

# UC Irvine

## UC Irvine Previously Published Works

### Title

SPICE: Simulation Package for Including Flavor in Collider Events

### Permalink

<https://escholarship.org/uc/item/3wh2d15g>

### Journal

Computer Physics Communications, 181(1)

### ISSN

0010-4655

### Authors

Engelhard, Guy  
Feng, Jonathan L  
Galon, Iftah  
[et al.](#)

### Publication Date

2010

### DOI

10.1016/j.cpc.2009.09.013

### Copyright Information

This work is made available under the terms of a Creative Commons Attribution License, available at <https://creativecommons.org/licenses/by/4.0/>

Peer reviewed

# SPICE: Simulation Package for Including Flavor in Collider Events

Guy Engelhard,<sup>1</sup> Jonathan L. Feng,<sup>2</sup> Iftah Galon,<sup>3</sup> David Sanford,<sup>2</sup> and Felix Yu<sup>2</sup>

<sup>1</sup>*Department of Particle Physics, Weizmann Institute of Science, Rehovot 76100, Israel*

<sup>2</sup>*Department of Physics and Astronomy,  
University of California, Irvine, CA 92697, USA*

<sup>3</sup>*Physics Department, Technion-Israel Institute of Technology, Haifa 32000, Israel*

(Dated: April 2009)

## Abstract

We describe **SPICE**: Simulation Package for Including Flavor in Collider Events. **SPICE** takes as input two ingredients: a standard flavor-conserving supersymmetric spectrum and a set of flavor-violating slepton mass parameters, both of which are specified at some high “mediation” scale. **SPICE** then combines these two ingredients to form a flavor-violating model, determines the resulting low-energy spectrum and branching ratios, and outputs **HERWIG** and **SUSY LesHouches** files, which may be used to generate collider events. The flavor-conserving model may be any of the standard supersymmetric models, including minimal supergravity, minimal gauge-mediated supersymmetry breaking, and anomaly-mediated supersymmetry breaking supplemented by a universal scalar mass. The flavor-violating contributions may be specified in a number of ways, from specifying charges of fields under horizontal symmetries to completely specifying all flavor-violating parameters. **SPICE** is fully documented and publicly available, and is intended to be a user-friendly aid in the study of flavor at the Large Hadron Collider and other future colliders.

PACS numbers: 11.30.Hv, 12.15.Ff, 14.60.Pq, 12.60.Jv, 13.85.-t

## I. PROGRAM SUMMARY

*Program title:* SPICE

*Programming language:* C++

*Computer:* Personal computer

*Operating System:* Tested on Scientific Linux 4.x

*Keywords:* Supersymmetry, flavor physics, lepton flavor violation

*PACS:* 11.30.Hv, 12.15.Ff, 14.60.Pq, 12.60.Jv, 13.85.-t

*External routines:* SOFTSUSY [1, 2] and SUSYHIT [3]

*Nature of problem:* Simulation programs are required to compare theoretical models in particle physics with present and future data at particle colliders. SPICE determines the masses and decay branching ratios of supersymmetric particles in theories with lepton flavor violation. The inputs are the parameters of any of several standard flavor-conserving supersymmetric models, supplemented by flavor-violating parameters determined, for example, by horizontal flavor symmetries. The output are files that may be used for detailed simulation of supersymmetric events at particle colliders.

*Solution method:* Simpson's rule integrator, basic algebraic computation.

*Additional comments:* SPICE interfaces with SOFTSUSY and SUSYHIT to produce the low-energy sparticle spectrum. Flavor mixing for sleptons and sneutrinos is fully implemented; flavor mixing for squarks is not included.

## II. INTRODUCTION

There are two flavor problems in particle physics. The standard model (SM) flavor problem is the unexplained pattern of SM fermion masses. In addition, there is the new physics flavor problem, that is, the difficulty of solving the gauge hierarchy problem, which inevitably requires the introduction of new particles at the weak scale, without generically violating well-known low-energy flavor constraints. Future colliders, including the Large Hadron Collider (LHC), may shed light on both the SM and the new physics flavor problems.

Unfortunately, the potential of the LHC in this regard is not well explored, in part because simulations of new physics typically focus on the simplest possibilities in which flavor violation is absent. In this paper, we describe a new tool, SPICE: Simulation Package for Including Flavor in Collider Events, which is intended as a first step toward addressing this problem.

In more detail, there are several motivations for developing collider event simulation tools for models with non-trivial flavor physics:

- In many new physics frameworks, such as supersymmetry or extra dimensions, new states are predicted that are governed by the same flavor or horizontal symmetries as the SM fermions. Understanding their masses and mixings may therefore lead to real progress in identifying the theory of flavor.
- Although the new physics may be minimally flavor violating, this is by no means required by current constraints; see, for example, Refs. [4, 5, 6, 7, 8, 9, 10, 11]. As LHC data become available, it is appropriate to relax theoretical constraints and consider more general frameworks, especially if they yield unusual signals.

- The exploration of new physics is often said to consist of three phases: particle discovery, measurement of particle masses, and measurement of flavor and other mixings. In reality, these stages are unlikely to be completely distinct, and the exploration of flavor effects may provide new opportunities for particle discovery or proceed in parallel with precision mass measurements.
- There are many promising probes of flavor violation at low energies. If a positive signal appears in one of them, for example,  $\mu \rightarrow e\gamma$ , there will be great interest in understanding whether new weak-scale physics can explain it. In this scenario, a program like **SPICE** will be helpful to understand what flavor-violating effects are observed or observable at colliders.
- Finally, searches for new physics often implicitly assume the (near) absence of flavor violation. It is important to test their robustness in the presence of flavor violation.

Although **SPICE** is flexible in several ways, in its typical form **SPICE** reads in parameters for one of the standard flavor-conserving supersymmetric models and lepton flavor-violating (LFV) masses specified by the user. **SPICE** uses the flavor-conserving model as a spine, then includes the corrections from LFV terms, determines the new superpartner masses and mass eigenstates, calculates branching ratios for all relevant flavor-conserving and flavor-violating decay modes, and outputs the results in **HERWIG** [12, 13, 14, 15] and SUSY Les Houches Accord (SLHA and SLHA2) [16, 17] format files, which may be used to generate collider events. **SPICE** uses **SOFTSUSY** [1, 2] to evolve supersymmetric parameters through renormalization group equations and uses **SUSYHIT** [3] to calculate flavor-conserving decay widths.

The flavor-conserving model may be any of the standard ones, for example minimal supergravity (mSUGRA), minimal gauge-mediated supersymmetry breaking (mGMSB), or anomaly-mediated supersymmetry breaking supplemented by a universal scalar mass (mAMSB). We review these and their input parameters below.

The LFV may be specified in a number of ways. At the “highest,” most model-dependent level, a user may specify a horizontal global symmetry [18, 19, 20], the charges of each of the supermultiplets under this symmetry, and the Froggatt-Nielsen expansion parameter  $\lambda$ . **SPICE** then populates each LFV mass matrix with entries that have the appropriate powers of  $\lambda$ , multiplied by randomly-chosen  $\mathcal{O}(1)$  coefficients. At a slightly lower level, the user may specify each of the  $\mathcal{O}(1)$  coefficients by hand. And finally, at the lowest and most model-independent level, a user may specify every LFV term by hand.

Supersymmetric models with LFV are, of course, just some of the many possibilities for non-trivial flavor physics at the LHC. We are led to consider LFV in supersymmetric models for a variety of reasons. Supersymmetry is well-motivated by the gauge hierarchy problem and force unification, both with and without gravity, and there exist supersymmetric flavor models that can both explain the observed charged lepton and neutrino masses and mixings and simultaneously solve the new physics flavor problem. In addition, and perhaps most important for the coming data-driven era, supersymmetry is flexible enough to encompass many diverse experimental signatures, and lepton flavor violation, as opposed to quark flavor violation, generically has obvious implications for observable experimental signatures and event topologies. That said, it would certainly also be interesting to study hadronic flavor violation, particularly that involving the 3rd generation, and to generalize these results to non-supersymmetric frameworks. A recent study of the observability of squark flavor violation at the LHC was done in Ref. [21].

The program and detailed installation instructions can be found on the web at the address

<http://hep.ps.uci.edu/~spice> .

In addition, App. A is a step-by-step guide to installing SPICE and gives helpful tips on getting started.

In the following sections, we explain how SPICE works. In Sec. III, we discuss the theoretical framework in more detail. In the presence of LFV, many new decay modes open up, but in most cases, the decay widths are simple generalizations of flavor-conserving ones. An exception is the case of “charge-flipping,” three-body slepton decays to like-sign leptons  $\tilde{\ell}_i^- \rightarrow \tilde{\ell}_j^+ \ell_k^- \ell_m^-$ . We have calculated these and discussed their interesting phenomenology elsewhere [22], but we summarize the main points in Sec. III.

In Sec. IV, we show how the program works, explaining how to install the program and specifying the relevant file names and input and output formats. To aid first-time users, we also present a simple example input file and the resulting output. Our conclusions are given in Sec. V. Further details regarding our conventions, Lagrangian terms, decay widths, and program details are given in a series of appendices.

### III. THEORETICAL FRAMEWORK

#### A. Flavor-Conserving Inputs

The user inputs to SPICE consist of a flavor-conserving SUSY spine supplemented by LFV terms. The flavor-conserving SUSY spine may be any model normally available in SOFTSUSY [1, 2], though in principle any model could be input with appropriate alterations to the SOFTSUSY code. The base options and their input parameters are the following:

- Minimal supergravity (mSUGRA) is specified by

$$\text{mSUGRA: } m_0, M_{1/2}, A_0, \tan \beta, m_{\text{GUT}}, \text{sign}(\mu) , \quad (1)$$

where  $m_0$ ,  $M_{1/2}$ , and  $A_0$  are the universal scalar, gaugino, and tri-linear scalar coupling parameters at the grand unified theory (GUT) scale  $m_{\text{GUT}}$ ,  $\tan \beta = \langle H_0^u \rangle / \langle H_0^d \rangle$ , and  $\text{sign}(\mu)$  is the sign of the Higgsino mass parameter  $\mu$ .

- Minimal gauge-mediated supersymmetry breaking (mGMSB) is specified by

$$\text{mGMSB: } N_5, M_{\text{mess}}, \Lambda, C_{\text{grav}}, \tan \beta, \text{sign}(\mu) , \quad (2)$$

where  $N_5$  is the number of  $\mathbf{5} + \bar{\mathbf{5}}$  multiplets in the messenger sector,  $M_{\text{mess}}$  is the messenger mass scale,  $\Lambda = F_S / \langle S \rangle$  sets the mass scale for the SM superpartners,  $C_{\text{grav}} = m_{\tilde{G}} / (F / \sqrt{3} m_{\text{Planck}})$  is the gravitino mass in units of its mass in the minimal case in which there is only one SUSY-breaking  $F$  term, and  $\tan \beta$  and  $\text{sign}(\mu)$  are as above.

- Minimal anomaly-mediated supersymmetry breaking (mAMSB) is specified by

$$\text{mAMSB: } m_0, m_{3/2}, \tan \beta, m_{\text{GUT}}, \text{sign}(\mu) , \quad (3)$$

where  $m_0$  is the universal scalar mass, motivated by the tachyonic slepton problem and added to all scalars, including the Higgs scalars,  $m_{3/2}$  is the gravitino mass, which sets the scale for all SM superpartners, and  $\tan \beta$ ,  $m_{\text{GUT}}$ , and  $\text{sign}(\mu)$  are as above.

## B. Flavor-Violating Inputs: Model-Independent Approach

The LFV terms are specified by a parameter  $x$  and three  $3 \times 3$  matrices  $m_E$ ,  $X_L$ , and  $X_R$ . All of these matrices are assumed real, and  $X_L$  and  $X_R$  are necessarily symmetric. In the most model-independent formulation accommodated by **SPICE**, LFV is therefore completely specified by 22 numbers:  $x$ , 9 for  $m_E$ , 6 for  $X_L$ , and 6 for  $X_R$ .

The matrix  $m_E$  is the SM charged lepton mass matrix.  $X_L$  and  $X_R$  determine the slepton mass matrices through

$$M_{\tilde{\nu}}^2 = m_L^2 \mathbf{1} + x \tilde{m}^2 X_L \quad (4)$$

$$M_{\tilde{E}_L}^2 = m_L^2 \mathbf{1} + m_E m_E^\dagger + x \tilde{m}^2 X_L \quad (5)$$

$$M_{\tilde{E}_R}^2 = m_R^2 \mathbf{1} + m_E^\dagger m_E + x \tilde{m}^2 X_R, \quad (6)$$

where  $m_L^2$  and  $m_R^2$  are the flavor-conserving contributions to the left- and right-handed sleptons, and  $\tilde{m}^2$  characterizes the size of flavor-conserving contributions to the slepton masses. For concreteness we take  $\tilde{m}^2$  to be equal to the average of the diagonal elements of  $m_L^2$ <sup>1</sup>. The parameter  $x$  specifies the size of the LFV effects relative to the flavor-conserving parameters.

