Title

Stoked Dynamos

Authors

Stone, Jennifer
Brummell, Nicholas
Byington, Ben
et al.

Publication Date

2010-09-01

Stoked Dynamos: Magnetic Feeding of Dynamos and Nondynamos

Jennifer M. Stone[1,2], Nicholas H. Brummell [3], Benjamin Byington [3]

ABSTRACT

In this project we address the question of whether a flow that is not a dynamo can be made to exhibit dynamo-like properties by feeding it with a small amount of magnetic field. This may be pertinent to the Solar dynamo and the processes that sustain it. We present a 3-D fully nonlinear magnetohydrodynamic simulation of the dynamo properties of a time-dependent ABC flow and discuss a method for leaking magnetic field into the computational domain. Our results suggest that sufficient magnetic feeding significantly boosts the magnetic energy of nondynamo flows and can maintain a magnetic field for long times.

1. INTRODUCTION

A dynamo is a hydromagnetic mechanism that converts kinetic energy into magnetic energy within the bulk of a conducting fluid (Moffatt 1978). It involves the amplification of a magnetic field from an initially weak state of magnetization and the subsequent saturation and maintenance of the field against the action of Ohmic dissipation. Dynamo processes have received a great deal of attention in astrophysics as an explanation for the origin of magnetic fields in the Universe and their persistence over cosmological timescales.

The amplification part of the dynamo process may be simplified as a kinematic problem in which the magnetic field is very weak such that the behavior of the flow is solely governed by the external forcing. The induction equation may then be solved as an eigenvalue problem using prescribed velocities. Since the induction equation is linear in the magnetic field, exponential solutions are obtained. Thus, flows giving rise to exponentially growing solutions are classified as kinematic dynamos, in that the magnetic energy does not decay to zero as a function of time.

[1]University of Wisconsin, Madison

[2]Center for Magnetic Self-Organization in Laboratory and Astrophysical Plasmas

[3]University of California, Santa Cruz

The growth of magnetic energy cannot continue unabated, as eventually the magnetic fluctuations become comparable to those in velocity and the resultant Lorentz force is able to back-react on the flow. This modifies the flow from its original state such that the problem becomes nonlinear and the magnetic field and velocity must be calculated self-consistently. A dynamic (nonlinear) dynamo describes the saturation and subsequent maintenance of the magnetic field against dissipation over long timescales.

Classifying nonlinear dynamo action is a nontrivial process since the velocities cannot be prescribed as in the kinematic regime. An alternative might be to classify dynamo action according to forcing functions, but this is challenging owing to the existence of multiple states (Childress et al. 1995). This was illustrated in a study of the dynamo properties of time-dependent ABC flows by Brummell, Cattaneo, & Tobias (1998, 2001; hereafter BCT98 and BCT01), in which it was shown that a kinematic dynamo is not necessarily also a dynamic (nonlinear) dynamo. Furthermore, it was suggested that MHD states might exist that are not dynamos, although finite observations might suggest otherwise. Such a scenario might be realized in stellar magnetism, in which cyclic activity could be produced by a dynamo or by an oscillating field, although the two mechanisms would probably be indistinguishable (Cattaneo et al. 1991).

These subtleties have been considered in the context of cyclic magnetic activity on the Sun. The question has been posed as to whether the Solar dynamo is truly a self-sustaining process, or relies on being continually fed by relic magnetic field from the interior (Gough, 2007). In the magnetically-fed scenario, upwelling flows at mid-latitudes drag magnetic field from the radiative interior such that it can penetrate the tachocline and so couple the interior to the convection zone (Gough & McIntyre 1998). The aim of this work is thus to ascertain whether a flow that is not a dynamo can be made to resemble a dynamo by feeding it with magnetic field. Although it is unlikely such a system could be distinguished from a true dynamo it may have impliciations for the evolution of stellar magnetic fields (Gough, 2007). This report is organized as follows: in §2 we provide details on the formulation of the problem, including a discussion of the flow properties and our method of magnetic feeding, in §3 we present preliminary results, in §4 we speculate about the physical mechanisms involved, and finally we summarize our findings in §5.

