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# Application of Voting Geometry to Multialternative Choice

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## Abstract

This paper presents an application of voting geometry to individual decision making. We demonstrate that a number of decision anomalies can arise as a natural consequence of the aggregation of preferences of different neural systems. We present a proof of existence of a set of voting procedures that can account for the attraction effect, the similarity effect and the compromise effect, and provide an example of one such procedure in the form of a modified Borda count. The result is an original closed form computational model of multialternative choice.

**Keywords:** Voting theory; decision-making; Borda count; multialternative choice;

## Introduction

For nearly 50 years, psychologists have been cataloguing violations of the standard assumptions of economic theory: that humans exhibit rational, stable, and ordered preferences. As the field has come to a consensus that the rational model is a poor descriptor of human behavior, researchers have moved from documenting anomalies to attempting to model them.

To date, the most successful attempt to model decision anomalies has been Decision Field Theory (DFT), introduced by Busemeyer and Townsend (1993). Using a sequential sampling approach, DFT can account for three of the most puzzling decision anomalies: compromise effects, attraction effects, and similarity effects. Busemeyer and his colleagues have proven that there exists no weighting function for a utility model that can effectively capture all three of these phenomena (Roe, Busemeyer, Townsend, 2001). While DFT remains the premier computational approach for multi-attribute choice, recent criticism about the biological plausibility of the model (Usher & McClelland, 2004) has led researchers to try and account for the "big three" decision anomalies using different techniques.

In this paper we propose a model for multi-attribute choice derived from the principles of voting geometry. We assume a number of neural systems within an individual's brain (i.e. "agents"), which differentially respond to different attributes of choice. For example, some agents

may attempt to maximize payoff while others prefer to minimize risk. We argue that the "big three" decision anomalies come about as a natural consequence of aggregating preferences across different agents.

## The Big Three

A very simple laboratory example will be used to represent human decision-making in real life (c.f. Roe et al., 2001). In this example, the choice set includes a limited number of choice options that vary on two attributes. This representation simplifies the demonstration of the anomalies and makes it easier to analyze decision-outcomes, but the findings are applicable to more complex real-life choice problems as well (Roe et al., 2001). The choice options are represented in terms of different pairs of shoes that vary on two attribute dimensions: comfort and style. The ideal shoe would be closest to the upper right hand corner of the diagram, where the shoes are most stylish and at the same time most comfortable. Figure 1 represents the "shoe" - choice set, which will be used throughout the entire paper.

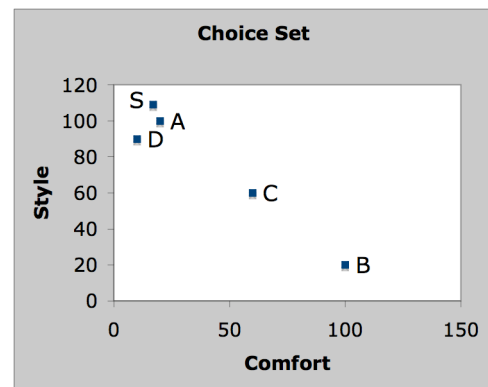


Figure 1: The choice set

Figure 1 provides a geometric representation of a hypothetical choice set. Options A, S, and D are in the upper left hand corner, which means that they are high on the style attribute but low on comfort. D is completely dominated by A, while S is similar to A but slightly better in style and

slightly worse in comfort. Option C lies exactly between A and B and is thus a compromise of the two dimensions. Choice B is high on comfort but low on the style attribute.

**Attraction effect:** The attraction effect, introduced to the literature by Huber, Payne and Pluto (1982), occurs when a choice set of two dissimilar options receives a new alternative that is completely dominated by one of the two options. This increases the attractiveness of the dominant option (Simonson, 1989). Consider, for instance, that a new brand is introducing a pair of stylish but uncomfortable shoes on the market. This new choice option D (Figure 1) is completely dominated by option A, which is both more stylish and more comfortable than D. In this case the probability of choosing the dominant option A will increase after D is added to {A, B} (Roe et al., 2001). Thus, the introduction of an asymmetrically dominated decoy leads to the following preference inconsistency:  $\Pr[A|\{A,B\}] < \Pr[A|\{A, D, B\}]$ . This violates the principle of regularity: that the preference for one option cannot be increased by the introduction of a new option (Simonson, 1989).

