Carbon-Optimal and Carbon-Neutral Supply Chains

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Abstract

Carbon footprinting is a tool for firms to determine the total greenhouse gas (GHG) emissions associated with their supply chain or with a unit of final product or service. Carbon footprinting efforts typically aim to identify where best to invest in emission reduction efforts, and/or to determine the proportion of total emissions that an individual firm is accountable for, whether financially and/or operationally. A major and under-recognized challenge in determining the appropriate allocation stems from the high degree to which GHG emissions (or emissions reductions) are the result of joint efforts by multiple firms.

In this paper we introduce a simple but effective model of joint production of GHG emissions in general supply chains, decomposing the total footprint into processes, each of which can be influenced by any combination of firms. A supply chain in which all firms exert their first-best emissions reduction effort levels is "carbon optimal", while a supply chain which offsets all emissions is "carbon neutral". With this structure, we examine conditions under which the supply chain can be carbon-neutral and/or carbon-optimal. We find that, in order to induce the carbon-optimal effort levels, the emissions need to be over-allocated. This means that the focus in the life-cycle assessment (LCA) and carbon footprinting literature on avoiding double-counting is, in the context of setting incentives, misguided. We also compare the situation where a single firm offsets all supply chain emissions with that where one powerful firm can enforce an emissions reduction target across all firms in the supply chain, and find that neither scenario is always preferred over the other. Our work aims to lay the foundation for a framework to integrate the economics- and LCA-based perspectives on supply chain carbon footprinting.

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1. Introduction

As concern over climate change continues to rise, the notion of "carbon footprinting" is becoming ever more widespread. Carbon footprinting refers to estimating the total greenhouse gas (GHG) emissions for a given entity, which could be a company, a product (with emissions measured over its entire life cycle) or a supply chain (in part or in full). For instance, Coca Cola reported that total GHG emissions in the Coca Cola system in 2009 were 5.4 million metric tons[1], and the carbon footprint of a 330ml glass bottle of Coca Cola is 360g.[2]

Beyond estimating their footprint, some firms go further and pledge to reduce emissions by specific amounts or percentages. Walmart's goal is to eliminate 20M tons of GHG from its supply chain by the end of 2015 (Birchall 2010). Tesco has committed to reduce the carbon footprint of the products they sell by 30% by 2020, jointly with their suppliers, and to help customers reduce their own footprints by 50% by 2020[3]. Reckitt Benckiser's Carbon20 initiative aims to reduce their products' total carbon footprint per dose by 20% by 2020[4]. Unilever has announced in 2010 that they plan to halve the footprint of their products across the lifecycle by 2020[5].

Yet other firms go further still, pledging to be or become carbon-neutral, which usually means purchasing carbon offsets for the remaining GHG emissions. Dole's operating subsidiary Standard Fruit de Costa Rica will develop a carbon-neutral supply chain for bananas and pineapples from Costa Rica to North America and Europe[6], in keeping with Costa Rica's goal to become the first carbon-neutral country by 2030[7]. FIJI Water used to claim to be carbon negative, offsetting its emissions by 120%[8], though those claims have been de-emphasized following a recently-filed class action lawsuit. Tesco aims to be a zero-carbon

[1] Coca Cola 2009/2010 Sustainability Review. Available at http://www.thecoca-colacompany.com/citizenship/pdf/SR09/SR09_Energy_ClimateProtct_20_23.pdf, last accessed February 20, 2011.

[2] Coca Cola 2008/2009 Corporate Responsibility Review. Available at http://www.cokecorporateresponsibility.co.uk/big-themes/energy-and-climate-change/product-carbon-footprint.aspx, last accessed February 20, 2011.

[3] http://cr2010.tescoplc.com/∼/media/Files/T/Tesco-Corporate-Responsibility-Report-2009/Tesco_CSR_2010.pdf, page 6, last accessed October 7, 2011.

[4] http://www.rb.com/Sites/carbon20/whatis.html, last accessed February 20, 2011.

[5] http://www.sustainable-living.unilever.com/the-plan/greenhouse-gases, last accessed October 12, 2011.

[6] http://www.doleorganic.com/programs/carbon-neutral-program.html, last accessed February 20, 2011.

[7] McPhaul, J. 2007. Costa Rica aims to win "carbon neutral" race. Reuters, May 24 2007. http://www.reuters.com/article/2007/05/24/environment-climate-costarica-dc-idUSN2438974220070526, last accessed February 20, 2011.

[8] See for instance http://www.fijiwater.com/2009/fiji-water-foundation-plants-forest/, last accessed February 20, 2011.

business by 2050[9]. Brazil's Natura Cosméticos is already carbon-neutral today, offsetting not only their own emissions but those of their entire supply chain; this is all the more noteworthy given that 95% of those total emissions are from Natura's supply chain rather than their own operations[10].

As is clear from these examples, many GHG emissions (or reductions in emissions) are the result of joint effort by multiple parties. For instance, in 2006 Dr Pepper Snapple Group redesigned its one-gallon Hawaiian Punch packaging to make it 19% lighter, which reduces the packaging supplier's carbon footprint as well as that of all firms involved in the product's distribution[11]. When Mattel redesigned its distribution system in China and in the US, adopting a full mix distribution center program in the US, total travel miles and hence carbon footprint decreased, potentially benefiting Mattel as well as the transportation providers[12].

Central to assessing the validity of any such claims or to the success of any footprint reduction efforts is a system for measuring and reporting carbon footprints. The "GHG protocol" (for companies, products, or value chains) is the dominant method for carbon accounting, which is closely related to life-cycle assessment (LCA). The existing literature on the GHG protocol and on LCA – which we describe further in the next section – focuses heavily on how to allocate emissions or reductions in emissions to each of the parties separately, aiming to avoid "double-counting" of emissions at all costs. Double-counting[13] occurs when the GHG emissions resulting from a particular activity are allocated to multiple parties in a supply chain, so that the total allocated emissions exceed the total actual emissions of that activity.

A critical question underlying LCA and the GHG protocol is: who is responsible for the emissions associated with a final product or service? A typical product goes through numerous manufacturing and transportation activities operated by a number of companies in a supply chain. Each actor in the supply chain can invest in projects that would result in lower emissions in her own operations. Furthermore, companies can also invest in projects

[9]http://cr2010.tescoplc.com/~/media/Files/T/Tesco-Corporate-Responsibility-Report-2009/Tesco_CSR_2010.pdf, page 6, last accessed October 7, 2011.

[10]Natura Cosméticos. 2009. Carbon Neutral 2009: Message from the President. http://www2.natura.net/Web/Br/Inst/CarbonoNeutro2009/src/EN/_PDF/NaturaCarbonNeutral2009_vPrint.pdf, last accessed February 20, 2011.

[11]http://www.drpeppersnapplegroup.com/sustainability/environmental-sustainability, last accessed February 20, 2011.

[12]Mattel Global Citizenship Report 2007, p24.

[13]We use double-counting to also refer to multiple-counting when more than two parties are involved in such collective carbon accounting.

that could lower the emissions of operations at upstream or downstream companies due to changes in the characteristics of the product such as its dimensions, form, flexibility, strength, required storage conditions, durability, etc. Some actors in the supply chain can affect others' emissions by collaboration, coordination, information sharing, or even simply by their economic power. For example, a manufacturer sharing his advance demand information with a supplier might smooth the supplier's operations and hence reduce the need for fast transportation, resulting in lower emissions.

In such contexts, where a number of firms in the supply chain jointly affect total emissions, firms face a challenge in greening their operations in the absence of regulatory requirements: how should responsibility for the total supply chain emissions be allocated to the various firms in order to encourage jointly optimal emissions abatement effort? Holding all parties responsible for their own emissions presupposes that emissions can be attributed to firms unambiguously, which is not the case due to the interactions discussed above. To tackle this question, we introduce a simple but effective model of joint production of GHG emissions in general supply chains with any number of firms, decomposing the total footprint into separate footprint components, each of which can be influenced by any combination of any number of firms in the supply chain. With that structure, we examine conditions under which the supply chain can be carbon-neutral, where all emissions are offset, and/or "carbon-optimal", where all firms exert their first-best emission reduction effort levels. We use the term "socially optimal" for a supply chain that does both: exerts optimal levels of emission reduction effort and fully offsets all remaining emissions. This classification of carbon footprint reduction approaches is summarized in Table 1.

	No offsetting	Total SC emissions are offset
No emission reduction (no effort)	status quo	carbon-neutral
First-best emission reduction (first-best effort level)	carbon-optimal	socially optimal

Table 1: Classification of Supply Chain (SC) Carbon Footprint Reduction Approaches.

A key finding of our analysis is that if the allocation mechanism is increasing – i.e., when any footprint component increases, the amount allocated to each firm cannot decrease, which holds for all the linear allocation rules used in practice – then over-allocation is necessary for the supply chain to be socially optimal (i.e. both carbon-neutral and carbon-optimal).

This means that the emphasis on avoiding double-counting in the carbon footprinting and LCA literatures is misguided in this context: sometimes double-counting is in fact necessary to provide each firm with the right incentive to reduce emissions.

