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Graviton cosmology in universal extra dimensions

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In models of universal extra dimensions, gravity and all standard model fields propagate in the extra dimensions. Previous studies of such models have concentrated on the Kaluza-Klein (KK) partners of standard model particles. Here we determine the properties of the KK gravitons and explore their cosmological implications. We find the lifetimes of decays to KK gravitons, of relevance for the viability of KK gravitons as dark matter. We then discuss the primordial production of KK gravitons after reheating. The existence of a tower of KK graviton states makes such production extremely efficient: for reheat temperature T_{RH} and d extra dimensions, the energy density stored in gravitons scales as $T_{\mathrm{RH}}^{2+3d/2}$. Overclosure and big bang nucleosynthesis therefore stringently constrain T_{RH} in all universal extra dimension scenarios. At the same time, there is a window of reheat temperatures low enough to avoid these constraints and high enough to generate the desired thermal relic density for KK weakly interacting massive particle (WIMP) and superWIMP dark matter.

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I. INTRODUCTION

Since the initial work of Kaluza and Klein [1], models of extra dimensions have played an important part in particle theory. These ideas have been revived in models of TeV-scale universal extra dimensions (UED) [2–4]. In these models, as in Kaluza-Klein (KK) theory, both gravity and standard model (SM) fields propagate in all $D=4+d$ dimensions.

In models where the extra dimensions are tori, every known particle has a tower of KK partner particles, each carrying KK number. Momentum conservation in the extra dimensions implies KK number conservation. Such models predict a plethora of unseen massless particles and a tower of stable KK particles, both of which are phenomenologically problematic.

For more general geometries, unwanted massless modes may be projected out, and there are fewer or no stable KK particles. A particularly interesting situation occurs when there is only a Z_2 symmetry (called KK parity) in the extra dimensions. An example of this situation is the case $D=5$ where the extra dimension is the orbifold S^1/Z_2, where S^1 is a circle of radius R [4]. For SM fields, the geometry projects out half of the massless modes, leaving only the 4D SM degrees of freedom, and also breaks KK number conservation, rendering almost all KK partners unstable. However, the geometry preserves KK parity, and so the lightest KK particle (LKP) is stable, making it a viable realization of dark matter with an extra-dimensional origin [5]. Among SM KK partners, the LKP is often B^1, the KK partner of the hypercharge boson [6]. For $m_{\mathrm{KK}} \sim$ TeV, B^1 is an excellent weakly-interacting massive particle (WIMP) dark matter candidate, with a thermal relic density consistent with observations and promising prospects for detection [7–13].

These studies, as well as those exploring the collider [6,14–18] and low-energy [4,19–25] implications of UED, have focused on the SM KK spectrum, typically neglecting the gravitational sector. (See, however, Refs. [15,26].) KK gravitons couple extremely weakly, so their effects are insignificant for colliders and low-energy experiments. Cosmologically, however, KK gravitons may play an important role. For example, the LKP may be the lightest KK graviton [27,28]. It is not easy to verify this, since loop diagrams involving gravitons are divergent. On general grounds, however, one expects the mass corrections to be $\delta m/m \sim (M_{\mathrm{Pl}}R)^{-2}$, since gravitons decouple as $M_{\mathrm{Pl}} \to \infty$. This is potentially very small, and, since loop corrections to other KK particle masses are typically positive [6], it is not unlikely that in UED theories, the lightest KK graviton is the LKP. In this case, all SM KK particles eventually decay to the KK graviton, and the KK graviton is the only possible KK dark matter candidate.

KK graviton dark matter interacts only gravitationally and is a superweakly interacting massive particle, or superWIMP, dark matter [27,28]. As such, it is impossible to detect in conventional dark matter experiments. However, its potential impact on big bang nucleosynthesis (BBN), the cosmic microwave background, and the diffuse photon flux provides alternatives for dark matter searches. In evaluating such signals, it is important to have accurate, rather than just order of magnitude, results for decay times of WIMPs to superWIMPs. We determine these in this study.

In addition, KK gravitons may be produced in the early universe during the reheating era following inflation. The phenomenon of gravitino production after reheating in supersymmetric scenarios is well studied [29,30]. The case of UED is qualitatively different, however, as there is an infinite tower of new particle states that may be populated at high temperatures, with the density of states growing rapidly at large masses, especially for large d. As we will see, in UED primordial KK graviton production is in fact very efficient, and constraints on dark matter abundances and BBN provide stringent bounds on early universe cosmology in all UED scenarios, irrespective of which particle is the LKP and other spectrum details.

We emphasize that the bounds we derive apply to UED scenarios in which all SM particles propagate in all extra dimensions. The acronym UED has also been used to refer to scenarios in which all SM particles propagate in some of the available extra dimensions, but there are additional very large extra dimensions in which only gravity propagates (see, e.g., Refs. [15,26]). In such scenarios, gravity may be strong not far from the weak scale, and upper bounds on the reheat temperature are extremely stringent and may be as low as ~MeV [31].

The paper is organized as follows. In Sec. II we derive the interactions of KK gravitons coupled to UED SM fields in general toroidal dimensions. In Sec. III we then obtain the KK graviton interactions for UED orbifold compactifications, specializing to the case of the S^1/Z_2 orbifold discussed above. For this case, we first show that boundary terms coming from the ends of the interval are typically negligible and then determine the spectrum and bulk interactions of the orbifold theory. In Sec. IV we find the decay rates for next-to-lightest KK particle (NLKP) gauge bosons $B^1 \to G^1 + \gamma$ and fermions $\psi^1 \to G^1 + \psi$, of relevance to superWIMP dark matter. We then estimate the primordial abundance for the tower of KK gravitons produced after reheating in Sec. V and derive constraints on the reheat temperature in Sec. VI. Our conclusions are given in Sec. VII.

II. GRAVITON INTERACTIONS FOR TORUS COMPACTIFICATIONS

To discuss graviton cosmology, we must first derive the interactions of KK gravitons in UED. Graviton interactions in extra dimensions have been discussed previously, particularly in the context of theories where SM fields are confined to four spacetime dimensions. (See, e.g., Refs. [32,33]; we will follow the analysis of Ref. [32] in discussing the linearized gravitational action.) Here we instead couple the gravitational sector to extra-dimensional SM fields and also implement the orbifold projection.

We begin in this section by determining the couplings of KK gravitons and KK SM fields in the case of torus compactifications. For D spacetime dimensions, the action for matter coupled to gravity is

$$S^D = M_D^{D-2} \int d^D x \sqrt{|g|}\, [R + \mathcal{L}(\bar{\phi})], \tag{1}$$

where M_D is the D-dimensional Planck scale and $\mathcal{L}$ is the Lagrangian for all matter fields, which we denote generically by $\bar{\phi}$. We use coordinates $x^M = (x^\mu, y^i)$, where $0 \leqslant y^i \leqslant 2\pi R_i \equiv L_i$.

Linearizing about flat space using $g_{MN} = \eta_{MN} + \bar{h}_{MN}$, where $\eta = (1,-1,-1,\ldots,-1)$, we find the Fierz-Pauli action [34]

$$S_{\rm lin}^D = M_D^{D-2} \int d^4x \left(\prod_i \int_0^{L_i} dy^i \right) \times \left[K(\bar{h}) + \mathcal{L}_0(\bar{\phi}) + \frac{1}{2} \bar{h}_{MN} T^{MN}(\bar{\phi}) \right], \tag{2}$$

where the graviton kinetic terms are contained in

$$K(\bar{h}) \equiv \frac{1}{4} (\bar{h}^{MN,P} \bar{h}_{MN,P} - \bar{h}^{,M} \bar{h}_{,M} - 2 \bar{h}^{MN}{}_{,M} \bar{h}_{PN}{}^{,P} + 2 \bar{h}_{,M} \bar{h}^{NM}{}_{,N}), \tag{3}$$

with $\bar{h} \equiv \bar{h}^M_M$ and $\bar{h}^{MN} \equiv \eta^{MP} \eta^{NQ} \bar{h}_{PQ}$. Also

$$\mathcal{L}_0(\bar{\phi}) \equiv \mathcal{L}(\bar{\phi})|_{g=\eta} \tag{4}$$

is the flat-space matter Lagrangian, and

$$T^{MN}(\bar{\phi}) \equiv \left(\eta^{MN} \mathcal{L} - 2 \frac{\partial \mathcal{L}}{\partial g_{MN}} \right) (\bar{\phi}) \Big|_{g=\eta} \tag{5}$$

is the flat-space stress-energy tensor. To reproduce the standard gravitational interactions among massless modes, we require $M_D^{D-2} \Pi_i L_i = (16\pi G_N)^{-1} \equiv M_4^2$. Defining $h = M_4 \bar{h}$ and $\phi = M_4 \bar{\phi}$, we find the linearized action

$$S_{\rm lin}^D = \int d^4x \left(\prod_i \int_0^{L_i} \frac{dy^i}{L_i} \right) \times \left[K(h) + \mathcal{L}_0(\phi) + \frac{1}{2M_4} h_{MN} T^{MN}(\phi) \right], \tag{6}$$

where the kinetic terms are now canonically normalized.

