UNIVERSITY OF CALIFORNIA,
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Decision and Theories in Ramsey's Philosophy

DISSERTATION

submitted in partial satisfaction of the requirements
for the degree of

DOCTOR OF PHILOSOPHY

in Philosophy

by

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DEDICATION

To Ronda.

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I would like to start by thanking my committee. In particular, special thanks goes to Brian Skyrms, Jeffrey Barrett, Kyle Stanford, and Jeremy Heis.

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Parts of chapter two on revising Ramsey's decision theory may appear in a forthcoming paper "Putting the Decision in Ramsey's "Theories"".

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ABSTRACT OF THE DISSERTATION

Decision and Theories in Ramsey's Philosophy

By

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Doctor of Philosophy in Philosophy

University of California, Irvine, 2023

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Among many things, Frank Ramsey is famous for his invention of a logical device called the Ramsey sentence. The Ramsey sentence of a scientific theory is a conjunction of the theory's propositions where the theoretical vocabulary is existentially quantified out. The sentence appears in an obscure paper titled "Theories". This paper is extremely difficult because Ramsey likely did not finish it before his death and it mostly consists of a mathematical model of scientific theories and a labyrinthine toy example. Consequently, philosophers have proposed many radically different and incoherent interpretations of the Ramsey sentence and Ramsey's philosophy of science. An important fact interpreters have ignored is that Ramsey had intended "Theories" to be a chapter in a book centered on a revised version of his formal theory for decision-making under uncertainty. The principal accomplishment of my dissertation is to leverage this fact to provide the first complete and philosophically satisfying interpretation of "Theories", including Ramsey's toy example and the Ramsey sentence.

Ramsey's theory for decision-making is incompatible with his philosophical commitments in "Theories" and contemporary papers. In my dissertation, I revise that theory to make it consistent with those commitments. I combine this with the formal model he describes in "Theories" to provide a unified reconstruction of Ramsey's intended model for scientific

theories. I use the reconstructed model to explain Ramsey's toy example, and I furnish a complete interpretation of Ramsey's arguments in "Theories". With these parts, I develop a detailed account of the laws and chances derived from scientific theories and a novel interpretation of the Ramsey sentence. This has several important philosophical consequences.

First, Ramsey did not use the Ramsey sentence in the ways philosophers have previously proposed. Philosophers view the Ramsey sentence either as a tool for eliminating theoretical terms or for asserting the existence of theoretical properties. Neither of these claims is correct. Instead, I argue that the Ramsey sentence is a communication and deliberation device aimed at representing the propositions an individual believes with her behavioral dispositions. Informally, it expresses as new propositions the mixture of rules a person uses to guide her behavior so that she may deliberate and communicate those rules with herself and other people. This suggests the Ramsey sentence can be used to read off the content of an agent's belief from their behavior.

Second, Ramsey formulated a novel and interesting type of non-reductionistic scientific anti-realism. That is, for Ramsey theoretical propositions are fictitious and yet not eliminable for observational propositions. Theoretical propositions are fictitious because their meaning depends on theoretical laws, which, being universally quantified, are also fictitious. But theoretical propositions cannot be eliminated because they entail conceptual possibilities that are richer than those given by the observation language.

Third, Ramsey held laws and chances to be rules of deference for degrees of belief. His account of laws and chances foreshadows pragmatic, subjectivist accounts of laws and chances from later in the twentieth century. Degrees of belief are representations of preferences over gambles involving observable propositions. A chance then is a rule for preferring some gambles over others, and a law is a chance that assigns unity to its degrees of beliefs. This prefigures the views that chances and subjunctive conditionals are reducible to properties of degrees of belief.

Chapter 1

Introduction

"It seems to me that in the process of clarifying our thought we come to terms and sentences which we cannot elucidate in the obvious manner by defining their meaning. For instance, variable hypotheticals and theoretical terms we cannot define, but we can explain the way in which they are used, and in this explanation we are forced to look not only at the objects which we are talking about, but at our own mental states."

Frank Ramsey, "Philosophy"

1.1 Introduction

Frank Ramsey only lived to the age of twenty-six, yet he has had an enormous influence on philosophy, economics, and mathematics. His work in metaphysics with the paper "Universals" attacks the core distinction between particulars and universals that has been of central importance from the ancients to Russell. Along with inventing a whole sub-discipline in combinatorics, he put forward and defended a philosophy of mathematics with a revised type theory, while also making a core distinction between the set-theoretic and semantic paradoxes. In the philosophy of language, his later views on laws pre-figured theories of subjunctive conditionals as inference tickets while proposing a test for the meaning of those conditionals. His views on meaning and truth married pragmatist ideas with modern logic, and these ideas strongly influenced Wittgenstein's evolving philosophy of language in 1929 and 1930. He revolutionized economics, probability theory, and logic by reviving the subjectivist conception of probability and inventing Bayesian decision theory. Finally, his invention of the Ramsey sentence, an existentially quantified sentence that replaces the theoretical terms of a scientific theory with bound variables, has had an immense impact on the realism and anti-realism debate in the philosophy of science and philosophy of mind for decades. It is perhaps an understatement to say that especially in the philosophy of probability and the philosophy of science that Ramsey's thoughts have been extraordinarily important.

Here I want to focus on Ramsey's philosophy of science. Why write about Ramsey's philosophy of science? There are historical and philosophical reasons. As mentioned earlier, Ramsey's purported ideas have played an important role in debates surrounding the claim that scientific theories are approximately true and their theoretical entities such as atoms exist as any other object in the world. It would be very useful to evaluate what Ramsey's actual views are to better place him with respect to those later debates. In addition, it would lead to a better, holistic understanding of Ramsey's philosophy while situating him better in the intellectual milieu of the 1920s and 1930s. Philosophically, Ramsey's views are valuable because of their relevance for assessing the approximate truth of scientific theories, the status of scientific entities, and as a possible method for formally modeling science. The first two in this list have been a long-vested interest of philosophers. The latter has gained prominence because of the desire to increasingly automate science via artificial intelligences; formal models of science, such as causal inference, increasingly have found application in scientific inquiry and industry. Ramsey's ideas might have something to contribute here. So it would be valuable for historical reasons as well as philosophical reasons to understand Ramsey's philosophy of science.

It is perhaps after forming such high expectations about Ramsey's philosophy of science that the reader finds himself utterly bewildered when reading Ramsey's core writings. The main paper "Theories"—the paper credited with the invention of the Ramsey sentence—is a complete and utter mess. Like the trails in an overgrown forest, arguments proliferate left and then right before terminating in thickets of unclear resolution. Landmarks such as a thesis are lost in the undergrowth of half-developed ideas and an obscure, gnarled mathematical toy model that twists its limbs into a labyrinth of formalisms. The famed Ramsey sentence appears towards the end of the paper as some strange crooked branch poking out at an odd angle and with hardly any discussion and attention one would think such an important idea needs. The paper's structure, if it could be called that, focuses on six questions before abruptly ending like a rabbit trail into the brushwood. This vexing paper has led one historian to remark that "Any claim to have extracted a philosophical theory from Ramsey's paper that captures his intentions would be tendentious" (Demopoulos, 2011, 190). With such primary source material, is any non-tendentious account of Ramsey's philosophy possible?

Historians have labored for such an account. It should not be surprising that this has produced a museum of curiosities. Some historians ascribe a reductionistic philosophy of science to Ramsey in the spirit of Russell and Carnap. Others propose Ramsey is a structural realist of some variety, where he is agnostic as to the identity of theoretical entities but humbly confident of the causal roles fulfilled by those entities. Still, others have found in Ramsey an application of an intuitionistic philosophy of logic and mathematics to the problem of scientific theories. And more have identified Ramsey as continuing the philosophical project of pragmatists such as C. S. Peirce. With such contradicting oddities, one might wonder whether the first historian's observation of tendentiousness has some truth to it.

An attractive strategy for solving the problem of interpreting Ramsey's philosophy of science is to identify whom he is responding to and how his response fits with his larger philosophical project. Fortunately, "Theories" provides significant evidence for identifying his targets. The largest section of the paper is in response to the following question:

> Can we reproduce the structure of our theory by means of explicit definitions within the primary system?
>
> [This question is important because Russell, Whitehead, Nicod, and Carnap all seem to suppose that we can and must do this.[1]]
>
> [1] Jean Nicod, *La Géométrie dans le Monde Sensible* (1924), translated in his *Problems of Geometry and Induction* (1930): Rudolf Carnap, *Der Logische Aufbau der Welt* (1928) (Ramsey, [1929] 1990m, 120).

Ramsey is responding to Russell, Whitehead, Nicod, and Carnap. What all of these projects have in common is what I will call a reductive project for scientific theories: they aim to eliminate theoretical propositions and terms. These particular projects do so via explicit definitions, as Russell had earlier eliminated cardinal numbers by defining them as classes of equinumerous classes. Fixing this target of Ramsey's philosophy of science, one can see how Ramsey's larger philosophy might produce a response. The spirit of that philosophy is summarized succinctly by Ramsey in the late essay "Philosophy": "Philosophy must be of some use and we must take it seriously; it must clear our thoughts and so our actions"

(Ramsey, [1929] 1990h, 1). For philosophy to be of use in action, it must guide decision-making, and for Ramsey, guidance in decision-making comes from his decision theory. So the strategy then is to see how Ramsey might respond to the reductive project by accounting for how scientific theories contribute to action through Ramsey's decision theory.

The goal of this introduction is to describe how other historians have understood Ramsey's philosophy of science, provide a short overview of the key paper "Theories", explore the reductive project in its various forms, and sketch for the reader the core pillars of Ramsey's response to the reductive project. I will not be able to answer every question, but I hope that I can help the reader with a basic map of the woods that are Ramsey's late philosophy. This will be important as there is no escaping the complexity and systematicity of Ramsey's broader philosophy in 1929.

1.2 Secondary Literature Review

The difficulty of Ramsey's paper "Theories" and affiliated notes has led to a menagerie of interpretations of Ramsey's philosophy of science. Those interpretations can be broken into four buckets: positivistic, functional realist, intuitionist, and pragmatist. While the contents of these buckets can be heterogenous in the sense that they may disagree about particulars, the contents concur about Ramsey's big-picture ideas. I go through each bucket here as a review, while pointing out significant weaknesses in these interpretations.

An early and enduring interpretation of Ramsey's philosophy of science is that Ramsey shows philosophers how to reduce or eliminate theoretical propositions through the Ramsey sentence. The most influential defender of this view is Carnap (Psillos, 2000; Carnap and Gardner, 1966) and a more recent example is Demopoulos (Demopoulos, 2003; Demopoulos, 2011).[1] They claim that Ramsey aims to somehow reduce theoretical propositions to observational propositions. In the classic case, the Ramsey sentence quantifies out the theoretical names and relations leaving only the observational vocabulary. So the positivist claim is that Ramsey's logical invention leads to a successful strategy for reducing theoretical propositions to observational propositions.

The positivist reading has three main weaknesses. First, it is in spirit, if not in letter, in conflict with some of Ramsey's conclusions. As noted earlier, the views he reacts to in "Theories" are the eliminativist theories via explicit definitions of Russell, Whitehead, Nicod, and Carnap. Ramsey states in multiple places that these projects fail because they result in arbitrary and complex definitions that arrest theory meaning and growth.[2] His conclusion is that scientific theories should not be reduced to observation. Second, he never mentions the Ramsey sentence as a solution to the problem of reducing theoretical to observational

[1]Carnap writes in a presentation he gave in 1958 that Ramsey's core idea was to eliminate "bothersome" theoretical terms with the eponymous sentence:

> Now Ramsey showed that this existential sentence—which we call now the Ramsey-sentence—is O-equivalent [equivalent in the observation language] to the theory TC [the conjunction of the theory's axioms and definitions (what Carnap calls the correspondence rules)]. And he made the following practical proposal. He said: The theoretical terms are rather bothersome, because we cannot specify explicitly and completely what we mean by them. If we could find a way of getting rid of them and still doing everything that we want to do in physics with the original theory, which contains these terms, that would be fine. And he proposes this existential sentence. You see, in the existential sentence the T-terms no longer occur. They are replaced by variables, and the variables are bound by existential quantifiers, therefore that sentence is in the language $L_{O'}$, i.e., in the extended observation language. And he said: let's just forget about the old formulation TC about the T-terms; let's just take this existential sentence, and from it we get all the observational consequences which we want to have, namely, all those which we can derive from the original theory (Psillos, 2000, 163).

Demopoulos argues in a similar spirit that Ramsey executed Russell's project from *Our Knowledge of the External World* by applying the supreme maxim of scientific philosophy: "Wherever possible, logical constructions are to be substituted for inferred entities" (Russell, [1914] 1951, p. 115). Demopoulos explicitly makes the connection to Russell:

> So understood, Ramsey's paper is a natural development of Russell's phenomenalism since it shows how to achieve the *effect* of the program of logical construction in the context of a theory of theories—namely, the elimination of theoretical vocabulary—without having to enter into the tendentious issue of what to count as an acceptable defining formula, and without having to construct suitable definitions, however the notion of definition is understood (Demopoulos, 2011, 191–192).

[2]Ramsey writes on the first point that while explicit definitions are always possible they are unwieldy:

propositions. It would be odd to claim that an existential statement is a successful reduction; as Hempel notes, it might satisfy the letter of the reductionist project while violating its spirit (Hempel, 1958, 81). Third, as other historians have noted, Ramsey displays very strong pragmatist tendencies in the direction of C.S. Peirce (see Misak, 2016). Those tendencies push strongly against an empiricist project aimed at reducing theories to observation.

A widely accepted interpretation of the Ramsey sentence is the functional realist view that understands the Ramsey sentence to describe the actual functions executed by theoretical entities. Theories are thus approximately true because they document the real causal roles captured in observation while remaining agnostic about the sort of things that act in those causal roles. Important functionalists include Braithwaite (Braithwaite, 1953), Hempel

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> [I]n general the definitions [of the theoretical terms] will have to be very complicated; we shall have, in order to verify that they are complete, to go through all the cases that satisfy the laws and consequences (together with any other propositions of the primary system we think right to assume) and see that in each case the definitions satisfy the axioms (Ramsey, [1929] 1990m, 129).

Even with these definitions, Ramsey rejects the idea of using them because they are at cross-purposes of the reasons for using the theory:

> To this the answer seems clear that it cannot be necessary or a theory would be no use at all. Rather than give all these definitions it would be simpler to leave the facts, laws and consequences in the language of the primary system. Also the arbitrariness of the definitions makes it impossible for them to be adequate to the theory as something in process of growth (Ramsey, [1929] 1990m, 130).

(Hempel, 1958), Lewis (Lewis, 1972), and Psillos (Psillos, 2004).[3] It is worth emphasizing

[3]Braithwaite, a friend and colleague of Ramsey, first articulated the view in 1953. He describes Ramsey's idea as picking out the important theoretical structures without fixing what satisfies those structures:

> One way of answering this question which is in essence the answer given by Ramsey is to say that the status of a theoretical concept (e.g. an electron) is given by the following propositions which specifies the status of an electron within the deductive system of contemporary physics: There is a property E (called "being an electron") which is such that certain higher-level propositions about this property E are true, and from these higher-level propositions there follow certain lowest-level propositions which are empirically testable. Nothing is asserted about the 'nature' of this property E; all that is asserted is that the property E exists, i.e. that there are instances of E, namely, electrons (Braithwaite, 1953, 79).

Hempel picks up Braithwaite's idea and ascribes the same thoughts to Ramsey while emphasizing the importance of leveraging the observational vocabulary in the Ramsey sentence to ascribe truth or falsehood to a theory:

> And indeed, Ramsey himself made no such claim [of avoiding theoretical concepts]. Rather, his construal of theoretical terms as existentially quantified variables appears to have been motivated by considerations of the following kind: If theoretical terms are treated as constants which are not fully defined in terms of antecedently understood observational terms, then the sentences that can formally be constructed out of them do not have the character of assertions with fully specified meaning, which can be significantly held to be either true or false; rather, their status is comparable to that of sentential functions, with the theoretical terms playing the role of variables. But of a theory, we want to be able to predicate truth or falsity, and the construal of theoretical terms as existentially quantified variables yields a formulation which meets this requirement and at the same time retains all the intended empirical implications of the theory (Hempel, 1958, 81).

Lewis identifies the logical relations in the Ramsey sentence as the causal roles fulfilled by the theoretical entities without the sentence explicitly naming those theoretical entities:

> [The Ramsey sentence] says of the entities—states, magnitudes, species, or whatever—named by the T-terms that they occupy certain *causal roles*; that they stand in specified causal (and other) relations to entities named by O-terms, and to one another [....] If I am right, T-terms are eliminable—we can always replace them by their definientia. Of course, this is not say that theories are fictions, or that theories are uninterpreted formal abacuses, or that theoretical entities are unreal. Quite the opposite! Because we understand the O-terms, and we can define the T-terms from them, theories are fully meaningful; we have reason to think a good theory true; and if a theory is true, then whatever exists according to the theory really *does* exist (Lewis, 1972, 253–254).

Psillos identifies the Ramsey sentence as Ramsey's reply to Russell's structural realism. Russell in *Analysis of Matter* tries to use isomorphisms to pick out the structures given by scientific theories, but it quickly was subject to an important objection of Newman's. Psillos argues by using an ordinary existential quantifier, Ramsey leaves open what sort of structures a theory is committed to while allowing theory growth. Psillos calls this a Ramseyean humility:

> We treat our theory of the world as a growing existential statement. We do that because we want our theory to express a judgement: to be truth-valuable. In writing the theory we commit ourselves to the existence of things that make our theory true and, in particular, to the existence of unobservable things that cause or explain the observable phenomena. We don't have to do this. But we think we are better off doing it, for theoretical, methodological and practical reasons. So we are bold. Our boldness extends a bit more. We take the world to have a certain structure (to have natural joints). We have independent reasons to think of it, but in

that this family of interpretation also ascribes immense importance to the Ramsey sentence and its existential quantifier. The quantifier allows Ramsey to capture the causal structure without a commitment to what sort of things have that structure.

