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R. G. Moorhouse and H. Oberlack

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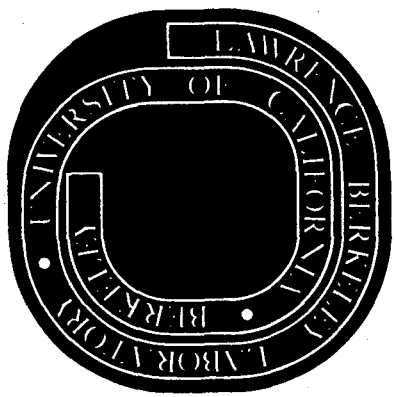

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ANALYSIS OF PION PHOTOPRODUCTION AND THE QUARK MODEL*

R. G. Moorhouse
Department of Natural Philosophy
Glasgow University, Glasgow, Scotland

and

H. Oberlack
Lawrence Berkeley Laboratory, University of California
Berkeley, California

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ABSTRACT

($N^*N\gamma$) couplings for resonances in the first, second, and third resonance region are determined in a continuous-energy partial wave analysis of single-pion photoproduction off protons and neutrons using fixed-t dispersion relations. Several couplings are determined for the first time. Most results, especially for the principal resonances, agree well with the predictions of a relativistic quark model, while the $p_{11}(1470)$ disagrees markedly with its present multiplet assignment.

We have undertaken a continuous-energy partial wave analysis of pion photoproduction from 240 to 1220 MeV photon laboratory energy, comprising nearly all the existing data on the reactions (i) $\gamma p \to \pi^+ n$, (ii) $\gamma p \to \pi^0 p$, (iii) $\gamma n \to \pi^- p$. The only previous analysis on all the reactions (i) - (iii) within the same large energy range was carried out five years ago by Walker [1] and subsequently updated in some respects for comparison with the quark model [2]. Our analysis employs a different method and a large amount of new data; consequently, while confirming some of the important features of Walker's analysis [1, 2] and other smaller analyses [3], we determine many partial waves and resonance couplings for the first time. Our determination of the ($N^*N\gamma$) couplings of the principal, and some of the other, resonances compares very favorably with the predictions of the L-excitation quark model of resonances, as we shall show.

The difficulty with the analysis of photoproduction is that for each process (i) - (iii) there are four independent complex amplitudes at each energy and angle, giving seven independent real quantities apart from the overall phase. Thus to make an independent determination at one energy and angle we need to make at least seven experimental measurements—say one differential cross section and six polarization measurements. In pion-nucleon scattering the corresponding number is three measurements. Because the amplitudes are continuous functions of energy, this complete set of measurements does not seem necessary in either case. Unfortunately the photoproduction data is of poor quality and relatively much less complete than the pion-nucleon data.

However, from the pion-nucleon partial wave analysis we already have the list of s-channel resonances that are active in photoproduction through the processes $\gamma N \to N^* \to \pi N$. Thus, as far as these processes are concerned, the only unknowns to be determined from the photoproduction data are the $(N^* N\gamma)$ couplings. Moreover there are indications from existing data and analyses [1-3] that resonances dominate the imaginary parts of the amplitudes and, insofar as this is true, the analysis of pion-photoproduction will be correspondingly simplified. Also it is a well-established fact that the real part of the amplitudes is important–in particular, the pion exchange in charged pion photoproduction that can be expressed gauge-invariantly by the Born approximation.

The preceding considerations have led us to parametrize the imaginary parts of the photoproduction amplitudes (in terms of resonances and background); but we calculate the real parts from these imaginary parts by fixed-t dispersion relations, which by definition include the Born terms in the real parts, and we determine the parameters by fitting the resulting complex amplitudes to experiment.

