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Dynamic Market for Distributed Energy Resources in the Smart Grid

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Abstract—Distributed Energy Resources (DER) are one of the distinguished features of Smart Grid. A combination of small-scale energy generator and storage unit can produce energy to serve the associated load, while at the same time store or sell excessive energy. Assuming there is an energy surplus, the system can choose to sell a portion of its available energy to the market. In this work, an oligopoly model is developed in order to study dynamic pricing in such a scenario. The problem can be characterized as a dynamic N -player differential game, where the optimal solutions correspond to the equilibria of the game. We provide a mathematical analysis for the solution of the game, where a complete characterization of the steady-state price and optimal strategies of the players can be obtained in the symmetric case. Extensive numerical studies are provided to demonstrate the behaviors of the proposed market model and to analyze the impacts of various market parameters on the system.

I. INTRODUCTION

With the help of advance information technology, the concept of Smart Grid is being developed and implemented in hopes that it can improve reliability and efficiency of the existing power system. It also encourages higher penetration of renewable energy sources in the current energy mix, resulting in lower GHG emission. One of the distinguished features of Smart Grid is the integration of distributed generation (DG) and distributed storage (DS) in the power system. These two elements, also known as Distributed Energy Resources (DER), are considered the largest “new frontier” for Smart Grid [1].

DG usually refers to some small-scale energy generators distributed over the power system. It brings the point of generation closer to the load, hence reduces loss during transmission and enhances the voltage profile [2]. The generators can be fossil fuel-based, or fuel cell-based. Photovoltaic panels and wind turbines are also gaining tremendous popularity due to growing environmental concern.

Renewable energy sources are, however, fluctuating and intermittent in nature. To mitigate this, energy storage devices can be deployed. They can discharge and augment the generation when the load peaks, or store any excess energy generated. They can even perform power arbitrage when time-of-use pricing is in place. A number of distributed energy storage technologies are available. For instance, superconducting materials, supercapacitor and flywheel are studied in [3].

For better management of the power system, one usually breaks it down into small clusters, or microgrids, of DERs and loads within a local area [2]. A microgrid can disconnect

itself from the grid at times when fault occurs. A number of experiments and test beds on microgrid technologies can be found around the world (e.g. [4]).

Apart from these, some researchers employ pricing and market-based strategies to operate DER-based systems. In order to match the power generation and demand in an islanded microgrid, Marzband et al. [5] use single side auction mechanism to determine the market clearing price. The use of distribution locational marginal price (DLMP) as the pricing signal to control a power distribution system with distributed renewable energy generation and storage is described in [6]. In the EcoGrid EU real-time market [7], the system operator carries the burden to settle the price. This is a huge optimization problem requiring vast information such as the capacity of each device, and responses from consumers and DERs. In [8], Ding et al. formulate a profit maximization problem for a single wind turbine system which can either sell the wind energy on a green energy market, or use it to serve the associated load. Although it is close to our problem, it does not consider the possibility of energy storage.

While most papers propose pricing schemes and design markets to integrate and optimize the management and control of DERs from the perspective of an individual DER system, there is little work in the literature that deals with the market competition among multiple systems, and the behaviors of players and market price in such a scenario. In this work, we look at this scenario where a player is considered to be a small-scale DER system which consists of a pair of distributed generator and storage units (DGS in short). While a DGS is serving its associated local load and any energy deficit can be sourced from the energy market, there will often be energy surplus which can be either stored or sold to the market. Subsequently, in order to develop such a market model, we will use techniques from differential game theory [11] to capture the dynamic nature of the price and players’ behaviors. Differential game has been applied in our previous work in communications engineering for the dynamic spectrum access problem in the form of an oligopolistic market competition among licensed spectrum holders [13]. The model of [13] can be naturally extended to deal with the problem of pricing and trading in the energy market. We seek to analyze the steady-state equilibrium of the energy price as well as the optimal strategies (i.e., amount of energy to sell) taken by each DGS.

