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# Non-linear wave-particle interactions and fast ion loss induced by multiple Alfvén eigenmodes in the DIII-D tokamak

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## Abstract

A new non-linear feature has been observed in fast-ion loss from tokamak plasmas in the form of oscillations at the sum, difference and second harmonic frequencies of two independent Alfvén eigenmodes (AEs). Full orbit calculations and analytic theory indicate this non-linearity is due to coupling of fast-ion orbital response as it passes through each AE—a change in wave-particle phase  $k \cdot r$  by one mode alters the force exerted by the next. The loss measurement is of barely confined, non-resonant particles, while similar non-linear interactions can occur between well-confined particles and multiple AEs leading to enhanced fast-ion transport.

Keywords: non-linear, energetic particles, multi-wave particle interactions

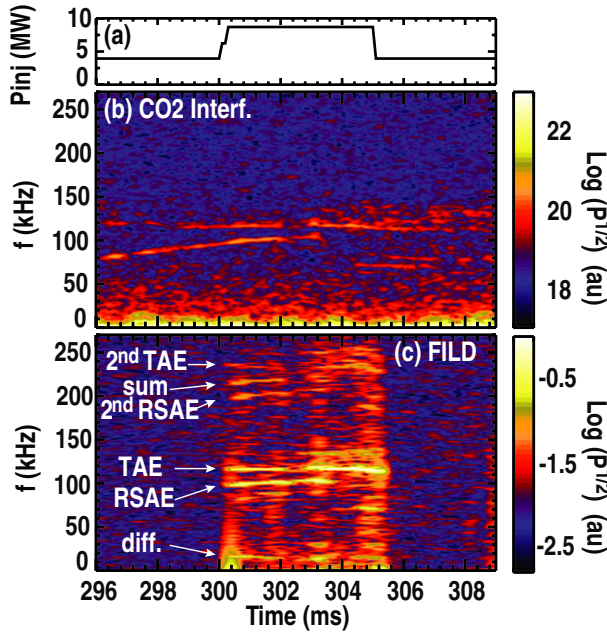
(Some figures may appear in colour only in the online journal)

By modifying the parameters of the medium, two source waves with frequencies  $w_1$  and  $w_2$  can non-linearly generate waves at the sum and difference frequencies,  $w_1 \pm w_2$ . The phenomenon occurs in water waves from fluctuating water depths [1], density variations in plasma waves [2], and in many other media. In this letter, we report sum and difference frequencies that arise from a new mechanism: a stream of energetic ions that traverses two distinct Alfvén eigenmodes (AEs) in a tokamak plasma is modulated at  $w_1 \pm w_2$  (as well as the second harmonics) on their first poloidal orbit before being collected outside the plasma by a fast-ion loss detector (FILD) [3]. Because the waves are relatively small amplitude ( $dB/B < 10^{-3}$ ) and have eigenfunctions with limited spatial overlap, perturbations at  $w_1 \pm w_2$  do not appear in the plasma. It will be shown that the FILD signal is modulated at  $w_1 \pm w_2$  because each wave deflects the ions, which alters the wave-particle phase when the ions traverse the other wave. An analytical model and full-orbit simulations explain the data. Controlling the redistribution and loss of alpha particles and fast ions is necessary for the practical realization of fusion energy. We report a previously unconsidered mechanism by which fusion products may be lost or redistributed in a reactor. Losses are observed from non-resonant, barely-confined ions, while it is likely that similar processes occur for well-confined, resonant fast ions [4] and can occur with other instabilities,

e.g., fishbone [5], energetic particle driven geodesic acoustic mode (EGAM) [6], etc., and external applied perturbations, e.g. resonant magnetic perturbation (RMP)s [7]. In addition, these observations are important for the development of complete, self-consistent (e.g., including non-linear dynamical processes) fast-ion transport models [8].