To write this in terms of 4D fields, we define

$$h_{MN} = \begin{pmatrix} h_{\mu\nu} + \eta_{\mu\nu}\phi & A_{\mu j} \\ A_{i\nu} & 2\phi_{ij} \end{pmatrix}, \tag{7}$$

$$T_{MN} = \begin{pmatrix} T_{\mu\nu} & T_{\mu j} \\ T_{i\nu} & T_{ij} \end{pmatrix}, \tag{8}$$

where $\phi = \delta^{kl} \phi_{kl}$. For simplicity, we will henceforth take $R_i = R$ for all i. Each component field is then conventionally decomposed into 4D fields through the Fourier expansion

$$h_{\mu\nu}(x,y) = \sum_{\vec{n}} h_{\mu\nu}^{\vec{n}}(x) e^{i\vec{n}\cdot\vec{y}/R},$$

$$A_{\mu i}(x,y) = \sum_{\vec{n}} A_{\mu i}^{\vec{n}}(x) e^{i\vec{n}\cdot\vec{y}/R}, \tag{9}$$

$$\phi_{ij}(x,y) = \sum_{\vec{n}} \phi_{ij}^{\vec{n}}(x) e^{i\vec{n}\cdot\vec{y}/R},$$

and similarly for the stress-energy tensor component fields. Here $\vec{n}\cdot\vec{y} \equiv \Sigma_i n_i y^i$, where $\vec{n} = (n_1, \ldots, n_d)$ and each n_i runs from $-\infty$ to ∞. We will also use the variable

$$m_n \equiv \left[\frac{\delta^{ij} n_i n_j}{R^2} \right]^{1/2}. \tag{10}$$

At each massive level $\vec{n}$, part of the fields $A_{\mu j}^{\vec{n}}$, $A_{i\nu}^{\vec{n}}$, and $\phi_{ij}^{\vec{n}}$ are absorbed to give mass to the graviton state $h_{\mu\nu}^{\vec{n}}$ in an extra-dimensional realization of spontaneous symmetry breaking. The resulting physical states are (defining $\vec{n}^2 \equiv \delta^{kl} n_k n_l$)

$$\tilde{h}^{\vec{n}}_{\mu\nu} = h^{\vec{n}}_{\mu\nu} + i\,\frac{\delta^{ij} n_i R}{\vec{n}^2}(\partial_\mu A^{\vec{n}}_{\nu j} + \partial_\nu A^{\vec{n}}_{\mu j}) - \left(\delta_{ij} + 2\frac{n_i n_j}{\vec{n}^2}\right)\left(\frac{2}{3}\,\frac{\partial_\mu \partial_\nu}{m_n^2} - \frac{\eta_{\mu\nu}}{3}\right)\phi^{\vec{n}}_{ij}, \quad (11)$$

$$\tilde{A}^{\vec{n}}_{\mu i} = P_{ij}\left(A^{\vec{n}}_{\mu j} + 2i\,\frac{\delta^{kl} n_k R}{\vec{n}^2}\,\partial_\mu \phi^{\vec{n}}_{jl}\right), \quad (12)$$

$$\tilde{\phi}^{\vec{n}}_{ij} = \sqrt{2}(P_{ik}P_{jl} + a P_{ij} P_{kl})\phi^{\vec{n}}_{kl}, \quad (13)$$

where we have defined

$$P_{ij} = \left(\delta_{ij} - \frac{n_i n_j}{\vec{n}^2}\right), \quad (14)$$

and a satisfies $3(d-1)a^2 + 6a = 1$.

The Lagrangian for the physical fields needs to be treated slightly differently for the zero modes versus the non-zero modes. For the zero modes, the action is

$$S^0_{\rm lin} = \int d^4x [\mathcal{L}^0_h + \mathcal{L}^0_{h\ \rm int}], \quad (15)$$

where

$$\mathcal{L}^0_h = \frac{1}{4}(h^{0\mu\nu,\rho} h^0_{\mu\nu,\rho} - h^0_{,\mu} h^{0,\mu} - 2h^{0\mu\nu}{}_{,\mu} h^0_{\rho\nu}{}^{,\rho} + 2h^0_{,\mu} h^{0\nu\mu}{}_{,\nu}) - \frac{1}{4}\sum_i F^0_{\mu\nu i} F^{0\mu\nu i} + \frac{1}{2}\partial^\mu \phi^0 \partial_\mu \phi^0 + \sum_{ij} \partial^\mu \phi^0_{ij} \partial_\mu \phi^0_{ij}, \quad (16)$$

$$\mathcal{L}^0_{h\ \rm int} = \frac{1}{2M_4} h^0_{\mu\nu} T^{0\mu\nu} + \frac{1}{M_4} A^0_{\mu i} T^{0\mu i} + \frac{1}{2M_4}\phi^0_{ij} T^{0ij}, \quad (17)$$

and $F^0_{\mu\nu\ i} \equiv \partial_\mu A^0_{\nu i} - \partial_\nu A^0_{\mu i}$.

The non-zero modes have the kinetic terms

$$S^{\vec{n}\ \rm K.E.}_{\rm lin} = \int d^4x \sum_{\vec{n}\neq\vec{0}} [\mathcal{L}^{\vec{n}}_h + \mathcal{L}^{\vec{n}}_A + \mathcal{L}^{\vec{n}}_\phi], \quad (18)$$

where

$$\mathcal{L}^{\vec{n}}_h = \frac{1}{4}(\tilde{h}^{\vec{n}\mu\nu,\rho}\tilde{h}^{-\vec{n}}_{\mu\nu,\rho} - \tilde{h}^{\vec{n}}_{,\mu}\tilde{h}^{-\vec{n},\mu} - 2\tilde{h}^{\vec{n}\mu\nu}{}_{,\mu}\tilde{h}^{-\vec{n}}_{\rho\nu}{}^{,\rho} + 2\tilde{h}^{\vec{n}}_{,\mu}\tilde{h}^{-\vec{n}\nu\mu}{}_{,\nu}) - \frac{1}{4}m_n^2 \tilde{h}^{\vec{n}\mu\nu}\tilde{h}^{-\vec{n}}_{\mu\nu} + \frac{1}{4}m_n^2 \tilde{h}^{\vec{n}}\tilde{h}^{-\vec{n}}, \quad (19)$$

$$\mathcal{L}^{\vec{n}}_A = -\frac{1}{4}\tilde{F}^{\vec{n}}_{\mu\nu i}\tilde{F}^{-\vec{n}\mu\nu i} + \frac{1}{2}m_n^2 \tilde{A}^{\vec{n}}_{\mu i}\tilde{A}^{-\vec{n}\mu i}, \quad (20)$$

$$\mathcal{L}^{\vec{n}}_\phi = \frac{1}{2}\partial_\mu \tilde{\phi}^{\vec{n}}_{ij}\partial^\mu \tilde{\phi}^{-\vec{n}ij} - \frac{1}{2}m_n^2 \tilde{\phi}^{\vec{n}}_{ij}\tilde{\phi}^{-\vec{n}ij}, \quad (21)$$

and $\tilde{F}^{\vec{n}}_{\mu\nu\ i} \equiv \partial_\mu \tilde{A}^{\vec{n}}_{\nu i} - \partial_\nu \tilde{A}^{\vec{n}}_{\mu i}$. The interaction terms of the non-zero modes are

