UCRL-16544

University of California

Ernest O. Lawrence Radiation Laboratory

CONNECTION BETWEEN REGGE POLES
AND ELEMENTARY PARTICLE POLES

Berkeley, California

DISCLAIMER

Submitted to Physical Review

UCRL-16544

UNIVERSITY OF CALIFORNIA

Lawrence Radiation Laboratory
Berkeley, California

AEC Contract No. W-7405-eng-48

CONNECTION BETWEEN REGGE POLES AND ELEMENTARY PARTICLE POLES

Takesi Saito

November 22, 1965

UCRL-16544

CONNECTION BETWEEN REGGE POLES AND ELEMENTARY PARTICLE POLES*

Takesi Saito+

Lawrence Radiation Laboratory
University of California
Berkeley, California

November 22, 1965

ABSTRACT

It is shown that if in a field theory the proper vertex function $\Gamma(s)$ vanishes under a condition that the proper vertex function poles are not the poles of scattering amplitudes, then the elementary particle lies smoothly on the Regge trajectory. (The condition $\Gamma(s) \equiv 0$ does not always mean the vanishing of the coupling constant, when the vertex function poles exist.) The bootstrap equations are immediate consequences of the above condition. We formulate our problem using multichannel theory. Other related results are: (a) It is found that in the multichannel case as well as in the single-channel case the proper vertex poles are not the poles of scattering amplitudes, if the poles come up from the second Riemann sheet. (b) The finite self-mass condition of the composite particle, due to Gerstein and Deshpande, is not applicable when the proper vertex pole appears below the elementary particle mass, i.e., the modified propagator needs one subtraction. (c) Vanishing renormalization constants, $Z_1 = Z_3 = 0$, are not sufficient to Reggeize the elementary particle. (d) All our conclusions remain true even when additional bound-state poles are included. In the latter case, however, we can also use a different condition, due to Kaus and Zachariasen, that the form factor (improper vertex function)

and Z_3 both vanish; then our main results again follow, that the elementary particle lies on the Regge trajectory and that the bootstrap equations are obtained. (The vanishing form factor does not always mean the vanishing of the coupling constant, when the bound-state poles exist.) But this last condition shows that the unsubtracted dispersion relation for the π-μ decay amplitude is not valid (to be discussed in a later publication).

I. INTRODUCTION AND SUMMARY

In conventional field theories, or in S-matrix theories with elementary particles, an elementary pion, for example, appears in the $\ell = 0$ N-$\bar{N}$ scattering state as a fixed pole. Its mass and coupling constant are regarded as arbitrary parameters. However, this type of formalism has come under criticism; it has been argued that perhaps none of the hadrons is specially distinguished as "elementary." Chew and Frautschi[1] suggested that all hadrons were composite on an equal footing, generating each other by the bootstrap mechanism. According to their ideas, each hadron should appear on a Regge trajectory, and there should be no fixed poles in ℓ, i.e., no Kronecker delta in the S matrix.

Before we abandon the notion of "elementary particle," we should like to explore the connection between the Regge pion and the elementary pion, for example. We shall consider multichannel scatterings, in which each amplitude includes the elementary pion pole in the S state. We shall find that if the proper vertex function with the elementary pion off the mass shell vanishes under a condition that the proper vertex function poles are not the poles of the scattering amplitudes, then the elementary pion disappears completely but the bootstrapped pion takes its place, lying on the Regge trajectory. The bootstrap equations of the pion are immediate consequences of the above condition. Our analysis is restricted by the assumption of two-particle multichannel unitarity, but if we regard the channel summations as the channel summations plus integrals over other continuous variables, due to manny particle states, all our conclusions are to be valid generally. (Sec. III.)

Here we have four remarks:

(a) In the one-channel case Jin and MacDowell[2] showed that if the proper vertex pole comes up from the second Riemann sheet as the coupling strength increases, then the pole is not the pole of the scattering amplitude. This interpretation is shown to be still true in the multichannel case, where by the second sheet we mean the sheet connected with the first sheet through the interval between the first threshold and the second threshold. However, for the pole appearing below μ^2 (the elementary pion mass) this interpretation is not valid, but we simply assume that this pole also is not the pole of the scattering amplitudes. (Sec. IV.)