The principal problems with the functionalist reading are that Ramsey is a fictionalist about theories and causes and likely a fictionalist about the existential quantifier. Universal propositions like "All men are mortal" and causal propositions like "Arsenic is poisonous" are not truth-apt; actual universal sentences, what Ramsey calls variable hypotheticals, are really "rules for judging" and not proper propositions. So it makes no sense to talk about "real" causal roles in the sense that the proposition about those causal rules is true. The same reasoning about universal propositions applies *mutatis mutandis* to existential propositions, and Ramsey hints as much in an affiliated note to "Theories".[4] So it would be very weird for Ramsey, an avowed fictionalist about general propositions and causes, to offer the Ramsey sentence as a solution to how scientific theories could be approximately true.

While the first two interpretations have had a longstanding impact on the philosophy of

> any case, we want to make our theory's claim to truth or falsity substantive. The theoretical superstructure of our theory is not just an idle wheel. We don't want our theory to be true just in case it is empirically adequate. We want the structure of the world to act as an external constraint on the truth or falsity of our theory. So we posit the existence of a natural structure of the world (with its natural properties and relations). We come to realise that this move is not optional once we made the first bold step of positing a domain of unobservable entities. These entities are powerless without properties and relations and the substantive truth of our theories requires that these are real (or natural) properties and relations
>
> That's, more or less, where our boldness ends. We don't want to push our (epistemic) luck too hard. We want to be humble too. We don't foreclose the possibility that our theory might not be uniquely realised. So we don't require uniqueness: we don't turn our growing existential statement into a definite description. In a sense, if we did, we would no longer consider it as growing. We allow a certain amount of indeterminacy and hope that it will narrow down as we progress (Psillos, 2004, 24).

[4]He writes about both universal and existential sentences as the clue to everything in understanding scientific theories:

> It is possible to have a 'realism' about terms in the theory similar to that about causal laws, and this is equally foolish. 'There is such a quality as mass' is nonsense unless it means merely to affirm the consequences of a mechanical theory. This must be set out fully sometime as part of an account of existential judgments. I think perhaps it is true that the theory of general and existential judgments is the clue to everything (Ramsey, [1929] 1990a, 138).

science, a less well-known and developed interpretation—that Ramsey pioneered an intuitionistic philosophy of science—has not been so important. Majer develops and defends this view in a paper (Majer, 1989).[5] He alleges that Ramsey adopted Weyl's philosophy of mathematics, which denies the law of excluded middle for quantified propositions because universal quantifiers lack an introduction rule and existential quantifiers lack an elimination rule. This has the knock-on effect of making it difficult to justify some iterated quantified sentences such as "every child has a father". The Ramsey sentence is a piece of mathematical logic that can justify these iterated quantifier sentences by witnessing the construction of a theoretical law. So the Ramsey sentence is an artifact of Ramsey's intuitionism and concern with the justification of certain laws.

Majer's interpretation is subtle and ingenious. Unfortunately, it has some fatal weaknesses. First, it assumes that Ramsey converted to Weyl's intuitionism, but the evidence for this is fairly thin. Second, Ramsey never mentions in "Theories" or other affiliated work a concern for iterated quantifier sentences. Third, Majer claims theories justify laws, yet he has no story for how the theories themselves are justified apart from the construction of more laws—laws that cannot be justified because of Ramsey's lack of an introduction rule for the universal

[5] Majer develops the view in great detail by first claiming that Ramsey converted to Weyl's intuitionism, and then claiming that some of Weyl's mathematical writings have a direct bearing on the philosophical problem of reductionism and theories. He argues that Ramsey was concerned with a problem Weyl considered, namely the justification of iterated quantified sentences like $\forall x \exists y R(x, y)$. Ramsey adopted Weyl's solution wholesale and this is what the Ramsey sentence is:

> Ramsey not only adopted Weyl's proposal entirely but made it the core conception of theories. This emerges when he explains the role of theoretical terms in "Theories." "The best way to write our theory seems to be this $(E\alpha, \beta, \gamma)$ dictionary • axioms." [(Ramsey, [1929] 1990m, 131)] If this is not immediately apparent, this is mainly due to the fact that the sentence just quoted only expresses the second and almost trivial part of Weyl's proposal: abstraction of the second order existential sentence from the general-law sentence: whereas the sentence omits the first and most important part of Weyl's proposal: the construction of a law or a theoretical function. It should, however, be crystal clear form what was said before, that according to Weyl the second step, the *abstraction* of the existence of theoretical functions α, β, and γ only makes sense if the first step, the *construction* of the appropriate functions α, β, and γ has already been established! Otherwise, the sentence "There exist theoretical functions α, β, γ satisfying the dictionary and axioms," which is now called the "Ramsey-sentence," would tell us *nothing*, it would be a mere verbal promise without any assurance that it indeed explains or justifies the facts (Majer, 1989, 248–249).

quantifier. Fourth, Majer's use of "justification" is a mysterious answer to a mysterious question about what role theories play; he never spells out what justification is here and the obvious candidate of logical justification fails to work because general propositions fail to be truth-apt. So while intriguing, the intuitionist interpretation has major lacunae that make it inadequate for interpreting Ramsey's philosophy of science.

The most fruitful tradition of scholarship on Ramsey has been provided by historians who interpret Ramsey as a pragmatist in the tradition of C. S. Peirce. Several historians fall into this camp, including Sahlin, Dokic and Engel, and Misak. A core feature of the pragmatist interpretation is the emphasis on how theories grow and change over time. Sahlin, Dokic, and Engel all treat the Ramsey sentence as providing a method by which theories can grow and develop over time; the Ramsey sentence allows the addition of new theoretical terms and relations under the scope of the old theory's quantifiers. But they argue that the theory is not true in the sense that the Ramsey sentence is a proper proposition like those in observation—rather like the functionalist they argue that the causal roles theoretical terms and relations play have some reality to them.[6] Misak adopts a similar emphasis on theory growth and change by way of the functionalist account, except she argues stringently against the claim that theories ever be viewed as fictions. Rather, she contends that Ramsey anticipated what Arthur Fine calls the Natural Ontological Attitude: theories are true in the mundane, everyday scientific sense of "true", and belief in a theory is a bet on its truth.[7] Theories, chances, and causes are all aimed at successful and reliable action just as ordinary beliefs. Consequently, it is proper to ascribe truth to theories and existence to theoretical entities because of their influence on action.[8]

[6] Sahlin for example takes there to be a distinction between theoretical terms having meaning independent of the theory and the theory itself failing to be true:

> In his view we cannot say anything meaningful about them outside the framework of the theory but notice that we have not said that electrons or Zeus do not exist; we say only that the entities we name 'electrons' or 'Zeus' gain their meaning because of their place in the theory. We must distinguish between a theory's terms and its concepts. If the terms 'electron' or 'Zeus' have a function in a particular theory, this does not directly answer the question whether electrons or Zeus really exists. If Ramsey's method is to be effective, the concepts and terms must be identified. It is thus quite conceivable that, using Ramsey sentences, one could argue in favour of both an instrumentalist and realist position [Sahlin then cites the David Lewis paper I discussed above] (Sahlin, 1990, 152).

Similarly, Dokic and Engel argue that Ramsey clearly believes theories to be fictions while still making room for the reality of the functional roles theoretical entities operate in:

> Ramsey's non-cognitivist or non-factualist conception of laws is even more salient when he deals with the classical problem of the nature of scientific theories in his article "Theories" (1929). Against the realist conception according to which theoretical terms such as "photon" or "gene" denote real unobservable entities, Ramsey stands clearly on the anti-realist side, holding that such terms do not have a separate denotation, but meaning only within statements which help to predict observations (Dokic and Engel, 2002, 35–36).
>
> For if theoretical entities themselves are only "fictions", the causal roles which they instantiate are real [....] In this sense they are anything but fictions (Dokic and Engel, 2002, 38).

[7] Misak writes that theory growth proceeds by way of a theory's excess "multiplicity" in the manner described by Sahlin. This commitment to theory growth and change reveals Ramsey's embrace of the NOA attitude to scientific theories:

> But there is an important, new point to excavate from this paper. Ramsey also suggests that what our theories are talking about can be taken to exist [....] [here she quotes Psillos, 2004] His [Ramsey's] position, rather than being reductionist or constructionist, is more along the lines of what Fine these days calls our natural ontological stance, in which the entities and relations that show up in our best theories are quite reasonably taken to exist [....] Ramsey's way of being a realist is by being a Peircean pragmatist. Causes will be in our best theory, we bet. But this is merely a bet (Misak, 2016, 227).

[8] Misak pushes back against other pragmatists who try to be more faithful to Ramsey's texts by emphasizing Ramsey's commitment to NOA, which she claims is the pragmatist theory of truth:

The main problem for the pragmatist readings of Ramsey is that they fail to reconcile Ramsey's objective of a non-reductive account of scientific theories with his avowed belief that theoretical and causal propositions fail to be truth-apt. Sahlin and Dokic and Engel are explicit about this tension without ever resolving it. Misak tries to finesse the tension by ascribing to Ramsey both a deflationary sense of truth and a pragmatic theory of truth, but these conflict with the text and both cannot be accurate. Ramsey does accept some version of the pragmatic theory of truth, but he seems to believe it only applies to singular propositions and not general propositions. This theory of truth, as Fine would describe it, however, conflicts with the Natural Ontological Attitude because it ascribes more to truth then the simple, everyday sense (see Fine, 1986). It is unlikely then that Ramsey had something like Fine's position, and his pragmatic theory of truth seems to only apply to singular propositions and not laws or theories. So the pragmatist reading fails to capture some core aspects of Ramsey's philosophy of science.

In summary, the four primary interpretations of Ramsey's philosophy of science have significant problems. The positivist reading conflicts with the spirit of Ramsey's argument in "Theories"; the functionalist reading ascribes to Ramsey a realism about theories and general propositions he plainly does not have; the intuitionist reading mistakenly attaches to Ramsey a concern about certain iterated quantifiers; and the pragmatist reading fails to marry Ramsey's anti-reductionism with his anti-realism. The task of my dissertation is to provide an account that leverages the strengths of these different interpretations without their weaknesses.

> My own view is that it is clear that significant elements in Ramsey's thought cut against [the view that general sentences are not truth-apt.] That is, he is not offering us a local pragmatism that says that only theoretical statements and other empirical-looking statements that we cannot in fact verify are to be assessed in terms of whether they work. *All beliefs* are habits with which we meet the future, and if they would meet the future well and be in our best

1.3 Ramsey's "Theories"

A good starting point for analyzing Ramsey's philosophy of science is to establish the structure of the principal paper "Theories" and to juxtapose that structure with the philosophies of science that are its target. In this section, I briefly provide the reader with an overview of "Theories" along with the main topic of discussion in the paper, the elimination of theoretical propositions for observational propositions.

Ramsey begins by considering scientific theories as if they are languages. He calls the theoretical language the secondary system, and he calls the propositions to be explained, i.e. observation, the primary system. The secondary system (theoretical language) consists of new vocabulary, axioms governing theoretical propositions, and a dictionary that defines primary system propositions in terms of secondary system propositions. His discussion of the two systems is fairly brief, with more attention paid to the primary system and its mathematical structure. The whole discussion lasts four pages.

Following the introduction of the basic apparatus of his philosophy of science, Ramsey states that his theory can be made clear with an example.[9] The example consists of a primary system that involves the perception of colors, the feeling of eyes closing and shutting, and feelings of movement, while the secondary system is a physical map of three locations that form a ring. It is evident from Ramsey's discussion of this toy secondary system that by scientific theories Ramsey includes things like the "physicalist language". Ramsey claims to deduce a series of five laws from his secondary system. The discussion of the example is exceedingly formal with both a rendition in first-order logic and in terms of equations.

theory, then they are true (Misak, 2016, 194).

[9]The example adds clarity to Ramsey's philosophy in the same way that a stroke adds to cognitive ability. Ramsey seems to agree when he provides a footnote that says "The example *seems* futile, therefore try to invent a better; but it in fact brings out several good points, which it would be difficult otherwise to bring out" (Ramsey, [1929] 1990m, 115). Given that I am helping his philosophy recover from a stroke, the title "doctor" may be well deserved.

Ramsey's discussion of this formal model also lasts four pages.

The bulk of the paper then follows with the discussion of six questions. Those questions are:

> 1. Can we say anything in the language of this theory that we could not say without it? (Ramsey, [1929] 1990m, 119)
>
> 2. Can we reproduce the structure of our theory by means of explicit definitions within the primary system? (Ramsey, [1929] 1990m, 120)
>
> 3. We have seen that we *can* always reproduce the structure of our theory by means of explicit definitions. Our next question is 'Is this *necessary* for the legitimate use of the theory?' (Ramsey, [1929] 1990m, 129)
>
> 4. Taking it then that explicit definitions are not necessary, how are we to explain the functioning of our theory without them? (Ramsey, [1929] 1990m, 130)
>
> 5. This mention of 'disputes' leads us to the important question of the relations between theories. What do we mean by speaking of equivalent or contradictory theories? or by saying that one theory is contained in another, etc.? (Ramsey, [1929] 1990m, 132)
>
> 6. We could ask: in what sort of theories does every 'proposition' of the secondary system have meaning in this sense [verifiability]? (Ramsey, [1929] 1990m, 134)

The question with the largest discussion is question two, which lasts ten pages of the twenty-four-page paper. By length alone, this is the primary topic of discussion for Ramsey; in comparison, the Ramsey sentence, which appears in Ramsey's answer to question four, only receives about two pages of discussion. So the bulk of the paper is devoted to how one can reduce the structure of a theory to observation with explicit definitions.

It is also this second question where Ramsey mentions Russell, Whitehead, Nicod, and Carnap. Wittgenstein should be added to this list because in responding to the question, Ramsey considers three approaches to explicitly defining theoretical propositions. Those approaches are:

1. Define the theoretical propositions with the laws and consequences of the theory.

2. Invert the theoretical dictionary to define theoretical propositions in terms of primary system propositions.

3. Define the theoretical propositions with their verification conditions.

The first of these naturally corresponds with Russell, Whitehead, and Nicod's philosophy of science where regularities and individual propositions are used to construct sets that stand in for theoretical propositions through either abstraction or extensive abstraction. The second corresponds roughly with Carnap's constructive program in the *Aufbau* of defining the theoretical through recollected similarity. The third, however, tracks none of these since Russell, Whitehead, Nicod, and Carnap do not appeal to verification for yielding the sense of theoretical propositions. As I discuss in the following section, the only natural candidate whom Ramsey interacted with and who held this view is Wittgenstein in 1929. So Ramsey's targets for the bulk of his discussion in "Theories" are Russell, Whitehead, Nicod, Carnap, and Wittgenstein.

1.4 Russell, Whitehead, Nicod, Carnap, and Wittgenstein

If the bulk of "Theories"—which is the bulk of Ramsey's philosophy of science—is a response to Russell, Whitehead, Nicod, Carnap, and Wittgenstein, then it would be good to review the views Ramsey argues against, and why Ramsey thinks they fail. This will be a cursory, high-level discussion of these positions; there are details that will be omitted that a more thorough historical assessment would include. However, I do not aim to recover their philosophies exactly and evaluate them justly on their truth, but to render them in a way that Ramsey seems to have understood them. So only the conclusion of Ramsey's arguments will be given.

Russell's philosophy of science is subtle and complicated. It starts from common knowledge, the body of beliefs that people accept as true (Russell, [1914] 2009, 51–52). Common knowledge beliefs, despite being true, are vague and ambiguous, and so they are hard to verify or falsify. The task of philosophy and with it science is to make these common knowledge beliefs exact so they can be tested (Russell, [1918] 1940, 37–38). This proceeds in two stages: analysis and synthesis. In order to avoid the "theft" of inferred entities, the proper method is the "honest toil" given by the Supreme Maxim of Scientific Philosophy: "Wherever possible, logical constructions are to be substituted for inferred entities" (Russell, [1914] 1951, 115). This method is recommended because of Occam's Razor; by minimizing the number of inferred entities, one can eliminate the number of propositions that might be false and so reduce the odds of error in the philosophical and scientific process (Russell, [1918] 1940, 153–154). Logical constructions are primarily crafted via the methods of abstraction and explicit definitions where the quintessential exemplar is Russell's own construction of the natural numbers as the class of equinumerous classes.[10] In the case of everyday objects and scientific entities, the fundamental building block of these constructions are collections of sense-data, particular patches of color, notes of sound, etc., because these are what peo-

[10]Russell places a great deal of utility on these methods for clearing "metaphysical lumber":

> In both these cases, and in many others, we shall appeal to a certain principle called "the principle of abstraction." This principle, which might equally well be called "the principle which dispenses with abstraction," and is one which clears away incredible accumulations of metaphysical lumber, was directly suggested by mathematical logic, and could hardly have been proved or practically used without its help. The principle will be explained in our fourth lecture, but its use may be briefly indicated in advance. When a group of objects have that kind of similarity which we are inclined to attribute to possession of a common quality, the principle in question shows that membership of the group will serve all the purposes of the supposed common quality, and that therefore, unless some common quality is actually known, the group or class of similar objects may be used to replace the common quality, which need not be assumed to exist (Russell, [1914] 2009, 33-34).

ple are ultimately directly acquainted with.[11] Collections of sense-data are taken to be the various "perspectives" that can be unioned together based on similarity relations to form aspects that characterize the various perspectival properties of objects such as visual size. The regularities that exist in and between series of these aspects define the physical objects themselves: "*things are those series of aspects which obey the laws of physics*" (Russell, [1914] 2009, 88).[12] So the physical laws, understood as particular series of aspects, are what ultimately characterize physical objects. With analysis finished, science begins with synthesis. In the synthetic mode, constructions are tested against particular experiences to verify and falsify hypotheses. When constructions turn out to not accurately predict future sense-data, they are to be rejected and the process of analysis begins anew to find scientific theories (constructions) that agree with the data. So philosophy and science proceed as a two-step dance of analysis to synthesis and back to analysis.