More explicitly, the fixed-t dispersion relations for the invariant amplitudes $A_i^{\pm,0}$, where $i = 1, 2, 3, 4$ and the superscripts refer to the charge of the produced pion, are

$$\mathrm{Re}A_i^{\pm,0}(s,t) = B_i^{\pm,0}(s,t) + \int_{(M+m)^2}^{\infty} ds' \left[\frac{\mathrm{Im}A_i^{\pm,0}(s',t)}{s'-s} + \xi_i \frac{\mathrm{Im}A_i^{\alpha,0}(s',t)}{s'-u} \right], \quad (1)$$

where $B_i^{\pm,0}$ are the Born terms, and M and m are the nucleon and pion masses respectively. For $i = 1, 2, 4$ the symbol α represents $\pm$ and $\xi_i = +1$, while for $i = 3$ the symbol α represents $\mp$ and $\xi_i = -1$. We denote the integrand in (1) by $I_i^{\pm,0}(s', s, t)$ and rewrite (1) as the sum of

a low-energy integral and a high-energy integral:

$$\mathrm{Re}A_i^{\pm,0}(s,t) = B_i^{\pm,0}(s,t) + \int_{(M+m)^2}^{\Lambda^2} ds' I_i^{\pm,0}(s',s,t) + \int_{\Lambda^2}^{\infty} ds' I_i^{\pm,0}(s',s,t), \quad (2)$$

where $(M+m)^2 < s < \Lambda^2$ is the region containing data that we are analyzing. The imaginary parts of the invariant amplitudes $\mathrm{Im}A_i(s',t)$ are expressed as sums of partial waves (we use parity-conserving helicity amplitudes) whose parameters, such as $(N^* N\gamma)$ resonance couplings or other parameters associated with background, are to be varied to fit the data.

Two points of potential difficulty arise. Firstly, the second integral of (2) contains parameters referring to the amplitude outside the data region. Obviously without direct data restriction on the high-energy imaginary part we cannot hope to determine these, in principle infinitely many, parameters. The problem is to find an adequate parametrization involving a minimum number of parameters; the smaller the contribution to $\mathrm{Re}A_i$ from the high-energy integral in (2), the more satisfactory is the fixed-t dispersion relation analysis. At this stage we have dealt with the problem rather arbitrarily by representing $\mathrm{Im}A_i^{\pm,0}(s',t)$ for $s' > \Lambda^2$ as a sum over a few pseudo-resonances, and we do <u>not</u> give physical significance to the final values of the parameters of these pseudo-resonances. One could envisage parametrizing $\mathrm{Im}A^{\pm,0}(s',t)$ for $s' > \Lambda^2$ in a Regge or other continuous form.

The second difficulty is that within the data region, $s' < \Lambda^2$, $\mathrm{Im}A_i^{\pm,0}(s',t)$ is represented as a sum of partial waves, cut off at some upper limit of angular momentum. The convergence of the partial wave series is not proved except for certain processes within certain regions (the Lehman ellipse of convergence). However, convergence

can hold outside this region, and it is likely that a cut off series provides a good approximation in a considerably extended region. Devenish, Lyth, and Rankin [5] have argued on the basis of the Mandelstam double spectral representation that the cut off series is good for $-t \lesssim 1.0$ (GeV/c)^2 in $\pi^\pm$ photoproduction and $-t \lesssim 1.5$ (GeV/c)^2 in π^0 photoproduction. We have introduced a parametrized correction with 12 parameters to the partial wave series, non-zero only for larger t.

The expansion of the Im $A_i^{\pm,0}(s,t)$ in terms of the helicity amplitudes is through standard formulae, which would be unilluminating to give here. We use the partial wave amplitudes $A_{1/2\,\ell\pm}(s)$ and $A_{3/2\,\ell\pm}(s)$ popularized by Walker [1], where $A_{1/2\ell\pm}$ correspond respectively to total helicities 1/2, 3/2 of the initial γN system and $\ell\pm$ is the usual orbital and total angular-momentum label of the final πN system. With these amplitudes,

$$\text{Im } A^{\pm,0}(s,t) = \text{linear function } \left[\text{Im } A^{\pm,0}_{1/2\,\ell\pm}(s),\ \text{Im } A^{\pm,0}_{3/2\,\ell\pm}(s)\right]. \quad (3)$$

