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NOISE IN 3-D IMAGING WITH LARGE-AREA POSITRON CAMERAS

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6. Noise in 3-D Imaging with Large-Area Positron Cameras, *B. Macdonald, J. P. Lenahan, V. Perez-Mendez (LBL)*

In the method of three-dimensional (3-D) reconstruction described below, we have investigated the statistical noise produced in the reconstruction due to a finite number of events.

The large-area type of positron camera considered here (Fig. 1) produces images of a volume distribution of radioactivity.[1] Images from these large-area cameras have generally been tomographic images, produced when the set of event-lines, lines connecting the positions of the two back-to-back gamma rays from a positron annihilation, are projected onto transverse planes through the object. One can see from Fig. 1 that these tomographic images have those sources on a given plane in focus while sources in all other planes contribute a blurred background superimposed on the in-focus information.

We can eliminate this background simultaneously from a number of these positron camera tomographic images $[t_j(\mathbf{r}),\ j = 1, \ldots, N]$ from a single camera exposure without rotation.[2] We let $h_{ij}(\mathbf{r} - \mathbf{r}')$ represent the response at point $\mathbf{r}$ of tomographic plane j to a unit point source in plane i at $\mathbf{r}'$ and let $o_i(\mathbf{r}')$ be the source distribution in each of these N planes. Each tomographic image has a contribution from each of the source planes given by

$$t_j(\mathbf{r}) = \sum_{i=1}^{N} h_{ij}(\mathbf{r}) * o_i(\mathbf{r}) \qquad j = 1, \ldots, N \ , \tag{1}$$

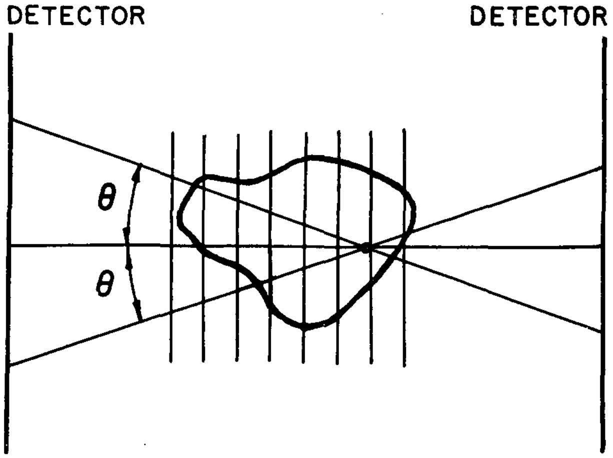

Fig. 1. Response of a large-area positron camera. All events between $-\theta$ and $+\theta$ are collected.

where * represents the convolution integral. These equations can be made into a set of linear equations by applying the Fourier transform, transforming the spatial functions $t_j(\mathbf{r})$, $o_i(\mathbf{r})$, $h_{ij}(\mathbf{r})$, to functions of spatial frequency $T_j(\mathbf{u})$, $O_i(\mathbf{u})$, $H_{ij}(\mathbf{u})$.

$$T_j(\mathbf{u}) = \sum_{i=1}^{N} H_{ij}(\mathbf{u})\, O_i(\mathbf{u}) \quad . \quad j = 1, \ldots, N \ . \qquad (2)$$

Solution of these equations for $O_i(\mathbf{u})$, and consequently for $o_i(\mathbf{r})$ by inverse Fourier transformation, requires finding the inverse matrix $H_{ij}^{-1}(\mathbf{u})$ for each value of $\mathbf{u}$.

When the determinant of H is zero at a given spatial frequency $\mathbf{u}$, $O_i(\mathbf{u})$ cannot be determined for any plane. The determinant for this type of positron camera is zero only at $\mathbf{u} = 0$, but it has small values near $\mathbf{u} = \mathbf{0}$. Since $O_i(0)$ is undetermined, the solutions $o_i(\mathbf{r})$ are indefinite by an additive constant which must be determined for each plane by some subsidiary condition; for instance, that $o_i(\mathbf{r})$ must be zero in the outer regions of the plane where there is known to be no activity.