We derive several further results using our modeling framework. Within the decentralized setting, we introduce the notion of a "carbon leader," a firm within the supply chain who decides on the carbon allocation rule for every firm in the network. We specifically consider two scenarios based on voluntary schemes observed in practice, one with a "carbon payer" (a carbon leader who offsets all emissions) and one with a "carbon enforcer" (a carbon leader who enforces a cap on each firm's emissions) and characterize the optimal allocation mechanisms and optimal effort levels for all firms under those scenarios. A carbon payer could be a firm such as Natura Cosméticos, offsetting all emissions for the entire supply chain, while a carbon enforcer could be a firm such as Walmart, pledging to eliminate 20M tons of GHG emissions from its supply chain; clearly any practical situation is likely to be a combination of the simple extreme cases captured by our scenarios. We show that, though these scenarios are the closest to actual practice, they fall short of the second-best solution where the allocation rule is determined by an external party.

We argue that a carbon leader would rather be a carbon enforcer than a carbon payer whenever possible (i.e., whenever the firm has the power to enforce targets), since then it can select an allocation rule so that most of the abatement effort, if not all, is made by the other firms in the supply chain. This is also better for society as long as the emission targets are stringent enough and the reduction efforts by the other firms are cost effective for society. If the targets are not stringent or if the reduction efforts that exclude the carbon leader are too costly, then society would be better off with a carbon payer, as e.g. carbon offset projects sponsored by the carbon payer would result in a higher overall emission reduction.

The contribution of this paper is twofold. The first contribution is the modeling framework itself, combining carbon footprints with joint production in a supply chain, hence relaxing the usual (implicit) assumption that carbon footprints are uniquely linked to individual firms. The second contribution lies in our results, most of all the observation that double-counting of emissions is often necessary to reach carbon optimality. We believe that the framework proposed here can be used to analyze many other theoretical and practical questions about allocating carbon footprints in supply chains.

The rest of this paper is structured as follows. We discuss relevant literature on LCA and carbon footprinting in Section 2. We present our model and elaborate on first- and second-best solutions in Section 3. We analyze the role of double-counting in Section 4, and discuss the scenarios with a carbon leader in Section 5. We offer our conclusions in Section 6.

2. Background and Literature

This paper draws on and aims to contribute to two main bodies of literature: that on sustainable supply chains, and on life-cycle assessment (LCA) and carbon footprinting in supply chains. We first review general background and literature on sustainable supply chains in Section 2.1, then do the same for LCA and carbon footprinting in Section 2.2.

2.1 Sustainable supply chains

Within the field of operations management, the notion of "greening the supply chain" has been gaining popularity, as illustrated by the reviews in Corbett and Klassen (2006) and Kleindorfer et al. (2005). Kolk and Pinske (2005) provide a framework for various ways in which firms can address climate change, either focusing internally on their own supply chain or going beyond that, and either merely compensating for their emissions or aiming to innovate. Our framework expands on their distinction between innovation vs. compensation, by allowing combinations of the two rather than treating them as distinct, and by embedding them in an analytical model in the context of joint carbon production.

The extensive literature review (citing 190 articles) on sustainable supply chains by Seuring and Müller (2008) suggests that carbon footprinting specifically has hardly been studied in the supply chain field so far. Benjaafar et al. (2009) show how carbon footprint parameters can be added to various optimization models. Cholette and Venkat (2009) find that different supply chain configurations can make a major difference in carbon intensity of wine distribution. Hoen et al. (2011) consider carbon emissions in transportation and find that plausible regulations and carbon costs will have minimal effect on the efficiency with which the transportation network is managed, assuming that the physical network does not change. Cachon (2011) studies a stylized supply chain design problem and concludes that the optimal solution is robust to misspecifications in the carbon cost. Keskin and Plambeck (2011) focus on how to allocate emissions from a process among its co-products.

Earlier work by Corbett and DeCroix (2001) and Corbett et al. (2005) studied how contracts could provide incentives for both buyer and supplier to exert effort to reduce consumption of undesirable materials in the presence of joint production. Though not in the context of green supply chains, this focus on joint production is continued in Roels et al. (2010).

We contribute to the literature on sustainable supply chains by developing a framework for joint production (of GHG emissions) in far more general supply chains than the common one-buyer one-supplier context, and by comparing various strategies for greening the supply chain.

2.2 Life-Cycle Assessment and carbon footprinting

Here we provide a brief introduction to LCA and carbon footprinting and then review the literature that relates most directly to the questions of allocation and joint production.

LCA is a method for determining total environmental impacts caused by products and processes, from “cradle to grave”, or “cradle to cradle” in case of recycling (Reap et al., 2008). LCA consists of several phases: goal definition and scoping, emissions inventory analysis, impact assessment, and interpretation; see, for instance, the EPA’s “LCA 101” document[14]. While LCA covers all environmental impacts, much attention has recently focused specifically on climate change and hence on “carbon footprinting”, which can be thought of as LCA limited to greenhouse gas (GHG) emissions. Due to the emergence of various carbon trading schemes (such as the European Trading Scheme or ETS), carbon accounting standards were needed. The most well-known is the Greenhouse Gas Protocol, jointly developed by the World Resources Institute and the World Business Council for Sustainable Development (WRI and WBCSD 2004). Even for firms that are not subject to carbon trading or other GHG regulations, there is increasing pressure from organizations such as the Carbon Disclosure Project (www.cdproject.net) to disclose their emissions.

In trading schemes and certain regulatory regimes, avoiding double-counting of GHG emissions is crucial. For that reason, the GHG protocol defines several “scopes” for the emissions inventory. Scope 1 is direct GHG emissions, i.e. all emissions that result from combustion or chemical processes in the company’s facilities or facilities controlled by the company. Scope 2 is electricity indirect emissions, i.e. emissions that result from electricity purchased by the company. Scope 3 is other indirect emissions, i.e. emissions that result from the activities of the company but from sources not owned or controlled by the company. Examples of Scope 3 emissions include the carbon embedded in purchased materials, employee

[14]Life cycle assessment: principles and practice. EPA/600/R-06/060, Reston; available at http://www.epa.gov/nrmrl/lcaccess/pdfs/600r06060.pdf, last accessed January 16, 2011.

travel, use of the company's products, and others. The existing carbon footprinting reporting infrastructure and literature treat Scope 1 and 2 emissions very differently from Scope 3, so the decision by a firm to spin off its transportation division into a separate entity would have a substantial impact on its reported footprint. The modeling framework we propose here is invariant under such outsourcing, as it focuses on a firm's influence on each carbon footprint component rather than its organizational boundaries.[15]

On average, only 14% of an industry's emissions are Scope 1, and only 26% are Scope 1 and 2 combined (Matthews et al., 2008). Using economic input-output life-cycle assessment, Figure 1 in Huang et al. (2009, p. 8513) shows how total GHG emissions are divided between Scope 1, Scope 2, and several subcategories within Scope 3 (commuting, top 10 suppliers, and other upstream emissions), finding that for several sectors the top 10 suppliers account for 30-50% of the sector's total footprint. Walmart estimates that over 90% of its emissions come from its supply chain rather than its own operations (Birchall, 2010). Together with the increasing focus on greening entire supply chains, this leads to a growing interest in measuring supply-chain wide carbon footprints, either at the supply-chain level or at the level of a single unit of final product or service. Two separate standards are under development for GHG accounting in a supply-chain context: the Corporate Value Chain (Scope 3) Accounting and Reporting Standard, and the Product Accounting and Reporting Standard, respectively (WRI and WBCSD 2010a, 2010b). The Carbon Disclosure Project has a separate Supply Chain Program aimed at encouraging firms to measure and report their supply-chain wide emissions; in the 2011 report[16] 55 member companies collected information from 1000 suppliers towards this goal.

Determining accurate product carbon footprints brings with it severe data requirements. The LCA literature contains several proposals that seek to address this. Schmidt (2009) proposes simplifying this process by requiring each firm in the supply chain to calculate the emissions "backpack" of their intermediate product, so that any firm wishing to estimate its products' carbon footprint would only have to add its own (Scope 1 and 2) emissions to the emissions backpacks from its purchased materials, rather than having to do a full LCA. Schmidt and Schwegler (2008) similarly propose calculating a recursive ecological indicator (the cumulative eco-intensity) for each stage in the supply chain, and allocating a supplier's emissions to a customer based on the share of that supplier's output the customer accounts for, departing from the usual physical allocation approach used in LCA.

[15] We are grateful to Nesim Erkip for this observation.

[16] Carbon Disclosure Project Supply Chain Report 2011. Available at https://www.cdproject.net/en-US/Results/Pages/reports.aspx, last accessed January 30, 2011.

All emissions that fall within Scope 3 for one company will fall within Scope 1 or 2 for some other firm. For instance, Natura employees' travel causes Scope 3 emissions for Natura but Scope 1 emissions for the airline used. The same GHG emissions can fall within the Scope 3 emissions of multiple companies: for instance, the emissions of the manufacturer of the paper used in Natura's catalogues will count towards Scope 3 for both Natura and the catalogue printer. The following quote from the draft GHG Protocol for Corporate Value Chain (Scope 3) emissions (WRI and WBCSD 2010, pp. 75-76) illustrates the challenge: "If GHG reductions take on a monetary value or receive credit in a GHG reduction program, companies should avoid any double counting of reductions within scope 3. To avoid double counting, companies should specify exclusive ownership of reductions through contractual agreements." As this quote illustrates, double-counting may be considered appropriate as long as there is no direct link between emissions and payments. We argue that double-counting can be necessary even when there is such a link.