Each partial wave amplitude $A^{\pm,0}_{1/2\,\ell\pm}(s)$, $A^{\pm,0}_{3/2\,\ell\pm}(s)$ is a T-matrix element whose imaginary part we obtain from the corresponding K-matrix through the formula $T^{-1} = K^{-1} - iq$, where q is the diagonal matrix of center-of-mass momenta and

$$K_{ij} = \sum_r s^{(r)} \frac{\gamma_i^{(r)} \gamma_j^{(r)}}{E^{(r)} - E}, \quad (4)$$

where i, j = 1, 2, 3. Channel 1 is πN, channel 3 is γN, and channel 2 is a pseudo-channel representing all other hadrons. The sum on the right-hand side of (4) comprises both resonance poles, where $E^{(r)}$ is within the data range, and background poles where $E^{(r)}$ is without the

data range. The factor $s^{(r)} = \pm 1$, but is restricted to be +1 when $E^{(r)}$ is within or near the data range so as not to violate causality. The $\gamma_i^{(r)}$ of (4) are partial widths that contain kinematic barrier factors, $\gamma_1^{(r)}$, $\gamma_2^{(r)}$ being previously determined from a fit to πN elastic scattering amplitudes and $\gamma_3^{(r)}$ being the parameters to be determined from fitting the photoproduction data as described above.

Using up to 4007 data points [8] from three reactions ($\gamma p \to \pi^+ n$, $\gamma p \to \pi^0 p$, $\gamma n \to \pi^- p$) in the energy range 240 to 1220 MeV, we obtained seven reasonable fits to the data. The best of these fits had a χ^2/data point = 5.4 with 75 variable parameters, and the worst had a χ^2/data point = 9.7 with 52 variable parameters. We were not surprised by these large χ^2. On the one hand there are obvious inconsistencies in the data, but we are mostly in agreement with the average of inconsistent experiments. On the other hand we use a small number of parameters to describe the data from three reactions over a very large energy range. So we would expect, from the stiffness of our parametrization, an inability to fit all details; this shows itself only in three or four places. In the case of pion photoproduction, where (except possibly in $\gamma p \to \pi^+ n$) there are appreciable systematic errors in the data, a certain degree of stiffness in the parametrization clearly is an advantage.

Detailed results as well as a more thorough discussion of the methods used will be given elsewhere [6]. As an example of the quality of both the experimental data and of our fits, we show in fig. 1 the differential cross sections for the three reactions fitted at k = 850 MeV/c.

Our parametrization contains all the established pion-nucleon resonances within our energy range. One of the chief interests of

pion photoproduction data and partial wave analysis is the level of agreement between the ($N^*N\gamma$) couplings and the prediction of the quark model. The predictions of the nonrelativistic quark model and a quark model with more relativistic features due to Feynman, Kislinger, and Ravndal [7] are similar.

Within the framework of the relativistic quark model of ref. 7 we have calculated the predicted resonance coupling for all the resonances used in the present analysis. Table 1 gives our partial wave analysis results [the amplitudes being in accordance with the definitions in ref. 2] together with the multiplet assignments and quark-model predictions. We indicate with an asterisk those cases where the sign of a calculated coupling can not depend on the particular form of the quark wave function.

Firstly, we comment on the $p_{33}(1236)$, $d_{13}(1520)$, and $f_{15}(1690)$, which are the most prominent resonances in pion-nucleon elastic scattering within their respective energy regions and are also nearly pure resonances with little background. For the p_{33} the helicity amplitudes disguise the fact that all the fits have a very small E2 amplitude, in agreement with the quark-model selection rule. The $f_{37}(1950)$ resonates outside the energy region covered by this analysis, but nevertheless has a strong influence on χ^2 largely through the dispersion relation and partly through the tail of its imaginary part, and we consider our determination of the coupling constant to be correct within about the errors given. In particular the sign determination can be regarded as firm. We regard the sign of all the larger couplings of the prominent resonances as completely firm; experience in other reactions shows that relative signs of resonant amplitudes are usually very well determined. All the couplings of these "pure" resonances

agree in sign with the quark model, and the predicted zero for the $A_{3/2}$ amplitude of f_{15}^0 is nearly attained.