$$S^{\vec{n}\ \rm int}_{\rm lin} = \int d^4x \sum_{\vec{n}\neq\vec{0}} [\mathcal{L}^{\vec{n}}_{h\ \rm int} + \mathcal{L}^{\vec{n}}_{A\ \rm int} + \mathcal{L}^{\vec{n}}_{\phi\ \rm int}], \quad (22)$$

where

$$\mathcal{L}^{\vec{n}}_{h\ \rm int} = \frac{1}{2M_4}\tilde{h}^{\vec{n}}_{\mu\nu} T^{-\vec{n}\mu\nu}, \quad (23)$$

$$\mathcal{L}^{\vec{n}}_{A\ \rm int} = \frac{1}{M_4} P_{ij}\tilde{A}^{\vec{n}\mu j} T^{-\vec{n}i}_{\mu}, \quad (24)$$

$$\mathcal{L}^{\vec{n}}_{\phi\ \rm int} = \frac{1}{2M_4}\tilde{\phi}^{\vec{n}}_{ij}\tilde{T}^{-\vec{n}ij}, \quad (25)$$

with

$$\tilde{T}^{\vec{n}}_{ij} = \left[\frac{2}{3}\eta^{\mu\nu} T^{\vec{n}}_{\mu\nu} + \frac{2}{3\partial_k^2}\partial^\nu(\partial^\mu T^{\vec{n}}_{\mu\nu}) + 2aP_{kl}T^{\vec{n}}_{kl}\right] \times \frac{P_{ij}}{\sqrt{2}(1 + a(d-1))} - \sqrt{2}P_{ik}P_{jl}T^{\vec{n}}_{kl}. \quad (26)$$

Note that in all expressions above, both $(n_1, n_2, \ldots, n_d)$ and $(-n_1, -n_2, \ldots, -n_d)$ are to be included in sums over $\vec{n} \neq \vec{0}$, and all d^2 pairs are to be included in sums over ij.

III. GRAVITON INTERACTIONS FOR ORBIFOLD COMPACTIFICATIONS

The analysis so far is valid for compactification on a torus S^d. For reasons described in Sec. I, we are most interested in the case where KK number is broken to KK parity. This may be achieved by compactifications on orbifolds.

For simplicity, we now concentrate on the $D=5$ case with coordinates $x^M = (x^\mu, y)$, where the extra dimension is an interval S^1/Z_2 of length πR [4]. This geometry can be described as an orbifold of a circle compactification, where we orbifold by the symmetry $y \to -y$. In the SM sector, this symmetry is accompanied by the transformations $V_\mu \to V_\mu$ and $V_5 \to -V_5$ for the gauge fields. The 4D scalar is projected out, and the massless sector includes only the 4D gauge field. A similar projection on fermions removes half of the degrees of freedom, leaving only the chiral fermions of the SM.

In the gravitational sector, we will project by the action $h_{\mu\nu} \to h_{\mu\nu}$, $h_{\mu 5} \to -h_{\mu 5}$, and $h_{55} \to h_{55}$ under $y \to -y$. At the massless level, this preserves the 4D graviton $h^0_{\mu\nu}(x)$, while removing the gravi-vectors $h^0_{\mu 5}(x)$ and $h^0_{5\nu}(x)$. The gravi-scalar $h^0_{55}(x)$ is preserved by this projection; we assume that some other physics stabilizes this mode and generates a large mass for it. (The phenomenology and cosmology of an extremely light gravi-scalar is discussed in Refs. [35].) The final massless sector is, then, just the SM plus the 4D graviton.

A. Boundary interactions

Prior to discussing the bulk interactions, we need to consider the boundary terms. These arise because the space we are considering is singular at $y=0,\pi R$, and hence we expect new interactions at these points. The nature of these interactions can only be derived from an underlying microscopic theory (such as string theory) that smooths out the singularity.

Nevertheless, we can make some qualitative statements about these terms. If the singular points are smoothed out to a size l_F, these interactions are localized near the singular points in a region of size l_F. We can formally write these interactions in the form

$$S=\int d^4x \int_0^{\pi R} \frac{dy}{\pi R}\left[f\left(\frac{y}{l_F}\right)+f\left(\frac{\pi R-y}{l_F}\right)\right]\mathcal{L}(x), \qquad (27)$$

where $f(w)$ goes to zero for $w\gg 1$. We have assumed that the resolution of the singularity is such that KK parity is preserved.

The basic assumption we will make is that $l_F \ll \pi R$. This is not unreasonable; for example, a natural guess for the scale l_F would be the (higher-dimensional) Planck scale. If we then consider $R^{-1}\sim 1$ TeV, we find $l_F^{-1}\sim 10^{10}$ TeV, so indeed $l_F\ll\pi R$ in this case. If $l_F\ll \pi R$, we can estimate the boundary terms as

$$S\sim \frac{l_F}{\pi R} f(0) \int d^4x \; \mathcal{L}(x), \qquad (28)$$

that is, the boundary terms are suppressed by a factor $l_F/\pi R$, which is small. Thus, to leading order, we can ignore these terms.

KK number-violating terms are generated at one loop [6,36]. The tree-level spectrum already breaks KK number, since certain modes are projected out by the orbifold action. These are translated at loop level to interactions that violate KK number. The typical decay widths of the higher KK modes are therefore $\sim \alpha m_n$.

There is another source of boundary terms: certain manipulations of the bulk terms require integration by parts, which produce boundary contributions. These terms are also suppressed for the reason given above.

B. Bulk interactions

Because we project out modes that are even or odd under $y\to -y$, it is more convenient to replace the conventional Fourier expansion of Eq. (9) by the expansion

$$h_{\mu\nu}(x,y)=h^0_{+\mu\nu}(x) + \sqrt{2}\sum_{n>0}\left[h^n_{+\mu\nu}(x)\cos\frac{ny}{R}+h^n_{-\mu\nu}(x)\sin\frac{ny}{R}\right], \qquad (29)$$

$$A_{5\nu}(x,y)=A^0_{+5\nu}(x) + \sqrt{2}\sum_{n>0}\left[A^n_{+5\nu}(x)\cos\frac{ny}{R}+A^n_{-5\nu}(x)\sin\frac{ny}{R}\right], \qquad (30)$$

$$\phi_{55}(x,y)=\phi^0_{+55}(x) + \sqrt{2}\sum_{n>0}\left[\phi^n_{+55}(x)\cos\frac{ny}{R}+\phi^n_{-55}(x)\sin\frac{ny}{R}\right], \qquad (31)$$

$$T_{\mu\nu}(x,y)=T^0_{\mu\nu}(x) + \sqrt{2}\sum_{n>0}\left[T^n_{+\mu\nu}(x)\cos\frac{ny}{R}+T^n_{-\mu\nu}(x)\sin\frac{ny}{R}\right], \qquad (32)$$

and similarly for the other component fields. All $h^n_{-\mu\nu}$, $A^n_{+5\nu}$, $A^n_{+\mu 5}$, and ϕ^n_{-55} fields are projected out by the orbifold.