Several simplifying assumptions are encoded in this formulation of the LFV effects:

1. In assuming  $m_E$ ,  $X_L$ , and  $X_R$  are real, we do not include CP violation from these matrices.
2. The flavor-violating contributions to left- and right-handed sleptons are of the same order, as set by  $x \tilde{m}^2$ . This is what one would expect of gravitational contributions, which are chirality-blind.
3. In general, there should also be LFV  $A$ -terms. We assume these are negligible at the mass scale where these SUSY-breaking masses are generated. Note, however, that  $A$ -terms are generated through RG evolution, and such effects are included.
4. In general, there should also be a non-trivial LFV neutrino mass matrix. However, since we are primarily interested in colliders, where neutrino flavor is unobservable, we simply assume that the neutrino masses vanish, and so the neutrino mass and gauge eigenstates are identical. Note that sneutrinos may have observable mixings, and these are included.

## C. Flavor-Violating Inputs: Flavor Symmetry Approach

An attractive possibility is that the LFV terms are determined by horizontal symmetries. In this approach, the 3 LFV matrices are

$$m_E = m_\ell \begin{pmatrix} c_1 \lambda^{n_1} & c_2 \lambda^{n_2} & c_3 \lambda^{n_3} \\ c_4 \lambda^{n_4} & c_5 \lambda^{n_5} & c_6 \lambda^{n_6} \\ c_7 \lambda^{n_7} & c_8 \lambda^{n_8} & c_9 \lambda^{n_9} \end{pmatrix} \quad (7)$$

---

<sup>1</sup> For a flavor-conserving spine where the slepton mass-squared matrices are not proportional to the identity,  $\tilde{m}^2$  is thus equal to the average left slepton mass-squared.

$$X_L = \begin{pmatrix} c_{10}\lambda^{n_{10}} & c_{11}\lambda^{n_{11}} & c_{12}\lambda^{n_{12}} \\ c_{11}\lambda^{n_{11}} & c_{13}\lambda^{n_{13}} & c_{14}\lambda^{n_{14}} \\ c_{12}\lambda^{n_{12}} & c_{14}\lambda^{n_{14}} & c_{15}\lambda^{n_{15}} \end{pmatrix} \quad (8)$$

$$X_R = \begin{pmatrix} c_{16}\lambda^{n_{16}} & c_{17}\lambda^{n_{17}} & c_{18}\lambda^{n_{18}} \\ c_{17}\lambda^{n_{17}} & c_{19}\lambda^{n_{19}} & c_{20}\lambda^{n_{20}} \\ c_{18}\lambda^{n_{18}} & c_{20}\lambda^{n_{20}} & c_{21}\lambda^{n_{21}} \end{pmatrix}, \quad (9)$$

where  $m_\ell$  is the lepton mass scale, the  $c_i$  are  $\mathcal{O}(1)$  coefficients,  $\lambda$  is the Froggatt-Nielsen expansion parameter, and the exponents  $n_i$  are determined by supermultiplet charges.

SPICE accommodates this possibility by providing an alternative way to specify the LFV parameters in the case of  $U(1)^n$  flavor models, with breaking parameters of the same size. For example, for  $U(1) \times U(1)$ , one sets  $N_{\text{charges}} = 2$ , and specifies the two  $U(1)$  charges for the six multiplets  $L_1, L_2, L_3, E_1, E_2$ , and  $E_3$ . SPICE then determines the correct exponent  $n_i$  for each entry of the three matrices. SPICE may also determine the coefficients  $c_i$  randomly or these may be set by the user.

#### D. Spectrum and Decay Calculations

Once the flavor-conserving and flavor-violating parameters are input, the model is completely specified. Formally, one should then evolve the mass parameters from the scale they are generated, adding in new contributions at the appropriate scale, until one reaches the weak scale. In SPICE, we instead add the LFV contributions to the flavor-conserving contributions at the scale at which the flavor-conserving contributions are generated, and then RG evolve the result to the weak scale. In the case that the flavor-conserving and LFV contributions are generated at the same scale, as in the case of mSUGRA or mAMSB spines with LFV generated at  $m_{\text{GUT}}$ , this is the correct prescription. If the generation scales differ, for example, as in the case of a mGMSB spine with gravity-mediated LFV effects, our prescription is not formally correct. Even in this case, however, we expect that the dominant difference resulting from this approximation may be absorbed into the LFV  $\mathcal{O}(1)$  parameters, which are not completely specified anyway.

Given the slepton masses and mixings determined above, we then calculate all branching ratios involving these LFV effects. Most of the flavor-violating decay widths are, at least to tree level, fairly simple generalizations of the flavor-conserving widths. New effects appear in the 3-body decays of sleptons both to like-sign leptons,  $\tilde{\ell}_i^- \rightarrow \tilde{\ell}_j^+ \ell_k^- \ell_m^-$ , and to opposite-sign leptons,  $\tilde{\ell}_i^- \rightarrow \tilde{\ell}_j^- \ell_k^- \ell_m^+$ . Previous work on these decays [23, 24] has assumed flavor conservation. In the presence of LFV, however, the lepton pair may be identical particles or charge conjugates of the same generation, leading to additional interference terms. We have calculated these decay widths in the presence of LFV and arbitrary left-right slepton mixing in another work [22] and included these in SPICE.

Currently SPICE discards decays to gravitinos. Such decays are typically either kinematically inaccessible or too slow to be relevant at colliders. Even in the cases where the model used for the spine predicts fast decays to gravitinos, as in the case of mGMSB with a low supersymmetry-breaking scale, the existence of significant LFV contributions implies that in the full LFV theory, supersymmetry is broken at a high scale, and decays to gravitinos are negligible.

1. User-supplied SPICEinit file is read by spice.x.
2. spice.x uses SOFTSUSY to generate the mass spectrum and calculates lepton flavor-violating decay widths.
3. SUSYHIT uses output from spice.x and calculates flavor-conserving decay widths.
4. FileCombine.x combines the output from spice.x and SUSYHIT to create HerwigFinal.out, SLHAFinal.out, and SLHA2Final.out, which can be used to generate collider events.

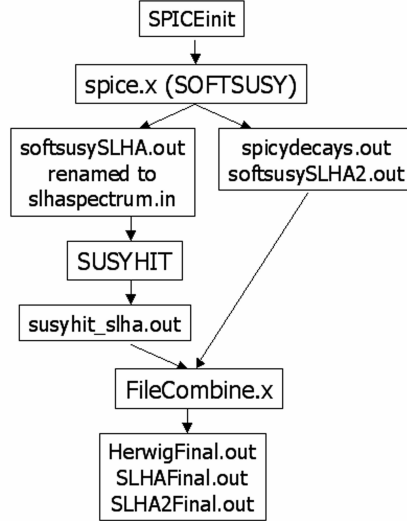


FIG. 1: Schematic flow chart for SPICE, showing the progression from the SPICEinit input file to the final output files HerwigFinal.out, SLHAFinal.out, and SLHA2Final.out. The explicit interfacing with SOFTSUSY [1, 2] and SUSYHIT [3] is also shown.

## IV. PROGRAM

### A. Procedure

The program is designed to take in one input file which specifies all of the input parameters. This file, by default, is called SPICEinit and is located in the `spice/` directory. The default input file is identical to the example input file presented below. These input parameters are used by SOFTSUSY [1, 2] to generate a particle mass spectrum, which is passed to SUSYHIT [3] to create a SUSY Les Houches Accord (SLHA) file with the traditional flavor-conserving decay table. Decay widths of the flavor-generalized decay channels involving sleptons and sneutrinos are calculated separately within the SOFTSUSY executable, which generates an intermediate file containing the flavor-generalized mass spectrum and relevant decays. The FileCombine subprogram merges the SUSYHIT output with this intermediate file to generate the file HerwigFinal.out, which is an input file for the Herwig event generator [12, 13, 14, 15] and both SLHAFinal.out and SLHA2Final.out, which are SLHA [16] and SLHA2 [17] formatted files appropriate for input to Monte Carlo event generators. The program flow is diagrammed schematically in Fig. 1.

The input file for the program contains two sets of input parameters — the flavor-conserving SUSY spine, and the flavor-violating effects. The first set specifies the SUSY breaking scenario parameters according to the SOFTSUSY [1, 2] specifications, which are briefly reviewed below. The second set describe high energy flavor mixing boundary conditions. The default input file is presented below as an example<sup>2</sup>.

<sup>2</sup> This example file uses the U(1) horizontal charges of Model B in Ref. [4].



*Example Input File*

```

gmsb 4 2.0e6 5.0e4 1.0 10 1
x 0.1
lambda 0.2
nCharges 2
L1 2 0
L2 0 2
L3 0 2
E1 2 1
E2 2 -1
E3 0 -1
Lep -0.13854 2.24310 -0.4399 1.84877 1.53904 -0.56145 0.89942 0.53290
1.45314
XL 0.98914 -1.2001 1.40588 -1.2001 -0.470473 2.67457 1.40588 2.67457
-0.22933
XR -0.5889 2.8415 -0.2321 2.8415 -0.43167 1.3267 -0.2321 1.3267 0.55438

```

- **Flavor conserving parameters:**

We incorporate the three SUSY breaking scenarios implemented by SOFTSUSY as options for the flavor-conserving SUSY spine. These are mSUGRA, mGMSB, and mAMSB, as described in Sec. III. The first line of the script file specifies which spine is used with an identifier of “sugra”, “gmsb”, or “amsb”, and then parameters needed for that given scenario. The ordering is consistent with that used in Sec. III and the SOFTSUSY manual [1, 2]. Note that the text labels for the SUSY breaking scenario is case sensitive. These are summarized as follows:

```

sugra <m0> <m12> <a0> <tanb> <mgut> <sgnMu>
gmsb <n5> <mMess> <lambda> <cgrav> <tanb> <sgnMu>
amsb <m0> <m32> <tanb> <mgut> <sgnMu>

```

- **Flavor mixing parameters**

The remainder of the input file is used to describe the slepton flavor mixing in the model in the context of U(1) horizontal charges as described in Ref. [4] and reviewed in Sec. III. The first two parameters are  $x$  and  $\lambda$ , and the next integer nCharges specifies the number of U(1) charges for the leptons. Note that text labels are required as placeholders before each variable, although the actual label text is arbitrary for all entries except the spine identifier and the “Lep” identifier which may be changed to “random” to generate random coefficients (see below). After the number of charges is passed, the charge assignments for each generation of left-handed sleptons and right-handed sleptons must be given. Lastly, the user can specify the  $\mathcal{O}(1)$  coefficients to be used in the lepton, left-handed slepton, and right-handed slepton mass matrices. The blocks “Lep”, “SLepXL”, and “SLepXR” define the nine  $\mathcal{O}(1)$  coefficients for each  $3 \times 3$  mass matrix, according to the pattern

$$\text{Lep } c_1 \ c_2 \ c_3 \ c_4 \ c_5 \ c_6 \ c_7 \ c_8 \ c_9 \rightarrow \begin{pmatrix} c_1 \lambda^{n_1} & c_2 \lambda^{n_2} & c_3 \lambda^{n_3} \\ c_4 \lambda^{n_4} & c_5 \lambda^{n_5} & c_6 \lambda^{n_6} \\ c_7 \lambda^{n_7} & c_8 \lambda^{n_8} & c_9 \lambda^{n_9} \end{pmatrix}. \quad (10)$$

“SLepXL” and “SLepXR” are symmetric matrices and must be defined with symmetric coefficients, while the “Lep” matrix has arbitrary coefficients. Alternatively, the randomly generated coefficients may be generated by replacing the last three lines with

```
random <rseed> <sigma>
```

where `<rseed>` is an integer, used as a random seed to generate  $\mathcal{O}(1)$  numbers of the form  $\pm \exp(a)$ , where  $a$  has a gaussian distribution with a mean of 0 and a  $\sigma$  of `<sigma>`. These coefficients have a 50% probability of being negative.

Besides the default example file, there are several example input files provided in the sub-directory `spice/Examples/` that demonstrate various charge assignments following the models A-D prescription in Ref. [4]. To use these, rename them to the default input file name `SPICEinit` in the `spice/` directory, or modify the `CalcDecays` command script appropriately.

## B. SOFTSUSY and SUSYHIT Details

SPICE relies upon SOFTSUSY [1, 2], using SOFTSUSY for renormalization with flavor-general boundary conditions. The lepton flavor mixing is rotated into slepton and sneutrino mass matrices at the high scale before renormalization is performed; the rotation of the neutrino masses is neglected along with the neutrino masses themselves. In general, diagonalizing the lepton mass matrix requires rotating the trilinear slepton couplings as well, but this is unnecessary in our case as we assume negligible trilinear contributions at the high scale. Using the flavor general boundary conditions in `spice.cpp`, SOFTSUSY iteratively solves the renormalization group equations (RGEs) and outputs a SUSY mass hierarchy in SLHA and SLHA2 format to `softsusySLHA.out` and `softsusySLHA2.out`, respectively; the SLHA file is passed to SUSYHIT as `slhaspectrum.in`. We do not force gauge coupling unification, the GUT scale is set at  $m_{\text{GUT}} = 2 \times 10^{16}$  GeV initially, we use 2-loop Higgs formulas for physical masses, and we generally follow SOFTSUSY program defaults and conventions as much as possible. For more details on the workings of SOFTSUSY, refer to the SOFTSUSY manual [1, 2].

