UC Merced

Proceedings of the Annual Meeting of the Cognitive Science Society

Title
Deriving a conclusion from relational premises

Permalink
https://escholarship.org/uc/item/3qw477fw

Journal
Proceedings of the Annual Meeting of the Cognitive Science Society, 24(24)

ISSN
1069-7977

Authors
Van der Henst, Jean-Baptiste
Schaeken, Walter

Publication Date
2002

Peer reviewed

eScholarship.org Powered by the California Digital Library
University of California

Deriving a Conclusion From Relational Premises

Jean-Baptiste Van der Henst (jvanderhenst@hotmail.com)
Laboratory of Experimental Psychology, K.U. Leuven, Tiensestraat 102
Leuven, 3000 Belgium

Walter Schaeken (Walter.Schaeken@psy.kuleuven.ac.be)
Laboratory of Experimental Psychology, K.U. Leuven, Tiensestraat 102
Leuven, 3000 Belgium

Abstract

Literature on relational reasoning mainly focuses on the performance question. It is typically argued that problem difficulty relies on the number of "mental models" compatible with the problem. However, no study has ever investigated the wording of conclusions that participants formulate. In the present work, we analyse the relational terms that people use in drawing conclusions from spatial relation problems (A is to the left of B, B is to the left C, D is in front of A, E is in front C, What is the relation between D and E?). We show that the linguistic form of premises, the presentation format, the orientation of the question and the internal inspection of the mental model are important factors in determining the wording of conclusions. Our study shows that the type of conclusion produced provides a key to identifying the mental processes involved in solving these problems.

Introduction

Consider the following relational reasoning problem:

A is to the left of B
B is to the left of C
D is in front of A
E is in front of C
What is the relation between D and E?

How might this problem be solved? One possibility consists of building in one's mind an analogical representation, which exhibits the relation between D and E:

A	B	C
D		E

Another is to use inference rules and apply these rules to the propositional form of the premises to derive the required relation. During the last forty years, numerous studies have attempted to discriminate between these two approaches. In the sixties and seventies, they were respectively labelled the 'analogical' approach (DeSoto, London & Handel, 1965; Huttenlocher, 1968) and the 'linguistic' approach (Clark, 1969a; b) and related to representational processes. In the eighties and nineties, they were labelled the "mental model" (Johnson-Laird, 1983; Byrne & Johnson-Laid, 1989) and the "mental logic" approaches (Hagert, 1984; Rips, 1994; Braine & O'Brien; 1998) and related to inferential processes.

To compare these two approaches, researchers have relied on performance results (see Evans, Newstead & Byrne, 1993 for review). They either consider the correctness of the conclusion or the response time. However, no study has ever investigated the type of conclusions that people formulate. It is rather surprising that psychologists of reasoning have ignored this question since in another field of deductive reasoning, namely reasoning involving quantifiers like "all", "none", and "some", this issue was one of the first to be investigated under the guise of the 'atmosphere' effect (Woodworth & Sells, 1935). This effect, which has since been extensively explored (see Evans, Newstead and Byrne, 1993 for review) refers to the fact that universal premises (All A are B) prompt universal conclusions, particular premises (Some A are B) prompt particular conclusions, affirmative premises prompt affirmative conclusions and negative premises prompt negative conclusions.

The question we will address is that of the wording of conclusions in relational reasoning. In the above example, the answers "D is to the left of E" and "E is to the right of D" are both correct. But which of these two conclusions do people actually draw? In this article, we want to show that taking into account the wording of the conclusions can reveal several important mechanisms that occur in representing and in reasoning from spatial relational descriptions. We now present several factors that may influence the wording of conclusions.