(b) We know that the proper vertex poles correspond to zeros of the modified pion propagator $\Delta'_F(s)$.[3] We also know that there can be at most one zero in $\Delta'_F(s)$ between the elementary pion pole μ^2 and the first threshold, and that there is one zero for $s < \mu^2$ if $\Delta'_F(s)$ needs one subtraction, but no zero for $s < \mu^2$ if no subtraction is needed. Gerstein and Deshpande[4] showed that if $Z_3 = 0$ but the self-mass $\delta\mu^2$ is finite, then the elementary pion becomes the Regge pion. However, their conditions are not applicable when the proper vertex pole appears below μ^2. (Sec. III.)

(c) From our condition it follows that $Z_1 = Z_3 = 0$.[5] But this inverse is not true, i.e., the vanishing renormalization constants are not sufficient to Reggeize the elementary pion. (Sec. III.)

(d) All our conclusions are still true even when the full amplitudes have additional bound state poles together with the elementary pion pole.

In this case, however, we can also have another Reggeized condition, i.e., if the form factor (improper vertex function) vanishes under a condition $Z_3 = 0$, then the elementary pion lies smoothly on the Regge trajectory. The bootstrap equations are also immediate consequences of this condition. This was considered by Kaus and Zachariasen.[6] But their condition contradicts the unsubtracted dispersion relation of the π-μ decay amplitude. This will be discussed in another publication. (Sec. III.)

II. SEPARATION OF THE KRONECKER DELTA TERM

Let us consider a two-particle (spinless) multichannel scattering, in which each amplitude includes in the S state an elementary particle pole representing a scalar pion of mass μ coupled to the channel α with a renormalized coupling constant g_α. In this system we regard μ and g_α as arbitrary parameters.

Following the usual N/D method we put the ℓth partial wave amplitude as

$$T_\ell(s)_{\alpha\beta} = \sum_\gamma N_\ell(s)_{\alpha\gamma} D_\ell^{-1}(s)_{\gamma\beta} ,$$

or in the sense of matrix product,

$$T_\ell(s) = N_\ell(s) D_\ell^{-1}(s) , \tag{2.1}$$

where

$$D_\ell(s) = 1 - \frac{s}{\pi} \int_R ds' \frac{\rho(s') N_\ell(s')}{s'(s' - s)} , \tag{2.2A}$$

$$N_\ell(s) = \delta_{\ell 0}\, g \frac{1}{\mu^2 - s} \left[g\, D_0(\mu^2) \right] + \frac{1}{\pi} \int_L ds' \frac{v_\ell(s';\mu,g) D_\ell(s')}{s' - s} . \tag{2.2B}$$

Here $\rho(s)$ is a phase-space volume with the asymptotic behavior $\rho(s) \sim$ const. The function $v_\ell(s; u,g)$ is a given input potential with the parameters μ and g, and has been assumed to be an analytic function of ℓ except for certain singularities. We have also assumed that $v_\ell(s; \mu,g)$ tends to zero in the appropriate order as $s \to -\infty$.[7] Note that the amplitude $T_\ell(s)$ is symmetric under time-reversal invariance, and also so is $v_\ell(s;\ \mu,g)$.

Now let us separate the $\delta_{\ell 0}$ term from Eqs. (2.2):

$$D_\ell(s) = d_\ell(s) + \delta_{\ell 0}\, G(s)\ , \tag{2.3A}$$

$$N_\ell(s) = n_\ell(s) + \delta_{\ell 0}\, H(s)\ , \tag{2.3B}$$

where

$$d_\ell(s) = 1 - \frac{s}{\pi} \int_R ds'\ \frac{\rho(s')\ n_\ell(s')}{s'(s' - s)}\ , \tag{2.4A}$$

$$n_\ell(s) = \frac{1}{\pi} \int_L ds'\ \frac{v_\ell(s';\ \mu,g)\ d_\ell(s')}{s' - s} \tag{2.4B}$$

and

$$G(s) = -\frac{s}{\pi} \int_R ds'\ \frac{\rho(s')H(s')}{s'(s' - s)}\ , \tag{2.5A}$$

$$H(s) = g\, \frac{1}{\mu^2 - s} \left[g \cdot D_0(\mu^2)\right] + \frac{1}{\pi} \int_L ds'\ \frac{v_0(s';\ \mu,g)G(s')}{s' - s}\ . \tag{2.5B}$$

