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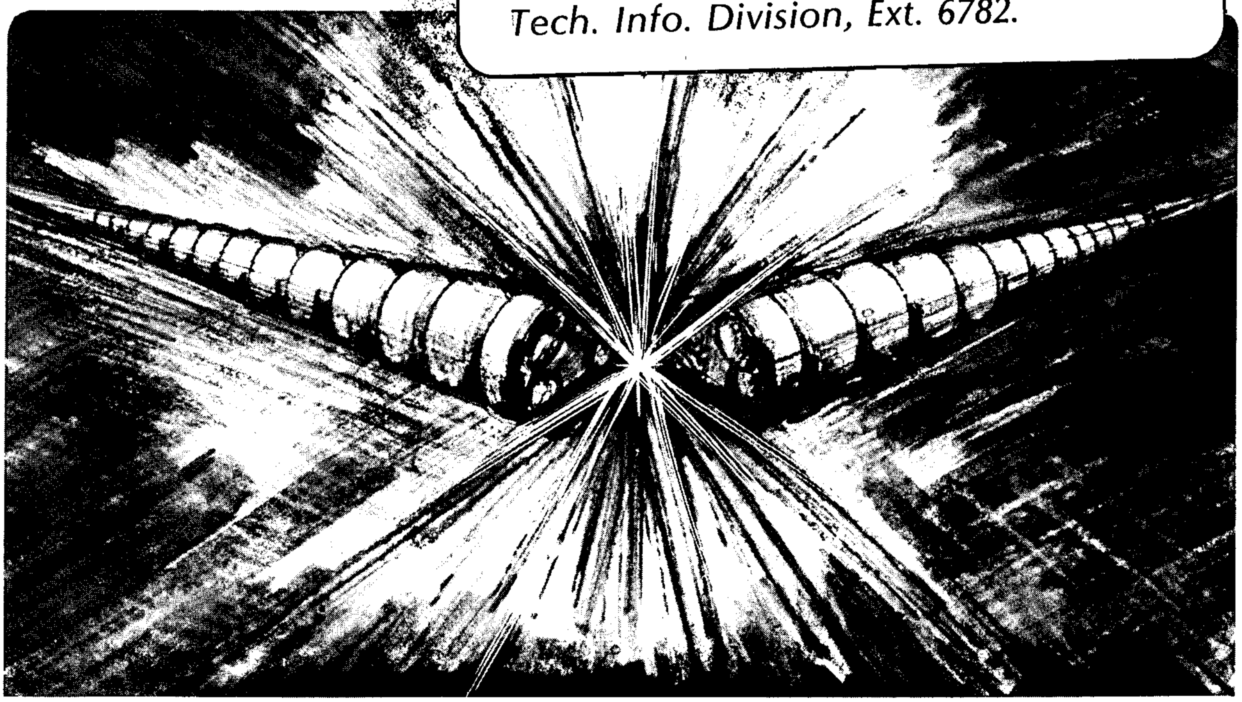

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SINGLE BUNCH INSTABILITIES IN AN SSC*

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In this note we estimate coherent instability thresholds for the SSC and discuss some of the subsequent design restrictions. In Fig. 1 the various instabilities are set out in a block diagram with the essential features of each. For the purposes of this paper we assume that long wavelength coupled bunch effects can be cured effectively by a feedback system (both longitudinal and transverse) and that the impedance of the feedback system is such as to cancel that of the environment (at low frequency). Alternatively, the long wake field is assumed to be exactly canceled, on the average, by a feedback wake field.

	Transverse	Longitudinal
Single Bunch Effects	Wavelength << bunch length Feedback N.G.	Wavelength << bunch length bunch lengthening
Single Bunch Effects	Wavelength ≃ bunch length Feedback may work	Wavelength ≃ bunch length bunch lengthening
Multi-Bunch Effects	rigid modes -- long wavelength Feedback works internal modes weaker	rigid modes -- long wavelength Feedback works internal modes weaker

Fig. 1

This leaves only single bunch effects. First we discuss thresholds for "fast-blowup" both in the longitudinal and transverse and then the transverse mode coupling instability more familiar in electron/positron storage rings. The impedances considered will be a broadband impedance and the resistive wall impedance.

Thresholds for Fast Blow Up [$Im(\omega) >> \omega_s$]

Consider the case of an instability with a wavelength much less than the bunch length. Then there is a coasting-beam-like instability both longitudinally[1] and transversely[2]. The thresholds are given by the coasting beam threshold with the current replaced by the peak current.