Whitehead's philosophy of science has some radical differences from Russell's. It does not conceptualize philosophy and science working in tandem through the methods of analysis and synthesis. Instead, Whitehead is more concerned with the metaphysical issue of whether the world is composed of interdependent processes or independent objects; he believes the former, which puts him at general odds with Russell during Russell's logical atomist phase. But like Russell, Whitehead wants to leverage the tools of mathematical logic to eliminate troublesome entities, i.e. points and other objects. Whitehead adopts the method of extensive abstraction, and he takes as basic spatiotemporal events (Whitehead, 1920, 24). Those events have numerical features—quantitative functions defined on the events such as temporal extension—that can then be used to define properties and objects from overlapping events. An abstractive set is an infinite series of events where each $i+1$-th member of the

[11] The actual method is even more subtle as collections are themselves incomplete symbols and so constructable. Russell proposes a no-class theory of classes, where sets are defined through definite descriptions and propositional functions (Russell, [1918] 1940, 136–138). Notably, this no-class theory employs a slightly different technique from explicit definitions: contextual definitions. See Linsky, 2003 and Hylton, 1993 for an extended discussion.

[12] See the same definition of physical objects in "The Relation of Sense-data to Physics": "*Physical things are those series of appearances whose matter obeys the laws of physics*" (Russell, [1914] 1951, 27).

series is a sub-event of the i-th member and the limit of their series of features is equal to zero. Notably, this set has no "smallest" member; it is atomless. Whitehead uses these sets to define things like points in geometrical physics or everyday objects such as tables or chairs.[13] This method of extensive abstraction can thus eliminate metaphysically suspect things such as points for more sound collections of processes. Similar to Russell, the regularities given by the numerical features can then be thought to adequately define the theoretical entities of physical objects through the corresponding abstractive sets (Whitehead, 1920, 61).[14] The difference between the two is that, according to Russell, the ultimate constituents of the world are logical atoms.[15] Thus, while Whitehead differs from Russell in terms of where to start when logically constructing physical theories and in the method of extensive abstraction, he still accepts the general project of logically constructing everyday objects from sense experience via the regularities in that experience.

Nicod is interested in the problem of how experience verifies the predictions of physics. He notes that verification is vague and ambiguous; to properly confirm or falsify a physical theory, one must be able to say precisely what has occurred in sensory experience. This necessitates the analysis of the geometry of experience—how the visual, auditory, and olfactory fields are ordered so as to apply the geometrical concepts like points, lines, and so on required for verification. In short, Nicod's project is to logically construct the geometry of experience from the primitive sense-data that compose experience (Nicod, 1930, 13–14).[16] The way he proceeds with the project is similar to Russell: use similarity or resemblance

[13]For a full discussion of this method, see Lawrence, 1950 and for problems with it see Grünbaum, 1953.

[14]Whitehead argues at length that the Newtonian laws of motion are an artifact of these numerical features over the more fundamental processes that are events. See his discussion in chapter six of Whitehead, 1920.

[15]Though this changes in the 1920s as Russell abandons the doctrine of logical atomism for something closer to Whitehead in *Analysis of Matter* (Russell, 1927).

[16]Nicod announces that the primary aim of his work is to see through the geometry of sense experience how the propositions of physics are verified:

> We propose in this work to ascertain in what way geometry is an aid to physics; how its propositions are applied to the order of the perceived world; how knowledge of them helps us in the formulation of experiments and laws. For every statement in physics teems with geometry: every prediction of a perceptual fact is dependent on a certain disposition of the objects and observers, which is expressible in geometrical form (Nicod, 1930, 14).

relations between the ultimate constituents of experience, sense-data, to logically construct points and other geometric features (Nicod, 1930, 51–54). These geometric features embody the laws of physics so the various logical constructions can be thought of as equivalent to the experiences given by those laws.[17] So like Russell and Whitehead, Nicod's project is a constructive one that builds upon the regularities in sense experience to explicitly define some theoretical entity, i.e. the geometry of that sense experience.

Ramsey's objection to this style of reductionism is that the logical constructions are *arbitrary* in the sense that multiple constructions are possible and *complicated* in the sense that they require massive disjunctions to be effective. He writes that the laws and consequences of a scientific theory are insufficient for explicitly defining the theory's propositions:

> We might, for instance, argue as follows. Supposing the laws and consequences

[17]Nicod concludes that the constructed geometry of experience embodies those laws given by the physical theories:

> That amounts to saying that geometry does not apply to the perceived world in only a limited domain of physics, such, for example, as that of the displacements of rigid solids, but really applies to all physics in each one of its branches. Does not every physical proposition contain places, directions, and distances? The geometrical structure of the world is the structure of all its laws embodied in a few formal characters.
>
> The application of geometry to nature is free of the limitations which we have imposed on ourselves. We have shown how certain bodies perceived in a certain way would present a perceptible spatial order. But in reality, this order embraces all bodies and all perceptions. We must guard against saying: Geometry does not apply to the universe except to the extent particular hypotheses of the solutions that have been exposed are satisfied. We must affirm on the contrary: These solutions are still only particular solutions of a general problem, which is no other than that of the empirical meaning of all physics (Nicod, 1930, 190–191).

Nicod's point is that the construction of the geometry of sense experience is a particular instance of finding the empirical meaning of all physical laws. The particular relations between sense-data encode the physical laws, and the physical laws of the theory should dictate the kind of relations that would obtain between sense-data. Nicod at the outset of his project admits this:

> This study might be a preface to the analysis of physics in terms of experience. It is also a beginning in it. For we shall find that the universal order of space to which every physical propositions seems to refer is, in truth, nothing but the very group of the laws of physics. The properties of space are already the most general schemata of physics and are nothing else. Thus—we shall be convinced of it as we proceed—the study of the spatial structure of a sensory universe is the study of the form and totality of all its laws (Nicod, 1930, 13–14).

> to be true, the facts of the primary system must be such as to allow functions to be defined with all the properties of those of the secondary system, and these give the solution of our problem. But the trouble is that the laws and consequences can be made true by a number of different sets of facts, corresponding to each of which we might have different definitions (Ramsey, [1929] 1990m, 120).

Ramsey's proposed solution is to disjoin material conditionals with particular facts and the laws to define the theoretical atomic propositions. However, this does not work because "what can be objected to it is complexity and arbitrariness, since [the laws] can probably be chosen each in many ways" (Ramsey, [1929] 1990m, 121). The complexity here refers to the disjunction while the arbitrariness refers to different laws that can be used. So Ramsey thinks this method is not ideal.

Like Russell, Carnap's philosophy of science in the *Aufbau* is sophisticated and subtle. The goal is to rationally reconstruct scientific language into a more perfect, ideal language for doing unified science; an important corollary of this construction is the elimination of metaphysics and pseudo-problems such as the nature of substance. Carnap proposes to reduce in successive fashion first-person psychological phenomena, the "autopsychological", to basic relations between gestalt experiences, the physical language to the autopsychological, the psychological phenomena of other people, the heteropsychological, to the physical language, and finally, the socio-economic and cultural language to the heteropsychological. While each person's reduction is unique due to the personal characteristics of their gestalt experiences, Carnap argues that the shared structural properties found in each reduction vouchsafe the inter-subjectivity of the reduced objects and concepts and so the objectivity of the unified science.[18] Importantly, this means that Carnap bets that scientific statements point to

[18] By "structure", Carnap means the logical characteristics of the relations in a logical language. He writes:

> There is a certain type of relation description which we shall call *structure description.* Unlike relation descriptions, these not only leave the properties of the individual elements of the range unmentioned, they do not even specify the relations themselves which hold between these

unique structures shared by people whose constructions differ in the elements and relations

> elements. In a structure description, only the *structure* of the relation is indicated, i.e, the totality of its form properties [....] By formal properties of a relation, we mean those that can be formulated without reference to the meaning of the relation and the type of objects between which it holds (Carnap, [1928] 1967, 21).

Carnap then gives classic examples such as symmetry, reflexivity, and transitivity. The idea of shared structure between two sentences then is an idea of isomorphism between the objects and relations that make those sentences true:

> In order to understand what is meant by the structure of a relation, let us think of the following arrow diagram: Let all members of the relation be represented by points. From each point, an arrows to those other points which stand to the former in the relation in question. A double arrow designates a pair of members for which the relation holds in both directions. An arrow that returns to its origin designates a member which has the relation to itself. If two relations of the same arrow diagram, then they are called *structurally equivalent*, or *isomorphic* (Carnap, [1928] 1967, 22).

Note that Carnap is operating before the distinction between a sentence and its model.

of their basis.[19] This uniqueness claim leads Carnap to assert that the content of scientific statements is strictly about the structures found in a rational reconstruction of scientific statements (Carnap, [1928] 1967, 29).[20] Carnap's proposal for recovering the claims of scientific theories as structural claims is through the method of explicit definitions and definitions

[19]The problem is that mathematical structures do not seemingly appeal to the content of the propositions that they are constructed from, but scientific structures being empirical must in some way. Carnap draws this contrast:

> Thus, our thesis, namely that scientific statements relate only to structural properties, amounts to the assertion that scientific statements speak only of form without stating what elements and the relations of these forms are. Superficially, this seems to be a paradoxical assertion. Whitehead and Russell, by deriving the mathematical disciplines from logistics, have given a strict demonstration that mathematics (viz., not only arithmetic and analysis, but also geometry) is concerned with nothing but structure statements. However, the empirical sciences seem to be of an entirely different sort: in an empirical science, one ought to know whether one speaks of persons or villages. This is the decision point: *empirical science must be in a position to distinguish these various entities*; initially, it does this mostly through definite descriptions utilizing other entities. But ultimately the definite descriptions are carried out with the aid of structure descriptions only (Carnap, [1928] 1967, 23).

Carnap's solution is to emphasize that empirical science has a structure unique enough to eliminate the alternatives. He argues this is an a priori precondition for the possibility of science, and then gives an example involving a railroad network to illustrate how it might be possible without recourse to something like ostensive definition (Carnap, [1928] 1967, 25–27). The basic thought is that if one exhausts every structure from empirical propositions without recourse to ostensive definition, then there is no objective empirical difference between proposed descriptions and that this is enough for unified science. From this example, he concludes that with empirical propositions, it is possible to record a unique structural description that, in modern terminology, pins down the models that satisfy that description:

> From the preceding example, we can see the following: on the basis of a structural description, through one or more only structurally described relations within a given object domain, we can frequently provide a definite description of individual objects merely through structure statements and without ostensive definitions, provided only that the object domain is not too narrow and that the relation or relations have a sufficiently variegated structure. Where such a definite description is not unequivocally possible, the object domain must be enlarged or one must have recourse to other relations. If all relations available to science have been used, and no difference between two given objects of an object domain has been discovered, then, as far as science is concerned, these objects are completely alike, even if they appear subjectively different [....] Thus, the result is that *a definite description through pure structure statements is generally possible to the extent in which scientific discrimination is possible at all*; such a description is unsuccessful for two objects only if these objects are not distinguishable at all by scientific methods (Carnap, [1928] 1967, 27).

[20]Carnap writes that scientific claims are structural claims:

> Now, the fundamental thesis of construction theory [...], which we will attempt to demonstrate in the following investigation, asserts that fundamentally there is only one object domain and that each scientific statement is about the objects in this domain. Thus, it becomes unnecessary to indicate for each statement the object domain, and the result is that *each scientific statement can in principle be so transformed so that it is nothing but a structure statement.* But this transformation is not only possible, it is imperative. For science wants to speak about what is

in use, where the definitions ultimately appeal to no material content of sentences but only to their formal content. In other words, if it is true that propositions in scientific theories are really structural claims, then there are definitions that capture those structural claims in the subjective, autopsychological language. Carnap's proposal is to use these definitions (transformations as he calls them) to reduce the cultural, heteropsychological, and ultimately physical to the autopsychological, where the autopsychological is logically constructed from a primitive basis of recollected similarity between gestalt experiences.

The core objection Ramsey has to Carnap's program is the uniqueness claim Carnap has for the structures that scientific theories make claims about. Ramsey tries to demonstrate how this is problematic by following a version of Carnap's proposal and "inverting" the dictionary of a scientific theory to uniquely define scientific functions and propositions with observation functions and propositions. After showing that such a unique definition is impossible with his toy example, he concludes that any unique definition is impossible because of the excess "multiplicity" ubiquitous in scientific theories:

> We conclude, therefore, that there is neither in this case nor in general any simple way of inverting the dictionary so as to get either a unique or an obviously preeminent solution which will also satisfy the axioms, the reason for this lying partly in difficulties of detail in the solution of the equations, partly in the fact that the secondary system has a higher multiplicity, i.e. more degrees of freedom, than the primary [....] and such an increase of multiplicity is, I think, a *universal* characteristic of useful theories (Ramsey, [1929] 1990m, 122).

Ramsey's objection is that the construction can proceed in multiple ways to the physical language. This means that multiple definitions are always possible, and the unique structure

objective, and whatever does not belong to structure but to the material (i.e., anything that can be pointed out in a concrete ostensive definition) is, in the final analysis, subjective (Carnap, [1928] 1967, 29).

Carnap desires is impossible. There is just more than one unique, empirically meaningful way a construction can proceed from the autopsychological. In short, scientific theories are not truth-functions of observation.

The last person Ramsey is responding to is Wittgenstein. There are several reasons for thinking that the last approach Ramsey considers for explicitly defining theoretical terms, relations, and propositions, through their verification conditions, is a proposal of Wittgenstein's. First, Russell, Whitehead, Nicod, and Carnap never mention the verification conditions of a theory. Second, the only person whom Ramsey was in communication with at that time who had this as a proposal is Wittgenstein. Third, Ramsey and Wittgenstein were interacting nearly daily in 1929 around the time Ramsey drafted "Theories", which would make it very probable that they did talk about the philosophy of science. And fourth, many of Wittgenstein's ideas in the *Philosophical Remarks* and in Waismann's notes bear a striking resemblance to Ramsey's own ideas. Consequently, a person whom Ramsey would have likely responded to in "Theories" would be Wittgenstein.

Wittgenstein's philosophy of science in 1929 and 1930 is a mishmash of earlier ideas from the *Tractatus*, some ideas that later came to dominate his thinking, and other temporary ideas included in none of the above. At its core, this philosophy of science marries the picture theory of meaning with a phenomenal verificationism while abandoning the thesis that the senses of elementary propositions are independent of one another. Wittgenstein still holds that propositions are pictures that depict facts through the sharing of a logical form.[21] The sense or meaning of a proposition is given by its verification conditions.[22]

[21]This comes in multiple places in the Waismann volume and *Philosophical Remarks*. Both of these are dated from 1929 to 1931. In Waismann, he states it clearly in conversation: "But it is the essential feature of a proposition that it is *a picture* and has compositeness" (Waismann, 1979, 90). This shows up in the same volume when Waismann enumerates a number of theses that Wittgenstein at the time seems to be committed to. Among those include a picture theory for propositions, though importantly those pictures may be given by the syntax of propositional systems. The syntax restricts the sort of pictures that can be constructed from a system artificially, so that the system has the right multiplicity to depict the facts it depicts:

> We can picture facts to ourselves.
>
> A picture represents the existence or non-existence of a state of affairs.
>
> What a picture represents is its *sense*.
>
> The agreement of its sense with reality constitutes the truth of a picture.
>
> A picture can be true or false only if it is different from what it depicts.
>
> [....]
>
> What a picture, even an incorrect one, must have in common with what it depicts is its *form*, i.e. the possibility of structure.
>
> A *true* picture also has its *structure* in common with what it depicts. A picture can depict everything that has the same form; it cannot, however, depict anything else.
>
> *Syntax* consists of rules which specify the combinations such that in them alone a word makes sense. It is by syntax that the construction of nonsensical combinations of words is excluded.
>
> Syntax hence becomes requisite where the nature of signs is not yet adjusted to the nature of things, where there are more combinations of signs than possible situations. This excessive multiplicity of language must be confined by artificial rules; and these rules are the syntax of language.
>
> The rules of syntax assign to combinations of signs the exact multiplicity they must possess in order to be pictures of reality (Waismann, 1979, 239–240).

Wittgenstein discusses how some propositions can provide incomplete pictures in *Philosophical Remarks*:

> There must be incomplete elementary propositions from whose application the concept of generality derives.
>
> This incomplete picture is, if we compare it with reality, right or wrong: depending on whether or not reality agrees with what can be read off from the picture (Wittgenstein, 1975, 115)

[22]Wittgenstein is explicit about this in multiple places. In the *Philosophical Remarks* he writes that "every proposition is the signpost for a verification" (Wittgenstein, 1975, 174). He explicitly ties this to sense:

> How a proposition is verified is what it says. Compare the generality of genuine propositions with generality in arithmetic. It is differently verified and so is of a different kind.
>
> The verification is not *one* token of the truth, it is *the* sense of the proposition. (Einstein: How a magnitude is measured is what it is.) (Wittgenstein, 1975, 200)

Similar strong claims appear in the Waismann volume in conversation:

> The other conception, the one I want to hold, says, 'No, if I can never verify the sense of a proposition completely, then I cannot have meant anything by the proposition either. Then proposition signifies nothing whatsoever.'
>
> In order to determine the sense of a proposition, I should have to know a very specific procedure for when to count the proposition as verified (Waismann, 1979, 47).