In the LCA and carbon footprinting literatures, various guidelines exist on how to allocate shared emissions, such as those of a ship, to the individual products carried on it (usually by weight, volume, or value). Allocating emissions when multiple parties share responsibility for those emissions is more challenging, as is clear from the quote above. Given that LCA is aimed at making product and process design decisions based on an accurate inventory of environmental impacts, it is natural that the LCA literature contains much discussion on avoiding double-counting of impacts; see for instance Lenzen (2008) for more on why double-counting is undesirable, particularly in the context of carbon offsetting.

More recently the LCA literature has started investigating how to reconcile allocating responsibility for impacts while avoiding double-counting. Lenzen et al. (2007) propose a scheme by which producers and consumers share responsibility for emissions in such a way that adding total emissions across all producers and consumers yields the correct economy-wide emissions. They do so building on Gallego and Lenzen (2005) who consider how responsibility can be shared across various agents in an economy. Our work shows that this focus on avoiding double-counting in allocating responsibility is misguided in the context of setting incentives. If the intention is purely to be able to aggregate emissions across subsystems in a manner that will lead to a correct estimate of system-wide emissions, then allocating responsibility is not necessary. Conversely, allocating responsibility is usually done in order to design regulatory mechanisms or incentive schemes; we argue that for such applications, whenever joint production is present, avoiding double-counting is not desirable.

3. Model

In this section we introduce our model in which we explicitly incorporate carbon emissions originating from different processes at individual firms that form a supply chain, as well as the efforts and costs associated with abating those emissions. We consider the realistic structure that the total footprint emanating from a certain process has multiple components that are possibly carried out at different firms, each of which can be affected by one or more firms if they exert costly effort. A key factor that differentiates the carbon emissions context from other joint production situations is the emergence of third-party verification mechanisms for carbon footprints, which fundamentally expands the set of practically conceivable allocation mechanisms relative to other contexts where no such verifiable information exists. For the rest, our model of joint production follows Battaglini (2006). We model the societal cost of the resulting carbon footprint as a cost p per unit of GHG emissions. With the term "societal cost" we refer to the socially accepted monetary equivalent of the environmental impact created by the emissions. Depending on the geographical location and context, the market price of carbon credits such as Certified Emission Reductions (CER) or the European Union Emissions Trading Scheme (ETS), or the average cost of accredited carbon offset projects could be considered as examples. This is in line with Bovenberg and Goulder (1996) who show that the optimal (carbon) tax is equal to the marginal damage (unless the tax is redistributed to households, in which case it can be much lower). How to determine p (and whether it is even a well-defined parameter) is a subject of debate, but outside the scope of our work. There is a wide range of estimates of p as shown by Tol (2005), whose meta-analysis of 28 estimates found a median of \$14 per ton CO2e and a mean of \$93 per ton. This includes Plambeck and Hope's (1996) estimate of \$21 per ton, with a 90% uncertainty range of \$10-48 per ton. Tol (2008) extends his meta-analysis to include 211 estimates. Weitzman (2009) uses probabilistic reasoning to argue that p can be arbitrarily large. As Pearce (2003) argues, it is better to include a parameter even in the presence of large uncertainty, than to ignore it altogether.

The societal cost p may be offset by one or more firms, or may not be offset at all (unless it is mandatory). We analyze and compare a number of approaches as to which allocation, abatement, and offset strategy is followed, in an attempt to shed light on the effectiveness of alternative carbon policies in supply chains.

We first introduce our notation. Throughout the paper, a vector is represented by a column matrix. Matrices are written in boldface, except for 0 and 1 which represent a matrix of zeros and ones of appropriate dimensions, respectively. Inequalities for matrices are componentwise. We use the superscript T to denote transpose. For functions, the terms "increasing" and "decreasing" are understood in the non-strict sense. We let $n \in \mathcal{N} = \{1, \ldots, N\}$ denote the firms and $i \in \mathcal{I} = \{1, \ldots, I\}$ denote the processes, which can be joint across multiple firms.

Assume that firm n can choose any of m_n alternative carbon abatement actions. Note that any action in that set $\{1, \ldots, m_n\}$ can influence any subset of the set of processes $\mathcal{I}$. The influence of any given action on process i can be explicit (changing transportation from truck to rail) or implicit (working with a transportation provider to use lighter packaging). The carbon abatement efforts that firm $n \in \mathcal{N}$ exerts associated with each action are given by $\mathbf{e}_n^{\mathrm{T}} = \{e_{n,1}, \ldots, e_{n,m_n}\}$, which are assumed to be non-negative and bounded, and we refer to the profit for firm $n \in \mathcal{N}$ as $V_n(\mathbf{e}_n)$, where we assume V_n to be differentiable and concave strictly decreasing (componentwise). With this last assumption we exclude any abatement efforts that are profitable and that the firm would implement regardless of any carbon considerations. If the firm's revenue does not depend on the abatement efforts, then $V_n(\mathbf{e}_n) = V_n - c_n(\mathbf{e}_n)$ where V_n is a constant and $c_n(\mathbf{e}_n)$ is the carbon abatement cost, which by assumption is convex and strictly increasing.

Let $\mathbf{e}^{\mathrm{T}} = \{\mathbf{e}_1^{\mathrm{T}}, \ldots, \mathbf{e}_N^{\mathrm{T}}\} \in [0, A]^M \subset \mathbb{R}^M$ be the effort vector, where $M = \sum_{n=1}^{N} m_n$ and A is a sufficiently large bound to guarantee interior solutions to the ensuing optimization problems. The resulting total footprint caused by process $i \in \mathcal{I}$ as a function of the collective effort $\mathbf{e}$ exerted by all firms is $f_i = f_i(\mathbf{e})$. We assume f_i to be differentiable and convex decreasing (componentwise). Let $\mathbf{f}^{\mathrm{T}} = \{f_1, \ldots, f_I\}$ be the footprint vector function that takes values in $\mathbb{R}^I$. We assume footprints to be non-negative, though this could be relaxed. We will refer to the vector $\mathbf{f}$ as the total footprint.

We now introduce a mapping between the firms and the processes (and hence the emissions) that they influence. Let $b_{n,i}$ be the indicator that equals one if and only if firm n influences the footprint of process i and zero otherwise. Formally, $b_{n,i} = 1 \iff \displaystyle\sum_{j=1}^{m_n} \frac{\partial f_i}{e_{n,j}} < 0$. Let $\mathbf{B}$ denote the $N \times I$ matrix of indicators $b_{n,i}$. For any process i, the sum $\sum_{n=1}^{N} b_{n,i}$ represents the number of firms that influence process i. Similarly, for any firm n, the sum $\sum_{i=1}^{I} b_{n,i}$ represents the number of processes that firm n influences. Firms that have no influence on any process, or conversely, processes that are not influenced by any firm are irrelevant for our analysis, and hence, with no loss of generality, we assume that each row and each column of $\mathbf{B}$ adds up to at least one, i.e., $\sum_{i=1}^{I} b_{n,i} \geq 1,\ \forall\, n \in \mathcal{N}$ and $\sum_{n=1}^{N} b_{n,i} \geq 1,\ \forall\, i \in \mathcal{I}$.

Consider the following simple example, with $I = 2$ processes and $N = 3$ firms. Assume process 1 is influenced by firms 1 and 2, and process 2 by all three firms. Then $\mathbf{B}^{\mathrm{T}} = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \end{pmatrix}$. We will refer back to this simple setting to illustrate several other definitions later. We say that the supply chain exhibits *joint carbon production* if $\mathbf{B}$ has at least one column that adds up to more than one, i.e, there exists $i^* \in \mathcal{I}$ such that $\sum_{n=1}^{N} b_{n,i^*} \geq 2$. We believe that most, if not all, supply chains exhibit joint carbon production; in the absence of joint carbon production, the footprint allocation problem becomes far simpler. Note that $\mathbf{B}$ is invariant under outsourcing decisions or other redrawing of firms' organizational boundaries, which is one way in which our modeling framework is more general than traditional contracting models.

In the context of product or supply-chain carbon footprinting, the total footprint $\mathbf{f}$ of the supply chain needs to be attributed (or allocated) to the firms in one way or another. This allocation is not trivial, as the 'share' of an individual firm in a given process (and hence its carbon emissions) is ambiguous due to the non-separability of the contributions to the joint processes. Let $\widehat{f}_n = \widehat{f}_n(\mathbf{f}\,)$ denote the rule that allocates emissions to firm n, with $\widehat{f}_n(0) = 0, \forall\, n \in \mathcal{N}$. Note that the allocated footprint depends indirectly on the effort vector $\mathbf{e}$ through the carbon footprint $\mathbf{f}$. When necessary, we write $\widehat{f}_n(\mathbf{e}\,)$ to make the dependency explicit. Finally, let $\widehat{\mathbf{f}}^{\,\mathrm{T}} = \{\widehat{f}_1, \ldots, \widehat{f}_N\}$ be the footprint allocation vector function, which takes non-negative values in $\mathbb{R}^N$. If the footprint allocation rule is linear, then $\widehat{\mathbf{f}} = \mathbf{A}\mathbf{f}$, where A is an $N \times I$ matrix. In our previous example, if the emissions of process 1 are fully allocated to firm 1, while the emissions of process 2 are allocated equally to firms 2 and 3, then $\mathbf{A}^{\mathrm{T}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0.5 & 0.5 \end{pmatrix}$.

