Symmetries of Model Construction in Spatial Relational Inference

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Abstract

This article studies spatial relational inference within the framework of mental model theory. It focuses on the phase of model construction for which two cognitive modelings currently exist (Berendt, 1996; Schlieder, 1995). Both refer to the aggregated results of a former experiment (Knauff, Rauh & Schlieder, 1995). However, conflicting evidence exists with respect to symmetry properties of model construction that makes the assessment of the cognitive adequacy of certain explanations impossible. We therefore conducted an experiment using computational tools provided by AI research on Qualitative Spatial Reasoning (QSR) to investigate whether the model construction process works the same from left to right and vice versa (symmetry of reorientation), and whether the processing of spatial relations depends on what was already processed (symmetry of transposition). Experimental results clearly indicate that the symmetry of transposition cannot be found in subjects' answers to indeterminate spatial *four-term series* problems and that the degree of reorientation symmetry is not perfect. The latter, however, can be entirely attributed to performance variation, since the responses of retested subjects to the same problems were only concordant to the same degree.

Introduction

Relational inferences are said to be very common, frequent and ubiquitous (Evans, Newstead & Byrne, 1993, chap. 6). An important subclass of these inferences concerns spatial relations. Consider the following example of a spatial relational inference about the (one-dimensional) position of two intervals on a line:

Interval A overlaps interval B from the left.
Interval B is completely in interval C.
How could interval A lie with respect to interval C?

In the psychology of reasoning, several proposals were made to explain how human reasoners solve such *three-term series* problems (the three terms are the intervals *A*, *B*, and *C*). According to Evans, Newstead and Byrne (1993), the most successful framework for describing spatial relational inference—and possibly also reasoning in general—is the mental model theory.

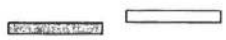
The mental model theory (Johnson-Laird, 1983; Johnson-Laird & Byrne, 1991) postulates that reasoning is a process during which reasoners first build an integrated representation of the situation described in the premises—the mental model—and then inspect this model for new, unstated information. In certain reasoning tasks such as deduction, alternative models of the premises are constructed to determine whether a model exists which contradicts a putative conclusion. In the domain of spatial relational inference, these three phases are called *model construction*, *model inspection*, and *model variation* (Rauh, Schlieder & Knauff, 1997). In this article, we concentrate on investigating the first phase—the phase of model construction.

Investigating Model Construction Using a System of Spatial Relations from AI Research

One–dimensional spatial relational inference is usually studied for the following systems of relations: *left–right, before–behind, east–west, north–south* (e.g. Byrne & Johnson-Laird, 1989; Ehrlich & Johnson-Laird, 1982; Johnson-Laird & Byrne, 1991; Maki, 1981; Mani & Johnson-Laird, 1982). As in our former experiments (Knauff, Rauh & Schlieder, 1995; Rauh, Schlieder & Knauff, 1997), we use the system of interval relations introduced by Allen (1983) providing the methodological advantages described below. Its 13 jointly exhaustive and pairwise disjoint relations describe the relative position of two intervals on a line. Originally, Allen (1983) proposed the system in order to represent relations between time intervals. Soon, however, the relations and the calculus based on them were applied to the spatial domain (e.g. Freksa, 1991). They have since been widely used in AI research on Qualitative Spatial Reasoning (QSR), and their computational properties have been thoroughly analyzed. Table 1 shows the 13 interval relations listing for each the relation's symbol, the natural language description of the relation used in our experiment, a graphical realization, and the ordering of the interval's startpoints and endpoints which defines the relation.

An example of a *three-term series* based on interval relations was given in the introduction. *Four-term series* problems can be formulated in a similar manner. A four-term series concerns the positioning of four intervals *A, B, C, D*, and consists of three *premises* $A\ r_1\ B$, $B\ r_2\ C$, and $C\ r_3\ D$, each of which specifies a relation between two intervals. The three remaining relations $A\ r_4\ C$, $B\ r_5\ D$, and $A\ r_6\ D$ are constrained by the premises. We call them the *conclusions* of the four-term series problem (strictly speaking, an abuse of logical terminology). In a reasoning task the premises are given and the subject is asked to provide the conclusions. We write this as $(A\ r_1\ B, B\ r_2\ C, C\ r_3\ D) \ \triangleright\ (A\ r_4\ C, B\ r_5\ D, A\ r_6\ D)$.

