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PHENOMENOLOGICAL APPROACH TO NUCLEAR FRAGMENTATION*

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ABSTRACT

The nova model of hadron-hadron scattering is extended to treat the fragmentation of heavy nuclei, and the results are in agreement with recent experiments.

The fragmentation of heavy nuclei has been studied recently in a series of heavy ion inclusive experiments,[1] and we present here a model that successfully describes the results. As in the nova or diffractive excitation model of hadron-hadron scattering,[2,3] we assume that nuclear fragmentation occurs in a two-step process: the diffractive production of two unstable novas, followed by their decay into the observed nuclear fragments. In Fig. 1a we depict the situation for the process A+B→C+anything, where the fragment C comes from nucleus B. We have assumed that double-nova production is the dominant process,[4,11] unlike the hadronic case, because the small nuclear binding energy makes it likely that both nuclei will fragment in such reactions.

By an obvious generalization of the arguments of Ref. 2, the inclusive cross section corresponding to the aforementioned reaction is given by

$$\frac{d^3\sigma_{AB}{}^C}{d^3q} = \iint \rho_{AB}(m_1, m_2)\, D_B^C(m_2, \vec{q})\, dm_1 dm_2, \qquad (1)$$

where $\vec{q}$ is the overall center-of-mass momentum of C; $\rho_{AB}(m_1, m_2)$ is the production cross section for novas of mass m_1 and m_2 in an A+B collision; and $D_B^C(m_2, \vec{q})$ is the decay rate of a nova of mass m_2, formed from nucleus B, into fragment C with momentum $\vec{q}$. The diffractive component of the total inelastic cross section is given by

$$\sigma_{inel}^D(s) = \int_{m_A} dm_1 \int_{m_B} dm_2 \;\; \rho_{AB}(m_1, m_2)\; \theta(\sqrt{s} - m_1 - m_2). \qquad (2)$$

To motivate the choice of the nova production spectrum ρ_{AB}, we recall the multi-Regge expression for double diffractive dissociation,[5] corresponding to Fig. 1b:

$$\frac{d^3\sigma}{dt\,dm_1dm_2} = \frac{1}{s^2}\ \beta_{AAR}(0)\,\beta_{BBR}(0)\,g^2_{PPR}(t)\left(\frac{s}{m_1^2m_2^2}\right)^{2\alpha_P(t)}$$

$$\times\,(m_1^2)^{\alpha_R(0)+\frac{1}{2}}\,(m_2^2)^{\alpha_R(0)+\frac{1}{2}}\ . \tag{3}$$

Here, $\beta_{AAR}(0)$ is the forward coupling of some Reggeon R to nucleus A; similarly, for $\beta_{BBR}(0)$, α_R and α_P are the trajectories of the Reggeon R and the Pomeron, and $g_{PPR}(t)$ is a triple-Regge coupling. Since the energies involved in the experiments of interest are not very high (2.1 GeV/c of beam momentum per nucleon), triple-Pomeron terms are unlikely to contribute appreciably and we assume R is the highest-intercept secondary Reggeon of appropriate quantum numbers, the f, with $\alpha_f(0) = \frac{1}{2}$. Using a linear trajectory $\alpha_P(t) = \alpha_P(0) + \alpha' t$ and exponential triple coupling $g_{PPf}(t) = g_{PPf}(0)\,e^{\gamma t}$ we can integrate over t to obtain a prototype missing mass distribution

$$\frac{\rho_{AB}(m_1,m_2)}{dm_1\ dm_2} = \beta_{AAf}(0)\,\beta_{BBf}(0)\ s^{2\alpha_P(0)-2}\left(\frac{1}{m_1^2m_2^2}\right)^{2\alpha_P(0)-\alpha_f(0)-\frac{1}{2}}$$

$$\times\left[2\alpha'\log\frac{s}{m_1^2m_2^2} + 2\gamma\right]^{-1}\exp\left[\left(2\alpha'\log\frac{s}{m_1^2m_2^2} + 2\gamma\right)t_{min}\right]\ , \tag{4}$$

where the minimum momentum transfer t_{min} is a known complicated function of s, m_A, m_B, m_1, and m_2 (Ref. 6) and its appearance will suppress the production of very massive novas. As in Ref. 2 we use instead of Eq. (4) a modified expression