The linguistic form of the premises

According to mental model theory, there are two stages in representing the premises. The first stage, which is compatible with a linguistic approach, consists of forming a propositional representation that is close to the surface form of the sentence. The second stage consists of using this propositional representation as a basis for constructing a mental model that is analogous to the situation described in the premises. Once the model is built, the linguistic details of the premises and their propositional representation tend to be forgotten (Mani & Johnson-Laird, 1982). The formulation of the

conclusion should then rely only on the mental model, which does not keep track of the comparative expression in the premises. Hence, the comparative used in the premises should not be used more often than its contrary in formulating the conclusion. 'Left-left' problems, whose first two premises contain the comparative 'left' (problems 1-2-9-10 in Table 1a) should not prompt more 'left' conclusions than 'right-right' problems (problems 7-8-15-16 in Table 1b).

On the other hand, according to the linguistic approach (Clark, 1969a; b; Hagert, 1984) the comparative used in the conclusion should be congruent with the comparative used in the premises. In the above example, the first two premises will be represented by two independent propositions: LEFT (A,B); LEFT (B,C). As a result, the predicates used in the inference rule will match those used in the premises: 'LEFT (X,Y) & LEFT (Y,Z) → LEFT (X,Z)'. Hence, the inferred relation between A and C will be stored with the predicate 'left': LEFT (A,C). Since the relation between D and E is identical to that between A and C, the D-E relation will be stored with the same predicate: LEFT (D,E). More generally, 'left-left' problems should prompt more 'left' conclusions than 'right-right' problems.

It also follows from the linguistic approach that the relevance of the premises may affect the wording of the conclusion. Consider Problem 11 from Table 1. The first premise is irrelevant (as for all problems in Table 1b) since it does not have to be taken into account to answer the question: The relation between D and E relies on the relation between A and C, which is explicitly given by the second premise. Hence, the comparative used in the conclusion is likely to be congruent with the comparative used in that premise. Given this assumption, Problems 11 and 12 (irrelevant premise with 'left', relevant premise with 'right') should prompt more 'right' conclusions than Problems 13 and 14 (irrelevant premise with 'right', relevant premise with 'left').

Table 1: The 16 spatial problems

Table 1a: One-model problems

Pb1	**Pb2**	**Pb3**	**Pb4**
A left B	A left B	A left B	A left B
B left C	B left C	C right B	C right B
D front A	E front C	D front A	E front C
E front C	D front A	E front C	D front A
Pb5	**Pb6**	**Pb7**	**Pb8**
B right A	B right A	B right A	B right A
B left C	B left C	C right B	C right B
D front A	E front C	D front A	E front C
E front C	D front A	E front C	D front A

Table 1b: Two-model problems (first premise always irrelevant)

Pb9	**Pb10**	**Pb11**	**Pb12**
A left B	A left B	A left B	A left B
A left C	A left C	C right A	C right A
D front A	E front C	D front A	E front C
E front C	D front A	E front C	D front A
Pb13	**Pb14**	**Pb15**	**Pb16**
B right A	B right A	B right A	B right A
A left C	A left C	C right A	C right A
D front A	E front C	D front A	E front C
E front C	D front A	E front C	D front A

Problem difficulty and presentation format

Most researchers currently agree that the difficulty of a relational reasoning problem is a function of the number of models it supports. For instance, the following problem,

A is to the left of B
A is to the left of C
D is in front of A
E is in front of C,
What is the relation between D and E?

is compatible with two models,

A	B	C	A	C	B
D		E		D	E

and is more difficult than a one-model problem since two models are harder to construct and store than a single one (Byrne & Johnson-Laird; 1989). It has also been shown that when the number of models increases, people are less likely to construct such models and are more prone to stay at the propositional level of representation (Mani & Johnson-Laird,1982). Mani and Johnson-Laird argued that the indeterminacy introduced by multiple-model problems disrupts the model construction process. They showed that people were more likely to recall linguistic details for indeterminate than for determinate descriptions. Hence, the premises may have a stronger influence over the wording of conclusions for two-model problems than for one-model problems: The comparative used in the conclusion should be more often congruent with the comparative used in the premises with two-model problems than with one-model problems.