SUSYHIT takes `slhaspectrum.in` and calculates non-flavor violating decays of the entire SUSY spectrum, according to its subroutines `SDECAY` and `HDECAY`. Details can be found in the SUSYHIT reference manual [3]. The output file appends the calculated decay table to the input file `slhaspectrum.in`, and this SUSYHIT output file is passed to `FileCombine`.

## C. Decay Calculation Files

The calculations of new flavor violating decays is performed within the SPICE executable with the code contained in `DecayFormat.cpp`, `DecayCalc.cpp` and `ThreeBodyIntegrals.cpp` and their respective `.h` header files. The content of these files is:

- The `DecayFormat` class:
  - Output of particle mass and lifetime data
  - Output of a standard format of two- and three-body decays
  - Output of SUSY parameters in HERWIG format

- A public method which chains these methods together to produce a pseudo-HERWIG file<sup>3</sup>
- The `DecayCalc` class:
  - Retrieval of the slepton/sneutrino mass matrices from `FlavourMssmSoftsusy`
  - Calculations for all decays involving sleptons and sneutrinos
  - Functions for Lagrangian coefficients and three-body decay integral multiplicative factors
  - Loop over all possible sleptons/leptons for the given decays and output via `DecayFormat` class
- The `ThreeBodyIntegrals` class:
  - Functions for the integrands of the three body decay integrals
  - A Simpson’s rule integration method
  - Methods to calculate the normal and interference term integrals given a vector containing multiplicative factors

The decay calculations (20 in total) are numbered according to Table II in App. D. These replace all the standard decays calculated by `SUSYHIT` that involve sleptons/sneutrinos as either decaying particle or product. The routine is called from `spice.cpp` using the `OutputHerwigFile` method of a `DecayFormat` object, which in turn creates a `DecayCalc` object and proceeds to call the other global output functions to generate a file in pseudo-HERWIG format containing the slepton/sneutrino decays. This method also generates the mass spectrum and model parameters for inclusion in the final output file appropriate for Monte Carlo event generators. There are also several decay modes listed in Table III in App. D that are irrelevant or not viable in the models we have studied but may be viable in other scenarios; instructions on how to insert new decay modes are presented in App. G.

#### D. Merging Flavor Violation and SUSYHIT Files

The final component of `SPICE` is the `FileCombine` subroutine, which takes the flavor-violating slepton/sneutrino decays file and merges it with the other decays calculated by `SUSYHIT`. `FileCombine` takes the particle mass info and model parameters from the `softsusySLHA.out` file, along with the slepton-sneutrino decay table. It then reads through the `SUSYHIT` decay file and stores all decays that do not involve sleptons or sneutrinos.

---

<sup>3</sup> This pseudo-HERWIG file differs from a true HERWIG file since it lacks SUSY decays beyond those involving sleptons and sneutrinos, it does not have the appropriate whitespace formatting, and it uses Particle Data Group (PDG) particle identification codes instead of HERWIG particle codes.

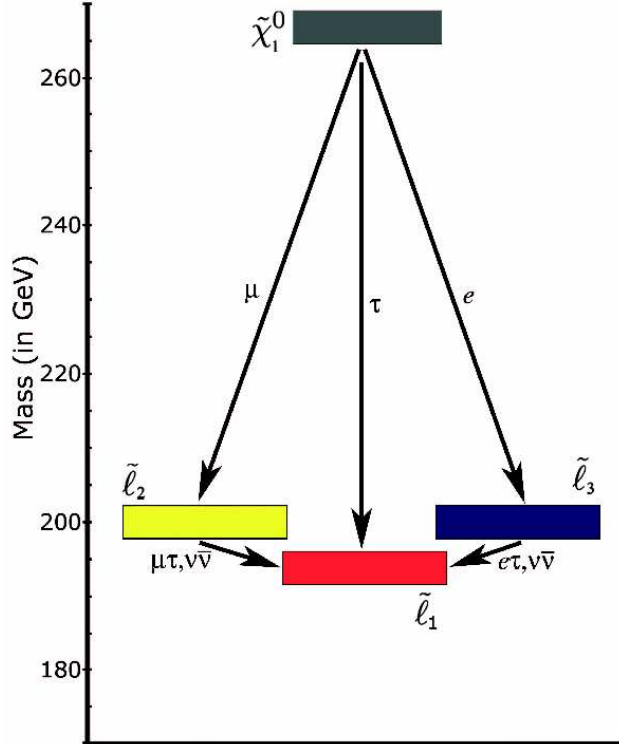


FIG. 2: Low-energy spectrum and decay modes for the flavor-conserving GMSB spine specified in the `SPICEinit` example input file. Flavor-violating effects were not included, and decays to gravitino are ignored. For the slepton shadings (colors), stau flavor is medium gray (red), smuon flavor is light gray (yellow), and selectron flavor is dark gray (blue). SUSY decays proceed along arrows and are labeled by the associated SM daughter particles. The relevant masses are  $\tilde{\chi}_1^0 = 266.7$  GeV,  $\tilde{\ell}_3^- = 200.1$  GeV,  $\tilde{\ell}_2^- = 200.1$  GeV, and  $\tilde{\ell}_1^- = 193.6$  GeV.

## E. Output

The program outputs the stored data as a standard `HERWIG` input file [12, 13, 14, 15] and both `SLHA`- and `SLHA2`-formatted files for input to other Monte Carlo event simulation packages. The first portion of the `HERWIG` file contains particle codes, masses, and lifetimes; the bulk contains branching fractions for particle decays, and the final section contains model parameters<sup>4</sup>. It contains all feasible (and large) flavor-violating slepton and sneutrino decays.

For the example input file in Subsection IV A, we provide some comments about the expected output and how our example model differs from traditional flavor-conserving models. Using

<sup>4</sup> The `SLHA` files are structured similarly; for details, see Refs. [16, 17].

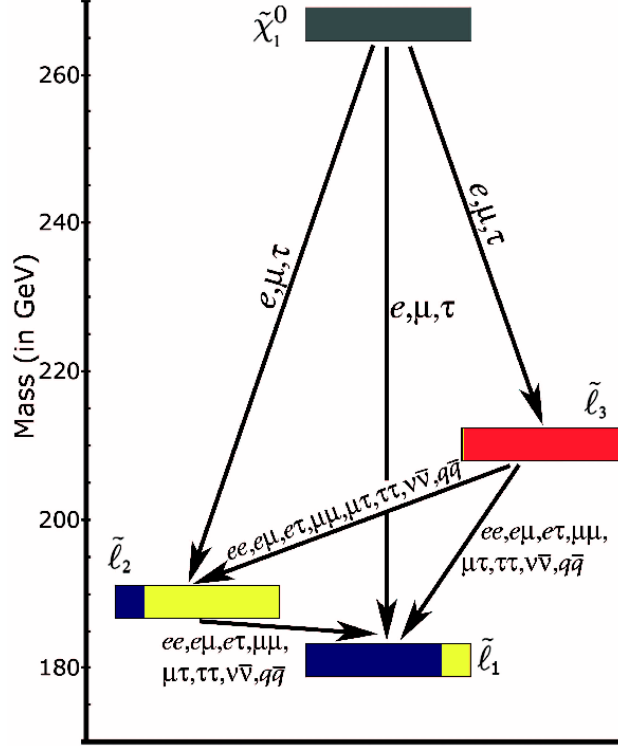


FIG. 3: Low-energy spectrum and decay modes for the full flavor-violating model specified in the `SPICEinit` example input file. Decays to gravitino are ignored. Flavor-violating effects are included and serve to modify slepton masses, mix the gauge eigenstates, and introduce many new decay modes. For the slepton shadings (colors), stau flavor is medium gray (red), smuon flavor is light gray (yellow), and selectron flavor is dark gray (blue). SUSY decays proceed along arrows and are labeled by the associated SM daughter particles. The relevant masses are  $\tilde{\chi}_1^0 = 266.7$  GeV,  $\tilde{\ell}_3^- = 210.0$  GeV,  $\tilde{\ell}_2^- = 188.7$  GeV, and  $\tilde{\ell}_1^- = 179.8$  GeV, and the branching ratios are listed in Table I.

only the flavor-conserving spine of the example model, for instance, we derive the traditional low-energy spectrum and straightforward decay channels depicted in Fig. 2. When `SPICE` calculates the full flavor-violating example model, however, the decays modes acquire new complexity and richness: these modes are diagrammed in Fig. 3 and listed in Table I. We

	Mass	Mode	$\tilde{\ell}_1^+ e^- \tau^-$	$\tilde{\ell}_2^+ \mu^- \tau^-$	$\tilde{\ell}_3^+ \tau^- \tau^-$	$\tilde{\ell}_1^+ \mu^- \tau^-$	$\tilde{\ell}_2^+ e^- \tau^-$				4 more
$\tilde{\chi}_1^0$	266.7	<b>B.R.</b>	35.4%	29.5%	20.6%	7.7%	6.5%				0.3%
$\tilde{\ell}_3^-$	210.0		$\tilde{\ell}_1^+ e^- \tau^-$ 44.3%	$\tilde{\ell}_1^- \tau^- e^+$ 23.9%	$\tilde{\ell}_1^+ \mu^- \tau^-$ 9.5%	$\tilde{\ell}_2^+ \mu^- \tau^-$ 8.4%	$\tilde{\ell}_1^- \tau^- \mu^+$ 5.2%	$\tilde{\ell}_2^- \tau^- \mu^+$ 4.8%	$\tilde{\ell}_2^+ e^- \tau^-$ 1.9%	$\tilde{\ell}_2^- \tau^- e^+$ 1.1%	50 more 0.9%
$\tilde{\ell}_2^-$	188.7		$\tilde{\ell}_1^+ e^- \mu^-$ 27.8%	$\tilde{\ell}_1^- \mu^- e^+$ 21.6%	$\tilde{\ell}_1^+ e^- e^-$ 20.0%	$\tilde{\ell}_1^+ \mu^- \mu^-$ 19.7%	$\tilde{\ell}_1^- e^- e^+$ 4.7%	$\tilde{\ell}_1^- \mu^- \mu^+$ 4.7%	$\tilde{\ell}_1^- e^- \mu^+$ 1.0%		22 more 0.5%

TABLE I: List of masses (in GeV) and branching ratios for decaying particles  $\tilde{\chi}_1^0$ ,  $\tilde{\ell}_3^-$ , and  $\tilde{\ell}_2^-$  using the `SPICEinit` example input file. Only branching ratios larger than 1.0% are individually listed, and decays to gravitino are ignored. This table refers to the spectrum depicted in Fig. 3.

note that the flavor mixing parameters for the  $X_R$  matrix in the example input file indicate that the two lightest sleptons are predominantly selectron and smuon, which is confirmed by the branching ratios for  $\tilde{\chi}_1^0 \rightarrow \tilde{\ell}_2^\pm \ell^\mp, \tilde{\ell}_1^\pm \ell^\mp$ . Thus, while traditional expectations favor the lightest slepton to be predominantly stau, given the large Yukawa coupling of the third generation, the lightest sleptons can instead generically be combinations of selectron and smuon in flavor-violating scenarios. More importantly, the wealth of new information in flavor-violating two- and three-body modes indicates it may be possible to measure flavor couplings from early collider data, if positive signals for SUSY are discovered.

## V. CONCLUSION

We have presented the program **SPICE**: Simulation Package for Including Flavor in Collider Events intended for use in studies of lepton flavor violation (LFV). **SPICE** simulates hybrid models where flavor-violating gravitational effects are added to a flavor-conserving SUSY spine. The program contains preset options for generating lepton flavor violation based on U(1) horizontal symmetries, but it also allows for input of more general flavor-violating effects. It interfaces with **SOFTSUSY**, which generates the low-energy mass hierarchy, and **SUSYHIT**, which calculates decays unconnected to LFV. **SPICE** automatically calculates all kinematically allowed decays involving sleptons and sneutrinos with LFV, and also calculates three-body slepton decays, which are important in models with a long-lived slepton. Given an input model with LFV, **SPICE** outputs the mass hierarchy and decay table with LFV effects, which can be used to generate collider events using Monte Carlo simulation packages. **SPICE** is freely available under the Gnu Public License, and can be downloaded at <http://hep.ps.uci.edu/~spice>.

There are several possible extensions to **SPICE**. Foremost among these is the inclusion of quark sector flavor violation, which, while not expected to impact collider signals as obviously as some possible LFV signals, also provides interesting effects. Another valuable extension would be the inclusion of low-energy effects, including neutrino masses and mixings and comparison to low-energy constraints such as  $\mu \rightarrow e\gamma$ . Another possibility is the flavor generalization and inclusion of three-body decays involving sneutrinos as detailed in Ref. [24]. In the case of a long-lived slepton, there is also the possibility of a decay of the form  $\tilde{\ell}_i \rightarrow \tilde{\ell}_j \gamma$  generated through LFV loops, which could be considered.