$$\frac{\rho_{AB}(m_1, m_2)}{dm_1 dm_2} = \beta_{AAf}(0)\, \beta_{BBf}(0)\, s^{2\alpha_P(0)-2}\, \frac{e^{-2\gamma t_{min}}}{2\gamma}\, G_B(m_2) G_A(m_1), \tag{5}$$

where

$$G_B(m_2) = \frac{n_B\, e^{-\beta/n_B(m_2-m_B)}}{[n_B(m_2-m_B)]^2} . \tag{6}$$

β is a constant, and we have departed slightly from previous work in adding several factors of n_B, the number of nucleons in B. The latter is necessary to obtain the correct nova mass dependence. This modified nova mass distribution has the following properties: (i) $m_1^{-2}\, m_2^{-2}$ fall off at high energy as expected from triple-Regge behavior when $m_i/\sqrt{s} << 1$; (ii) diffractive total nova production cross section is independent of the parent mass: in other words it is completely determined by the effective nucleus-Reggeon residues $\beta_{AAf}(0)\, \beta_{BBf}(0)$ at the high energy limit. The value of β depends the type of nova:[2] 2.1 GeV for pion-produced novas and 1.8 GeV for proton novas; in the present case of heavy nuclei we use the value $\beta < 1.8$ GeV. In Fig. 2 we plot G_B as a function of $X = m_2 - m_B$ (for B = ^{12}C); note that the maximum occurs at $m_2 = m_B + \beta_B/2n_B$. The actual maximum nova mass will be somewhat lower than this, due to the t_{min} dependence in Eq. (5). For the triple-Regge coupling we use the results of some recent phenomenology[7] and choose $g_{PPf}(t) = 3.5\, e^{4.4t}$. The Regge residue functions $\beta_{AAf}(0)$ would be expected to have an $A^{1/3}$ dependence on nucleon number due to the peripheral character of the reaction, and this behavior has been found consistent in an analysis of K_L - K_S regeneration by charge-exchange degeneracy on nuclear targets.[8]

In Fig. 3 we compare the experimental inelastic cross section with the results of a calculation using Eq. (2, 5, 6) normalized at lead, and the agreement is satisfactory.

It is interesting to notice here that there are two pieces of preliminary experimental results on target dependence to be investigated further: The target dependence of total inelastic cross section, σ_{incl}^{AB}, is $A^{0.42}$ (Fig. 3) while that of partial cross section $\frac{d\sigma_{AB}^{C}}{d^3q}$ is $A^{0.256}$ which was observed for γ_A assuming factorizability of target and beam

$$\frac{d\sigma_{AB}^{C}}{d^3q}(\theta_L^C < 12\text{mv}) \equiv \gamma_B^C\,\gamma_A$$

These two information on target dependence implies that the average multiplicity should decrease as target mass increases as

$$n_C \equiv \frac{1}{\sigma_{incl}^{AB}}\int \frac{d\sigma_{AB}^{C}}{d^3q}\, d^3q \propto A^{-0.16}$$

if we assume that the target dependence is factorizable and has the same dependence at all angles.

The decay distribution of a nova is conventionally[2] considered to be a Gaussian in the nova rest frame

$$D_B^C(m_2, \vec{k}) = N_C(m_2)\exp\left\{-\frac{(k_L^2 + k_T^2)}{[K_C(m_2)]^2}\right\}, \qquad (7)$$

where we have resolved k into longitudinal (k_L) and transverse (k_T)

components with respect to the incident nucleus B. This form is inferred from a cascade decay picture, but similar decay distributions can be derived from a bootstrap model.[9] $N_C(m_2)$ is a normalization factor which is related to the average number of fragments C into which a nova of mass m_2 decays. $K_C(m_2)$ gives the width of the momentum distribution of fragments C, and we make the simplest reasonable assumption that it is a constant independent of m_2. For pions produced from proton novas it is found[2] that $K_C^2 \approx 1/9\ (\text{GeV/c})^2$, while in the nuclear case[1] the form of Eq. (7) fits the data roughly with $K_C^2 \approx 1/25\ (\text{GeV/c})^2$. (There is C-dependence, as discussed later.) It is noted that for heavy nuclei, the nova rest frame is found experimentally[10] to coincide with that of the parent nucleus B; the energy transferred to the nucleus presumably just dissociates it into sub-nuclei with small relative momenta. However, there is an indication of the existence of the recoil momentum ($P_{recoil} \lesssim 40$ MeV/c) for a carbon target with ^{16}O beam at 2.1 GeV/c. The fact that the momentum distribution is narrower for nuclei is understandable from the nova mass distribution in Fig. 2.