Then $T_\ell(s)$ can be written as[6]

$$T_\ell = n_\ell\, d_\ell^{-1} + \delta_{\ell 0}\, d_0^{-1T} \left[d_0^T H - n_0^T G\right] D_0^{-1}\ ,$$

after a minimal amount of algebra. The bracket term, $d_0^T H - n_0^T G$, has no singularities other than a pole at $s = \mu^2$,[8] as is easily seen from Eqs. (2.4) and (2.5), so that

$$d_0{}^T H - n_0{}^T G = \left[d_0{}^T(\mu^2) g\right] \frac{1}{\mu^2 - s} \left[g D_0(\mu^2)\right] . \qquad (2.6)$$

Therefore $T_\ell(s)$ reduces to

$$T_\ell(s)_{\alpha\beta} = t_\ell(s)_{\alpha\beta} + \delta_{\ell 0}\, \Gamma_\alpha(s) \frac{1}{\mu^2 - s} K_\beta(s)$$

$$= t_\ell(s)_{\alpha\beta} + \delta_{\ell 0}\, K_\alpha(s) \frac{1}{\mu^2 - s} \Gamma_\beta(s) , \qquad (2.7)$$

where

$$t_\ell(s) \equiv n_\ell(s)\, d_\ell{}^{-1}(s) , \qquad (2.8)$$

$$\Gamma(s) \equiv g\, d_0(\mu^2)\, d_0{}^{-1}(s) , \qquad (2.9)$$

$$K(s) \equiv g\, D_0(\mu^2)\, D_0{}^{-1}(s) . \qquad (2.10)$$

These formulae coincide exactly with those derived by several authors[9] from a quite different method. The first term $t_\ell(s)$ obviously satisfies a unitarity relation by itself above the threshold, and may be expected to be an analytic function of ℓ, because $t_\ell(s)$ is generated by the potential $v_\ell(s; \mu, g)$ only. The function $K(s)$ defined by (2.10) is just the form factor with the pion off the mass shell, and its unitarity relation is

$$\mathrm{Im}\, K(s) = K(s)\, \rho(s)\, T_0{}^\dagger(s)$$

$$= T_0(s)\, \rho(s)\, K^\dagger(s) \quad \text{(above threshold)}. \qquad (2.11)$$

The similar function $\Gamma(s)$ defined by (2.9) has no left-hand cut, and satisfies the unitarity relation

$$\text{Im}\,\Gamma(s) = \Gamma(s)\,\rho(s)\,t_0^{\dagger}(s)$$

$$= t_0(s)\,\rho(s)\,\Gamma^{\dagger}(s) \quad \text{(above threshold).} \qquad (2.12)$$

This equation implies that $\Gamma(s)$ is composed of proper vertex graphs only, and therefore can be regarded as a proper vertex function with the pion off the mass shell. In fact, the above authors[9] showed that $\Gamma(s)$ in Eq. (2.7) is just the proper vertex function.

III. DERIVATION OF THE BOOTSTRAP CONDITIONS

We assume that $t_\ell(s)_{\alpha\beta}$ in Eq. (2.7) has at least one pole at $s = s_\ell$ with the residue, $-(g_\ell)_\alpha(g_\ell)_\beta$, given by the equations

$$|d_\ell(s)| = 0 , \tag{3.1A}$$

$$-(g_\ell)_\alpha(g_\ell)_\beta = \lim_{s\to s_\ell} (s - s_\ell)t_\ell(s)_{\alpha\beta} . \tag{3.1B}$$

Here s_ℓ and g_ℓ depend on μ, g, and ℓ, since $d_\ell(s)$ and $n_\ell(s)$ are generated from $v_\ell(s;\, \mu,g)$. Therefore, it may be convenient to write them as

$$s_\ell(\mu,g) \quad \text{and} \quad g_\ell(\mu,g) , \tag{3.2}$$

if necessary. When $\ell = 0$, the pole s_0 is also the pole of $\Gamma(s)$, as is seen from Eq. (2.9). But we set the condition that s_0 is not the pole of $T_0(s)$. This condition is

$$-(g_0)_\alpha(g_0)_\beta + \gamma_\alpha \frac{1}{\mu^2 - s_0} K_\beta(s_0) = 0 , \tag{3.3A}$$

or

$$\left[\frac{(g_0)_\alpha}{K_\alpha(s_0)}\right]^2 = \frac{\gamma_\alpha/K_\alpha(s_0)}{\mu^2 - s_0} \equiv c , \tag{3.3B}$$

where

$$\begin{aligned} \gamma_\alpha &= \lim_{s\to s_0} (s - s_0)\Gamma_\alpha(s) \\ &= \lim_{s\to s_0} (s - s_0) \left[g\, d_0(\mu^2)d_0^{-1}(s)\right]_\alpha . \end{aligned} \tag{3.4}$$

The pole s_0 is unobservable, while s_ℓ with $\ell \neq 0$ is observable when ℓ is an integer. In Sec. IV the physical meaning of the condition (3.3) will be considered.

