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Quarkyonic or Baryquark Matter

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ABSTRACT

It has been proposed that at high densities nuclear matter will consist of a Fermi sea of quarks surrounded by a small shell of confined baryon at the large momenta, so called Quarkyonic matter. In this contribution we will discuss an alternative configuration, dubbed Baryquark matter, which in a sense is a complement of Quarkyonic matter. Baryquark matter consists of a Fermi sea of confined baryons surrounded by a shell of deconfined quarks. Following Ref. [12] we will show that for certain (simplified) implementations Baryquark matter is energetically favored over Quarkyonic matter. We will then briefly discuss how the inclusion of the quark structure of nucleons will lead to a configuration which resembles the picture of Quarkyonic matter.

1. Introduction

The concept of quarkyonic matter [16], inspired by the expected QCD properties in the large N_c limit, is a realization of quark-hadron duality where both degrees of freedom appear as quasiparticles. Quarkyonic matter may be viewed as a mixed phase in momentum space consistent with the Pauli exclusion principle. Its main feature is that excitations around the Fermi surface are baryonic, which is realized by imposing a shell structure in the momentum space: a Fermi sea of quasi-free, "deconfined" quarks surrounded by a shell of confined baryons (see Fig. 1a).

Such a momentum shell structure may occur dynamically [17]: at small baryon densities, where nuclear interactions play a small role, the matter is purely baryonic since at fixed baryon density the energy per baryon of a free gas of nucleons is smaller than that of constituent quarks. At higher baryon densities, however, the hard-core repulsive interactions between nucleons become important and eventually make the existence of a quark Fermi sea more favorable relative to pure nucleon matter [10]. And, as the density increases, the quark Fermi sea becomes more and more dominant with the confined baryons residing in an increasingly thin baryon shell [10, 17]. The resulting equation of state exhibits a soft-hard evolution, with the speed of sound containing a peak exceeding the conformal limit, in line with neutron star phenomenology [1, 6, 22, 23]. Various extensions and applications of this picture were studied in recent years [2–5, 8, 9, 20, 21, 24], as well as how such a state of matter may emerge [13, 14, 18].

It is noteworthy, that the dynamical generation of the momentum space structure considered in Refs. [10, 17] has been performed under the *assumption* that a baryon Fermi shell on top of a quark Fermi sea is the only possible realization of the Pauli exclusion principle. However, the Pauli exclusion principle alone also permits other momentum space configurations and other configurations may very well be energetically favored over quarkyonic matter. And indeed, as shown in detail in Ref. [12], a complementary or opposite configuration with baryons at low momenta surrounded by quarks (see: 1(b)), subsequently dubbed baryquark matter, turns out to be energetically favored over the quarkyonic configuration. This picture is preserved also in the presence of both attractive and repulsive nuclear interactions [19]. In the following we will sketch the arguments and calculation of Ref. [12] and we will discuss how a more refined treatment of the Pauli principle leads to a quarkyonic configuration at high densities.

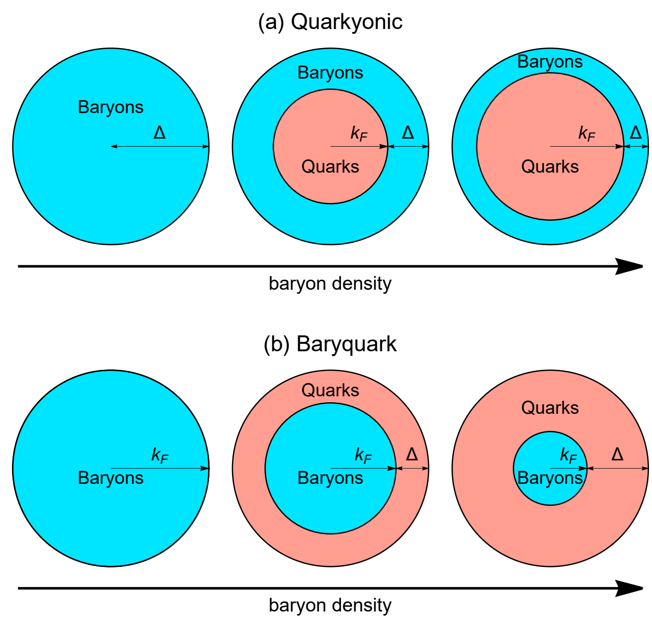

Figure 1: Evolution of the momentum shell structure in isospin symmetric (a) quarkyonic and (b) baryquark matter as a function of baryon number density at zero temperature. The momentum scales k_F and Δ are given for baryon degrees of freedom, i.e. for quarks they should be divided by N_c. Figure adapted from [12].