We can make a more quantitative statement about the dependence of K_c on C with the aid of a simple model. Suppose a nova decays into just two fragments. This is probably not true in general, but the actual fragmentation could simulate this simpler case. If a nova produced from nucleus B decays into two nuclear fragments C and D, then by baryon number conservation and the approximate constancy of binding energy per nucleon we have $m_B = m_C + m_D + \delta$. (The mass defect $\delta \ll m_C, m_D$.) Furthermore, if P is the momenta of C (and D) in the rest frame of the nova (of mass m_2), then conservation of energy implies

$$m_2 = \sqrt{m_C^2 + P^2} + \sqrt{m_D^2 + P^2} .$$

In the case of interest, $P \ll m_C, m_D$ so we can rewrite the last equation as

$$P^2 = 2m_C m_D[(m_2 - m_B + \delta)/m_B] .$$

Averaging over nova mass, we obtain

$$K_C^2 \equiv P_C^2 = 2\frac{m_C m_D}{m_B^2} \int m_B(m_2 - m_B + \delta) \, \rho(m_1,m_2) dm_1 dm_2/N$$

$$\equiv 2\left(\frac{m_C}{m_B}\right)\left(\frac{m_B - m_C}{m_B}\right)\bar{Q}^2, \qquad (8)$$

where N is normalization and

$$N = \int \rho(m_1,m_2) \, dm_1 \, dm_2,$$

where the constant Q^2 is independent of C. Thus K_c^2 is a parabola as a function of m_C, with a maximum at $m_C = m_B/2$. This formula is compared to experiment in Fig. 4, for an ^{16}O nova, and the agreement of the shape is quite good for its simplicity. The parabolic shape of the target dependence is also predicted from the quantum mechanical sudden approximation[12] and the statistical model.[13]

At the peaks of the nova mass spectrums, $m_{1,2} = m_{A,B} + \beta/2n_{A,B}$, the momentum P of the fragment C at mass number 8 for ^{16}O nova, and the target recoil momenta are calculated as a function of β and shown in Table I. The best result is obtained at $\beta = 0.5 \pm 0.1$. The fragment momentum is rather sensitive on δ at small β and could have a factor as large as $1/\sqrt{3}$.

We conclude by summarizing some of the predictions of the diffractive double-nova production model:

1) To the extent that diffraction is dominated by a Pomeron with intercept near 1, we expect little or no energy dependence of total inelastic production cross sections.

2) The dependence on the target nucleus occurs mainly through the Regge residue function $\beta_{AAf}(0)$, with small corrections due to the appearance of t_{min} in Eq. (5), and phase space. At high energy limit it reaches to $A^{1/3}$ dependence.

3) With further approximations, the width of the momentum distribution of secondaries is found to have a parabolic dependence on the mass of the secondary.

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TABLE I. The target nova recoil momenta are calculated at the peak points of nova mass distributions. The momentum P of the fragment is also listed for the case when ^{16}O breaks into two mass number 8 fragments.

	Recoil momentum (MeV/c)				Fragment momentum at mass number 8
Parameter \ Target	C	Au	Cu	Pb	$P(\delta = 8\ MeV)$
$\beta = 0.4$	22	10	7	5	183
$\beta = 1.0$	54	27	18	12	400
$\beta = 1.8$	97	47	33	22	602

Figure Captions

FIG. 1a. Diffractive double-nova production and subsequent decay into fragments.

1b. Multi-Regge approximation to double-diffractive nova production.

FIG. 2. Nova mass spectrum, Eq. (6) for carbon beam. $\beta = 1.8$.