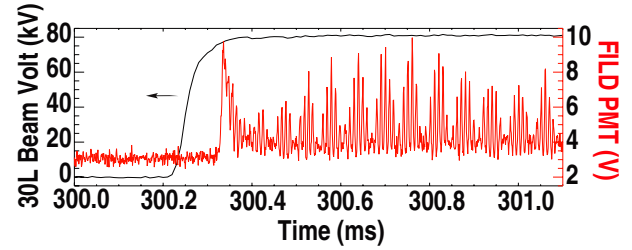
The primary loss diagnostic for this experiment, FILD, is a scintillator-based magnetic charged-particle spectrometer [3, 9]. It uses the tokamak magnetic fields to measure the energy and pitch of fast ions that are lost to the low field side. Escaping ions entering through a collimating aperture impinge on the scintillator. The illumination light is captured by a charge-coupled device (CCD) camera and by fast photomultipliers (PMTs). The pitch and energy of the lost particles can be resolved along with the frequency features (up to 500 kHz).

These experiments study the effect of AEs on fast ion confinement in the DIII-D tokamak [10] during the early current rise phase of plasmas with reversed magnetic shear (i.e., minimum in safety factor  $q_{\min}$  is off-axis). Plasma conditions are plasma current  $I_p = 0.6$  MA,  $q_{\min} \sim 3.7$  at normalized square root of toroidal flux  $r = 0.45$ , toroidal field  $B_T = 2.06$  T, central electron temperature  $T_e = 3.1$  keV, and central electron density  $n_e = 1.45 \times 10^{13}$  cm<sup>-3</sup>. Starting from  $t = 250$  ms, two auxiliary heating neutral beams with total power of 4 MW are continuously injected at 75 kV with

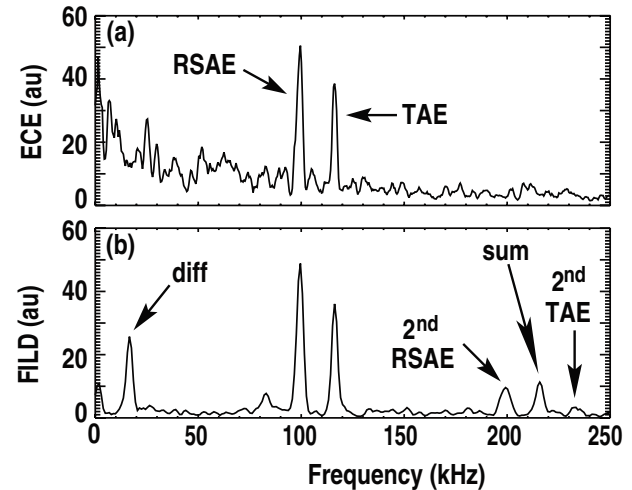


**Figure 1.** Time evolution of (a) total neutral beam injected power, spectrogram of cross power between vertical and radial CO<sub>2</sub> interferometer chords (b) and spectrogram of FILD data (c) in a reversed magnetic shear plasma 146095.

2.3 MW electron cyclotron heating (ECH) deposited at  $r = 0.1$ – $0.4$  at the same time. Another two neutral beams are pulsed (first pulse at 300 ms, pulse length 5 ms, repetition time 100 ms, at 77 kV, total 4.5 MW) for diagnostic purposes (figure 1(a)). Fast-ions from the sub-Alfvénic deuterium beams drive multiple toroidal Alfvén eigenmodes (TAE) [11–13] and reversed shear Alfvén eigenmodes (RSAE) [14–16] as can be seen on spectrogram of CO<sub>2</sub> interferometer (figure 1(b)), ECE radiometer, beam emission spectroscopy (BES), and edge magnetic sensors signals. Coherent losses due to some of these modes are observed on FILD during the short 5 ms diagnostic beam pulse (figure 1(c)) and are identified as prompt beam-ion losses from one (named ‘30L’) of the two pulsed beams. FILD detects losses within 100 ms after the 30L beam is turned on, that is on the same order of the fast-ion poloidal transit time (figure 2). The prompt non-resonant loss mechanism is similar to [17]. The FILD detects coherent losses induced by a pair of RSAE and TAE at not only the fundamental frequencies but also the sum, difference and 2nd harmonic frequencies (figure 3). As shown in figure 1(c), losses synchronized with an RSAE (with frequency up-chirping from 97 to 104 kHz) and a TAE (with nearly constant frequency of 117 kHz) are detected on the FILD. Those modes are also clearly visible on the interferometer (figure 1(b)) and on the ECE (figure 3(a)) and BES diagnostics. But more interestingly, the FILD also detects losses at the difference, sum and 2nd harmonic frequency of the RSAE and TAE (figure 3(b)). Those sum, difference, and 2nd harmonic frequencies are not observed on other diagnostics such as the interferometer (figure 1(b)), magnetic probes, BES, and the ECE diagnostics as shown in figure 3 where the ECE and FILD spectra at  $t = 300.8$  ms are compared. The peaks at  $f = 100$  kHz and  $f = 117$  kHz in the two power spectra correspond to the RSAE and the TAE respectively. The FILD amplitude at the difference frequency is higher than that at the sum frequency.