In addition, the presentation format might have an effect on the type of representation involved (see Potts & Scholz, 1975; Ormrod; 1979; Schaeken & Johnson-Laird, 2000; Roberts, 2000) and, consequently, on the type of conclusion formulated. One can distinguish between two ways of presenting the premises. With <u>simultaneous</u> presentation, all premises are presented together

with the question and remain available when one solves the problem. Sequential presentation places more load on working memory: the premises are presented one after the other and disappear each time a new premise or the question occurs. It has been argued (Ormrod, 1979) that an analogical representation is more likely to occur with a sequential presentation. The reason is that in a linguistic representation, each relation – given in the premises or inferred from them – is stored separately whereas in a model representation, all premises are integrated within a single representational format. Thus, when working memory load increases it becomes harder to keep track of the premises and inferences separately and a mental model becomes a more efficient and concise mode of representation. Hence, we should observe fewer conclusions with a relational comparative congruent with that of the premises in the sequential presentation than in the simultaneous presentation.

Scanning the mental model

The wording of the conclusions might reveal how individuals scan the model they constructed. If people scan their model in a 'left-to-right' direction, they will be likely to make a 'left' conclusion since the first element they encounter, which is likely to be the first element mentioned in the conclusion, is on the left part of the model; alternatively; if people scan their model in a 'right-to-left' direction, they will be likely to make a 'right' conclusion. One might suppose that the direction of scanning is driven by left-to-right reading habits (Cicirelli, 1977), leading to 'left-to-right' inspections of mental models and to 'left' conclusions.

Another factor that might govern the direction of model-inspection is the question. The question given in the above examples ("What is the relation between D and E?") initially directs attention to the left side of the models since D is mentioned first in the question and is located on the models' left side. Consequently, such a question is likely to induce a 'left-to-right' inspection of the model and a 'left' conclusion. Inversely, the question "What is the relation between E and D?" is likely to induce a 'right' conclusion.

Finally, the order in which the items are inserted within the model might direct inspection of the model. If the premise containing the D item is provided before the premise containing the E item, D will be inserted before E in the model and the construction of the D-E line will proceed from left to right (granted that D is to the left of E as in all the problems of Table 1). Payne (1993) has shown that people keep track of the construction process. One can extend this approach and assume that keeping track of the construction process may induce people to scan their model in the direction of its construction.

Experiment

Before describing the method of the experiment let us first recall the predictions:

- The linguistic form of the premises should influence the wording of the conclusion according to a linguistic approach but not according to an analogical approach.
- Problems conveying an indeterminacy (i.e. two-model problems) could favor more the occurrence of linguistic processes than determinate problems (i.e. one-model problem).
- A sequential presentation could favor more the occurrence of analogical processes than a simultaneous presentation.
- 'Left-to-right' reading habits could prompt 'left-to-right' inspections of mental models.
- 'Left-to-right' questions could prompt 'left-to-right' inspections of mental models.
- 'Left-to-right' constructions of mental models could prompt 'Left-to-right' inspections of mental models.

Method

Participants The participants were 174 first-year psychology students from the University of Leuven.

Design Each participant received 16 relational reasoning problems (Table 1). Half of these problems were one-model problems (Table 1a) and the other half two-model problems (Table 1b). In half of the problems, the first two premises had the same relational term (four 'left-left' problems and four 'right-right' problems); in the remaining half, the first two premises had different relational terms (four 'left-right' problems and four 'right-left' problems). Moreover for half of the problems (Type-1 problems), the premise introducing the item located on the left (i.e. the item D) was given before the premise introducing the item located on the right (i.e. the item E; Problems 1-3-5-7-9-11-13-15 in Table 1). For the other half (Type-2 problems), this presentation order was reversed (Problems 2-4-6-8-10-12-14-16).

There were two between-participant manipulations: 2 types of presentation format × 2 types of question. First, participants received the premises and question in a simultaneous presentation format or in a sequential presentation format. Second, the first item mentioned in the question was either the item mentioned in the third premise (i.e. "what is the relation between D and E?" for Type-1 problems and "what is the relation between E and D?" for Type-2 problems) or the item mentioned in the fourth premise (i.e. "what is the relation between E and D?" for Type-1 problems or "what is the relation between D and E?" for Type-2 problems).

