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Heavy Quark Jets

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Besides its theoretical interest, fragmentation of heavy quarks is an important information needed to experimentalists. Fragmentation functions appear everywhere when one wants to estimate experimental signatures of heavy quarks at high energies. The energy and angular distributions of decay products of heavy quarks are critically dependent on how heavy flavored hadrons fragment from heavy quarks as well as how heavy quarks are produced.

I will review our present understanding of heavy quark fragmentation and discuss on expected behaviors of heavy quark jets.

1. Short-distance vs long-distance parts of fragmentation

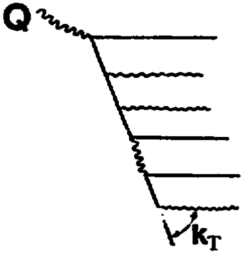
Since the leading logarithm summation technique was established in QCD,[1] it has been known that a fragmentation function involves a scale breakin~ dependent on Q^2, the energy-momentum scale of production. For the light quarks, the moments of fragmentation functions should show the Q^2 dependence as

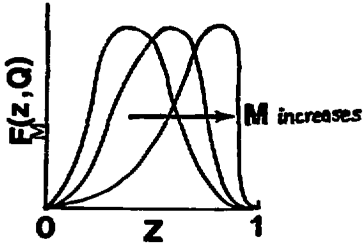
$$\int_0^1 z^{n-1} F(z,Q)\, dz \simeq c_n (\log Q)^{-\gamma_n} \qquad (1)$$

where z is the fraction of energy given to the observed hadron, γ_n $(n = 1,2\cdots)$ are related to anomalous dimensions of relevant operators or probability functions of parton emission.[2] The same leading log summation works for heavy hadrons if Q is much larger than the masses of heavy quarks. Repeating the derivation leading to (1) above, we find for the fragmentation of heavy quark of mass M

$$\int_0^1 z^{n-1} F_M(z,Q)\, dz \simeq c_n \left(\frac{\log Q}{\log M}\right)^{-\gamma_n} \qquad (2)$$

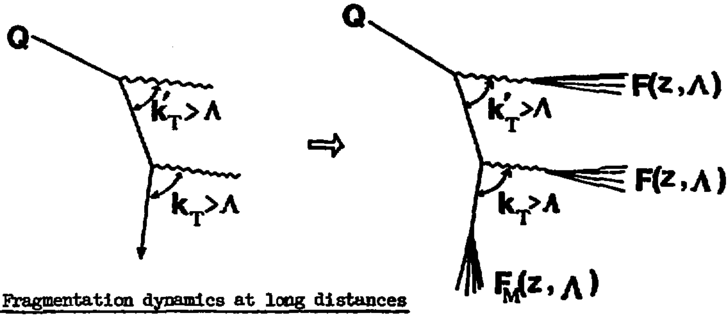
in the limit of $\log Q/\log M \to \infty$ with $\log M/\log \Lambda \gg 1$. Here Λ is the scale of strong interactions, normally chosen to be $0.5 \sim 1$ GeV. The fractional powers γ_n are identical to those in (1) and describe the short distance dynamics of QCD, while its coefficient c_n involves all of the long-distance dynamics for which a perturbative calculation fails and therefore we have no rigorous way to calculate at present. In the leading log ladder summation using the axial gauge of QCD, $(\log Q/\log M)^{-\gamma_n}$ results from the most singular terms in the region where the transverse momenta of emitted partons extend from $O(\Lambda)$ to $O(Q)$, while c_n picks up the contribution from the region of k_T no larger than $O(\Lambda)$.





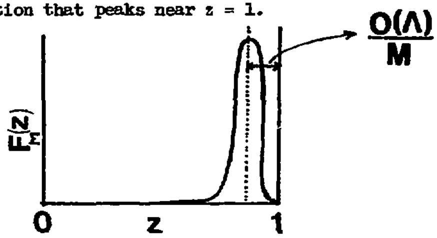
The short-distance part $(\log Q/\log M)^{-\gamma_n}$ shows an interesting behavior as a function of M. It deforms the shape of $F_M(z,Q)$ towards the region of large values of z as M increases with Q fixed. However, testing such an M dependence may not be more than of theoretical interest unless we know the M dependence of c_n and even when c_n is known, w^ will not go to energies high enough to measure accurately the short-distance effect in the next decade.