**Figure 2.** The loss signal (red) rapidly increase when the corresponding neutral beam turns on (consistent with a single poloidal transit time).



**Figure 3.** Spectra of (a) ECE measurements and (b) FILD measurements at  $t = 300.82$  ms.

From the fact that only the fundamental AE frequencies are observed on magnetic probes, interferometers, BES and ECE diagnostics, we conclude that the modes are independent of each other. Bi-coherence [18] analysis confirmed this. While the bi-coherence between the two AEs is very weak as measured inside the plasma by interferometer and ECE diagnostics, the bi-coherence in the FILD signal is strong, indicating that the expelled fast ions have interacted non-linearly with these two independent AEs on their single poloidal bounce. A strong normalized bi-coherence of 0.6–0.75 was found between the AEs and the sum and difference peaks. Similar high bi-coherence was also found for each mode with its second harmonic. Another issue to address is that a non-linear detector response might produce spurious sum and difference frequencies that are not present in the original signal [19, 20]. From a survey of FILD data and from synthetic modelling of the FILD intensity response function [21], it was found that the detector system does not introduce spurious signals at sum and difference frequencies of the modes. For example, there are many cases when the FILD prompt coherent losses at fundamental frequencies are at comparable or higher magnitudes and yet no peaks at the sum and difference frequencies are observed.

The observed losses at the sum, difference and 2nd harmonic frequencies can be understood from a theoretical model in which two independent waves interact with a particle. The electric field of each wave accelerates the particle, but because the wave fields are spatially varying, the response of

the particle becomes non-linear. The combined acceleration on the particles by the two waves is given by:

$$\mathbf{a}(\mathbf{r}, t) = \hat{\mathbf{E}}_1 \sin(\omega_1 t - \mathbf{k}_1 \cdot \mathbf{r}) + \hat{\mathbf{E}}_2 \sin(\omega_2 t - \mathbf{k}_2 \cdot \mathbf{r} + \phi) \quad (1)$$

whereby  $\hat{\mathbf{E}} = q\mathbf{E}/m$  is the electric field,  $\omega$  is the wave frequency,  $\mathbf{k}$  is the wave number, and  $\phi$  is a phase factor. The particle acceleration depends on the particle location, which makes the problem non-linear. One wave pushes the particle leading to a modification of the force on the particle due to the other wave, and vice versa. In order to calculate the displacements due to the waves, assume first that the terms  $\mathbf{k} \cdot \mathbf{r}$  are negligibly small, which then gives, after integrating the acceleration twice in time, the zeroth order displacement:

$$\mathbf{r}_0 = -\frac{\hat{\mathbf{E}}_1}{\omega_1^2} \sin(\omega_1 t) - \frac{\hat{\mathbf{E}}_2}{\omega_2^2} \sin(\omega_2 t + \phi) \quad (2)$$

whereby, without the loss of generality the integration constants, initial velocity and particle location were taken to be zero. This displacement is then used in equation (1) to calculate the first order correction to the displacement. Writing the sine function as complex quantities (i.e.,  $\sin x = \text{Im}\{e^{ix}\}$ ) to simplify the following math and applying the Jacobi–Anger expression, we obtain the following expression for the first order acceleration:

$$\begin{aligned} \mathbf{a}_1 = & \hat{\mathbf{E}}_1 \sum_n \sum_m J_n \left( \frac{\hat{\mathbf{E}}_1 \cdot \mathbf{k}_1}{\omega_1^2} \right) J_m \left( \frac{\hat{\mathbf{E}}_2 \cdot \mathbf{k}_1}{\omega_2^2} \right) \\ & \times \text{EXP}\{i m \phi\} \text{EXP}\{i [(n+1)\omega_1 + m\omega_2] t\} \\ & + \hat{\mathbf{E}}_2 \sum_j \sum_k J_j \left( \frac{\hat{\mathbf{E}}_1 \cdot \mathbf{k}_2}{\omega_1^2} \right) J_k \left( \frac{\hat{\mathbf{E}}_2 \cdot \mathbf{k}_2}{\omega_2^2} \right) \\ & \times \text{EXP}\{i (k+1)\phi\} \text{EXP}\{i [j\omega_1 + (k+1)\omega_2] t\}. \end{aligned} \quad (3)$$

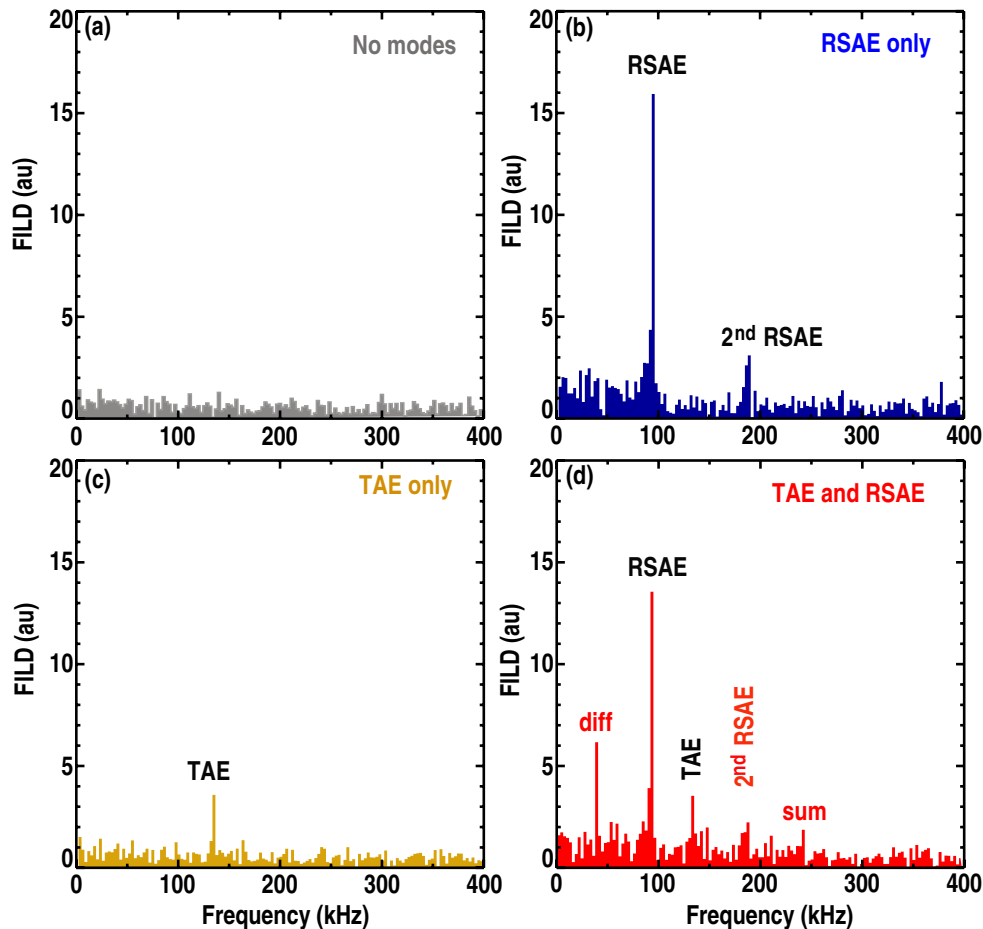
After using the small argument expansion for the Bessel functions:  $J_n(x) \approx \frac{1}{n!} \left(\frac{x}{2}\right)^n$  and keeping only the terms that are linear in the Bessel function arguments, we obtain an expression for the accelerations which gives, after integration, a first order corrected displacement of:

$$\begin{aligned} \mathbf{r}_1 = & -\frac{\hat{\mathbf{E}}_1}{\omega_1^2} \sin(\omega_1 t) - \frac{\hat{\mathbf{E}}_2}{\omega_2^2} \sin(\omega_2 t + \phi) \\ & - \frac{\hat{\mathbf{E}}_1 (\hat{\mathbf{E}}_1 \cdot \mathbf{k}_1)}{8\omega_1^4} \sin(2\omega_1 t) \\ & - \frac{\hat{\mathbf{E}}_2 (\hat{\mathbf{E}}_2 \cdot \mathbf{k}_2)}{8\omega_2^4} \sin(2\omega_2 t + 2\phi) \\ & - \frac{1}{(\omega_1 + \omega_2)^2} \left[ \frac{\hat{\mathbf{E}}_1 (\hat{\mathbf{E}}_2 \cdot \mathbf{k}_1)}{2\omega_2^2} \sin((\omega_1 + \omega_2) t) \right. \\ & \left. + \frac{\hat{\mathbf{E}}_2 (\hat{\mathbf{E}}_1 \cdot \mathbf{k}_2)}{2\omega_1^2} \sin((\omega_1 + \omega_2) t + \phi) \right] \\ & + \frac{1}{(\omega_1 - \omega_2)^2} \left[ \frac{\hat{\mathbf{E}}_1 (\hat{\mathbf{E}}_2 \cdot \mathbf{k}_1)}{2\omega_2^2} \sin((\omega_1 - \omega_2) t) \right. \\ & \left. + \frac{\hat{\mathbf{E}}_2 (\hat{\mathbf{E}}_1 \cdot \mathbf{k}_2)}{2\omega_1^2} \sin((\omega_1 - \omega_2) t - \phi) \right] \\ & + \left( \frac{\hat{\mathbf{E}}_1 (\hat{\mathbf{E}}_1 \cdot \mathbf{k}_1)}{4\omega_1^4} + \frac{\hat{\mathbf{E}}_2 (\hat{\mathbf{E}}_2 \cdot \mathbf{k}_2)}{4\omega_2^4} \right) t \end{aligned} \quad (4)$$

In addition to the linear (or zeroth order) displacements, the change in phase term  $\mathbf{k} \cdot \mathbf{r}$  due to the orbit deflection at a fundamental frequency introduces non-linear displacements at twice of the wave frequencies, and at the sum and difference frequencies. Consistent with experiments, the predicted displacements at frequencies other than fundamental are non-linear and generally small as they are proportional to the wave amplitude squared and inversely proportional to the 4th power of the wave frequency. The analytical model also predicts the displacement at the difference frequency is larger than that at the sum frequency, which is qualitatively consistent with the experimental trend. The second harmonic displacements are predicted to be of the same order as the sum displacement, which is also in line with the observations.

Although the analytical model gives insight into the mechanism of the non-linear orbit response, numerical simulations are needed to model the experiments in toroidal geometry including the magnetic perturbations. Furthermore, the magnitude of the fluctuating loss signal is proportional to not only the displacement but also the ionization gradient of beam-ions [17], at the source of the detected lost particles. All these aspects can be included in simulations with the full orbit-following SPIRAL code [22], in which particles at injected energy are followed in the toroidal field geometry with electric and magnetic field perturbations as calculated with the NOVA code [23] together with the full three dimensional beam deposition profile code [24] and realistic DIII-D machine walls. The non-linear interaction between beam-ions and a RSAE and TAE was simulated by SPIRAL in a standard reversed shear plasma equilibrium similar to figure 1 with  $q_{\min} = 3.6$  at  $r = 0.4$ . The measured density and temperature profiles were used together with the ionization cross-sections from the ADAS database. SPIRAL simulations were done for a single  $n = 4$  TAE at 135 kHz, a single  $n = 2$  RSAE at 95 kHz, and in a run with the two modes combined. The mode amplitudes were set to  $\tilde{n}/n = 1\%$ . A run without any modes was also made as a control. An ensemble of particles was selected that came within 10 cm of the FILD probe and with a pitch within detection range ( $v_{\parallel}/v \leq 0.8$ ) in the equilibrium fields present; this reduced the number of particles that can reach the FILD aperture due to the AEs in a few number of poloidal transits from  $10.5 \times 10^6$  to  $0.75 \times 10^6$ . The particles were then loaded uniformly in a 390 ms interval which ensures that at least 13 periods of the difference frequency are present in the loss signal, and followed until they were lost to the wall or until the end simulated time (390 ms).