## Acknowledgements

We are grateful to Y. Nir and Y. Shadmi for many helpful discussions and careful readings of the manuscript. We would also like to thank S. French and C. Lester for helpful advice and comments about program applications. The work of JLF, DS, and FY was supported in part by NSF grants PHY-0239817 and PHY-0653656 and the Alfred P. Sloan Foundation. IG thanks the UC Irvine particle theory group for their hospitality while this work was in progress. This research was supported in part by the United States-Israel Binational Science Foundation (BSF) under grant No. 2006071. The research of IG was also supported by the Israel Science Foundation (ISF) under grant No. 1155/07.

## APPENDICES

This section of the paper provides all details relevant to running `SPICE`. Appendix A outlines the installation procedure and provides general tips on getting started, as well as basic troubleshooting solutions. Detailed command-line instructions can be found in the `README` file on the website. Appendix B covers the notation and conventions used in `SPICE`. Appendix C defines the relevant portions of the Lagrangian for flavor-general sleptons and defines Lagrangian coefficients to simplify the forms of the decay modes. Appendix D enumerates the flavor-generalized decays used and calculated by `SPICE`, as well as the decays `SPICE` ignores. Appendix E gives formulas for flavor-generalized two-body decays. Appendix F discusses the three-body decays; the full formulas are presented in Ref. [22]. Appendix G, finally, discusses the structure of the program and details about the integration between `SPICE`, `SOFTSUSY`, and `SUSYHIT`.

### APPENDIX A: INSTALLATION INSTRUCTIONS AND TROUBLESHOOTING

#### 1. Installing and Running `SPICE`

`SPICE` can be downloaded from <http://hep.ps.uci.edu/~spice>. `SPICE` requires both `SOFTSUSY` (<http://projects.hepforge.org/softsusy>) and `SUSYHIT` (<http://lappweb.in2p3.fr/~muehlleitner/SUSY-HIT>) to be installed in order to function properly. After obtaining and unzipping all three packages, use the `SPICE` Makefile in the `spice/` directory to build the executables `spice.x` and `FileCombine.x`, and also run the default `SUSYHIT` Makefile to compile `SUSYHIT`<sup>5</sup>. The command script `CalcDecays`, located in the `spice/` directory, handles program execution and movement of input and output files: by default, it uses the file `SPICEinit` for input parameters. The output of `SPICE` is a Herwig input file `HerwigFinal.out`, a SUSY LesHouches Accord file `SLHAFinal.out`, and a SLHA2 file `SLHA2Final.out`, all located in `spice/FileCombine/`. When running the default `SPICEinit` file, the program output should match the example output files in the `spice/FileCombine/` directory. Additional input files, which demonstrate the versatility of `SPICE`, can be found in `spice/Examples/`.

#### 2. Troubleshooting

By default, `SPICE` assumes a particular directory structure, with the `spice/`, `softsusy/`, and `susyhit/` directories all located in the same parent directory, *i.e.*, `Desktop/spice/`, `Desktop/softsusy/`, and `Desktop/susyhit/`. Any other directory structure or different directory names can be accommodated by making the appropriate changes to the directory path names in the `SPICE` Makefile and command script `CalcDecays`.

`SPICE` was tested with the standard `SOFTSUSY` 3.0.2. and `SUSYHIT` 1.3 builds, so a customized build of either program's core files may cause errors. In addition, `SPICE` was compiled using the `g++-3.4` compiler, so use of a different compiler may require changes to

---

<sup>5</sup> Please note that `spice.x` contains the relevant portions of `SOFTSUSY`, so a separate `SOFTSUSY` build is not required.

the SPICE Makefile. In particular, for the gcc-4.2 compiler, the user should remove the `-ff90` option in the Makefile and replace the `-lg2c` inline tag with `-lgfortran`. Further installation details are provided in the README file included with SPICE.

## APPENDIX B: CONVENTIONS

In this section we use the spacetime metric  $(+---)$  and fermionic propagators of the form  $(\not{p} - m) / (p^2 - m^2)$ . Our projection operators are  $P_{L,R} = (1 \mp \gamma_5)/2$ . The constants  $g$  and  $g'$  are the standard weak scale SU(2) and U(1) gauge couplings; we break from SOFTSUSY convention and use the typical standard model value for  $g'$  rather than the gauge unification value.

Throughout this appendix, greek indices refer to gauge eigenstates while roman indices refer to mass eigenstates for clarity. This convention is dropped in later sections to avoid confusion with Lorentz indices.

### 1. Higgs Bosons

Our convention for the Higgs doublets is

$$H_u = \begin{pmatrix} H_u^+ \\ H_u^0 \end{pmatrix} \quad H_d = \begin{pmatrix} H_d^0 \\ H_d^- \end{pmatrix}, \quad (\text{B1})$$

and we set their vacuum expectation values (VEVs) to be

$$\langle H_u \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_u \end{pmatrix} \quad \langle H_d \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} v_d \\ 0 \end{pmatrix}. \quad (\text{B2})$$

We set the SM Higgs VEV  $v$  to be  $v^2 \equiv v_u^2 + v_d^2$ , and also we define  $\tan \beta = v_u/v_d$ . Correspondingly, the SM gauge boson masses are

$$m_W^2 = \frac{1}{4}g^2v^2 \quad m_Z^2 = \frac{1}{4}(g^2 + g'^2)v^2, \quad (\text{B3})$$

and the SM lepton masses are

$$m_{\ell_{ij}} = \frac{1}{\sqrt{2}}y_{ij}^{(\ell)}v_d. \quad (\text{B4})$$

After electroweak symmetry breaking, the neutral Higgs doublet is given by

$$\begin{pmatrix} H_u^0 \\ H_d^0 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} v_u \\ v_d \end{pmatrix} + \frac{1}{\sqrt{2}}R_{\theta_H} \begin{pmatrix} h^0 \\ H^0 \end{pmatrix} + \frac{i}{\sqrt{2}}R_{\beta} \begin{pmatrix} G^0 \\ A^0 \end{pmatrix}, \quad (\text{B5})$$

and the charged Higgs doublet is

$$\begin{pmatrix} H_u^+ \\ H_d^{-*} \end{pmatrix} = R_{\beta} \begin{pmatrix} G^+ \\ H^+ \end{pmatrix}, \quad (\text{B6})$$

where

$$R_{\theta_H} = \begin{pmatrix} \cos \theta_H & \sin \theta_H \\ -\sin \theta_H & \cos \theta_H \end{pmatrix} \quad \text{and} \quad R_{\beta} = \begin{pmatrix} \sin \beta & \cos \beta \\ -\cos \beta & \sin \beta \end{pmatrix}. \quad (\text{B7})$$



## 2. Neutralinos

The neutralino gauge eigenstates are defined by  $\tilde{\psi}^0 = (-i\tilde{B}, -i\tilde{W}, \tilde{\psi}_d^0, \tilde{\psi}_u^0)^T$ , with the ordering of the up/down higgsinos chosen to conform with **SOFTSUSY**. All the above fields are two-component Weyl spinors.

The neutralino mass term in the Lagrangian is given by

$$\mathcal{L} = -\frac{1}{2}\tilde{\psi}_\alpha^{0\dagger} M_{\tilde{\psi}^0\alpha\beta} \tilde{\psi}_\beta^0, \quad (\text{B8})$$

so diagonalizing according to  $O_{i\alpha}^\dagger M_{\tilde{\psi}^0\alpha\beta} O_{\beta j} = m_{\chi_i^0} \delta_{ij}$  gives neutralino mass eigenstates of  $\chi_i^0 = O_{i\alpha}^\dagger \tilde{\psi}_\alpha^0$ . The corresponding Dirac spinors are  $\tilde{\chi}_i^0 = (\chi_i^0, \bar{\chi}_i^0)^T$ . Here we follow **SOFTSUSY** convention for the neutralino mass matrix by keeping the matrix real and allowing the neutralino masses to be negative.

## 3. Charginos

Similarly, the chargino gauge eigenstates are defined by  $\tilde{\psi}^+ = (-i\tilde{W}^+, \tilde{\psi}_u^+)^T$  and  $\tilde{\psi}^- = (-i\tilde{W}^-, \tilde{\psi}_d^-)^T$ . Again, these are two-component Weyl spinors.

The Lagrangian mass term is given by

$$\mathcal{L} = -\tilde{\psi}^{-\dagger} M_{\tilde{\psi}^\pm} \tilde{\psi}^+, \quad (\text{B9})$$

so diagonalizing with the rotation angles  $\theta_L$  and  $\theta_R$  gives

$$\begin{pmatrix} \cos\theta_L & \sin\theta_L \\ -\sin\theta_L & \cos\theta_L \end{pmatrix} M_{\tilde{\psi}^+} \begin{pmatrix} \cos\theta_R & -\sin\theta_R \\ \sin\theta_R & \cos\theta_R \end{pmatrix} = \begin{pmatrix} m_{\chi_1^+} & 0 \\ 0 & m_{\chi_2^+} \end{pmatrix}. \quad (\text{B10})$$

The chargino mass eigenstates thus become

$$\begin{pmatrix} \chi_1^+ \\ \chi_2^+ \end{pmatrix} = \begin{pmatrix} \cos\theta_R & \sin\theta_R \\ -\sin\theta_R & \cos\theta_R \end{pmatrix} \begin{pmatrix} -i\tilde{W}^+ \\ \tilde{\psi}_u^+ \end{pmatrix} \quad (\text{B11})$$

and

$$\begin{pmatrix} \chi_1^- \\ \chi_2^- \end{pmatrix} = \begin{pmatrix} \cos\theta_L & \sin\theta_L \\ -\sin\theta_L & \cos\theta_L \end{pmatrix} \begin{pmatrix} -i\tilde{W}^- \\ \tilde{\psi}_d^- \end{pmatrix}. \quad (\text{B12})$$

The corresponding Dirac spinors are  $\tilde{\chi}_i = (\chi_i^+, \bar{\chi}_i^-)^T$ .

## 4. Charged Sleptons

The mass term for charged sleptons is

$$\mathcal{L} = \tilde{\ell}_\alpha^* (M_\ell^2)_{\alpha\beta} \tilde{\ell}_\beta. \quad (\text{B13})$$

Here  $\tilde{\ell}_\alpha = (\tilde{e}_L, \tilde{\mu}_L, \tilde{\tau}_L, \tilde{e}_R, \tilde{\mu}_R, \tilde{\tau}_R)^T$  is a column vector containing the left-handed sleptons in the first three components and the right-handed sleptons in the last three components.

The  $6 \times 6$  mass-squared matrix for sleptons is

$$M_{(\tilde{\ell})}^2 = \begin{pmatrix} M_L^2 & A \\ A^\dagger & M_R^2 \end{pmatrix}. \quad (\text{B14})$$

The mass matrix is diagonalized according to  $(M_{\tilde{\ell}}^2)_{\alpha\beta} = U_{\alpha i}^{(\tilde{\ell})} (M_{\tilde{\ell}}^2)_{ii} U_{i\beta}^{(\tilde{\ell})\dagger}$ , so the slepton mass eigenstates are defined by  $\tilde{\ell}_i = U_{i\alpha}^{(\tilde{\ell})\dagger} \tilde{\ell}_\alpha$  and  $\tilde{\ell}_\alpha = U_{\alpha i}^{(\tilde{\ell})} \tilde{\ell}_i$ , with the mass eigenstates ordered in terms of increasing mass.

Note that the first index of the matrix  $U$  is a gauge index and the second is a mass index; almost all interactions treat the left- and right- handed sleptons differently, so typically the first index will be summed from either 1 to 3 or 4 to 6, while the second will be summed over all six lepton mass eigenstates. Note also that in our convention the rotation matrix is the hermitian conjugate of the rotation matrix presented in Ref. [17].

## 5. Sneutrinos

The sneutrino mass term is

$$\mathcal{L} = \tilde{\nu}_\alpha^* (M_{\tilde{\nu}}^2)_{\alpha\beta} \tilde{\nu}_\beta, \quad (\text{B15})$$

with  $\tilde{\nu}_\alpha = (\tilde{\nu}_e, \tilde{\nu}_\mu, \tilde{\nu}_\tau)^T$ . Diagonalizing gives  $(M_{\tilde{\nu}}^2)_{\alpha\beta} = U_{\alpha i}^{(\tilde{\nu})} (M_{\tilde{\nu}}^2)_{ii} U_{i\beta}^{(\tilde{\nu})\dagger}$ , and thus  $\tilde{\nu}_i = U_{i\alpha}^{(\tilde{\nu})\dagger} \tilde{\nu}_\alpha$  and  $\tilde{\nu}_\alpha = U_{\alpha i}^{(\tilde{\nu})} \tilde{\nu}_i$ . Again our rotation matrix is the hermitian conjugate of the one in [17].

## 6. Leptons

As previously mentioned, we work in the basis of charged leptons in which the mass and flavor eigenstates are in correspondence. This is possible because there is a freedom of a single superfield rotation, and we choose to use this rotation to rotate away any mixing in the charged lepton sector (this was done for both notational simplicity and convenient coding). Also, as mentioned previously, we have neglected neutrino mixing, as neutrino flavor is not observable at colliders.