The verification conditions of a proposition are given by the phenomenal experiences one would expect were the proposition true or false.[23] Non-elementary propositions can be

Wittgenstein repeats this again in the Waismann volume, where he states "The sense of a proposition is the method of its verification" (Waismann, 1979, 79). Waismann also records it in his theses, with a whole section titled "Verification":

> A person who utters a proposition must know under what conditions the proposition is to be called true or false; if he is not able to specify that, he also does not know what he has said.
>
> To understand a proposition means to know how things stand if the proposition is true.
>
> [....]
>
> In order to get an idea of the sense of a proposition, it is necessary to become clear about the procedure leading to the determination of its truth. If one does not know that procedure, one cannot understand the proposition either.
>
> [....]
>
> *The sense of a proposition is the way it is verified* (Waismann, 1979, 243–244)

[23]The "procedures for verification" that Wittgenstein considers are those that are executed with respect to the senses. He writes in the *Philosophical Remarks* that while he no longer believes in a primary system, he does admit that a phenomenal language is necessary to see what is essential in language: "a recognition of what is essential and what inessential in our language if it is to represent, a recognition of which parts of our language are wheels turning idly, amounts to the construction of a phenomenological language" (Wittgenstein, 1975, 51). This means that the essential features of language—the sense of its propositions—have to be tied with the phenomenological:

> A phenomenon isn't a symptom of something else: it is the reality.
>
> A phenomenon isn't a symptom of something else which alone makes the proposition true or false: it itself is what verifies the proposition (Wittgenstein, 1975, 283).

He ties this to the elementary propositions. In Waismann's theses, the elementary propositions are those that "deal with reality immediately" (Waismann, 1979, 248), and they describe phenomenal experiences:

> To specify the elementary propositions means to specify the states of affairs in the world.
>
> It is clear that statements about bodies (tables, chairs) are not elementary propositions. Nor will anybody believe that in talking about bodies we have reached the ultimate elements of description.
>
> Phenomena (experiences) are what elementary propositions describe (Waismann, 1979, 249)

analyzed using definitions so that they are reduced to their verification conditions.[24] So the theory is that propositions are pictures of experience whose meaning is given by its verification conditions in sense experience, and a successful definition of one sign in terms of another is to provide the verification conditions for all the propositions the defined sign occurs in. Theoretical propositions—what Wittgenstein calls "hypotheses"—like "The chair is brown", fall into this theory as logical structures that act as prediction engines whose sense, if any, is given by how they are verified.[25] They are inductions, laws, that provide the propositions that verify them.[26] So a natural position might be to define hypotheses

[24]Wittgenstein provides a number of remarks about definitions. He divides them into two cases: ostensive and sign-to-sign. The sign to sign is a definition of a sign in terms of another sign. It is successful in so far as it provides a complete specification of the verification conditions of one proposition in terms of another. This is most fully presented in Waismann's theses:

> You know the meaning of a sign if you understand the sense of the propositions in which it occurs.
>
> To define a sign thus means to explain the sense of propositions in which it occurs.
>
> A definition thus consists in the specification of a rule which tells how to express by means of other signs the sense of a proposition in which the sign in question occurs.
>
> A definition is a translation rule—it translates a proposition into other signs.
>
> Translation preserves the sense of a proposition (Waismann, 1979, 246–247).

[25]Wittgenstein discusses hypotheses in multiple places, but the most prolonged discussion is in the *Philosophical Remarks*. He seems to suggest that all propositions, including hypotheses, must ultimately be connected with some process of verification to have sense:

> A proposition, an hypothesis, is coupled with reality—with varying degrees of freedom. In the limit case there's no longer any connection, reality can do anything it likes without coming into conflict with the proposition: in which case the proposition (hypothesis) is senseless!
>
> [....]
>
> All that's required for our propositions (about reality) to have sense, is that our experience *in some sense or other* either tends to agree with them or tends not to agree with them ()

He then states that hypotheses are structures that allow certain rules to hold where how the rules connect with the experience determine the truth or falsity of the hypothesis:

> An hypothesis is a logical structure. That is, a symbol for which certain rules of representation hold.
>
> The point of talking of sense-data and immediate experience is that we're after a description that has nothing hypothetical in it. If an hypothesis can't be definitively verified, it can't be verified at all, and there's no truth or falsity for it (Wittgenstein, 1975, 283).

Here Wittgenstein seems to oscillate between lumping propositions and hypotheses together. He is much clearer to distinguish between 'statements' and 'hypotheses' in his discussion with Waismann: "An hypothesis is not a statement, but a law for constructing statements" (Waismann, 1979, 99).

[26]Wittgenstein writes in multiple places about how hypotheses extend into the future and particular

through the verifications they produce as laws. The meaning of a hypothesis would just be the meaning given by the intersection of its verifications. Hypotheses would be reduced to their verifications.

Notably, this natural conclusion Wittgenstein fails to endorse. He explicitly rejects the meaning of a hypothesis to be its verifications.[27] Instead, Wittgenstein proposes that hypotheses can only be judged through their relative probability, by which he means simplicity. Why does Wittgenstein believe this despite suggesting in some places that hypotheses can be

---

propositions are "cross-sections" of the hypothesis:

> You could obviously explain an hypothesis by means of pictures. I mean, you could, e.g., explain the hypothesis, 'There is a book lying here', with pictures showing the book in plan, elevation and various cross-sections.
>
> Such a representation gives *a law*. Just as the equation of a curve gives a law, by mean of which you may discover the ordinates, if you cut at different abscissae.
>
> In which case the verification of particular cases correspond to cuts that have actually been made (Wittgenstein, 1975, 284).

The thought is that hypotheses provide laws that consist of various propositions that verify the laws. For example, the verifications for the book example Wittgenstein uses consist of the sense experiences that would occur by examining the book from different perceptual angles and so on. Similar language is used in discussions with Waismann, where Wittgenstein states "what we observe are always merely 'sections' through the connected structure of the law" (Waismann, 1979, 100).

[27] Wittgenstein states in multiple places that hypotheses are never verified and are not true or false in the way ordinary propositions happen to be. For example, in *Philosophical Remarks*, he states hypotheses are never confirmed and extend past available experience:

> What is essential to an hypothesis is, I believe, that it arouses an expectation by admitting of future confirmation. That is, it is of the essence of an hypothesis that its confirmation is never completed.
>
> When I say an hypothesis isn't definitively verifiable, that doesn't mean that there is a verification of it which we may approach ever more nearly, without ever reaching it. That is nonsense—of a kind into which we frequently lapse. No, an hypothesis simply has a different formal relation to reality from that of verification. (Hence, of course, the words 'true' and 'false' are also inapplicable here, or else have a different meaning (Wittgenstein, 1975, 285).

Wittgenstein also says in conversation with Waismann that laws cannot be verified and hypotheses are laws—so hypotheses are not verified:

> A natural law cannot be verified or falsified. Of a natural law you say that is neither true nor false but 'probable,' and here 'probable' means: simple, convenient. A statement is true or false, never probable. Anything that is probable is not a statement (Waismann, 1979, 100).

Waismann records similar views in his theses where he writes that "only particular statements can be true or false, an hypothesis cannot" and also that "a law of nature is not constructed by means of the *sense* of particular descriptions—it is hence not a truth-function of those propositions" (Waismann, 1979, 255).

analyzed through their verifications?[28]

This is precisely the question Ramsey considers in "Theories". Ramsey considers a possible reduction of theoretical terms, relations, and propositions to their verification conditions. He concludes after an extended, in-depth argument with multiple considered objections that such a reduction while possible would always result in arbitrary and complicated definitions:

> But in general the definitions will have to be very complicated; we shall have, in order to verify that they are complete, to go through all the cases that satisfy the laws and consequences (together with any other propositions of the primary system we think right to assume) and see that in each case the definitions satisfy the axioms, so that in the end we shall come to something very like the general disjunctive definitions with which we started this discussion (Ramsey, [1929] 1990m, 129).

Ramsey concludes that definitions through verification conditions fail for the same reason definitions through laws and consequences fail: they are arbitrary and complicated. Now it stands to reason that this is precisely the reason why Wittgenstein rejects equating hypotheses with their verifications despite edging close to the view. He and Ramsey were in constant conversation, and Ramsey's discussion in "Theories" could only possibly be about Wittgenstein. So it is likely that Ramsey's complicated argument here is a direct representation of discussions he had with Wittgenstein in 1929.

[28]For instance, Wittgenstein often complains about how hypotheses can have "idle wheels" and so not be meaningful. This would only make sense if the hypothesis's meaning came from its verification conditions as Wittgenstein seems to hold of all propositions. For instance, he writes that laws can be senseless if they fail to have verification conditions:

> Of course this is only a natural law if it can be confirmed by a particular experiment, and also refuted by a particular experiment. This isn't the case on the usual view, for if *any* event can be justified throughout an arbitrary interval of time, then *any* experience *whatever* can be reconciled with the law. But that means the law is idling; it's senseless (Wittgenstein, 1975, 290).

In summary, Ramsey's arguments in "Theories" are responses to a series of philosophies of science endorsed by members of his social circles. He explicitly targets Russell, Whitehead, and Nicod when considering explicitly defining theories through their laws and consequences; he attacks Carnap when evaluating the potential of inverting a theory's dictionary; and he effectively directs Wittgenstein's philosophy of science away from a dead end. The negative part of Ramsey's argument is clear then.

1.5 Ramsey's Core Beliefs

Ramsey has a number of negative arguments against the various reductionist philosophies of science from his contemporaries. While he is clear about what goes wrong with those philosophies, he is less transparent about his own positive theory of science. The task of this dissertation is to provide the clarity Ramsey omits. The story is complicated, but at its core is the same theme that pervades Ramsey's philosophy generally:

> Philosophy must be of some use and we must take it seriously; it must clear our thoughts and so our actions. Or else it is a disposition we have to check, and an inquiry to see that this is so; i.e. the chief proposition of philosophy is that philosophy is nonsense. And again we must then take seriously that it is nonsense, and not pretend, as Wittgenstein does, that it is important nonsense! (Ramsey, [1929] 1990h, 1)

Ramsey's core theme is that philosophy must have use through how it clears thoughts and guides actions. If it fails to do so, then it is nonsense and should be treated as such without mystical admiration. This commitment leads Ramsey to embrace a number of core theses that underpin his philosophy of science.

Those core theses are *1) belief is fundamentally a disposition to act, 2) beliefs are to be guided by Ramsey's decision theory, 3) that decision theory measures beliefs through gambles over a privileged partition of propositions unique up to utility, 4) the meaning of a belief in a proposition is given by the effects the belief would have on the person's actions, 5) the objectivity of the content in beliefs is given by how successful people are when acting on those beliefs, i.e. how the world pushes back on the actions a person embarks upon, and 6) the philosophical picture of the world must be able to place people and their decisions in that world.*

Ramsey holds that belief is a disposition to act. This means that belief is a causal, counterfactual concept; belief in a proposition is not equivalent to the actions a person in fact takes but would take were the circumstances right. In particular, belief is a disposition to accept or reject gambles on propositions where gambles include most decisions among options. My belief that traffic to the airport is bad amounts to my willingness to gamble on taking the train instead of driving.

Beliefs are analyzed and guided by Ramsey's decision theory. His decision theory analyzes belief in the sense that when people reflect on what to do it can describe their actions as the result of certain beliefs and desires. It can guide action as a prescriptive means of self-control. Importantly, decision theory allows both analysis and guidance for full beliefs as well as for partial beliefs by treating action as the mathematical expectation of those beliefs and associated desires. So it is supported by a belief and desire psychology where action is the result of maximizing expected desires.

An important feature of this decision theory is that it measures beliefs through gambles over a privileged partition of propositions that are unique up to utility. There are two parts to this. First, it uses gambles—causal propositions linking conditions with consequents like "I get a heifer if an ace is drawn and a goat otherwise"—to measure a person's beliefs and partial beliefs. Second, the outcomes of gambles must ultimately bottom out in non-gamble propositions that are intrinsically valuable to the person. This is important because it means that gambles have only derivative value; a person is not interested in gambling for its own sake but only for the consequences it might ultimately bring. Those consequences partition possible worlds into classes of equally valuable worlds, i.e. worlds the person assigns the same utility to. The upshot is that only particular propositions happen to be intrinsically valuable for people.

Ramsey analyzes the content of beliefs—their propositional reference as he calls it—through their truth conditions, where truth conditions are given by an interaction between beliefs, actions, and consequences of those actions. Roughly, he holds the content of a belief to be given by its effects on actions. Because his decision theory holds a person's actions to be given by the act with the highest expected desirability, the truth conditions of a belief are given by the consequences of the action chosen on that belief. This yields the sense or meaning of the propositional content of that belief: a person's belief is true just in case when he obtains the desired consequence in acting on the belief and false otherwise. My belief that traffic is bad happens to be true just in case my decision to take the train and not the car really does lead me to arrive at the airport faster, as I desired. Importantly, this ties the sense of a proposition to its utility. It restricts what propositions can be true or false based in part on the desirability of those proposition's consequences since belief in the truth of a proposition will lead to the actions whose expected consequences are the most desirable. Sense is thus tied to preference, but only insofar as preference leads to actions whose expected consequences are in fact realized.

The pragmatist conception of meaning adopted by Ramsey exposes him to worries about the subjectivity of propositional content. The reference of beliefs, for them to be about the world, must in some sense be objective in that they do not only depend on whether the person thinks them to be true or desires them to be true. To avoid this conclusion, Ramsey emphasizes how beliefs can be in error. His theory for error is that a belief is incorrect when the consequence obtained by an action due to that belief is different from what is desired. The chicken that believes a caterpillar is edible acts by eating the caterpillar but discovers, much to its chagrin, that it has an upset stomach as opposed to a delicious meal. The chicken's belief is then in error. So beliefs can be more or less wrong based on the success and error they bring in action. They can be objective precisely in the sense that success in action is not determined by the belief alone but requires pushback from the world.

By making philosophy useful for action, Ramsey aims to place humans front and center in the world. That is, Ramsey wants to explain how beliefs can incorporate the believer in their representation. Clarity in thought should entail clarity of the thinker in the world and guidance in action should show how the person is a part of the world and acts in the world. In contrast, Ramsey's contemporaries, such as Russell, Wittgenstein, and Carnap, paint pictures of people as looking out into the world without situating those people as part of the world pictured. This proves important because a key aspect of theorizing about the world is theorizing about the self in action. Accounting for agency in the world proves to be an important philosophical project of Ramsey's.

These views have ramifications for Ramsey's philosophy of science. Most importantly, they mean that belief in a theory must be understood through Ramsey's decision theory. That decision theory determines what role scientific theories play in peoples' cognitive economy. Furthermore, the questions about the content of theoretical propositions and the alethic status of those propositions depends importantly on the relationship between the actions they inspire and the consequences those actions lead to. Scientific theories are about things in the world only insofar as they lead to changes in action; they are only true or false if they realize or fail to realize consequences an agent cares about. So Ramsey's solutions to classical philosophical problems about scientific theories run through these core commitments.

The reader should be ready to make recourse to these fundamental theses when thinking about Ramsey's philosophical claims. They cannot be separated from the particulars of

Ramsey's philosophy of science as they underpin his entire philosophical project.

1.6 Map of the Dissertation

This dissertation is broken into a series of chapters. Each chapter builds upon the previous ones, so they should be taken sequentially. My starting point is how a reconstructed decision theory fits with Ramsey's philosophy of science. This chapter solves the problem of interpreting singular theoretical propositions with that decision theory. This naturally leaves the status of general propositions in the theoretical language as expressed by the Ramsey sentence. Chapters three and four begin addressing this issue by first describing how general propositions fit in Ramsey's theory of cognitive psychology and what non-theoretical laws happen to be. Because general theoretical propositions are intimately tied with the Ramsey sentence, chapter five provides a unique interpretation of that sentence. An important lacuna from that interpretation is the role of existential quantification in Ramsey's philosophy of logic. Chapter six explores this issue. Finally, I end with a discussion of a big philosophical issue that Ramsey's philosophy of science impinges upon: scientific realism.

Chapter two focuses on reconstructing Ramsey's decision theory given his philosophical commitments in 1929. Ramsey never published his seminal essay "True and Probability". The principal reason is that he was dissatisfied with several aspects of the theory, including its reliance on the *Tractatus*'s theory of propositions. Instead, he hoped to remedy these defects in a book. This chapter aims to reconstruct what Ramsey might have done had he lived long enough. The principal accomplishment is a decision theory that abandons Wittgenstein's logical atomism. I then show how singular theoretical propositions function in this reconstituted decision theory. This leaves open the problem of how to understand general theoretical propositions, which are given in part by the Ramsey sentence. To address this problem, I have to explore the more general question about the status of universal and existential propositions in Ramsey's philosophy. That requires first sketching a picture of how human psychology works for Ramsey.

Chapter three explores the psychological underpinnings for Ramsey's decision theory and philosophy of logic. Ramsey states that his theory of science is a "forecasting theory"; I aim to address what he means by this remark by way of his theory of human cognitive psychology. I show he operates with a primitive dual process theory that produces psychological expectations. The role of logic in this psychology is as a method of self-control, and the relevant logic for Ramsey is decision theory. With this account in hand, I can then address how universal propositions work.

Chapter four produces an account of universal propositions such as laws and chances. I focus on an unresolved debate in the secondary literature over the extent to which Ramsey's two accounts of laws are one and the same and the extent of his commitment to the pragmatist theory of truth, i.e. the theory that what is true is what is fated to be believed. Ramsey's theory of laws is given through his theory of chances. I then show how methodological assumptions about the learning of chances account for his seeming belief in the pragmatist theory of truth but commit him to the Principle of Indifference. With this project complete, I can then turn to the question of general propositions in theories by way of the Ramsey sentence.