The remainder of the paper is organized as follows. In Sec-

tion II, the problem formulation is introduced. The dynamic market is described in Section III. In Section IV, we present the mathematical analysis for the model and in Section V, we provide numerical results to validate our model. Finally, Section VI concludes the paper.

II. SYSTEM MODEL

The system consists of multiple DGSs, each of which consists of a pair of energy generator and storage unit as shown in Fig. 1. Due to the intermittent nature of renewable energy sources, forecast of energy generation has to be done within short timeframes to achieve reasonable accuracy [9]. Hence, we can divide the horizon into periods of T time units. In each period, depending on the total amount of energy available, the pair can consider either to sell its excessive energy to an energy market if there is an energy surplus, or to buy more energy from the market if there is an energy shortage. Assume that for the i -th DGS, its available energy stock at time t is $E_i(t)$. In reality, $E_i(t)$ can be expressed as

$$E_i(t) = G_i(t) + S_i - L_i \quad (1)$$

where

- $G_i(t)$ is the total amount of energy generated during the period $[0, t)$ ($t \leq T$). Naturally, $G_i(t)$ should be a continuous, non-negative and non-decreasing function in $[0, T]$. If the energy forecast is accurate enough, the shape of $G_i(t)$ in the interval can be known *a priori*.
- S_i is the amount of energy available in the storage unit or its state of charge (at $t = 0$). We assume there is no loss in charging/discharging of the storage and also the rate of charging/discharging is fast compared to the length of the interval T so S_i is known.
- L_i is the total loads or amount of energy consumed during the time interval. It is assumed that loads can be predicted so L_i is also known.

In the following analysis, we focus on the case where the DGSs have energy surplus during the interval (i.e., $E_i(t) > 0$, $\forall t$). As such, there is an incentive for the DGS to sell part of its available energy to the energy market for monetary benefits. Assume that during this period, there are N such DGSs having energy to sell, who will compete among each other as the N oligopolists in the market (i.e., sellers in a market dominated by a small number of firms). In Fig. 1, the said energy market can be thought of as a common platform implemented at a centralized controller, who acts as a market broker. The broker can provide several necessary market functions, such as player coordination, price announcement and price update.

A reasonable assumption in this model is the homogeneous goods assumption, i.e., the energy offered by each DGS is equally preferred by the broker, without any priority given to a particular DGS. Under this assumption, a unit energy is sold at a common market price p which is set and monitored by the market. Each player i will need to determine its strategy in terms of the amount of energy offered or market output, e_i , with the goal of maximizing its own payoff or utility function.

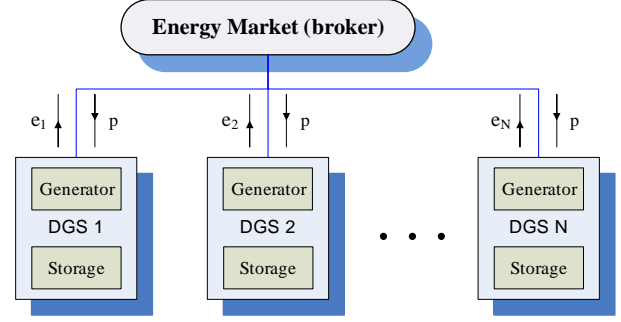


Fig. 1. Energy market model.

Note that e_i will be constrained by $E_i(t)$. Also of interest is the (steady-state) value of the market price p under the dynamics of competition among the players.

In the following section, an oligopolistic differential game framework will be proposed in order to analyze such dynamics and study the outcomes of the energy market.