The physical graviton field G^n at KK level $n>0$ is

$$G^n_{\mu\nu}=(h^n_{+\mu\nu}+\eta_{\mu\nu}\phi^n_{+55})+\frac{R}{n}(\partial_\mu A^n_{-5\nu}+\partial_\nu A^n_{-5\mu}) - \frac{R^2}{n^2}\partial_\mu\partial_\nu(2\phi^n_{+55}). \qquad (33)$$

In this new basis, the linearized action for the G fields is

$$S^5_{\rm lin}=\int d^4x\left[\mathcal{L}^0_G+\mathcal{L}_0(\overline{\phi})+\mathcal{L}^0_{G\ \rm int}+\sum_{n>0}(\mathcal{L}^n_G+\mathcal{L}^n_{G\ \rm int})\right], \qquad (34)$$

where

$$\mathcal{L}^0_G=\frac{1}{4}(G^{0\mu\nu,\rho}G^0_{\mu\nu,\rho}-G^0_{,\mu}G^{0,\mu} - 2G^{0\mu\nu}{}_{,\mu}G^0_{\rho\nu}{}^{,\rho}+2G^0_{,\mu}G^{0\nu\mu}{}_{,\nu}), \qquad (35)$$

$$\mathcal{L}^0_{G\ \rm int}=\frac{1}{2M_4}G^0_{\mu\nu}T^{0\mu\nu}, \qquad (36)$$

$$\mathcal{L}_G^n = \frac{1}{4}(G^{n\mu\nu,\rho}G^n_{\mu\nu,\rho} - G^n_{,\mu}G^{n,\mu} - 2G^{n\mu\nu}{}_{,\mu}G^n_{\rho\nu}{}^{,\rho} + 2G^n_{,\mu}G^{n\nu\mu}{}_{,\nu}) - \frac{1}{4}m_n^2 G^{n\mu\nu}G^n_{\mu\nu} + \frac{1}{4}m_n^2 G^n G^n, \tag{37}$$

$$\mathcal{L}_{G\ \mathrm{int}}^n = \frac{1}{2M_4}G^n_{\mu\nu}T_+^{n\mu\nu}. \tag{38}$$

The KK graviton propagator is[1]

$$\langle G^m_{\mu\nu}G^n_{\rho\sigma}\rangle = \frac{i\delta^{mn}B_{\mu\nu\rho\sigma}(k)}{k^2 - m_n^2 + i\varepsilon}, \tag{39}$$

where

$$B_{\mu\nu\rho\sigma}(k) = 2\left(\eta_{\mu\rho} - \frac{k_\mu k_\rho}{m_n^2}\right)\left(\eta_{\nu\sigma} - \frac{k_\nu k_\sigma}{m_n^2}\right) + 2\left(\eta_{\mu\sigma} - \frac{k_\mu k_\sigma}{m_n^2}\right)\left(\eta_{\nu\rho} - \frac{k_\nu k_\rho}{m_n^2}\right) - \frac{4}{3}\left(\eta_{\mu\nu} - \frac{k_\mu k_\nu}{m_n^2}\right)\left(\eta_{\rho\sigma} - \frac{k_\rho k_\sigma}{m_n^2}\right), \tag{40}$$

and $m_n = n/R$.

To determine the KK graviton couplings, we must determine the matter stress-energy tensor components $T_+^{n\,\mu\nu}$. For gauge fields,

$$T_{MN} = F_M{}^P F_{PN} - \frac{1}{4}\eta_{MN}F_{PQ}F^{PQ}. \tag{41}$$

We expand the gauge field in harmonics

$$a_\mu(x,y) = a^0_{+\mu}(x) + \sqrt{2}\sum_{n>0}\left[a^n_{+\mu}(x)\cos\frac{ny}{R} + a^n_{-\mu}(x)\sin\frac{ny}{R}\right], \tag{42}$$

$$a_5(x,y) = a^0_{+5}(x) + \sqrt{2}\sum_{n>0}\left[a^n_{+5}(x)\cos\frac{ny}{R} + a^n_{-5}(x)\sin\frac{ny}{R}\right]. \tag{43}$$

All $a^n_{-\mu}$ and a^n_{+5} fields are projected out by the orbifold.

The physical gauge field A^n_μ at KK level $n>0$ is

$$A^n_\mu = a^n_{+\mu} - \frac{R}{n}\partial_\mu a^n_{-5}, \tag{44}$$

and identifying coefficients of $\cos ny/R$, we find

$$T^n_{+\mu\nu} = \sum_{m=0}^{n}\left[F_\mu{}^{m\rho}F^{n-m}_{\nu\rho} - \frac{1}{4}\eta_{\mu\nu}F^m_{\rho\sigma}F^{n-m\rho\sigma} + m_n m_{n-m}\left(A^m_\mu A^{n-m}_\nu - \frac{1}{2}\eta_{\mu\nu}A^n_\rho A^{n-m\rho}\right)\right], \tag{45}$$

where $F^m_{\mu\nu} \equiv \partial_\mu A^m_\nu - \partial_\nu A^m_\mu$. The $G^n_{\mu\nu}(q)A^m_\alpha(k_1)A^{n-m}_\beta(k_2)$ vertex is therefore

$$X_{\mu\nu\alpha\beta} = \frac{i}{2M_4}\Big[\eta_{\alpha\beta}k_{1\mu}k_{2\nu} - \eta_{\mu\alpha}k_{1\beta}k_{2\nu} - \eta_{\nu\beta}k_{1\mu}k_{2\alpha} + \eta_{\mu\alpha}\eta_{\nu\beta}(k_1\cdot k_2) - \frac{1}{2}\eta_{\mu\nu}(\eta_{\alpha\beta}(k_1\cdot k_2) - k_{1\beta}k_{2\alpha}) + m_n m_{n-m}\left(\eta_{\mu\alpha}\eta_{\nu\beta} - \frac{1}{2}\eta_{\mu\nu}\eta_{\alpha\beta}\right) + (\alpha\leftrightarrow\beta)\Big]. \tag{46}$$

The fermion couplings may be calculated similarly. The stress-energy tensor for 5D Dirac fermions is

$$T_{MN} = \eta_{MN}(\bar\psi i\gamma^P D_P\psi - m_{\psi^0}\bar\psi\psi) - \frac{1}{2}\bar\psi i\gamma_M D_N\psi - \frac{1}{2}\bar\psi i\gamma_N D_M\psi - \frac{1}{2}\eta_{MN}\partial^P(\bar\psi i\gamma_P\psi) + \frac{1}{4}\partial_M(\bar\psi i\gamma_N\psi) + \frac{1}{4}\partial_N(\bar\psi i\gamma_M\psi), \tag{47}$$

where m_{ψ^0} is the 5-dimensional mass of the fermion. We will consider a Z_2 action on the fermion which acts as $\psi(x,y) = -\gamma_5\psi(x,-y)$. This preserves the zero mode of the left-handed fermion, but not that of the right-handed one. In this case, the expansion in harmonics is of the form

$$\psi(x,y) = \psi^0_L(x) + \sqrt{2}\sum_{n>0}\left[\psi^n_L(x)\cos\frac{ny}{R} + i\psi^n_R(x)\sin\frac{ny}{R}\right], \tag{48}$$

$$i\partial_5\gamma_5\psi(x,y) = \sqrt{2}\sum_{n>0} i m_n\left[\psi^n_L(x)\sin\frac{ny}{R} + i\psi^n_R(x)\cos\frac{ny}{R}\right]. \tag{49}$$

The stress-energy tensor for fermions is

[1]This analysis differs by a factor of 2 from the journal version of Ref. [32], but agrees with the later e-print version hep-ph/9811350v4.

$$T^n_{+\mu\nu}=\sum_{m=0}^{n}\bigg[\eta_{\mu\nu}(\overline{\psi^m_L}i\gamma^\rho D_\rho\psi^{n-m}_L-m_{n-m}\overline{\psi^m_R}\psi^{n-m}_L)-\frac{1}{2}\overline{\psi^m_L}i\gamma_\mu D_\nu\psi^{n-m}_L-\frac{1}{2}\overline{\psi^m_L}i\gamma_\nu D_\mu\psi^{n-m}_L-\frac{1}{2}\eta_{\mu\nu}\partial^\rho(\overline{\psi^m_L}i\gamma_\rho\psi^{n-m}_L)+\frac{1}{2}\eta_{\mu\nu}(m_m+m_{n-m})\times(\overline{\psi^m_R}\psi^{n-m}_L)+\frac{1}{4}\partial_\mu(\overline{\psi^m_L}i\gamma_\nu\psi^{n-m}_L)+\frac{1}{4}\partial_\nu(\overline{\psi^m_L}i\gamma_\mu\psi^{n-m}_L)+(R\leftrightarrow L)\bigg], \tag{50}$$

where $\psi^0_R(x)=0$. The KK graviton-fermion-fermion vertex for $G^n_{\mu\nu}(q)\overline{\psi^m}(k_1)\psi^{n-m}(k_2)$ interactions is then

$$Y_{\mu\nu}=\frac{i}{4M_4}\bigg[\eta_{\mu\nu}[2(\gamma^\rho k_{2\rho}-m_{n-m})-(\gamma^\rho k_{1\rho}-m_m)-(\gamma^\rho k_{2\rho}-m_{n-m})]-\frac{1}{2}(k_1+k_2)_\mu\gamma_\nu-\frac{1}{2}(k_1+k_2)_\nu\gamma_\mu\bigg]. \tag{51}$$

Note that if the fermions in this vertex are on shell, only the last two terms remain.