Chapter five analyzes the famed Ramsey sentence, and its role in Ramsey's philosophy of science. I show that the Ramsey sentence is about the latent variables people use for deliberation and communication when formulating laws. Informally, the Ramsey sentence of a theory expresses as new propositions the mixture of rules a person uses to guide his actions so that he can deliberate and communicate those rules with other people. More formally, the Ramsey sentence identifies latent variables that are projections of a person's conditional betting preferences into their propositional algebra. They are in an important sense properties of an agent's beliefs in observational propositions. However, this leaves an important question about the status of existential propositions. For solving that, I have to turn to Ramsey's philosophy of mathematics and logic.

Chapter six addresses Ramsey's philosophy of mathematics and logic by looking at how he interpreted the existential quantifier. I show that he adopted a view of existential propositions as descriptions of witnessing propositions in the sense they are arbitrary finite disjunctions including the vouchsafed witness. This places the entire theoretical content of an existential proposition in its witness. With this done, I have a complete account of Ramsey's philosophy of science in the context of a revised decision theory. That leaves the problem of seeing how this account factors into the realism and anti-realism question.

Chapter seven explores Ramsey's relation to the scientific realism debate. I show that Ramsey is a two-level anti-realist: he is an anti-realist at one level insofar as the Ramsey sentence is not truth-apt via its existential quantifier, and he is an anti-realist at another level insofar as the witness for the Ramsey sentence is a universal proposition and so also not truth-apt. I explore why fundamentally general propositions are not truth-apt by connecting them to an important technical limitation of Ramsey's decision theory and his theory of a proposition's truth conditions. I end by connecting this to Ramsey's interest in situating agents in the world.

Chapter 2

Decision Theory in Ramsey's "Theories"

2.1 Introduction

Despite living to the age of twenty-six, Frank Ramsey's work has had a lasting impact on philosophy, economics, and mathematics. Among those influences include the invention of the Ramsey sentence. The Ramsey sentence has received special attention in the philosophy of science since it was introduced to philosophers by Ramsey's friend and collaborator Braithwaite (Braithwaite, 1953), named by Hempel (Hempel, 1958), and exposed to the broader philosophical community by Carnap (Carnap and Gardner, 1966). Ramsey invents the famed sentence in a little-read paper named "Theories", which was likely written in 1929 and initially published posthumously by Braithwaite in a collection of essays (Ramsey, [1931] 2013). This piece and several others were intended to be combined with the essay "Truth and Probability" (Ramsey, [1926] 1990n) in a larger, book-length project titled *Truth and Probability* (Misak, 2020, 383–384). This suggests that "Theories" should be read as an application of Ramsey's decision theory and his subjective interpretation of probability.

Leveraging "Truth and Probability" might help with "Theories" abstruseness. Demopoulos writes for many a Ramsey reader that

> 'Theories' ([1929]) is a detailed working out of an artificial example of a theory which describes the temporal series of perceptions of color one would have if one occupied one of three possible locations in a very simple world. The point of the paper is far from transparent, and the philosophical considerations motivating the discussion are not nearly as explicit as one would like. The notion of a Ramsey sentence is introduced only toward the end of the paper and without the emphasis on its possible significance that has accompanied its introduction by subsequent advocates of the notion's utility. Any claim to have extracted a philosophical theory from Ramsey's paper that captures his intentions would be tendentious (Demopoulos, 2011, 190).

There have been many tendentious interpretations of "Theories". Carnap and Demopoulos view Ramsey as providing a tool for the elimination of theoretical terms (Carnap and Gardner, 1966; Demopoulos, 2011). Braithwaite, Hempel, and Lewis think that Ramsey has provided a way to understand theoretical entities through the roles they fulfill in scientific theories (Braithwaite, 1953; Hempel, 1958; Lewis, 1972). Psillos believes that Ramsey invented a novel type of scientific realism (Psillos, 2004). Sahlin, Dokic and Engel, and Misak argue that Ramsey expanded upon the philosophy of science of pragmatist C. S. Peirce (Sahlin, 1990; Dokic and Engel, 2002; Misak, 2016). And Majer describes Ramsey as developing an intuitionistic philosophy of science (Majer, 1989).

None of these have tried to understand how "Theories" fits with Ramsey's decision theory. Ramsey's plan to include "Theories" in a larger book with a revised "Truth and Probability" suggests this would be a good strategy for understanding "Theories". I aim to execute that strategy in this chapter.

The place to start is with what historians have identified as a fundamental idea of Ramsey's:

that belief is a disposition to act. This idea lies at the heart of Ramsey's decision theory. He writes about partial beliefs that "we are driven therefore to the second supposition that the degree of a belief is a causal property of it, which we can express vaguely as the extent to which we are prepared to act on it" (Ramsey, [1926] 1990n, 65). Full beliefs too must lead to action. In "Facts and Propositions", he argues that "it is evident that the importance of beliefs and disbeliefs lies not in their intrinsic nature, but in their causal properties, i.e. their causes and more especially their effects" (Ramsey, [1927] 1990d, 44). All beliefs should in some way lead to action—including scientific beliefs: "Variable hypotheticals and theoretical terms we cannot define, but we can explain the way in which they are used" (Ramsey, [1929] 1990h, 5). Beliefs in laws (variable hypotheticals) and theoretical entities are not reducible to definitions from observation but unique dispositions to act.

Belief is a disposition to act. What a disposition to act happens to be is made precise by Ramsey's decision theory. In that theory, decisions are made on wagers.[1] Belief in a proposition is always a willingness to bet on that proposition. When discussing his method for measuring credences, Ramsey writes

> [I]t is based fundamentally on betting, but this will not seem unreasonable when it is seen that all our lives we are in a sense betting. Whenever we go to the station we are betting that a train will really run, and if we had not a sufficient degree of belief in this should decline the bet and stay home. The options God gives us are always conditional on our guessing whether a certain proposition is true (Ramsey, [1926] 1990n, 79).

Humans are always betting on the truth of propositions through their actions. Thus every belief can be understood in terms of preferences over possible gambles. Importantly, these gambles need not involve money. What exactly they are and what outcomes do they involve

[1] I will use wager, bet, and gamble interchangeably throughout this chapter.

will be discussed further in this essay.

Despite the centrality of a dispositional account of belief explicated in terms of preference over wagers, Ramsey does not mention either when discussing scientific theories in “Theories”. Instead, he describes scientific theories as if they are languages:

> Let us try to describe a theory simply as a language for discussing the facts the theory is said to explain. This need not commit us on the philosophical question of whether a theory is only a language, but rather if we knew what sort of language it would be if it were one at all, we might be further towards discovering if it is one (Ramsey, [1929] 1990m, 112).

His interpretation of theories as if they are languages is problematic. First, Ramsey does not elaborate on what he means by “language,” though his extended discussion indicates he is thinking of formal languages. Second, he never answers the question of whether theories are languages. And third, he writes elsewhere that when looking at how theories are used “in this explanation we are forced to look not only at the object which we are talking about, but at our own mental states”, which suggests that when one believes a theory, one’s “own mental state” is of more importance than its linguistic properties (Ramsey, [1929] 1990h, 5).

While he does not do so in “Theories”, Ramsey does explicitly talk about forming credences over theoretical propositions in the companion piece “Causal Qualities”. He writes that “Singular propositions in the secondary system we believe with such and such degrees of probability just as in the primary system” (Ramsey, [1929] 1990a, 137). Just like everyday propositions (the aforementioned primary system), one can have probabilities over singular theoretical propositions. He calls such an approach a “forecasting theory”: “As opposed to a purely *descriptive* theory of science, mine may be called a *forecasting* theory” (Ramsey, [1929] 1990e, 163). Thus credences may be formed over theoretical propositions. Different credences in singular theoretical propositions result in different forecasts. Given that Ramsey defines credences through bets, one can infer that theoretical propositions can be wagered on just as observational propositions.

A key question then is what sort of bets does one make with scientific theories? This question can be addressed in three parts: what are wagers on the singular propositions of a theory, what are wagers on the general propositions of a theory, and what are wagers on theories themselves?

My goal in this chapter is to address what wagers are on the singular propositions of a theory. I will not address how one wagers on the general propositions of theories or theories themselves; this will occur later in the dissertation. Informally, my argument is that the test conditions of a singular, theoretical proposition provide the conditions for the satisfaction of a bet on that proposition, and the payoff of a bet is just some specification of observable propositions plus the hypothesized "truth" of the theoretical propositions that is wagered. For example, if the observable propositions are the presence or absence of a newspaper on my doorstep during the days of the week and the theoretical proposition is that there is a paper boy who delivers the newspaper on Mondays, then the test conditions are just that I see a newspaper only on Mondays and not on other days while the payoff of a gamble on the newspaper boy is just that I get a newspaper on Monday and not on the other days with the supposed "truth" of there being a paperboy. More formally, I argue that wagers over singular, theoretical propositions are determined by the verification conditions of those propositions and where the outcomes of these bets are sets of possible worlds generated by a theory that form a partition over an individual's possibility space.

Here is how I proceed. I review Ramsey's decision theory and what are wagers in that decision theory. Then I will provide a detailed sketch of what scientific theories are and connect this description to an important component of Ramsey's argument in "Theories" involving theoretical propositions' verification conditions. Then I introduce and explain how verification conditions work with the concept of truth-possibilities. I argue that truth-possibilities are sets of possible worlds. Finally, I show that truth-possibilities fit the description of Ramsey's outcomes and are, therefore, what singular theoretical propositions are wagered over.

The reader will also find an extensive appendix at the end of the chapter. This appendix is a thorough and complete reconstruction of Ramsey's toy example from "Theories". It includes extensive code in Python recreating the example along with graphics exploring how the model works. As an appendix, it is strictly optional and there for the reader if he or she wishes to explore Ramsey's toy example in detail.

2.2 Ramsey's Decision Theory

In order to explain how singular propositions in theories are wagered, I need to explain how exactly wagers work in Ramsey's decision theory and what they are over. It should be emphasized immediately that what I mean by "Ramsey's decision theory" is the version of his decision theory consistent with his philosophical beliefs. The exact presentation in "Truth and Probability" explicitly relies upon Wittgenstein's theory of propositions, which Jeffrey and Bradley identify as having both technical and philosophical problems for Ramsey (Jeffrey, 1990, 55–57; Bradley, 2001, 21). However, Ramsey writes in a footnote to "Truth and Probability" that though he assumes Wittgenstein's theory, "it would probably be possible to give an equivalent definition in terms of any other theory" (Ramsey, [1926] 1990n, 73). My goal in this section is to describe that "equivalent definition" where the framework given in "Truth and Probability" coheres with Ramsey philosophical commitments at the time of "Theories". This likely reflects what Ramsey might have written had he lived to revise "Truth and Probability". There will be two proposed reconstructions. The first takes some liberty by relaxing one of the structural conditions Ramsey proposes in his axioms—making Ramsey's decision theory closer to a suggested revision given by Bradley. The second will adhere very closely to Ramsey's original decision theory in the sense that it allows his representation theorem to go through with minimum modification. Both make the same key innovation in jettisoning Wittgenstein's theory of propositions: a movement to an atomless algebra of propositions that contains a privileged partition an agent intrinsically cares about.

There are three key ideas used by Ramsey in his decision theory. First, an agent's preferences over gambles are determined by the mathematical expectation of their utility. This reflects the psychological assumption that agents choose actions they think most likely to realize their desires (Ramsey, [1926] 1990n, 69). Second, an interval scale of the agent's utilities can be measured through the agent's preferences over gambles involving what Ramsey calls ethically neutral propositions. Third, once utilities are measured, probabilities can be inferred through the agent's indifference over wagers and outcomes.

Ramsey's gambles are a type of conditional. If α and β are outcomes and P is a proposition, then a wager Γ is of the form: α if P; β if $\neg P$. What this says is that if I accept Γ and P happens to be true, then I will receive α, but if P is false, then I will receive β. Since the outcomes are dependent on a condition holding, these are sometimes referred to as "conditional prospects" (Bradley, 2001, 8). The utility of a gamble Γ is given by the mathematical expectation. If $\overline{\alpha}$ is the utility of the outcome α and $\overline{\beta}$ is the utility of the outcome β, then the utility of Γ is:

$$U(\Gamma) = \mathbb{E}[\Gamma] = \Pr(P)\overline{\alpha} + (1 - \Pr(P))\overline{\beta}$$

For example, if I assign utility 1 to α, 0.5 to β, and believe P to probability 0.2, then the utility of Γ would be 0.6.[2] I will say more about what α and β are later.

[2] Technically, the expected value of a gamble is actually given by the following expectation where P is logically compatible with α and P^c is logically compatible with β:

The measurement of an agent's utilities proceeds through ethically neutral propositions. Intuitively, an ethically neutral proposition is one whose truth value the agent is indifferent to. Ramsey needs such a contrivance because measuring ordinary preferences over outcomes does not automatically result in an interval scale of utilities (see Bradley, 2001 and Elliott, 2017 for a discussion). While I can specify that α is preferred to β, I cannot say to what extent it is preferred. Ramsey defines ethically neutral propositions as follows. If P is an atomic proposition, then P is ethically neutral if and only if for any two outcomes α and β that differ only in the value of P, the agent is indifferent between α and β, i.e. $\overline{\alpha} = \overline{\beta}$. If P is non-atomic, then it is ethically neutral if and only if every atomic truth-function of P is ethically neutral. Examples of ethically neutral propositions for most people include whether a comet in Alpha Centari orbits its star every seventy years, my computer lost an electron just now, or whether this next coin toss lands heads.

Crucially, Ramsey needs there to be ethically neutral propositions believed to probability one-half. He defines this through his concept of a gamble. Let P be an ethically neutral proposition and α and β two outcomes compatible with P where α is preferred to β, i.e. $\overline{\alpha} > \overline{\beta}$. P is believed to probability one-half if and only if the agent is indifferent between the gamble Γ, α if P; β if $\neg P$, and the gamble Γ', β if P; α if $\neg P$. The correctness of this definition can be seen by setting the utilities of Γ and Γ' equal and solving for $\Pr(P)$.

With ethically neutral propositions believed to probability one-half, Ramsey can then measure an agent's utilities through the process of offering nested gambles. He can do this because a gamble on the ethically neutral proposition believed to one-half will always have utility halfway between its two outcomes. There are two possible ways of doing this: by iterating gambles on different ethically neutral propositions believed to probability one-half

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$$U(\Gamma) = \mathbb{E}[\Gamma] = \Pr(P)\overline{\alpha \cap P} + (1 - \Pr(P))\overline{\beta \cap P^c}$$

where P^c is the complement of P. One can ignore this requirement by simply specifying that outcomes α and β include P and P^c respectively and the gamble function only maps to compatible outcomes. Hence my or by requiring as a property of a person's preferences that for any two outcomes α and β, there exists another outcome δ such that the person is indifferent between δ and the gamble α if P; β if not P, where P is an ethically neutral proposition believed to one-half. I go through each method in turn.

For the first method, consider the following example. Suppose I preferred flying to visit my relatives over driving to visit them. I could then measure my exact utility of any outcome whose preference lay between these two options of travel. Here is how. Suppose I believe the series of ethically neutral propositions "the ith coin toss will be heads", P_i, to probability one-half. Then if α represents the outcome of me flying and β represents the outcome of me driving, then the utility of the gamble Γ, α if P_i; β if $\neg P_i$ will be $(\overline{\alpha}+\overline{\beta})/2$ or exactly halfway between the utility of flying and driving. I can now construct another gamble between flying and Γ to figure out my three-quarters utility point. I offer myself the wager Γ': α if P_{i-1}; Γ if $\neg P_{i-1}$. Solving for the expected utility of it, I find that the utility of Γ' is $(3\overline{\alpha}+\overline{\beta})/4$. This is half-way between the utilities of α and Γ. I can repeat the trick on the other side of Γ. I can iterate these gambles to any desired level of precision. Any other outcome, like I take the train to visit my relatives, can be assessed by comparing it to such gambles. If I am indifferent between that outcome and such a gamble, then I have found my relative utility of that outcome.

For the second method, consider the same example as before where I prefer flying to visit my relative over driving to visit them. Let α and β stand for the former and the latter again. Now consider the sole ethically neutral proposition "this one toss of the coin lands heads" and the gamble Γ "α if this one toss of the coin lands heads and β otherwise". By assumption, my preferences are such that I am indifferent between this gamble and the outcome of taking the bus. Call the bus option δ. Now I can construct another gamble Γ', α if the coin toss lands heads; δ otherwise, that will be exactly half-way between α and δ

presentation.

(three-quarters between α and β). I can then iterate on this method with the same ethically neutral proposition because, for each gamble I construct, I can find some outcome that is equivalent to the newly constructed gamble. The upside is that I do not need to have an infinite set of ethically neutral propositions believed to probability one-half; the downside is I must have a considerable "richness" to my preferences that may be unrealistic. It is this method that Ramsey in fact uses in "Truth and Probability", though the technique of ethically neutral propositions works just as well for the first method.