FIG. 3. Target dependence of the total inelastic cross section. The data points are from Ref. 11. ● = ^{16}O beam, △ = ^{12}C beam at 2.1 GeV/nucleon. The continuous curves are the results from calculation which is normalized at Pb for ^{16}O beam.

FIG. 4. Momentum width distribution, Eq. (8), along with the experimental result from Ref. 10. The value of σ normalized to be 160 MeV/c at mass number A = 8.

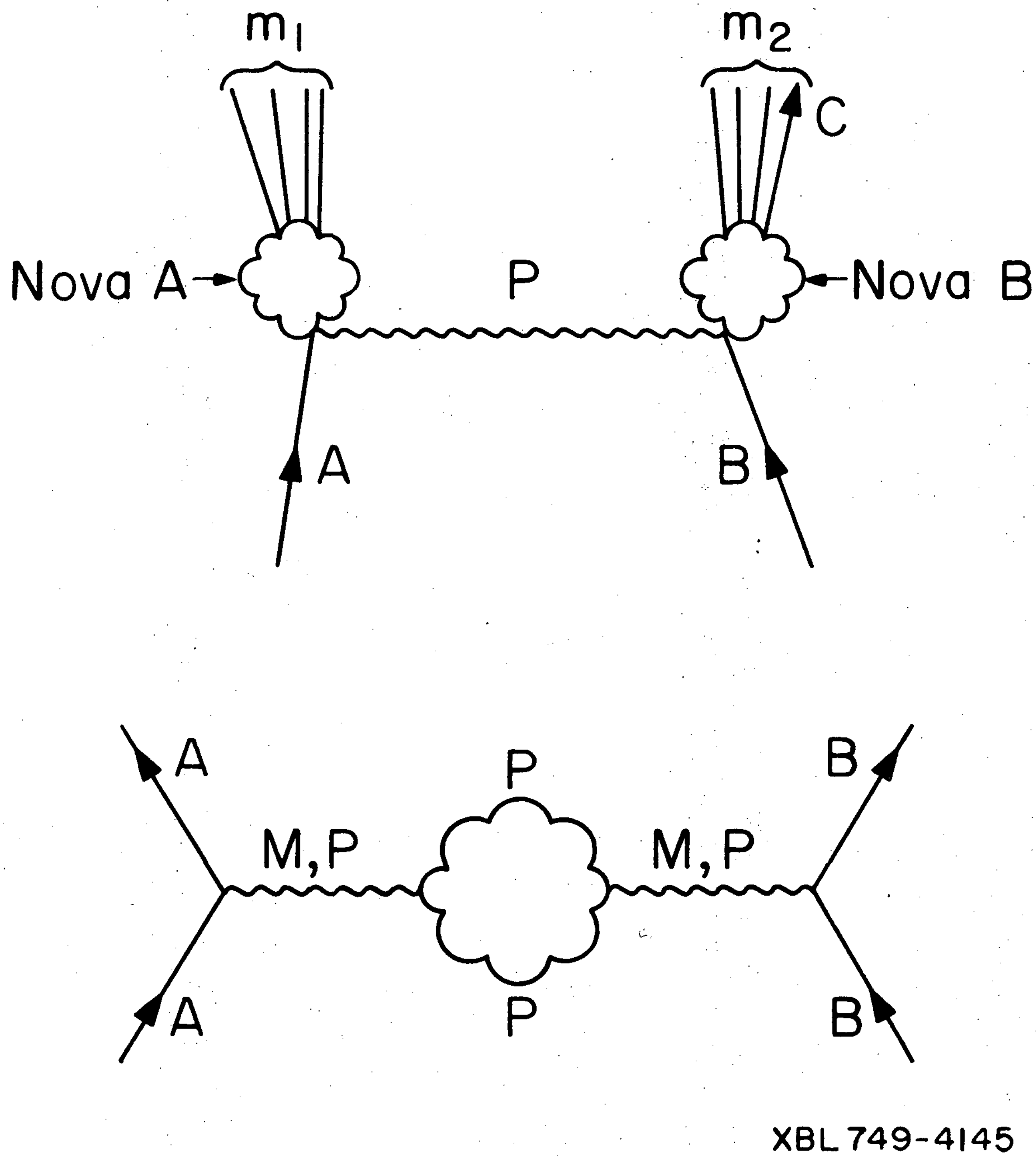


Fig. 1.