2. FORMULATION

Our aim is to investigate the effect of magnetic feeding on dynamo action in both the kinematic and dynamic regimes. We seek to introduce magnetic field into a periodic domain with 2π periodicity in all three directions that contains a fluid with constant electrical conductivity and viscosity. The evolution of this system is given by the equations of magnetohydrodynamics (MHD), which can be written in nondimensional form as:

$$(\partial_t - R_e^{-1}\nabla^2)\boldsymbol{U} + \boldsymbol{U}\cdot\boldsymbol{\nabla}\boldsymbol{U} = -\boldsymbol{\nabla}p + \boldsymbol{J}\times\boldsymbol{B} + \boldsymbol{F} \quad (1)$$

$$(\partial_t - R_m^{-1}\nabla^2)\boldsymbol{B} = \boldsymbol{\nabla}\times(\boldsymbol{U}\times\boldsymbol{B}) + \boldsymbol{F}_B \quad (2)$$

$$\boldsymbol{\nabla}\cdot\boldsymbol{B} = \boldsymbol{\nabla}\cdot\boldsymbol{U} = 0 \quad (3)$$

where $\boldsymbol{U}$ is the velocity, $\boldsymbol{B}$ is the magnetic field, p is the pressure, $\boldsymbol{J} = \boldsymbol{\nabla}\times\boldsymbol{B}$ is the electric current density, $\boldsymbol{F}$ and $\boldsymbol{F}_B$ are forcing functions, and R_e and R_m are the kinetic and magnetic Reynolds numbers, respectively. The magnetic forcing function, $\boldsymbol{F}_B$, is added as a source term in the induction equation to enable magnetic feeding, and is discussed further in §2.2. These equations are solved using a pseudospectral method (Byington 2010). The initial conditions are given by $\boldsymbol{U}(0) = \boldsymbol{U}_0(0)$ and we choose a weak magnetic perturbation in the $\hat{y}$ direction (containing no mean field) with an amplitude of 10^{-5}. All runs discussed in this paper have a resolution of 96^3 Fourier modes, chosen to match the maximum resolution of BCT01, and we set $R_e = R_m = 100$.

2.1. Flow Properties

In this work we consider the time-dependent ABC flow of BCT98 and BCT01. The original ABC flow is a 3-D steady, incompressible, periodic flow, and is maximally helical with chaotic streamlines (Arnold 1965; Beltrami 1889; Childress 1970). The flow is made time-dependent in order to alter its Lagrangian statistics and improve the propensity for dynamo action. The flow is given by:

$$\begin{aligned}\boldsymbol{U}_0(\boldsymbol{x},t) \;=\; & [\sin(\mathrm{z}+\epsilon\sin\Omega\mathrm{t}) + \cos(\mathrm{y}+\epsilon\sin\Omega\mathrm{t}),\\ & \sin(\mathrm{x}+\epsilon\sin\Omega\mathrm{t}) + \cos(\mathrm{z}+\epsilon\sin\Omega\mathrm{t}),\\ & \sin(\mathrm{y}+\epsilon\Omega\mathrm{t}) + \cos(\mathrm{x}+\epsilon\sin\Omega\mathrm{t})],\end{aligned} \quad (4)$$

where ϵ is the driving amplitude and Ω is the driving frequency. This corresponds to an ABC flow with its origin moved sinusoidally along the diagonal $x = y = z$. In the case $\epsilon = 0$ the original ABC flow is recovered.

The properties of this flow have been discussed at length in BCT98 and BCT01, so here we simply summarize those that are salient to this project. The finite-time Lyapunov exponents for this flow have a monotonic dependence on the driving amplitude, ϵ (BCT01). For the case $\epsilon = 1$, chaotic streamlines occupy almost all of the volume, so we set $\epsilon = 1$ for all the runs discussed in this paper, as was done in BCT98 and BCT01. The chaotic properties of the modified ABC flow are also sensitive to the driving frequency, Ω. The degree of chaoticity initially increases as Ω is increased, but then decreases as Ω becomes larger (BCT98, BCT01), as shown in Figure 1. In this work we investigate the effect of magnetic feeding for various values of Ω.

The flow can be maintained as a solution of the Navier-Stokes equation with a suitable choice of forcing function, $\boldsymbol{F}$, for which we use:

$$\boldsymbol{F}_0(\boldsymbol{x}, t) = (\partial_t - R_e^{-1}\nabla^2)\boldsymbol{U}_0(x, t). \tag{5}$$

This function drives the time-dependent ABC flow, $\boldsymbol{U}_0$, in the kinematic regime in the absence of magnetic effects.