III. DYNAMIC OLIGOPOLISTIC ENERGY MARKET

In the dynamic market, one assumes that the price $p(t)$ and the players' strategies $e_i(t)$ are time-dependent and that the dynamics of $p(t)$ follow a trajectory which is controlled by its past values as well as the values of each $e_i(t)$, reflected by a differential equation. Thus, the system is a differential game [11] where the price $p(t)$ is the state variable and each $e_i(t)$ is the control variable of each player. Following [13], we adopt a standard inverse demand function in Cournot oligopoly as

$$\hat{p}(t) = a - \lambda \sum_{i=1}^N e_i(t), \quad (2)$$

where constants $a, \lambda > 0$ are the intercept and slope of the inverse demand curve, respectively. We additionally require that $\lambda < 2N$ in order to ensure market stability, as will be shown later. The quantity $\hat{p}(t)$ is called the desirable price level, i.e., the price level that meets the current total supplies given by $\sum_{i=1}^N e_i(t)$. The feasibility of linear demand in electricity market has been studied by some authors, e.g., [10].

Hence, the dynamics of the market price is such that it reacts to the difference between the desirable and the current price levels $\Delta p(t) = \hat{p}(t) - p(t)$, i.e., $\dot{p}(t) = dp(t)/dt = G(\Delta p(t))$. We let $G(\cdot)$ be a linear function in $\Delta p(t)$, i.e., $G(\Delta p(t)) = k\Delta p(t)$, so that

$$\dot{p}(t) = k[a - \lambda \sum_{i=1}^N e_i(t) - p(t)], \quad p(0) = p_0. \quad (3)$$

Eq. (3) is called the price trajectory. p_0 is the initial price at time $t = 0$. Note that negative price can also be allowed, i.e., $p \in \mathbb{R}$. The goal of player i is to maximize its utility function, which is defined in terms of its revenue earned, subtracted by the cost incurred. The instantaneous utility of player i at time t is given by

$$U_i(p, e_i, e_{-i}) = \underbrace{e_i(t)p(t)}_{\text{revenue}} - \underbrace{(\alpha e_i(t) + \beta e_i^2(t))}_{\text{cost}} \quad (4)$$

where $\alpha, \beta > 0$ are positive constants; and e_{-i} refers to the joint strategies of player i 's opponents.

The revenue $e_i(t)p(t)$ refers to the money gained from selling $e_i(t)$ units of energy. The term $\alpha e_i(t)$ accounts for miscellaneous cost, including a commission fee paid to the market broker, which is assumed to be linear to the amount of energy sold; while the term $\beta e_i^2(t)$ is a second-order adjustment cost which serves as a penalty in case the DGs sell out too much energy, which may result in undesirable consequences, e.g., reduced storage lifetime due to excessive battery discharging.

Thus, each player seeks to maximize its accumulated payoff given by

$$J_i(p, e_i, e_{-i}) = \int_0^T e^{-rt} [e_i(t)p(t) - \alpha e_i(t) - \beta e_i^2(t)] dt \quad (5)$$

where e^{-rt} ($r > 0$) is the standard discount factor.

The overall oligopolistic differential game can be stated as the following optimization, $\forall i$

$$\begin{aligned} & \max_{e_i} J_i(p, e_i, e_{-i}) \\ \text{s.t.} & \begin{cases} \dot{p}(t) = k \left[a - \lambda \sum_{i=1}^N e_i(t) - p(t) \right], & p(0) = p_0. \\ 0 \leq e_i(t) \leq E_i(t) \quad \forall t. \end{cases} \end{aligned} \quad (6)$$

IV. ANALYSIS OF THE OLIGOPOLISTIC MARKET

A. Equilibrium in the General Case

Before providing a characterization of the proposed game's equilibrium solution, we first comment that the time needed for the price and players' strategies to converge to its steady-state values (i.e., the point where the market is stable and the supplies have matched the demands) might be relatively smaller than the duration T . This assumption will be numerically verified in a subsequent section. Thus, in order to simplify the mathematical analysis, one may allow an infinite time horizon where $T \rightarrow \infty$. Under this assumption, we will look for the feedback Nash equilibrium (NE) of the game where each player employs its stationary Markov strategies. These concepts are defined as follows [11], [12].