Binning the particles that were lost to the FILD location resulted in a synthetic time trace that can be compared to the experimentally observed one (cf figure 2). After taking a Fourier transform of the lost particle time trace, the TAE and RSAE loss peaks are clearly visible (figures 4(b) and (c)) in the runs with single modes while in the run with the two modes combined (figure 4(d)) the losses at the difference, sum and 2nd harmonic RSAE frequencies appear. Although losses at the 2nd harmonic TAE frequency is not found in the simulated spectrum, a bi-coherence analysis of the SPIRAL output revealed a significant bi-coherence between the TAE frequency and its second harmonic, indicating that the 2nd harmonic TAE is present above the noise in these simulations. It would be, however, computationally too expensive to increase the



**Figure 4.** Spectra of the SPIRAL simulated FILD signal (a) without any mode, in the presence of (b) a single  $n = 2$  RSAE, (c) a single  $n = 4$  TAE, and (d) combined  $n = 2$  RSAE and  $n = 4$  TAE.

statistics in the simulations to a point where the small second harmonic TAE peak becomes visible to the eye in the simulated spectrum. This SPIRAL spectrum reproduces the measured ones well with the loss amplitude at the difference frequency much higher than that at the sum frequency. It is also in qualitative agreement with the analytical model.

In conclusion, oscillations in energetic particle losses in the DIII-D tokamak are observed for the first time at the difference, sum and harmonic frequencies for multiple independent AEs. These additional fluctuations are not plasma modes since they are not observed on interferometer, ECE, BES or magnetic sensors. An analytical model and numerical simulations suggest that these losses result from non-linear interactions of non-resonant ions on their first poloidal bounce with two separate AE modes. Data is understood with an analytical model and numerical simulations. The non-linearity is not due to the mode-mode coupling but due to the orbit response to the phase-dependent force from the AE modes: orbit deflection by each mode alters the phase of the particle relative to the other mode with the coupling of the modulations at fundamental frequencies generating the modulations at additional frequencies. Full-orbit following SPIRAL simulations which include the plasma and beam birth profiles, the Alfvén eigenmodes, and DIII-D geometry agree qualitatively with the experiment, including features such as the prompt non-resonant loss feature and the loss

amplitude at the difference frequency being higher than at the sum frequency. However, a complete description of the conditions needed to generate the non-linear loss does not exist yet. These phenomena have only been observed when AE modes are strongly driven in the plasma, but strong modes do not always cause non-linear loss signals on the FILD. It is more often observed when there is an RSAE present, which might be because of its relatively large wavenumber. The less frequent observation during two TAEs may be due to the smaller difference frequency, which can be lower than the typical poloidal transit frequency of beam-ions ( $\sim 10$ – $15$  kHz for these plasmas).

Although it is well known that multiple modes cause larger fast-ion transport than individual modes, the underlying physics isn't always clear or well tested. With the pitch and energy measurements using the FILD, the full-orbit-following simulation SPIRAL code and an analytical model, we have been able to, for the first time, obtain a clear detection and understanding of one type of multi-mode fast-ion interaction. The prompt loss mechanism for these energetic ions enables the clear observation of the non-linear modulation at the edge loss detector that otherwise might be washed away or complicated by other modes after several poloidal transits. Similar non-linear interactions can happen for both trapped and passing particles, for both resonant and non-resonant particles, at both the edge and core of plasma, and in both fusion and space

plasmas. In addition, the non-linear features in the data provide a unique opportunity for code validation that is crucial for making reliable predictions for ITER and future reactors.

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