We know that the proper vertex function $\Gamma_\alpha(s)$ related to the form factor $K_\alpha(s)$ through the modified pion propagator $\Delta'_F(s)$:

$$Z_3(s) \equiv \Delta_F(s)/\Delta'_F(s) = \Gamma_\alpha(s)/K_\alpha(s) , \tag{3.5}$$

where $\Delta_F(s)$ is the free pion propagator: $\Delta_F(s) = 1/(\mu^2 - s)$. Therefore the pole s_0 of $\Gamma_\alpha(s)$ is just the zero of $\Delta'_F(s)$. We also know that, from the Herglotz property of $\Delta'_F(s)$, there can be at most one zero in $\Delta'_F(s)$ between μ^2 and the threshold and that there is one zero for $s < \mu^2$ if $\Delta'_F(s)$ needs one subtraction, but no zero for $s < \mu^2$ if no subtration is needed:

$$\Delta'_F(s) = \frac{1}{\mu^2 - s} + \frac{1}{\pi}\int_R ds' \frac{K(s')\rho(s')K^\dagger(s')}{(s' - \mu^2)^2(s' - s)} \quad \text{(no subtraction)}, \tag{3.6}$$

$$\Delta'_F(s) = \frac{1}{\mu^2 - s} + \text{const.} + \frac{s - \mu^2}{\pi}\int_R ds' \frac{K(s')\rho(s')K^\dagger(s')}{(s' - \mu^2)^3(s' - s)} \quad \text{(one subtraction)}. \tag{3.7}$$

When $s_0 < \mu^2$, the finite self-mass condition of the composite pion, due to Gerstein and Deshpande,[4] is not applicable, because their discussions are based on the condition $s_0 > \mu^2$. In either case, (3.6) or (3.7), the renormalization function $Z_3(s)$ defined by (3.5) is

$$Z_3(s) = 1 + \frac{s-\mu^2}{\pi} \int_R ds' \frac{\Gamma(s')\rho(s')\Gamma^\dagger(s')}{(s'-\mu^2)^2(s'-s)} - \frac{s-\mu^2}{s-s_0} C , \tag{3.8}$$

where C is given by (3.3B) and has a positive sign, consistent with the Herglotz property of $Z_3(s)$. Here we have used the following:

(a) The unitarity relations (2.11) and (2.12), and

(b) The form factor K(s) has no zeros (assumption of no CDD zeros).

The wave function renormalization constant of the pion is given by the limit

$$Z_3 = \lim_{s\to\infty} Z_3(s) . \tag{3.9}$$

Now, the $\delta_{\ell 0}$ term in $T_\ell(s)$ given by Eq. (2.7) represents the existence of the elementary pion. In the Reggeized world, such a $\delta_{\ell 0}$ term should disappear for any α and β , i.e.,

$$K_\alpha(s)\Gamma_\beta(s) \equiv 0 . \tag{3.10}$$

Therefore, we have two cases: $\Gamma_\alpha(s) \equiv 0$ for any α , and $K_\alpha(s) \equiv 0$ for any α . No other cases occur, as is easily seen.

Case A. $\Gamma_\alpha(s) \equiv 0$ for any α

From the condition[10]

$$\Gamma(s) = g\, d_0(\mu^2) d_0^{-1}(s) \equiv 0 , \tag{3.11}$$

it follows that

$$g\, d_0(\mu^2) = 0 , \tag{3.12}$$

and hence

$$| d_0(\mu^2) | = 0 . \tag{3.13}$$

Therefore, the parameter μ^2 must be

$$\mu^2 = s_0(\mu, g) . \tag{3.14}$$

On the other hand, the condition (3.3) yields

$$g = g_0(\mu, g) , \tag{3.15}$$

under Eq. (3.14), as is easily seen.