IV. NLKP DECAY WIDTHS INTO LKP GRAVITONS

Using the vertices of Sec. III B, we can now calculate Feynman diagrams involving gravitons and SM fields. We are particularly interested in the decay widths for applications to models of dark matter [27,28], where an NLKP SM field decays to a stable LKP graviton. The NLKP may either be a gauge boson, such as B^1, or a fermion, such as τ^1. These decay widths may be calculated using the usual trace techniques or by helicity amplitude methods. We have done both and checked that they yield identical answers. We present the helicity amplitude analysis here.

The polarization vectors of a massive graviton, given the normalization conventions of Sec. III B, are

$$e^{\pm 2}_{\mu\nu}=2\epsilon^\pm_\mu\epsilon^\pm_\nu,$$

$$e^{\pm 1}_{\mu\nu}=\sqrt{2}(\epsilon^\pm_\mu\epsilon^0_\nu+\epsilon^0_\mu\epsilon^\pm_\nu),$$

$$e^0_{\mu\nu}=\sqrt{\frac{2}{3}}(\epsilon^+_\mu\epsilon^-_\nu+\epsilon^-_\mu\epsilon^+_\nu-2\epsilon^0_\mu\epsilon^0_\nu).$$

Here $\epsilon^\pm_\mu$ and ϵ^0_μ are the polarization vectors of a massive gauge boson; for a massive vector boson with $p^\mu=(E,0,0,p)$ and mass m,

$$\epsilon^+_\mu(p)=\frac{1}{\sqrt{2}}(0,1,i,0), \tag{52}$$

$$\epsilon^-_\mu(p)=\frac{1}{\sqrt{2}}(0,-1,i,0), \tag{53}$$

$$\epsilon^0_\mu(p)=\frac{1}{m}(p,0,0,-E). \tag{54}$$

The graviton polarization vectors satisfy the normalization and polarization sum conditions

$$e^{s\mu\nu}e^{s'*}_{\mu\nu}=4\delta^{ss'}, \tag{55}$$

$$\sum_s e^s_{\mu\nu}e^{s*}_{\rho\sigma}=B_{\mu\nu\rho\sigma}, \tag{56}$$

where $B_{\mu\nu\rho\sigma}$ is as given in Eq. (40).

With these conventions, the helicity amplitudes for the decay of a KK hypercharge gauge boson to a KK graviton are

$$|\mathcal{M}(B^{1\pm}(k)\to G^{10}(p)+\gamma^\pm(q))| = \cos\theta_W X^{\mu\nu\alpha\beta}\epsilon^\pm_\alpha(k)\epsilon^{\pm *}_\beta(q)e^{0*}_{\mu\nu}(p) = \frac{\cos\theta_W}{\sqrt{6}M_4}\frac{m^4_{B^1}}{m^2_{G^1}}\bigg[1-\frac{m^2_{G^1}}{m^2_{B^1}}\bigg], \tag{57}$$

$$|\mathcal{M}(B^{10}(k)\to G^{1\pm 1}(p)+\gamma^\mp(q))| = \cos\theta_W X^{\mu\nu\alpha\beta}\epsilon^0_\alpha(k)\epsilon^{\mp *}_\beta(q)e^{\pm 1*}_{\mu\nu}(p) = \frac{\cos\theta_W}{\sqrt{2}M_4}\frac{m^3_{B^1}}{m_{G^1}}\bigg[1-\frac{m^2_{G^1}}{m^2_{B^1}}\bigg], \tag{58}$$

$$|\mathcal{M}(B^{1\pm}(k)\to G^{1\pm 2}(p)+\gamma^\mp(q))| = \cos\theta_W X^{\mu\nu\alpha\beta}\epsilon^\pm_\alpha(k)\epsilon^{\pm *}_\beta(q)e^{\pm 2*}_{\mu\nu}(p) = \frac{\cos\theta_W}{M_4}m^2_{B^1}\bigg[1-\frac{m^2_{G^1}}{m^2_{B^1}}\bigg]. \tag{59}$$

The total squared amplitude, averaged over the initial three polarizations and summed over final states, is

$$\frac{1}{3}\sum_{h,h',h''}|\mathcal{M}(B^{1h}(k)\to G^{1h'}(p)+\gamma^{h''}(q))|^2 = \frac{\cos^2\theta_W}{9}\frac{m^4_{B^1}}{M^2_4}\bigg[1-\frac{m^2_{G^1}}{m^2_{B^1}}\bigg]^2\bigg[6+3\frac{m^2_{B^1}}{m^2_{G^1}}+\frac{m^4_{B^1}}{m^4_{G^1}}\bigg], \tag{60}$$

and the decay width of a KK hypercharge gauge boson to a KK graviton is therefore

$$\Gamma(B^1\to G^1\gamma)=\frac{\cos^2\theta_W}{144\pi M_4^2}\frac{m_{B^1}^7}{m_{G^1}^4}\left[1-\frac{m_{G^1}^2}{m_{B^1}^2}\right]^3\times\left[1+3\frac{m_{G^1}^2}{m_{B^1}^2}+6\frac{m_{G^1}^4}{m_{B^1}^4}\right]. \quad (61)$$

The decay width of a chiral KK fermion to a KK graviton may be calculated similarly in the helicity amplitude formalism. For $p^\mu=(E,0,0,p)$, the helicity spinors are

$$u^+(p)=\begin{pmatrix}\sqrt{E+m}\ \xi^+\\ \sqrt{E-m}\ \xi^+\end{pmatrix},\quad u^-(p)=\begin{pmatrix}\sqrt{E+m}\ \xi^-\\ -\sqrt{E-m}\ \xi^-\end{pmatrix}, \quad (62)$$

where $\xi^{+T}=(1,0)$ and $\xi^{-T}=(0,1)$, and we take the Dirac representation.

The helicity amplitudes for a chiral fermion decaying to a KK graviton are, then,

$$|\mathcal{M}(f_{L,R}^{1\pm}(k)\to G^{1\pm1}(p)+f_{L,R}^{0\mp}(q))| = Y^{\mu\nu}\overline{u^{\mp}}(q)u^{\pm}(k)e_{\mu\nu}^{\pm1}*(p) = \frac{1}{2M_4}\frac{m_{f^1}^3}{m_{G^1}}\left[1-\frac{m_{G^1}^2}{m_{f^1}^2}\right]^{3/2}, \quad (63)$$

$$|\mathcal{M}(f_{L,R}^{1\pm}(k)\to G^{10}(p)+f_{L,R}^{0\pm}(q))| = Y^{\mu\nu}\overline{u^{\pm}}(q)u^{\pm}(k)e_{\mu\nu}^{0}(p) = \frac{1}{\sqrt{6}M_4}\frac{m_{f^1}^4}{m_{G^1}^2}\left[1-\frac{m_{G^1}^2}{m_{f^1}^2}\right]^{3/2}, \quad (64)$$

where we have assumed a massless zero mode fermion. The squared amplitude, averaged over the initial two polarizations and summed over final states, is

$$\frac{1}{2}\sum_{h,h',h''}|\mathcal{M}(f_{L,R}^{1h}(k)\to G^{1h'}(p)+f_{L,R}^{0h''}(q))|^2 = \frac{1}{6}\frac{m_{f^1}^4}{M_4^2}\left[1-\frac{m_{G^1}^2}{m_{f^1}^2}\right]^3\left[3+2\frac{m_{f^1}^2}{m_{G^1}^2}\right]\frac{m_{f^1}^2}{m_{G^1}^2}. \quad (65)$$

The decay width of a chiral fermion to a KK graviton is

$$\Gamma(f_{L,R}^1\to f_{L,R}^0+G^1)=\frac{1}{96\pi M_4^2}\frac{m_{f^1}^7}{m_{G^1}^4}\left[1-\frac{m_{G^1}^2}{m_{f^1}^2}\right]^4\left[2+3\frac{m_{G^1}^2}{m_{f^1}^2}\right]. \quad (66)$$

The decay widths of Eqs. (61) and (66) provide accurate expressions for NLKP lifetimes in the LKP graviton scenario. For NLKP and LKP masses at the weak scale $M_{\rm Weak}$, and assuming mass splittings of the same order, the naive expectation of lifetimes $\tau=\Gamma^{-1}\sim 4\pi M_4^2/(M_{\rm Weak})^3\sim 10^5$ s -10^8 s is born out. NLKPs therefore decay after BBN, and their decay products are subject to rather stringent BBN constraints.