Ramsey states a series of axioms for an agent's preferences. If those axioms are obeyed by an agent, there is a unique (up to affine linear transformation) utility function representing their preferences.[3]

From an agent's measured utilities, Ramsey can construct that agent's unique probabilities. The method proceeds by using the expected utility of wagers. Suppose one wanted to know one's credence in the proposition P (P need not be ethically neutral). One's probability in P can be ascertained if one is indifferent between some outcome α and the wager β if P;

[3] Ramsey gives eight proposed axioms, but unfortunately in their original rendition Ramsey is not careful to specify them in purely qualitative terms about preferences (for the original eight, see Ramsey, [1926] 1990n, 74–75). Bradley rectifies this by translating Ramsey's axioms into a more legible and appropriate form as follows:

> Let Δ be a non-empty set of ethically neutral propositions of probability one-half and suppose that P belongs to Δ. Then Ramsey postulates:
>
> R1 If $Q \in \Delta$ and (α if P)(β if $\neg P$) $\geq$ (γ if P)(δ if $\neg P$), then: (α if Q)(β if $\neg Q$) $\geq$ (γ if Q)(δ if $\neg Q$).
>
> R2 If (α if P)(δ if $\neg P$) $\approx$ (β if P)(γ if $\neg P$) then:
>
> (i) $\alpha > \beta \iff \gamma > \delta$ (ii) $\alpha \approx \beta \iff \gamma \approx \delta$
>
> R3 If $\Phi \geq \Psi$ and $\Psi \geq \Theta$, then $\Psi \geq \Theta$ [where Φ, Ψ, Θ are prospects like worlds and gambles].
>
> R4 If (α if P)(δ if $\neg P$) $\geq$ (β if P)(γ if $\neg P$) and (γ if P)(ζ if $\neg P$) $\geq$ (δ if P)(η if $\neg P$), then (α if P)(ζ if $\neg P$) $\geq$ (β if P)(η if $\neg P$).
>
> R5 $\forall(\alpha, \beta, \gamma)[\exists(\delta) :$ (α if P)(γ if $\neg P$) $\approx$ (δ if P)(β if $\neg P$)]
>
> R6 $\forall(\alpha, \beta)[\exists(\delta) :$ (α if P)(β if $\neg P$) $\approx$ (δ if P)(δ if $\neg P$)]
>
> R7 Axiom of Continuity.
>
> R8 Archimedean Axiom (Bradley, 2001, 14).

Importantly, axiom R6 is the structural condition on preferences that ensures a richness that allows Ramsey to use a single ethically neutral proposition believed to probability one-half. Axiom R5 is a similar condition ensuring a "matching" of gambles. This allows one to always find the value of $\overline{\beta} \cdot \overline{\gamma}$ from $\overline{\alpha}$ (Ramsey writes this axiom as $\overline{\alpha\gamma} = \overline{\beta\delta}$).

γ if $\neg P$. Setting $\overline{\alpha}$ equal to the utility of the aforementioned gamble, one solves for $\Pr(P)$, which will be the ratio of the difference between the utility of α and γ and β and γ, i.e. $\Pr(P) = (\overline{\alpha} - \overline{\gamma})/(\overline{\beta} - \overline{\gamma})$. Ramsey asserts and then proves that with additional constraints, that these measured partial beliefs are in fact probabilities (they satisfy the probability axioms).

Missing from the above discussion is what exactly α, β, and γ are supposed to be. Ramsey describes them as possible worlds:

> To begin with we shall suppose, as before, that our subject has certain beliefs about everything; then he will act so that what he believes to be the total consequences of his action will be the best possible. If then we had the power of the Almighty, and could persuade our subject of our power, we could, by offering him options, discover how he placed in order of merit all possible courses of the world. In this way all possible worlds would be put in an order of value [....] (Here and elsewhere we use Greek letters to represent the different possible totalities of events between which our subject chooses—the ultimate organic unities) (Ramsey, [1926] 1990n, 72–73).

Ramsey proposes that the outcomes wagered on (apart from other gambles) are exhaustive totalities of ways the world could be. This is not very clear. While he describes them as possible worlds, the surrounding discussion does not mean they are automatically possible worlds as discussed in contemporary philosophy.

To figure out what Ramsey exactly means here and what outcomes are supposed to be, one needs to observe four desiderata on outcomes that follow from Ramsey's decision theory and philosophy.

First, he needs there to be possible worlds that settle the truth of every proposition. Call this the *determination* desideratum. It is evident from his above comment that the agent is evaluating "all possible courses of the world" and that there are "ultimate organic entities". His remark elsewhere in "Truth and Probability" that he assumes Wittgenstein's theory of propositions also suggests that there are worlds that settle the truth of every proposition.

Second, he needs there to be worlds that are compatible with a possibly infinite number of ethically neutral propositions. Call this the *compatibility* desideratum. Ramsey believes he needs ethically neutral propositions to measure agent's utilities along an interval scale.[4] The reason is due to the fact that a proposition wagered on in a gamble might influence the outcome of such a gamble and that would not allow him to establish a proposition believed to probability one-half (Ramsey, [1926] 1990n, 73). So, there need to be ethically neutral propositions.[5] Furthermore, those ethically neutral propositions need to be compatible with every world: "α and β must be supposed so far undefined as to be compatible with p and not-p" (Ramsey, [1929] 1990m, 74). Bradley refers to this desiderata as conditionalism (Bradley, 2001, 21). As many have noted (Jeffrey, 1990, 56; Bradley, 2001, 21; Elliott, 2017, 5), this immediately conflicts with the determination desideratum.

Third, there should be no presupposition that there are independent elementary propositions. Call this the *non-independence* desideratum. This does not follow from Ramsey's stated decision theory, but from Ramsey's other philosophical commitments. He explicitly rejects the thesis that there are independent elementary propositions. In his review of the *Tractatus* (written before "Truth and Probability"), he raises the infamous color exclusion problem and thinks it is unsolvable. He finds Wittgenstein's attempted explanation of this unconvincing (as did Wittgenstein eventually) (Ramsey, 1923, 473). More importantly, Ramsey argues in "Universals" that even if there are elementary propositions, because we infer them from other propositions (Ramsey, [1925] 1990o, 19), "the truth is that we know and can know

[4]As Elliot shows, his decision theory can be amended to do without them. See Elliott, 2017 for a full discussion.

[5]Elliott shows that this is strictly not true (Elliott, 2017).

nothing whatever about the forms of atomic propositions" (Ramsey, [1925] 1990o, 29). The properties of the elementary propositions are beyond one's kin. In a note written after "Universals", Ramsey softens this stance to allow for the possibility of discovering facts about the elementary propositions but states that such knowledge cannot be gained *a priori* (Ramsey, [1926] 1990g, 31). This means that, at a minimum, his decision theory should not presume that there are independent elementary propositions.

Fourth, the agent must intrinsically care about the worlds that are the outcomes of gambles. Call this the *utility* desideratum. Like the non-independence desideratum, this follows in part from Ramsey's broader philosophical commitments after "Truth and Probability". The principal thesis driving this is Ramsey's commitment to a particular theory of truth and sense of propositions that he describes in "Facts and Propositions". This theory holds that a person's belief in a proposition is true just in case when that person acts on that belief, they choose the action whose consequences are most desirable and in fact, receive the desired consequence of that action, and that the person's belief is false when doing the same action they in fact receive something different from what they desired. The sense of proposition—its truth conditions—is then given by the consequences that would be realized by the agent's acts on belief in the proposition and the realized consequences of those acts when the agent is mistaken. An important upshot of this view about truth is that the consequences of an action have to be differentiated by their desirability. This is required for differentiating the content of beliefs, what Ramsey calls a belief's propositional reference, since the actions chosen are precisely those most desirable given the truth of the believed proposition. The same lesson applies to Ramsey's decision theory. Action for Ramsey is just a type of gamble. When I choose to take the train to the airport on the belief that the traffic is congested, I am gambling on taking the train if traffic is congested and taking my car otherwise. In other words, I am acting on the belief of the proposition. For me to individuate that proposition from another, say the price of rice in China is one yuan, I have to specify the consequences I would take when presented with the proposition for sure *solely on their desirability*. Otherwise, why would I act? That is just what it means for me to be maximizing my expected utility. So the worlds have to be differentiated by their desirability to determine the degree to which an agent believes the propositions of a gamble. This would allow Ramsey to specify the exact proposition the agent has a partial belief in because it ties the sense of the gambled proposition with the relative utilities of the consequences of that gamble. As a result, worlds need to be individuated by the extent to which the agent cares about them.[6]

There have been several interpretations of Ramsey's decision theory by secondary source authors. Most authors are well aware of the desiderata mentioned above. I review those interpretations and evaluate them with the four desiderata described above.

Richard Jeffrey provides an account of Ramsey's worlds that stays very close to the text of "Truth and Probability" (Jeffrey, 1990, 55–57). He argues that Ramsey's possible worlds are Tractarian worlds composed of independent atomic propositions: "Following Ludwig Wittgenstein, *Tractatus Logic-Philosophicus*, Ramsey postulated the existence of *atomic* propositions, each of which can be true or false quite independently of any or all of the others, and which have the characteristic that any proposition whatever is some truth functional compound of atomic ones" (Jeffrey, 1990, 55). Jeffrey uses this feature of the Tractarian account to solve the compatibility desideratum. If one restricts the "worlds" in wagers to be near worlds, which omit certain atomic propositions, then one can use those omitted atomic propositions to function as ethically neutral ones. Jeffrey writes that this "makes an abstract sort of sense if you think the business of specifying a possible world as simply a matter of specifying, for each atomic proposition, whether it is to be true or false in the world in question. Each such specification can be made in either way, independently of how the

[6] This stands in contrast to "Truth and Probability" where Ramsey largely ignores this issue because he works with Wittgenstein's theory of propositions. In fact, Ramsey does leverage this by constructing the value of a Tractarian world as just the equivalence class of worlds where the agent assigns similar utility. He could have used these values as the outcome of wagers, but refrained from doing so because he wanted to leverage a Tractarian world's ability to determine the truth of all propositions. As it turns out when proving the representation theorem, the Tractarian worlds do little work: instead the equivalence classes defined by their values provide the entire heavy lifting. See Bradley, 2001 for the representation theorem proof.

truth value of the other atomic propositions have been or will be specified" (Jeffrey, 1990, 56). The picture then is that determination and compatibility can (mostly) be satisfied by taking each Tractarian world and restricting them to near worlds for the propositions the agent cares about. The remaining atomic propositions and their truth-functions constitute the ethically neutral propositions used for measuring utilities. The outcomes of those wagers with ethically neutral propositions are specified to be said near worlds plus the conditional ethically neutral proposition (Jeffrey, 1990, 56–57).

Jeffrey's account satisfies both the determination and compatibility desiderata. But it fails the non-independence and pragmatic desiderata. It fails non-independence because it requires Ramsey to assume that there are some atomic propositions and those propositions are independent of one another. What is worse, it assumes that all atomic propositions are independent. This runs afoul of the non-independence desideratum, which requires that no assumptions be made of the nature of atomic propositions, if there are any. Furthermore, it individuates worlds by their logical relations among elementary propositions and not based on their intrinsic desirability for the agent. So it largely ignores the importance of the desirability of worlds in construing the content of the gambles. This means it may not be a good fit for the utility desiderata.

Jordan Sobel amends Jeffrey's account in an attempt to more-closely capture the non-independence desideratum. He notes, correctly, that Ramsey is leery of the Tractarian account of propositions and worlds. But he needs some method to allow for ethically neutral propositions to be compatible with any outcome world. Sobel's proposal is a thin logical atomism:

> He needs to speak of ethically neutral propositions that are believed to degree 1/2 in order to say, in his way, when differences between the values of worlds are equal. And to identify an ethically neutral proposition p in terms of comparisons

> of worlds, he needs to speak of worlds that differ "only in regard to the truth of p" (p. 33). But worlds cannot differ only with regard to the truth of a single proposition. The closest possibility is of worlds that differ with regard to the truth of a single proposition of a class of independent propositions that determine worlds uniquely.
>
> A thin logical atomism is sufficient for Ramsey's purposes that says that there are propositions—this theory's 'atomic propositions'—"each of which can be true or false quite independently of any or all of the others" (Jeffrey 1990, p. 55) such that no two worlds are exactly alike with regard to these propositions [....] It is important for his theoretical purposes not *how many* there are—as far as he is concerned, there can even be innumerably many that do not constitute a set or any kind of collecting One. Nor does it matter to him *which* they are, or whether they are epistemically special. All that is important is *that* they are (Sobel, 1998, 236–237).

The idea is that there is a special, privileged class of independent atomic propositions that can specify worlds. These propositions would be the ones from which the ethically neutral propositions are drawn. Not all propositions need to be in this class. Worlds could have many atomic propositions and truth-functions of them that are not independent of one another. This class of independent atomic propositions need not be infinite nor be epistemically privileged. All that matters is that they exist and can be used to specify worlds uniquely as the outcome of wagers.

This account is still too strong for Ramsey. While determination, compatibility, and (possibly) utility are satisfied, the claim that there exist some atomic propositions that are independent of one another and all others runs afoul of non-independence. Ramsey's claim with non-independence is that even if atomic propositions exist, one should not make any presupposition about them. Sobel's account clearly does: he claims that they are independent and uniquely determine worlds.

Lastly, Richard Bradley offers a revision of Ramsey's theory without Tractarian worlds (Bradley, 2001, 25–27). Bradley's account initially defines worlds as being near maximal, though not independent in the Tractarian sense. Gambles are then defined as functions going from partitions to worlds. For example, if $X_1, X_2, \ldots, X_n$ form a partition over outcomes, then a gamble $f : \{X_1, X_2, \ldots, X_n\} \to W$ uniquely outputs some Tractarian world for each element of the partition. Bradley notes that the problem with this setup is that it can violate compatibility or what he calls ethical conditionalism: "For any propositions P and Q, there are worlds $\alpha \in P$ and $\beta \in Q$ such that $\alpha \approx \beta$", where $\alpha \approx \beta$ says one is indifferent between α and β (Bradley, 2001, 21). This is equivalent to compatibility since two propositions containing worlds one is indifferent over allows the specification of at least one ethically neutral proposition (just take the pair of α and β). He then points out that without worlds being defined by independent atomic propositions, there is a conflict between determination and compatibility:

> Defensible thought it may be, Ethical Conditionalism is not consistent with Ramsey's atomistic framework. For consider worlds α and β such that $\alpha \not\approx \beta$ and the proposition—call it A— that α is the actual world. Then since worlds are (nearly) maximally specific it follows that any world in which A is true is ranked with α. But then there is no world in which A is true which is equally preferred to β (Bradley, 2001, 21).

The solution is to eliminate determination by removing maximal worlds. This requires a rework of Ramsey's decision theory so that outcomes are now propositions, gambles are functions from partitions to propositions, and there are no atomic propositions. This makes the revised decision theory closer to Jeffrey's (Bradley, 2001, 26).

This elegant account meets the compatibility and non-indepenence desiderata. Compatibility is satisfied by requiring that there be no propositions that settle the truth of every other proposition. And importantly, no assumptions about independent atomic propositions is required because there are no atomic propositions. But this all comes at the cost of giving up on determination: there are no "worlds" that determine the truth of every proposition. It also leaves unsettled the utility desiderata, though this might be easily fixable.

No available secondary source interpretation best matches the desiderata necessary for Ramsey's decision theory, though Bradley's comes the closest. The main frustration is the conflict between determination and compatibility. The only known way for the same worlds to satisfy both desiderata is via logical atomism. But this automatically leads to conflict with non-independence and leaves unsolved utility.

The solution is to make a distinction. As Jeffrey, Sobel, and Bradley make clear, Ramsey cannot maintain that his worlds be maximal with respect to all propositions. This would disallow the value of the ethically neutral ones he needs to construct the utility scales. So in all accounts I have described, an implicit distinction must be made between the worlds that determine the truth of all propositions and the worlds that are wagered on. Call the former $worlds_1$ and the latter $worlds_2$. The distinction to be made is that determination only applies to $worlds_1$ and compatibility and utility only apply to $worlds_2$. Therefore, there are really two types of "worlds" necessary for Ramsey's decision theory.

This leads to two possible constructions. As noted previously, the device of an ethically neutral proposition believed to probability one-half can be used to infer a calibrated utility scale in at least two different ways. The first makes use of an infinite set of such propositions to construct a scale to arbitrary precision. The second uses only one ethically neutral proposition believed to probability one-half but assumes the agent has sufficiently rich preferences such that for any gamble on that ethically neutral proposition there is an outcome the agent is indifferent to with respect to that gamble. I can use the distinction between $worlds_1$ and $worlds_2$ in both cases to recover a decision theory more compatible with Ramsey's later philosophical commitments.

Starting with the first proposed construction, an agent's epistemology is a triple:

$$\langle \Omega, \mathcal{A}, \Pr \rangle \tag{2.1}$$

where Ω is a possibility (sample) space and $\mathcal{A}$ is an algebra on Ω.[7] Importantly, I follow Richard Bradley (Bradley, 2001) and make $\mathcal{A}$ atomless. In this picture, the elements of Ω are $worlds_1$—they determine the truth of every proposition. Propositions are then just sets of worlds. Note that their singletons are not in the agent's algebra $\mathcal{A}$. For $worlds_2$, one specifies that there is some partition $G \subseteq \mathcal{A}$ whose elements are defined by an equivalence relation on Ω that specifies worlds whose utilities are identical to the agent. That is, $w, w' \in \Omega$ are in the same $g \in G$ if and only if $\overline{w} = \overline{w}'$.[8] These are the outcomes of wagers for Ramsey and constitute $worlds_2$. In order to ensure that there are always ethically neutral propositions that are compatible with $worlds_2$, ethically neutral propositions are redefined as follows. Let the set of ethically neutral propositions E be the propositions e from $\mathcal{A}$ such that for every g in G, e is compatible with g and e is valued the same as tautology[9] or

$$E = \{e \in \mathcal{A} | \forall g \in G, g \cap e \neq \emptyset \text{ and } \overline{e} = \overline{\Omega}\} \tag{2.2}$$

[7]Recall that an algebra on a set X is a set of subsets that includes the set X and is closed under finite union and complement. A sigma-algebra is an algebra closed under countable union.

[8]An important complication is that an agent's utility function is defined on both the points in the sample space and the algebra. Note that this may seem odd since the points are not in the agent's algebra. Philosophically, I justify this by saying that if the agent were presented two $worlds_1$ in the same $g \in G$, she would assign the same utility to them even though she will in fact never encounter those worlds among her propositions.

[9]This condition is necessary to force ethically neutral propositions to not provide evidence for propositions one might care about. See Bradley, 2017 for an extended discussion.

