ABSTRACT OF THE DISSERTATION

Multi-Level Micromechanical Modeling of Bone Tissues

by

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In this thesis, we aim to develop robust multi-level micromechanical constitutive models for human bone tissues. First, the hierarchical microstructure of human bones is considered, and a multi-scale micromechanical homogenization scheme is proposed in Chapter 3. The proposed framework predicts that the pattern of mineralization and the shape of the mineral crystals serve to improve the mechanical function of collagen fibrils along the longitudinal axis. The numerical results in comparison to the experimental data from nanoindentation tests reflect a predictive precision and demonstrate the capability of the current micromechanical model to simulate the key behavior at this scale.

The effects of collagen fibril orientations upon the elastic properties of bone lamellae or lamellar units are studied in the following chapter. The homogenized anisotropic elastic properties of bone lamellae or lamellar units are estimated using an innovative method: multiscale homogenized model of ultrastructure combined with a micromechanical framework of layered composite for lamellar units and fibril orientation patterns observed in the experiments. Five fibril orientation patterns are compared in this study – the orthogonal and twisted plywood pattern, the five sublayer pattern, an X-ray diffraction-based pattern, and a microscopy observed pattern. The model results show the deviation of fibrillar orientation from the anatomical axis (osteon axis) and demonstrate that the elastic mechanical behaviors of bone lamellae vary with different fibril orientation patterns. The proposed method opens new possibilities in the exploitation of fibrillar orientation data and provides a better understanding of the mechanical properties of bone lamellae.

The effects of accumulation of diffuse damage in human bone upon the material properties of lamellae are investigated in Chapter 5. A three-parameter formulation is adopted to characterize the evolution of density of microcracks. An effective elastic micromechanical damage formulation for bone with evolutionary matrix cracking is then proposed based on Ju and Tseng's micromechanical damage formulation. It is demonstrated that microdamage exists in bone in vivo, accumulates with age and contributes to the degradation of bone's material properties.

In Chapter 6, on the basis of constant-shear model, the rising R-curve behavior of human cortical bone at the lamella level is studied with the framework of linear-elastic

fracture mechanics combined with multi-scale effective elastic micromechanical model for lamella. By taking advantage of cumulative probability function or Weibull distribution, the effect of collagen fiber breakage is simulated. The distribution and evolution of fiber bridging stress, crack mouth opening, and stress intensity factor then are analyzed by considering the probable fiber breakage. Crack bridging by collagen fibrils in toughening cortical bone was first time investigated systematically.

A three-dimensional structural model composed of two different unit cells, which have distinct mechanical behavior in the vertical direction, with doubly tapered struts for human vertebral cancellous bone is proposed in Chapter 7. The ensemble-volume and orientation averaging procedures are employed to derive the equations. The effects of age-related changes in vertebral cancellous bone on effective stiffness and collapse stress in both horizontal and vertical directions are studied. The predicted mechanical behaviors of human vertebral cancellous bone are generally consistent with experimental observations in the literatures.

The dissertation of Yu-Kai Wang is approved.

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To my beloved parents

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