2.2. Magnetic Feeding

We introduce the magnetic forcing function, $\boldsymbol{F}_B$, as a source term in the induction equation in order to leak additional magnetic field into the domain. This is similar to how the momentum equation is forced, in that we choose a forcing function so as to drive a particular magnetic field in the absence of induction:

$$\boldsymbol{F}_{B0}(\boldsymbol{x}, t) = (\partial_t - R_m^{-1}\nabla^2)\boldsymbol{B}_F(x, t), \tag{6}$$

where $\boldsymbol{B}_F$ is the magnetic field to be added. The constraints on this field are that it must be solenoidal and periodic, and it must be added in such a way that it provides zero net flux. We adopt the following choice for the forcing magnetic field:

$$\boldsymbol{B}_F(\boldsymbol{x}, t) = B_0 \,\mathrm{sin\omega t}\,\mathrm{sinkz}\,[\mathrm{sinky}, 0, 0], \tag{7}$$

where B_0 is the amplitude, ω is the frequency and k is the wavenumber. Appropriate choices for the parameters B_0, ω, and k are required so that the added field does not diffuse away too rapidly and so that it does not overwhelm the system, because in order to investigate dynamo action we require the fluid itself to be dominant in driving the system.

The diffusive properties of the forcing field are controlled by the wavenumber, k, such that low k implies slower diffusion. For all runs discussed in this work we set $k = 1$. The rate at which the field is added is controlled by ω, and the sinusoidal form of the time-dependent term allows the possibility of adding and subtracting field in a cyclic fashion. In this work we only report on the effects of varying the amplitude, B_0, so set $\omega = 1 \times 10^{-4}$ in all our runs, so that the forcing field is added more slowly than the rate at which the magnetic field would otherwise decay in the case of a nondynamo. Given the long period of this frequency and the run times presented here, our forcing is approximately steady.

3. RESULTS

We report on three case studies that are designed to investigate the effect of magnetic feeding in the various regimes of dynamo action. In §3.1 we choose a flow that is a marginal kinematic dynamo and add magnetic field as described in §2.2 and examine the magnetic energy. In §3.2 we feed a flow that is a kinematic dynamo but is not a dynamo in the dynamic regime, that is, a flow that is a nonlinear nondynamo. We discern whether the decay of magnetic energy for a such a flow can be arrested by magnetic feeding. In §3.3 we add magnetic field to a flow that is a dynamo in both the kinematic and dynamic regimes (i.e. a traditional dynamo) to see if additional effects can be uncovered.

The time-dependent ABC flow, given by Equation (4), with $\epsilon = 1$ and $\Omega = 1$ is a dynamo in both the kinematic and dynamic regimes (BCT98 and BCT01). However, a flow with $\epsilon = 1$ and $\Omega = 2.5$, is a kinematic dynamo, but in the dynamic regime it is modified by the Lorentz force such that the flow can no longer sustain the magnetic energy and hence is no longer a dynamo. We call such a flow a nonlinear nondynamo. The magnetic energies of these flows are plotted as a function of time in Figure 2. If these systems are evolved for longer times the magnetic energy of the $\Omega = 2.5$ continues to decay below 10^{-12} (BCT98, BCT01). This case eventually relaxes into a purely hydrodynamic state that is not the original time-dependent ABC flow. At this point the magnetic forces are again negligible, but this new flow is not a kinematic dynamo because the magnetic field is never regenerated (BCT98, BCT01). Flows having large values of Ω are marginal kinemtic dynamos in that their growth rates are very small (refer to bottom panel of Figure 1). In this work we regard a flow having $\Omega = 8.0$ as being a marginal kinematic dynamo. In the subsections that follow we discuss the effect of magnetic feeding on the above cases.

3.1. Magnetic Forcing of Marginal Kinematic Dynamos

It would be most interesting to see the effects of magnetic feeding on a flow that is not a kinematic dynamo (i.e. a flow with a negative growth rate). Extrapolation of the kinematic growth rate shown in the bottom panel of Figure 1 suggests that a time-dependent ABC flow with $\Omega > 5$ should have a negative growth rate. However, given the nonmonotonic behavior at low Ω, it is perhaps not surprising that this is found not to be the case. Ideally, we would like to find the critical value of Ω at which kinematic growth rates become negative, and then test the effects of magnetic feeding on either side of this value. For the time being however, we report on the case of a time-dependent flow with $\Omega = 8.0$, which does have a positive growth rate, albeit a very small one.

The magnetic energies for the marginal kinematic dynamo with $\epsilon = 1$ and $\Omega = 8.0$ are shown in Figure 3, with and without magnetic feeding. We have used a small magnetic forcing amplitude of $B_0 = 1 \times 10^{-3}$ which proves to be sufficient to boost the magnetic energy of the marginal kinematic dynamo by around four orders of magnitude. While this is a potentially significant result, we will only gain more insight upon magnetically feeding a true kinematic nondynamo.