I will concentrate in the present talk on the long-distance part of fragmentation dynamics, represented by c_n in the formula (2) above. Though the separation of the long-distance part from the short-distance part may look rather theoretical, the long-distance part of $F_M(z,Q)$ will be sufficient to carry out all practical calculations on the heavy quark fragmentation accompanying no more than a few wide angle sub-jets at energies of our interest. With such $F_M(z,Q)$ given, our prescription is to draw first skelton jet diagrams by treating all sub-jets of $k_T > \Lambda$ as independent jets and then to apply $F_M(z,\Lambda)$ to the well-collimated low k_T jets of heavy quarks.



2. Fragmentation dynamics at long distances

We have no established prescription to calculate the long-distance QCD at present. Naturally, we are to be guided by models based on experimental observations relevant to such dynamics and/or theoretical considerations drawn from general QCD framework. I will present here three different models. All of them unanimously lead to the fragmentation function that peaks near z = 1.



This conlusion is almost kinematical. It is very difficult to get anything different from the one drawn above for a heavy hadron of mass M as long as Q/M is much larger than unity.

(a) Universal hadronization in rapidity plot[3]

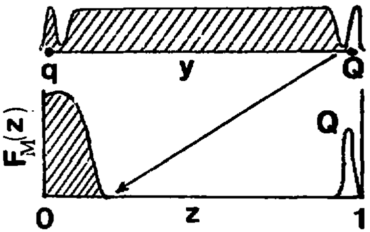
We have known the low p_T physics in the light hadron production in hadron-hadron collisions. The low p_T dynamics is best characterized in the rapidity plot. Produced light hadrons fill in uniformly the rapidity separation between two leading particles going back to back. The way how produced hadrons populate in the rapidity gap does not depend on what the leading particles are.

Let us apply this picture to the process where a fast heavy quark is moving away with energy E from a light quark at rest. To make the argument more persuasive and plausible,[3] go to the frame where the heavy quark Q is at rest and the light quark q is moving away from Q.

Since the rapidity is given by

$$y = \frac{1}{2} \log \frac{E + p_{\parallel}}{E - p_{\parallel}} \simeq \log(E/M) ,$$

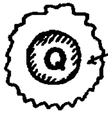
the distance of the rapidity gap between q and Q is $\log(E/M)$, which is shorter by $\log(M/m_T)$ than the gap between two light quarks of laboratory energy E. Here m is the light quark mass and $m_T = \sqrt{m^2 + p_T^2}$. When Q is struck by q, a bunch of light hadrons are emitted in the direction of q rather than in the direction opposite to the incident q. These emitted hadrons fill in the rapidity gap uniformly and universally (independent of flavors of q and Q).



It is shown above how the leading particles and hadronized light particles are located in the rapidity plot. Let us replot this rapidity distribution into the energy distribution, or more precisely, the z-distribution. Note that hadrons filling in the gap are light particle of $p_T \lesssim 1$ GeV . Even when the leading hadron of heavy flavor and a light hadron are found at the extreme left end of the rapidity plot, the heavy hadron remains near $z = 1$ and the light hadron is transformed down to the neighborhood of $z = 0$, at $z = O(m_T/M)$. The entire spectrum of light hadrons are thrown into the small neighborhood of $z \lesssim m_T/M$.

(b) A heavy quark as a sizzling fire ball[4]

When a heavy quark is produced in a deep inelastic process, it is in an excited state with a cloud of light hadrons around it. It keeps fragmenting the light hadrons as it moves away fast. The parton model (restricted to low p_T according to our opening remark in the preceding section) implies that the excited heavy quark state is not so far off the mass shell. Then ask what the invariant mass of the excited heavy quark state is at the time of production.



The invariant mass of such a state, which is the sum of the heavy quark mass and the invariant mass of the light hadron cloud, is expected to be larger than the heavy quark mass by only a small finite amount independent of the heavy quark mass,

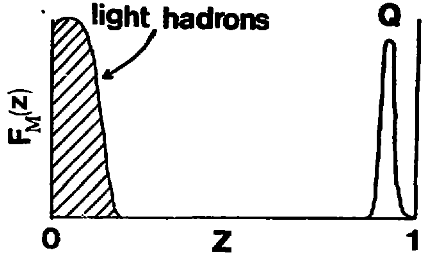
$$M_{\text{fire ball}} = M + m_{\text{cloud}} , \tag{3}$$

$$m_{\text{cloud}} \text{ independent of } M \text{ and } \ll M. \tag{4}$$

Recall that we are interested in $F_M(z,Q)$ when emitted transverse momentua of light hadrons are restricted to be $k_T < O(\Lambda)$. The invariant mass of the cloud m_{cloud} is indeed of the order of Λ under this restriction. The assumption that m_{cloud} is independent of M is consistent with the current picture of strong interaction, QCD. The strong interaction coupling is the universal gauge coupling independent of flavors. Whether a quark is heavy or light is determined not by self-energy due to strong interactions, but by its coupling to

Higgs particles that is a weak coupling having nothing to do with strong interactions. There is a small (logarithmic) asymptotic freedom effect dependent on M when k_T becomes larger than Λ. This tends to weaken the strength of the heavy quark coupling for a given value of k_T. This mild M dependence is precisely the log M effect at short distances as is seen in Equation (2). After we separate the short-distance part, the M independence of the cloud mass is fully consistent with QCD, if not proven by it.