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Consider the effect of a broadband impedance at the cutoff frequency, ω_c,

$$\omega_c = c/2b \equiv \omega_0 n_c \quad . \tag{1}$$

Then the sufficient condition for no fast blowup for a Gaussian bunch is given by

Transverse:[2]

$$\frac{e I_p |Z_\perp(\omega_c)| \bar{\beta}}{4\sqrt{2\pi}\, E\, \alpha\, \sigma_\epsilon\, n_c} \leq 1 \tag{2}$$

Longitudinal:[1]

$$\frac{e I_p |Z_\parallel(\omega_c)|}{4\pi E\, \alpha\, \sigma_\epsilon^2\, n_c} \leq 1 , \tag{3}$$

where

$\sigma_\epsilon = \frac{\Delta p}{p}$ rms
I_p = peak current = $\frac{\sqrt{2\pi}}{\sigma} I \cdot R$
I = bunch current = $eN_b f_o$
σ = bunch length (rms)
E = energy
α = freq. slip factor
$Z_\perp$ = transverse impedance
$Z_\parallel$ = longitudinal impedance
$\bar{\beta}$ = average beta function

The chromaticity for the transverse case is assumed to be zero. The word "threshold" refers to the equality in Eq. (2) and (3).

In order to compare the two thresholds we relate the longitudinal to the transverse with the commonly used formula,

$$|Z_\perp(\omega)| = \frac{2c}{b^2} \frac{|Z_\parallel(\omega)|}{\omega}$$

$$= \frac{2R}{b^2} \frac{|Z_\parallel(n\omega_0)|}{n} . \tag{4}$$

This is exact for the resistive wall, however, it should be used with caution in the general case. Substituting into Eq. (2) and (3) yields the thresholds,

Transverse:

$$1 = \frac{e I_p |Z_\parallel(n_c)|}{4\pi E\, n_c\, \alpha\, \sigma_\epsilon} \left(\frac{\sqrt{2\pi}\, R\, \bar{\beta}}{n_c\, b^2}\right) \tag{5}$$

Longitudinal:

$$1 = \frac{e I_p |Z_\parallel(n_c)|}{4\pi E\, n_c\, \alpha\, \sigma_\epsilon} \left(\frac{1}{\sigma_\epsilon}\right) . \tag{6}$$

Thus the two thresholds would be equal if

$$\sigma_\epsilon = \frac{b^2 n_c}{\sqrt{2\pi}\, R\, \bar{\beta}} = \frac{b}{\sqrt{8\pi}\, \bar{\beta}} .$$

If σ_ε is larger, the transverse threshold is lower than the longitudinal. For the parameters in Table 1 the transverse threshold occurs at

$$\sigma_\varepsilon = 1.2 \times 10^{-4} >> \frac{b}{\sqrt{8\pi}\ \bar{\beta}} , \qquad (7)$$

thus at the transverse threshold the longitudinal is quite stable.

The thresholds just calculated are not very restrictive; the transverse mode coupling instability has a lower threshold.

Table 1

$N_B = 2.86\text{x}10^{10}$
$f_0 = 3.3134$ KHz
$I = eN_Bf_0 = 1.52\text{x}10^{-5}$ a
$B = 6000$ bunches
$I_T = 91.3$ ma
$\sigma = .192$ m
$I_p = \sqrt{2\pi}\ I\ R/\sigma = 2.86$ a
$S_b = 15.08$ m
$\alpha = 1.14\text{x}10^{-4}$
$\nu = 97$
$Z(n)/n = 1\Omega$ at $n\omega_0 = c/2b$
$\bar{\beta} = 180$ m
$b = 1.5$ cm
$R = 1.44\text{x}10^4$ m

Thresholds for the Transverse Mode Coupling Instability[3,4]

If the wavelength is the order of the bunch length, it is still possible to have instabilities which have growth times the order of or somewhat longer than the synchrotron period. This instability is due to mode coupling and is sometimes referred to as the strong head tail effect. It has been observed both at PETRA[5] and PEP[6] and may also have been observed much earlier at SPEAR[7]. In this section we assume again that the chromaticity is zero so that the normal head tail effect is absent.

Then at small current the bunch (in the moving frame) oscillates in a stable way at frequencies given approximately by

$$\omega \simeq \omega_y + m\omega_s ;$$

in the lab this is

$$n\omega_0 + \omega_y + m\omega_s .$$

As the current increases these modes move, and if any two become degenerate, there is an instability. Note that, in principle, all modes must be kept because we don't know which two will become degenerate. For electron positron storage rings the bunches are so short that it is mode $m = 0$, the rigid mode, which shifts the most. The instability occurs when modes with $m = 0$ and -1 collide.

