A SIMPLE MODEL FOR UNDERSTANDING THE DAY-NIGHT TEMPERATURE CONTRAST ON HOT JUPITERS

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ABSTRACT

1. INTRODUCTION

Hot Jupiters are tidally locked extrasolar planets. Due to their synchronous rotation about their host star, they have a permanent day- and night-side.

Their X day synchronous rotation periods are slow enough that Coriolis forces do not dominate the flow as in the case of the giant planets in the solar system.

The intense heating contrast between the day and nightside drives global wind circulation. $\sim 10^5$ W/m^2 irradiate the dayside of these planets and their nightsides cool via infrared emission.

From Spitzer observations of the lightcurves emitted by the Star + planet, we can determine the temperature contrast of hot Jupiters (as a function of the phase angle). Variation as a function of the orbital phase. From these you can infer a temperature. Maximum occurs before the secondary eclipse which is interpreted as the hotspot being shifted eastwards of the substellar point. Knutson et al. (2007).

This result is generally interpreted as the result of heat advection being the primary mechanism for tranporting heat from the day to the nightside which requires individual air parcels to

This scenario leads to the not well justified rule of thumb used by workers that the temperature contrast can be estimated by simply comparing the radiative timescale to the advective timescale. (If radiative timescale is much less than advective timescale you expect the contrast to be high, whereas in the opposite regime you expect a globally uniform temperature). Indeed hot Jupiter simulations develop a superrotating jet with speeds on the order of 1 km/s which can advect heat.

However there is another mechanism for planets to adjust their temperature and this is due to gravity waves. Gravity waves can adjust to buoyancy forces caused by the lateral heating gradient. Isentropes in the planet. On the dayside there is heating, which causes lowering of the isentropes with respect to isobars. And in the nightside you obtain higher values. The discontinuity will create pressure gradient forces that will radiate gravity waves that go in both directions which will tend to flatten the isentrope (i.e., erase the temperature gradient). This process is analogous to water waves in a pond.

This mechanism does not require large displacements of air parcels, it is purely a wave mechanism. It acts on a different timescale than advection. This is the accepted mechanism for setting the temperature on the Earth's tropics, but has yet to be considered in the hot Jupiter literature.

So which one is the relevant dynamical timescale that sets the temperature contrast on hot Jupiters (is it the advective timescale or the gravity wave timescale)?

And also what it should it be compared against? The radiative timeconstant is one option but there could be a frictional timeconstant involved. We would like to interpret data like.

Phase amplitude over the secondary eclipse (related to temperature contrast). As you increase the global mean temperature the amplitude increases.

2. SHALLOW-WATER MODEL

To study the heat transport mechanisms in the most simple possible context, we adopt a highly idealized two-layer shallow-water model. The buoyant upper layer, constant in density, represents the meteorologically active atmosphere of the extrasolar planet, while the infinitely deep bottom layer with a higher density represents the convective interior of the planet. In the limit where the lower-layer pressure gradients are in steady state (i.e, the upper layer is in isotatic balance), this system reduces to the 2D shallow water equations for the flow in the upper layer (Vallis 2006):

$$\frac{d\mathbf{v}}{dt} + g\nabla h + f\mathbf{k} \times \mathbf{v} = \mathbf{R} - \frac{\mathbf{v}}{\tau_{\rm drag}}, \tag{1}$$

$$\frac{\partial h}{\partial t} + \nabla \cdot (\mathbf{v}h) = \frac{h_{\rm eq}(\lambda, \phi) - h}{\tau_{\rm rad}}, \tag{2}$$

where $\mathbf{v}(\lambda, \phi, t)$ is the horizontal velocity, $h(\lambda, \phi, t)$ is the thickness of the the upper layer, t is time, g the reduced gravity[1], $f = 2\Omega \sin \phi$ is the Coriolis parameter, $\mathbf{k}$ is the upward unit vector, Ω is the planetary rotation frequency, and (λ, ϕ) are longitudinal and latitudinal angles. Here $d/dt = \partial/\partial t + \mathbf{v} \cdot \nabla$ is the material derivative.