## APPENDIX C: LAGRANGIANS

Indices in Lagrangian coefficients are ordered by “gaugino, sfermion, fermion” if there is a gaugino present. Our Lagrangian assumes no neutrino masses and thus that the charged leptons can freely be rotated into mass eigenstates. Thus all the mixing terms will be parameterized by only the slepton and sneutrino mixing matrices (and the standard neutralino/chargino mixing matrices).

For the mixing matrices, the left and right bases are different, with one being the gauge basis and the other the mass basis. We have chosen the first index to correspond to a gauge eigenstate and the second to correspond to a mass eigenstate. Thus, in a term like  $U_{c,b}^{(\tilde{\ell})}$ ,  $c$  is the gauge index and  $b$  is a slepton mass index. Note that in summations over these indices, the gauge summations are always taken from 1 to 3 and the mass summations are taken from 1 to 6 for sleptons and 1 to 3 for sneutrinos.

In this section, we only consider phenomenologically viable decays to lowest order; thus we neglect four scalar couplings and couplings for modes that are unlikely to be kinematically allowed.

## 1. Neutralino Terms

The neutralino interaction terms are

$$\mathcal{L}_{\text{Neutralino}} = \left[ \alpha_{abc} \tilde{\nu}_b^* \tilde{\chi}_a^0 P_L \nu_c + \tilde{\ell}_b^* \tilde{\chi}_a^0 \left( \beta_{abc}^{(1)} P_L + \beta_{abc}^{(2)} P_R \right) \ell_c \right] + \text{h.c.}, \quad (\text{C1})$$

where

$$\alpha_{abc} = -\frac{1}{\sqrt{2}} \left( g O_{2,a}^* - g' O_{1,a}^* \right) U_{c,b}^{(\tilde{\nu})^*} \quad (\text{C2})$$

$$\beta_{abc}^{(1)} = \frac{1}{\sqrt{2}} \left( g O_{2,a}^* + g' O_{1,a}^* \right) U_{c,b}^{(\tilde{\ell})^*} - y_c^{(\ell)} O_{3,a}^* U_{c+3,b}^{(\tilde{\ell})^*} \quad (\text{C3})$$

$$\beta_{abc}^{(2)} = -\sqrt{2} O_{1,a} g' U_{c+3,b}^{(\tilde{\ell})^*} - y_c^{(\ell)} O_{3,a} U_{c,b}^{(\tilde{\ell})^*}. \quad (\text{C4})$$

For these terms, the matrix  $O_{i,j}$  is the neutralino mixing matrix and  $y_c^{(\ell)}$  is the  $c$  component of the diagonal lepton Yukawa matrix; the gauge components correspond to 1 - bino, 2 - wino, 3 - down Higgsino<sup>6</sup>.

## 2. Chargino Terms

The chargino interaction terms are

$$\mathcal{L}_{\text{Chargino}} = \left\{ \gamma_{abc} \tilde{\ell}_b^* \tilde{\chi}_a^{\pm} P_L \nu_c + \tilde{\nu}_b^* \tilde{\chi}_a^{\pm} \left( \delta_{abc}^{(1)} P_L + \delta_{abc}^{(2)} P_R \right) \ell_c \right\} + \text{h.c.}, \quad (\text{C5})$$

where

$$\gamma_{abc} = \begin{cases} -g \cos \theta_L U_{c,b}^{(\tilde{\ell})^*} + y_c^{(\ell)} \sin \theta_L U_{c+3,b}^{(\tilde{\ell})^*} & a = 1 \\ g \sin \theta_L U_{c,b}^{(\tilde{\ell})^*} + y_c^{(\ell)} \cos \theta_L U_{c+3,b}^{(\tilde{\ell})^*} & a = 2 \end{cases} \quad (\text{C6})$$

$$\delta_{abc}^{(1)} = \begin{cases} -g \cos \theta_R U_{c,b}^{(\tilde{\nu})^*} & a = 1 \\ g \sin \theta_R U_{c,b}^{(\tilde{\nu})^*} & a = 2 \end{cases} \quad (\text{C7})$$

$$\delta_{abc}^{(2)} = \begin{cases} y_c^{(\ell)} \sin \theta_L U_{c,b}^{(\tilde{\nu})^*} & a = 1 \\ y_c^{(\ell)} \cos \theta_L U_{c,b}^{(\tilde{\nu})^*} & a = 2 \end{cases}. \quad (\text{C8})$$

Here  $\theta_L$  and  $\theta_R$  are the rotation angles associated with rotating the negatively and positively charged (respectively) gauginos and higgsinos into the mass eigenstates.

---

<sup>6</sup> The up Higgsino corresponds to  $a = 4$ , but this is never needed.

### 3. Higgs Terms

The neutral Higgs interaction terms are

$$\mathcal{L}_{\text{Neutral Higgs}} = \sigma_{ab}^{(1)} \tilde{\nu}_a^* \tilde{\nu}_b H^0 + \sigma_{ab}^{(2)} \tilde{\ell}_a^* \tilde{\ell}_b h^0 + \sigma_{ab}^{(3)} \tilde{\ell}_a^* \tilde{\ell}_b H^0 + i\sigma_{ab}^{(4)} \tilde{\ell}_a^* \tilde{\ell}_b A^0 + i\sigma_{ab}^{(5)} \tilde{\ell}_a^* \tilde{\ell}_b G^0. \quad (\text{C9})$$

There are five different  $\sigma$  coefficients since the two slepton couplings to  $h^0$  and  $H^0$  are different:

$$\sigma_{ab}^{(1)} = -\frac{gm_W}{2 \cos^2 \theta_W} \cos(\theta_H + \beta) \delta_{ab} \quad (\text{C10})$$

$$\begin{aligned} \sigma_{ab}^{(2)} = & - \left[ \left( \frac{gm_W}{2} (1 - \tan^2 \theta_W) \sin(\theta_H + \beta) - \frac{gm_{\tilde{\ell}_c}^2 \sin \theta_H}{m_W \cos \beta} \right) U_{c,a}^{(\tilde{\ell})*} U_{c,b}^{(\tilde{\ell})} \right. \\ & + \left( gm_W \tan^2 \theta_W \sin(\theta_H + \beta) - \frac{gm_{\tilde{\ell}_c}^2 \sin \theta_H}{m_W \cos \beta} \right) U_{c+3,a}^{(\tilde{\ell})*} U_{c+3,b}^{(\tilde{\ell})} \\ & \left. - \frac{gm_{\tilde{\ell}_c}}{2m_W \cos \beta} (\mu \cos \theta_H + A_c^{\tilde{\ell}} \sin \theta_H) \left( U_{c,a}^{(\tilde{\ell})*} U_{c+3,b}^{(\tilde{\ell})} + U_{c+3,a}^{(\tilde{\ell})*} U_{c,b}^{(\tilde{\ell})} \right) \right] \end{aligned} \quad (\text{C11})$$

$$\begin{aligned} \sigma_{ab}^{(3)} = & \left[ \left( \frac{gm_W}{2} (1 - \tan^2 \theta_W) \cos(\theta_H + \beta) - \frac{gm_{\tilde{\ell}_c}^2 \cos \theta_H}{m_W \cos \beta} \right) U_{c,a}^{(\tilde{\ell})*} U_{c,b}^{(\tilde{\ell})} \right. \\ & + \left( gm_W \tan^2 \theta_W \cos(\theta_H + \beta) - \frac{gm_{\tilde{\ell}_c}^2 \cos \theta_H}{m_W \cos \beta} \right) U_{c+3,a}^{(\tilde{\ell})*} U_{c+3,b}^{(\tilde{\ell})} \\ & \left. + \frac{gm_{\tilde{\ell}_c}}{2m_W \cos \beta} (\mu \sin \theta_H - A_c^{\tilde{\ell}} \cos \theta_H) \left( U_{c,a}^{(\tilde{\ell})*} U_{c+3,b}^{(\tilde{\ell})} + U_{c+3,a}^{(\tilde{\ell})*} U_{c,b}^{(\tilde{\ell})} \right) \right] \end{aligned} \quad (\text{C12})$$

$$\sigma_{ab}^{(4)} = \frac{gm_{\tilde{\ell}_c}}{2m_W} (\mu + A_c^{\tilde{\ell}} \tan \beta) \left( U_{c,a}^{(\tilde{\ell})*} U_{c+3,b}^{(\tilde{\ell})} - U_{c+3,a}^{(\tilde{\ell})*} U_{c,b}^{(\tilde{\ell})} \right) \quad (\text{C13})$$

$$\sigma_{ab}^{(5)} = \frac{gm_{\tilde{\ell}_c}}{2m_W} (\mu \tan \beta + A_c^{\tilde{\ell}}) \left( U_{c,a}^{(\tilde{\ell})*} U_{c+3,b}^{(\tilde{\ell})} - U_{c+3,a}^{(\tilde{\ell})*} U_{c,b}^{(\tilde{\ell})} \right). \quad (\text{C14})$$

Here  $\tan \theta_W = g'/g$  as usual,  $\tan \beta = v_u/v_d$ , and the angle  $\theta_H$  is the rotation angle for the neutral Higgs scalars  $H_u$  and  $H_d$  into the real neutral Higgs mass eigenstates. The light and heavy real Higgs bosons are denoted  $h^0$  and  $H^0$ , and the pseudoscalar Higgs is denoted  $A^0$ . The parameter  $\mu$  is the Higgsino mass parameter, and  $A_{\ell_i}$  is the left-right mixing term for  $\ell_i$  in the diagonal lepton basis.

It is useful to note that several of the neutral Higgs terms are proportional to  $U_{c,a}^{(\tilde{\ell})*} U_{c,b}^{(\tilde{\ell})}$ ,  $U_{c+3,a}^{(\tilde{\ell})*} U_{c+3,b}^{(\tilde{\ell})}$ , or  $U_{c+3,a}^{(\tilde{\ell})*} U_{c,b}^{(\tilde{\ell})}$ . The first two are ‘‘block flavor diagonal’’ in left and right sleptons and the third has explicit left-right mixing, so flavor mixing decays of these modes should be significantly suppressed relative to chargino/neutralino modes by the size of left-right mixing, as discussed in Ref. [22].

Using the conventions above, the charged Higgs interaction terms are

$$\mathcal{L}_{\text{Charged Higgs}} = \left[ \rho_{ab}^{(1)} \tilde{\nu}_a^* \tilde{\ell}_b H^+ + \rho_{ab}^{(2)} \tilde{\nu}_a^* \tilde{\ell}_b G^+ \right] + \text{h.c.}, \quad (\text{C15})$$

where

$$\rho_{ab}^{(1)} = -g \left[ \left( \frac{m_W}{\sqrt{2}} \sin 2\beta - \frac{m_{\tilde{\ell}_c}^2 \tan \beta}{\sqrt{2} m_W} \right) U_{c,a}^{(\tilde{\nu})*} U_{c,b}^{(\tilde{\ell})} \right]$$

$$-\frac{m_{\ell_c}}{\sqrt{2}m_W}(\mu + A_{\ell_c} \tan \beta)U_{c,a}^{(\tilde{\nu})*}U_{c+3,b}^{(\tilde{\ell})}] \quad (\text{C16})$$

$$\rho_{ab}^{(2)} = g \left[ \left( \frac{m_W}{\sqrt{2}} \cos 2\beta - \frac{m_{\ell_c}^2}{\sqrt{2}m_W} \right) U_{c,a}^{(\tilde{\nu})*}U_{c,b}^{(\tilde{\ell})} + \frac{m_{\ell_c}}{\sqrt{2}m_W}(\mu \tan \beta - A_{\ell_c})U_{c,a}^{(\tilde{\nu})*}U_{c+3,b}^{(\tilde{\ell})} \right]. \quad (\text{C17})$$

Note there is no additional rotation angle since  $\beta$  serves as the rotation angle for the charged Higgs.

#### 4. Gauge Bosons

The Lagrangian interaction terms for the gauge bosons are

$$L_{\text{Gauge Boson}} = \left[ i\zeta_{ab}^{(1)} (\tilde{\nu}_a^* \partial_\mu \tilde{\ell}_b - \tilde{\ell}_b \partial_\mu \tilde{\nu}_a^*) W^{+\mu} + \text{h.c.} \right] + i\zeta_{ab}^{(2)} (\tilde{\ell}_a^* \partial_\mu \tilde{\ell}_b - \tilde{\ell}_b \partial_\mu \tilde{\ell}_a^*) Z^\mu + i\zeta_{ab}^{(3)} (\tilde{\ell}_a^* \partial_\mu \tilde{\ell}_b - \tilde{\ell}_b \partial_\mu \tilde{\ell}_a^*) A^\mu, \quad (\text{C18})$$

where

$$\zeta_{ab}^{(1)} = -\frac{g}{\sqrt{2}}U_{c,a}^{(\tilde{\nu})*}U_{c,b}^{(\tilde{\ell})} \quad (\text{C19})$$

$$\zeta_{ab}^{(2)} = \frac{g}{2 \cos \theta_W} \left[ U_{c,a}^{(\tilde{\ell})*}U_{c,b}^{(\tilde{\ell})} - 2 \sin^2 \theta_W \delta_{ab} \right] \quad (\text{C20})$$

$$\zeta_{ab}^{(3)} = e\delta_{ab}. \quad (\text{C21})$$

Here,  $e$  is the electromagnetic charge. As above for neutral Higgs bosons, the flavor mixing modes involving  $Z$  bosons are heavily suppressed by the left-right mixing. The photon vertex remains flavor diagonal even in the flavor mixing case, as expected.