Definition 1. *The strategy function e_i of player i can be classified as stationary Markov if $e_i \equiv e_i(p(t))$, which is solely a function of the current state $p(t)$.*

Definition 2. *Consider the differential game (6) with an infinite horizon, where all players' strategies $e_i \equiv e_i(p(t))$ are of the stationary Markov type defined above. Then, the strategy profile $(e_1^*, e_2^*, \dots, e_N^*)$ is a stationary Markov feedback NE if for any player i , any $p(t)$ at any time t ,*

$$J_i(p, e_i^*, e_{-i}^*) \geq J_i(p, e_i', e_{-i}^*), \quad \forall e_i' \neq e_i^*. \quad (7)$$

Using known results from optimal control and differential game theory, the feedback Markov NE for the game can be characterized by solving the Hamilton-Jacobi-Bellman (HJB) equations [12], which are a set of N partial differential equations (PDE), stated in the following theorem.

Theorem 1. *The differential game (6) with an infinite horizon admits a stationary Markov strategy profile $(e_1^*, e_2^*, \dots, e_N^*)$, $e_i^* \equiv e_i^*(p)$ as a stationary Markov feedback NE if for any player i , there exists a continuously differentiable function $V_i(p) : \mathbb{R} \mapsto \mathbb{R}$ that satisfies¹*

$$\begin{aligned} rV_i(p) = & \max_{e_i} \{ (p e_i - \alpha e_i - \beta e_i^2) \\ & + \frac{\partial V_i(p)}{\partial p} \cdot k [a - \lambda e_i - \lambda \sum_{j=1, j \neq i}^N e_j^* - p] \} \end{aligned} \quad (8)$$

The solution of (8) is the set of N functions $V_i(p)$, $i = 1, \dots, N$, commonly known as the *value functions*. As the maximand in (8) is quadratic in e_i , one can carry out the maximization by taking the (partial) derivative with respect to e_i . Subsequently, we denote Φ_i this derivative, given by

$$\Phi_i = p - \alpha - 2\beta e_i - k\lambda \frac{\partial V_i}{\partial p}. \quad (9)$$

Due to the energy constraint $0 \leq e_i \leq E_i$, the solution to $\Phi_i(e_i) = 0$ will be the optimal strategy e_i^* only when this constraint is satisfied; otherwise the optimal point should occur at the boundaries, i.e., either at 0 or E_i , conditioned on the sign of Φ_i . In general, at equilibrium, if there are $M < N$ players whose optimal strategies are either 0 or E_i , then for the remaining players, the problem becomes an unconstrained $(N - M)$ -player linear-quadratic differential game model [11], in which the corresponding value function of a player i , if exists, takes the quadratic form $V_i = \frac{1}{2} X_i p^2 - Y_i p + Z_i$. Thus, e_i^* can be obtained by substituting $\frac{\partial V_i}{\partial p} = X_i p - Y_i$ in (9) and solving $\Phi_i(e_i) = 0$. In summary,

$$e_i^* = \begin{cases} \frac{1}{2\beta} [(1 - k\lambda X_i)p + (k\lambda Y_i - \alpha)] & \Phi_i = 0, \\ 0 & \Phi_i < 0, \\ E_i & \Phi_i > 0. \end{cases} \quad (10)$$

Here, X_i , Y_i and Z_i are constant and depend on N, r, k, α, β and λ . However, determining the conditions for their existence and finding their closed forms are generally a mathematically intractable problem [13], as one will need to solve N simultaneous non-linear PDEs. Moreover, due to the 3 separate conditions for each player, a complete analysis of the equilibrium may need to exhaustively include up to 3^N different market scenarios.

Nonetheless, some conclusions can be made on the general solutions for a particular player i :

- $\Phi_i < 0$ implies $p < \alpha + 2\beta e_i + k\lambda \frac{\partial V_i}{\partial p}$. The market price p can be seen as the marginal revenue (MR), i.e., earnings from selling one extra unit of energy. On the other hand, the right-hand side is the marginal cost (MC). As $\text{MR} < \text{MC}$, player i has no incentives to sell energy.
- Similarly, $\Phi_i > 0$ implies that $\text{MR} > \text{MC}$. That is, there are enough profits in the market for player i to sell at maximum quantities.