Once we are given this picture, it is a matter of Lorentz transformation to obtain the z-distribution of heavy flavored hadron and light hadrons. The heavy flavored hadron carries $z = M_H/(M + m_{cloud})$, where M_H ($\simeq M$) is the mass of the physical hadron with heavy flavor. All the light hadrons can carry only as much as $z < m_T/(M + m_{cloud})$, even if they are moving as fast as the heavy hadron.



(c) QCD ladder summation in the region of $k_T < \Lambda$.

The ladder summation in the axial gauge can be justified only in the leading log Q approximation to $F_M(z,Q)$. The origin of powers of log Q is in the k_T integrals extending up to $O(Q)$. We adopt this calculation in the region of $k_T < \Lambda$ without justification. It is a bremsstrahlung model of gluons and light quarks with $k_T < \Lambda$ from a heavy quark. When k_T is restricted to be $\lesssim$ 1 GeV, we can ignore heavy quark pair production in the middle of ladder and consider only gluon emission from a heavy quark.

It is then easy to observe that if a gluon of k_T is emitted, the heavy quark propagator by one vertex before acquires a large damping effect

$$\frac{1}{(p+k)^2 - M^2} \simeq \frac{1}{\frac{(1-z)}{z} M^2 + \frac{z}{1-z} k_T^2}$$

as compared with the case of a light quark propagator. Such a damping is ineffective if an emitted gluon carries away only a small fraction of energy $z_G = 1 - z = O(k_T/M)$ from the heavy quark. Namely, the emitted gluons, which eventually hadronize into light hadrons, are confined in the small region of $z \lesssim k_T/M$.

By replacing the kernel

$$\frac{g^2}{k_T^2 \log k_T^2} \int_{z_{min}}^{1} \frac{dz}{(1-z)_+} \frac{1+z^2}{z}$$

by

$$g^2 \int_{z_{min}}^{1} \frac{dz}{(1-z)_+} \cdot \frac{z^3(1+z^2)\, k_T^2 + (1-z)^4 M^2}{[(1-z)^2 M^2 + z^2 k_T^2]^2 \log[k_T^2 + \frac{(1-z)^2}{z^2} M^2]}$$

in the nonsinglet channel calculation of $e^+e^- \to$ light hadron + anything[5], we obtain for the moments of $F_M(z)$

$$\int_0^1 z^n \times F_M(z)\, dz \simeq [\, 1 - O\left(\frac{n\Lambda}{M}\right)]\, f(\Lambda) \qquad \text{for } n \ll M/\Lambda , \quad (5)$$

where $f(\Lambda)$ is a function of Λ . This leads to $F_M(z)$ that peaks at $z = 1 - O(\Lambda/M)$ like a δ-function.

We have thus shown that the three models, all consistent with experimental and theoretical observations at hand, lead to the fragmentation function peaking near $z = 1$ like a δ-function. What would we have to assume if we would like to obtain fragmentation functions in which a heavy hadron does not appear at the high z end ?

In the universal hadronization model (a), it must happen that when a heavy hadron is struck by a light quark, the heavy quark starts moving fast in the direction of the light incident, emitting energetic light particles into the backward cone or else leaving behind an enormous number of soft light hadrons. Then the heavy hadron would be buried in the middle of the rapidity plot. In the fire ball model (b), the invariant mass of the light hadron cloud must increase proportionally as the heavy quark mass increases. As it was remarked, this is highly implausible in the light of the QCD dynamics. In the ladder summation, it looks impossible to draw diagrams which allow a heavy quark to lose its significant fraction of energy without causing the propagator damping of $O(k_T/M)$.