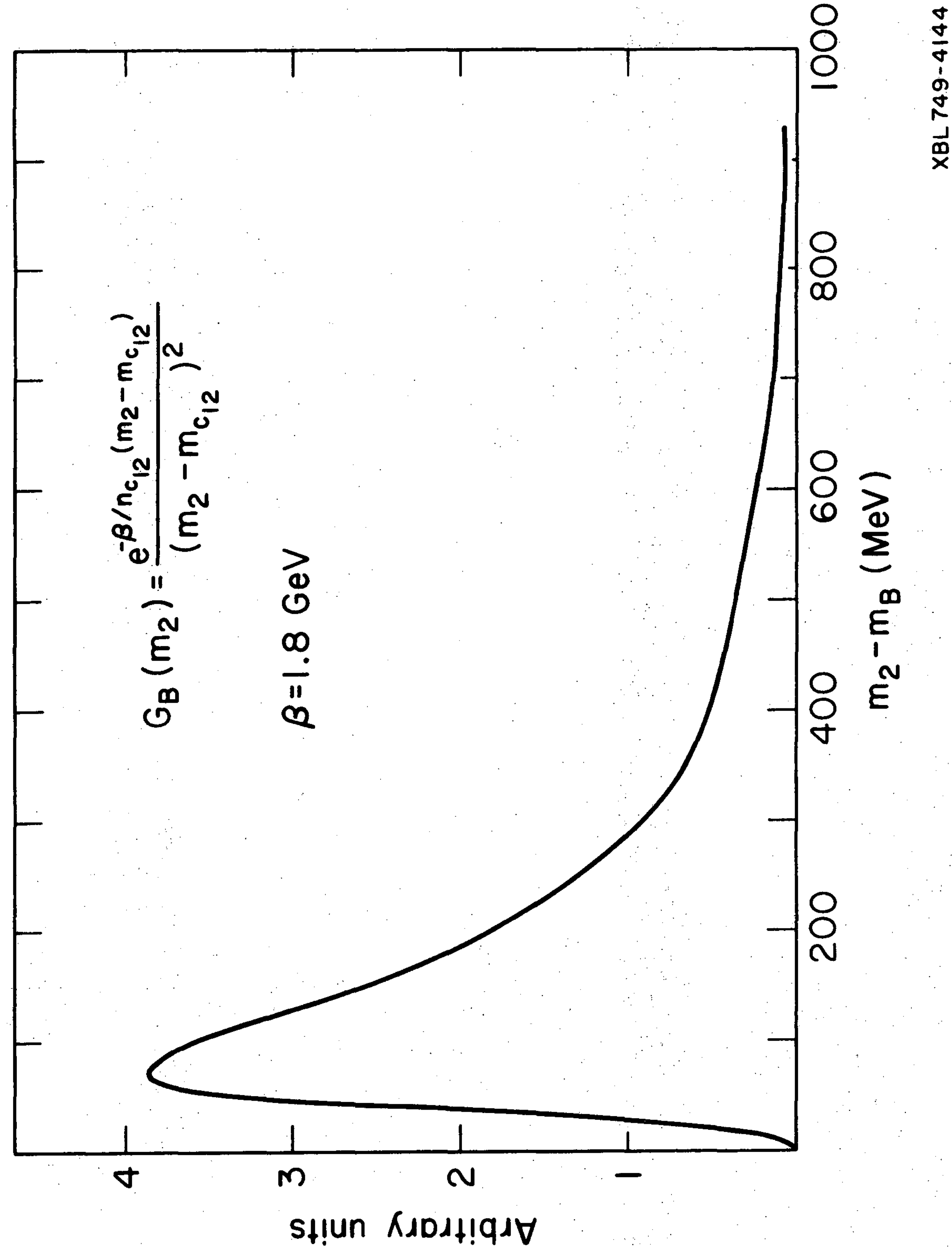
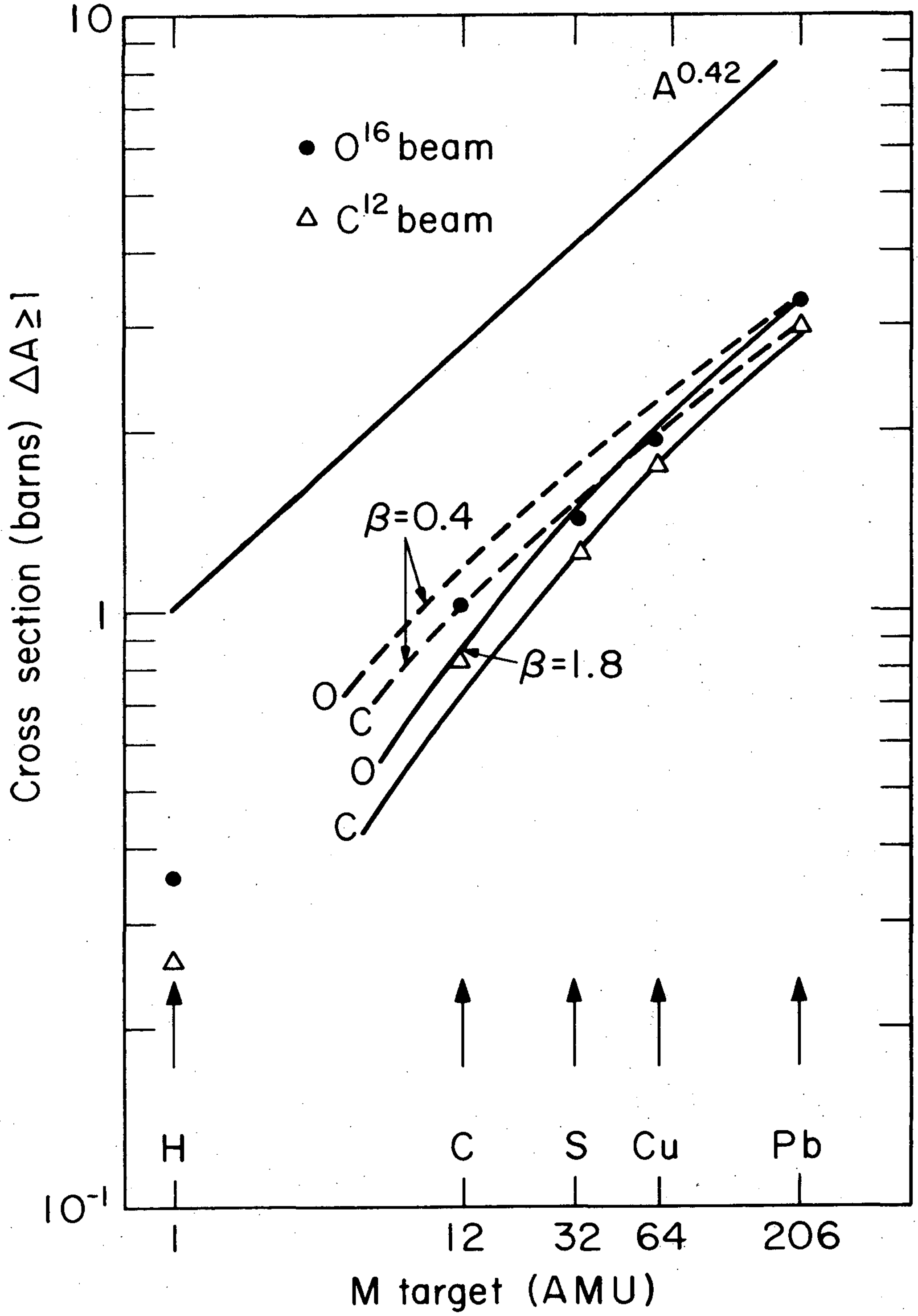
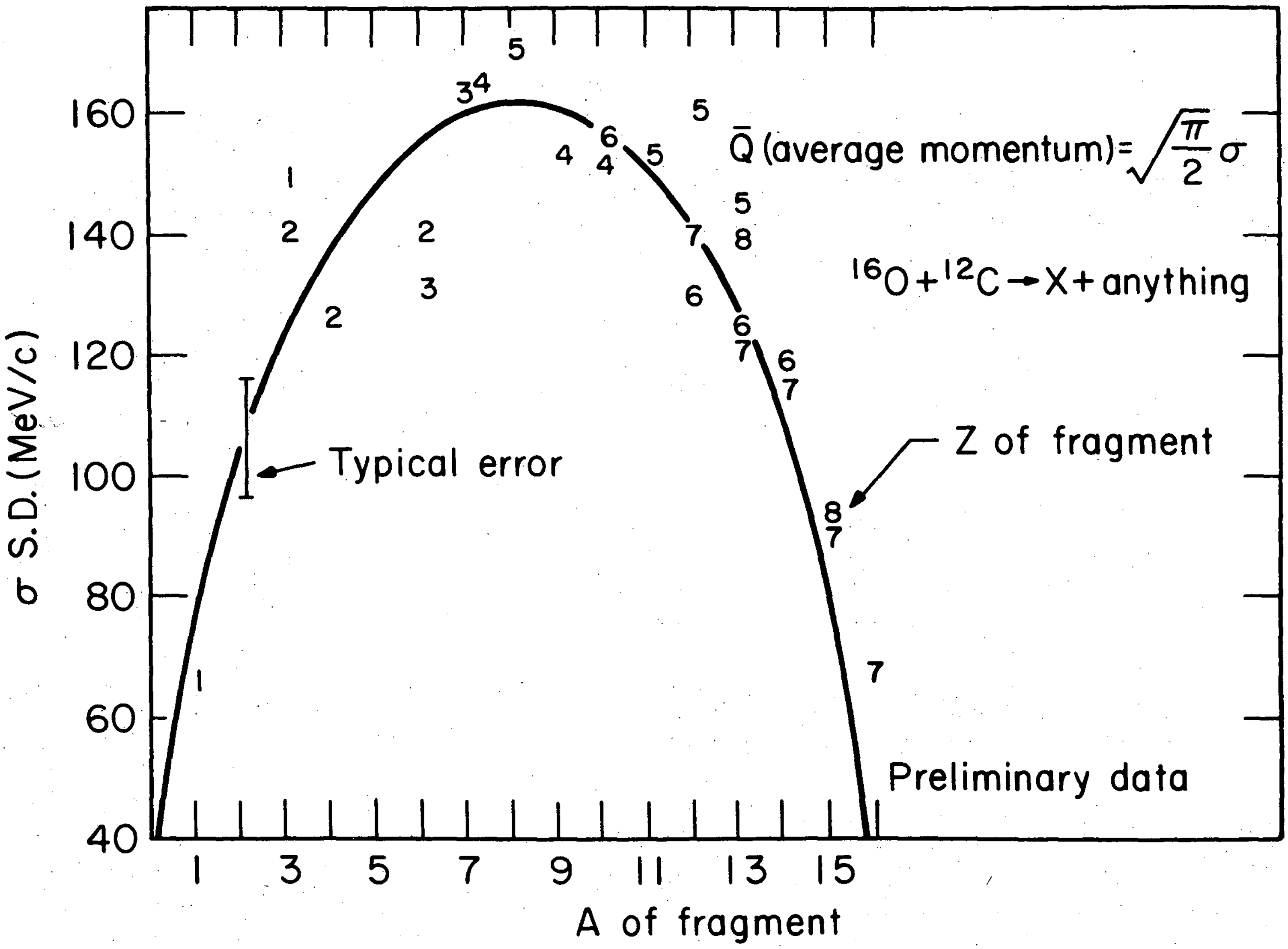

$G_B(m_2) = \frac{e^{-\beta/n_{c_{12}}(m_2 - m_{c_{12}})}}{(m_2 - m_{c_{12}})^2}$
β=1.8 GeV
Arbitrary units
4
3
2
1
200
400
600
800
1000
$m_2 - m_B$ (MeV)
XBL 749-4144

Fig. 2.



XBL 749-4147

Fig. 3.



σ S.D. (MeV/c)
A of fragment
Q̄ (average momentum) = $\sqrt{\frac{\pi}{2}}\sigma$
$^{16}O + ^{12}C \rightarrow X +$ anything
Z of fragment
Typical error
Preliminary data
XBL749-4146

Fig. 4.

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