APPENDIX D: TABLE OF DECAY MODES

Decay Number	Decay Mode	Corresponding Program Method
1	$\tilde{\chi}_i^0 \rightarrow \tilde{\ell}_j^- \ell_k^+$	CalcChi0SlepLepBar
2	$\tilde{\chi}_i^0 \rightarrow \tilde{\nu}_j \bar{\nu}_k$	CalcChi0SnuNuBar
3	$\tilde{\chi}_i^- \rightarrow \tilde{\ell}_j^- \bar{\nu}_k$	CalcChiMinusSlepNuBar
4	$\tilde{\chi}_i^+ \rightarrow \tilde{\nu}_j \ell_k^+$	CalcChiPlusSnuLepBar
5	$\tilde{\ell}_i^- \rightarrow \tilde{\chi}_j^0 \ell_k^-$	CalcSlepChi0Lep
6	$\tilde{\ell}_i^- \rightarrow \tilde{\chi}_j^- \nu_k$	CalcSlepChiMinusNu
7	$\tilde{\ell}_i^- \rightarrow \tilde{\ell}_j^- h^0$	CalcSlepSlepHiggs
8	$\tilde{\ell}_i^- \rightarrow \tilde{\ell}_j^- Z^0$	CalcSlepSlepZ
9	$\tilde{\ell}_i^- \rightarrow \tilde{\nu}_j W^-$	CalcSlepSnuWMinus
10	$\tilde{\nu}_i \rightarrow \tilde{\chi}_j^0 \nu_k$	CalcSnuChi0Nu
11	$\tilde{\nu}_i \rightarrow \tilde{\chi}_j^+ \ell_k^-$	CalcSnuChiPlusLep
12	$\tilde{\nu}_i \rightarrow \tilde{\ell}_j^- W^+$	CalcSnuSlepWPlus
13	$H^0 \rightarrow \tilde{\ell}_i^- \tilde{\ell}_j^+$	CalcHSlepSlepBar
14	$H^0 \rightarrow \tilde{\nu}_i \tilde{\nu}_j$	CalcHSnuSnuBar
15	$A^0 \rightarrow \tilde{\ell}_i^- \tilde{\ell}_j^+$	CalcASlepSlepBar
16	$H^- \rightarrow \tilde{\ell}_i^- \tilde{\nu}_j$	CalcHMinusSlepSnuBar
17	$\tilde{\ell}_i^- \rightarrow \tilde{\ell}_j^- \ell_k^- \ell_m^+$	CalcSlepSlepLepLepBar
18	$\tilde{\ell}_i^- \rightarrow \tilde{\ell}_j^+ \ell_k^- \ell_m^-$	CalcSlepSlepBarLepLep
19	$\tilde{\ell}_i^- \rightarrow \tilde{\ell}_j^+ \nu_k \bar{\nu}_m$	CalcSlepSlepNuNuBar
20	$\tilde{\ell}_i^- \rightarrow \tilde{\ell}_j^+ q_k \bar{q}_k$	CalcSlepSlepQQBar

Table II: List of included decay modes with corresponding program methods in order of appearance.

Neglected Decay Modes	Justification
$\tilde{\ell}_i^- \rightarrow \tilde{\ell}_j^- H^0$	Kinematics — $H^0$ assumed heavier than $\tilde{\ell}$
$\tilde{\ell}_i^- \rightarrow \tilde{\ell}_j^- A^0$	Kinematics — $A^0$ assumed heavier than $\tilde{\ell}$
$\tilde{\ell}_i^- \rightarrow \tilde{\nu}_j H^-$	Kinematics — $H^-$ assumed heavier than $\tilde{\ell}$
$\tilde{\nu}_i \rightarrow \tilde{\nu}_j Z$	Kinematics — $Z$ assumed heavier than $\tilde{\nu}$ splitting
$\tilde{\nu}_i \rightarrow \tilde{\nu}_j h^0$	Kinematics — $h^0$ assumed heavier than $\tilde{\nu}$ splitting
$\tilde{\nu}_i \rightarrow \tilde{\nu}_j H^0$	Kinematics — $H^0$ assumed heavier than $\tilde{\nu}$
$\tilde{\nu}_i \rightarrow \tilde{\ell}_j^- H^+$	Kinematics — $H^+$ assumed heavier than $\tilde{\nu}$
$h^0 \rightarrow \tilde{\ell}_i^- \tilde{\ell}_j^+$	Kinematics — multiple $\tilde{\ell}$ 's assumed heavier than $h^0$
$h^0 \rightarrow \tilde{\nu}_i^- \tilde{\nu}_j^+$	Kinematics — multiple $\tilde{\nu}$ 's assumed heavier than $h^0$
Any with emitted $\gamma$	QED interactions still flavor neutral
Scalar three-body decays	Suppression by couplings and phase space factors

Table III: List of neglected decay modes, and reason for omission.

## APPENDIX E: TWO BODY DECAYS

This section contains the matrix elements and decay widths for all two-body decays. Charge conjugate modes are not included; we work in a non-CP violating lepton sector, so charge conjugate modes in all cases have the same decay widths as the original modes. Again, neutrinos are taken to be massless.

Our index notation is that the index  $i$  refers to the decaying particle and the indices  $j$  and  $k$  to the decay products. Further, if the decaying particle is a slepton, sneutrino, chargino, or neutralino then the index  $j$  refers to the resultant SUSY particle while  $k$  refers to the resultant Standard Model particle; this convention does not apply to Higgs decays, which have two SUSY daughters.

The general formula for the decay width of a two-body decay in the center of mass frame of the decaying  $i$  particle with daughters  $j$  and  $k$  is

$$\Gamma_{2 \text{ body decay}} = \frac{|\mathcal{M}|^2}{16\pi m_i^3} \lambda^{1/2} (m_i^2, m_j^2, m_k^2), \quad (\text{E1})$$

where  $\lambda(x, y, z) = x^2 + y^2 + z^2 - 2(xy + xz + yz)$ , and the matrix element squared has an implicit average/sum over fermion spins if the decay involves fermions. For our purposes, there are three general cases of matrix element forms: those containing two fermions (lepton and neutralino/chargino), those containing gauge bosons, and those containing only sleptons and Higgs scalars. The scalar case is trivial, since the matrix element is equal to the Lagrangian term with no momentum dependence, and the decay width is simply the square of the Lagrangian term multiplied by a phase space factor. For fermionic modes with fermions  $f_1$  and  $f_2$  and a scalar  $s$ , the matrix element takes the form of

$$\mathcal{M}_{\text{fermionic 2-body}} \sim i \begin{pmatrix} \bar{u}_1(p_{f_1}) \\ \bar{v}_1(p_{f_1}) \end{pmatrix} (AP_R + BP_L) \begin{pmatrix} u_2(p_{f_2}) \\ v_2(p_{f_2}) \end{pmatrix}, \quad (\text{E2})$$

which gives a decay width proportional to  $\pm(|A|^2 + |B|^2)(m_{f_1}^2 + m_{f_2}^2 - m_s^2) \pm 4m_{f_1}m_{f_2} \text{Re}(AB^*)$  times the standard phase space factor (with the possibility of  $m_{f_2} = 0$  if a neutrino is a decay product). The sign on the second term will be positive if both spinors are  $u$  or  $v$ , and negative if they are different spinors.

Finally, gauge boson matrix elements with scalars  $s_1$  and  $s_2$  and a gauge boson  $b$  are proportional to  $(p_{s_1} + p_{s_2})^\mu \epsilon_\mu$ , so the decay width will be proportional to  $\lambda^{\frac{1}{2}}(m_{s_1}^2, m_{s_2}^2, m_b^2) / m_b^2$ .

### 1. Neutralino Decays

For a neutralino two-body decay to charged slepton and lepton, the flavor-generalized matrix element is

$$\mathcal{M}(\tilde{\chi}_i^0 \rightarrow \tilde{\ell}_j^- \ell_k^+) = i \bar{u}_{\ell_k}(p_k) \left( \beta_{ijk}^{(1)*} P_R + \beta_{ijk}^{(2)*} P_L \right) u_{\tilde{\chi}_i^0}(p_i), \quad (\text{E3})$$

and the decay width is

$$\Gamma(\tilde{\chi}_i^0 \rightarrow \tilde{\ell}_j^- \ell_k^+) = \frac{1}{32\pi m_{\tilde{\chi}_i^0}^3} \lambda^{1/2}(m_{\tilde{\chi}_i^0}^2, m_{\tilde{\ell}_j^-}^2, m_{\ell_k^+}^2) \left[ \left( |\beta_{ijk}^{(1)}|^2 + |\beta_{ijk}^{(2)}|^2 \right) \left( m_{\tilde{\chi}_i^0}^2 + m_{\ell_k^+}^2 - m_{\tilde{\ell}_j^-}^2 \right) + 4m_{\tilde{\chi}_i^0} m_{\ell_k^+} \text{Re}(\beta_{ijk}^{(1)*} \beta_{ijk}^{(2)}) \right]. \quad (\text{E4})$$

For a neutralino two-body decay to sneutrino and neutrino, the matrix element is

$$\mathcal{M}(\tilde{\chi}_i^0 \rightarrow \tilde{\nu}_{\ell_j} \bar{\nu}_k) = i\alpha_{ijk}^* \bar{\nu}_{\nu_k}(p_k) P_R v_{\tilde{\chi}_i^0}(p_i), \quad (\text{E5})$$

and the decay width is

$$\Gamma(\tilde{\chi}_i^0 \rightarrow \tilde{\nu}_{\ell_j} \bar{\nu}_k) = \frac{1}{32\pi m_{\tilde{\chi}_i^0}^3} |\alpha_{ijk}|^2 (m_{\tilde{\chi}_i^0}^2 - m_{\tilde{\nu}_j}^2)^2. \quad (\text{E6})$$

## 2. Chargino Decays

The matrix element for a chargino decay to slepton and neutrino is

$$\mathcal{M}(\tilde{\chi}_i^- \rightarrow \tilde{\ell}_j^- \bar{\nu}_{\ell_k}) = \gamma_{ijk}^* \bar{\nu}_{\nu_k}(p_k) P_R v_{\tilde{\chi}_i^-}(p_i), \quad (\text{E7})$$

and the decay width is

$$\Gamma(\tilde{\chi}_i^- \rightarrow \tilde{\ell}_j^- \bar{\nu}_{\ell_k}) = \frac{|\gamma_{ijk}|^2}{32\pi m_{\tilde{\chi}_i^-}^3} (m_{\tilde{\chi}_i^-}^2 - m_{\tilde{\ell}_j}^2)^2. \quad (\text{E8})$$

The matrix element for a chargino decay to sneutrino and lepton is

$$\begin{aligned} \mathcal{M}(\tilde{\chi}_i \rightarrow \tilde{\nu}_j \ell_k^+) &= i\bar{\nu}_{\ell_k}(p_k) (\delta_{jik}^{(1)*} P_R + \delta_{jik}^{(2)*} P_L) \mathcal{C} [\bar{u}_{\tilde{\chi}_i}(p_i)]^T \\ &= i\bar{\nu}_{\ell_k}(p_k) (\delta_{jik}^{(1)*} P_R + \delta_{jik}^{(2)*} P_L) v_{\tilde{\chi}_i}(p_j), \end{aligned} \quad (\text{E9})$$

and the decay width is

$$\begin{aligned} \Gamma(\tilde{\chi}_i \rightarrow \tilde{\nu}_j \ell_k^+) &= \frac{1}{32\pi m_{\tilde{\chi}_i}^3} \lambda^{1/2}(m_{\tilde{\chi}_i}^2, m_{\tilde{\nu}_j}^2, m_{\ell_k}^2) \\ &\left[ \left( |\delta_{ijk}^{(1)}|^2 + |\delta_{ijk}^{(2)}|^2 \right) (m_{\tilde{\chi}_i}^2 + m_{\ell_k}^2 - m_{\tilde{\nu}_j}^2) + 4m_{\tilde{\chi}_i} m_{\ell_k} \text{Re}(\delta_{ijk}^{(1)*} \delta_{ijk}^{(2)}) \right]. \end{aligned} \quad (\text{E10})$$

Note that  $\mathcal{C}$  is the charge conjugation operator.