The boundary between both layers represents an atmospheric isentrope, across which mass flows in the presence of heating and cooling. Thus, heat transfer is represented as mass sources and sinks in the shallow water model. We model heat transfer with Newtonian relaxation of the height filed h towards $h_{\rm eq}$—set by radiative equilibrium—over a radiative time scale $\tau_{\rm rad}$, which we treat as a free parameter.

The momentum Equation (1) includes the drag timescale $\tau_{\rm drag}$ mentioned previously and the $\mathbf{R}$ term, which represents the effect of momentum advected with mass transfer from the lower layer into the upper layer introduced by Showman & Polvani 2011. In this work we set $\mathbf{R} = 0$ for simplicity.

We will study heat transport processes with shallow-water models of ever increasing complexity. In Section 2.1 we begin by study a even more idealized system that still captures all the relevant physics. We derive this model from Equations (1) & (2), with two further simplifying approximations: First, we drop all ϕ-dependence from the system (including the Coriolis term), focusing on the tropical belt. We present linear analytic solution to this system. In Section 2.2, we compare our results from our simple 1D model with the linear solutions of Equations (1) & (2) given by Showman & Polvani (2011). Finally, in Section 2.3 we compare the results with the fully non-linear solutions determined numerically.

2.1. Linearized 1D Shallow Water Model at Planetary Equator

Our tropical 1D-version of the shallow water model in cartesian geometry reads

$$\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x} + g\frac{\partial h}{\partial x} = -\frac{v}{\tau_{\rm drag}}, \tag{3}$$

[1] $g = \frac{\rho_{\rm lower} - \rho_{\rm upper}}{\rho_{\rm upper}} \frac{GM_{\rm planet}}{R^2_{\rm planet}}$

$$\frac{\partial h}{\partial t} + \frac{\partial}{\partial x}(vh) = \frac{h_{\rm eq}(\lambda) - h}{\tau_{\rm rad}}, \tag{4}$$

where x is the eastward distance.

We linearize Equations (3) & (4) about a constant reference height H and a constant eastward zonal wind speed $\bar{u}$.

$$\frac{\partial u}{\partial t} + \bar{u}\frac{\partial u}{\partial x} + g\frac{\partial \eta}{\partial x} = -\frac{u}{\tau_{\rm drag}} - \frac{\bar{u}}{\tau_{\rm drag}}, \tag{5}$$

$$\frac{\partial \eta}{\partial t} + \bar{u}\frac{\partial \eta}{\partial x} + H\frac{\partial u}{\partial x} = \frac{\eta_{\rm eq}(\lambda) - \eta}{\tau_{\rm rad}}, \tag{6}$$

where u is the deviation of the flow velocity from $\bar{u}$, such that $v = \bar{u} + u$. Analogously, η is the deviation of the thickness from H, such that $h = H + \eta$ and $\eta_{\rm eq} = h_{\rm eq} - H$.

We drop the term $\bar{u}/\tau_{\rm drag}$ from Equation (5) as it is balanced by...

Most three-dimensional models of tidally locked exoplanets relax to steady circulation patterns (Showman & Guillot 2002; Cooper & Showman 2005, 2006), and we therefore seek solutions in the presence of external forcing and damping.

We nondimensionalize Equations (5) & (6) with a lengthscale $L = 2\pi/k$, a velocity scale $U = \sqrt{gH}$, and a timescale $\mathcal{T} = (k\sqrt{gH})^{-1}$, which correspond respectively to the wavelength of the thermal forcing ($\sim$ planetary circumference), the gravity wave speed, and the time for a gravity wave to cross L. The thickness is nondimensionalized with H and the thermal and drag time constants with $\mathcal{T}$. In steady state we obtain the equations,

$$\bar{u}\frac{\partial u}{\partial x} + \frac{\partial \eta}{\partial x} = -\frac{u}{\tau_{\rm drag}}, \tag{7}$$

$$\bar{u}\frac{\partial \eta}{\partial x} + \frac{\partial u}{\partial x} = \frac{\eta_{\rm eq} - \eta}{\tau_{\rm rad}}, \tag{8}$$

where all quantities are now nondimensional.