**Compromise effect:** The compromise effect (Simonson, 1989) occurs when a new option is introduced into a choice set of two dissimilar options, and falls in between those two options on all relevant dimensions, thus acting as a “compromise” option. Typically, the probability of choosing the compromise option is greater than the probability of choosing either of the extremes. Consider the introduction of a shoe that is moderately stylish and comfortable, represented as C in Figure 1. When all three options A, B, and C are available, the probability of choosing the compromise option C is greater than the probability of choosing either of the two extremes:  $\Pr[A|\{A, B\}] = \Pr[A|\{A, C\}] = \Pr[B|\{B, C\}]$  but  $\Pr[C|\{A, B, C\}] > \Pr[A|\{A, B, C\}]$  and  $\Pr[C|\{A, B, C\}] > \Pr[B|\{A, B, C\}]$  (Roe et al., 2001). Thus, the attractiveness of option C is enhanced by the presence of A and B.

**Similarity effect:** The similarity effect, first noted by Tversky (1972), occurs when a new option is introduced to a choice set containing two dissimilar options. This new option is very similar to one of the original options but neither dominates it, nor is dominated by it. In the case of the two options A and B in Figure 1, the similarity effect is produced by the introduction of option S. Shoe S closely resembles A, but it is better on the style attribute and worse on the comfort attribute. The decision-maker has to decide between the stylish yet uncomfortable shoes A and S and the very comfortable, but unstylish shoe B. The empirical finding in this case is that the introduction of the shoe S will take away more buyers from shoe A than from shoe B. This can lead to the following preference reversal:  $\Pr[A|\{A, B\}] > \Pr[B|\{A, B\}]$  but  $\Pr[B|\{A, S, B\}] > \Pr[A|\{A, S, B\}]$  (Roe et al., 2001). This is an anomaly because the preference order of options A and B should be independent from the presence of option S.

## Voting Theory

### Basic characteristics

Just as different neurons and neural systems in the visual system can preferentially respond to stimuli of a particular shape or orientation, we posit that different neurons and neural systems in the frontal cortex (and other decision centers; a.k.a. “agents”) might prefer different attributes among a set of options. Some may attempt to reduce risk, others might respond to the maximum possible payoff, etc. The decision outcome for the organism as a whole depends on the aggregation and evaluation of the preferences of the individual agents. The collection of this information from multiple, non-dominant agents can be modeled using voting geometry (Saari, 1995). There are an infinite number of different procedures for the aggregation of preferences, but voting geometry can constrain the space of possible procedures and explain how individually rational agents can lead to election outcomes that correspond to decision-making anomalies.

### Procedure Lines

With a choice set of three choices A, B, and C, there are six possible preference profiles for each agent:

1.  $A > B > C$
2.  $A > C > B$
3.  $C > A > B$
4.  $C > B > A$
5.  $B > A > C$
6.  $B > C > A$

These profiles can be represented geometrically in a triangle, with each vertex representing a choice option. The triangle can then be divided into six equally large regions, which represent the specific profiles.

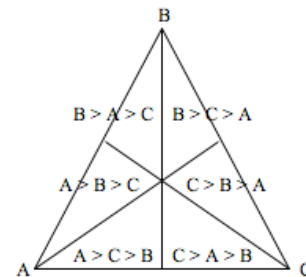


Figure 2: Geometrical representation of profiles

Procedure lines are the geometrical representation of voting outcomes when employing different aggregation procedures for a set of preference profiles (Saari, 1995). A positional election with the three candidates A, B, and C is defined by the voting vector  $w_s = (w_1, w_2, w_3) = (1, s, 0)$  where  $0 \leq s \leq 1$ . For a given voting procedure, each choice option receives a number of points reflecting its ranking. For example, the plurality vector assigns one point to the top ranked choice, and no points to any other option:  $w_p^3 = (1, 0, 0)$ . The antiplurality vector assigns points to all

but the lowest ranked option:  $W_{AP}^3 = (1,1,0)$ . For voting procedures which allow fractional points to middle options (e.g. the Borda Count), voting vectors are normalized (Saari, 1995).

The procedure line – a geometric representation of the possible outcomes that can be generated across all voting procedures for a given preference distribution – is defined by the line segment connecting the plurality vector and the antiplurality vector. Intuitively, since the plurality and antiplurality procedures represent the two extremes (antiplurality gives full credit to the 2<sup>nd</sup> place option while plurality gives none), every other procedure will lie between those two points.