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2. Baryquark matter wins

To understand why baryquark matter is energetically favored can already be seen by considering non-interacting quarks and nucleons which are only subject to the Pauli exclusion principle. In the simple constituent quark picture, the mass and momentum of a nucleon is N_c times that of a constituent quark, $m_N = N_c m_Q$, $p_N = N_c p_Q$ The baryon density for a Fermi gas of nucleons with Fermi momentum p_f is given by and that for quarks with Fermi momentum q_f are given by

$$
\begin{aligned}
n_{B;N} &= \frac{4}{2\pi^2}\int_0^{p_f} k^2 dk = D\frac{p_f^3}{3} \\
n_{B;Q} &= \frac{1}{N_c}\frac{4}{2\pi^2}N_c\int_0^{q_f} q^2 dk = D\frac{q_f^3}{3}
\end{aligned} \tag{1}
$$

where $D = 4/(2\pi^2)$ is an overall constant taking into account the spin-isospin degeneracy which is the same for quarks and nucleons. Similarly, the energy densities are given by

$$
\begin{aligned}
\epsilon_N &= D\int_0^{p_f} k^2 dk\sqrt{m_N^2 + k^2} \\
&\simeq m_N n_{B;N} + D\frac{1}{2m_N}\frac{p_f^5}{5} \\
\epsilon_Q &= DN_c\int_0^{q_f} q^2 dq\sqrt{m_Q^2 + q^2} \\
&\simeq m_N n_{B,Q}(q_f) + D\frac{N_c^2}{2m_N}\frac{q_f^5}{5}
\end{aligned} \tag{2}
$$

Here we used that $m_N = N_c m_Q$ and assumed the Fermi momenta to be no too large.

Let us now consider a non-interacting system at a given baryon density with nucleons only, $n_B = D\frac{k_f^3}{3}$, where k_f is the Fermi momentum of the nucleons. Next, we replace some nucleons with quarks while keeping the baryon density fixed. In order to get quarkyonic matter we need to replace nucleons at the center of the Fermi sphere, whereas for baryquark matter we need to replace them at the Fermi surface. For a meaningful comparison of the two scenarios, it is useful to express the energy in terms of the quark fraction, f_Q,

$$
f_Q = \frac{\Delta n_{B;Q}}{n_B} \tag{3}
$$

where $\Delta n_{B,Q}$ denotes the contribution of quarks to the baryon density and $n_B = Dk_f^3/3$ is the total baryon density, which we will keep constant. Let us consider baryquark matter first. In this case, we add quarks and remove baryons, both on top of the Fermi sphere. Since all nucleon up to k_f are occupied, we can add quarks starting at $q = k_f/N_c$. Thus we have to leading order in Δq:

$$
\Delta n_{B;Q} = D\int_{k_f/N_c}^{k_f/N_c+\Delta q} q^2 dk \simeq D\left(\frac{k_f}{N_c}\right)^2\Delta q
$$

Thus the quark fraction is

$$
f_Q = \frac{3}{N_c^2}\frac{\Delta q}{k_f}. \tag{4}
$$

The corresponding shift in the energy density is then

$$
\begin{aligned}
\Delta\epsilon_Q &= DN_c\int_{k_f/N_c}^{k_f/N_c+\Delta q} q^2 dq\sqrt{m_Q^2 + q^2} \\
&\simeq m_N\Delta n_{B;Q} + D\frac{N_c^2}{2m_N}\left(\frac{k_f}{N_c}\right)^4\Delta q \\
&= m_N\Delta n_{B;Q} + \frac{D}{6m_N}k_f^5 f_Q.
\end{aligned} \tag{5}
$$

Next we need to remove nucleons from the Fermi surface to ensure that the total baryon density, $n_B = n_{B;N} + n_{B;Q}$, remains the same. For given interval Δk we have to leading order

$$
\Delta n_{B;N} = D\int_{k_f-\Delta k}^{kf} k^2 dk \simeq Dk_f^2\Delta k \tag{6}
$$

with the corresponding reduction in the energy density

$$
\begin{aligned}
\Delta\epsilon_N &= -D\int_{k_f-\Delta k}^{kf} k^2 dk\sqrt{m_N^2 + k^2} \\
&\simeq -\left(m_N\Delta n_{B,N} + \frac{D}{2m_N}k_f^4\Delta k\right).
\end{aligned} \tag{7}
$$

