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This problem was recently considered by Parzen[1] who concluded that quantum corrections to the classical results of Schwinger[2] and Schiff[3] should be appreciable at an electron energy of 200 mc^2 in a field of 10^4 gauss. The form of his correction is such that for this magnetic field the energy loss per turn by radiation would only increase as $R^{1/3}$ with increasing energy, instead of increasing as E^4/R, thus removing the stringent radiation limitation on synchrotron design.

An examination of Parzen's calculation reveals an invalid approximation[4]. More significantly, the assertion that only $\ell = 0$ to $\ell = 0$ transitions are appreciable can be shown to be incorrect by directly summing the series (eq. (36)). This yields

$$I(n'\ell \mid n0) = \frac{(-\alpha^2/2)^{\ell/2}}{\sqrt{\ell!}} \quad I(n'0 \mid n0) \tag{1}$$

in which the important values of $(\alpha^2/2)$ are of order unity. This

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1 G. Parzen, Phys. Rev. 84, 235 (1951).

2 J. Schwinger, Phys. Rev. 75, 1912 (1949).

3 L. I. Schiff, Rev. Sci. Inst. 17, 8 (1946).

4 Following his eq.(23) we should have $\left[(n+\lambda)!/n!\right]^{\frac{1}{2}} \simeq n^{\frac{1}{2}\lambda} \exp(\lambda^2/4n)$, from the Stirling approximation; terms of order $\lambda^3/n^2 \approx \alpha^3/n^{\frac{1}{2}}$ are neglected since $n \sim 10^{14}$.

result may be verified more easily by use of the energy eigenfunctions in cartesian coordinates[5], by the use of which the summation over ℓ is implicitly performed.

If one now examines the formula for the power radiated in the orbital plane and makes use of these results the exponential correction factor in eqs. (26) cancels out and the classical result is obtained as sketched below.

$$I(n'0 \mid n0) = \sqrt{\frac{n'!}{n!}} \left(-\frac{\alpha^2}{2}\right)^{\lambda/2} e^{-\alpha^2/2} L^{\lambda}_{n'}\left(\frac{\alpha^2}{2}\right) \tag{2a}$$

$$= \frac{(-1)^{\lambda/2}}{\sqrt{n!\, n'!}} \int_0^{\infty} dt\, e^{-t}\, t^{n'+\lambda/2}\, J_{\lambda}(\alpha\sqrt{2t}). \tag{2b}$$

Combining eq. (2b) with the relation[6]

$$J_{\lambda}(\alpha\sqrt{2t}) = \left(1+\frac{\Upsilon}{n}\right)^{\lambda/2} \sum_{m=0}^{\infty} \frac{\left(-\frac{\alpha\Upsilon}{\sqrt{2n}}\right)^m}{m!} J_{\lambda+m}(\alpha\sqrt{2n}) \quad , \tag{3}$$

where $t = n + \Upsilon$, one obtains

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[5] M. H. Johnson and B. A. Lippman, Phys. Rev. 76, 828 (1949), eq. (43).

[6] G. N. Watson, Theory of Bessel Functions (The MacMillan Co., New York, 1944), p. 141, eq. (5)'.

$$I(n'0 \mid n0) \simeq \frac{(-1)^{\lambda/2}}{\sqrt{2\pi n}} e^{-\lambda^2/4n'} \int_{-n}^{\infty} d\tau \, e^{-\tau^2/2n} \sum_{m=0}^{\infty} \frac{\left(-\frac{\alpha\tau}{\sqrt{2n}}\right)^m}{m!} J_{\lambda+m}(\alpha\sqrt{2n}) \tag{4}$$

The major contribution to this integral comes in the region $|\tau| < \sqrt{2n}$. Also, $\alpha\sqrt{2n} = \lambda\beta$ in the orbital plane. In the summation only about α terms contribute so that the relevant values of m are of order $\alpha = \frac{\lambda\beta}{\sqrt{2n}} << \lambda$. For these m, $J_{\lambda+m}(\lambda\beta) \cong J_\lambda(\lambda\beta)$ for $1 - \beta^2 << 1$, so that (4) becomes

$$\frac{(-1)^{\lambda/2} J_\lambda(\lambda\beta) \, e^{-\lambda^2/4n'}}{\sqrt{2\pi n}} \int_{-\infty}^{\infty} e^{-\tau^2/2n - \alpha\tau/\sqrt{2n}} \, d\tau \simeq e^{-\alpha^2/4} J_\lambda(\lambda\beta).$$

The power radiated in the orbital plane is proportional to the sum

$$\sum_{\ell=0}^{\infty} \left| I(n'\ell \mid n+1,0) + I(n'\ell \mid n-1,0) \right|^2$$

which may now be evaluated using eq. (1) to give $\left[2J'_\lambda(\lambda\beta)\right]^2$, the classical result.

It is a pleasure to acknowledge helpful discussions with Professor E. M. McMillan, who brought this matter to our attention.