Consider an example in which 1/2 of the voters prefer  $A > B > C$ , 1/3 prefer  $B > C > A$  and 1/6 prefer  $B > A > C$ . The plurality vote assigns one point to the top-ranked choice, which is A in the first profile and B in the second and third profiles. C does not receive any points. The points are multiplied with the fraction of votes received by the choice option, thus yielding the following normalized plurality vector:  $w_p = (\frac{1}{2}, \frac{1}{3} + \frac{1}{6}, 0) = (\frac{1}{2}, \frac{1}{2}, 0)$ . The antiplurality is

computed in the same way, but by assigning half a point to all the options except for the bottom ranked one. Thus, the normalized antiplurality vector in this case is  $w_{AP} = (\frac{1}{4} + \frac{1}{12}, \frac{1}{4} + \frac{1}{6} + \frac{1}{12}, \frac{1}{6}) = (\frac{1}{3}, \frac{1}{2}, \frac{1}{6})$ . The formula

developed by Saari (1995) to compute the procedure line is  $(1 - 2s)$  (plurality vector) +  $2s$  (antiplurality vector), with  $s \in [0, \frac{1}{2}]$ . This yields a line that can be geometrically represented inside the triangle represented in Figure 2. The voting regions crossed by the procedure line reflect all the possible voting outcomes employing different procedures. Using the computational approach of procedure lines, it is now possible to look at the different predictions of the voting model for the similarity, attraction, and compromise effects, depending on the voting procedure.

**Procedure line reflecting the attraction effect:** Consider the choice set A, B, and D in Figure 1. A and D are similar but A is a dominant alternative. Thus, no matter what dimension an agent cares about, A will always be preferred to D. The attraction effect refers to the fact that the introduction of D will increase the probability of choosing A. For simplicity and clarity, in this demonstration it is assumed that the two profiles  $A > B$  and  $B > A$  are equiprobable (although the logic holds even without this assumption). The introduction of D into choice set {A, B} will divide the agents who previously had the preference  $A > B$  into two groups:  $A > B > D$  and  $A > D > B$ . The agents that had the previous profile  $B > A$  only have one possible profile:  $B > A > D$ , and consequently all the agents that chose  $B > A$  will now choose  $B > A > D$ . This is because A dominates D and therefore the profile  $B > D > A$  is impossible because  $D > A$  will never be chosen. This leads to the following preference distribution:

1.  $\frac{1}{4} A > D > B$
2.  $\frac{1}{4} A > B > D$
3.  $\frac{1}{2} B > A > D$

This yields the plurality vector:  $w_p = (\frac{1}{2}, \frac{1}{2}, 0)$  and the

antiplurality vector  $w_{AP} = (\frac{1}{2}, \frac{3}{8}, \frac{1}{8})$ . Thus

$$PL = (1 - 2s)(\frac{1}{2}, \frac{1}{2}, 0) + 2s(\frac{1}{2}, \frac{3}{8}, \frac{1}{8}) = (\frac{1}{2}, \frac{2-s}{4}, \frac{s}{4}) | s \in [0, \frac{1}{2}]$$

The plurality outcome lies exactly between A and B on the edge of the triangle (Figure 3).

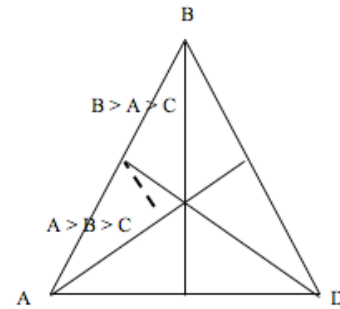


Figure 3: Procedure line for the attraction effect

This indicates that, depending on the voting procedure employed, the outcome will be either  $A \sim B > D$  or  $A > B > D$ . The procedure line (Figure 3) indicates that with the plurality procedure, A and B will be chosen equally often, whereas with any other procedure, the outcome of the vote will be  $A > B > D$ , which is the preference structure of the attraction effect. Thus, the attraction effect arises naturally from a voting model, nearly independent of the voting procedure adopted.

**Procedure line reflecting the compromise effect:** Now consider the choice set A, B, and C in Figure 1. C lies exactly between options A and B, reflecting the fact that it is a compromise between the two. Although C is inferior to one of the other options in each attribute, according to the characteristics of the compromise effect, the probability of choosing C should be higher than the probability of choosing either A or B.