To make a stringent comparison with the quark model we may take only the larger couplings of the prominent resonances, and only those where quark-model predictions are "starred" in Table I. We find seven such cases where experimentally the sign is completely sure and which theoretically depend on, and only on, the Clebsch-Gordan coefficients of the quark model (the same in either the "relativistic" or "nonrelativistic" models). Furthermore, those couplings which the quark model predicts to be large are also large experimentally.

We emphasize that the agreement in sign is highly nontrivial. For example SU3, with the addition of the g_D/g_F ratio and vector dominance, would only give values for the ratios $A_{1/2}^+/A_{1/2}^0$ and $A_{3/2}^+/A_{3/2}^0$ for an isospin-1/2 resonance decay. At the Daresbury Conference, Walker [2] already drew attention to the agreement of some of the d_{13} and f_{15} numbers from his revised analysis with the non-relativistic quark model. He did not have helicity-1/2 values for $d_{13}^0(1520)$ or $f_{15}^0(1690)$, which, according to our fits, also agree in sign. (We may note that in the non-relativistic quark model the spring coupling can be adjusted to give a small coupling for the A of d_{13}^+ and f_{15}^+. The model of ref. 7 seems more rigid in this respect.)

We confirm previous findings that the $s_{11}(1545)$ is predominantly excited via isovector coupling. Mixing of the $s_{11}[8,2]$ and $s_{11}[8,4]$ with a mixing angle of 35° as given by Faiman and Hendry [9] will reduce the predicted coupling and give better agreement with experiment. The $p_{11}(1470)$ disagrees markedly with its present assignment, but there may be strong mixing here involving $p_{11}(1750)$.

Because the recently discovered [10, 11] $d_{13}(1700)$ is very inelastic, we would expect the mixing with $d_{13}(1520)$ to be small, and so will not disturb the conclusion we have drawn on the agreement with the quark model for the $d_{13}(1520)$. The non-principal resonances above the second resonance region are less well determined, perhaps partly because of background. However, some results can be regarded as indicatory.

We find that the $d_{15}(1670)$ is only weakly photoproduced on proton targets, in agreement with the quark model. The photon coupling constants of the $d_{33}(1640)$ are determined in this analysis for the first time. Although we find a remarkable agreement with the quark-model predictions, further investigation is needed, since strong interferences between the s_{31} and the d_{33} can be observed in our fits. More data in the energy region 1600-1650 MeV are clearly needed to resolve the ambiguities [12]. Our results for $p_{11}(1750)$ disagree with the quark-model predictions. As remarked above, this may be a sign of mixing among different multiplets. We have tried to look for a signal of a $d_{13}(1700)$ and $p_{33}(1680)$, but do not find compelling evidence.

In conclusion, we emphasize that our analysis agrees very well with many of the firm predictions of the quark model and has no significant disagreement at the present level of experiment and theory. Similar checks on the quark-model predictions can be obtained through partial wave analyses of the reactions $\pi N \to \rho N$ and $\to \pi\Delta$ within the resonance region. They yield results on the relative signs of resonance couplings such as $(N^* N\rho)$ and $(N^* \Delta\pi)$, which are not predicted by SU3 but are predicted by the quark model. Herndon et al. [11] have made such an isobar-model analysis from experiments on $\pi N \to \pi\pi N$, but full results are not available yet. Similar experiments and analyses on

Y^* decays such as $\bar{K}N \to \Sigma(1385)\pi$, or $\bar{K}N \to \Sigma(1385)\pi$, or $\bar{K}N \to \rho Y$, ωY are important, particularly those involving relative signs of resonances which are likely to be unmixed, such as $d_{05}(1820)$, $f_{05}(1815)$, $f_{15}(1910)$, and the possible $f_{07}(2030)$ belonging to the possible {70} $L = 2^+$ multiplet.

FOOTNOTES AND REFERENCES

*Work done under the auspices of the U. S. Atomic Energy Commission.