For longer bunches the higher modes are important, however, for simplicity we will estimate the threshold using only mode 0. The bunches will also be quite short in the SSC, however, this method may give a somewhat pessimistic answer.

The shift of mode 0 for a Gaussian bunch is given approximately by

$$\frac{\Delta\nu}{\nu_s} \simeq - \frac{ie\ I\ \beta}{4\pi\ E\ \nu_s} \int_{-\infty}^{\infty} Z_\perp(p\omega_0)\ e^{-p^2\sigma_0^2}\ dp$$

$$\sigma_0 \equiv \sigma/R . \qquad (8)$$

The threshold occurs when $-\Delta\nu/\nu_s$ is the order of unity.

$$1 \simeq \frac{e\ I\ \bar{\beta}}{4\pi\ E\ \nu_s} \int_{-\infty}^{\infty} i\ Z_\perp(p\omega_0)\ e^{-p^2\sigma_0^2}\ dp . \qquad (9)$$

Due to the symmetry properties of $Z_\perp$ it is only the reactive part which contributes to this integral. The bunch factor cuts off the integral at $p \sim 1/\sigma_0$ or at

$$p\omega_0 \simeq c/\sigma . \qquad (10)$$

However, if we assume broadband impedance at the cutoff frequency, then the impedance varies little in this range. Thus, the threshold is given approximately by

$$1 \simeq \frac{e\ I\ \bar{\beta}\ i\ Z_\perp(0)}{4\pi\ \nu_s\ E} \left(\frac{\sqrt{\pi}}{\sigma_0}\right)$$

$$1 = \frac{e\ I_p\ \bar{\beta}\ i\ Z_\perp(0)}{4\pi\ \nu_s\ E\sqrt{2}} \qquad (11)$$

$$1 = \frac{e\ I\ \bar{\beta}\ i\ Z_\perp(0)}{4\sqrt{\pi}\ \alpha\ (\Delta E)_{rms}} .$$

For the broadband impedance assumed ($Q = 1$ resonator) the reactive part of the transverse impedance at $\omega = 0$ is related to the resistive part at $\omega = \omega_c$,

$$i\ Z_\perp(0) = Z_\perp(\omega_c) . \qquad (12)$$

Therefore, using Eq. (4) to relate the transverse impedance, we find

$$1 = \frac{e\ I_p}{4\pi\ E\ \nu_s} \left(\frac{Z_\parallel(n)}{n}\right) \frac{\sqrt{2}\ R\ \bar{\beta}}{b^2}$$

$$1 = \frac{e\ I}{2\sqrt{\pi}\ (\Delta E)_{rms}} \frac{Z_\parallel(n)}{n} \frac{R\ \bar{\beta}}{\alpha\ b^2} . \qquad (13)$$

Note that this instability is driven by $Z_\perp(\lesssim c/\sigma)$; this has been converted to an effective $Z(n)/n$.

Thus, given we know all quantities except $(\Delta E)_{rms}$, this threshold tells us how much energy spread we need at injection. For the parameters in Table 1 this yields at 1 TeV

$$\sigma_\varepsilon = 4\text{x}10^{-4} , \qquad (14)$$

which yields

$$\nu_s = 3.7\text{x}10^{-3}\ [.19\ \text{m}/\sigma] \qquad (15)$$

at injection. These are threshold values. During acceleration $(\Delta E)_{rms}$ must be kept larger than or equal to its value at injection.

Scaling Laws, R vs. b

For the purposes of scaling it is useful to rewrite the threshold conditions in terms of the total current

$$I_T = I\,B = I\,\frac{2\pi R}{S_b} \qquad (16)$$

where B is the number of bunches separated by a distance S_b. Rewriting the thresholds for transverse mode coupling and transverse fast blowup we have

Mode Coupling:

$$1 = \frac{e\ I_T(Z_{\parallel}(n)/n)}{4\pi(\Delta E)_{rms}\sqrt{\pi}}\,\frac{\gamma_T^{\,2} S_b \bar{\beta}}{b^2} \qquad (17)$$

Transverse Fast Blowup:

$$1 = \frac{e\ I_T(Z\ (n)/n)}{4\pi(\Delta E)_{rms}\sqrt{\pi}}\,\frac{\gamma_T^{\,2} S_b \beta}{b^2}\left(\frac{\sqrt{\pi}}{\sigma_0 n_c}\right) \qquad (18)$$

$\sigma_0 n_c$ must be large in order to apply the transverse fast blowup threshold.[2] In our case

$$\sigma_0 n_c = \frac{\sigma}{R}\,\frac{R}{2b} = \frac{\sigma}{2b} = 6.3\ . \qquad (19)$$

This means that the fast blowup threshold is much less restrictive than the mode coupling threshold. This makes physical sense because for fast blowup $Im(\omega) \gg \omega_S$. Thus more coherent force is necessary to drive this instability and thus a larger (peak) current is necessary.