The presence of C in choice set {A, B, C} will divide the previous two preference profiles  $A > B$  and  $B > A$  in the following way: The agents that preferred  $A > B$  care more about the “style” dimension than the “comfort” dimension. Since C is greater than B on style, those agents will always prefer C to B. Agents that strongly prefer style to comfort will prefer A to C and will now have the profile  $A > C > B$  while others will prefer a balance of style and comfort and will have the profile  $C > A > B$ . The analog occurs for agents that had preferred  $B > A$ : the two possible profiles are  $B > C > A$  and  $C > B > A$ . The most conservative approach to this demonstration is to consider an equal number of agents that value each dimension. This distribution leads to the following preference profiles:

1.  $\frac{1}{3} A > C > B$
2.  $\frac{1}{3} B > C > A$

3.  $1/6 C > A > B$
4.  $1/6 C > B > A$

The plurality vector is  $w_P = (\frac{1}{3}; \frac{1}{3}; \frac{1}{3})$  and the antiplurality vector is  $w_{AP} = (\frac{1}{4}; \frac{1}{4}; \frac{1}{2})$ . Thus,

$$PL = (1-2s)(\frac{1}{3}; \frac{1}{3}; \frac{1}{3}) + 2s(\frac{1}{4}; \frac{1}{4}; \frac{1}{2}) = (\frac{2-s}{6}; \frac{2-s}{6}; \frac{1+s}{3}) | s \in [0; \frac{1}{2}]$$

The plurality outcome lies at the barycenter of the triangle, thus yielding the outcome  $A \sim B \sim C$  (indifference). The antiplurality outcome indicates that  $C > A \sim B$ . The line segment representing all the different election outcomes lies on the midline between A and B (Figure 4).

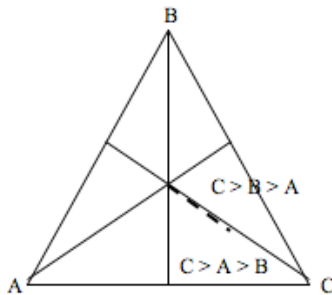


Figure 4: Procedure line for the compromise effect

Thus, if the voting model employs plurality vote, then the outcome will be complete indifference between the three choice options, and each of them will get selected equally often. With any other voting procedure, the outcome is  $C > A \sim B$ , which is the preference structure of the compromise effect. Thus, the compromise effect also arises naturally from a voting model, nearly independent of voting procedure adopted.

**Procedure line reflecting the similarity effect:** Finally, consider the choice set A, S, and B (Figure 1). A and S are both very similar, but A is better than S on the comfort attribute and S is better than A on the style attribute. According to the definition of the similarity effect, the introduction of S in the choice set {A, B} should decrease the probability of choosing A relative to the probability of choosing B. The introduction of S to choice set {A, B} will divide the agents who previously had the profile  $A > B$  into two groups:  $A > S > B$  and  $S > A > B$ . The fraction of votes received by each of these profiles depends on the distance between A and S. For simplification, it will be assumed that the votes are split evenly between them. The agents who previously had the profile  $B > A$  will be split into  $B > A > S$  and  $B > S > A$ , reflecting the fact that A and S are very similar to each other. This leads to the following preference distribution among the agents:

1.  $1/4 A > S > B$
2.  $1/4 S > A > B$
3.  $1/4 B > A > S$
4.  $1/4 B > S > A$

The plurality vector is  $w_P = (\frac{1}{4}; \frac{1}{2}; \frac{1}{4})$  and the antiplurality vector is  $w_{AP} = (\frac{3}{8}; \frac{1}{4}; \frac{3}{8})$ . Thus,

$$PL = (1-2s)(\frac{1}{4}; \frac{1}{2}; \frac{1}{4}) + 2s(\frac{3}{8}; \frac{1}{4}; \frac{3}{8}) = (\frac{1+s}{4}; \frac{1-s}{2}; \frac{1+s}{4})$$

The plurality outcome is  $B > A \sim S$  and the antiplurality outcome is  $A \sim S > B$ . The procedure line lies on the midline between A and S, thus indicating that no matter what procedure employed, the votes will be evenly split between these two options (Figure 5). This makes sense, since A and S are very similar.

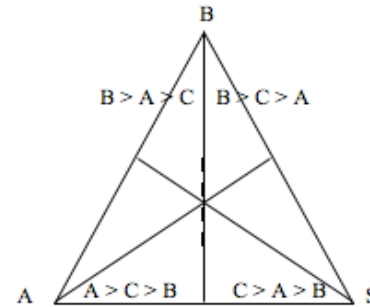


Figure 5: Procedure line for the similarity effect

The plurality vote is better at representing the fact that the introduction of choice option S into the choice set {A, B} will take away votes from A, but will not affect votes for B. The votes for A would thus be split in half by the introduction of S, and therefore the preference profile  $B > A \sim S$  best reflects the similarity effect.