3.2. Magnetic Forcing of Nonlinear Nondynamos

The magnetic energies for the nonlinear nondynamo case with $\epsilon = 1$ and $\Omega = 2.5$, subjected to various magnetic forcing amplitudes, are plotted as a function of time in Figure 4. It can be seen that as the amplitude of the magnetic forcing is increased the rate at which the magnetic energy of the $\Omega = 2.5$ nonlinear nondynamo decays becomes slower. In fact, for the $B_0 = 0.5$ case shown in blue in Figure 4 the magnetic energy is essentially sustained, albeit at a lower magnetic energy than the $\Omega = 1$ case that is plotted for comparison.

3.3. Magnetic Forcing of Dynamos

The magnetic energies for the dynamo case with $\epsilon = 1$ and $\Omega = 1$, subjected to various magnetic forcing amplitudes, are plotted in Figure 5. It appears that as the amplitude of the magnetic forcing is increased for $t < 80$ in the kinematic regime, the magnetic energy grows more quickly such that saturation is reached at progressively earlier times. This may be due to the structure of the magnetic eigenfunctions, which in the magnetically-fed cases are quite different from the original $\Omega = 1$ case. However, each case attains a similar growth rate for $t > 80$ until saturation. In the dynamic regime the magnetically forced flows have approximately similar magnetic energies to the original $\Omega = 1$ dynamo.

4. DISCUSSION

Our results suggest that sufficient magnetic feeding of a flow that is only a marginal kinematic dynamo can boost the growth rate of the magnetic field by several orders of magnitude. We have also shown that the exponential decay of magnetic energy of a nonlinear nondynamo can be halted by magnetic feeding of large enough amplitude. Finally, we showed that magnetic feeding has little to no effect on the properties of a traditional in the dynamic regime, and maybe only a small effect at early times in the kinematic regime due to the structure of the magnetic eigenfunctions.

In the case of magnetic feeding of a nonlinear nondynamo, we showed in Figure 4 that the linear addition of magnetic field is able to arrest the exponential decay of the magnetic energy. We propose that the added field must undergo some type of amplification, perhaps by stretching of the fieldlines by the background flow, or by the action of a Lorentz-type force due to the forcing field. The amount of stretching may be quantified by calculating the components of $\boldsymbol{B}_F \cdot \boldsymbol{\nabla U}$, and the action of a Lorentz force may be gauged by calculating $\boldsymbol{J}_F \times \boldsymbol{B}_F$. Our future work will include a spectral analysis of the magnetic field for both the magetically-fed and non-fed nonlinear nondynamo cases, which will enable assessment of the roles of a Lorentz force versus stretching in modifying the added magnetic field.

5. CONCLUSIONS

In this work we have addressed the question of whether a system that is not a dynamo can be made to resemble a dynamo by supplying it with magnetic field. This is motivated by speculation about the Solar dynamo process and whether its existence depends on it being refuelled with magnetic field from the interior (Gough, 2007). To that end we have presented a 3-D fully nonlinear magnetohydrodynamic dynamo simulation in which magnetic field is leaked into the domain. We have shown that linear addition of magnetic field can be sufficient to arrest the exponential decay of magnetic energy in systems that are not dynamos. We suggest that this is due to some type of amplification of the introduced field, although further work is required to quantify and understand such a mechanism. We also plan to investigate whether the signature of the original forcing field can be extracted from the ensuing properties of the systems discussed.

We are very grateful to Douglas Gough for originally suggesting this problem and for his input during the ISIMA program. JMS would like to thank Nic Brummell and Ben Byington for their guidance and encouragement, and also the ISIMA organizers and participants for an outstanding summer.

REFERENCES

Arnold, V. I. 1965, C. R. Acad. Sci. Paris, 261, 17

Beltrami, E. 1889, Opera Matematiche, 4, 304

Brummell, N. H., Cattaneo, F. & Tobias, S. M. 1998, Physics Letters A, 249,437 (BCT98)

Brummell, N. H., Cattaneo, F. & Tobias, S. M. 2001, Fluid Dynamics Research, 28, 237 (BCT01)

Byington, B. 2010, private communication

Cattaneo, F., Hughes, D. W., & Weiss, N. O. 1991, MNRAS, 253, 479

Childress, S. 1970, J. Math. Phys., 11, 3063

Childress, S. & Gilbert, A. D. 1995, Stretch, Twist, Fold: The Fast Dynamo, Springer, Berlin

Gough, D. O. & McIntyre, M. E. 1998, Nature, 394, 755

Gough, D. O. 2007, in The Solar Tachocline, ed. Hughes, D. W., Rosner, R., & Weiss, N. O., Cambridge University Press, Cambridge

Moffatt, H. K. 1978, Magnetic Field Generation in Electrically Conducting Fluids, Cambridge University Press, Cambridge