We refer to the firm that, by choice or otherwise, imposes a footprint allocation rule on the whole supply chain as the *carbon leader*. This is a firm that has the necessary means, motivation, and power to do so.

Firms can purchase carbon offsets to offset any emissions allocated to them, at a price p per unit (usually expressed in tons of CO2e emissions). A footprint allocation rule $\widehat{\mathbf{f}}$ is said to satisfy *limited liability* if it does not require firms to use external sources (e.g., endowment) to pay for all carbon emissions allocated to it. Formally, $p\,\widehat{f}_n(\mathbf{f}) \leq V_n(0),\ \forall\, n \in \mathcal{N},\ \mathbf{f} \geq 0$, where $V_n(0)$ is the profit of firm n when it exerts no effort. Note that the total footprint allocated to all firms does not necessarily have to be exactly equal to the actual supply chain wide footprint, which yields the following definitions: A footprint allocation rule $\widehat{f}_n, n \in \mathcal{N}$, satisfies *footprint balance* if it allocates the total footprint exactly once, that is:

$$\text{footprint balance:} \sum_{n=1}^{N} \widehat{f}_n(\mathbf{f}\,) = \sum_{i=1}^{I} f_i, \quad \forall\, \mathbf{f} \geq 0. \tag{1}$$

In the event of a linear allocation rule $\widehat{\mathbf{f}} = \mathbf{Af}$, this corresponds to requiring that the sum over each column in $\mathbf{A}$ equals 1. A supply chain is *carbon-neutral* if it offsets all emissions associated with its processes and possibly more, which is formally expressed as:

$$\text{carbon neutrality: the supply chain offsets at least } p \sum_{i=1}^{I} f_i. \tag{2}$$

If each firm offsets all emissions that are allocated to it, and $\sum_{n=1}^{N} \widehat{f}_n(\mathbf{f}\,) \geq \sum_{i=1}^{I} f_i,\ \forall\, \mathbf{f} \geq 0$, then the supply chain is carbon-neutral.

If for some $\mathbf{f} \geq 0$ the supply chain offsets more than the sum of all emissions, then we say that there is *over-allocation* of footprint (colloquially known as "double-counting") since some emissions would be allocated to multiple firms. When this occurs, the supply chain could be referred to as carbon-negative, though throughout the paper we use the more general term "carbon neutral". If all firms fully offset the footprint allocated to them, then footprint balance implies carbon-neutrality, but not the converse. If firms do not fully offset their allocated emissions, footprint balance means that all emissions are allocated, but as they may not be offset, there is no guarantee of carbon neutrality.

How best to allocate the emissions among firms in the supply chain depends on the context, including whether or not the supply chain is obliged to offset emissions, and whether or not one firm in the supply chain can impose an allocation rule on the others. In the first-best and second-best scenarios below, we take the perspective of a social planner who seeks to maximize social welfare, which in our model consists of maximizing supply chain profits net of the societal costs associated with total supply chain emissions. In the first-best case, the social planner can optimize directly over all firms' effort levels, in the second-best case the social planner can only indirectly influence firms' effort levels through the emission allocation rule she imposes.

We then turn to two scenarios where there is no social planner but which are closer to situations encountered in practice. In both cases, we assume that there is a firm in the supply chain that takes the initiative with respect to reducing the supply chain carbon footprint, i.e., there is a carbon leader. In the first scenario, a single firm – the carbon leader that acts as a "carbon payer" – is held accountable (by choice or by regulation) for all emissions in the supply chain. In the second scenario, there is a firm – again the carbon leader who now acts as a“carbon enforcer” – that can impose emission reduction targets for all firms in the supply chain, while offsetting is not required. For each of these scenarios we determine the implications of various allocation schemes for supply-chain profits and emissions.

3.1 First-best Solution

The centralized solution or *first-best* represents the socially optimal effort and offset levels that maximize the value of the centralized supply chain net of the societal costs associated with the total emissions. It is obtained from solving the following concave problem:

$$z^C = \max_{\mathbf{e}_1,\ldots,\mathbf{e}_N} \quad \sum_{n=1}^{N} V_n(\mathbf{e}_n) - p \sum_{i=1}^{I} f_i(\mathbf{e}). \tag{3}$$

Let $\mathbf{e}^*$ denote the carbon-optimal effort levels that solve (3) which for simplicity we assume are unique. Let $\mathbf{f}^*$ denote the socially optimal emission levels, i.e., $f_i^* = f_i(\mathbf{e}^*),\ \forall\, i \in \mathcal{I}$. Any regulation or allocation scheme under which firms choose effort levels $\mathbf{e}^*$ and the total supply chain transfers $p \sum_{i=1}^{I} f_i(\mathbf{e}^*)$ to society to offset the remaining emissions is socially optimal.

Going back to Table 1, we say that the supply chain is *carbon-optimal* if all firms exert the socially optimal abatement effort levels as defined in (3). Note that a supply chain can be carbon-optimal but not carbon-neutral, e.g., if the firms exert the socially optimal effort but do not offset the remaining emissions. The converse is also true. That is, the supply chain can be carbon-neutral but not carbon-optimal, e.g., if all emissions are offset but no (or suboptimal) abatement efforts are exerted to reduce those emissions.

3.2 Second-best Solution

In the second-best scenario, the regulator cannot control firms' effort levels directly, only indirectly by setting appropriate incentives and regulation, in this case deciding a footprint allocation rule and requiring that all firms offset all emissions allocated to them. For any given allocation rule $\widehat{\mathbf{f}}$, we call D-offset the decentralized solution in which all firms offset the emissions allocated to them. This is given by the following Nash equilibrium:

$$\text{incentive-compatibility: } \mathbf{e}_n \in \arg\max \big\{ V_n(\mathbf{e}_n) - p\, \widehat{f}_n(\mathbf{f}(\mathbf{e})) \big\}, \quad \forall\, n \in \mathcal{N}. \tag{4}$$

Consistent with Holmstrom (1982), we define the second-best solution as the allocation rule $\widehat{\mathbf{f}}$ that induces effort and offset levels that maximize the value of the decentralized supply chain net of the societal costs of emissions, subject to footprint balance. This solution is obtained from solving

$$z^D = \max_{\widehat{f}_1,\dots,\widehat{f}_N} \quad \sum_{n=1}^{N} V_n(\mathbf{e}_n) - p\sum_{i=1}^{I} f_i(\mathbf{e}) \qquad \text{subject to (1) and (4).} \tag{5}$$

We assume that $z^D \geq 0$. Note that in the absence of joint carbon production, allocating each footprint component to the single firm that causes it and requiring each firm to offset those emissions will yield the carbon-optimal effort levels. When there is joint production, the second-best solution is carbon-neutral but in general not carbon-optimal.

In general, there is an optimality gap between the first and second-best solution, i.e., $z^C \geq z^D$. However, the following result due to Battaglini (2006) shows that if there are "enough" processes, then there exists an allocation rule (albeit somewhat contrived) such that the inequality holds as an equality. In other words, it is possible to achieve first-best in a decentralized supply chain where firms offset the emissions allocated to them. Moreover, this can be done with an allocation rule that satisfies footprint balance and limited liability.

Proposition 1 (Battaglini 2006, Theorem 1, p. 149)**.** *Let $1 \leq m_n \leq I,\ \forall n \in \mathcal{N}$. An allocation rule $\widehat{\mathbf{f}}$ that satisfies footprint balance, limited liability and achieves first-best generally exists if and only if $\sum_{n=1}^{N} m_n \big/ (N-1) < I$.*

Corollary 2 (Holmstrom 1982, Theorem 1, p. 326)**.** *If there is a single process, then there does not exist an allocation rule $\widehat{\mathbf{f}}$ that satisfies footprint balance, limited liability and achieves first-best.*

The Battaglini result relies on being able to distinguish firms that may have deviated from the first-best effort levels from those which definitely complied, and then only penalizing the former group. To understand the result, let $\mathbf{e}^*$ and $\mathbf{f}^*$ be the efforts and footprints in the first-best solution respectively. Let $Y_n := \left\{\mathbf{f} \in \mathbb{R}^I \,\middle|\, \exists\, \mathbf{e}_n \in [0, A]^{m_n} \text{ s.t. } f_i = f_i(\mathbf{e}_n, \mathbf{e}^*_{-n}),\ \forall\, i \in \mathcal{I}\right\}$ be the carbon emission levels that can be achieved if all firms except n exert the first-best effort, and firm n exerts any level of effort, and let the intersection $Y := \bigcap_{n\in\mathcal{N}} Y_n$, the set of emissions levels that can be achieved with a unilateral deviation from first-best by any one firm. For a given footprint $\mathbf{f}$, let $G(\mathbf{f}) := \left\{n \in \mathcal{N} \,\middle|\, \mathbf{f} \in Y_n\right\}$ be the subset of firms such that the supply chain can achieve $\mathbf{f}$ after a unilateral deviation from the first-best solution by any firm $n \in G(\mathbf{f})$. Note that if $Y = \{\mathbf{f}^*\}$, there is no single footprint that could be achieved irrespective of which firm n deviates unilaterally from first-best. In that case, if the observed emissions $\mathbf{f}$ are different from $\mathbf{f}^*$, then $G(\mathbf{f})$ contains the suspected firms that might have deviated from the first-best. In particular, if firm n deviates unilaterally, then $n \in G(\mathbf{f}) \subsetneq \mathcal{N}$. Hence, it would be possible to distinguish between firms that for sure did not deviate, $\mathcal{N} \backslash G(\mathbf{f})$, and those that might have, $G(\mathbf{f})$, and one can penalize only the latter. For this to work, Y must be a singleton, i.e., it must not contain other footprint levels different from the first-best level $\mathbf{f}^*$. Otherwise, if the actual footprint was in Y but not equal to $\mathbf{f}^*$, it would be impossible to distinguish between firms that may have deviated and firms that definitely did not, as any firm could have deviated in that case. So let $\mathcal{F}$ be the set of carbon footprint vector functions that satisfy our definition above and let $\mathcal{F}_1$ be the subset containing those vector functions such that $|\,Y| = 1$. Battaglini shows that $\mathcal{F}_1$ is dense in $\mathcal{F}$ if and only if $\sum_{n=1}^{N} m_n/(N-1) < I$. Here "density" means that there might be footprint functions for which Y is not a singleton, but they can be approximated as closely as necessary by another function for which $|\,Y| = 1$.