## 3. Slepton Decays

The slepton decay to neutralino and lepton has a matrix element given by

$$\mathcal{M}(\tilde{\ell}_i^- \rightarrow \tilde{\chi}_j^0 \ell_k^-) = i\bar{u}_{\ell_k}(p_k) (\beta_{jik}^{(1)} P_R + \beta_{jik}^{(2)} P_L) v_{\tilde{\chi}_j^0}(p_j), \quad (\text{E11})$$

and the corresponding decay width is

$$\begin{aligned} \Gamma(\tilde{\ell}_i^- \rightarrow \tilde{\chi}_j^0 \ell_k^-) &= \frac{1}{16\pi m_{\tilde{\ell}_i}^3} \lambda^{1/2}(m_{\tilde{\ell}_i}^2, m_{\tilde{\chi}_j^0}^2, m_{\ell_k}^2) \\ &\left[ \left( |\beta_{jik}^{(1)}|^2 + |\beta_{jik}^{(2)}|^2 \right) (m_{\tilde{\ell}_i}^2 - m_{\tilde{\chi}_j^0}^2 - m_{\ell_k}^2) - 4m_{\tilde{\chi}_j^0} m_{\ell_k} \text{Re}(\beta_{jik}^{(1)*} \beta_{jik}^{(2)}) \right]. \end{aligned} \quad (\text{E12})$$



For a slepton decay to chargino and neutrino, the matrix element is

$$\mathcal{M}(\tilde{\ell}_i^- \rightarrow \tilde{\chi}_j^- \nu_{\ell_k}) = i\gamma_{jik}\bar{u}_{\nu_k}(p_k)P_R v_{\tilde{\chi}_j^-}(p_j), \quad (\text{E13})$$

and the decay width is

$$\Gamma(\tilde{\ell}_i^- \rightarrow \tilde{\chi}_j^- \nu_{\ell_k}) = \frac{|\gamma_{jik}|^2}{16\pi m_{\tilde{\ell}_i}^3} \left( m_{\tilde{\ell}_i}^2 - m_{\tilde{\chi}_j^-}^2 \right)^2. \quad (\text{E14})$$

A slepton decay to light (neutral) higgs and slepton has a matrix element of

$$\mathcal{M}(\tilde{\ell}_i^- \rightarrow \tilde{\ell}_j^- h^0) = i\sigma_{ji}^{(2)}, \quad (\text{E15})$$

and the decay width is

$$\Gamma(\tilde{\ell}_i^- \rightarrow \tilde{\ell}_j^- h^0) = \frac{|\sigma_{ji}^{(2)}|^2}{16\pi m_{\tilde{\ell}_i}^3} \lambda^{1/2} \left( m_{\tilde{\ell}_i}^2, m_{\tilde{\ell}_j}^2, m_h^2 \right). \quad (\text{E16})$$

The slepton to slepton and  $Z$  boson matrix element is

$$\mathcal{M}(\tilde{\ell}_i^- \rightarrow \tilde{\ell}_j^- Z^0) = i\zeta_{ji}^{(2)}(p_i + p_j)^\mu \epsilon_\mu^*(p_Z), \quad (\text{E17})$$

and the decay width is

$$\Gamma(\tilde{\ell}_i^- \rightarrow \tilde{\ell}_j^- Z^0) = \frac{|\zeta_{ji}^{(2)}|^2}{16\pi m_Z^2 m_{\tilde{\ell}_i}^3} \lambda^{3/2} \left( m_{\tilde{\ell}_i}^2, m_{\tilde{\ell}_j}^2, m_Z^2 \right). \quad (\text{E18})$$

The slepton to sneutrino and  $W$  boson matrix element is

$$\mathcal{M}(\tilde{\ell}_i^- \rightarrow \tilde{\nu}_j W^-) = i\zeta_{ji}^{(1)}(p_i + p_j)^\mu \epsilon_\mu^*(p_W), \quad (\text{E19})$$

and the decay width is

$$\Gamma(\tilde{\ell}_i^- \rightarrow \tilde{\nu}_j W^-) = \frac{|\zeta_{ji}^{(1)}|^2}{16\pi m_W^2 m_{\tilde{\ell}_i}^3} \lambda^{3/2} \left( m_{\tilde{\ell}_i}^2, m_{\tilde{\nu}_j}^2, m_W^2 \right). \quad (\text{E20})$$

#### 4. Sneutrino Decays

The two-body mode for sneutrino decaying to neutralino and neutrino has a matrix element of

$$\mathcal{M}(\tilde{\nu}_i \rightarrow \tilde{\chi}_j^0 \nu_k) = i\alpha_{jik}\bar{u}_{\nu_k}(p_k)P_R v_{\tilde{\chi}_j^0}(p_j), \quad (\text{E21})$$

and a decay width

$$\Gamma(\tilde{\nu}_i \rightarrow \tilde{\chi}_j^0 \nu_k) = \frac{|\alpha_{jik}|^2}{16\pi m_{\tilde{\nu}_i}^3} \left( m_{\tilde{\nu}_i}^2 - m_{\tilde{\chi}_j^0}^2 \right)^2. \quad (\text{E22})$$

The sneutrino to chargino and lepton two-body mode has a matrix element of

$$\begin{aligned} \mathcal{M}(\tilde{\nu}_i \rightarrow \tilde{\chi}_j^+ \ell_k^-) &= i\bar{u}_{\ell_k^-}(p_k) \left( \delta_{jik}^{(1)} P_R + \delta_{jik}^{(2)} P_L \right) \mathcal{C} \left[ \bar{v}_{\tilde{\chi}_j^+}(p_i) \right]^T \\ &= i\bar{u}_{\ell_k^-}(p_k) \left( \delta_{jik}^{(1)} P_R + \delta_{jik}^{(2)} P_L \right) u_{\tilde{\chi}_j^+}(p_j), \end{aligned} \quad (\text{E23})$$

and a decay width of

$$\Gamma(\tilde{\nu}_i \rightarrow \tilde{\chi}_j^+ \ell_k^-) = \frac{1}{16\pi m_{\tilde{\nu}_i}^3} \lambda^{1/2} \left( m_{\tilde{\nu}_i}^2, m_{\tilde{\chi}_j^+}^2, m_{\ell_k^-}^2 \right) \left[ \left( |\delta_{jik}^{(1)}|^2 + |\delta_{jik}^{(2)}|^2 \right) \left( m_{\tilde{\nu}_i}^2 - m_{\tilde{\chi}_j^+}^2 - m_{\ell_k^-}^2 \right) + 4m_{\tilde{\chi}_j^+} m_{\ell_k^-} \operatorname{Re} \left( \delta_{jik}^{(1)*} \delta_{jik}^{(2)} \right) \right]. \quad (\text{E24})$$

The matrix element for sneutrinos decaying to sleptons and  $W$  bosons is

$$\mathcal{M}(\tilde{\nu}_i \rightarrow \tilde{\ell}_j^- W^+) = i\zeta_{ij}^{(1)} (p_i + p_j)^\mu \epsilon_\mu^*(p_W), \quad (\text{E25})$$

and the decay width is

$$\Gamma(\tilde{\nu}_i \rightarrow \tilde{\ell}_j^- W^+) = \frac{|\zeta_{ij}^{(1)}|^2}{16\pi m_W^2 m_{\tilde{\nu}_i}^3} \lambda^{3/2} \left( m_{\tilde{\nu}_i}^2, m_{\tilde{\ell}_j^-}^2, m_W^2 \right). \quad (\text{E26})$$

## 5. Higgs Decays

Finally, the Higgs decays have trivial matrix elements, so we only present their widths. The decay width for heavy Higgs boson to two sleptons is

$$\Gamma(H^0 \rightarrow \tilde{\ell}_j^- \tilde{\ell}_k^+) = \frac{|\sigma_{jk}^{(3)}|^2}{16\pi m_H^3} \lambda^{1/2} \left( m_H^2, m_{\tilde{\ell}_j^-}^2, m_{\tilde{\ell}_k^+}^2 \right), \quad (\text{E27})$$

while the width to two sneutrinos is

$$\Gamma(H^0 \rightarrow \tilde{\nu}_j \tilde{\nu}_k) = \frac{|\sigma_{jk}^{(1)}|^2}{16\pi m_H^3} \lambda^{1/2} \left( m_H^2, m_{\tilde{\nu}_j}^2, m_{\tilde{\nu}_k}^2 \right). \quad (\text{E28})$$

The decay width for pseudoscalar Higgs to two sleptons is

$$\Gamma(A^0 \rightarrow \tilde{\ell}_j^- \tilde{\ell}_k^+) = \frac{|\sigma_{jk}^{(4)}|^2}{16\pi m_{A^0}^3} \lambda^{1/2} \left( m_{A^0}^2, m_{\tilde{\ell}_j^-}^2, m_{\tilde{\ell}_k^+}^2 \right), \quad (\text{E29})$$

and the decay width for charged Higgs to slepton and sneutrino is

$$\Gamma(H^- \rightarrow \tilde{\ell}_j^- \tilde{\nu}_k) = \frac{|\rho_{jk}|^2}{16\pi m_{H^-}^3} \lambda^{1/2} \left( m_{H^-}^2, m_{\tilde{\nu}_k}^2, m_{\tilde{\ell}_j^-}^2 \right). \quad (\text{E30})$$

## APPENDIX F: THREE BODY DECAYS

The general three-body decay width is

$$\Gamma_{3 \text{ body decay}} = \frac{1}{64\pi^3 m_i} \int_{m_k}^{\frac{1}{2m_i}(m_i^2 + m_k^2 - (m_j + m_m)^2)} dE_k \int_{E_m^-}^{E_m^+} dE_m |\mathcal{M}|^2, \quad (\text{F1})$$

where the squared matrix element includes a sum over the outgoing fermion spins. The limits are

$$E_m^\pm = \frac{1}{2(p_i - p_k)^2} \left[ (m_i - E_k) \left( m_i^2 - m_j^2 + m_k^2 + m_m^2 - 2m_i E_k \right) \pm p_k \lambda^{\frac{1}{2}} \left( (p_i - p_k)^2, m_j^2, m_m^2 \right) \right], \quad (\text{F2})$$

and  $\lambda(x, y, z) = x^2 + y^2 + z^2 - 2(xy + xz + yz)$  as in the two-body decays.

For our purposes, we analytically reduce this to a one-dimensional integral over  $E_k$  by integrating out  $E_m$ , and then we evaluate the integral numerically. We only evaluate the three-body modes if there are no kinematically allowed two-body decays<sup>7</sup>. We could technically apply the three-body formulas to the entire slepton sector, but this would in effect double count the on-shell decays of heavier sleptons.

The full form of these slepton three-body modes is too lengthy to include here; complete expressions are given in Ref. [22]. At a qualitative level, there are four distinct decay modes:

$$\tilde{\ell}^- \rightarrow \tilde{\ell}^- \ell_k^- \ell_m^+, \quad \tilde{\ell}^- \rightarrow \tilde{\ell}^+ \ell_k^- \ell_m^-, \quad \tilde{\ell}^- \rightarrow \tilde{\ell}^+ \nu_k \bar{\nu}_m, \quad \tilde{\ell}^- \rightarrow \tilde{\ell}^+ q_k \bar{q}_k. \quad (\text{F3})$$

The first three decays exist in the flavor-conserving case, occurring through an off-shell neutralino or chargino; the addition of flavor violating effects introduces flavor-violating couplings and an interference term between the diagrams in the  $\tilde{\ell}^- \rightarrow \tilde{\ell}^+ \ell_k^- \ell_m^-$  mode. Flavor violating effects also introduce the possibility of radiating an off-shell  $Z$  or Higgs, which contribute both to the decays to a slepton and a same-generation lepton and anti-lepton pair and to a slepton and two quarks.

## APPENDIX G: PROGRAM DETAILS

Here we outline the structure of the **SPICE** program. There are three distinct components: the code to impose generalized flavor-violating boundary conditions for **SOFTSUSY** and calculate sleptonic decay widths, an instance of **SUSYHIT** to calculate non-sleptonic decay widths, and a subroutine which merges the information from both files into an output file. As previously mentioned, the instance of **SUSYHIT** is unchanged from its original form.

### 1. SOFTSUSY Customization and Sleptonic Decay Widths

Our customization of the **SOFTSUSY** code has two components — renormalization boundary conditions with both gauge and gravity mediated contributions and calculation of decay widths for slepton flavor violating decays.

We have not altered the **SOFTSUSY** renormalization algorithms; however, since the lepton mass matrix is in general non-diagonal, the mixing between lepton gauge and mass eigenstates must be rotated out of the lepton sector at the high scale to conform to **SOFTSUSY**'s boundary conditions. This is done by first diagonalizing the lepton mass matrix

$$\ell_{L\alpha} m_{E_{\alpha\beta}} \ell_{R\beta} = \ell_{L\alpha} U_{\alpha i}^{(\ell_L)} m_{E_{ii}}^D U_{i\beta}^{(\ell_R)\dagger} \ell_{R\beta} = \ell_{L_i} m_{E_{ii}}^D \ell_{R_i}, \quad (\text{G1})$$

where  $m_{E_{\alpha\beta}}$  is the original lepton mass matrix and  $m_{E_{ii}}^D$  is the diagonalized lepton mass matrix.