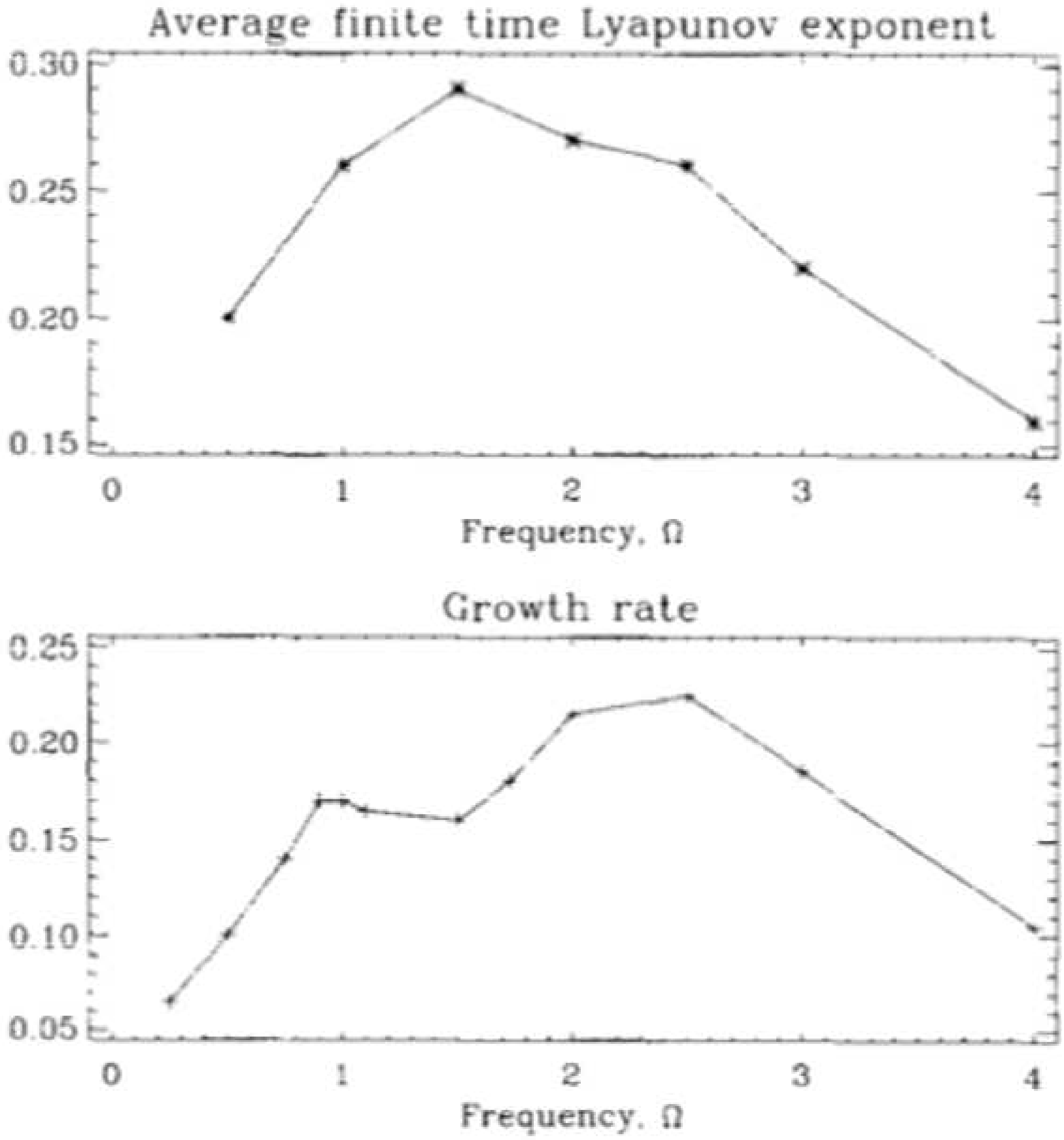

Fig. 1.— Variation of finite-time Lyapunov exponent (top panel) and kinematic dynamo growth rate (bottom panel) as a function of the driving frequency, Ω, for the time-dependent ABC flow given by Equation (4) (reproduced from BCT98).