Equations (7) & (8) can be combined to yield a single second-order differential equation for η or u. We seek solutions for the unknowns η and u for a given thermal forcing $\eta_{\rm eq}/\tau_{\rm rad}$. For tidally locked exoplanets we expect thermal forcing to be an oscillating function corresponding to the day-night variation in heating/cooling of the planet. For simplicity we choose a pattern of heating and cooling pattern that is sinusoidally varying in longitude $\eta_{\rm eq} = \Delta\eta_{\rm eq}\cos(kx)$ with the maximum centered at the substellar point and the minimum at its antipode.

Appendix A gives the details

This background velocity which will play the role of advection and give the advective timescale.

In this simple model there are four relevant timescales: The radiative timescale for heating and cooling, the drag timescale which we parametrize and can represent MHD friction, turbulent mixing. There is also the advection timescale and the wave timescale.

We keep the wave timescale $1/\sqrt(gh)$ constant and normalize all other time constants to this value.

First show solutions in limits familiar. Solution for very strong drag, meaning that gravity wave transport is suppressed and the only heat transport is due to advection. Initially we have a very low advection speed and the solution relaxes to the radiative equilibrium curve. As we increase the advection speed, the hotspot is shifted towards the east and the day-night temperature contrast is reduced (amplitude of curve).

In the low drag limit, even without advection, the gravity waves already have adjusted the temperature contrast of the atmosphere. Large increases in amplitude and excursions of the hotspot, both in the eastern and western direction occur when the advection velocity is increased.

When the advection velocity is very high, the system reaches a state where advection completely dominates over gravity waves and drag becomes irrelevant.

Rainbow plot: Solutions to our system of equations as a function of the radiative and drag timescales. Each frame is for a different advection time. Contours: Red: Temperature set by radiative equilibrium (amplitude is high). When low, temperature has been homogenized by gravity waves (because there is no advection in this frame). In this limit the radiative and drag timeconstants play complimentary roles (if you decrease one and increase the other by the same amount, you obtain the same solution).

Advancing the advection timescale we observe a complex series of behaviors. We separate the solution into four regimes.

Advection regime: Advection speed less than the wave speed. You get weird three thoghed behavior.

Advection speed is equal to the wave speed: Solution shows hangover pattern. The solution is in phase with the forcing and that raises the amplitude.

Advection timescale much grater than the wave timescale: Regime where drag becomes