Table 1: The 13 qualitative interval relations, associated natural language expressions, graphical realization, and ordering of startpoints and endpoints (adapted and augmented according to Allen, 1983).

Relation symbol	Natural language description	Graphical realization	Point ordering (s=startpoint, e=endpoint)
X < Y	X lies to the left of Y		$s_X < e_X < s_Y < e_Y$
X m Y	X touches Y at the left		$s_X < e_X = s_Y < e_Y$
X o Y	X overlaps Y from the left		$s_X < s_Y < e_X < e_Y$
X s Y	X lies left-justified in Y		$s_Y = s_X < e_X < e_Y$
X d Y	X is completely in Y		$s_Y < s_X < e_X < e_Y$
X f Y	X lies right-justified in Y		$s_Y < s_X < e_X = e_Y$
X = Y	X equals Y		$s_X = s_Y < e_Y = e_X$
X fi Y	X contains Y right-justified		$s_X < s_Y < e_Y = e_X$
X di Y	X surrounds Y		$s_X < s_Y < e_Y < e_X$
X si Y	X contains Y left-justified		$s_X = s_Y < e_Y < e_X$
X oi Y	X overlaps Y from the right		$s_Y < s_X < e_Y < e_X$
X mi Y	X touches Y at the right		$s_Y < e_Y = s_X < e_X$
X > Y	X lies to the right of Y		$s_Y < e_Y < s_X < e_X$

Correct and Model-Consistent Solutions

A conclusion is correct if it is consistent with the premises, i.e. if an interval configuration exists in which the premises and the conclusion in question hold. Three conclusions form a *correct solution* of the four-term series if each conclusion is correct. In general, there are several correct solutions to a four-term series. For instance, each of the following, *(A di B, B > C, C m D)* ▷ *(A m C, B d D, A = D)* and *(A di B, B > C, C m D)* ▷ *(A > C, B f D, A = D)* is correctly solved. A computer-based analysis reveals that there are 275 correct solutions to this task; however, the correct solutions fall into two fundamentally different classes: Those which are *model-consistent* and those which are not. For a model-consistent solution, a single interval configuration exists in which all three conclusions are correct. The first solution given above is of this type. A correct solution must not be model-consistent as correctness requires only that each conclusion be consistent with the premises, but *not* with the other two conclusions. The second solution, for example, is correct but not model-consistent. In fact, only 35 out of the 275 correct solutions to this problem are model-consistent.

Being able to distinguish between correct and model-consistent solutions constitutes an important methodological advantage. In investigating spatial relational inference, it is usually assumed that if a problem is correctly solved, the construction of the mental model succeeded. With four-term series based on interval relations, however, correct solutions can be identified for which model construction must either have failed or never been attempted (e.g. through the application of a different reasoning strategy). Such solutions are not model-consistent. Computational tools for this new type of control for model construction have been provided by the AI community in the form of efficient algorithms for computing the correct as well as the model-consistent solutions.

Symmetry Analysis of Solutions

Computational analysis of the interval relations has shown that a group of symmetry transformations plays an important role (Ligozat, 1990). A symmetry transformation relates one four-term series problem (premises and conclusions) to another. Two fundamental symmetry transformations, called *transposition* and *reorientation*, generate the whole group. Both have a simple geometrical interpretation as is illus-

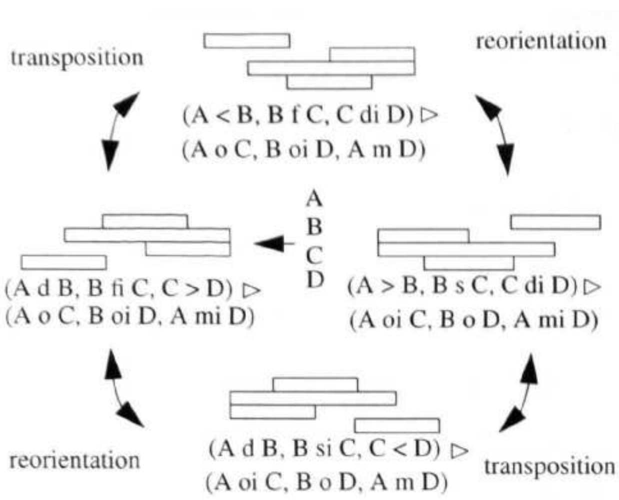


Figure 1: Symmetry transformations on Allen-based four-term series problems constituting an *orbit* of 4 inference tasks.