3. Relevance or irrelevance to super high energy experiment

There is a subtle difference between Model (a) and Model (b) in the treatment of light hadrons produced in jets. In Model (a) all the light hadrons are considered including vees and therefore the multiplicity of light hadrons is linearly related to the length of the hadronization plateau in the rapidity plot, while in Model (b) only the light hadrons of $z \neq 0$ are considered as the light hadron cloud. Separation of vees from nonvees is not unambiguous in real experiment. We include here in a jet only those hadrons for which $p_{\parallel}/m_T$ is no smaller than of the order of the Lorentz factor $\gamma_Q = P_Q/M$ of the heavy quark. (It can not be larger than that.) In other words, a jet is defined to consist of particles within a cone of half-angle $\theta = O(1/\gamma_Q)$. To do such an analysis, we have to know event by event whether or not a heavy quark is produced and what the heavy quark energy is. The former is probably possible by geometrical observations such as sharpness of jet cones, multiplicity of all final hadrons (mentioned later), and so forth. The latter is hard for the associated production in hadron-

hadron collisions, but trivial for the production in e^+e^- annihilation.

With the definition of jets as given above, our theoretical models predict that the invariant masses of heavy quarks are very close to the heavy quark masses themselves. This would not be true if a heavy hadron is produced in the middle of the z distribution. With light hadrons only within a cone of half-angle $\theta = O(1/\gamma_Q)$, the invariant mass of a heavy quark jet is given by

$$M_{jet} = M + \sum_i \sqrt{m_i^2 + p_{Ti}^2} \quad , \tag{6}$$

where the summation over i is for the light hadrons and it accounts for the hadron cloud around the heavy quark in Model (b), namely, nonvee hadrons. The number of light hadrons that are grouped together in a jet according to our definition of jet is presumably a small finite number. It should certainly be much smaller than the total multiplicity of light hadrons in the hadronization plateau of the rapidity plot. To be most generous, however, we will use the multiplicity in the hadronization plateau in our estimate of invariant mass below. The informations from the low p_T physics to be used are as follows:

The multiplicity in the hadronization plateau;

$$\langle n \rangle \approx 2y \; .$$

The average transverse momentum of light hadrons;

$$\left\langle \sqrt{m^2 + p_T^2} \right\rangle \approx 0.8 \text{ GeV} \; .$$

The fluctuation in multiplicity;

$$\sqrt{\langle n^2 \rangle / \langle n \rangle^2 - 1} \approx 0.6 \; .$$

We set the light hadron contribution in (6) equal to $\langle n \rangle \left\langle \sqrt{m^2 + p_T^2} \right\rangle$. It pushes up slightly the jet mass from the heavy quark mass, typically no more than a few GeV. Since the multiplicity and the transverse momenta have distributions, the jet masses are not sharp, but have widths. With the standard deviation in n as given above and the exponential falloff in p_T^2, the widths of these two origins

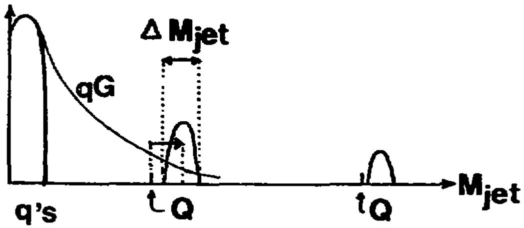
$$\Delta M_{jet}(p_T) = (M_{jet} - M)/\sqrt{\langle n \rangle} \quad ,$$

$$\Delta M_{jet}(n) = \sqrt{\langle n^2 \rangle / \langle n \rangle^2 - 1} \; (M_{jet} - M)$$

can be estimated easily. The results are tabulated below.

γ (=E/M)	M_{jet}-M	$\Delta M_{jet}(p_T)$	$\Delta M_{jet}(n)$
5	2.6 GeV	1.4 GeV	1.5 GeV
10	3.7 GeV	1.7 GeV	2.2 GeV
20	4.8 GeV	2.0 GeV	2.5 GeV

The charmed particles produced at the highest energies of PETRA and PEP have $\gamma \simeq 12$ for which ΔM_{jet} is around 2.5 GeV and therefore the M_{jet} distribution of the charmed quark will overlap with those of the light quark jets. For the bottom flavored particles (M = 5 GeV) produced at $\sqrt{s}$ = 36 GeV, we find that $M_{jet} \simeq$ 7.0 GeV and $\Delta M_{jet} \simeq$ 1.5 GeV. These jets will be clearly separated in the M_{jet} distributions from the u, d, s, and c jets. A top flavored hadron of mass 30 GeV produced with γ = 3.3 at the highest LEP energy will stand out from the rest even more clearly.