Next, there needs to always be additional ethically neutral propositions that can be utilized in forming new gambles from old. This is not guaranteed by the previous definition of ethically neutral propositions because the intersection of ethically neutral proposition may not itself be ethically neutral. To ensure this, I specify there is some countably infinite subset F of the set of ethically neutral propositions E such that for any finite subset of F, the intersection of the elements of that subset is also in the set of ethically neutral propositions or:

$$\exists F \subseteq E \text{ s.t. } |F| = \aleph_0 \text{ and } \forall X \subseteq F, |X| < \aleph_0 \Rightarrow \bigcap_{x \in X} x \in E \tag{2.3}$$

One then requires that there be a generating set of ethically neutral propositions believed to probability one-half. This set would be ethically neutral per the above definition and axiom with the added requirement that it be believed to probability one-half. The intersection condition would allow other ethically neutral propositions to probability one-half still be ethically neutral, though the probability of the intersection of both those would in fact be one-quarter. This ensures there are a sufficient number of ethically neutral propositions to measure utilities to any degree of precision.

Intuitively, one can think of the relationship between $worlds_1$, $worlds_2$, and the ethically neutral propositions through binary sequences. In the case where the number of $worlds_2$ is finite (corresponding to a finite partition on Ω), each infinite binary sequence is a $worlds_1$. Indexing each one and zero with the naturals $\{0, 1, 2, \ldots, n, \ldots\}$, the set of $worlds_2$ correspond with all possible finite subsequences up to place n with n being the number of propositions one cares about. The set of ethically neutral propositions is any finite subsequence from $n+1$ on. They "extend" the initial sequence given by $worlds_2$ while allowing any such finite extension to be compatible with the initial sequence. In the case where the number of $worlds_2$ is infinite, one can think of $worlds_1$ being pairs of infinite binary sequences and the introduction of ethically neutral propositions to be the first n digits for n ethically neutral propositions in the second sequence.

This description satisfies each of the desiderata in Ramsey's decision theory. First, it satisfies determination in that there are worlds, the elements of Ω, that determine the truth of every proposition (for each $w \in \Omega$ and each $a \in \mathcal{A}$, either $w \in a$ or $w \notin a$). Second, compatibility is satisfied since the members of G that are the outcomes of wagers do not have atoms and there are always finite sets of ethically neutral propositions whose intersection are compatible with the members of G. Third, non-independence is met since there is no desideratum that the underlying point in Ω is defined by independent elementary propositions. And fourth, it satisfies utility because $worlds_2$ are all unique up to utility per the agent. The result is very close to Bradley's suggested revision and close to a suggestion given by Skyrms for modifying the *Tractatus* (Skyrms, [1993] 2012).

Such a reconstruction has a desirable philosophical property, an undesirable philosophical property, and an undesirable historical property. The desirable philosophical property is that it does away with Ramsey's axiom six, which requires the agent to have some outcome they value indifferently with any possible gamble involving an ethically neutral proposition believed to probability one-half. This axiom might be violated if I care about only finitely many outcomes, where I could "run out" of outcomes that I would be indifferent over with respect to the gambles I could construct using my ethically neutral proposition. The undesirable philosophical property is that it requires there be an infinite number of propositions whose truth I am indifferent to and I believe to probability one-half. While less implausible than Ramsey's axiom six, this has received considerable philosophical criticism (see Elliott, 2017 for an extended discussion). Finally, the undesirable historical property is that Ramsey himself does not pursue this. His decision theory does assume the agent's preferences to be sufficiently rich. And that is how he proves his representation theorem. So this construction is less historically faithful in at least one important respect.

If historical faithfulness in the sense of preserving as much of Ramsey's original theory and representation theorem as possible is desired, then an alternative construction can be given that jettisons the use of infinitely many ethically neutral propositions. Here an agent's epistemology still consists of a sample space, an atomless algebra, and a probability function, and there is still a partition G whose members specify equivalence classes of $worlds_1$ by the latters' utility.[10] There are two main differences. First, one only requires there exists a proposition e such that e is compatible with every element of G and for all refinements of G, G',[11] for any $g, h \in G'$, the agent is indifferent between g if P; h if not P and h if P; g if not P. This is just the requirement there exists an ethically neutral proposition believed to probability one-half. Second, one requires that for any gamble involving P, there is some element $g \in G$ such that the agent is indifferent between that gamble and g. Basically, the agent's $worlds_2$ needs to be rich enough to allow the representation of their utilities with a real-valued function. With this in place, one can provide Ramsey's original decision theory minus the Tractarian theory of propositions. It allows one to recover the original representation theorem with minimal mutilation of the underlying theory.

It also enables me to satisfy all four desiderata like the first reconstruction. Determination, compatibility, and independence are automatically satisfied like in the previous reconstruction. Utility is also satisfied here because again, the outcomes of wagers, $worlds_2$, are defined as just a partition whose elements are unique up to utility. This enables any proposition's truth conditions to be given strictly in terms of the outcomes the agent ultimately cares about.

While this construction is historically faithful and philosophically advantageous in that it

[10] Atomless is still necessary because of compatibility. One wants to avoid counterexamples to what Bradley calls ethical conditionalism, where for any two propositions, there are worlds in both of those propositions an agent ends up being indifferent to. If the algebra has atoms, then obviously this condition can be violated (Bradley considers the example where two worlds might be strictly ranked in terms of preferences but then a proposition that specifies one of those worlds as actual would violate ethical conditionalism). See Bradley, 2001, 21.

[11] A refinement here is just any partition G' whose elements are subsets of one and only one member of G.

requires only one ethically neutral proposition believed to probability one-half, it has one serious downside. Namely, it requires the $worlds_2$ partition to be infinite outside the trivial case of an agent indifferent over all outcomes and infinite in a way so as to match the utility of any possible gambles involving the ethically neutral proposition e.[12] This means that Ramsey is requiring the agent to care about infinitely many propositions and also to have a certain degree of richness in the things he cares about to match many gambles. There is an interesting asymmetry then between the two constructions: the first has to posit infinitely many propositions an agent is indifferent over while the second has to posit infinitely many propositions an agent cares about. I leave it to the reader to make their choice as to what is more plausible.

Summing up the previous discussion, there are two types of worlds in Ramsey's decision theory. The first type, $worlds_1$ determine the truth of every proposition in an agent's algebra. But these worlds are not accessible to the agent. One does not know if they are *Tractarian* or not. The second type, $worlds_2$ are members of some partition in their algebra that is specified by the agent's utilities. They are the outcomes of wagers. They are compatible with many propositions. Consequently, a decision theory compatible with Ramsey's framework in "Truth and Probability" and his philosophical commitments at the writing of "Theories" is possible.

2.3 Ramsey's Theory of Scientific Theories

From Ramsey's decision theory, we know wagers are over propositions of an atomless algebra with outcomes being propositions in that algebra that are maximal up to utility. I need to explain how Ramsey's account of scientific theories fits with this description. In particular, I need to explain what a wager on a singular theoretical proposition is. Since wagers over

[12]To see why it would have to infinite, suppose $|G| = n$ for some finite n and there exists $g, h \in G$ such

theoretical propositions are still wagers, they must still be conditionals. So what are the propositions people bet on and the outcomes they receive? Ramsey's philosophy of science is given in "Theories" and in sporadic places elsewhere in his writings. The account in "Theories" has three parts: a primary system, a secondary system, and a dictionary linking said systems. Critically, Ramsey considers the existence of the dictionary of principal importance. It is important because Ramsey believes the secondary system to be essentially a fiction: its propositions are not properly true or false. Consequently, there needs to be some other way to assess the satisfaction of the conditions in wagers over theoretical propositions. Ramsey's dictionary allows the specification of verification conditions that can meet that requirement.

The primary system is what other philosophers term the observation language. For Ramsey, the primary system is a "universe of discourse" that describes "the facts to be explained" (Ramsey, [1929] 1990m, 112). The vocabulary of the system consists of integers as terms, functions as predicates, and propositions as assertions of those functions' values on arguments (Ramsey, [1929] 1990m, 112–114). The reason why he believes the primary system is mathematical in nature is because he thinks that "the terms of our primary system have a structure, and any structure can be represented by numbers (or pairs or other combinations of numbers)" (Ramsey, [1929] 1990m, 113). Furthermore, from his discussion of the primary system and the example he uses, he considers it to be in some sense a language of experience.

This can be seen in his toy model. Throughout "Theories" Ramsey uses an elaborate example involving colors and eye and body movements to illustrate a primary system. The vocabulary of this example can be seen in figure 2.1. It consists of three functions and terms that are the integers representing time instants. Furthermore, he presents it in two forms: a mathematical one and a logical one. As the previous discussion indicated, the mathematical form is more basic.

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that g is strictly preferred to h. Then find two members $i, j \in G$ preferred between g and h such that there exists no third member $k \in G$ ranked between them. Then the gamble i if P; j if not P will lie exactly between i and j but will not have any $g \in G$ the agent has an indifference to.

Mathematical Form	Logical Form	English Translation
$\phi(n) = 1$	$A(n)$	I see blue at n.
$\phi(n) = -1$	$B(n)$	I see red at n.
$\phi(n) = 0$	$\neg A(n) \wedge \neg B(n)$	I see nothing at n.
$\chi(n) = 1$	$C(n)$	Between $n-1$ and n I feel my eyes open.
$\chi(n) = -1$	$D(n)$	Between $n-1$ and n I feel my eyes shut.
$\chi(n) = 0$	$\neg C(n) \wedge \neg D(n)$	Between $n-1$ and n I feel my eyes neither open nor shut.
$\psi(n) = 1$	$E(n)$	I move forward a step at n.
$\psi(n) = -1$	$F(n)$	I move backward a step at n.
$\psi(n) = 0$	$\neg E(n) \wedge \neg F(n)$	I do not move at n

Figure 2.1: The vocabulary of Ramsey's toy model primary system.

Mathematical Form	Logical Form	English Translation
$\alpha(n) = 1$	$\alpha(n, 1)$	At time n I am at place 1.
$\alpha(n) = 2$	$\alpha(n, 2)$	At time n I am at place 2.
$\alpha(n) = 3$	$\alpha(n, 3)$	At time n I am at place 3.
$\beta(n, 1) = 1$	$\beta(n, 1)$	At time n place 1 is blue.
$\beta(n, 1) = -1$	$\neg\beta(n, 1)$	At time n place 1 is not blue.
$\beta(n, 2) = 1$	$\beta(n, 2)$	At time n place 2 is blue.
$\beta(n, 2) = -1$	$\neg\beta(n, 2)$	At time n place 2 is not blue.
$\beta(n, 3) = 1$	$\beta(n, 3)$	At time n place 3 is blue.
$\beta(n, 3) = -1$	$\neg\beta(n, 3)$	At time n place 3 is not blue.
$\gamma(n) = 1$	$\gamma(n)$	At time n my eyes are open.
$\gamma(n) = 0$	$\neg\gamma(n)$	At time n my eyes are closed.

Figure 2.2: The vocabulary of Ramsey's toy model secondary system.

The secondary system is what philosophers typically name the theoretical language. It introduces new vocabulary in the form of functions and a set of axioms that specify the value ranges of those functions. In Ramsey's toy model, this secondary system specifies when the agent's eyes are open or closed and introduces a ring of places that display colors. Figure 2.2 lists the vocabulary and figure 2.3 lists the axioms of his toy model.

Finally, the dictionary connects the primary system functions with the secondary system functions. It is a set of structural equations that specify the behavior of each primary system function in terms of its secondary system counterparts. When viewed as a logical calculus, Ramsey states these definitions are logical equivalences. The example dictionary from his toy model can be found in figure 2.4.

Mathematical Form	English Translation
$\forall n(\alpha(n) = 1 \vee \alpha(n) = 2 \vee \alpha(n) = 3)$	I am at only one of three places at n.
$\forall n(\beta(n, 1) = 1)$	Place 1 is always blue.
$\forall n(\beta(n, 2) \neq \beta(n + 1, 2))$	Place 2 alternates colors.
$\forall n, m(\beta(n, m) = 1 \vee \beta(n, m) = -1)$	Every place is red or blue at all times.
$\forall n(\gamma(n) = 0 \vee \gamma(n) = 1)$	My eyes are either open or closed.

Figure 2.3: Ramsey's axioms in mathematical form and English translation.

Mathematical Form	English Translation
$\phi(n) = \gamma(n) \times \beta(n, \alpha(n))$	The color I see is a function of my eyes being open and the color of where I am at.
$\chi(n) = \gamma(n) - \gamma(n - 1)$	My eyes opening or closing is a function of my eyes being opened or closed at n and $n - 1$.
$\psi(n) = (\alpha(n) - \alpha(n - 1)) \mod 3$	My stepping forward or backward is a function of the places I have been.

Figure 2.4: Ramsey's dictionary in mathematical form and English translation.

With the secondary system's axioms and the dictionary, one can deduce a set of laws in the primary system. These laws take the form of universal propositions. One can also deduce singular propositions in the primary system from the theory, which Ramsey calls consequences. Ramsey asserts that "it is the totality of laws and consequences which our theory asserts to be true" (Ramsey, [1929] 1990m, 115). These are the propositions actually asserted by one's theory.

An important question that confronts Ramsey is what enables the primary and secondary systems to be connected by a dictionary. He writes in his notes that the two *must* be connected:

> There must be a link between physics and experience. The truth seems to be every prop [proposition] of experience is equivalent to a prop [proposition] of physics including 'normal' or 'non-hallucination' as a physical idea. But not conversely. There is a dictionary for experience into physics, not for physics into experience.
>
> There must be a dictionary; what is the relation between the props [propositions] on two sides of it? In a sense ideally, in a sense complete incomparability according to how we conceive the secondary system (FPRP Realism, 5).

Ramsey's concern is about how one relates experience with the world of physics (scientific theories). Why can one show one to be equivalent to propositions of the other? Furthermore, Ramsey says that the dictionary has to go from the secondary system (physics) to the primary system (experience). What should warrant this? What warrants the existence of the dictionary at all?

An answer is given by the alethic status of the secondary system propositions. In multiple locations, Ramsey states that the secondary system is a fiction. When asked to explain how theories function without explicit definitions, he writes that "in such a theory judgment is involved, and the judgments in question could be given by the laws and consequences, the theory being simply a language in which they are clothed, and which we can use without working out the laws and consequences" (Ramsey, [1929] 1990m, 131). Ramsey states that the laws and consequences of the theory in the primary system are what we really assert with the secondary system. This matches with his explicit description of theoretical propositions as fictions in his other work. He writes that

> The truth is that we deal with our primary system as part of a fictitious secondary system. Here we have a fictitious quality, and we can also have fictitious individuals [....] Fictitiousness [of singular theoretical propositions] is simply ignored; we speculate about a body's weight just as much as about its position, without for a moment supposing that it has not one exact weight. The only difference is that we are not ultimately interested in fictitious propositions, but use them merely as intermediaries: we do not care about them for their own sake (Ramsey, [1929] 1990a, 137).

The anti-realism here is evident. This explains the need for the dictionary. Wagers on fictional propositions need to be settled somehow. By being settled, I mean just the satisfaction of the wager's conditions. Ramsey believes secondary system propositions are not truth-apt.

So something else has to "stand-in" for them to permit the settlement of the bet. Fictional propositions need to be "translated" into proper propositions. The dictionary allows that bridge; it enables one to move from fictional intermediaries to the propositions one cares about. What supports that bridge is still unclear but the purpose of the dictionary is clear: it allows one to evaluate the fictional propositions in terms of the real ones.

In other words, the dictionary allows the construction of verification conditions. What Ramsey means by verification conditions is given in his argument against explicitly defining theoretical propositions in terms of observational propositions. The largest portion of "Theories" is devoted to answering the question of whether it is possible to explicitly define a theory in terms of observation. Ramsey's answer is that it can only be done at the cost of such definitions being arbitrary, in the sense that there are multiple possible definitions, and complicated, in the sense that such definitions are disjunctive. Using verification conditions is one such method:

> [T]he next hopeful method is to use both dictionary and axioms in a way which is referred to in many popular discussions of theories when it is said that the meaning of a proposition about the external world is what we should ordinarily regard as the *criterion* or *test* of its truth. This suggests that we should define propositions in the secondary system by their criteria in the primary (Ramsey, [1929] 1990m, 122–123).

Ramsey's answer will be negative here. But importantly for the current discussion, the verification conditions can help address how secondary system propositions are wagered. What Ramsey means by the criterion or test of a secondary system proposition's truth should be directly related to how secondary system propositions are wagered. Because gambles are decided on the truth of propositions, it would be important to know when those propositions are true. But recall that those propositions themselves are not strictly true or false. So how are they evaluated? The answer must be their verification conditions. And since the verification conditions are tied in with the dictionary, understanding how they work will help in addressing the question of what underpins the dictionary. Consequently, Ramsey's discussion of verification conditions is directly relevant to understanding how secondary system propositions are wagered.

2.4 Verification Conditions and Truth-Possibilities

The story thus far is that gambles over secondary system singular propositions must be assessed on those propositions' verification conditions. But what are the verification conditions exactly? And what are the outcomes being wagered in these gambles? I aim to answer the first question in this section.

Ramsey provides two definitions of verification conditions in "Theories". The first is a logical definition. Ramsey defines what he calls the necessary and sufficient conditions for the truth of a secondary system proposition through logical consequence. He then proceeds to make no use of this definition and provides an alternative one.

His alternative definition utilizes the concept of truth-possibilities. If one lets $\sigma(p)$ be the sufficient conditions for propositions p and $\tau(p)$ be the necessary conditions for p, then Ramsey writes

> We can elucidate the connection of $\sigma(p)$ and $\tau(p)$ as follows. Consider all truth-possibilities of atomic propositions in the primary system which are compatible with the dictionary and axioms. Denote such a truth-possibility by r, the dictionary and axioms by a. Then $\sigma(p)$ is the disjunction of every r such that
>
> $r\bar{p}a$ is a contradiction,

$\tau(p)$ the disjunction of every r such that

rpa is not a contradiction.