¹When dealing with the HJB equation and its subsequent analysis, it is understood that p is treated as a variable and the results should hold for all t ; so $p, e_i(p)$ and $V_i(p)$ can be used instead of $p(t), e_i(p(t))$ and $V_i(p(t))$.

- $\Phi_i = 0$ implies that MR = MC, i.e., player i faces perfect competition and will have to select an output that maximizes his profits. The optimal solution corresponds to the Markov NE strategy, which appears to be a linear feedback function of price p .

B. Players with Similar Constraints

In the following section, we investigate the special case, i.e., symmetric game where a complete characterization of the Markov feedback NE is obtainable. We assume that all players have identical constraints, i.e., $E_i(t) \equiv E(t), \forall t, \forall i$. As a result, the game becomes a symmetric oligopoly. Following the analysis from [13], at the feedback NE, players' strategies are also symmetric with $e_i^* = e_i$ and $V_i = V, \forall i$. Thus, due to the symmetric condition, the general 3^N scenarios are reduced to 3 distinct market regions, depending on the value of $p(t)$. Next, we will discuss each of the region.

1) *Region 1 - True Oligopoly*: If the price $p(t)$ is such that the optimal feedback strategy fulfills the constraint $0 \leq e_i \leq E_i(t)$, then all players are able to maximize their profits by choosing this optimal strategy as the solution to $\Phi_i(e_i) = 0$. This scenario is therefore seen as the true-oligopoly case. The closed-form solution can be obtained as follows.

Proposition 1. *In region 1, the value function and its derivative have the following forms*

$$\frac{\partial V(p)}{\partial p} = Xp - Y, \quad V(p) = \frac{1}{2}Xp^2 - Yp + Z, \quad (11a)$$

where

$$X = \frac{(2\beta + N)k + \beta r - \sqrt{((2\beta + N)k + \beta r)^2 - (2N - \lambda)k^2\lambda}}{(2N - \lambda)k^2\lambda} \quad (11b)$$

$$Y = \frac{2\beta kaX + kN\alpha X - \alpha}{(2N - \lambda)k^2\lambda X - (2\beta + N)k - 2\beta r} \quad (11c)$$

$$Z = \frac{\alpha^2 + (2N - \lambda)k^2\lambda Y^2 - 2NkY\alpha - 4\beta kaY}{4\beta r}. \quad (11d)$$

Furthermore, the optimal feedback strategy taken by each player will be linear in p , given by

$$e^*(p) = \frac{1}{2\beta} [(1 - k\lambda X)p + (k\lambda Y - \alpha)]. \quad (12)$$

Next, the exact price trajectory $p(t)$ can be readily obtained by solving (3) with e^* given by (12). The general solution is given by

$$p(t) = \Gamma + Ce^{-k\left(1 + \frac{N(1 - k\lambda X)}{2\beta}\right)t}, \quad C = \text{const}, \quad (13)$$

where Γ is the steady-state price in region 1, which equals

$$\Gamma = \frac{2\beta a + N(\alpha - k\lambda Y)}{2\beta + N(1 - k\lambda X)}. \quad (14)$$

The constant C can be found by applying initial condition for $p(t)$. In particular, with $p(0) = p_0$, we obtain $C = p_0 - \Gamma$.

In addition, under the original assumption that $\lambda < 2N$, it can be shown that $X < \frac{1}{k\lambda}$ (details omitted). It follows that the

exponential term of (13) approaches 0 and consequently $p(t)$ approaches Γ as $t \rightarrow \infty$, provided that Γ lies within region 1. Γ is also called the *market equilibrium*, which marks the point where the supplies and demands are balanced.