trated in Figure 1, in which four tasks with related solutions are shown, each having an interval configuration in which the premises and the conclusions are correct and model-consistent. A set of four tasks related by symmetries is called an *orbit*. Reorientation of a task is achieved by reflecting the interval configuration around the vertical axis. The transposition of a task is obtained by reflecting the interval configuration around the horizontal axis. Solutions given to four-term series problems can be analyzed in order to determine whether they respect or violate the symmetries. If, for instance, the problem (*A < B, B f C, C di D*) is answered with (*A o C, B oi D, A m D*) then there is only one solution to the problem (*A > B, B s C, C di D*) which respects reorientation symmetry, namely (*A oi C, B o D, A mi D*). Valuable information about the reasoning process can be obtained by analyzing symmetry violations.

The Construction of Preferred Mental Models

Investigating the phase of model construction, Knauff, Rauh and Schlieder (1995) were able to show that a significant majority of people constructed identical first solutions to spatial three-term series problems that have multiple solutions. The solutions most frequently constructed were termed *preferred mental models*. From a logical perspective, there is no reason in preferring one solution to another. The reason for model preference is psychological, and lies in the cognitive process that constructs the preferred mental model. One way of gaining further insight into this process is to look at violations of reorientation symmetry and transposition symmetry. Both symmetry transformations possess a functional interpretation in terms of model construction. A complete absence of violations of reorientation symmetry means that model construction follows the same layout strategy working from left to right as it does working from right to left. Similarly, a complete absence of violations of transposition symmetry indicates that the positioning of intervals in the mental model does not depend on the order in which previous intervals were integrated into the mental model.

Conflicting Evidence Concerning Symmetries

The presence or absence of one or both of these symmetry properties can lead to the discarding of numerous explanations and provides minimal restrictions that must be fulfilled by cognitive modelings of the model construction process. Subsequent symmetry analyses of the aggregated data of preferred mental models in Knauff et al. (1995) revealed 4 reorientation asymmetries but 13 transposition asymmetries, out of a possible 28 for each. This gave rise to the conclusion that accounts demanding strictly transposition symmetries are empirically not tenable, and that the number of reorientation symmetries is low enough that a cognitive modeling need not account for them (Schlieder, 1995).

Based on this data, Schlieder (1995) proposed a cognitive modeling in which model construction works identically from left to right and right to left (it constructs preferred mental models that are symmetrical with respect to reorientation). It is however also able to account for asymmetries with respect to transposition. The same—reorientation symmetry, transposition asymmetry—is true of the imagery-based cognitive modeling of the model construction process proposed by Berendt (1996).

In a replication of this experiment (Kuß, Rauh & Strube, 1996), the number of reorientation symmetries was the same, but the occurrence of transposition asymmetries decreased to the absolute number of 4. The question of whether model construction is sensitive to the information already processed was thus re-opened.

In the following, we describe an experiment which was planned to resolve this issue. Being not just another replication of the former two, it was tailored to answer the questions (i) whether model construction is necessarily symmetric with respect to transposition, and (ii) whether model construction shows the same degree of symmetry with respect to reorientation and transposition. To avoid the possibility of an artifact of aggregation where a pattern of effects can arise that never occurs on the individual level because of changing majorities, symmetry analysis was conducted on an individual level.

Experimental Investigation of Symmetries of Preferred Mental Models

Each subject read referentially continuous *four-term series* problems (Ehrlich & Johnson-Laird, 1982) according to the separated-stages paradigm (Potts & Scholz, 1975), and had to then determine one possible relation between the first and fourth, first and third, and second and fourth interval separately.

Subjects:

20 paid students, 10 female and 10 male, of the University of Freiburg participated in the experiment which took approximately two hours.