The effect of statistical variation in the tomograms is of critical importance and we have investigated this using a Monte Carlo simulation with a PDP 11/45 computer. The system modeled used two square detectors, 32 cm on a side, separated by 32 cm, to image a positron-emitting object in the shape of an octahedron (two pyramids joined base to base; 16 cm square, 16 cm total height, with an inner area having $\frac{1}{4}$ the intensity of the outer area). Figure 2a shows the phantom's activity in the immediate region of seven planes parallel to the detectors.

Two million positron events were used and projected onto each of seven tomographic planes (Fig. 2b). Reconstruction on these seven planes was made (Fig. 2c) using inverse matrices expected from geometrical considerations. The additive constants used were obtained as stated previously, and some slight improvement has been made by requiring that the first spatial frequency components were small in the outermost regions.

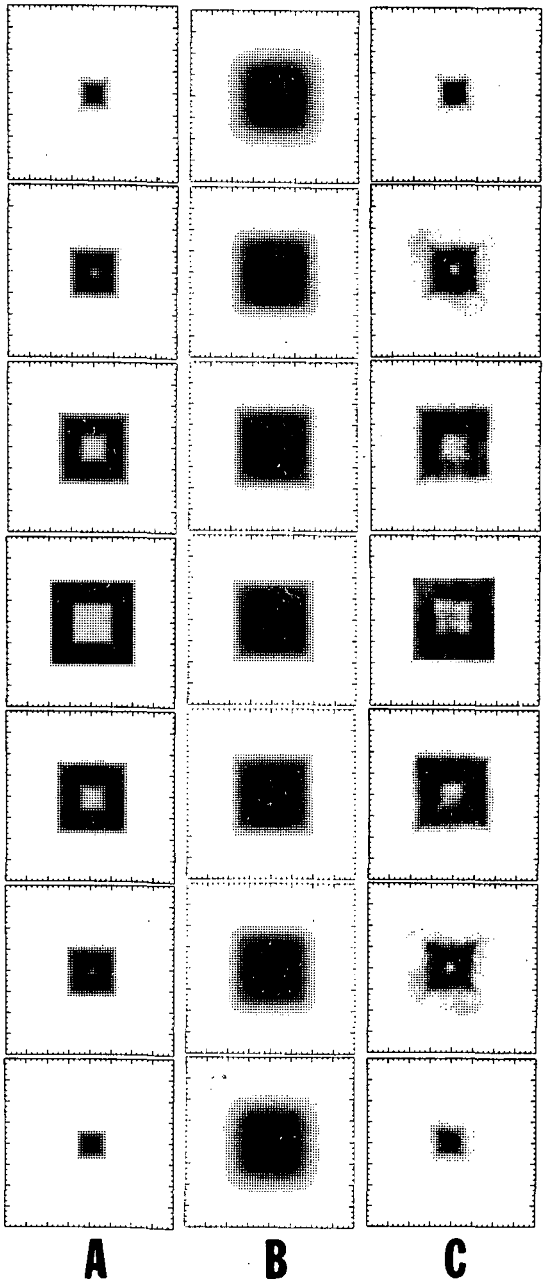

Fig. 2. (a) Activity from the immediate neighborhood of seven planes through the octahedron phantom. Plane separation 2 cm, $\frac{1}{2}$-cm bins, $\theta = \pm 26$ deg. (b) Tomographic images from two million octahedron positron events projected onto each of these seven planes. (c) Reconstructions from these tomograms.

The difference between these results and the ideal result (Fig. 2a) is noise introduced by the reconstruction method due to random placement in the detectors of the limited number of positron events from a given source point. As expected from the smallness of the determinant in the region of $\mathbf{u} = 0$, this reconstruction noise is mainly of low frequency. These results show that this noise is appreciable in nuclear medicine applications. This noise is expected to be reduced when generalized inverse solutions are made for an overdetermined system and results from this approach will also be reported.

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