Procedure and Materials. Participants were tested in groups of 12 to 20 individuals. The instructions

and the problems were given in Dutch and were displayed on a screen via a data projector. Participants received 2 training problems and 16 randomly ordered test problems with contents relating to fruits and vegetables (see Table 1). In the simultaneous conditions, each problem was displayed for 50 seconds. In the sequential conditions, each premise and the question appeared for 10 seconds. Participants wrote their answers on a sheet of paper.

Results. Performance was in line with the relational reasoning literature since one-model problems were easier to solve than two-model problems (83% vs. 73%, Wilcoxon's $\underline{T} = 1787$, $\underline{n} = 131$, $\underline{p} < .00001$). We now turn to the analysis of the conclusions that people expressed. We discarded incorrect answers and 0.1% of correct but imprecise answers (i.e. 'D is next to E'). We will now discuss the relevant main effects and interactions.

First, participants had a clear preference for 'left' conclusions. Overall, there were 68.6% of 'left' conclusions and 31.4% of 'right' conclusions, suggesting a tendency for mental models to be scanned in a 'left-to-right' direction.

Second, the type of question influenced the wording of the conclusion. With 'left-to-right' questions (D-E?) there were 83.1% of 'left' conclusions (and consequently 16.9% of 'right' conclusions) whereas with 'right-to-left' questions (E-D?) there were only 54.2% of 'left' conclusions (Wilcoxon's $\underline{T} = 1082$, $\underline{n} = 135$, $\underline{p} < .00001$).

Third, the extent to which the question influenced the incidence of 'left' and 'right' conclusions depended on presentation format. Given the 'left-to-right' question, which prompted a 'left-to-right' inspection of the model, simultaneous presentation gave rise to 78.5% of 'left' conclusion while sequential presentation led to 88.5% of 'left' conclusions (Mann-Whitney $\underline{U}$'s $= 3029.5$, $\underline{n1} = 92$, $\underline{n2} = 82$, $\underline{p} < .01$). A similar, but non-significant trend was obtained for the 'right-to-left' question, which prompted a 'right-to-left' inspection of the model. Here, simultaneous presentation gave rise to 57% of 'left' conclusions, but sequential presentation resulted in 50.7% of 'left' conclusions. The directional scanning induced by the question is apparently enhanced by sequential presentation, in accordance with the notion that analogical processes are more likely under such conditions.

Fourth, the wording of the premises influenced the wording of the conclusions in the simultaneous condition since 'left-left' problems elicited more 'left' conclusions than 'right-right' problems. Whereas, 75.5 % of 'left' conclusions were observed for 'left-left' problems, there were 57.4% of 'left' conclusions for 'right-right' problems (Wilcoxon's $\underline{T} = 385.5$, $\underline{n} = 61$, $\underline{p} < .00005$). In contrast, in the sequential condition there was no influence of the linguistic form of the premises since the rate of 'left' conclusions was essentially the same in each type of problem (71.5% for 'left-left' problems, and 71.1 % for 'right-right' problems). This indicates that when the premises were not available, participants were not inclined to use the comparative introduced by the premises. The results suggest that participants tend to adopt a linguistic representation given simultaneous presentation and an analogical representation given sequential presentation.

Similarly, in the simultaneous presentation condition, the findings obtained for two-model problems, which all had an irrelevant first premise, indicated that participants were prone to formulating conclusions congruent with the relevant premise. When the relevant premise contained the comparative 'left' and the irrelevant one contained the comparative 'right', 77.9% of 'left' conclusions occurred; when the relevant premise contained the comparative 'right' and the irrelevant one contained the comparative 'left', 57.4% of 'left' conclusions occurred (Wilcoxon's $\underline{T} = 236$, $\underline{n} = 49$, $\underline{p} < .0001$). However, given sequential presentation, participants were not really inclined to formulate a conclusion congruent with the relevant premise. When the relevant premise contained the comparative 'left', 69.1% of conclusions were 'left' and when it contained the comparative 'right', the percentage of 'left' conclusions was 67.6.