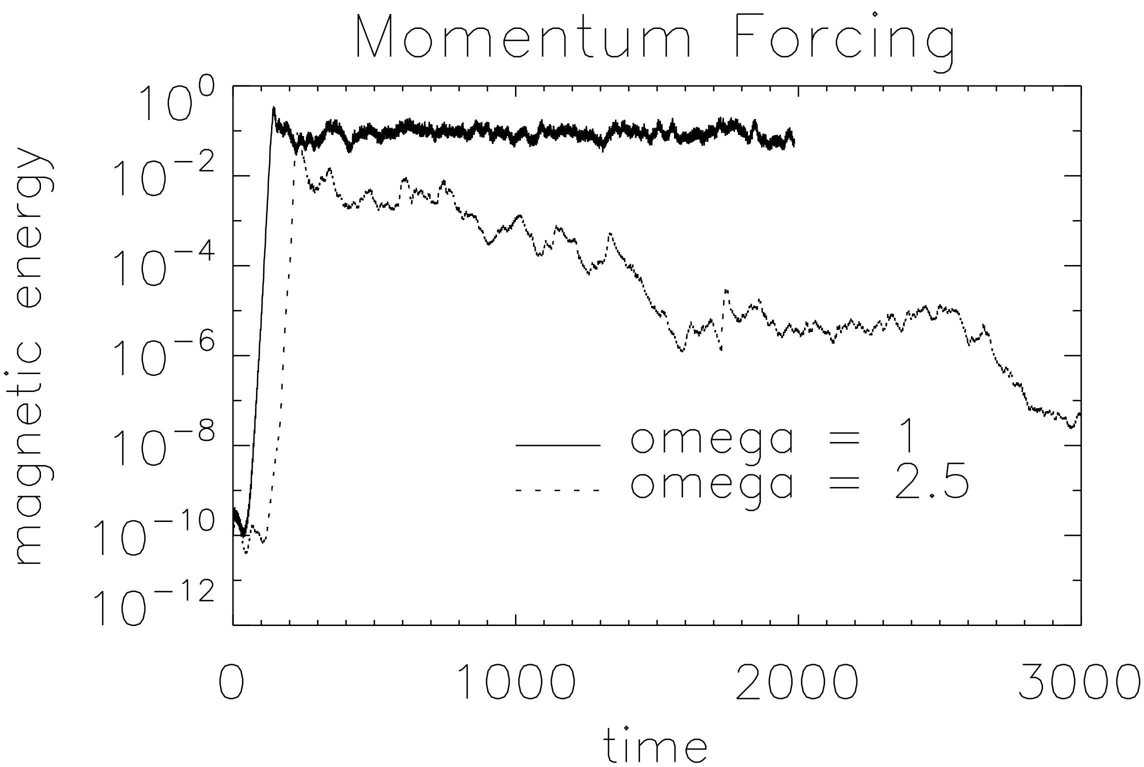

Fig. 2.— Magnetic energies of time-dependent ABC flows with $\epsilon = 1$ and $\Omega = 1$ (solid line) and $\Omega = 2.5$ (thin, dashed line). The $\Omega = 1$ case is a traditional dynamo in that the magnetic energy grows exponentially in the kinematic regime and then saturates and is sustained for long times. The $\Omega = 2.5$ case is a kinematic dynamo, but in the nonlinear regime the magnetic energy decays with time so it is ultimately not a dynamo. Note, the magnetic energy, $< B^2 >$, is given in velocity units.