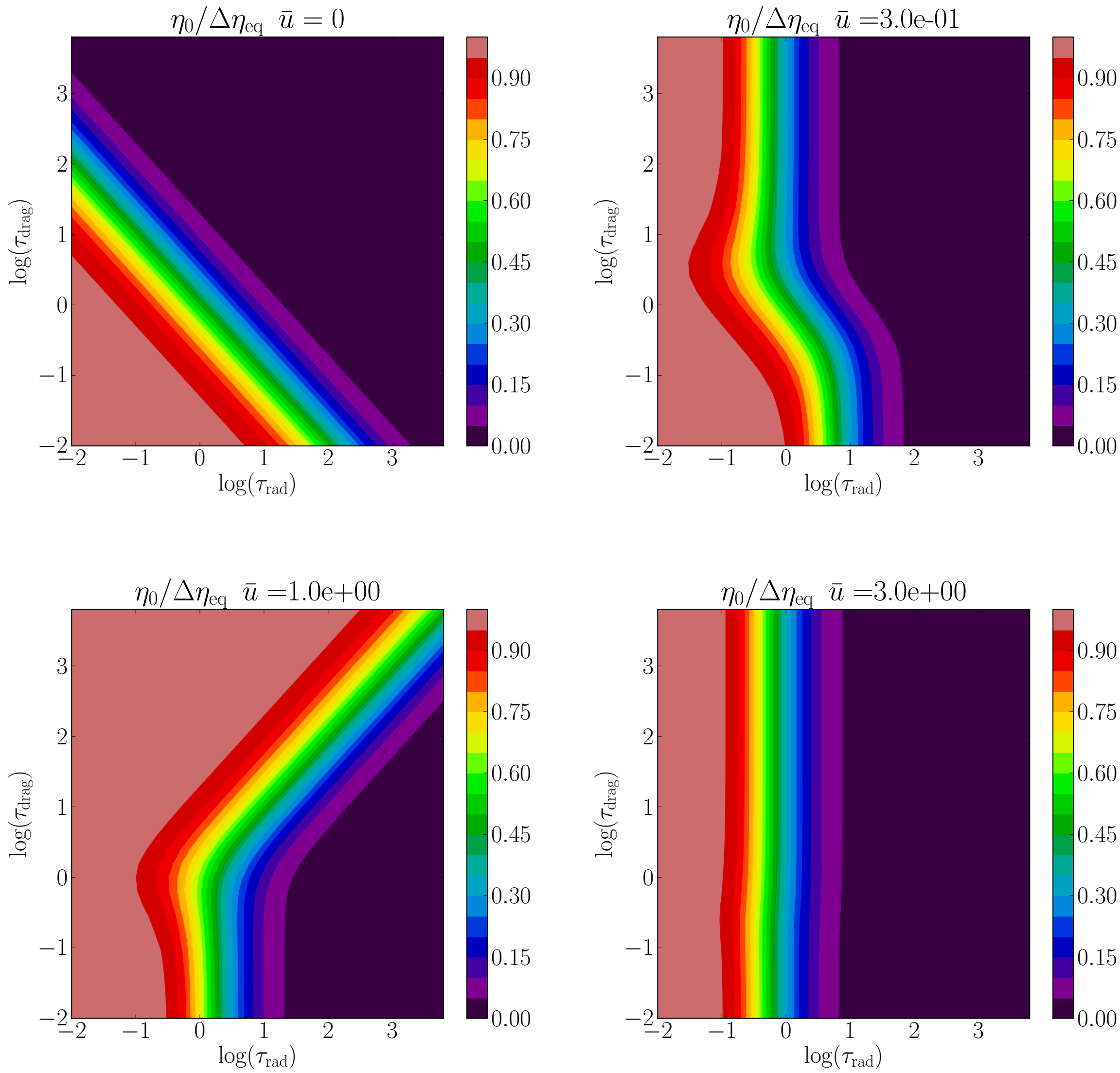

Fig. 1.— my caption

irrelevant.

Which one applies to hot Jupiters? Advection jet speeds are between 1 and 3 km/s and the wave speed for the same planets is 2 km/s. We are in the regime where both terms are important.

New plot: Advection over the radiative timescale: Each plot for a different radiative timescale (all three reasonable assumptions for hot Jupiters). All three limits can be present:

advetion dominated regime, phase maching resonant behaviour, regime where wave transport dominates. (solution depend strongly on drag)

2.2. Linearized 2D Shallow Water Model

Compare toy model with simulations of ever increasing complexity. First with 2D shallow water model in the linear regime. In the linear regime, the shallow water equations can be solved analytically Showman & Polvani (2011).

If the raditive and drag time are very short, the solution matches the forcing. (Forcing hotspot on the center of the substellar point and cooling on the nightside). As you increase either drag or the radiative timescale, you can see nice rosby and kelvin patterns developing, which cause a superrotating jet. When both timescales are high, the solution dominated by off-equator cyclones and anticyclones.The amplitude is small.

If you do a plot in the same parameter space as we did before, you can see it compares well. We can only compare these two frames, because the know solution is linearized about a state of rest. For the zero-advection speed the qualitative match is quite good.

2.3. Fully Non-Linear 2D Shallow Water Model

We can also solve the full shallow water equations numerically. There solution differs substantially from the linear behavior and you can make the same comparison. (with finite mean motions). You see the kink appearing.

Make a cut through the data and you can see solution saturating. You can qualitative comparisions with previous plot.