To implement Battaglini's result in the carbon context, let $\alpha_n > 0,\ \forall\, n \in \mathcal{N}$ be such that $\sum_{n=1}^{N} \alpha_n = 1$. Adapting the suggested allocation rule to our context yields:

$$p\,\widehat{f}_n(\mathbf{f}) = \begin{cases} V_n & n \in G(\mathbf{f}) \text{ and } |\,G(\mathbf{f})| < N \\ V_n - \alpha_n\big(\sum_{n'=1}^{N} V_{n'} - p\sum_{i=1}^{I} f_i\big)\big/\big(\sum_{n' \in \mathcal{N}\backslash G(\mathbf{f})} \alpha_{n'}\big) & n \notin G(\mathbf{f}) \text{ and } |\,G(\mathbf{f})| < N \\ V_n - \alpha_n\big(\sum_{n'=1}^{N} V_{n'} - p\sum_{i=1}^{I} f_i\big) & \mathbf{f} \neq \mathbf{f}^* \text{ and } |\,G(\mathbf{f})| = N \\ V_n(\mathbf{e}_n^*) - \alpha_n\big(\sum_{n'=1}^{N} V_{n'}(\mathbf{e}_{n'}^*) - p\sum_{i=1}^{I} f_i\big) & \mathbf{f} = \mathbf{f}^* \text{ and } |\,G(\mathbf{f})| = N, \end{cases} \tag{6}$$

where $V_n = V_n(0),\ \forall n \in \mathcal{N}$. Note that the allocation rule (6) is non-differentiable, and more importantly, it is not monotone increasing. Therefore, an allocation rule of this form is unlikely to be implementable in practice. Note also that Battaglini's result requires the profit functions $V_n(\mathbf{e}_n), n \in \mathcal{N}$, to be known to the entity imposing the carbon allocation. This is essential to identify the socially optimal effort levels $\mathbf{e}^*$ and to construct the allocation rule (6).[17] In the next section we show that for any practical allocation rule (e.g., linear), double-counting plays a key role in achieving a carbon-optimal supply chain.

4. The Role of Double-Counting

To begin with, consider an allocation rule $\widehat{\mathbf{f}}$ which is differentiable with respect to the footprint $\mathbf{f}$ and let $J_{\widehat{\mathbf{f}}}(\mathbf{f})$ denote the Jacobian, i.e., $J_{\widehat{\mathbf{f}}}(\mathbf{f})_{n,i} = \dfrac{\partial \widehat{f}_n}{\partial f_i}$. In the case of a linear

[17]Legros and Matthews (1993) show that first-best can be achieved for a single process when the action space is discrete and the outputs are non-differentiable. Similarly, first-best is achievable if firms determine their efforts sequentially instead of simultaneously (see Strausz 1999).

rule $\widehat{f} = \mathbf{A}\mathbf{f}$, then $J_{\widehat{\mathbf{f}}}(\mathbf{f})_{n,i} = a_{n,i}$, where $\mathbf{A} \equiv \{a_{n,i}\}_{(n,i)\in\mathcal{N}\times\mathcal{I}}$. We first state the following proposition, which is a key insight of this paper. (All proofs are provided in the Appendix.)

Proposition 3. *Consider a decentralized supply chain where firms offset emissions allocated according to the rule* $\widehat{\mathbf{f}}$ *that is differentiable and increasing, i.e.,* $J_{\widehat{\mathbf{f}}}(\mathbf{f}) \geq 0$. *If there is joint carbon production, then over-allocating is necessary to attain a carbon-neutral and carbon-optimal solution.*

This necessity to over-allocate emissions occurs because a carbon-neutral supply chain without double-counting must, by definition, be footprint balanced. That restriction, combined with the monotonicity of the allocation rule, prevents players from internalizing the full consequences of their abatement decisions, so they have insufficient incentive to exert effort. Note that the monotonicity of the allocation rule is crucial since otherwise the result by Battaglini (2006) would allow for a footprint balanced solution when the number of processes is large (Proposition 1). The statement of Proposition 3 also requires the allocation rule to be differentiable. This is satisfied by the linear rules used in practice and simplifies the proof, but in general it is not a necessary condition for the over-allocation result to hold. The next proposition considers the case $0 \leq J_{\widehat{\mathbf{f}}}(\mathbf{f}) \leq 1$, i.e., no single firm is allocated more than 100% of the footprint from any process i. (This does not preclude that footprint being allocated to multiple firms.)

Proposition 4. *Consider a decentralized supply chain where firms offset emissions allocated to them according to the differentiable rule* $\widehat{\mathbf{f}}$ *such that* $0 \leq J_{\widehat{\mathbf{f}}}(\mathbf{f}) \leq 1$. *Then,* $J_{\widehat{\mathbf{f}}}(\mathbf{f}) \geq \mathbf{B}$ *is necessary to attain a carbon-optimal solution. If* $\widehat{\mathbf{f}}$ *is convex, then* $J_{\widehat{\mathbf{f}}}(\mathbf{f}) \geq \mathbf{B}$ *is also sufficient.*

Note that the condition $J_{\widehat{\mathbf{f}}}(\mathbf{f}) \geq \mathbf{B}$ (more on which below) is sufficient for carbon neutrality, but not necessary. In practice, allocation rules will usually be linear: if the total footprint to be allocated is doubled, the footprint allocated to each firm will also be doubled. Corollary 5 applies Proposition 4 to the case of linear allocation rules, using the linear counterpart $0 \leq \mathbf{A} \leq 1$ to the condition $0 \leq J_{\widehat{\mathbf{f}}}(\mathbf{f}) \leq 1$ in Proposition 4.

Corollary 5. *Let the allocation rule* $\widehat{\mathbf{f}}$ *be linear, i.e.,* $\widehat{\mathbf{f}} = \mathbf{A}\,\mathbf{f}$, *where* $\mathbf{A}$ *is a* $N \times I$ *matrix such that* $0 \leq \mathbf{A} \leq 1$. *Then,* $\mathbf{A} \geq \mathbf{B}$ *is necessary and sufficient to attain carbon optimality in a decentralized supply chain where firms offset the emissions allocated to them.*

Corollary 5 implies that a simple way to define a linear allocation rule that will lead to carbon-optimality is to use the $\mathbf{B}$ matrix (which captures which firms influence which processes) as the allocation matrix $\mathbf{A}$. In the simple example we used earlier, this means setting $\mathbf{A}^\mathrm{T} = \mathbf{B}^\mathrm{T} = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \end{pmatrix}$ is sufficient to achieve carbon optimality. Joint production immediately leads to over-allocation. In this example, the footprint of process 1 is allocated twice (to firms 1 and 2), while the footprint of process 2 is allocated three times (to firms 1, 2 and 3).

Next we consider allocation rules that depend only on the sum of the footprints $\sum_{i=1}^{I} f_i$, not on the full footprint vector $\mathbf{f}$.

Proposition 6. *Let $\widehat{\mathbf{f}}$ be a differentiable allocation rule. If $\widehat{\mathbf{f}}$ only depends on the total sum of the footprints, i.e., $\widehat{f}_n(\mathbf{f}) = \widehat{h}_n\big(\sum_{i=1}^{I} f_i\big)$, for some scalar function $\widehat{h}_n$, $\forall\, n \in \mathcal{N}$, then $J_{\widehat{\mathbf{f}}}(\mathbf{f}) = 1$ is necessary and sufficient to attain carbon optimality in a decentralized supply chain where firms offset the emissions allocated to them. In other words, N-counting is necessary and sufficient to attain carbon optimality in a decentralized supply chain where firms offset the emissions allocated to them.*

Proposition 6 implies that, when the footprint allocated to an individual firm can only depend on the sum of the footprints $\sum_{i=1}^{I} f_i$, then that total footprint must be allocated in full to each firm, i.e., $\mathbf{A} = 1$, a very extreme form of over-allocation. Next, for completeness, we recall the original result in Holmstrom (1982) for the single-process case.