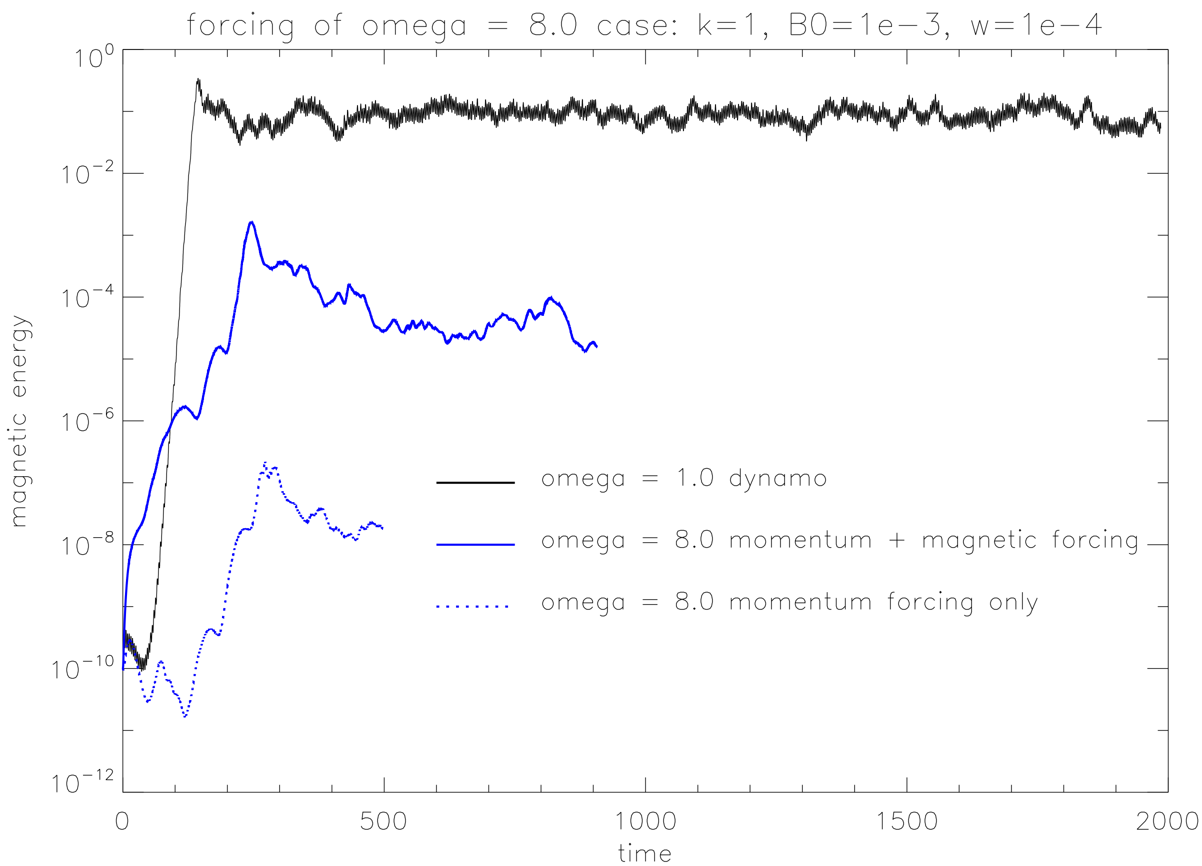

Fig. 3.— Magnetic energies of a marginal kinematic dynamo. The blue lines show the growth rates for the time-dependent ABC flow having $\epsilon = 1$ and $\Omega = 8$ with (solid blue line) and without (dotted blue line) magnetic feeding. The $\epsilon = 1$, $\Omega = 1$ true dynamo case is also presented (black line) for comparison.