Inversely, if the parameters μ and g satisfy Eqs. (3.14) and (3.15), then Eq. (3.11) immediately follows, because s_0 is not the pole of $n_0(s)$ so that

$$\lim_{s \to s_0} (s - s_0) \, t_0(s) \, d_0(s) = g_0[g_0 \, d_0(s_0)] = 0 ,$$

and hence

$$g_0 \, d_0(s_0) = g \, d_0(\mu^2) = 0 .$$

The condition (3.3) is automatically satisfied.

The coupled equations, (3.14) and (3.15), thus obtained are nothing but the bootstrap equations for the pion. The amplitude $T_\ell(s)$ is now equal to $t_\ell(s)$.[11] The elementary pion has disappeared but the bootstrapped pion, on the trajectory given by the equations

$$\mu^2 = s_\ell(\mu, g) , \tag{3.16}$$

$$g = g_\ell(\mu, g) , \tag{3.17}$$

has taken its place.

From the condition $\Gamma(s) \equiv 0$ alone it follows that $Z_3(s)$ given by Eq. (3.8) must disappear,[12] and hence

$$Z_3(s) = 0 = Z_3 . \tag{3.18}$$

Note that, in order to obtain this, the condition (3.3), i.e., Eq. (3.15), is not necessary. But this inverse is not true, i.e., $\Gamma(s) \equiv 0$ does not follow from $Z_3 = 0$. The condition $\Gamma(s) \equiv 0$ means $Z_1 = 0$, because

$$\lim_{s\to\infty} \Gamma(s) = Z_1 \, g . \tag{3.19}$$

But this inverse is also not true, i.e., $\Gamma(s) \equiv 0$ does not follow from $Z_1 = 0$, because there may be a case, $d_0(s) \to \infty$ as $s \to \infty$.[13] Therefore, $Z_1 = Z_3 = 0$ is not sufficient to Reggeize the elementary pion.

In conclusion we showed that the condition (3.3) and $\Gamma(s) \equiv 0$ yield the bootstrap Eqs. (3.14) and (3.15) of the pion, then the elementary pion lies smoothly on the Regge trajectory. When other bound-state poles exist in $T_0(s)$, Eqs. (3.6) and (3.7) should have these poles, which yield new zeros in $\Delta'_F(s)$. But our conclusions never change.

Case B. $K_\alpha(s) \equiv 0$ for any α

From the condition

$$K(s) = g \, D_0(\mu^2) D_0^{-1}(s) \equiv 0 , \tag{3.20}$$

it must follow that

$$g\, D_0(\mu^2) = 0 , \tag{3.21}$$

hence

$$| D_0(\mu^2) | = 0 . \tag{3.22}$$

This equation is valid only when $T_0(s)$ has bound-state poles $s_B(\mu,g)$ given by $|D_0(s)| = 0$, and only when

$$\mu^2 = s_B(\mu,g) . \tag{3.23}$$

If there exist no bound states in $T_0(s)$, the Case B does not occur. Therefore, this case is nothing but the case considered by Kaus and Zachariasen.[6] They concluded that $K(s) \equiv 0$ and $Z_3 = 0$ Reggeize the elementary pion and give the bootstrap equation of the pion. But their conditions contradicts the unsubtracted dispersion relation of the π-μ decay. This will be discussed in another publication.

IV. PHYSICAL MEANING OF THE CONDITION (3.3)

At first we suppose that there is no pole in $\Gamma(s)$. Then Eq. (3.8) is

$$Z_3(s) = \Gamma_\alpha(s)/K_\alpha(s) = 1 + \frac{s-\mu^2}{\pi}\sum_\beta \int_{s_\beta}^{\infty} ds' \frac{\Gamma_\beta(s')\rho_\beta(s')\Gamma_\beta^\dagger(s')}{(s'-\mu^2)^2(s'-s)} . \tag{4.1}$$

Here we have set

$$s_1 < s_2 < s_3 < \cdots . \tag{4.2}$$

The proper vertex function $\Gamma_\alpha(s)$ has several Riemann sheets, in which by the second sheet we mean the sheet connected with the first sheet through the interval between s_1 and s_2 .

