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The experiments related with the n-p interaction cross section can be classified as follows:[1] 1) Measurements of the total cross section; 2) Measurements of differential cross sections (angular distributions) from 3 MeV on. The energy region between 0 and 3 MeV has been studied only through experiments of type 1, due to the obvious experimental difficulties involved in experiments of type 2. The radiative capture cross section is of the order of 1.5% of the total cross section at zero energy and drops with a 1/v law, and therefore the total cross section is essentially a measure of the elastic cross section. The information on p-p interaction is obtained in the same energy region through experiments of type 2, because of the experimental difficulties in performing experiments of type 1, and also because of problems intrinsic to the Coulomb interaction in the limit of zero energy. It is possible that the scattering parameters extracted for the n-p interaction at low energies, depend on implicit assumptions about the processes that take place during the interaction.

It is well known that when the n-p scattering parameters were first obtained, there was no question about the absolute distinguishability of neutron and proton. It can then be assumed safely that the interaction can be described by a wave equation with a short range potential $V = V(r)$ and a wave function $\psi(\vec{r})$ without any defined symmetry, satisfying Schrödinger's equation

$$[T + V(r)]\ \psi(\vec{r}) = E\ \psi(\vec{r})$$

for ${}^1S_0$-wave scattering $\quad \psi(\vec{r}) = e^{i\vec{k}_i\vec{r}} + \frac{1}{k} e^{i\delta_0} \sin \delta_0 \frac{e^{i\vec{k}_f \cdot \vec{r}}}{r}$

$$\frac{\vec{k}_i}{|\vec{k}_i|} \equiv \vec{z} \text{ axis} \qquad |\vec{k}_i| \equiv |\vec{k}_f|$$

so that $\quad f(\theta) = \frac{1}{k} e^{i\delta_0} \sin \delta_0 \quad$ and $\quad \sigma(\theta) = \frac{4\pi}{k^2} \sin^2 \delta_0$

Nevertheless, when higher energy angular distributions became available it was observed that n-p scattering between 10 and 100 MeV exhibited a persistent symmetry around 90° CM, similar to the p-p scattering symmetry. The "explanation" of this striking phenomenon of the interaction of distinguishable particles, which nevertheless exhibits features of the scattering of indistinguishable particles, was attributed to a Serber[2] "force" and to a potential that can be written as

$$V = V(r)\ (\alpha + \beta\, M_{12})$$

M_{12} is the Majorana exchange operator and $\alpha + \beta = 1$. The n-p scattering symmetry requires $\alpha \simeq \beta \simeq 1/2$. This explanation has the well known property of eliminating the odd waves. The Schrödinger equation would be

$$T\ \psi(\vec{r}) + V(r)\ \tfrac{1}{2}[\psi(\vec{r}) + \psi(-\vec{r})] = E'\ \psi(\vec{r}) \qquad (1)$$

It is meaningless as it stands, but it can be reduced to a pair of equivalent equations writing

$$\psi(\vec{r}) = \tfrac{1}{2}[\psi(\vec{r}) + \psi(-\vec{r})] + \tfrac{1}{2}[\psi(\vec{r}) - \psi(-\vec{r})] = \Phi_S + \Phi_A \qquad (2)$$

Substituting into (1) we can separate the equation into the two following equations

$$[T + V(r)]\,\Phi_S = E\Phi_S \tag{3}$$

$$T\Phi_A = E\Phi_A \tag{4}$$

The sum of (3) and (4) regenerates (1). The conclusion is also well known, the scattering potential affects only even waves, and odd waves remain in the beam. The scattering amplitude corresponding to Φ_S is of course

$$\hat{f}(\theta) = \frac{1}{2}[f(\theta) + f(\pi - \theta)] \tag{5}$$

and therefore we obtain the same scattering amplitude for S-wave scattering as if the scattering potential were simply $V = V(r)$, without the exchange operator. The choice $\alpha \simeq \beta \simeq 1/2$ is indicated by the symmetry of the experimental angular distributions. It is also apparent that it involves equal probabilities for no exchange and for exchange when any pair n-p interacts. If this is the case it would be impossible to attribute certainty to the nature of an outgoing nucleon (neutron or proton) throughout the interaction. What are the consequences of a loss of identity of the interacting nucleons? Is there a significant difference between this loss of identity and the indistinguishability of identical particles? Quoting Landau and Lifschitz[3] "in quantum mechanics, identical particles entirely lose their individuality," and we might say that what holds for identical particles, holds for the unidentifiable pair n-p, if an exchange process can occur during the interaction. Of course, with this assumption we obtain a different normalization of the wave function, and of the scattering amplitudes, in agreement with the classical limit,[4] whereas the current Serber force treatment does not agree with it.

In the general case $\alpha \neq \beta$ and then the Schrödinger equation is

$$[T + V(r)\ (\alpha + \beta\, M_{12})]\,\psi(\vec{r}) = E\,\psi(\vec{r}) \tag{6}$$

An equation of type (6) requires eigenfunctions of well defined symmetry, as

$$\psi_S = \frac{1}{\sqrt{2}} [\psi(\vec{r}) + \psi(-\vec{r})] \tag{7}$$

$$\psi_A = \frac{1}{\sqrt{2}} [\psi(\vec{r}) - \psi(-\vec{r})] \tag{8}$$

the normalization is consistent with the analogy to the case of identical particles. The Eq. (6) gives rise to the following equations through the use of (7) and (8)

$$[T + V(r)]\, \psi_S = E\, \psi_S$$

and (9)

$$[T + V(r)\ (\alpha-\beta)]\, \psi_A = E'\, \psi_A$$

Obviously $\psi_{S,A} = \sqrt{2}\, \Phi_{S,A}$. The scattering amplitude corresponding to ψ_S in

$$\hat{f}(\theta) = \frac{1}{\sqrt{2}} [f(\theta) + f(\pi - \theta)] \tag{10}$$

Let us consider classically, a charge exchange scattering process particles distinguishable only by their charges, and assume that there are equal amplitudes for the direct and exchange processes. The cross-section is

$$\hat{\sigma}(\theta) = \frac{\sigma(\theta)}{2} + \frac{\sigma(\pi-\theta)}{2} \tag{11}$$

Half of the particles are scattered "directly" through θ (compared to the same dynamical process without exchange), while half are scattered through θ with an exchange of the "label" charge. Therefore, classically, we would consider exchange (spin or charge) as a non-dymanical process that simply exchanges the labels on incoming and target particles. However, quantum mechanically, there is, of course, interference of the amplitudes for the direct and exchange processes. These interference terms vanish in the classical limit. The normalization of the scattering amplitude which gives (11) in the limit that interference terms vanish in precisely (10).

The consequences of the alternate normalization obtained through our assumption is straightforward, the scattering length of the $^1S_0$ n-p interaction, analog to the s-wave p-p interaction, becomes

$$a_S^{np} = \frac{-23.7}{\sqrt{2}}\ f = -16.7\ f$$

in excellent agreement with the p-p scattering length $a_S^{pp} = -17$ (corrected for Coulomb effects) and with the most recent value for the n-n scattering length $a_S^{nn} = -16.4$ f.[5]

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[*] This work performed under the auspices of the U. S. Atomic Energy Commission.

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