Corollary 7 (Holmstrom 1982, Equation (6), p. 326)**.** *If there is a single process, the allocation rule is differentiable, and firms offset the emissions allocated to them, then $\widehat{f}_n(f) = f$, $\forall\, n \in \mathcal{N}$, is necessary and sufficient to attain a decentralized carbon-optimal solution.*

This last corollary has an important implication as it shows that a decentralized supply chain with an allocation rule that only depends on the aggregate footprint $\sum_{i=1}^{I} f_i$ can only be carbon-optimal if it allocates that entire aggregate footprint to each one of the N firms in the supply chain. This is an extreme case of double-counting which is not necessary if the footprint can be disaggregated by process and each firm is only allocated the footprint of the processes it can affect. In other words, decomposing the supply chain into distinct processes allows more fine-grained allocation of footprint, which reduces the amount of double-counting that is necessary to attain carbon optimality. Mathematically, this follows from the fact that setting $\mathbf{A} \geq \mathbf{B}$ is weaker than having to set $\mathbf{A} = 1$.

5. Decentralized Supply Chains with a Carbon Leader

So far we have focused on scenarios with a social planner and, in some cases, allocation rules that are impractical or implausible. This served to highlight that any supply chain with a carbon footprint allocation rule that avoids double-counting will not be carbon-optimal. We now turn to more practical scenarios, where a single firm in the supply chain acts as the *carbon leader*, i.e., that firm decides the footprint allocation rule $\widehat{\mathbf{f}}$. For convenience, we use $n = N$ to denote the carbon leader when it exists.

5.1 Carbon Payer's Problem

A supply chain has a *carbon payer* if there is a single firm that bears the carbon offsetting cost of the entire supply chain, voluntarily (as in the case of Natura Cosméticos) or imposed by a regulator. A supply chain with a carbon payer is carbon-neutral.

Consider a supply chain with a carbon leader who also acts as the carbon payer. Let F_i be the footprint of process $i \in \mathcal{I}$ if no abatement effort is made by any of the firms, i.e., $F_i = f_i(0)$, and let $\mathbf{F}^{\mathrm{T}} = \{F_1, \ldots, F_I\}$. Here the carbon leader decides an allocation rule $\widehat{\mathbf{f}}$ in order to incentivize the other firms to exert effort by sharing the offset savings. The share of savings that the carbon leader allocates to firm n is $\widehat{f}_n(\mathbf{F}) - \widehat{f}_n(\mathbf{f})$, so the carbon leader pays $p\left(\widehat{f}_n(\mathbf{F}) - \widehat{f}_n(\mathbf{f})\right)$ to firm n in exchange for firm n choosing the right effort level. The total savings will not be allocated more than once as that would never be in the carbon payer's interest, which means that the allocation rule must be footprint-balanced.[18] Clearly, no sharing is a special case where $\widehat{f}_N(\mathbf{f}) = \sum_{i=1}^{I} f_i$, $\widehat{f}_n(\mathbf{f}) = 0$, $\forall\, n \neq N$, and the carbon leader is the only firm that exerts effort. However, by sharing some of the offset savings it might do better. Then, the carbon leader and the other firms play a Stackelberg game similar to (5) except for the objective function:[19]

$$\max_{\widehat{f}_1,\ldots,\widehat{f}_N} \quad V_N(\mathbf{e}_N) - p\sum_{i=1}^{I} F_i + p\left(\widehat{f}_N(\mathbf{F}) - \widehat{f}_N(\mathbf{f}(\mathbf{e}))\right) \qquad \text{subject to (1) and (4).} \tag{7}$$

[18] In principle, the allocation rule must satisfy $\sum_{n=1}^{N}\left(\widehat{f}_n(\mathbf{F}) - \widehat{f}_n(\mathbf{f})\right) = \sum_{i=1}^{I}(F_i - f_i)$, $\forall\, \mathbf{F}, \mathbf{f} \geq 0$. However, this is equivalent to the footprint balance condition (1) when $\widehat{f}_n(0) = 0$, $\forall\, n \in \mathcal{N}$.

[19] Note that $\mathbf{e}_n \in \arg\max\left\{V_n(\mathbf{e}_n) + p\left(\widehat{f}_n(\mathbf{F}) - \widehat{f}_n(\mathbf{f})\right)\right\}$, $\forall\, n \in \mathcal{N}$, is equivalent to (4).

5.2 Carbon Enforcer's Problem

A supply chain has a *carbon enforcer* if there is a firm in the chain that can impose an emissions reduction target on the other firms. The target may be set externally by a regulator or voluntarily by the carbon enforcer, as in the case of Walmart. Note that the carbon enforcer can induce the other firms to exert abatement efforts (e.g., by threatening to switch suppliers otherwise), but offsetting the resultant emissions is not required. Hence, the supply chain is not carbon-neutral. Formally, let $\overline{F}_i$ be the target emissions for process i, e.g., $\overline{F}_i = F_i(1-\beta),\ \forall\, i \in \mathcal{I}$, where β is a reduction target. We assume that the set $\big\{\mathbf{e} \in [0,A]^M \big|\ f_i(\mathbf{e}) = \overline{F}_i,\ \forall\, i \in \mathcal{I}\big\}$ is non-empty to ensure that the target is achievable. Let $\widehat{f}_n(\mathbf{f}),\ \forall\, n \in \mathcal{N}$, be the allocation rule in place. Then, in the presence of a carbon enforcer, the firms in the supply chain select their efforts by playing the following decentralized game, which we call D-target (in contrast with D-offset):

$$\mathbf{e}_n \in \arg\max \big\{ V_n(\mathbf{e}_n) \ \big|\ \widehat{f}_n(\mathbf{f}(\mathbf{e})) = \widehat{f}_n(\overline{\mathbf{F}}\,) \big\}, \quad \forall\, n \in \mathcal{N}. \tag{8}$$

One could think of replacing the emission constraints with the weaker condition $\widehat{f}_n(\mathbf{f}\,) \leq \widehat{f}_n(\overline{\mathbf{F}}\,),\ \forall\, n \in \mathcal{N}$. However, if there were an equilibrium with $\widehat{f}_n(\mathbf{f}\,) < \widehat{f}_n(\overline{\mathbf{F}}\,)$ for some firm n, firm n would be exerting more costly effort than needed to meet its emissions target, which is not possible by our earlier assumptions.

Let the carbon leader N also act as the carbon enforcer. Then the carbon enforcer's problem is as follows: the carbon enforcer has a pre-determined emissions reduction target β and selects an allocation rule $\widehat{f}_n(\mathbf{f}),\ \forall\, n \in \mathcal{N}$, and can force firms to stay within the allocated target. Each firm will then select its effort subject to this constraint. Hence, the carbon enforcer and the other firms play the following Stackelberg game:

$$\begin{aligned} \max_{\widehat{f}_1,\ldots,\widehat{f}_N} \quad & V_N(\mathbf{e}_N) \\ & \text{subject to (1) and (8).} \end{aligned} \tag{9}$$

In this case, society bears the cost of any remaining emissions. Unless additional conditions are imposed on the allocation rule, any rational carbon enforcer would choose to allocate all the emissions to the other players in order to exert no effort himself.

5.3 Analysis and Discussion of Carbon Leader Scenarios

Given that the carbon enforcer can avoid exerting any costly effort, it is the preferred strategy for a carbon leader in our limited context, as long as it has the power to impose carbon targets on the other firms in the supply chain. This could lead the carbon enforcer to delegate carbon reductions to others that it could achieve itself at lower cost. Such diversion of effort would not be desirable from a social perspective, as is shown in the following proposition.

Proposition 8. *Let $\mathbf{F}^P$ be the optimal carbon emissions in the carbon payer's problem. Let z^P denote the social value of the supply chain in the carbon payers's problem, and $z^E(\mathbf{F})$ the social value in the carbon enforcer's problem for some emissions target $\mathbf{F}$. If the effort in the carbon enforcer's problem $\mathbf{e}^E$ is such that $\mathbf{f}(\mathbf{e}^E) = \mathbf{F}^P$, then*

$$z^C \geq z^D \geq z^P \geq z^E(\mathbf{F}^P). \tag{10}$$

Proposition 8 shows that, if the carbon payer and enforcer both end up with the same footprint $\mathbf{F}^P$, then both are worse than the second-best solution – i.e., when an external regulator sets the allocation rule – but it is still better for society to have a carbon payer who reallocates offset costs to other firms than a carbon enforcer who achieves the same aggregate footprint but with a suboptimal distribution of efforts.

The rationale for inequality (10) is the fact that the carbon enforcer will exert less effort than the carbon payer, and therefore the other firms have to make up for it. Note that the condition $\mathbf{f}(\mathbf{e}^E) = \mathbf{F}^P$ is guaranteed if for instance the number of processes is no greater than the number firms, $N \geq I$, and the Jacobian of the allocation rule $\widehat{\mathbf{f}}^E$ in the carbon enforcer's problem has full range at $\mathbf{f}(\mathbf{e}^E)$. If the allocation rule is linear, $\widehat{\mathbf{f}}^E = \mathbf{A}\,\mathbf{f}$, then it suffices that $rank(\mathbf{A}) = I \leq N$. Note also that the condition $\mathbf{f}(\mathbf{e}^E) = \mathbf{F}^P$ is sufficient, but not necessary. Indeed, the inequality (10) also holds if for instance the effort in the carbon payer's problem $\mathbf{e}^P$ weakly Pareto-dominates $\mathbf{e}^E$ in the subgame (4), i.e., $V_n(\mathbf{e}_n^P) - p\widehat{f}_n^P(\mathbf{f}(\mathbf{e}^P)) \geq V_n(\mathbf{e}_n^E) - p\widehat{f}_n^P(\mathbf{f}(\mathbf{e}^E))$, $\forall n \in \mathcal{N}$, where $\widehat{\mathbf{f}}^P$ is the allocation rule chosen by the carbon payer.