The interaction terms in the Lagrangian have the form

$$\tilde{\ell}_{L\alpha}^* \ell_{L\alpha} = U_{i\alpha}^{(\tilde{\ell}_L)\dagger} U_{\alpha j}^{(\ell_L)} \tilde{\ell}_{L_i}^* \ell_{L_j}, \quad (\text{G2})$$

---

<sup>7</sup> We always ignore two-body decays to gravitino. See Sec. IV for details.

with similar expressions for the right-handed slepton and sneutrinos. Then the lepton mixing may be absorbed into the slepton mixing at the high scale, whereby

$$\begin{aligned}
 U_{i\alpha}^{(\tilde{\ell}_L)^\dagger} &\rightarrow U_{i\beta}^{(\tilde{\ell}_L)^\dagger} U_{\beta\alpha}^{(\ell_L)} \\
 \left(M_{\tilde{\ell}_L}^2\right)_{\alpha\beta} &\rightarrow U_{\alpha\sigma}^{(\tilde{\ell}_L)^\dagger} \left(M_{\tilde{\ell}_L}^2\right)_{\sigma\rho} U_{\rho\beta}^{(\ell_L)},
 \end{aligned}
 \tag{G3}$$

again with similar expressions for right sleptons and sneutrinos.

Here we take advantage of the negligible left-right mixing terms at the high scale to decouple mixing of left and right sleptons, though the procedure does work in general if the  $3 \times 3$  matrix associated with left-right mixing is also rotated in a similar fashion. The diagonalized lepton mass matrix and the rotated slepton and sneutrino mass matrices are then used as the high-scale boundary conditions for `SOFTSUSY`'s renormalization routine, and is found in the `gaugegravityBcs` method.

## 2. Sleptonic Decay Calculations

The new calculation of flavor violating decays is encompassed in three classes: `DecayFormat`, `DecayCalc`, and `ThreeBodyIntegral`. Calculations of decay widths is performed in the `DecayCalc` class, with the help of the `ThreeBodyIntegrals` class for three-body decays. The `DecayFormat` class handles output to the intermediate file and interfaces with the main program.

### a. `DecayCalc` class

Method	Purpose
<code>Lambda</code>	Common phase space function
<code>DecayCalc</code>	Loads the slepton masses and mixing matrices from a passed <code>FlavourMssmSoftsusy</code> object.
<code>Alpha</code> , <code>Beta[1-2]</code> , <code>Gamma</code> , <code>Delta[1-2]</code> , <code>Sigma[1-4]</code> , <code>Rho</code> , <code>Zeta[1-2]</code>	Methods which return the value of the corresponding coupling coefficient for a given set of mass eigenstates. The methods are in direct correspondence to the couplings in the Lagrangians presented in App. C.
<code>Calc[...]</code>	Calculates the decay width (in GeV) of the given decay for a given set of mass eigenstates. The calculation is performed according to the decay widths given in App. E and App. F.
<code>Output[...]</code> <code>Decays</code>	Methods which loop through all decays of a particular particle type and passes decay widths to the <code>DecayFormat</code> object for output.
<code>IntegralCoefficient</code> <code>CrossCoefficient</code>	Methods to produce the beta coefficients which weight the three-body decay integrals.

`DecayCalc` loads the slepton mass and mixing matrices from a passed `FlavourMssmSoftsusy` object, which has already undergone the renormalization routine. The slepton and sneutrino masses are contained in six and three component vectors, respectively, and the corresponding rotation matrices are  $6 \times 6$  and  $3 \times 3$  orthogonal matrices. The rest of the class is dedicated to calculations of decay widths. The individual decay widths are calculated from the methods named `Calc[...]`, which take in the integer index of the mass eigenstates associated with the decaying and daughter particles. Here  $i$  is the index of the decaying particle, while  $j$ ,  $k$ , and  $m$  are the indices of the daughter particles in order. Finally the decays are organized in the methods named `Output[...]Decays`, which each contain all the decays of the appropriate parent particle with loops over all decaying and daughter particle mass indices. The remainder of the class is composed of helper functions, including functions corresponding to the functions Lagrangian coefficients in App. C. Additional helper methods are included in the file `DecayMath.cpp`, which are primarily functions which perform the integration for three-body decays.

*b. ThreeBodyIntegrals Class*

<b>Variable</b>	<b>Description</b>
<code>start, end</code>	Upper and lower integration limits calculated from the currently saved mass ratios
<code>r[...]</code>	Dimensionless ratios of the masses of the daughter particles and the mass of the initial slepton
<b>Method</b>	<b>Purpose</b>
<code>Lambda</code>	Common phase space function
<code>F, F1, F2</code>	Common functions used in the integrals
<code>Simpsons</code>	Method to perform an integral of a given one-dimensional function. The integration limits are stored class variables.
<code>Integral[1-6]</code>	Six integrals corresponding to the standard three body decay mode and eight corresponding to the interference term.
<code>CrossIntegral[1-8]</code>	
<code>setMasses</code>	Sets the mass ratios and integration limits.
<code>evaluateIntegrals</code>	Methods which group together sets of integrals and then take a sum using a vector of coefficients as weights
<code>evaluateCrossIntegrals</code>	
<code>evaluateCharginoIntegral</code>	

The `ThreeBodyIntegrals` helper class is used to calculate the values of integrals for the three body decays. The class takes in particle masses using the function `setMasses`, which sets common integration limits and mass ratios to use for multiple integrals. A summation over several integrals (or just one for the chargino) can then be performed using the methods `evaluateIntegrals`, `evaluateCrossIntegrals`, and `evaluateCharginoIntegral`. Coefficients for the integrals are passed into these methods to properly weight the sum.

c. `DecayFormat`

<b>Method</b>	<b>Purpose</b>
<code>PDGCode</code>	Function which returns the PDG code of a particle given its type and mass eigenstate
<code>OutputParticle</code>	Output particle code, mass, and lifetime
<code>OutputParticleData</code>	Output a table of particle masses and lifetimes
<code>TwoBodyDecay</code>	Output the decay width for a two-body decay
<code>ThreeBodyDecay</code>	Output the decay width for a three-body decay
<code>OutputDecayTable</code>	Output a table of decay modes with decay widths for decays involving sleptons and sneutrinos
<code>OutputSusyParams</code>	Outputs SUSY parameters to be appended to the end of <code>HERWIG</code> input file
<code>OutputIntermediateFile</code>	Outputs the full intermediate file containing sleptonic decays

Finally, the class `DecayFormat` is devoted to constructing the intermediate file to send to the `FileCombine` part of the code. The `DecayFormat` class creates an output stream at declaration time, and its two methods `TwoBodyDecay` and `ThreeBodyDecay` are used by the `DecayCalc` class to output two- and three-body decays, respectively. The remaining methods are used to output the particle decay table and `HERWIG` and `SLHA` model parameters.

d. *Adding a Decay*

The easiest possible alteration to the program is the addition of another decay mode. To add a new decay mode, first add a new function to produce the appropriate decay width in corollary to the functions `Calc[...]`; this functions should take in the appropriate indices for the mass eigenstates. Then place a set of loops in the appropriate `Output[...]Decays` function, which calculates the decay width and, if it is non-zero, outputs the decay width through either the `TwoBodyDecay` or `ThreeBodyDecay` method of the stored `DecayFormat` object in the `DecayCalc` class. From this, the decay mode will be added to the intermediate file and be automatically included in the final `HERWIG` and `SLHA` output files.

### 3. `FileCombine` Program

The `FileCombine` program merges the intermediate file from the sleptonic decay calculation and the `SUSYHIT` output. The program is composed of three classes: `DecayMode` stores the particle ID's and decay width of one decay mode. `Particle` stores all the information, including a vector of all decay modes, for a single particle. `Model` contains a map of all particles, various model parameters, and both input and output functions.

a. *DecayMode Class*

Method	Purpose
IsSleptonic	Returns true if the parent or any daughter particle is a slepton or sneutrino
addDaughter	Add another daughter particle
OutputHerwigDecayMode	Output the decay mode in HERWIG format
OutputSLHADecayMode	Output the decay mode in SUSY Les Houches format
OutputSLHA2DecayMode	Output the decay mode in SUSY Les Houches 2 format

The `DecayMode` class stores all information for one decay. This includes the parent particle HERWIG ID, the number of daughter particles, the PDG and HERWIG ID's of the daughter particles, and the decay width of the decay mode (in GeV). The class also contains methods which output the mode in HERWIG and SLHA(2) format.

b. *Particle Class*

Method	Purpose
AddDecayMode	Add given decay mode to the decays for the particle
FixWidth	Determine the total decay width based on all currently stored decay modes and produce branching fractions
OutputHerwigDecays	Outputs the decay table in HERWIG format
OutputSLHADecays	Outputs the decay table in both SLHA and SLHA2 format

The `Particle` class contains the mass, total decay width, and decays for a given particle. The decays are stored as a vector of `DecayMode` objects.

c. *Model Class*

Variable	Description
particleMap	Map object containing stored particle objects. The map is indexed by HERWIG Id's
Method	Description
PDGToHerwig	Converts from PDG to HERWIG particle code
OnShell	Checks whether a passed decay mode is on shell
ReadIntermediate	Read in the model and decay information from the intermediate file produced by SPICE
ReadLesHouchesFile	Read in decays from the Les Houches file produced by SUSYHIT
OutputHerwigFile	Produces a HERWIG input file from the stored model parameters and decays
OutputLesHouchesFile	Produces a SLHA and SLHA2 output file. This method copies the previous decay file up the decays, then produces a new decay table.

The `Model` class contains a map of particle objects, stores the generic model parameters, and handles the input/output functions of `FileCombine`. The `ReadIntermediate` function reads in particle mass information, decays, and model parameters from the intermediate file; `ReadLesHouchesFile` fills the rest of the decay table from the `SUSYHIT` output file, discarding off-shell decays. The `Combine` function is called from outside the class to force a consistent order (since the particle mass information is read from the intermediate file). The two output functions produce `HERWIG` input and SLHA and SLHA2 format files.

- 
- [1] B. C. Allanach, *Comput. Phys. Commun.* **143**, 305 (2002) [arXiv:hep-ph/0104145].
  - [2] B. C. Allanach and M. A. Bernhardt, arXiv:0903.1805 [hep-ph].
  - [3] A. Djouadi, M. M. Muhlleitner and M. Spira, *Acta Phys. Polon. B* **38**, 635 (2007) [arXiv:hep-ph/0609292].
  - [4] J. L. Feng, C. G. Lester, Y. Nir and Y. Shadmi, *Phys. Rev. D* **77**, 076002 (2008) [arXiv:0712.0674 [hep-ph]].
  - [5] S. Bar-Shalom and A. Rajaraman, *Phys. Rev. D* **77**, 095011 (2008) [arXiv:0711.3193 [hep-ph]].
  - [6] G. D. Kribs, E. Poppitz and N. Weiner, *Phys. Rev. D* **78**, 055010 (2008) [arXiv:0712.2039 [hep-ph]].
  - [7] Y. Nomura, M. Papucci and D. Stolarski, *Phys. Rev. D* **77**, 075006 (2008) [arXiv:0712.2074 [hep-ph]].
  - [8] Y. Nomura, M. Papucci and D. Stolarski, *JHEP* **0807**, 055 (2008) [arXiv:0802.2582 [hep-ph]].
  - [9] S. Bar-Shalom, A. Rajaraman, D. Whiteson and F. Yu, *Phys. Rev. D* **78**, 033003 (2008) [arXiv:0803.3795 [hep-ph]].
  - [10] Y. Nomura and D. Stolarski, *Phys. Rev. D* **78**, 095011 (2008) [arXiv:0808.1380 [hep-ph]].
  - [11] G. Hiller, Y. Hochberg and Y. Nir, *JHEP* **0903**, 115 (2009) [arXiv:0812.0511 [hep-ph]].
  - [12] G. Marchesini, B. R. Webber, G. Abbiendi, I. G. Knowles, M. H. Seymour and L. Stanco, *Comput. Phys. Commun.* **67**, 465 (1992).
  - [13] G. Corcella *et al.*, *JHEP* **0101**, 010 (2001) [arXiv:hep-ph/0011363].
  - [14] G. Corcella *et al.*, arXiv:hep-ph/0210213.
  - [15] S. Moretti, K. Odagiri, P. Richardson, M. H. Seymour and B. R. Webber, *JHEP* **0204**, 028 (2002) [arXiv:hep-ph/0204123].
  - [16] P. Skands *et al.*, *JHEP* **0407**, 036 (2004) [arXiv:hep-ph/0311123].
  - [17] B. Allanach *et al.*, *Comput. Phys. Commun.* **180**, 8 (2009) [arXiv:0801.0045 [hep-ph]].
  - [18] C. D. Froggatt and H. B. Nielsen, *Nucl. Phys. B* **147**, 277 (1979).
  - [19] Y. Nir and N. Seiberg, *Phys. Lett. B* **309**, 337 (1993) [arXiv:hep-ph/9304307].
  - [20] Y. Grossman and Y. Nir, *Nucl. Phys. B* **448**, 30 (1995) [arXiv:hep-ph/9502418].
  - [21] G. D. Kribs, A. Martin and T. S. Roy, arXiv:0901.4105 [hep-ph].
  - [22] J. L. Feng, I. Galon, D. Sanford, Y. Shadmi and F. Yu, arXiv:0904.1416 [hep-ph].
  - [23] S. Ambrosanio, G. D. Kribs and S. P. Martin, *Nucl. Phys. B* **516**, 55 (1998) [arXiv:hep-ph/9710217].
  - [24] S. Kraml and D. T. Nhung, *JHEP* **0802**, 061 (2008) [arXiv:0712.1986 [hep-ph]].