Materials:

As described in Schlieder (1995), the group of symmetry transformation decomposes the set of all 12x12 three-term series (omitting the "=" relation) into 14 orbits. One member of each orbit was chosen randomly, augmented by one additional interval relation yielding a *four-term series* problem. To obtain the other three corresponding symmetric *four-term series* problems, the symmetry operations of reorientation, transposition, and reorientation together with transposition were applied to the original problem. One such orbit with four *four-term series* problems can be seen in Figure 1. The four members of one orbit were compiled in four different blocks, yielding blocks of 14 referentially continuous four-term series problems plus one practice block in the beginning consisting of 5 other simple *four-term series* problems. The sequence of experimental blocks was counterbalanced across subjects according to a sequentially balanced Latin square.

Procedure:

The computer-aided experiment was separated into three phases. To refer the obtained results to the inferential aspects (see Evans, 1972; Knauff, Rauh & Schlieder, 1995), we attempted to keep constant pre-experimental differences with respect to linguistic and/or conceptual aspects of Allen's relations by applying a definition phase and a criterion-based learning phase before the actual inference phase.

In the *definition phase*, subjects read descriptions of the spatial relation of a red and a blue interval using the 13 qualitative interval relations. Each verbal description was presented with a short commentary about the location of the startpoint and endpoint of the red and the blue interval and a pictorial example.

The *learning phase* consisted of trial blocks during which subjects were presented with a one-sentence description of the red and blue intervals. They had to then determine the startpoints and endpoints of a red and a blue interval using mouse clicks (analogous to "SCREEN 4" in Figure 2). After having confirmed her/his final choices, the subject was told whether her/his choices were correct or false. If they were false, additional information about the correct answer was given, i.e. displaying verbal descriptions of the ordering of startpoints and endpoints of the intervals. Trials were presented in blocks of all 13 relations in random order. The learning criterion for one relation was accomplished if the subject gave correct answers in 3 consecutive blocks of the corresponding relation. The learning phase stopped as soon as the subject reached the learning criterion for the last remaining relation. Subjects required between 15 and 30 minutes to accomplish the learning phase.

In the *inference phase*, subjects had to solve 61 (5 + 56) spatial *four–term series* generation problems. Premises were presented successively in a self-paced manner, each on an extra screen as shown in Figure 2.

On the next screen, subjects had to specify one possible relation between the red and the blue intervals, on the following screen between the red and the green, and finally between the yellow and the blue intervals by applying mouse clicks. Each time, they specified the startpoints and endpoints of both intervals in lightly colored rectangular regions, as they had done in the learning phase (see Figure 2 for details of the experimental display).

Of the recorded dependent measures of premise processing times, error rates, and type of constructed interval relations, we will report only on the latter, excluding practice trials from subsequent statistical analyses. A level of significance of 5% will be adopted in case of reporting on inferential statistics.

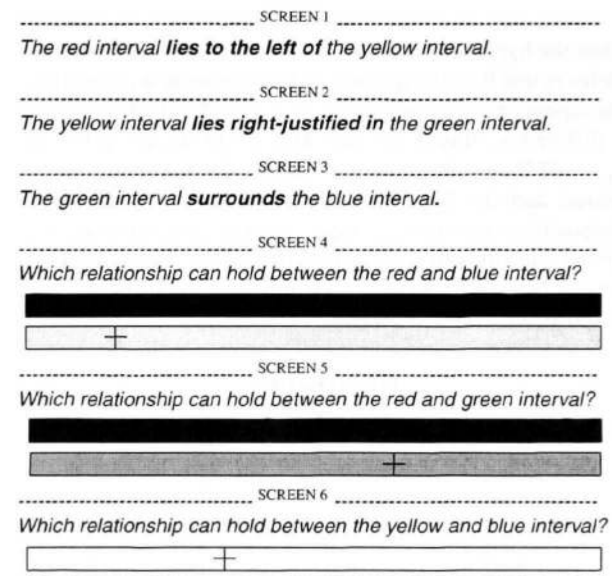


Figure 2: Successive verbal presentation of referentially continuous *four-term series* problems (screen 1 to 3), and three separate screens for generation of solutions (screen 4 to 6), as displayed to the subjects in the inference phase.

