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ABSTRACT

It has recently been shown that one can explain the collective enhancement of the quadrupole transition in C^{12} by invoking correlations in the ground state of C^{12}. The effect of these ground state correlations on the single particle transition rates in C^{13} have been studied.

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INTRODUCTION

During recent years, successful attempts have been made in accounting for collective effects in nuclei starting from the basis of the shell model. Ferrell and Visscher[1] and Fallieros and Ferrell[2] have shown how the hole-particle interactions can build up correlations in doubly closed shell nuclei. The large numerical work that is required in making a complete calculation has been overcome by the schematic model proposed by Brown and Bolsterli[2] and the above method has been applied successfully to interpret the dipole, quadrupole and octupole states of doubly closed shell nuclei.[3]

The shell model calculations of the collective states of closed shell nuclei have stressed the fact that it is essential to introduce correlations in the ground state in order to interpret the enhancement of the collective transitions. An explicit calculation has been made for the amount of correlation in the ground state of C^{12} by Goswami and Pal[4]. They have shown that there is a large correlation in the ground state, the major part of which is due to the quadrupole vibration.

In the present work, we have attempted to study the effect of the correlated ground state of C^{12} on the single particle transition rates in

C^{13}. In view of the fact that the amount of correlation in C^{12} calculated by Goswami and Pal[4] is likely to be an overestimate, our calculations are likely to give an upper limit on the effect on the transition rates.

THE FORMALISM

Let us denote the correlated ground state of C^{12} by ket $|\Phi_o\rangle$, and let $|\Phi_n\rangle$ denote all the other states of C^{12}. We assume that the low lying states of C^{13} can be treated as single particle states and designate them by $|\psi_n\rangle$ where

$$|\psi_n\rangle = a^{+}_{j_n m_n} |\Phi_o\rangle \qquad (1)$$

where $a^{+}_{j_n mn}$ is the creation operator for a single particle in the state of total angular momentum j_n and its projection m_n. The electromagnetic transition operators are single particle operators and can be designated by

$$O = \sum_{\alpha\beta} \langle\alpha| O|\beta\rangle \; a_{\alpha} + a_{\beta} \; . \qquad (2)$$

We are interested in matrix elements of the type $\langle\psi_p|O|\psi_n\rangle$ in:

$$\langle\psi_p|O|\psi_n\rangle = \sum_{\alpha\beta} \langle\Phi_o| a_{j_p m_p} a_{\alpha} + a_{\beta} \; a^{+}_{j_n m_n} |\Phi_o\rangle \langle\alpha|O|\beta\rangle \qquad (3)$$

It can easily be seen that the above matrix element reduces to the single particle value $\langle j_p m_p|O|j_n m_n\rangle$ if the state $|\Phi_o\rangle$ was uncorrelated. If one of the states $(j_p m_p)$ or $(j_n m_n)$ is absent in $|\Phi_o\rangle$.

Eq. (3) becomes:

$$\langle\psi_p|O|\psi_n\rangle = \sum_{\beta} \langle\Phi_o| a_{\beta} \; a^{+}_{j_n m_n}|\Phi_o\rangle \langle j_p m_p|O|\beta\rangle$$

$$=\sum_{\beta} (1-\nu_{j_n m_n})\, \delta_{\beta j_n m_n} \langle j_p m_p | O | \beta \rangle \qquad (4)$$

where $\nu_{j_n m_n}$ is the occupation probability of the state $j_n m_n$ in the ground state $|\Phi_o\rangle$. If the shell model treatment is valid, $\nu_{j_n m_n}$ should be negligible if it refers to a particle and again the matrix element takes on the single particle value.

