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COMMENTS ON A THREE BODY MODEL OF STRIPPING†

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The three body model of stripping proposed by Pearson and Coz is discussed. It is shown that in a (d,p) reaction the assumption that the neutron-proton interaction does not affect the state of the proton and only provides an average potential for neutron capture leads to a prediction of vanishing of the transition amplitude.

Introduction

Recently, Pearson and Coz [1] proposed a new theory of stripping reactions. The theory has since been extended by Bang and Pearson [2] and its predictions compared with experimental angular distributions and polarization in (d,p) reactions. Unlike the conventional DWBA description of the deuteron stripping reactions [3], the new theory assumes that in the entrance channel, the proton and neutron scatter independently off the target nucleus, their scattering amplitudes being weighted by the momentum distribution of the bound state of the deuteron. In the exit channel, the proton elastically scatters off the target while the neutron is captured under the influence of the averaged neutron-proton interaction.

In actual calculations, an additional assumption has been made in order to further simplify the integrals appearing in the expression of the transition

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amplitude. The assumption is that, in a (d,p) reaction, the neutron-proton interaction allows only forward scattering of the proton. In this paper, we analyze the consequence of the above assumption.

The Formalism

We shall, for simplicity, consider the case where the target nucleus can be treated as an inert core. We are then dealing with a three body system. The Hamiltonian of the system is given by

$$H = K_n + K_p + V_n + V_p + V_{np} \tag{1}$$

where K_n and K_p are respectively the kinetic energy operators for the neutron and proton, V_n and V_p represent their respective interaction with the core, and V_{np} is their mutual interaction. We shall consider the case where the initial state comprises of a free proton of momentum $\hbar\vec{k}_p$ incident on a bound state of the neutron and the core. The asymptotic initial state $|\Phi_{op}\rangle$ has the coordinate representation given by

$$\begin{aligned} \Phi_{op}(\vec{r}_n,\vec{r}_p) &= \langle\vec{r}_n\vec{r}_p|\Phi_{op}\rangle \\ &= (2\Lambda)^{-3/2} \exp(i\vec{k}_p\cdot\vec{r}_p)\phi_n(\vec{r}_n) \end{aligned} \tag{2}$$

where $\phi_n(\vec{r}_n)$ represents the bound state of the neutron-core system, and satisfies the equation of motion

$$(\epsilon_n + K_n + V_n)\ \phi_n(\vec{r}_n) = 0 \tag{3}$$

where $-\epsilon_n$ is the binding energy of the neutron. The total energy of the system is given by

$$E = -\epsilon_n + \frac{\hbar^2 k_p^2}{2m} .$$

$|\Phi_{op}\rangle$ is an eigenstate of the asymptotic channel Hamiltonian H_{op} given by

$$H_{op} = K_n + K_p + V_n \tag{4}$$

One can define channel Hamiltonians corresponding to the various rearrangements, i.e.;

$$H_{od} = K_n + K_p + V_{np}$$

describes the deuteron channel, and

$$H_{on} = K_n + K_p + V_p$$

describes the neutron channel. The exact scattering wavefunction $|\Psi_p^{(+)}\rangle$ is an eigenstate of the total Hamiltonian and satisfies the many channel Lippman-Schwinger equation

$$|\Psi_p^{(+)}\rangle = |\Phi_{op}\rangle + \frac{1}{E^+ - H_{op}} (V_p + V_{np})|\Psi_p^{(+)}\rangle \tag{5a}$$

$$= \frac{1}{E^+ - H_{of}} (H - H_{of})|\Psi_p^{(+)}\rangle , \; f \neq p \tag{5b}$$

i.e.; it is a solution of an inhomogeneous integral equation in the entrance channel and of homogeneous equations in all the other channels.