Since the total baryon density should remain unchanged we have $\Delta n_{B;Q} - \Delta n_{B;N} = 0$ so that

$$
\Delta k = \frac{\Delta q}{N_c^2} = \frac{k_f}{3}f_Q. \tag{8}
$$

As a result, the shift in the total energy density vanishes to leading order in f_Q

$$
\Delta\epsilon = \Delta\epsilon_Q + \Delta\epsilon_N = 0. \tag{9}
$$

Next let us consider quarkyonic matter. In this case we have to put the quarks at the center of the Fermi sphere. The baryon density of the quarks is then

$$
\Delta n_{B;Q} = D\int_0^{\Delta q} q^2 dk \simeq D\frac{(\Delta q)^3}{3}, \tag{10}
$$

so that in this case the quark fraction is

$$
f_Q = \left(\frac{\Delta q}{k_f}\right)^3 \tag{11}
$$

and the corresponding energy density

$$
\begin{aligned}
\Delta\epsilon_Q &= DN_c\int_0^{\Delta q} q^2 dq\sqrt{m_Q^2 + q^2} \\
&\simeq m_N\Delta n_{B;Q} + \frac{D}{2m_N}N_c^2\frac{k_f^5}{5}f_Q^{5/3},
\end{aligned} \tag{12}
$$

where $\Delta n_{B;Q}$ refers here to expression Eq. 10. Since the momentum of a nucleon is N_c times that of a constituent quark inside it, $p_N = N_c p_q$, we need to remove all nucleon up to $\Delta k = N_c \Delta q$. The corresponding baryon density

$$\Delta n_{B;N}^{(-)} = D \int_0^{N_c \Delta q} k^2 dk = D \frac{\left(N_c \Delta q\right)^3}{3} > \Delta n_{B;Q} \quad (13)$$

is larger than that for the quarks replacing the nucleons. As a consequence, we have to add additional nucleon on top of the Fermi surface to ensure that the total baryon density remains the same,

$$\begin{aligned} \Delta n_{B;N}^{(+)} &= \Delta n_{B;Q}^{(-)} - \Delta n_{B;Q} = D \frac{\left(N_c^3 - 1\right)(\Delta q)^3}{3} \\ &= D \int_{k_f}^{k f + \Delta k^{(+)}} k^2 dk \simeq D k_f^2 \Delta k^{(+)} \end{aligned} \quad (14)$$

Hence,

$$\Delta k^{(+)} = \frac{1}{3}\left(N_c^3 - 1\right) \frac{(\Delta q)^3}{k_f^2} = \frac{1}{3}\left(N_c^3 - 1\right) k_f f_Q. \quad (15)$$

Removing the nucleons at small momenta results in a gain of energy

$$\begin{aligned} \Delta \epsilon_N^{(-)} &= -D \int_0^{N_c \Delta q} k^2 dk \sqrt{m_N^2 + k^2} \\ &\simeq -\left(m_N \Delta n_{B,N}^{(-)} + \frac{D}{2 m_N} \frac{\left(N_c k_f\right)^5}{5} f_Q^{5/3} \right) \end{aligned} \quad (16)$$

whereas adding the nucleons on top of the Fermi surface cost energy

$$\begin{aligned} \Delta \epsilon_N^{(+)} &= D \int_{k_f}^{k_f + \Delta k^{(+)}} k^2 dk \sqrt{m_N^2 + k^2} \\ &\simeq m_N \Delta n_{B,N}^{(+)} + \frac{D}{2 m_N} k_f^4 \Delta k^{(+)} \\ &= m_N \Delta n_{B,N}^{(+)} + \frac{D}{6 m_N}\left(N_c^3 - 1\right) k_f^5 f_Q. \end{aligned} \quad (17)$$

Thus the total shift in the energy density is

$$\Delta \epsilon = \Delta \epsilon_Q + \Delta \epsilon_N^{(+)} + \Delta \epsilon^{(-)} = \frac{D}{6 m_N}\left(N_c^3 - 1\right) k_f^5 f_Q + \mathcal{O}(f_Q^{5/3}) \quad (18)$$

and thus positive to leading order in the quark fraction f_Q. Therefore, baryquark matter is energetically favored. Adding some mean field type of interaction which depends on the density of the nucleons will not change the picture because in both cases the density of the nucleons is the same by construction. This finding is also borne out in the detailed calculation of Ref. [12] where a hard core repulsion has been used similar to Refs. [10, 17], as well as Ref. [19] where an attractive mean field for nucleons was considered

