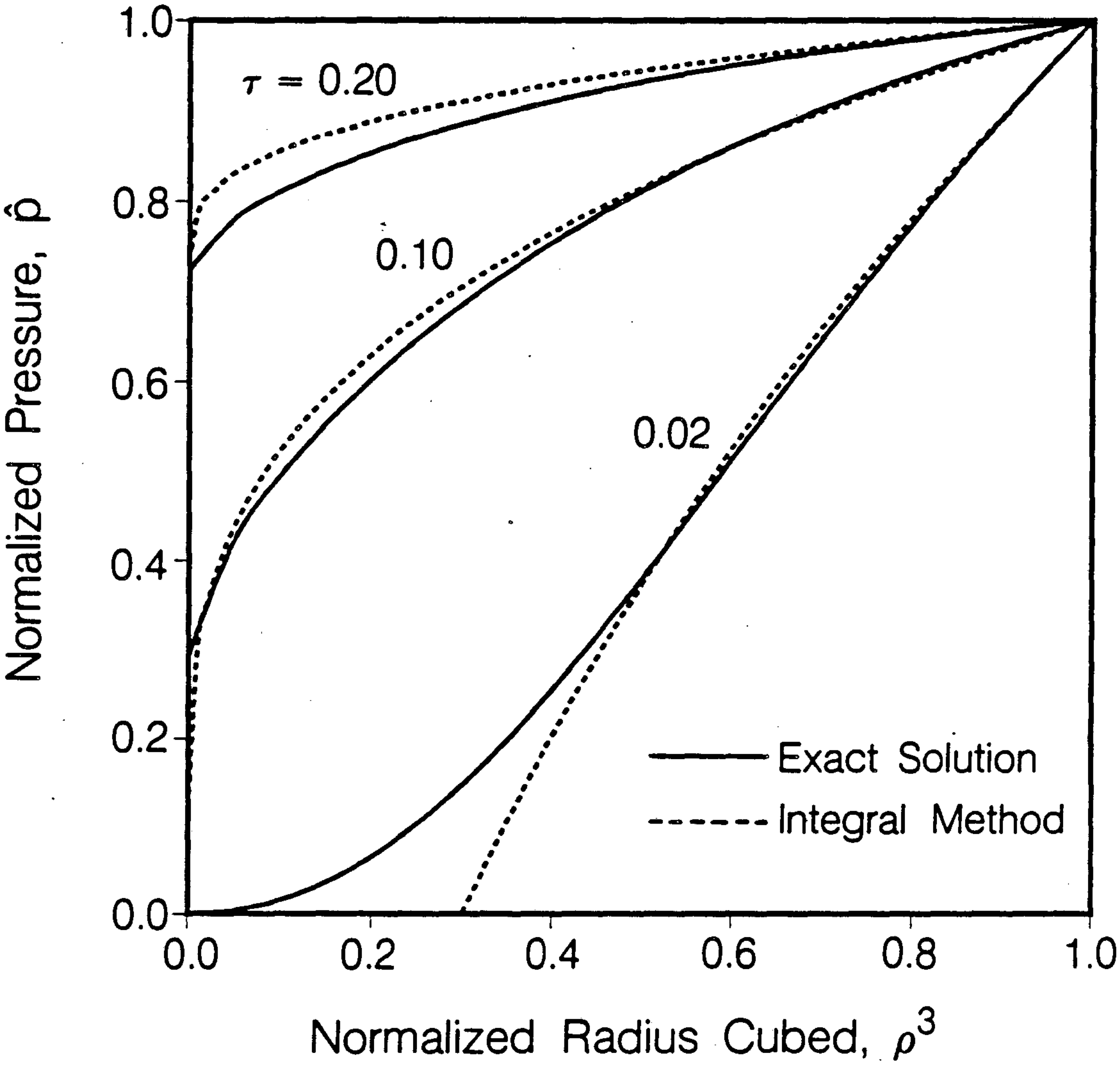
XBL 888-10400

Fig. 3. Instantaneous flux as a function of dimensionless time, according to both the exact solution and the approximate solution obtained from the integral method using a linear pressure profile.



XBL 888-10401

Fig. 4. Normalized cumulative flux as a function of dimensionless time, according to both the exact solution and the approximate solution obtained from the integral method using a linear pressure profile.



XBL 888-10402

Fig. 5. Pressure profiles at various values of dimensionless time, according to both the exact and approximate solutions.

LAWRENCE BERKELEY LABORATORY
TECHNICAL INFORMATION DEPARTMENT
UNIVERSITY OF CALIFORNIA
BERKELEY, CALIFORNIA 94720