### An example voting procedure: Modified Borda Count

The exploration of the procedure line not only allows an intuitive visualization of different voting outcomes across procedures, but also provides a valuable mathematical tool to determine what type of preference aggregation can account for the three decision-making anomalies. For example, a voting model cannot use a plurality vote, since the plurality procedure cannot account for the attraction or compromise effects. A Borda count procedure, which assigns 2 points to the top option, 1 point to the middle option and 0 point to the bottom option, seems to be well suited for the attraction and compromise effects, but it cannot fully account for the similarity effect. However a procedure that lies between Borda count and plurality vote will be able to account for the big three anomalies.

One such procedure would be a modification of the Borda count, assigning 3 points to the top ranked option (instead of 2 as in Borda count), 1 point to the middle-ranked option, and 0 points to the lowest ranked option. This procedure assures that the second-ranked option still receives points, but that the distribution places more emphasis on the top-ranked choice option. Looking at the procedure lines, this

modified Borda count can account for the three anomalies of decision-making. The procedure lies between the Borda count and a plurality vote, because it assigns more weight to the top-ranked option than Borda count, but not as much weight to the top option as the plurality vote (which does not give any points to the second-ranked option at all). Of course, this procedure is only one example of an infinite number of aggregation processes lying between Borda count and plurality. However, this procedure will serve as a demonstration of how voting models can account for the three effects.

**How Voting Theory accounts for the Attraction effect:**

Consider the choice set A, D, and B (Figure 1) with the three possible profiles  $A > D > B$ ,  $B > A > D$ , and  $A > B > D$ . These profiles reflect the fact that A always dominates D. As discussed earlier the preference distribution for this case is as follows:

Borda	1/4	1/4	1/2
3	A	A	B
1	D	B	A
0	B	D	D

With n agents:  
 $A = 2n$   
 $B = 7/4n$   
 $D = 1/4n$

For  $n$  agents, the modified Borda Count procedure yields  $2n$  points for option A ( $.25*3+.25*3+.5*1$ ),  $1.75n$  points for option B ( $.5*3+.25*1+.25*0$ ), and  $.25n$  points for D ( $.25*1+.25*0+.5*0$ ). This shows that the presence of D increases the attractiveness of option A in contrast to B. Thus,  $\Pr[A|\{A,B\}] < \Pr[A|\{A,B,D\}]$ , characteristic of the attraction effect, which violates the principle of regularity.

**How Voting Theory accounts for the Compromise effect:**

Now, consider the choice set A, B, and C in Figure 1, with C being the compromise option between A and B. The profiles in this case are  $A > C > B$ ,  $C > A > B$ ,  $C > B > A$  and  $B > C > A$ . As discussed earlier, the preference distribution among the four possible profiles is as follows:

Borda	1/3	1/6	1/6	1/3
3	A	C	C	B
1	C	A	B	C
0	B	B	A	A

With n agents:  
 $A = 7/6n$   
 $B = 7/6n$   
 $C = 10/6n$

Assuming that there are  $n$  agents contributing to the decision outcome, the voting procedure using the modified Borda count will assign  $7/6n$  points to options A and B ( $0*1/3+0*1/6+1*1/6+3*1/3$ , for each) and  $10/6n$  points to option C ( $1*1/3+1*1/3+3*1/6+3*1/6$ ), reflecting the fact that C will get chosen more often than either A or B. The profiles also reflect the fact that in the absence of option C, options A and B will receive the same amount of points. Thus, the organism is not biased toward either one of the options at the beginning of the vote, so  $\Pr[A|\{A, B\}] = \Pr[A|\{A, C\}] = \Pr[B|\{B, C\}]$  but  $\Pr[A|\{A, B, C\}] < \Pr[C|\{A,$

$B, C\}]$  and  $\Pr[B|\{A, B, C\}] < \Pr[C|\{A, B, C\}]$ . This violation of the independence of irrelevant alternatives (Tversky & Simonson, 1993) is characteristic of the compromise effect.

The fractions of agents representing each profile were chosen in the most conservative way possible. However, their manipulation will allow us to make predictions about the behavior of the compromise effect when those fractions change. In the case when A and B move farther away from C, the relative fraction of agents representing C should increase as well. The increase in distance between the extreme options will thus lead to a stronger compromise effect. This is the opposite prediction of multialternative decision field theory (Roe et al., 2001) and suggests an empirical method for dissociating the models.