If the target is treated as an inert core, one could rewrite $|\Psi_p^{(+)}\rangle$ in a distorted wave series by defining the distorted wavefunction by

$$|\Psi_{op}^{(+)}\rangle = |\Phi_{op}\rangle + \frac{1}{E^+ - H_{op}} V_p |\Psi_{op}^{(+)}\rangle \tag{6}$$

$|\Psi_{op}^{(+)}\rangle$ is a product of the elastic scattering state of the proton and the bound state of the neutron, i.e.;

$$\Psi_{op}^{(+)}(\vec{r}_n,\vec{r}_p) = \chi_{\vec{k}_p}^{(+)}(\vec{r}_p)\ \phi_n(\vec{r}_n) \tag{7}$$

where $\chi_{\vec{k}_p}^{(+)}(\vec{r}_p)$ is the elastic scattering wavefunction of the proton due to the V_p interaction. The exact wavefunction $|\Psi_p^{(+)}\rangle$ can be expressed as a solution of the modified integral equation

$$|\Psi_p^{(+)}\rangle = |\Psi_{op}^{(+)}\rangle + \frac{1}{E^+ - H_{op} - V_p} V_{np} |\Psi_p^{(+)}\rangle \tag{8}$$

The Green's function $(E^+ - H_{op} - V_p)^{-1}$ describes the propagator of a neutron and a proton in interaction with the core and not with each other. It has the form

$$G(\vec{r}_n\vec{r}_p|\vec{r}_n{}'\vec{r}_p{}') = \langle \vec{r}_n\vec{r}_p|(E^+ - H_{op} - V_p)^{-1}|\vec{r}_n{}'\vec{r}_p{}'\rangle$$

$$= \mathop{\mathcal{L}t}_{\eta\to 0^+} \iint d^3q_p\ d^3q_n \frac{\chi_{q_p}^{(+)}(\vec{r}_p)\ \chi_{q_n}^{(+)}(\vec{r}_n)\ \chi_{\vec{q}_p}^{(+)*}(\vec{r}_p{}')\ \chi_{\vec{q}_n}^{(+)*}(\vec{r}_n{}')}{E - \frac{\hbar^2}{2m}(q_p{}^2 + q_n{}^2) + i\eta} \tag{9}$$

where the integral also includes the sum over discrete bound states of the neutron and proton. Levin [4] has studied the structure of the above Green's function and shown how to treat the limit $\eta \to 0^+$.

If we now use the approximation made by Pearson and Bang,

$$\langle \chi^{(+)}_{\vec{q}_p} | V_{np} | \chi^{(+)}_{\vec{q}_p{}'} \rangle = \delta(\vec{q}_p - \vec{q}_p{}') \, U_n(\vec{q}_p, \vec{r}_n) \tag{10}$$

it can be shown that the solution of eq. 8 is of the form

$$\langle \vec{r}_n \vec{r}_p | \Psi^{(+)}_p \rangle = \chi^{(+)}_{\vec{k}_p}(\vec{r}_p) \, \Phi_n(\vec{r}_n) \tag{11}$$

where $\Phi_n(\vec{r}_n)$ satisfies the equation of motion

$$(\epsilon_n + K_n + V_n + U_n(\vec{k}_p)) \, \Phi_n(\vec{r}_n) = 0 \tag{12}$$

$\Phi_n(\vec{r}_n)$ represents the bound state of the neutron under a combined influence of its interaction with the core and an averaged neutron-proton interaction. Comparing eqs. 3 and 12, we find that ϕ_n and Φ_n have the same eigenvalue even though they see different potentials. This is possible only for very specific values of U_n. In other words, for arbitrary forms of U_n one would not obtain normalizable solutions of eq. 12.

In conclusion, unless both the neutron and proton are scattered in nonforward directions by their mutual interaction, no scattering or rearrangement is possible. If the mass of the core is finite, the above conclusion is not valid. However, it is possible to show that the corresponding transition amplitudes will be of the order of magnitude $\mathfrak{M}/M$ where $\mathfrak{M}$ is the nucleon mass and M is the mass of the core. Hence, we would be calculating a small recoil correction and not the main transition amplitudes. All of the above conclusions remain valid for a physical target nucleus if we neglect its inelastic excitations due to its interaction with the neutron and proton.

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