Fifth**,** one might have expected that when the insertion of the last two items in the mental model proceeds from left to right (Type-1 problems), more 'left-to-right' inspections and 'left' conclusions would occur than when it proceeds from right to left (Type-2 problems). This was not the case and the results even show a tendency in the opposite direction. There were 66.6% of 'left' conclusions for Type-1 problems and 71% of 'left' conclusions for Type-2 problems.

Sixth, it could be argued that when the number of models increases participants should be more prone to relying on the linguistic form of the premises. However, the differences in 'left' and 'right' conclusions were almost identical in one and two-model problems. Indeed, 'left-left' one-model problems gave rise to 73.9% of 'left' conclusions and 'left-left' two-model problems gave rise 73.6% of left conclusions. Similarly, 'right-right' one-model problems gave rise to 62.6% of 'left', and 'right-right' two-model problems gave rise to 64.9% of 'left' conclusions. Hence, people who gave a correct answer to two-model problems did not rely more on the linguistic form of the premises than in the case of one-model problems.

General discussion

Our study is the first to analyze the wording of the conclusions people draw in relational reasoning. We have shown that the wording of conclusions exposes several psychological mechanisms. Two of the effects we have demonstrated are compatible with the analogical approach to reasoning and provide new insight in the way people inspect their

mental model: First, the preference for 'left' conclusions is nicely explained by the fact that people construct mental models and inspect them from left to right. Second, the nature of the question is an important factor in determining the direction of model inspection**:** a 'left-to-right' question prompts 'left-to-right' inspection of the model and a 'left' conclusion. On the other hand, the congruence effect is compatible with the linguistic approach to reasoning. It shows that linguistic details of the premises, like the type of comparative, are stored in memory and are used in the inferential phase.

However, the occurrence of analogical and linguistic processes and the degree to which they are involved largely depend upon the presentation format. Sequential presentation increases the incidence of analogical processing: the "preference-for-left"-effect and the question-effect were stronger in the sequential condition than in the simultaneous condition. Simultaneous presentation induces linguistic processes: the congruence effect was present in the simultaneous condition but not in the sequential condition.

Interestingly, two effects were not observed. First, the number of models did not influence the wording of conclusions: the congruence effect was not greater in indeterminate problems. At first sight this seems to contradict the results of Mani & Johnson-Laird (1982), who found that people more often recalled linguistic details when the description was indeterminate. But they also reported that recall of the gist was lower when the description was indeterminate. Hence, having a weak representation of the gist of the description was related to high retention of the linguistic details of the sentences supporting the description. However, in the results taken into account here the representation was not weak since only correct conclusions were considered. This might explain the absence of a greater congruence effect with 2-model problems, and shows that when the representation of the description is correct people do not rely more on the linguistic level given indeterminate vs. determinate problems.

Second, the direction of model inspection was not congruent with the direction of model construction. People did not scan their model in the direction they constructed it. When construction of the D-E part of the model proceeded from left to right, it did not prompt more frequent 'left-to-right' inspection than when it proceeded from right to left.

In conclusion, some of the data presented here support the analogical framework and others support the linguistic framework. This contrasts with many reasoning experiments in which the data are considered to be entirely compatible with one approach and entirely incompatible with the other (see also Roberts, 1993)

Our findings show that both linguistic and analogical processes do contribute to the wording of conclusions in spatial reasoning. However, the impact of the different kind of processes is unequal: whereas the simultaneous presentation format provides evidence supporting both approaches (i.e. the congruence effect and effects related to model scanning were observed), the sequential presentation condition provides empirical evidence supporting the analogical approach and not the linguistic approach (i.e. the congruence effect was not observed for this condition). This pattern of data seems to indicate that analogical processes are pre-eminent in reasoning from spatial premises.