Though Proposition 8 might imply a preference for a carbon payer over a carbon enforcer, it should be interpreted with care since it does not rule out the possibility that $z^E(\overline{\mathbf{F}}) > z^P$ for some target $\overline{\mathbf{F}} < \mathbf{F}^P$. Indeed, suppose there is a subset of actions $\mathcal{M}$ such that

$$-p\sum_{i\in\mathcal{I}} \frac{\partial f_i(\mathbf{e}^E)}{\partial e_m} \geq -\sum_{n\in\mathcal{N}} \frac{\partial V_n(\mathbf{e}_n^E)}{\partial e_m},\ \forall\, m \in \mathcal{M},$$

then a marginal tightening of the abatement efforts e_m, $m \in \mathcal{M}$, is socially cost efficient, so a marginal increase in the emission targets would increase the social value of the supply chain with a carbon enforcer. Hence, if the emissions target is stringent enough, then the social value of the carbon enforcer scenario $z^E(\overline{\mathbf{F}})$ could eventually be greater than the value z^P achieved with a carbon payer. Moreover, in practice a carbon enforcer is likely to also exert some effort, at least to have credibility as a leader, which could further make this approach more desirable for society.

6. Discussion and Conclusions

What do our findings mean for firms, governments and NGOs hoping to use voluntary approaches to reduce supply chain carbon footprints? If a supply chain has a firm that is powerful enough to enforce aggressive emissions reduction targets, it would be best to focus on programs that identify and support such firms to be carbon leader and enforcer. Major retailers such as Walmart and Tesco come to mind. Conversely, if no such candidate for the carbon enforcer role exists, encouraging an individual firm to step up as carbon payer is a better approach, as Natura has been doing. Although Natura does not use explicit contracts to encourage its vendors to reduce emissions, it does actively work with its vendors, as reducing emissions anywhere in the supply chain reduces the total cost of offsets that Natura needs to purchase. Empirical work could help identify more precisely the type of situation in which a carbon payer or enforcer is preferred, and how suboptimal either would be relative to the first-best and second-best outcomes.

A related implication is that, even in the absence of an optimal allocation rule (which would require double-counting), firms who do have an interest in overall supply chain efficiency should at a minimum include the full cost of all GHG emissions that they can influence when they decide where to focus their efforts. The fact that double-counting is unlikely to be implemented on any meaningful scale in practice should not preclude firms from identifying where their efforts may have the greatest effect. If the greatest return on firm 1's effort is on emissions currently allocated to firm 2, then firm 1 could explore mechanisms to share the costs and benefits associated with emissions reduction efforts with firm 2. Without at least allowing double-counting in a pro-forma fashion, many valuable opportunities for joint improvement will go unexploited. This is of course analogous to the double marginalization problem in price-setting which in turn is a manifestation of a fundamental problem of decentralized decision-making in general.

In the context of a carbon leader, this implies that the carbon leader should include double-counting in their footprint allocation methodology, even if they do not actually require firms to offset the emissions allocated to them. E.g., if Walmart conducts a supply chain carbon footprinting exercise and attempts to allocate the resulting footprint to each firm in the supply chain, and they do so without double-counting, then when any given firm in that supply chain conducts their internal capital allocation decision-making process, it will undervalue the total emissions reduction it can achieve.

To summarize, even though we fully realize that regulators and firms are unlikely to implement incentive mechanisms based on double-counting in practice, failing to allow for double-counting in conducting supply chain carbon footprinting efforts will misinform all firms about the best improvement opportunities available to the supply chain. Moreover, given the likelihood that any practical carbon price p will be well below the marginal damage (if implemented at all), allowing double-counting could be seen as at least a partial remedy against the otherwise inevitable underinvestment.

In this paper, we compared two approaches to reducing a supply chain's carbon footprint: offsetting emissions vs. investing in reducing emissions. To do so, we used a simple modeling framework that decomposes the total footprint into separate footprint components, each of which can be influenced by any combination of firms. Our modeling framework helps supply chains choose the "best" strategy for them, taking emissions and costs into account. For example, a seemingly environmentally responsible action of offsetting supply chain emissions by the most downstream firm could be improved both in terms of total costs and pre-offset emissions if suppliers are given incentives by the downstream firm to improve their own processes that also affect the downstream firm's emissions, where these incentives are financed by the avoided emissions offset costs. Our analysis revealed that under fairly general conditions over-allocation is necessary for the supply chain to be carbon-neutral and carbon-optimal. In other words, double-counting is necessary to provide each firm with the right incentive to reduce emissions, which is in contrast to the emphasis on avoiding double-counting in the carbon footprinting and LCA literatures. Furthermore, we show that the current practice in many supply chains, with a carbon payer or carbon enforcer, is inferior not only to the centralized solution, but also to the decentralized solution. Consequently, our results imply that policy makers need to encourage mechanisms by which entire supply chains jointly take responsibility for their emissions, rather than relying on a carbon leader emerging and taking the initiative in any given supply chain.

Finally, our modeling framework can also be applied to other contexts than carbon footprinting. For instance, the literature on quality and warranties deals with an analogous situation where (possibly joint) efforts by a buyer and a supplier can reduce external failures. Baiman et al. (2000, 2001), Balakrishnan and Radhakrishnan (2003) and Zhu et al. (2007) study one-buyer one-supplier systems where both parties' quality may be unobservable, and examine how different forms of warranty contracts affect their efforts to improve quality. Similarly, Chao et al. (2009) compare different forms of cost sharing on a single buyer and supplier's efforts to reduce costs associated with product recalls. In their 'no cost-sharing' scenario, the manufacturer internalizes all costs of recalls, even though the supplier may sometimes be at fault. This is analogous to our carbon payer scenario. Our modeling approach would help extend the one-buyer one-supplier setting that dominates in the quality and warranty literature to more general supply chains.

This paper aims to provide a first step towards dealing with the profound challenge of joint carbon production in supply chains. Several extensions are worth exploring in future work. First, we have assumed that the cost of emissions can be captured by a parameter p which is known and equal for firms and society. One could extend this analysis to allow for a market price of carbon p which could differ from (and presumably be lower than) the societal cost of emissions. Second, we have assumed that the abatement cost functions are common knowledge; future work could consider the case with private information about emissions reduction opportunities. Third, empirical work or field research could be used to better understand what types of emissions reductions opportunities are likely to be missed in practice due to the emphasis on avoiding double-counting. Fourth, one could build on our modeling framework and explore the use of cooperative game theory in allocating carbon footprints, also building on Shubik (1962) and subsequent work.

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A. Proofs

We first state the following lemma, which provides a condition for achieving first-best in a decentralized supply chain.

Lemma 9. *Consider a decentralized supply chain where firms offset emissions allocated to them according to the differentiable rule $\widehat{\mathbf{f}}$. A necessary condition for the supply chain to be carbon-optimal is*

$$\sum_{i\in\mathcal{I}} \frac{\partial f_i}{\partial e_{n,j}}\left(\frac{\partial \widehat{f}_n}{\partial f_i} - 1\right) = \sum_{i\in\mathcal{I}} \frac{\partial f_i}{\partial e_{n,j}}\left(\frac{\partial \widehat{f}_n}{\partial f_i} - b_{n,i}\right) = 0, \quad \forall\, n \in \mathcal{N}, j \in \{1,\ldots,m_n\}. \tag{11}$$

If $\widehat{\mathbf{f}}$ is increasing and convex, then (11) *is also sufficient.*

Proof. Consider the decentralized supply chain where firms play D-offset with the allocation rule $\widehat{\mathbf{f}}$. Suppose that the supply chain is carbon-optimal. Then, in equilibrium, the effort levels in the decentralized game must be the same as in the centralized solution. This means that the first-order conditions of Equations (3) and (4) yield the same effort levels. Differentiating both equations with respect to $e_{n,j}$ and equating the common term $\dfrac{\partial V_n}{\partial e_{n,j}}$ we obtain the necessary condition:

$$\sum_{i\in\mathcal{I}} \frac{\partial f_i}{\partial e_{n,j}} = \sum_{i\in\mathcal{I}} \frac{\partial \widehat{f}_n}{\partial f_i}\frac{\partial f_i}{\partial e_{n,j}}, \quad \forall\, n \in \mathcal{N}, j \in \{1,\ldots,m_n\}. \tag{12}$$

The equation above can be re-written as

$$\sum_{i\in\mathcal{I}} \frac{\partial f_i}{\partial e_{n,j}}\left(\frac{\partial \widehat{f}_n}{\partial f_i} - 1\right) = \sum_{i\in\mathcal{I}} \frac{\partial f_i}{\partial e_{n,j}}\left(\frac{\partial \widehat{f}_n}{\partial f_i} - b_{n,i}\right) = 0, \quad \forall\, n \in \mathcal{N}, j \in \{1,\ldots,m_n\}, \tag{13}$$

where the second equality follows from the definition of $b_{n,i}$ and the fact that $b_{n,i} = 0$ implies $\dfrac{\partial f_i}{\partial e_{n,j}} = 0\ \forall\, j \in \{1,\ldots,m_n\}$. That proves the necessary part. For sufficiency, note that if $\widehat{\mathbf{f}}$ is convex increasing, then the payoff functions in D-offset are concave in the effort levels (this follows from the composition of convex functions, see Bazaraa and Shetty 1979). Then, from