If both $(j_n m_n)$ and $(j_p m_p)$ are present in the ground state $|\Phi_o\rangle$. We could rewrite Eq. (3) in the form

$$\langle \psi_p | O | \psi_n \rangle = \sum_{\substack{\alpha\beta \\ \ell}} \langle \Phi_o | a_{j_p m_p} a^{+}_{\alpha} | \Phi_\ell \rangle \langle \Phi_\ell | a_\beta a^{+}_{j_n m_n} | \Phi_o \rangle \langle \alpha | O | \beta \rangle \qquad (3')$$

where we have introduced a complete set of intermediate states $|\Phi_\ell\rangle$. The first term in the sum of Eq. (3') has a simple form

$$\langle \psi_p | O | \psi_n \rangle \simeq \sum_{\alpha\beta} \langle \Phi_o | a_{j_p m_p} a_{\alpha}^{+} | \Phi_o \rangle \langle \Phi_o | a_\beta a^{+}_{j_n m_n} | \Phi_o \rangle \langle \alpha | O | \beta \rangle$$

$$=\sum_{\alpha\beta} (1-\nu_{j_p m_p})\, \delta_{\alpha j_p m_p} (1-\nu_{j_n m_n})\, \delta_{\beta j_n m_n} \langle \alpha | O | \beta \rangle \qquad (5)$$

In order to account for the measurable departure of the transition rates from the single particle value, it is possibly essential to consider a large number of intermediate states and this will have the same effect as a collective enhancement.

In view of the fact that, in the first order, the ground state correlations in C^{12} are primarily due to the quadrupole vibration, we have considered the quadrupole state at 4.43 MeV as another possible intermediate state. The quadrupole state $|\Phi_2\rangle$ can be expressed as

$$|\Phi_2\rangle = Q^+|\Phi_o\rangle \qquad (6)$$

where Q^+ is the creation of an oscillator quantum. Thus the first correction to Eq. (5) is

$$\sum_{\alpha\beta} \langle \Phi_o | a_{j_p m_p} \, a_\alpha^{+} Q^+ | \Phi_o \rangle \, \langle \Phi_o | Q \, a_\beta \, a^{+}_{j_n m_n} | \Phi_o \rangle \, \langle \alpha | O | \beta \rangle . \qquad (7)$$

The single particle transition matrix elements are calculated for the $1f_{7/2}$, $1d_{5/2}$ and $2s_{1/2}$ states of C^{13} at 4.55 MeV, 3.85 MeV and 3.09 MeV respectively. The results are tabulated in Table 1.

RESULTS AND CONCLUSIONS

The results tabulated show the effect of ground state correlations on the single particle transitions in C^{13}. It is seen that there is negligible correction in transitions leading to the 1/2+ level at 3.09 MeV. The reason for this is that the ground state correlations in C^{12} do not contain the state $2s_{1/2}$. The corrections in the other transitions are small but not negligible. For a complete calculation, one possibly would have to consider the effect of the octupole and dipole states in the ground state correlation.

It is known that the single particle transition from the 1/2+ level to the 5/2+ ground state of O^{17} has a much larger probability than the single particle estimate. This has been explained by a weak coupling of the last odd particle to the collective oscillations of O^{16}. It would be of interest to study the effect of the ground state correlations of O^{16} on these transitions.

The presence of ground state correlation in C^{12} also allows for a mixing of multipole transitions in C^{13}. For instance, in the transition from the 3.09 MeV, 1/2+ state of C^{13} to its ground state, one would expect a pure E1 transition on the shell model picture. Because of the correlations in C^{12} ground state, we have also mixtures of E2 and E3 transitions, which, in this particular case, are of negligible importance. It would be interesting if one could find such examples in closed shell + one nucleon nuclei, for it would be an indication of a strong ground state correlation.

In conclusion, one should also study the effect of ground state correlation of closed shell nuclei on the energy levels of the neighboring nuclei. The ground state correlation could strongly influence the ordering as well as the energy spacing of these levels.

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FOOTNOTES AND REFERENCES

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Table 1. Matrix Elements Of Single Particle Transitions.

Initial Single Particle State	Final Single Particle State	Matrix Element in Units of $\langle f \vert O \vert i \rangle$
2s1/2	1p1/2	0.985
1d5/2	2s1/2	0.990
1f7/2	2s1/2	0.990
1d5/2	1p1/2	1.215 + other multiple mixtures
1f7/2	1p1/2	1.185 + other multiple mixtures
1f7/2	1d5/2	1.145 + other multiple mixtures