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Table 1. Average resonance couplings from seven fits to the data compared with quark-model predictions. The result from the partial wave analysis is an average over seven fits and the error is the spread over the seven fits; directly underneath the partial wave analysis result we give the quark-model result for the usual assignment of the resonance to an {SU6} L, [SU3, 2S+1] multiplet. An asterisk denotes that the quark-model result does not involve a difference of two terms. Table 1a comprises resonances assigned to the $\{56\}$ $L = 0^+$ and $\{70\}$, $L = 1^-$ multiplets and I(b) the $\{56\}$ $L = 2^+$, $\{56\}_2$ $L = 0^+$, $\{70\}_2$, $L = 0^+$ multiplets where the suffix denotes radial excitation. In table 1b we also give quark-model results for some resonances for which we do not have partial wave results since they are outside our data range. $A_{1/2}$ and $A_{3/2}$ denote decays through helicity-1/2 and helicity-3/2 states respectively, and superscripts + and 0 denote decays of charge +1 and charge 0 particles respectively. Units are $GeV^{-12} \times 10^{-3}$.

Table 1a

	N^*(mass) / [SU3, $2S_{quark}+1$] J^P	$A^+_{1/2}$	$A^+_{3/2}$	$A^0_{1/2}$	$A^0_{3/2}$
$\{56\}L=0^+$	$p_{33}(1230)$	-142 ± 6	-259 ± 16		
	[10, 4] $3/2^+$	-108*	-187*		
$\{70\}L=1^-$	$s_{11}(1545)$	53 ± 20		-48 ± 21	
	[8,2] $1/2^-$	156		-108	
	$d_{13}(1512)$	-26 ± 15	194 ± 31	-85 ± 14	-124 ± 13
	[8,2] $3/2^-$	-34	109*	-31	-109*
	$s_{31}(1620)$	90 ± 76			
	[10,2] $1/2^-$	47			
	$d_{33}(1635)$	68 ± 42	22 ± 52		
	[10, 2] $3/2^-$	88	84*		
	$s_{11}(1690)$	66 ± 42		-72 ± 66	
	[8, 4] $1/2^-$	0		-30	
	$d_{13}(1700)$	3 ± ?	20 ± ?	-28 ± ?	27 ± ?
	[8, 4] $3/2^-$	0*	0*	10*	-40*
	$d_{15}(1670)$	11 ± 12	21 ± 20	10 ± 40	-35 ± 14
	[8, 4] $5/2^-$	0*	0*	38*	-53*

Table 1b

	N^*(mass) / $[SU3, 2S_{quark}+1]J^P$	$A^+_{1/2}$	$A^+_{3/2}$	$A^0_{1/2}$	$A^0_{3/2}$
$\{56\}\ L=2^+$	$[8,2]3/2^+$	-11	30	30	0*
	$f_{15}(1690)$	-8 ± 4	100 ± 12	17 ± 14	-5 ± 18
	$[8,2]5/2^+$	-10	60*	30*	0*
	$[10,4]1/2^+$	-30			
	$[10,4]3/2^+$	-30	50		
	$f_{35}(1870)$	-60 ± ?	-100 ± ?		
	$[10,4]5/2^-$	-20	-90		
	$f_{37}(1950)$	-133 ± 46	-100 ± 41		
	$[10,4]7/2^+$	-50*	-70*		
$^2\{56\}\ L=0^+$	$p_{11}(1470)$	-55 ± 28		2 ± 25	
	$[8,2]1/2^+$	27		-18	
$^2\{70\}\ L=0^+$	$p_{11}(1750)$	26 ± 28		27 ± 22	
	$[8,2]1/2^+$	-40		10	

Figure Caption

Fig. 1. Differential cross sections at k = 850 MeV/c for the reactions (a) $\gamma p \to \pi^+ n$, (b) $\gamma p \to \pi^0 p$, (c) $\gamma n \to \pi^- p$. The solid curves are from a fit of this analysis. The data are described in ref. 9.