The boundary values for region 1 occur at price levels where constraints $0 \leq e_i \leq E(t)$ start to be violated. From (12), one can see that region 1 thus corresponds to an interval $p \in [p_1, p_2(t)]$ where

$$p_1 = \frac{\alpha - k\lambda Y}{1 - k\lambda X}, \quad p_2(t) = \frac{\alpha - k\lambda Y + 2\beta E(t)}{1 - k\lambda X}. \quad (15)$$

2) *Region 2 - No Participation*: In region 2, the optimal strategy occurs at the boundary $e^* = 0$ for all players due to $\Phi_i(e_i) < 0$. This happens if the price level p drops below the lower threshold p_1 as given above. Thus, in this region, it is implied that all DGSs do not participate in the market due to inadequately low price which does not give them profits. The detailed solution is given as follows.

Proposition 2. *In region 2, the players' equilibrium strategy is $e^* = 0$ for all $p < p_1$. The steady-state price of this region is $p = a$, following a price trajectory given by*

$$p(t) = a + Ce^{-kt}, \quad C = \text{const}. \quad (16)$$

Moreover, the value function $V(p)$ in this region has the following form

$$V(p) = V_1 \frac{(a - p_1)^{r/k}}{(a - p)^{r/k}} \quad (17)$$

where $V_1 = \frac{1}{2}Xp_1^2 - Yp_1 + Z$.

For initial condition $p(0) = p_0$, the constant C is given by $p_0 - a$. Additionally, the condition $p(t) < \alpha + k\lambda \frac{\partial V}{\partial p}$ which results from $\Phi_i(e_i) < 0$ can be verified.

3) *Region 3 - Output Saturation*: In region 3, as opposed to region 2, equilibrium strategy occurs at the upper boundary, i.e., $e^* = E(t)$ for all $p > p_2(t)$. It is seen that as price level goes above the upper threshold value, all DGSs will maximize their profits by selling the maximum amount of energy, i.e., their outputs become saturated. The solution is given as follows.

Proposition 3. *In region 3, the players' equilibrium strategy is $e^* = E(t)$ for all $p > p_2(t)$. The actual price trajectory will depend on the form of the function $E(t)$, with the general solution given by*

$$p(t) = e^{-kt} \left[\int k(a - NE(t))e^{kt} dt + C \right], \quad C = \text{const}. \quad (18)$$

Moreover, the value function $V(p)$ in this region has the following form

$$V(p) = Rp + S + (V_2 - Rp_2 - S) \frac{(a - \lambda NE(t) - p_2)^{r/k}}{(a - \lambda NE(t) - p)^{r/k}} \quad (19)$$

where $V_2 = \frac{1}{2}Xp_2^2 - Yp_2 + Z$, $R = \frac{E(t)}{r+k}$, and $S = \frac{1}{r} \left[\frac{kE(t)}{r+k} (a - \lambda NE(t)) - \alpha E(t) - \beta E^2(t) \right]$.

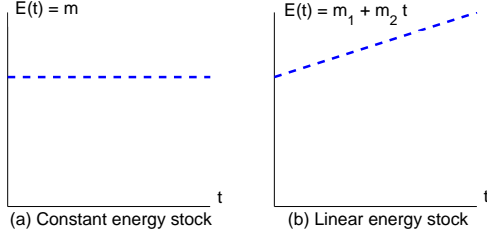


Fig. 2. Two examples of the function $E(t)$.

From $\Phi_i(e_i) > 0$, the condition is $p(t) > \alpha + 2\beta E(t) + k\lambda \frac{\partial V}{\partial p}$ which can also be verified.

As we can see, the shape of the available energy function $E(t)$ can affect the solution if region 3 occurs. $E(t)$ depends on the energy generation function $G(t)$ according to (1). As previously stated, the exact form of $G(t)$ is difficult to obtain due to the random nature of renewable energy sources; however in general, $G(t)$ can be approximated as continuous, non-negative and non-decreasing functions.