However, a comprehensive theoretical account of these results has to take into account both types of processes. Such a mixed model has been adopted previously by several researchers like Shaver, Pierson & Lang, (1974) Sternberg (1980) and Johnson-Laird (1983; Mani & Johnson-Laird, 1982). According to Sternberg and Johnson-Laird, the premises are first decoded into a linguistic format and are subsequently represented by a spatial mental model. However, this view concerns only the representational phase but not the inferential phase. Accordingly, it seems then that the inferential phase, during which the reasoner produces a conclusion, relies only on the inspection of the mental model. However the data we obtained indicate that the formulation of a conclusion is influenced by both analogical and linguistic factors in the simultaneous presentation, and support the idea that both factors influence not only the representational phase but also the inferential one. A possible explanation is that when people have achieved the construction of the mental model, they go back to the premises, when they are available, and use the premises as a guide to inspect the mental model. According to this view, linguistic factors play a role, but it does not necessarily imply that inferences rules are used while our data clearly indicate that mental models are constructed.

References

Byrne, R.M.J. & Johnson-Laird, P.N. (1989). Spatial reasoning. *Journal of Memory and Language*, *28*, 564-575.

Clark, H.H. (1969a). Linguistic processes in deductive reasoning. *Psychological Review, 76,* 387-404.

Clark, H.H. (1969b). Influence of language on solving three-term series problems. *Journal of Experimental Psychology, 82,* 205-215.

Cicirelli, V.G. (1977). Children's problem solving in relation to perceptual habit. *Perceptual and Motor Skills, 44*, 883-888.

De Soto, C.B., London, M., & Handel, S. (1965). Social reasoning and spatial paralogic. *Journal of Personality and Social Psychology, 2,* 293-307.

Evans, J.St.B.T., Newstead, S.E., & Byrne, R.M.J. (1993). *Human reasoning. The Psychology of Deduction.* Hove, UK: Lawrence Erlbaum Associates.

Hagert, G. (1984). Modelling mental models: Experiments in cognitive modelling spatial reasoning. In T. O'Shea (Ed.), *Advances in*

artificial intelligence, pp. 389-398. Amsterdam: North-Holland.

Huttenlocher, J. (1968). Constructing spatial images: a strategy in reasoning. *Psychological Review, 75,* 550-560.

Johnson-Laird, P.N. (1983). *Mental Models.* Cambridge: Cambridge University Press.

Johnson-Laird, P.N. & Byrne, R.J.M. (1991). *Deduction*. Hove, U.K: Lawrence Erlbaum Associates.

Mani, K., & Johnson-Laird, P.N. (1982). The mental representation of spatial descriptions. *Memory and cognition*, *10*, 181-187.

Ormrod, J.E. (1979). Cognitive processes in the solution of three-term series problems. *American Journal of Psychology, 92*, 235-255

Payne, S.J. (1993). Memory for mental models of spatial descriptions: An episodic-construction-trace hypothesis. *Memory and Cognition, 21*, 591-603

Potts, G.R., & Scholz, K.W. (1975). The internal representation of three terms series problem. *Journal of verbal Learning and Verbal behavior, 14,* 439-452.

Roberts, M.J. (1993). Human reasoning: deduction rules or mental models, or both? *The Quarterly Journal of Experimental Psychology, 46A*, 569-589.

Roberts, M.J. (2000). Strategies in relational reasoning. *Thinking and Reasoning, 6*, 1-26.

Rips, L.J. (1994). *The psychology of proof. Deductive reasoning in human thinking.* Cambridge, Massachusetts, MIT Press, Bradford

Schaeken, W. & Johnson-Laird, P.N. (2000). Strategies in temporal reasoning. *Thinking and Reasoning, 6*, 193-219.

Shaver, P., Pierson, L., & Lang, S. (1974). Converging evidence for the functional role of imagery in problem solving. *Cognition, 3*, 359-375.

Sternberg, R.J. (1980). Representation and process in linear syllogistic reasoning. *Journal of Experimental Psychology: General, 109*, 119-159.

Woodworth, R.S. & Sells, S.B. (1935). AN atmosphere effect in syllogistic reasoning. *Journal of Experimental Psychology, 18*, 451-460.