It might look that a whole new field of "jet spectroscopy of heavy quarks" will be coming up on the horizen. But the nature is probably not as generous as it might look at the first sight. First, the background. When light quark jets are produced, it is also possible to produce double jets that consist of a light quark jet and a gluon jet emitted from the light quark with a finite or wide angle. As energies go up, triple jets and so on will become nonnegligible. Let us call thse jets as qG jet, qGG jet and so on. Since the invariant masses of these jets are continuous and extend over to large values, they act as backgrounds for the jet spectroscopy of heavy quark jets. Likewise, QG jets, QGG jets etc will act as backgrounds for the jet spectroscopy of an even heavier quark. If the production rate of the Q quark is suppressed by orders of magnitude as compared with that of the light quark q, as is anticipated in the associated productionin hadron-hadron collision, the rate of wide-angle qG jets will overwhelm Q jets completely. It may be possible to separate wide-angle qG jets by an acoplanarity cut, but even wide-angle qGG jets may well compete. In this respect, the electron-positron annihilation provides an ideal setup since the production rates are determined by electric charge squares. The qG backgroun will be easily separated from the genuine Q jets in the e^+e^- annihilation if everything else goes right. It should be commented that though the situation may not be as promising as in the e^+e^- annihilation, there may be some chance in neutrino reactions.

The next obstacle may be serious. When heavy flavored hadrons decay weakly, can we possibly recognize heavy quark jets as such ? This pessimism has its origin in that heavy quarks presumably decay through weak interactions following a certain pattern. If we believe the present theory of weak interactions, a heavy quark first decays to the next heavy quark plus a (ud) pair, a (cs) pair, or a lepton pair through the charged current interactions. If the next heavy quark is much lighter than the initial quark (which is probably the case with the present lower limit on the t-quark mass), the decay will result in three jets associated with three quarks (or else one quark jet, one charged lepton and a large missing momentum).[3] Unless an initial heavy quark is moving very fast, it is difficult to disentangle the three daughter jets from jet products coming from the other pair-produced heavy quark. Some speculation has been made that some kind of statistical may work for final hadrons of weak nonleptonic decays of a very heavy quark. If it should be true, heavy quark jets would still look like well-collimated jets in spite of weak cascade decays. To my best understanding of strong interaction physics at our hand, however, statistical models are wrong for weak nonleptonic decays of heavy quarks of mass ≥ 15 GeV. See Reference 3 on this subject.

When weak decay produces three jets of hadrons, the multiplicity of final hadrons is much higher than in events involving no heavy hadron.[3] When a heavy quark of mass M is produced with energy E and it decays through three jets, the expected multiplicity is given by

$$\langle n \rangle_H \simeq \left\{ a \log(E/M) + b \right\} + \sum_{i=1,2,3} \left\{ a \log(M/3m_{iT}) + b \right\} , \quad (7)$$

where a and b are the constants that characterize the density of hadronized light particle population in the rapidity plot ($\langle n \rangle = ay + b$). The first term is due to the initial heavy quark and the remainder comes from the daughter jets. If the decay proceeds by cascade, the multiplicity increases even more since

$$\left\{ a \log(m_{iT}/3m_T) + b \right\}$$

is to be added to $\langle n \rangle_H$ for each jet in subsequent cascade decays. This should be compared with the multiplicity of a light quark jet of the same energy E, $\langle n \rangle_L \simeq a \log(E/m_T) + b$. For example, with M = 30 GeV and E = 100 GeV , the cascade decay

$$t \longrightarrow b + (u\bar{d}) \longrightarrow c + (\bar{u}d) \longrightarrow s + (u\bar{d})$$

leads us with a = 2 and b = 0 to

$$\langle n \rangle_H = 18$$

while $\langle n \rangle_L = 9$. Multiplicity measurement will certainly help to separate heavy quark production events from the rest as well as geometrical observation on the sharpness of jets will. For the decay lepton signature, I suggest that one should refer to the talk by D. M. Scott of this Workshop and the paper by Pakvasa, Dechantsreiter, Halzen, and Scott.[6]

To conclude this talk, I must admit experimental obstacles for us to do the spectroscopy of heavy quark jets in super high energy experiment. I hope, however, that you will feel far more comfortable from now on in using the $\delta(1-z)$ fragmentation function for very heavy quarks.

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References

1. A large number of stadard references exist. Most theorists know about them. If you are experimentalists, please ask your colleagues in theory. If I write them here, I have to include all of them.

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4. M. Suzuki, Phys. Lett. 71B (1977) 139.

5. J.F. Owens, Phys. Lett 76B (1978) 85; T. Uematsu, Phys. Lett. 79B (1978) 97. Some people have been aware of the heavy quark propagator damping leading to $\delta(1-z)$. See D. M. Scott, Phys. Rev. D18 (1978) 210.

6. S. Pakvasa, M. Dechantsreiter, F. Halzen, and D.M. Scott, University of Hawaii/University of Wisconsin Preprint, UH-511-349-79 and COO-881-96, July, 1979.