We can consider two simple cases of $E(t)$, i.e., the constant energy stock and the linear energy stock. In Fig. 2(a), it is implicitly assumed that $G(t) = \text{const}$ or there is no generation during the period (e.g., no sunlight during night time). This results in a constant energy stock $E(t) = m > 0$. In Fig. 2(b), we implicitly assume a linear $G(t)$, i.e., energy is generated at a steady rate m_2 over time, which results in a linear stock $E(t) = m_1 + m_2 t$ ($m_1, m_2 > 0$). This might correspond to a daytime period with steady wind or solar power.

For constant energy stock, the final trajectory equation (with $p(0) = p_0$) is given by

$$p(t) = a - Nm + (a - Nm - p_0)e^{-kt}. \quad (20)$$

On the other hand, for linear energy stock, the trajectory equation becomes

$$p(t) = a - Nm_1 + \frac{Nm_2}{k} - Nm_2 t + (a - Nm_1 + \frac{Nm_2}{k} - p_0)e^{-kt} \quad (21)$$

V. PERFORMANCE EVALUATION

In this section, we present the numerical results in order to evaluate the performance of the proposed game and study the behaviors of the energy market. In our simulation, the key parameters are listed in Table I. Notably, it is assumed that the energy stock function for all players takes the linear form $E(t) = 30 + 2.5t$, where the time t is measured in hours and $E(t)$ in MWh. The settings may not reflect all the real-time data of an actual energy market; but the example could be useful to illustrate the model's behaviors.

Using the previous settings, we obtain the dynamics of the price trajectory $p(t)$ and the path of the equilibrium strategies taken by the players $e(p(t))$ for 3 different initial market price $p_0 = 1, 35, 70$. The results are shown in Fig. 3. In accordance with our theoretical analysis, the price $p(t)$ is convergent and approaches its steady-state value as time progresses. Hence, $e(p(t))$ also converges as a result. (Note that the interval

TABLE I
ENERGY MARKET PARAMETERS.

Parameters	Values
N	4
a	90 \$/MWh
λ	1 \$(/MWh) ²
k	0.5
r	0.3
α	4 \$/MWh
β	0.5 \$(/MWh) ²
$E(t)$	30 + 2.5t (MWh)
T	4 hrs

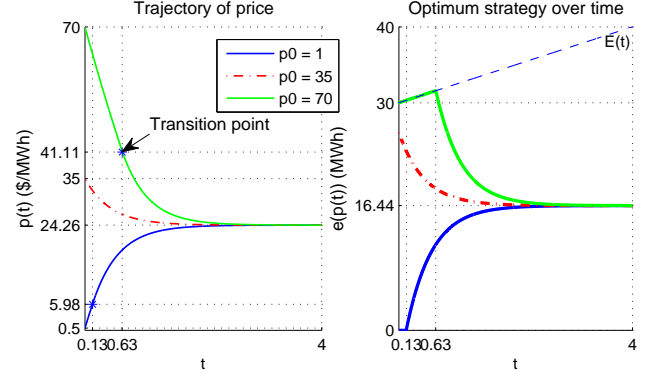


Fig. 3. Dynamics of price and players' strategies.

duration $T = 4$ hrs in this example is sufficiently large for the equilibrium to happen). It is clear that the equilibrium price is independent of the initial price p_0 , because the dynamical system is linear in p which admits a unique steady-state point. The steady-state value is $p_{steady} = \Gamma = 24.26$ which lies in region 1. Region 2 occurs if $p < p_1 = 5.98$ and region 3 occurs if $p > p_2 = 41.11$. Different initial prices affect the starting regions of the market, as it starts in region 2 and 3 for $p_0 = 1$ and $p_0 = 70$, respectively. Along the trajectory, when $p(t)$ reaches the boundary p_1 or p_2 , a transition from one region to another will occur. This is evident in the abrupt change in the shapes of $e(p(t))$ at $t = 0.13$ and $t = 0.63$. Note that $p(t)$ should remain continuous and differentiable at the transition points (which is mathematically provable).