Now, if a pole s_0 comes up from the second Riemann sheet, then the integral path C_1 in Eq. (4.1) will be deformed as shown in Fig. 1. The integral along the deformed path yields

$$\frac{s-\mu^2}{\pi}\oint ds' \frac{\Gamma_1(s')\rho_1(s')\Gamma_1^{II}(s')}{(s'-\mu^2)^2(s'-s)} = -\frac{s-\mu^2}{s-s_0}\frac{1}{(\mu^2-s_0)^2}\left(\frac{\gamma_1}{g_{01}}\right)^2 , \tag{4.3}$$

where $\Gamma_1^{II}(s)$ is the vertex function continued into the second Riemann sheet, given by

$$\Gamma_1^{II}(s) = \Gamma_1(s)/[1 + 2i\ \rho_1(s)t_{11}(s)] . \tag{4.4}$$

Other continued functions of $\Gamma_\alpha(s)$ are

$$\Gamma_\alpha^{II}(s) = \Gamma_\alpha(s) - 2i\, \Gamma_1^{II}(s)\, \rho_1(s)\, t_{\alpha 1}(s) . \qquad (4.5)$$

Since s_0 is not the pole of $\Gamma_\alpha^{II}(s)$, we have

$$\gamma_1/g_{01} = \gamma_\alpha/g_{0\alpha} \qquad (4.6)$$

from Eq. (4.5). On the other hand, s_0 is the pole of $\Gamma_\alpha(s)/K_\alpha(s)$,[14] and hence we have

$$\left(\frac{1}{\mu^2 - s_0}\,\frac{\gamma_1}{g_{01}}\right)^2 = \left(\frac{1}{\mu^2 - s_0}\,\frac{\gamma_\alpha}{g_{0\alpha}}\right)^2 = \frac{1}{\mu^2 - s_0}\,\frac{\gamma_\alpha}{K_\alpha(s_0)} ,$$

or

$$-g_{0\alpha}\, g_{0\beta} + \gamma_\alpha \frac{1}{\mu^2 - s_0} K_\beta(s_0) = 0 .$$

This relation is just the condition (3.3). Therefore, it is true also in the multichannel case that if the proper vertex pole comes out from the second Riemann sheet, then the pole is not the pole of the scattering amplitudes.

Note that this pole thus coming out can exist only in the interval $\mu^2 \leqq s \leqq s_1$, but not in the region $s < \mu^2$. Therefore, for the pole appearing below μ^2 the above interpretation is not valid. We have simply assumed that the pole below μ^2 also is not the pole of the scattering amplitudes.[15]

ACKNOWLEDGMENT

The author would like to thank Professor Geoffrey F. Chew, Dr. G. Konisi, and Dr. W. Rarita for their kind interests and valuable comments. Thanks are also due to Dr. David L. Judd for his hospitality at the Lawrence Radiation Laboratory.

FOOTNOTES AND REFERENCES

* Work done under auspices of the United States Atomic Energy Commission.

† Formerly Takesi Ogimoto. On leave of absence from Department of Physics, Osaka University, Osaka, Japan.

1. G. F. Chew and S. C. Frautschi, Phys. Rev. Letters 7, 394 (1961).
2. Y. S. Jin and S. W. MacDowell, Phys. Rev. 137, B688 (1965).
3. C. J. Goebel and B. Sakita, Phys. Rev. Letters 11, 293 (1963).
4. I. S. Gerstein and N. G. Deshpande, Phys. Rev. (to be published).
5. That an elementary particle may be made to look dynamical by imposing $Z_1 = Z_3 = 0$ has been discussed by several authors. Most recent works are, for example, M. Ida, Progr. Theoret. Phys. (Kyoto) 34, 92 (1965); T. Akiba, S. Saito and F. Takagi, Nuovo Cimento 39, 316 (1965); and Ref. 6. Other references are given in these papers.
6. P. E. Kaus and F. Zachariasen, Phys. Rev. 138, B1304 (1965).
7. When $v_\ell \sim (1/\ln s)(1/\ln \ln s)$, for example, then we have $N_\ell \sim 1/\ln s$ and $D_\ell \sim \ln \ln s$ as $s \to \infty$.
8. The bracket term tends to zero as $s \to \infty$.
9. S. D. Drell and F. Zachariasen, Phys. Rev. 105, 1407 (1957); R. Blankenbecler, M. L. Goldberger, N. N. Khuri, and S. B. Treiman, Ann. Phys. (N. Y.) 10, 62 (1960); M. Ida, Phys. Rev. 135, B499 (1964); B. W. Lee, K. T. Mahanthappa, I. S. Gerstein, and M. L. Whippman, Ann. Phys. (N. Y.) 28, 466 (1964); and Ref. 2.
10. This does not always mean $g = 0$ when "bound-state poles" given by $|d_0(s)| = 0$ exist. To show this we study especially the behavior of $\Gamma(s)$ near $s = \mu^2$. The dispersion relation of $\Gamma(s)$ is