Even relatively mild degeneracies may disrupt these expectations; however, as for $\Delta m\equiv m_{\rm NLKP}-m_{\rm LKP}\ll m_{\rm LKP}$ the lifetimes scale as $\tau\propto(\Delta m)^{-3}$ and $\tau\propto(\Delta m)^{-4}$ in the gauge boson and fermion cases, respectively. This behavior is mirrored in the analogous case in supersymmetry of superpartners decaying to gravitinos. The supersymmetric case was analyzed in Ref. [28], where alternative signals of superWIMP dark matter were identified. The similarity of Eqs. (61) and (66) to the supersymmetric case indicates that KK graviton dark matter is also a viable superWIMP candidate with promising possibilities for detection.

V. REHEATING AND KK GRAVITON PRODUCTION

KK gravitons are produced copiously after the big bang. Inflation dilutes these gravitons away, but their number density may be regenerated during reheating. The situation is analogous to the case of supersymmetry, where gravitinos may be produced after reheating, and constraints bound the reheat temperature to $T_{\rm RH}\lesssim 10^8-10^{10}$ GeV [29,30]. As we will see, however, the presence of a tower of KK levels leads to much stronger production of gravitons, and much stronger bounds on reheat temperature in UED models. In this section we estimate the density of KK gravitons produced after reheating for UED models with an arbitrary number of extra dimensions d. In the next section, we discuss the cosmological significance of these results. Throughout we assume that the extra dimensions remain fixed in size.

UED models are characterized by two hierarchically separated scales: the KK mass scale $m_{\rm KK}\sim$TeV, and the 4D Planck scale $M_4\simeq 1.7\times 10^{18}$ GeV. For this analysis, cosmology introduces another scale, the reheat temperature $T_{\rm RH}$. As UED models are 4D effective theories of some higher-dimensional theory, they are valid only up to some cutoff scale M_s, which is much smaller than M_4 [37]. We therefore assume $T_{\rm RH}<M_s$, and so $T_{\rm RH}\ll M_4$.

In terms of these scales, the expansion rate of the universe is $H\sim T^2/M_4$. The interaction rate of SM particles and their KK partners with each other is $\sigma_{\rm SM}n\sim\alpha^2 T$. The decay rate of SM particles at KK level n may also be estimated to be $\Gamma_{\rm SM}\sim\alpha m_n$. Given the hierarchy between T and M_4,

$$\sigma_{\rm SM}n,\Gamma_{\rm SM}\gg H, \quad (67)$$

and so SM particles and their KK partners remain in thermodynamic equilibrium as the universe cools after reheating. In contrast, the time scale for graviton interactions is very long. For example, the interaction rate for $\phi_a^{\vec{k}}+\phi_b^{\vec{l}}\to\phi_c^{\vec{m}}+G^{\vec{n}}$, where $\vec{k}$, $\vec{l}$, $\vec{m}$, and $\vec{n}$ specify KK levels and a, b, and c label SM degrees of freedom, is $\sigma_G n\sim T^3/M_4^2$. Similarly, as discussed in Sec. IV, the decay rates of KK gravitons are $\Gamma_G\sim m_n^3/M_4^2$. These rates are therefore far below the expansion rate,

$$\sigma_G n,\Gamma_G\ll H, \quad (68)$$

and gravitons never reach thermodynamic equilibrium.

The qualitative picture, then, is that after reheating, the SM degrees of freedom exist in thermal equilibrium. Occasionally, they produce KK gravitons, which are metastable and decay long after big bang nucleosynthesis. If overproduced, they may overclose the universe or their eventual decay products may destroy the predictions of big bang nucleosynthesis.

We now work toward a more quantitative estimate of graviton abundances. During the era of graviton production, the expansion rate H is given by

$$H^2=\frac{8\pi G_N}{3}\rho_R=\frac{\pi^2}{180M_4^2}g_*(T)T^4, \tag{69}$$

where ρ_R and $g_*(T)$ are the total energy density and the effective number of light degrees of freedom, respectively. The entropy density is

$$s=\frac{2\pi^2}{45}g_*(T)T^3, \tag{70}$$

and the number density of a massless bosonic degree of freedom is

$$n_0=\frac{\zeta(3)}{\pi^2}T^3. \tag{71}$$

For UED theories,

$$g_*(T)=g_*^{KK}D_d(T). \tag{72}$$

Here g_*^{KK} is the effective number of degrees of freedom per KK level, and is a model-dependent constant. In all models, however, there are more degrees of freedom at excited KK levels than at the zero mode level, where many degrees of freedom are projected out. For the SM, the effective number of degrees of freedom, organized by spin, is $g_*^{\rm SM}=g_0^{\rm SM}+\frac{7}{8}g_{1/2}^{\rm SM}+g_1^{\rm SM}=4+\frac{7}{8}90+24=106.75$. For the S^1/Z_2 UED model, at each KK level $n>0$ a gauge boson field has 3 degrees of freedom and a fermion field has 4 degrees of freedom, so the total effective number of degrees of freedom is $g_*^{\rm KK}=g_0^{\rm KK}+\frac{7}{8}g_{1/2}^{\rm SM}\frac{4}{2}+g_1^{\rm SM}\frac{3}{2}=197.5$. A similar counting may be done for other UED models once the number of additional dimensions and the orbifold is specified.

The function $D_d(T)$ counts the number of excited modes in the thermal bath at temperature T. $D_d(T)$ may be approximated by counting all modes with mass below T. This yields, for d extra dimensions,

$$D_d(T)=\frac{1}{2^d}V_d\left[\frac{T}{m_{\rm KK}}\right]^d, \tag{73}$$

where

$$V_d=\frac{\pi^{d/2}}{\Gamma\left(1+\frac{d}{2}\right)}=2,\pi,\frac{4}{3}\pi,\frac{1}{2}\pi^2,\ldots, \tag{74}$$

for $d=1,2,3,4,\ldots$, is the volume of a unit spherical ball in d dimensions, and the factor of $1/2^d$ in Eq. (73) accounts for the restriction to $\vec{n}$ with non-negative components.

The number density of KK gravitons at level $\vec{n}$, $n_{G^{\vec{n}}}$, is determined by the Boltzmann equation

$$\frac{dn_{G^{\vec{n}}}}{dt}+3Hn_{G^{\vec{n}}}=C_{G^{\vec{n}}}, \tag{75}$$

where

$$C_{G^{\vec{n}}}=\sum \langle \sigma_{\phi_a^{\vec{k}}\phi_b^{\vec{l}}\to\phi_c^{\vec{m}}G^{\vec{n}}}v\rangle n_{\phi_a^{\vec{k}}}n_{\phi_b^{\vec{l}}} \tag{76}$$

is the collision operator. All graviton destruction processes are negligible, given the low abundances and decay rates of gravitons. We will parametrize the collision operator as

$$C_{G^{\vec{n}}}=C\sigma[g_*(T)n_0]^2, \tag{77}$$

where

$$\sigma=\frac{\alpha_3}{4\pi M_4^2}. \tag{78}$$

Here α_3 is the strong coupling constant and C is a constant. If every light degree of freedom could interact with every other light degree of freedom with cross section σ, $C_{G^{\vec{n}}}$ with $C=1$ would be a reasonable estimate. However, global and local symmetries restrict which reactions are possible, and not all interactions involve strongly interacting SM particles. A detailed calculation of C can be done once a specific UED model is chosen and its spectrum determined. Here we keep the analysis general by leaving C as a free parameter. Based on the results of detailed studies of gravitino abundances from reheating in supersymmetric models [30], we expect values of $C\sim\mathcal{O}(0.01)$, and we will consider a range of $0.001\lesssim C\lesssim 0.1$ in present numerical results below. Given the high power dependence of the graviton abundance on $T_{\rm RH}$, we will see that bounds on $T_{\rm RH}$ are rather insensitive to even this generous range for C. Note also that, as $C=1$ is certainly an overestimate, if a scenario passes all constraints even with $C=1$, it is certainly allowed.