The impacts of the number of players on the market behaviors, especially the steady-state values, are also investigated and shown in Fig. 4. It can be observed that in this example, $p_{steady} = \Gamma$ and e_{steady} both drop as the number of players N grows larger. Economically, it can be explained that more competitors increase the aggregate supplies which brings down the equilibrium price. However, each individual player gets a smaller share of the market; hence, e_{steady} also drops. Mathematically, one can prove that as $N \rightarrow \infty$, $p_{steady} \rightarrow \alpha$ and $e_{steady} \rightarrow 0$ (assuming final equilibrium in region 1).

Next, the proposed market is studied in terms of the aggregate profits gained by a DGS over time. For a better assessment, we compare our scheme with alternative schemes in which each DGS decides to sell a fixed quantity of energy

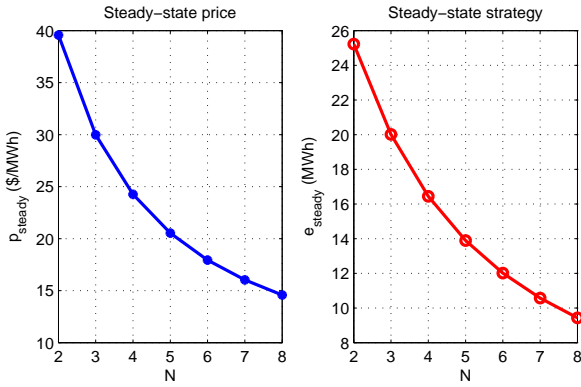


Fig. 4. Steady state values for different numbers of players

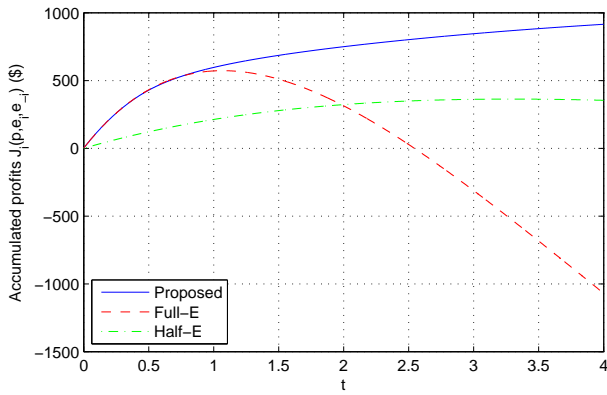


Fig. 5. Comparison of accumulated profits

to the market without dynamic, feedback strategy adaptation. In the first scheme, the DGSs choose to fully sell its energy stock (Full-E in short); while in the second scheme, the DGSs sell half of its energy stock and store the rest (Half-E). In both schemes, the price is determined by the inverse demand function (2). Their accumulated profits can be estimated by (5) and are plotted in Fig. 5. It can be seen that the proposed solution optimally maximizes the profits of the DGSs over time and is clearly better than any of the fixed-quantity schemes. Another interesting observation is that for fixed-quantity schemes, the DGSs may even incur loss (in the case of Full-E, the profit curve becomes negative after certain point) as revenues cannot cover the growing costs.

VI. CONCLUSION & FUTURE WORKS

In this short paper, we have developed an oligopolistic market model based on differential game to allow the DGSs to dynamically trade energy and optimize their profits over time. The Markov feedback NE solution for the game is mathematically analyzed; and a complete characterization is obtained for symmetric players with similar constraints. Through analytical and numerical evaluations, it can be seen that players adaptively determine their energy quantities to sell

based on feedback functions of price level which will lead to different market regions. The proposed solution is shown to maximize the DGSs' aggregate profits over fixed-quantity schemes. However, this work has only considered the case where DGSs have energy surplus and a hybrid scenario will be a topic of future studies, where certain DGSs have energy deficit and need to buy back from the grid. Furthermore, the loads, energy generation and storage functions of a DGS of adjacent intervals are interdependent and may not always be accurately predicted; and how this influences the cost function and strategies will be studied. A discrete-time method for the proposed scheme will also be proposed, which can bring together the DGSs and the market broker and pave the way for its implementation in practice.

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