From the numerical solutions, we can make lightcurves of how the planet would look like when viewed from Earth. Phase shift of the hotpsot in the parameter space. Negative (purple) corresponds to an eastward shift in the jet and high values correspond to westward shifted hotspots. For westward shifts, the contrast between the day and nightside temperature is extremely low. Eastward hotspots are easier to detect because they generally have larger amplitudes.

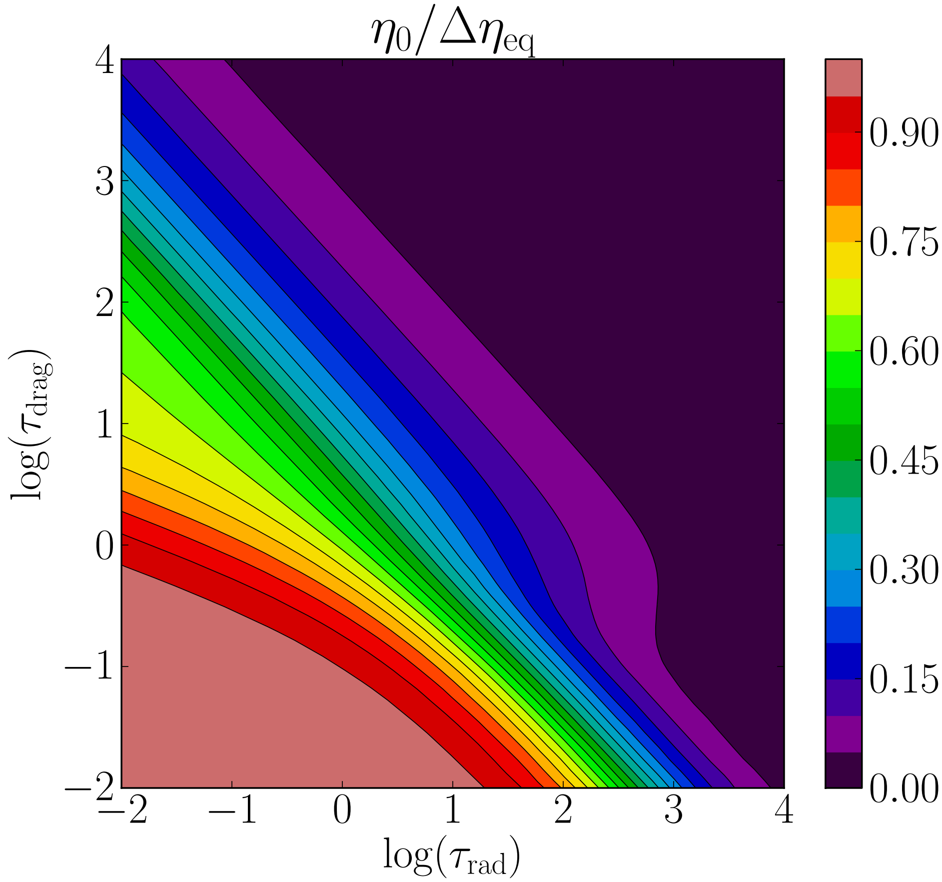
$\eta_0/\Delta\eta_{eq}$
4
3
2
1
0
−1
−2
$\log(\tau_{drag})$
−2
−1
0
1
2
3
4
$\log(\tau_{rad})$
0.90
0.75
0.60
0.45
0.30
0.15
0.00

Fig. 2.—

3. OUTLOOK

We are working on how to best digest our solution into meaningful statements.

Talk about rossby deformation radius:

Natural length scale over which flows can occur on planets. Wave behavior occurs on these length scales.

4. Acknowledgments

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REFERENCES

Cooper, C. S., & Showman, A. P. 2005, ApJ, 629, L45

—. 2006, ApJ, 649, 1048

Knutson, H. A., et al. 2007, Nature, 447, 183

Showman, A. P., & Guillot, T. 2002, A&A, 385, 166

Showman, A. P., & Polvani, L. M. 2011, ApJ, 738, 71

Vallis, G. K. 2006, Atmospheric and Oceanic Fluid Dynamics, ed. Vallis, G. K.