$$\Gamma(s) = g + \frac{s - \mu^2}{s - s_0} \frac{\gamma}{s_0 - \mu^2} + \frac{s - \mu^2}{\pi} \int_R ds' \frac{t_0^{\dagger}(s')\rho(s')\Gamma(s')}{(s' - \mu^2)(s' - s)} .$$

Now let us suppose $\Gamma(s) = g\, d_0(\mu^2)d_0^{-1}(s) \equiv 0$ except at $s = \mu^2$. Then the integral term in the above equation vanishes, μ^2 must go to s_0 because of $|d_0(s_0)| = 0$, and hence $\gamma/(s_0 - \mu^2) \to -g$. Therefore, the first term g is canceled by the second term, so that $\Gamma(s)$ is equal to zero even at $s = \mu^2$ (consistent with the property of an analytic function), but $g \neq 0$. We shall assume $g \neq 0$. Note that when $\mu^2 = s_0$ the formula $\Gamma(s) = g\, d_0(\mu^2)d_0^{-1}(s)$ is indefinite at $s = \mu^2$, so $\Gamma(\mu^2)$ should be defined by $\lim_{s\to\mu^2} \Gamma(s) = \Gamma(\mu^2)$.

11. This does not mean $D_0 = d_0$ and $N_0 = n_0$. Equation (2.6) is now $d_0^T H - n_0^T G = 0$. Therefore T_0 can be written as

$$T_0 = N_0 D_0^{-1} = (n_0 + H)(d_0 + G)^{-1} = n_0 d_0^{-1} = HG^{-1} .$$

12. The form factor, $K(s) = \Gamma(s)/Z_3(s)$, generally depends on s, in spite of the limits, $\Gamma(s) \to 0$ and $Z_3(s) \to 0$. Consider an example, $\Gamma(s) = a\, f(s)$ and $Z_3(s) = b\, g(s)$. In the limits, $a \to 0$ and $b \to 0$, in such a way that $a/b \to$ const, we have $K(s) \to \text{const}\, f(s)/g(s)$. The asymptotic behavior of $K(s)$ is restricted by the equations, $Z_3 = 0$ and

$$Z_3^{-1} = 1 + \int ds\, K(s)\, \rho(s)\, K^{\dagger}(s)/(s - \mu^2)^2 .$$

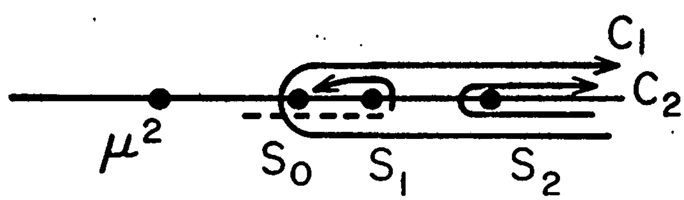
13. See footnote 7. In the pseudoscalar theory, $Z_1 = 0$ is an identity, as was pointed out by M. Ida in Refs. 5 and 9.

14. We have already shown in Eq. (4.3) that $\Gamma_\alpha(s)/K_\alpha(s)$ should have the pole s_0, and hence $|D_0(s_0)| \neq 0$. Nevertheless $T_0(s)$ may have the pole s_0 due to the pole of $N_0(s)$. The condition (3.3) assures that the residue of the pole of $N_0(s)$ at $s = s_0$ vanishes.

15. Let us suppose that $\Gamma(s)$ has two poles, s_0 and s'_0. If we put $\mu^2 = s_0$ and $g = g_0$, then the other pole s'_0 disappears by virtue of the condition (3.3).

FIGURE CAPTION

Fig. 1. Deformation of integral path in Eq. (4.1). The proper vertex pole s_0 moves on the real axis. The dotted line lies on the second Riemann sheet, while the solid lines lie on the first Riemann sheet.



MUB-8740

Fig. 1