To scale to rings of different magnetic field strength, consider a fixed interaction point with nearly head-on collisions of short bunches spaced equally by a distance S_B. If we scale the optics to yield a dispersion which is independent of radius, R, then I_T and S_b are independent of radius while $\gamma_T \propto R^{1/2}$ and $\beta \propto R^{1/2}$. If in addition the effective $Z(n)/n$ is scaled similarly to the resistive wall,

$$Z(n)/n = \Big[Z(n)/n\Big]_0\left(\frac{b_0}{b}\right), \qquad (20)$$

and ΔE is scaled to fill a fixed fraction of the chamber, $\Delta E \propto b$, then we find that the threshold for mode coupling scales as

$$1 = (\text{constant})\ \frac{R^{3/2}}{b^4}\ . \qquad (21)$$

Thus if we scale $b \propto R^{3/8}$, then the threshold condition remains invariant.

If we allow the geometry of the interaction point to vary with radius (for example large crossing angle and very long bunches), then the scaling law above should not be applied. For very long bunches ($\nu_S \sim 0$) all modes must be kept, and it is the "fast blowup" threshold that is important. In addition the impedance at lower frequencies is also important in this case.

However, if again we fix a different interaction point geometry and bunch length, this sets a frequency at which the impedance becomes important ($n\omega_0 \gtrsim 1/\sigma$). Thus Eq. (18) yields a scaling law identical to Eq. (21) except the constant may be quite different.

The Resistive Wall, Single Bunch Effects

The resistive wall will drive transverse coupled bunch modes; however, since at higher frequency the impedance falls off as $\omega^{-1/2}$ the effects for single bunch instabilities are weaker. In addition if a cold chamber is envisaged, this decreases the resistivity and the skin depth. However, for the frequencies of interest here ($\omega \sim 1/\sigma$), the anamolous skin effect plays a role also. Thus, the enhancement of the skin depth at low frequency cannot be simply translated to a similar enhancement at high frequency. In addition the frequency dependence at high frequency of the effective skin depth is altered at low temperature.

With these caveats consider just the normal skin effect. To include qualitatively the low temperature effects, the skin depth can be replaced by an effective skin depth. The transverse impedance is given by

$$Z_{\perp}(\omega) = -i\ RZ_0(1+i)\ \frac{\delta(\omega)}{b^3}$$

where $\delta(\omega)$ is the skin depth and Z_0 is the impedance of the free space. Using only a resistive wall to calculate the transverse mode coupling threshold (Eq. (9)) yields

Mode Coupling:

$$1 = \frac{e\ I_p\ Z_0\ \Gamma(1/4)}{4\pi\ \nu_s\ E\sqrt{2\pi}}\ \frac{R\ \bar{\beta}\ \delta(1/\sigma)}{b^3}\ . \qquad (22)$$

where Γ is the gamma function and δ is evaluated at $1/\sigma$. Rewriting in terms of the total current for the purpose of scaling yields

$$1 = \frac{e\ I_T\ Z_0\ \Gamma(1/4)\ \gamma_T^{\,2}\ S_B\ \bar{\beta}\ \delta(1/\sigma)}{4\pi\Delta E_{rms}\ (2\pi)\ b^3}\ . \qquad (23)$$

If we compare this threshold to that obtained with the broadband impedance, then for a copper wall at room temperature we find

$$\frac{I^{th}(\text{Resis. Wall})}{I^{th}(\text{b. band})} \simeq 8\ . \qquad (24)$$

Thus the resistive wall does not pose a problem for the parameters given in Table 1.

However, it does yield a much more fundamental constraint and may become important for larger machines in spite of improvements due to a cold vacuum chamber.

Conclusion

To conclude note that the above considerations are based on estimates of the transverse and longitudinal impedances and on estimates of the thresholds. Since there exist methods both to calculate impedances and thresholds much more precisely, the numbers calculated must be considered provisional. However, using the estimates above it is clear that for the parameters in Table 1:

1. There will be no fast blowup either longitudinally or transversely.

2. The transverse mode coupling instability can be controlled by adjusting the energy spread in the beam.

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