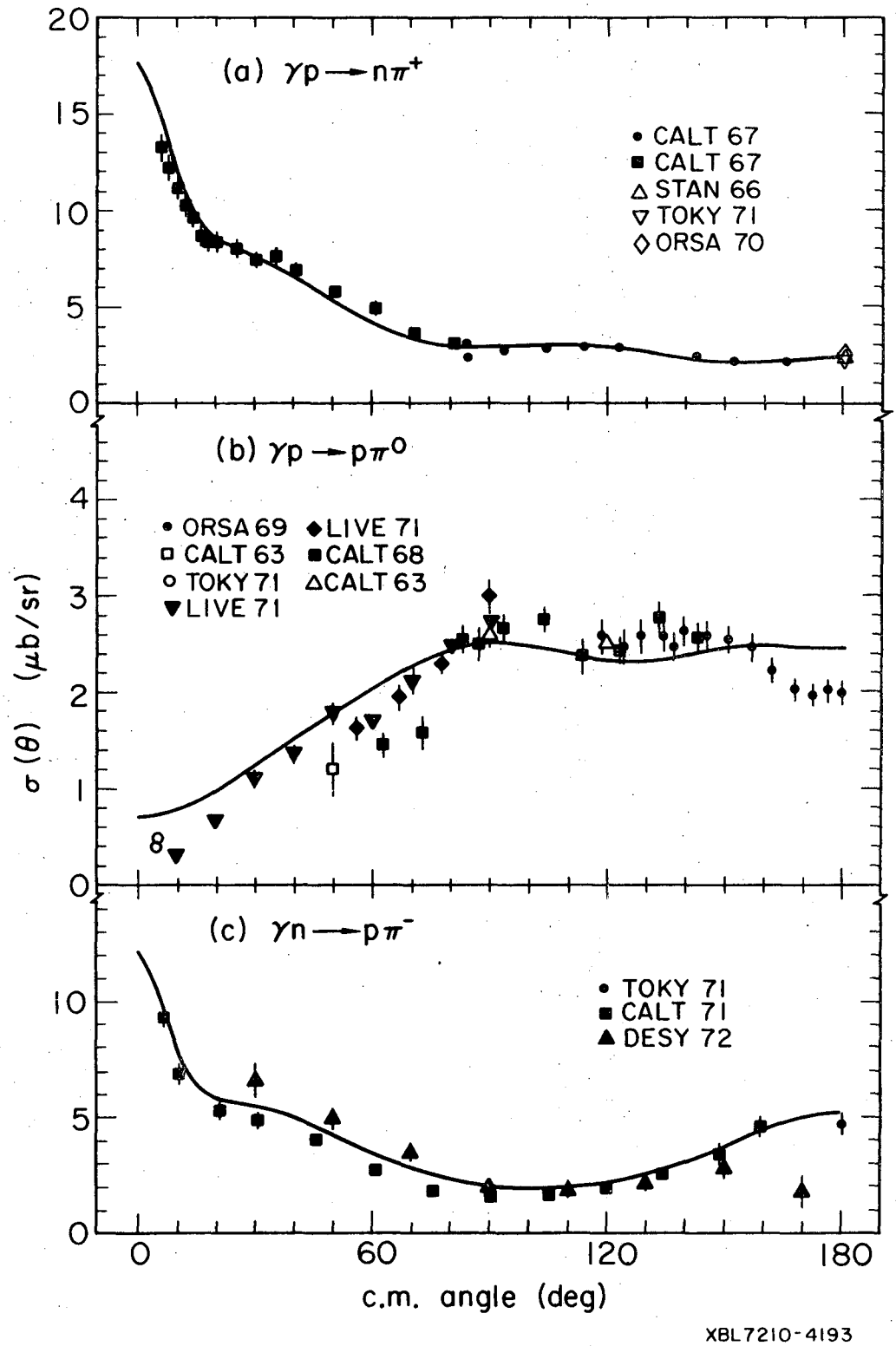
(a) γp → nπ+
• CALT 67
■ CALT 67
△ STAN 66
▽ TOKY 71
◇ ORSA 70
(b) γp → pπ0
• ORSA 69
□ CALT 63
○ TOKY 71
▼ LIVE 71
◆ LIVE 71
■ CALT 68
△ CALT 63
8
(c) γn → pπ−
• TOKY 71
■ CALT 71
▲ DESY 72
σ (θ) (μb/sr)
c.m. angle (deg)
0
60
120
180
XBL7210-4193

Fig. 1

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LAWRENCE BERKELEY LABORATORY
UNIVERSITY OF CALIFORNIA
BERKELEY, CALIFORNIA 94720