**How Voting Theory accounts for the Similarity effect:**

Finally, consider the choice set A, B and S (Figure 1), with S being very similar to A. The possible profiles in this case are  $A > S > B$ ,  $S > A > B$ ,  $B > A > S$  and  $B > S > A$ . As discussed earlier, the preference distribution among the four possible profiles is as follows:

Borda	1/4	1/4	1/4	1/4
3	A	S	B	B
1	S	A	A	S
0	B	B	S	A

With n agents:  
 $A = 5/4n$   
 $B = 6/4n$   
 $S = 5/4n$

Assuming  $n$  agents, the modified Borda count procedures will assign  $5/4n$  points to options A and S ( $3*1/4+1*1/4+1*1/4+0*1/4$ , for each), and  $6/4n$  to option B ( $3*1/4+3*1/4+0*1/4+0*1/4$ ). This vote reflects the characteristics of the similarity effect: the introduction of S into the choice set  $\{A, B\}$  takes away points from A and thus creates the following preference reversal:  $\Pr[B|\{A, S, B\}] > \Pr[A|\{A, S, B\}]$ . The modified Borda count procedure unlike the traditional Borda count is thus able to model the similarity effect accurately.

When a compromise option such as option C (discussed previously) moves in the direction of one of the two options and thus becomes more similar to it, the compromise effect turns into the similarity effect, as the two similar options will have to share the votes. Thus, moving an option from position C to position S (Figure 1) cancels out the compromise effect. B will receive more points than either A or S, as shown here. The fractions of agents representing each profile were chosen in the most conservative way possible for the present demonstration and the results tend to be robust across preference ratios. However, their manipulation will allow us to make predictions about exactly when a compromise option becomes similar enough to another option to cease invoking the compromise effect. By analyzing the fraction of agents representing a profile and the influence of manipulating distances on this fraction, it is possible to model the transition between different

effects and make predictions about the decision strategies employed. In other words, voting models can be used to make novel predictions about the boundary conditions and relationships between different decision anomalies.

### Conclusion

The mathematical analysis of procedure lines proved the existence of voting procedures that can account for the attraction, the similarity and the compromise effect. One example, a modified Borda Count has been presented here. Voting geometry provides a novel approach to individual decision-making and is attractive for various reasons. First, a connectionist model based on voting geometry seems plausible from a biological perspective. The assumption of multiple independent agents parallels the literature on multi-agent systems, which have been broadly applied to a number of problems of cognition (e.g. Sun, 2001). Additionally, the theory is easily extendable and could accordingly provide explanations for other phenomena, such as the employment of different decision-making strategies under time constraints.

Consider for example the procedure line representing the compromise effect (Figure 4). Decision Field Theory predicts that the difference of choice probability between the compromise option C and the extreme options A and B increases with deliberation time (Roe et al., 2001). In other words, the longer one deliberates, the stronger the preference of C over A and B. This effect can be modeled by moving toward option C on the procedure line in Figure 4, which means that the procedure employed becomes more and more like the antiplurality vote. In other words, under strong time constraints, only an agent's top choice is considered; as more time is spent in deliberation, secondary options are given more weight. This logic provides a computational account of the model and empirical findings of Payne, Bettman, and Johnson's (1993) notion of the adaptive decision maker.

Future extensions of the model could come in many forms. Currently the model has no way to take into account memories of previous preference states. Additionally, implementations of this model should consider the stochastic nature of choice (Busemeyer & Townsend, 1993). Ideally the model would predict probability distribution across choices as opposed to simply determining the most frequently chosen option. It would also be worthwhile to look at additional anomalies aside from the big three, and see if there might be further constraints on the space of possible voting procedures that could account for human decision making. Future research might also focus on exploring the effects of different assumptions about the distributions of preferences of agents, and considering alternative voting procedures.

Although the present voting model is very simplified, it can nonetheless account for the three main anomalies found in decision-making. The next step in the exploration of the voting model would thus be to construct a connectionist model of the voting procedure and to look at the predictions

made by the model for other phenomena, such as strategy switching and the evolution of decision-outcomes over time. Future work should also attempt to empirically tease apart the predictions of a voting model and those from other computational approaches to multi-attribute choice (Roe et al, 2001; Usher & McClelland, 2004).

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