Results

All 20 subjects successfully passed the learning phase. Therefore, all data collected in the inference phase could be further analyzed. Individual performance showed considerable variation, ranging from 23.2% to 92.9% correct answer triples. As outlined above, symmetry analysis is only meaningful for model-consistent answers. From the overall number of observations of 1120 answer triples, 648 (= 57.9%) were correct. Of these 648 correct answer triples, a total of 484 (= 74.7%) were model-consistent and could be analyzed with respect to symmetries.

Looking for model-consistent pairs of four-term series problems belonging to the same orbit, there were 135 pairs left for analysis of reorientation and 130 pairs for analysis of transposition. 52 of the 135 (= 38.5%) reorientation pairs matched the reoriented answer triple of the other, whereas only 14 of the 130 (= 10.8%) transposition pairs matched each other (see Table 2 for an overview).

Table 2: Degree of symmetries in solving related four-term series problems.

	n	absolute	%
Symmetry			
Reorientation	135	52	38.5
Transposition	130	14	10.8

To test the hypothesis that the number of reorientation asymmetries is less than the number of transposition asymmetries, a chi-square test was conducted, based on the H_0 that there is no difference. Based on this H_0, a chi-square value of $\chi^2_{(1)} = 27.26, p < .00005$ was obtained. Thus, the H_0 can be rejected and the hypothesis adopted that there are more transposition asymmetries than reorientation asymmetries. This also justifies the conclusion that model construction is necessarily asymmetrical at least in some particular cases of transposition.

Discussion

Concerning the conflicting evidence found in the data in Knauff et al. (1995) compared to the data in Kuß et al. (1996), a clear decision can now be made in favor of the hypothesis that there are more transposition asymmetries than reorientation asymmetries, and that there are massive *order effects* in spatial relational inference. This rules out all explanations that necessarily demand transposition symmetry of model construction.

With respect to the symmetry of reorientation, however, results are less decisive. Since the percentage of reorientation asymmetries is far from zero, this would, at first glance, create problems for all accounts that necessarily demand reorientation symmetry in model construction, like the cognitive modelings of Schlieder (1995) and Berendt (1996). Referring to the competence–performance distinction, however, one could argue that other factors, extrinsic to model construction, led to the considerable amount of reorientation asymmetries, especially in such demanding reasoning tasks like indeterminate four-term series problems.

In fact, evidence exists which indicates that no "perfect" reorientation symmetry could be found in empirical data. The within-subject concordance of constructed relations of two subjects that had previously participated in two versions of the same experiment with indeterminate Allen-based *three-term series* problems was not perfect, and reached only 83.0% and 58.3% respectively. Since these results on the robustness of solutions to indeterminate three-term series problems cannot serve as an adequate estimate of the robustness of solutions to the far more demanding four-term series problems, we re-tested those subjects (n=7) with the highest number of correct answer triples some weeks later again. The outcome should give an adequate estimate on the amount of reorientation symmetries one can expect if one subject solves four-term series problems and their symmetrical problems.

The results of the re-test compared to the first test showed an overall concordance of 17.1%, i.e. the percentage of identical answer triples given the same four-term series problems[1]. Restricting the analysis on those 3 subjects[2] that showed nearly perfect model-consistency in their answer triples (percentage of model-consistent answer triples within correct answer triples: > 94.0%), the concordance rose to 28.6%.

These results show that the outcome of solving four-term series problems and specifying three answers to implicit relations between intervals as in our experiment is very brittle, probably because of putting a heavy load on working memory resources. But, far more importantly, the number of reorientation asymmetries can now be explained solely by performance variation. Based on the outcome of these robustness analyses, a justified answer can now be given to the question of whether accounts strictly demanding reorientation symmetry are tenable or have to be revised in order to account for reorientation asymmetries as well. Since the percentage of reorientation symmetries is about as high (or descriptively even higher) as the test-retest robustness of answer triples of identical four-term series problems, this corroborates the assumption that model construction works the same from left to right and right to left.

[1] Interestingly enough, nearly all concordant answer triples were also model-consistent (89.6%).

[2] Those were also the subjects that had the highest percentage of correct answer triples, i.e. higher than 89.2%.

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