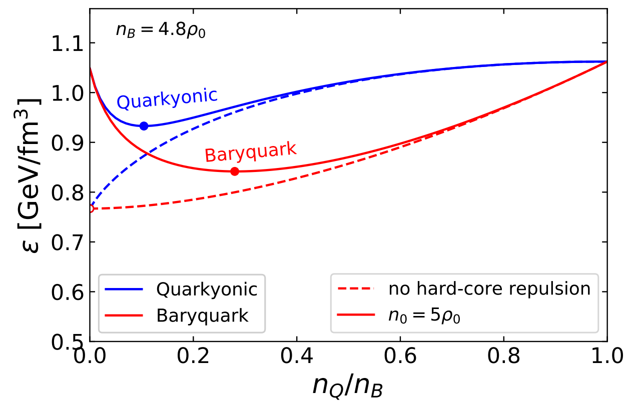

Figure 2: Dependence of the energy density in the excluded volume (blue) quarkyonic and (red) baryquark matter on the quark fraction n_Q/n_B at a fixed baryon density of $n_B = 4.8\rho_0$. The limiting nucleon density due to excluded volume in both cases is $n_0 \equiv b^{-1} = 5\rho_0$. Figure adapted from [12].

additionally to model nuclear liquid-gas transition. The resulting energy density is shown in Fig. 2 with and without the hard core repulsion. In both cases the energy density for baryquark matter is below that for quarkyonic matter. In Fig. 3 we also show the results for the equation of state and speed of sound for both quarkyonic and baryquark matter. We note that baryquark matter other than quarkyonic matter does not need a regulator to avoid unphysical singularities for the speed of sound (full blue line in Fig. 3).

To conclude this section, we have shown that baryquark matter is energetically favored over quarkyonic matter, at least if a simple mean field type of interaction is utilized to generate the desired shell structure of the nucleon momentum distribution of quarkyonic matter. Conceptually, of course, quarkyonic matter is arguably more appealing. Consider the long-wavelength quark interactions, which one would typically associate with confinement. These interactions are Pauli-blocked in the Fermi sea but permitted on the Fermi surface. One would thus identify the states in the Fermi shell with confined baryons rather than free quarks, as in quarkyonic matter.

One could imagine salvaging the quarkyonic picture by introducing some kind of momentum dependent interaction. However, as we shall discuss next there is a much more appealing and physically well founded way which naturally leads to a nucleon momentum space configuration as originally envisioned [16].

3. Quarkyonic strikes back

In the previous section, we have shown that using simple interactions among nucleons leads to quarkyonic matter being energetically disfavored. However, most of these models for quarkyonic matter account for the effect of the Pauli principle only by not allowing nucleon in states that are occupied by fermions and vice versa. They do not account

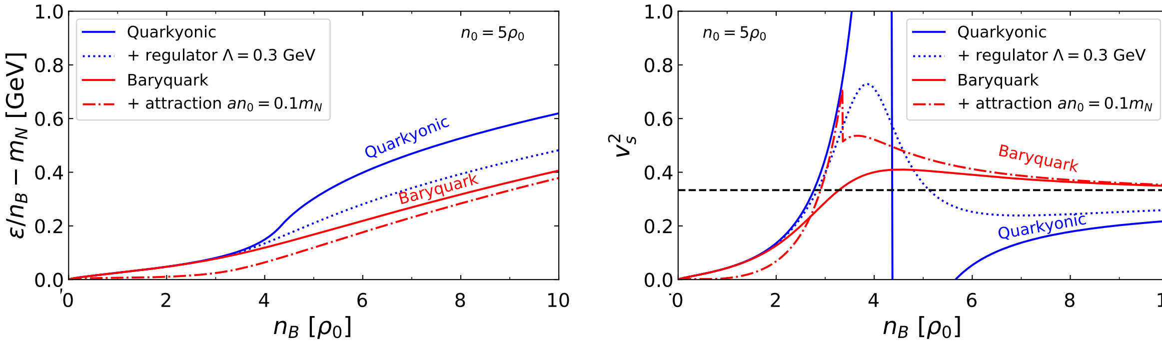

Figure 3: Baryon density dependence of energy per baryon (left) and speed of sound squared (right) evaluated in excluded volume quarkyonic (blue lines) and baryquark (red lines) matter. The excluded volume parameter in all cases corresponds to the limiting nucleon density of $n_0 \equiv b^{-1} = 5\rho_0$. The dotted blue lines correspond to quarkyonic matter with an infrared regulator $\Lambda = 0.3$ GeV while the dash-dotted red lines depict baryquark matter with attractive nucleon mean-field, $an_0 = 0.1m_N$. Figure adapted from [11].