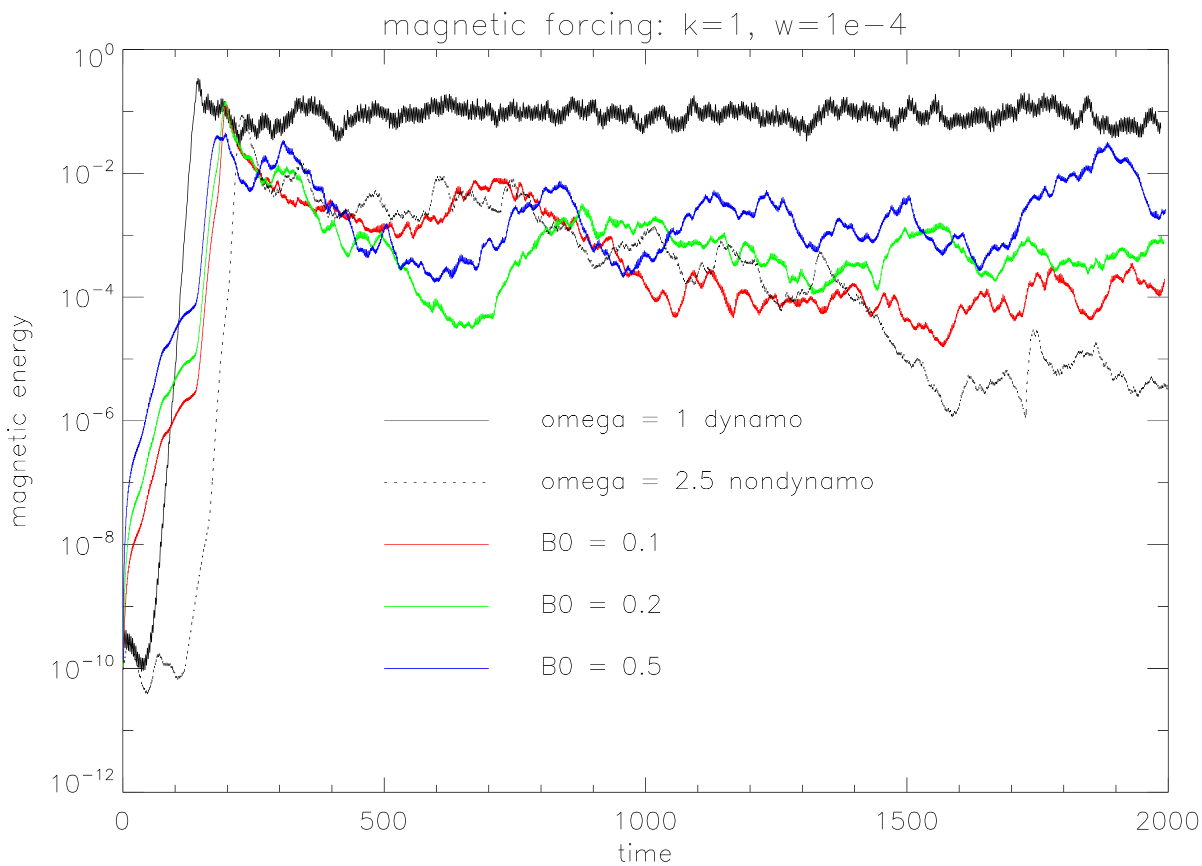

Fig. 4.— Magnetic energies of a nonlinear nondynamo. The dotted black line shows the magnetic energy of a time-dependent ABC flow with $\epsilon = 1$ and $\Omega = 2.5$ that was also shown in Figure 2. The colored lines show the cases in which magnetic fields of various amplitudes are introduced into the domain. The $\epsilon = 1$, $\Omega = 1$ true dynamo case is also presented (solid black line) for comparison.

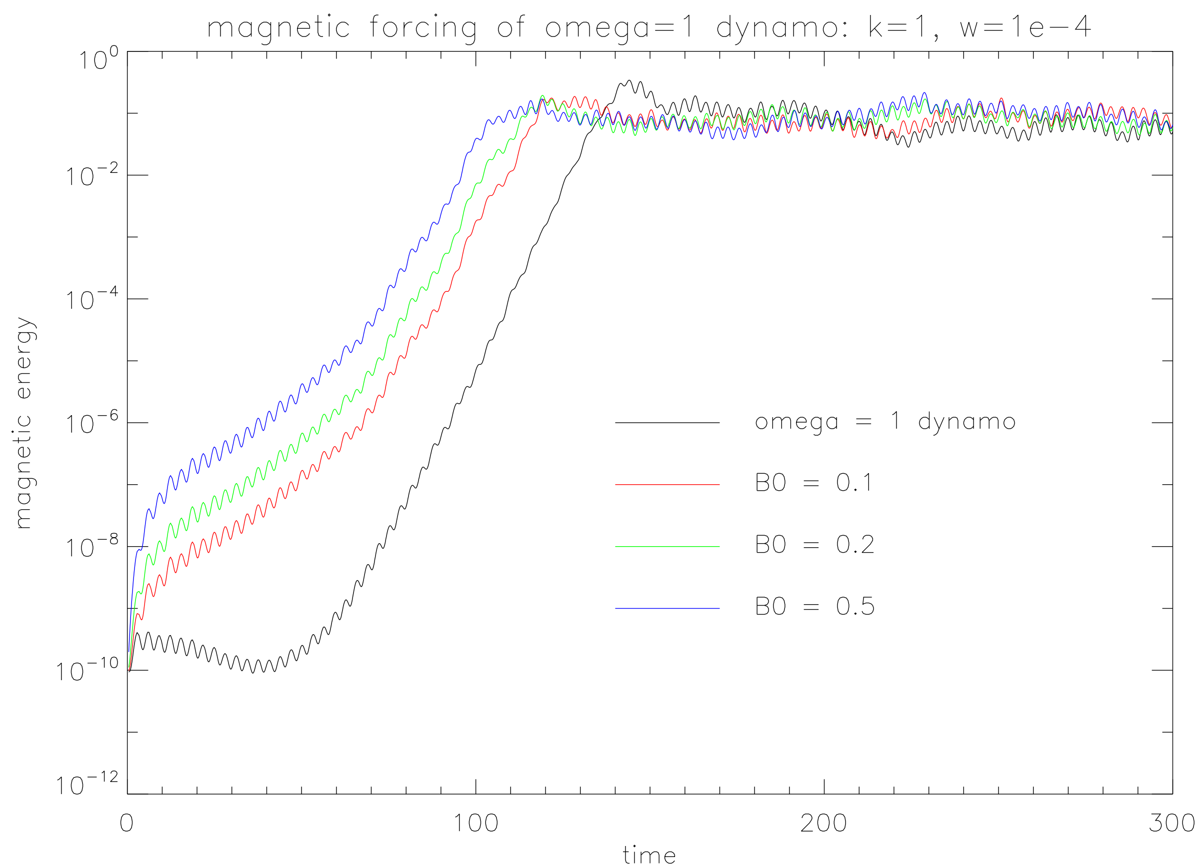

Fig. 5.— Magnetic energies of a traditional dynamo. The solid black line shows the original $\epsilon = 1$, $\Omega = 1$ dynamo and the colored lines represent magnetic forcings of various amplitudes.