With these definitions, the KK graviton number density satisfies

$$\frac{dn_{G^{\vec{n}}}}{dt}+3Hn_{G^{\vec{n}}}=C\sigma[g_*(T)n_0]^2. \tag{79}$$

This is most conveniently solved by changing variables $n_{G^{\vec{n}}}\to Y_{G^{\vec{n}}}\equiv n_{G^{\vec{n}}}/s$ and $t\to T$. Adiabaticity implies that entropy $S=sR^3\propto g_*(T)T^3R^3\propto T^{3+d}R^3$ is conserved, and so

$$\frac{1}{s}\frac{ds}{dt}=-3\frac{1}{R}\frac{dR}{dt}=-3H,\quad \frac{dT}{dt}=-\frac{3}{3+d}HT. \tag{80}$$

With the relations of Eq. (80), the Boltzmann equation becomes

$$\frac{dY_{G^{\vec{n}}}}{dT} = -\frac{3+d}{3}\frac{1}{HTs}C\sigma[g_*(T)n_0]^2. \tag{81}$$

$Y_{G^{\vec{n}}}$ changes until $G^{\vec{n}}$ production stops at temperature $T \sim m_n$ and then remains constant until gravitons begin to decay. After BBN and before KK gravitons decay, then,

$$\begin{aligned} Y_{G^{\vec{n}}} &= \int_{m_n}^{T_{\rm RH}} dT\, \frac{3+d}{3}\frac{1}{HTs}C\sigma[g_*(T)n_0]^2 \\ &= \frac{45\sqrt{5}\zeta^2(3)}{2\pi^8}\alpha_3\frac{m_{\rm KK}}{M_4}C\sqrt{g_*^{KK}}\frac{3+d}{2+d} \\ &\quad \times \sqrt{\frac{V_d}{2^d}}\left[\left(\frac{T_{\rm RH}}{m_{\rm KK}}\right)^{1+(d/2)} - |\vec{n}|^{1+(d/2)}\right]. \end{aligned} \tag{82}$$

For comparison with cosmological constraints, it is convenient to determine the graviton energy density, normalized to the background photon number density $n_\gamma = 2n_0$, at the time of BBN:

$$\begin{aligned} \zeta_{G^{\vec{n}}} &\equiv \left.\frac{m_n n_{G^{\vec{n}}}}{n_\gamma}\right|_{\rm BBN} = m_n Y_{G^{\vec{n}}}\left.\frac{s}{n_\gamma}\right|_{\rm BBN} \\ &= \frac{\pi^4}{45\zeta(3)} g_*^{\rm BBN}|\vec{n}| m_{\rm KK} Y_{G^{\vec{n}}}. \end{aligned} \tag{83}$$

The total graviton energy density is then determined by summing over $\vec{n}$. For T significantly larger than $m_{\rm KK}$, we may take the continuum limit

$$\sum_{\vec{n}} \to \int d^d n = \int \frac{1}{2^d} A_d n^{d-1} dn, \tag{84}$$

where

$$A_d = \frac{2\pi^{d/2}}{\Gamma\left(\frac{d}{2}\right)} = 2, 2\pi, 4\pi, 2\pi^2, \ldots, \tag{85}$$

for $d = 1,2,3,4, \ldots$ is the area of a unit sphere in d dimensions, and the factor of $1/2^d$ in Eq. (84) again accounts for the restriction to $\vec{n}$ with non-negative components. Integrating over all KK levels up to $m_n = T_{\rm RH}$, the total energy density in KK gravitons is

$$\begin{aligned} \zeta_G &= \sum_{\vec{n}} \zeta_{G^{\vec{n}}} \approx \int_0^{T/m_{\rm KK}} \frac{1}{2^d} A_d n^{d-1} \zeta_{G^{\vec{n}}}\, dn \\ &= \frac{\sqrt{5}\zeta(3)}{2\pi^4}\alpha_3 \frac{m_{\rm KK}^2}{M_4} C\sqrt{g_*^{KK}} g_*^{\rm BBN} \frac{3+d}{(1+d)(4+3d)} \\ &\quad \times \sqrt{\frac{V_d A_d^2}{2^{3d}}} \left(\frac{T_{\rm RH}}{m_{\rm KK}}\right)^{2+(3d/2)}. \end{aligned} \tag{86}$$

If KK gravitons decay to SM particles, ζ_G gives the total energy density, normalized to the background photon density, deposited during the era of graviton decay.

Alternatively, one may take the opposite limit, and assume that KK gravitons decay predominantly through KK number preserving interactions $G^n \to n$ LKP$+X$, and nearly all of the energy stored in KK gravitons exists now in the form of KK dark matter. In this scenario, the current dark matter energy density, normalized to the critical density, is

$$\Omega_G = \zeta_G \frac{n_\gamma^0}{\rho_c^0} \simeq \frac{\zeta_G}{13 \text{ eV}}, \tag{87}$$

where $n_\gamma^0 \simeq 410$ cm^{-3} is the present photon number density, and $\rho_c^0 \simeq 5300$ eV cm^{-3} is the critical density.

Numerically, given $g_*^{\rm BBN} \simeq 3.36$ and $\alpha_3 \approx 0.1$, we find for $D=5$:

$$\zeta_G = 1.1\times 10^{-14}\text{ GeV}\times C\left[\frac{g_*^{KK}}{200}\right]^{1/2}\left[\frac{m_{\rm KK}}{1\text{ TeV}}\right]^2\left[\frac{T_{\rm RH}}{m_{\rm KK}}\right]^{7/2} \tag{88}$$

$$\Omega_G = 8.4\times 10^{-7}\times C\left[\frac{g_*^{KK}}{200}\right]^{1/2}\left[\frac{m_{\rm KK}}{1\text{ TeV}}\right]^2\left[\frac{T_{\rm RH}}{m_{\rm KK}}\right]^{7/2}, \tag{89}$$

for $D=6$:

$$\zeta_G = 8.9\times 10^{-15}\text{ GeV}\times C\left[\frac{g_*^{KK}}{200}\right]^{1/2}\left[\frac{m_{\rm KK}}{1\text{ TeV}}\right]^2\left[\frac{T_{\rm RH}}{m_{\rm KK}}\right]^{5} \tag{90}$$

$$\Omega_G = 6.8\times 10^{-7}\times C\left[\frac{g_*^{KK}}{200}\right]^{1/2}\left[\frac{m_{\rm KK}}{1\text{ TeV}}\right]^2\left[\frac{T_{\rm RH}}{m_{\rm KK}}\right]^{5}, \tag{91}$$

and for $D=7$:

$$\zeta_G = 5.0\times 10^{-15}\text{ GeV}\times C\left[\frac{g_*^{KK}}{200}\right]^{1/2}\left[\frac{m_{\rm KK}}{1\text{ TeV}}\right]^2\left[\frac{T_{\rm RH}}{m_{\rm KK}}\right]^{13/2} \tag{92}$$

$$\Omega_G = 3.9\times 10^{-7}\times C\left[\frac{g_*^{KK}}{200}\right]^{1/2}\left[\frac{m_{\rm KK}}{1\text{ TeV}}\right]^2\left[\frac{T_{\rm RH}}{m_{\rm KK}}\right]^{13/2}. \tag{93}$$

Note that for each D, these expressions, derived from Eqs. (86) and (87), are not mutually consistent — the energy density in gravitons is deposited in SM particles, or in KK dark matter (WIMP or superWIMP), or a mixture. These expressions are the maximal energy deposited in SM particles, and the maximal primordial energy density of KK dark matter. Which, if either, is realized depends on the specific UED model, and detailed considerations of higher KK mode cascade decays. On the other hand, these expressions are useful, as, if both are within existing bounds, the model is guaranteed to be consistent with current constraints.