Theorem 1.2 in Fudenberg and Tirole (1991) there exists a pure-strategy Nash equilibrium for D-offset. Such effort levels must satisfy the first-order conditions of (4), which are

$$\frac{1}{p}\frac{\partial V_n}{\partial e_{n,j}} = \sum_{i\in\mathcal{I}} \frac{\partial \widehat{f}_n}{\partial f_i}\frac{\partial f_i}{\partial e_{n,j}} = \sum_{i\in\mathcal{I}} \frac{\partial f_i}{\partial e_{n,j}}, \quad \forall n \in \mathcal{N}, j \in \{1,\dots,m_n\}, \tag{14}$$

where the last equality follows from (12). Hence, the effort levels in the decentralized game also satisfy the first-order conditions of Equation (3), which means that the effort levels are socially optimal since the first-order conditions are sufficient in the centralized game, as we assumed uniqueness of the optimal effort levels. This completes the proof. □

A.1 Proof of Proposition 3

Proof. Consider a supply chain with joint carbon production. Hence, there exist a process $i^* \in \mathcal{I}$ and firms $n_1, n_2 \in \mathcal{N}$ such that $b_{n_1,i^*} = b_{n_2,i^*} = 1$. This implies that there exist actions $j_1 \in \{1,\dots,m_{n_1}\}$ and $j_2 \in \{1,\dots,m_{n_2}\}$ such that $\frac{\partial f_{i^*}}{\partial e_{n_1,j_1}} < 0$ and $\frac{\partial f_{i^*}}{\partial e_{n_2,j_2}} < 0$. Let $\widehat{\mathbf{f}}$ be differentiable and increasing such that the decentralized supply chain is carbon-neutral and carbon-optimal. Suppose that $\widehat{\mathbf{f}}$ does not allow for double-counting. Hence, $\widehat{\mathbf{f}}$ is footprint-balanced since otherwise the supply chain is not carbon-neutral. From Equation (1), we must have that

$$\sum_{n=1}^{N} \frac{\partial \widehat{f}_n}{\partial f_i} = 1, \quad \forall\, i \in \mathcal{I}, \tag{15}$$

which together with the fact that $\widehat{\mathbf{f}}$ is (componentwise) increasing implies that $\frac{\partial \widehat{f}_n}{\partial f_i} \leq 1$, $\forall n \in \mathcal{N}, i \in \mathcal{I}$. Then, from Equation (13) it follows that

$$\frac{\partial f_i}{\partial e_{n,j}}\left(\frac{\partial \widehat{f}_n}{\partial f_i} - 1\right) = 0,\ \forall i \in \mathcal{I}, n \in \mathcal{N}, j \in \{1,\dots,m_{n_i}\}. \tag{16}$$

In particular, for process i^* we must have $\frac{\partial \widehat{f}_{n_1}}{\partial f_{i^*}} = \frac{\partial \widehat{f}_{n_2}}{\partial f_{i^*}} = 1$, because $\frac{\partial f_{i^*}}{\partial e_{n_1,i^*}} < 0$ and $\frac{\partial f_{i^*}}{\partial e_{n_2,i^*}} < 0$ which implies that $\sum_{n=1}^{N} \frac{\partial \widehat{f}_n}{\partial f_{i^*}} > 1$. This is a contradiction with Equation (15). Therefore, the allocation rule $\widehat{\mathbf{f}}$ must induce double-counting. □

A.2 Proof of Proposition 4

Proof. Suppose that the decentralized supply chain can attain a carbon-optimal solution. Then, from Lemma 9, Equation (13) must hold. This, together with $J_{\widehat{\mathbf{f}}}(\mathbf{f}) \leq 1$, implies

Equation (16). We now show that $J_{\widehat{\mathbf{f}}}(\mathbf{f}) \geq \mathbf{B}$, and to do so we consider the two possible values $b_{n,i}$ can take. If $b_{n,i} = 1$, then there exists $j \in \{1, \ldots, m_n\}$ such that $\dfrac{\partial f_i}{\partial e_{n,j}} < 0$, which from (16) implies that $\dfrac{\partial \widehat{f}_n}{\partial f_i} = 1 = b_{n,i}$. If $b_{n,i} = 0$, then $\dfrac{\partial \widehat{f}_n}{\partial f_i} \geq b_{n,i}$ because by assumption $J_{\widehat{\mathbf{f}}}(\mathbf{f}) \geq 0$. Hence, $J_{\widehat{\mathbf{f}}}(\mathbf{f}) \geq \mathbf{B}$ is necessary. For the sufficient part, if $b_{n,i} = 1$, then $\dfrac{\partial \widehat{f}_n}{\partial f_i} \geq b_{n,i}$ together with $J_{\widehat{\mathbf{f}}}(\mathbf{f}) \leq 1$ implies that $\dfrac{\partial \widehat{f}_n}{\partial f_i} = 1$. On the other hand, if $b_{n,i} = 0$, then by definition $\dfrac{\partial f_i}{\partial e_{n,j}} = 0$, $\forall j \in \{1, \ldots, m_n\}$. Hence, Equation (16) holds, which implies Equation (13) and the result follows from Lemma 9. □

A.3 Proof of Proposition 6

Proof. Note that if $J_{\widehat{\mathbf{f}}}(\mathbf{f}) = 1$, then $\widehat{\mathbf{f}}$ is convex increasing. Hence, the sufficient part follows directly from Lemma 9. For the necessary part, note that in this case, Equation (11) can be written as

$$\left(\widehat{h}'_n - 1\right) \sum_{i \in \mathcal{I}} \frac{\partial f_i}{\partial e_{n,j}} = 0, \quad \forall\, n \in \mathcal{N}, j \in \{1, \ldots, m_n\}.$$

By assumption, for any process $i \in \mathcal{I}$ there must exist a firm $n \in \mathcal{N}$ and $j \in \{1, \ldots, m_n\}$ such that $\dfrac{\partial f_i}{\partial e_{n,j}} < 0$. So the inequality above implies that $\widehat{h}'_n = 1, \forall\, n \in \mathcal{N}$, which proves the result. □

A.4 Proof of Proposition 8

Proof. Let $\mathbf{e}^D$ and $\mathbf{f}^D$ be the effort and emissions induced by the second-best solution. Similarly, let $\widehat{\mathbf{f}}^P$ and $\mathbf{e}^P$ be the allocation rule and the effort in the solution to the carbon payer's problem. Also, let $\mathbf{F}^P = \mathbf{f}(\mathbf{e}^P)$. Then, $z^P = \sum_{n=1}^{N} \left(V_n(\mathbf{e}_n^P) + p\,(\widehat{f}_n^P(\mathbf{F}) - \widehat{f}_n^P(\mathbf{F}^P))\right) - p \sum_{i=1}^{I} F_i = \sum_{n=1}^{N} V_n(\mathbf{e}_n^P) - p\, \sum_{i=1}^{I} f_i(\mathbf{e}^P) \leq z^D$, where the middle equality follows from the fact that $\widehat{\mathbf{f}}^P$ is footprint-balanced and the last inequality follows from the fact that problems (5) and (7) have the same feasible region. Finally, for $z^P \geq z^E(\mathbf{F}^P)$, let $\widehat{\mathbf{f}}^E$ and $\mathbf{e}^E$ denote the optimal allocation rule and the equilibrium efforts in problem (9). Then, we have that:

$$
\begin{aligned}
z^P &= \sum_{n=1}^{N} V_n(\mathbf{e}_n^P) - p\sum_{i=1}^{I} F_i^P \\
&= \sum_{n=1}^{N} \big(V_n(\mathbf{e}_n^P) + p\,(\widehat{f}_n^P(\mathbf{F}) - \widehat{f}_n^P(\mathbf{F}^P))\big) - p\sum_{i=1}^{I} F_i \\
&\geq \sum_{n=1}^{N} \big(V_n(\mathbf{e}_n^E) + p\,(\widehat{f}_n^P(\mathbf{F}) - \widehat{f}_n^P(\mathbf{F}^P))\big) - p\sum_{i=1}^{I} F_i \\
&= \sum_{n=1}^{N} V_n(\mathbf{e}_n^E) - p\sum_{i=1}^{I} F_i^P \\
&= z^E(\mathbf{F}^P).
\end{aligned}
$$

The first equality is the definition of z^P and the second equality follows from the fact that $\widehat{\mathbf{f}}^P$ is footprint-balanced. The inequality is because the effort level $\mathbf{e}^P$ is a Nash equilibrium with the additional constraint $\mathbf{f}(\mathbf{e}) = \mathbf{F}^P$, and therefore $V_n(\mathbf{e}_n^P) \geq V_n(\mathbf{e}_n^E),\ \forall n \in \mathcal{N}$, since no firm can do better by a unilateral deviation. Finally, the last two equalities revert the first two, which completes the proof. $\square$