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DETERMINATION OF THE DENSITY PROFILE IN A PLASMA SLAB *

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ABSTRACT

A simple formula is derived, for the direct determination of the electron density profile of a symmetric plasma slab, in terms of the ac resistance of the slab as a function of frequency. The cold plasma approximation is used.

In this note we present an extremely simple and direct method for the determination of the density profile in a plasma slab, under the conditions that:

(1) the density $n(x)$ is an <u>even</u> function of the single coordinate x;

(2) the applied magnetic field, if any, is in the y-z plane, and is uniform in magnitude, although not necessarily in direction;

(3) the <u>cold</u> plasma approximation is valid;

(4) the plasma slab is accessible for the measurement of its frequency-dependent impedance as a capacitor.

Under these conditions, the Poisson equation reads

$$\frac{\partial}{\partial x}\left[\epsilon(\omega;\, x)\, E_x\right] = 0 \quad , \tag{1}$$

where the dielectric constant is

$$\epsilon(\omega;\, x) = 1 - \frac{\omega_p^{\,2}(x)}{\omega^2 - \Omega^2} \quad , \tag{2}$$

and $\omega_p^{\,2}(x) \equiv 4\pi\, n(x)\, e^2/m$; $\Omega \equiv eB/mc$.

The complex impedance is then, for unit surface area,

$$Z(\omega) = \frac{8\pi i}{\omega} \int_0^L \frac{dx}{\epsilon(\omega;\, x)} \quad , \tag{3}$$

where $x = 0$ is the slab center, and $x = L$ is its edge (where $\epsilon = 1$). Allowing $\epsilon(\omega;\, x)$ to have an infinitesimal positive imaginary part to allow for weak dissipative processes, we see that the real part of the impedance, or resistance, is

$$R(\omega) = (8\pi/\omega) \int_0^L dx \, \pi \, \delta[\epsilon(\omega; x)]$$

$$= (8\pi^2/\omega) \left| \partial\epsilon(\omega; x)/\partial x \right|^{-1}_{x(\omega)} \quad , \qquad (4)$$

where $x(\omega)$ is determined by the zero of ϵ:

$$\omega^2(x) = \Omega^2 + \omega_p^2(x) \quad . \qquad (5)$$

From Eq. (2), we see that

$$\frac{\partial\epsilon(\omega; x)}{\partial x} = -(\omega^2 - \Omega^2)^{-1} \frac{d\omega_p^2(x)}{dx}$$

$$= -(\omega^2 - \Omega^2)^{-1} \frac{d\omega^2(x)}{dx} \quad . \qquad (6)$$

Substituting (6) into (4), we obtain

$$x(\omega) = \frac{1}{4\pi^2} \int_{\omega}^{\omega(0)} d\omega' \frac{\omega'^2}{\omega'^2 - \Omega^2} R(\omega') \quad . \qquad (7)$$

From Eqs. (4) and (6), we see that $R(\omega)$ is infinite at $\omega(0)$, within the cold-plasma approximation. This serves to determine $\omega(0)$ experimentally, and thereby $n(0)$. The singularity in the integrand of (7), is however, integrable.

Thus, from the measurement of $R(\omega)$, one can immediately obtain $x(\omega)$ from (7), and then $n(x)$ from (5).

To illustrate the use of Eq. (7), we offer an example allowing analytic quadrature. Suppose

$$R(\omega) = A\,\omega^{-1}(\omega^2 - \Omega^2)(\omega_0^{\,2} - \omega^2)^{-\frac{1}{2}} \quad , \qquad (8)$$

for $\Omega < \omega < \omega_0$, and $R(\omega) = 0$ elsewhere. We then find

$$n(x) = (4\pi e^2)^{-1}\, m[\omega_0^{\,2} - \Omega^2 - (4\pi^2 x/A)^2] \quad . \qquad (9)$$

The validity of the cold-plasma assumption has been investigated by one of us[1,2] for the case of no magnetic field. By including thermal effects, it was shown that the energy absorbed at the resonance $x(\omega)$ is carried away by plasma waves. If the energy in these waves is absorbed by collisions or by Landau damping, the theory including finite temperature yields the same expression for the externally observed impedance as that obtained using the cold plasma approximation. However, if these plasma waves can be reflected from a region removed from the resonance point and thus converted to standing waves, the expression (3) for the impedance becomes modified. The increased fine structure in the observed impedance due to trapping of plasma waves between the plasma resonance and the sheath region has been used by several authors (see Ref. 1) to explain Tonks-Dattner resonances in positive columns.

In the presence of a magnetic field, the energy is carried away from the resonance region by electron cyclotron waves propagating perpendicular to the magnetic field. Generally, these are absorbed, giving the cold plasma result for the impedance. However, when the frequency is near a harmonic of Ω, a trapping of these waves can occur through the center of the plasma, again leading to a fine structure in the impedance.[3]

FOOTNOTES AND REFERENCES

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2. D. E. Baldwin and D. W. Ignot, to be published in Phys. of Fluids

3. S. J. Buchsbaum and A. Hasegawa, Phys. Rev. Letters 12, 685 (1964); Phys. Rev. A143, 303 (1966).