VI. COSMOLOGICAL CONSTRAINTS

There are two sets of constraints on the KK graviton abundance. First, gravitons $G^{\vec{n}}$ may decay to stable LKPs, such as G^1 or B^1. The fraction of initial energy that winds up in LKPs is highly dependent on the UED model and the spectrum of KK modes, which determines the form of cascade decays. Clearly the energy density in LKPs cannot exceed the energy density of the initial KK gravitons. We may therefore impose the constraint

$$\Omega_G < \Omega_{\rm DM} \approx 0.23. \tag{94}$$

If this constraint is satisfied, KK dark matter will not overclose the universe, irrespective of details of the UED model.

The second set of constraints follows from requiring that SM particles produced in late decays of KK gravitons not destroy the successful predictions of BBN. These constraints are complicated, depending sensitively on the kind of energy deposited and the time at which it is released. For electromagnetic decays, the constraint on $\zeta_G^{\rm EM} = B^{\rm EM}\zeta_G$ has been studied in detail [38–40], most recently in Ref. [41]. These constraints are very weak for decay times $\tau < 10^5$ s, increase in stringency from $\zeta_G^{\rm EM} \lesssim 10^{-9}$ GeV at $\tau \sim 10^6$ s to $\zeta_G^{\rm EM} \lesssim 10^{-12}$ GeV at $\tau \sim 10^9$ s, and then remain roughly constant up to $\tau \sim 10^{12}$ s.

For hadronic cascades, the picture is at present less clear. For $\tau \lesssim 10^2$ s, hadronic constraints are relatively weak, but they become $\zeta_G^{\rm had} = B^{\rm had}\zeta_G \lesssim 10^{-12}$ GeV for 10^2 s $\lesssim \tau \lesssim 10^4$ s [42]. For longer lifetimes, there are no detailed recent analyses. The constraint may become weaker, but we assume conservatively that it remains at the $\zeta_G^{\rm had} \lesssim 10^{-12}$ GeV level.

From Eqs. (88)–(93), we see that both the overclosure and BBN constraints may be satisfied for any D and $T_{\rm RH} \sim m_{\rm KK} \sim 1$ TeV, even for $C \sim 1$ and taking the extreme cases in which all energy is deposited in either stable LKPs, EM cascades, or hadronic cascades. Although there are many uncertainties in this analysis, there is surely an allowed window in which the KK graviton abundance satisfies all existing constraints. This is necessary to establish the viability of KK WIMP and superWIMP dark matter. For LKPs to achieve the desired thermal relic density, either in the form of WIMPs such as B^1 or superWIMPs such as G^1, the universe must be reheated to a temperature $T_{\rm RH} \gtrsim m_{\rm KK}/25$. The region with ideal KK WIMP thermal abundance [7] is also shown. This result shows that there are consistent cosmologies in which the reheat temperature is low enough to suppress the primordial KK graviton abundance appropriately, but high enough to generate the desired thermal relic abundance for WIMP or superWIMP dark matter.

At the same time, given the rather strong power law dependence of the graviton energy density, overclosure and BBN stringently constraint $T_{\rm RH}$. In Fig. 1 we plot the bounds on $T_{\rm RH}$ as functions of $m_{\rm KK}$ for $D = 5,7$ from constraints on overabundance and energy release. Roughly we find constraints $T_{\rm RH} \lesssim 1 - 10$ TeV for 100 GeV $< m_{\rm KK} < 1$ TeV. In a given model, one of the constraints may be inapplicable, but they cannot both be completely avoided.

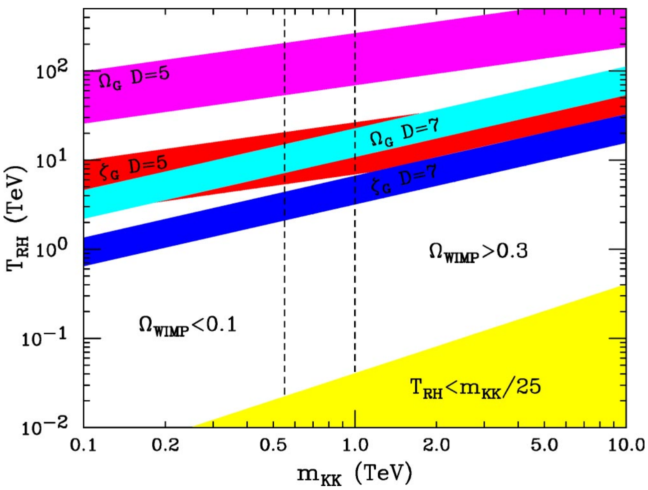

FIG. 1. Bounds on the reheat temperature $T_{\rm RH}$ as a function of $m_{\rm KK}$ from the overclosure constraint $\Omega_G < 0.23$ and the BBN constraint $\zeta_G < 10^{-12}$ GeV for $D = 5,7$, as indicated. We assume $g_*^{KK} = 200$; the range in each bound arises from varying C from 0.001 to 0.1 (see text). In the region with $T_{\rm RH} < m_{\rm KK}/25$, $T_{\rm RH}$ is too low to generate the thermal relic abundance for WIMPs. The vertical bands delimit regions where the B^1 thermal relic abundance is too low ($\Omega_{\rm WIMP} < 0.1$), approximately right, and too high ($\Omega_{\rm WIMP} > 0.3$).

The range in each constraint results from varying C in the range $0.001 < C < 0.1$. Note that the $T_{\rm RH}$ limits are rather insensitive to the substantial uncertainties encoded in C, given the extreme sensitivity of graviton abundances to $T_{\rm RH}$; for C varying by two orders of magnitude, the bounds on $T_{\rm RH}$ vary by only factors of 2 to 4, depending on the number of extra dimensions.

VII. SUMMARY

We have provided a general formalism for analyzing the dynamics of gravitons in UED theories. In particular, we found the couplings of KK gravitons to fermions and gauge bosons and presented the widths for decays of excited fermions and gauge bosons into KK gravitons in Eqs. (61) and (66). These results are of special relevance when a KK graviton is the LKP and a superWIMP candidate, as they determine the observable implications of KK graviton dark matter for, for example, BBN, the cosmic microwave background, and the diffuse photon flux.

We have also determined the abundance of KK gravitons produced after reheating in a general manner applicable to UED models for arbitrary numbers of extra dimensions, and also more generally to other models of extra dimensions. The possibility of populating a large number of graviton states at different KK levels implies that the production of gravitons after reheating is extremely efficient and extremely sensitive to the reheat temperature $T_{\rm RH}$. For d extra dimensions, the energy density in gravitons, given in Eq. (86), is

$$\zeta_G = \sum_{\vec{n}} m_{G^{\vec{n}}} \frac{n_{G^{\vec{n}}}}{s} \sim \frac{m_{\rm KK}^2}{M_4}\left(\frac{T_{\rm RH}}{m_{\rm KK}}\right)^{2+(3d/2)}. \tag{95}$$

This is to be contrasted with the case of gravitinos [29,30], for which

$$\zeta_{\tilde{G}}=m_{\tilde{G}}\frac{n_{\tilde{G}}}{s}\sim\frac{m_{\rm SUSY}^2}{M_4}\frac{T_{\rm RH}}{m_{\rm SUSY}}. \tag{96}$$

The constraints on $T_{\rm RH}$ are therefore extremely stringent. They are presented in Fig. 1 and are of the order of $T_{\rm RH} \lesssim 1-10$ TeV for 100 GeV$<m_{\rm KK}<1$ TeV.

The constraints derived here are robust, being independent of the gravi-scalar mass [35] and applicable irrespective of which KK particle is the LKP. These constraints also apply in the presence of KK parity violating interactions, as these will only serve to increase the primordial graviton production and lead to the decay of gravitons to SM particles. They supplement the requirement that the reheat temperature be below the cutoff of the 4D effective theory [37] and rather severely constrain ideas for leptogenesis. Such low reheat temperatures also constrain inflation scenarios, requiring, for example, that inflaton decay to SM particles be suppressed by extremely small couplings or kinematically through enhanced SM plasma masses at high temperatures [43].

At the same time, we have found that there exists a range of reheat temperature with $T_{\rm RH}\sim m_{\rm KK}$ such that the primordial production of KK gravitons is within bounds, but a thermal relic density of WIMPs may be produced. KK WIMP and superWIMP candidates are therefore viable, despite the stringent graviton constraints applicable to these extra dimensional theories.

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