for the internal quark structure of the nucleon. This has been first addressed in Ref. [7] (see also the contribution by L. McLerran to these proceedings). There, the authors took into account the momentum distribution $\phi(p)$ of quarks inside a nucleon. They observed that above a certain density of nucleons, the momentum space density of quarks at vanishing momentum, $f_Q(p = 0)$ become unity, at which point the Pauli principle does not allow to add more nucleons and a rearrangement of the nucleons is needed. To be specific, given the momentum distribution of quarks inside a nucleon $\phi(p)$, the overall momentum distribution of, $f_Q(p)$ is related to that of the nucleon, $f_N(k)$ via

$$f_Q(p) = \int \frac{d^3k}{(2\pi)^3}\varphi(|p - k/N_c|)f_N(k). \tag{19}$$

The Pauli principle in the quark sector requires a rearrangement of the nucleons once the Fermi momentum is above a "critical" momentum, $k_f > k_F^{crit}$ given by the solution of

$$1 = f_Q(0) = \int_{k<k_F^{\text{crit}}} \frac{d^3k}{(2\pi)^3}\,\varphi(k/N_c)f_N(k). \tag{20}$$

To get a rough idea about the magnitude of the critical momentum, let us consider the constituent quark model. In this case the quark wave-function is a Gaussian so that the quark momentum distribution inside a nucleon is

$$\varphi_{\text{gauss}}(p) = 8\pi^{3/2}R^3e^{-p^2R^2} \tag{21}$$

where the "size" parameter R is related to root mean square radius, r_{RMS}, of the nucleon by

$$r_{RMS} = \sqrt{\frac{2}{3}}R \tag{22}$$

Solving Eq. 20 one finds $k_F^{crit} = 1.01\,k_F^0$ for $r_{RMS} = 1\,\text{fm}$ and $k_F^{crit} = 1.3\,k_F^0$ for $r_{RMS} = 0.8\,\text{fm}$, with $k_F^0 = 265\,\text{MeV}$ the Fermi momentum of ground state nuclear matter. In other words, nuclear matter density is already very close to the critical density where the Pauli principle of the quark sector becomes relevant. Of course this is only a simple estimate and, thus, should be taken with grain of salt. The actual density when the Pauli principle becomes relevant depends, among other things, on the detailed momentum distribution of the quarks inside a nucleon. A recent study using realistic models of unintegrated quark distributions found this density to be $n_{crit} = 0.17 \pm 0.04\ \text{fm}^{-3}$ [15]. Ultimately, however, it may have to be determined by experiment. The authors of Ref. [7] devised a solvable model and found that above the critical density the Pauli blocking of the quarks leads to a nucleon momentum distribution with exhibits a "hole" at small momenta which increase in size as the density is increased. Also, and not surprisingly, the Pauli principle leads to additional repulsion for the nucleons. Indeed, Ref. [11] showed that this repulsion may replace the conventional repulsion from vector mesons so that nuclear matter can be bound with reasonable values for the incompressibility with pion and sigma exchange interactions only. In addition, the authors showed that, even though there is a "hole" in the nucleon distribution at low momenta, quasi-elastic scattering data can be well reproduced. If indeed nuclear matter and thus nuclei show such a new momentum space structure needs to be further checked by comparing with, for example, data from $(e, e'p)$ reactions as well as it implications for nuclear structure. However, even if this is not the case it is difficult to see how the Pauli exclusion principle cannot be relevant at high densities such as encountered in neutron stars.

4. Summary

We have shown that simple implementations of the concept of quarkyonic matter lead to system which are not energetically favored. To rescue them one either needs to introduce additional interactions, such as momentum dependent forces. A more appealing approach in our view is that of Ref. [7] where the internal quark structure of nucleon is taken into account. It naturally leads to the proposed shell-like momentum distribution originally envisioned for quarkyonic matter [16] and it remains to be seen if such a structure is already in place for ordinary nuclear matter and nuclei.

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