(Ramsey, [1929] 1990m, 123).

Ramsey's "elucidation" on the necessary and sufficient verification conditions is clear as mud. To evaluate it, one needs to discuss what are the truth-possibilities of a set of propositions.

The phrase "truth-possibility" is first used in Ramsey's translation of the *Tractatus* (Wittgenstein, 1961). Ramsey discusses them in his review of the *Tractatus* where he defines them in the context of the sense of a proposition:

> The sense of propositions in general is explained by reference to elementary propositions. With regard to n elementary propositions there are 2^n possibilities of their truth and falsehood, which are called the truth-possibilities of the elementary propositions; similarly there are 2^n possibilities of existence and non-existence of the corresponding atomic facts (Ramsey, 1923, 470).

Intuitively, the truth-possibilities of a set of propositions are the rows in a truth-table that enumerate all possible combinations of truth assignments to those propositions. In Ramsey's jargon, they are the possibilities where those propositions are true or false. This device is used frequently throughout Ramsey's writing. It shows up not only in "Theories" but also in notes associated with the drafting of "Theories" and more importantly in "Facts and Propositions" (FPRP Theories, 21). In the latter case, Ramsey uses truth-possibilities to explain the content of complex or multiple atomic propositions:

> Any such attitude can, however, be defined in terms of the truth-possibilities of atomic propositions with which it agrees and disagrees. Thus, if we have n atomic

TP #	$\phi(0) = 1$	$\phi(0) = -1$	$\alpha(0) = 1$
1	1	1	1
2	1	1	0
3	1	0	1
4	1	0	0
5	0	1	1
6	0	1	0
7	0	0	1
8	0	0	0

Figure 2.5: An example set of truth-possibilities from Ramsey's toy model primary and secondary system. Only functions ϕ and α are used.

> propositions, with regard to their truth and falsity there are 2^n mutually exclusive possibilities, and a possible attitude is given by taking any set of these and saying that it is one of this set which is, in fact, realized, not one of the remainder [....] To say that feeling belief towards a sentence expresses such an attitude is to say that it has certain causal properties which vary with the attitude, i.e. with which possibilities are knocked out and which, so to speak, are still left in (Ramsey, [1927] 1990d, 45–46).

Truth-possibilities are not merely a device of Wittgenstein's but also one that Ramsey intends to use for an account of the content of beliefs understood as dispositions to act. That is to say that truth-possibilities are instrumental in understanding the content of a proposition in relation to what gambles one would accept on that proposition, i.e., the different possibilities or outcomes preferred or not.

Returning to "Theories", Ramsey's discussion of verification conditions asks one to consider the truth-possibilities of the primary system compatible with the secondary system's axioms and the dictionary. The presence of the dictionary allows one to go further and consider the joint truth-possibilities over the primary *and* secondary system. An example of this applied to some of the propositions in Ramsey's toy model can be seen in figure 2.5. This is just the truth-table over the propositions $\phi(0) = 1, \phi(0) = -1$, and $\alpha(0) = 1$.

TP #	$\phi(0) = 1$	$\phi(0) = -1$	$\alpha(0) = 1$
3	1	0	1
4	1	0	0
6	0	1	0
7	0	0	1
8	0	0	0

Figure 2.6: Examples of compatible joint truth-possibilities and verification conditions. The green rows in the verification conditions represent the sufficient conditions for $\alpha(0) \neq 1$ and the red rows in the verification conditions represent the necessary conditions for $\alpha(0) = 1$.

In his definition of verification conditions, Ramsey says to consider the truth-possibilities of the primary system compatible with the axioms of the secondary system and the dictionary. One can recover those from the joint truth-possibilities by considering the rows of the expanded table that are compatible with the axioms and dictionary. In figure 2.6, there is an example of compatible truth-possibilities taken from the full table given in figure 2.5. Because one of Ramsey's laws eliminates primary system functions from having more than one value, truth-possibilities one and two are eliminated. And since the second axiom and dictionary specify that one cannot be at place one and see blue, the fifth truth-possibility is eliminated.

The joint truth-possibilities also enable one to describe what the verification conditions are. Consider a secondary system proposition, such as $\alpha(0) \neq 1$. Its sufficient conditions can be found by examining the joint truth-possibilities compatible with the axioms and the dictionary whose primary system fragment are also incompatible with its negation. Translated into the example from above, one needs to find a row where $\alpha(0) = 1$ is false but its dual (same assignments except for $\alpha(0) = 1$) is missing. This would be row six in figure 2.6, since truth-possibility six's dual where $\alpha(0) = 1$, row five, is deleted. The sufficient conditions themselves would just be the primary system fragment from row six, i.e. $\phi(0) \neq 1$ and $\phi(0) = -1$. The same trick applies to the necessary conditions except one now examines the rows whose primary system fragment are compatible with the proposition itself. Consider $\alpha(0) = 1$. This proposition is true in rows three and seven in figure 2.6. Since rows four and eight also share the same primary system truth-value assignments, they are included in the necessary conditions. So the necessary conditions for $\alpha(0) = 1$ are truth-possibilities three, four, seven, and eight. From this example, it is evident that the disjunction of the sufficient conditions of a proposition and the necessary conditions of its negation yield the full truth-possibilities, which Ramsey asserts are the laws of the theory (Ramsey, [1929] 1990m, 123–124).

To summarize, the verification conditions can be found by examining the joint truth-possibilities of the primary and secondary system that are compatible with the axioms and the dictionary. Sufficient and necessary conditions can be yielded by considering just the primary system fragment of those joint truth-possibilities. This then explains how a gamble on a singular, theoretical proposition can be settled. Finding the sufficient conditions or a violation of the necessary conditions in conjunction with the theory's laws will verify a singular secondary proposition. However, these verification conditions make use of truth-possibilities. What exactly are truth-possibilities? There are several hypotheses to consider. I argue that the most likely hypothesis is that truth-possibilities are sets of possible worlds in the sense of $worlds_1$.

First, truth-possibilities are the representation of the existence and non-existence of Tractarian facts. The evidence for this comes from Ramsey's review of the *Tractatus*. There he writes that the sense of a proposition is its agreement and disagreement with the truth-possibilities of elementary propositions, whose senses are given by their agreement and disagreement with atomic facts. Similar descriptions occur in Ramsey's notes on the *Tractatus*. Unfortunately, the evidence against this hypothesis is much stronger. In that same review, Ramsey expresses puzzlement over Wittgenstein's theory of facts (Ramsey, 1923, 466) and over Wittgenstein's account of sense (Ramsey, 1923, 471). Furthermore, Ramsey rejects the particulars and universal distinction and with them *any* account of the true nature of elementary propositions and the facts they represent in "Universals". Finally, Ramsey endorses the main idea of Russell's multiple relation theory of judgment that belief is not a relation to one thing such as a fact, which casts further doubt on the need for there to be facts that beliefs express agreement or disagreement with (Ramsey, [1927] 1990d, 38).

Second, truth-possibilities are purely syntactic entities, i.e. they are sets of consistent sentences. The evidence for this hypothesis comes directly from "Theories". Ramsey discusses scientific theories as if they were languages and he does not appeal to reference. In addition, in "Facts and Propositions" he eliminates talk of propositions altogether—suggesting a syntactic approach. But this suggestion fails to square with Ramsey's broader philosophy of meaning. When discussing primary system propositions and truth-possibilities, Ramsey explicitly describes them as being true or false. Sentences cannot be truth-bearers, however, because in *On Truth* he says they cannot (Ramsey, 1991b, 7). A syntactic view also conflicts with how Ramsey discusses languages as being interpreted in "Theories". Finally, when Ramsey intends truth-possibilities to be purely syntactic, he uses the alternative phrase "alternatives" (Ramsey, [1928] 2009, 272).

Third, Ramsey has no account for what truth-possibilities are. Evidence for this hypothesis includes the fact that Ramsey never succeeded at providing an account of the content of beliefs, i.e. propositional reference. This would mean that there just is no story for what truth-possibilities are supposed to be, since they specify when propositions are true or false. It would be anachronistic to believe they could involve possible worlds or any modern semantic concepts. But as I have argued, Ramsey did in fact have possible worlds in two senses. Furthermore, even if Ramsey had not filled in every detail, he did have a story about the contents of beliefs. That content was supposed to be the belief's causes and, in particular, its effects. Those causes and effects are the wagers one would endorse or not endorse. And at least in the context of the primary system, wagers are defined over sets of $worlds_1$. Finally, this story should cohere with the story given in "Theories" because *they were meant to be part of the same book*. It is the same system.

Fourth, Ramsey is just inconsistent: truth-possibilities are about theories, languages, and full beliefs and do not fit with Ramsey's broader philosophy. The primary evidence for this is that Ramsey does not mention wagers or probabilities anywhere in "Theories". But like the previous hypothesis, evidence from other sources pushes against this claim. Ramsey does explicitly say that credences can be had over singular secondary system propositions. And as previously mentioned, "Theories" was meant to be a chapter in the same book with a revised "Truth and Probability". Finally, unless he is explicitly being inconsistent, charity should dictate that views in "Theories" should be consistent in some way with his decision theory.

Finally, that leaves the hypothesis that truth-possibilities just are what Ramsey says they are: they are sets of possible worlds or more specifically, $worlds_1$. The evidence for this hypothesis includes Ramsey's own definition, the presence of worlds in "Truth and Probability", the fact that "Theories" and "Truth and Probability" were part of the same book, and his claim that one can assign credences to secondary system propositions—making such propositions members of an agent's algebra, i.e. sets of worlds. The primary evidence against this view is that Ramsey does not explicitly say it in "Theories". But the majority of the evidence—the ratio of likelihoods between this hypothesis and the others—make it by far the most likely one to be true.

If truth-possibilities are sets of $worlds_1$, that makes them collectively a partition on one's possibility space. This means that verification conditions, being disjunctions of truth-possibilities, are effectively unions of elements from this partition. So the conditions of a wager—when one condition of a wager is decided to obtain—are just the union of relevant partition elements. Truth-possibilities explain what the conditions of wagers on singular secondary propositions amount to.

Truth-possibilities being sets of $worlds_1$ also explains what underpins the dictionary. A common set of possibilities from which primary and secondary system propositions are formed

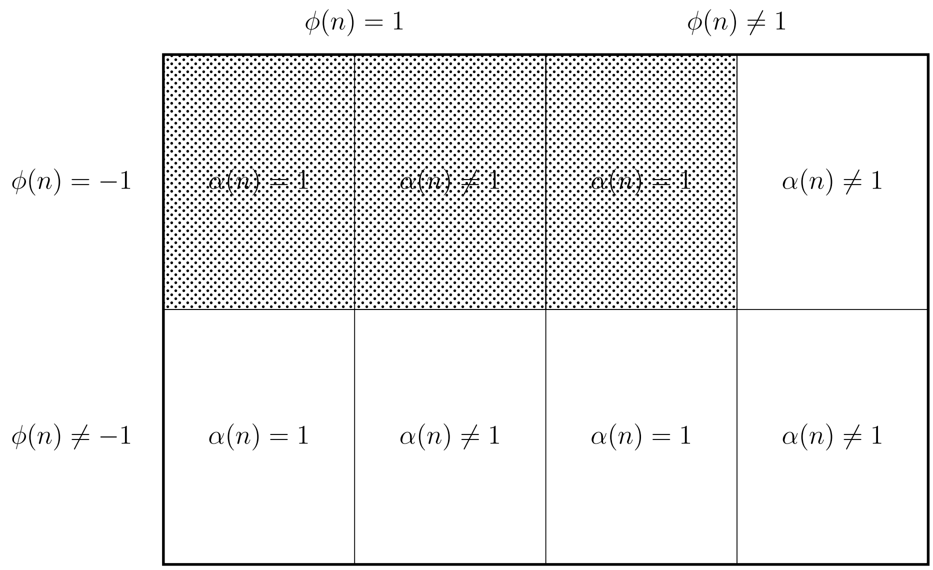

Figure 2.7: A diagram of an agent's possibility space using the truth-possibilities described in the previous figures. Shaded regions are truth-possibilities not compatible with a theory.

allows one to connect them by equivalences. This can be seen in figure 2.7. Here the joint truth-possibilities are given, which are just sets of points in the underlying rectangle. Propositions in both primary and secondary system can thus be formed by the unions of those underlying truth-possibilities. The truth-possibilities incompatible with the theory of Ramsey's toy model are shaded out. A bet on a singular, theoretical proposition is a bet on its regions. And the criterion of truth of a proposition (when the bet is realized) are the primary system truth-possibilities defined by that proposition's verification conditions.

Importantly, one might observe that the regions in figure 2.7 used for specifying verification conditions are those that are *unshaded*. This is because the verification conditions are defined through the truth-possibilities that are compatible with the theory's axioms and the dictionary. And the shaded regions are the areas not compatible with the theory. Ramsey remarks on this fact by observing that the meaning of a theoretical proposition always has to occur in the context of a theory:

> When we ask for the meaning of e.g. $\alpha(0,3)$ it can only be given when we know to what stock of 'propositions' of the *first and second* systems $\alpha(0,3)$ is

to be added. Then the meaning is the difference in the first system between $[\exists\alpha, \beta, \gamma(\text{stock} \wedge \alpha(0,3))]$ and $[\exists\alpha, \beta, \gamma(\text{stock})]$ [....] This account makes $\alpha(0,3)$ mean something like what we called above $\tau\{\alpha(0,3)\}$, but it is really the difference between $\tau\{\alpha(0,3) + \text{ stock}\}$ and $\tau(\text{stock})$ (Ramsey, [1929] 1990m, 131).

Here the Ramsey sentence makes an appearance to describe the fact just discussed. The meaning of a theoretical proposition, its truth conditions, are always given relative to the theory that defines that proposition. This corresponds to the fact that in considering the verification conditions of $\alpha(0) = 1$ or $\alpha(0) \neq 1$, I did not include rows one, two, and five from figure 2.6 since they were incompatible with Ramsey's toy model secondary system. What the Ramsey sentence effectively means in this context is to locate what truth-possibilities one takes live if one accepts the theory: it says to ignore the shaded regions in the possibility space given in figure 2.7. The remaining areas can then be used to specify the meaning of theoretical propositions, i.e. to determine when they might be verified.

Summarizing, the verification conditions of a proposition are important for determining the satisfaction of conditions in wagers. Those verification conditions are given by the primary system fragment of the joint truth-possibilities compatible with a theory. Truth-possibilities are just sets of $worlds_1$. This enables the existence of the dictionary. And it results in the meaning of a theoretical proposition always being relative to the theory in which it occurs.

2.5 Outcomes in Theories

There are three parts to a wager in Ramsey's decision theory. The first part, wagers are a type of conditional, is common to both wagers in the primary system and wagers in the secondary system. The second part, wagers are over the truth of propositions, requires a modification for the fictional secondary system propositions. Because secondary system propositions are fictitious, their cognitive values must come from their verification conditions. Those verification conditions are the primary system fragment of sets of truth-possibilities formed from primary and secondary system propositions. Each truth-possibility is just a set of points from an agent's possibility space. That leaves the third part: the outcomes of wagers. These are members of some privileged partition in the agent's algebra that the agent cares about. What are they in the context of bets on singular theoretical propositions?

A quick observation is that the only candidates are just $worlds_2$, i.e., the elements of a privileged partition that consists of equivalence classes of $worlds_1$ unique up to utility. This is required to yield the sense of the propositions wagered. For reasons I will not discuss until chapter seven, the $worlds_2$ here would just be the truth-possibilities of Ramsey's primary system. These characterize the consequences of gambles on the truth of a primary system proposition. But they have a big problem if they are the consequences of gambles involving secondary system propositions: they leave undetermined the "truth" of the secondary system proposition. I use the word "truth" here in quotation because what is really undetermined is the proposition of the agent's algebra they would receive should their gamble pay off. Each $worlds_2$ allows a secondary system proposition and its complement to be still live in terms of an agent's possibility space. So what would be required is a refinement of the $worlds_2$ that returns at a minimum the secondary system proposition along with the observable consequences the agent actually cares about.

It is insufficient, however, to just declare that the wagered secondary system proposition is "true" in the outcome. The theory, because of the axioms and the dictionary, restricts what combinations of secondary system and primary system propositions can be live; these propositions are not independent of one another like the elementary propositions of Wittgenstein's *Tractatus*. Instead, to specify the "truth" of a wagered secondary system proposition in the outcome, one must specify the "truth" of every other secondary system proposition and compatible $worlds_2$. This is just to yield as an outcome a joint truth-possibility from the primary and secondary system.

So the hypothesis on the table for the outcome of wagers involving secondary system propositions is that those outcomes are just joint primary and secondary system truth-possibilities compatible with the theory. Two facts highly suggest lend credence to this hypothesis: the outcomes need to be compatible with the propositions being wagered and truth-possibilities fit nicely with the account of worlds given in section 2.2.

First, outcomes in wagers need to be compatible with the propositions being wagered. Otherwise the wager is empty in the sense that it is contradictory. Consider the wager where I live in the world where I receive a slice of cheesecake if the proposition "I will not receive a slice of cheese cake" is true; or I live in the world where I go for a drive if the proposition "I will receive a slice of cheese cake" is true. One side of a wager is impossible due to the condition and its world not both being possible simultaneously. So this wager is broken in an important sense. To avoid such wagers over secondary system propositions, the outcomes have to be sensitive to the truth of their conditions. This means that even if one only considers a $worlds_2$ in the primary system as an outcome, one has to specify what corresponding joint truth-possibilities would allow that $worlds_2$ to avoid an illicit gamble. So one is already operating with the joint truth-possibilities in the outcomes.

Second, joint truth-possibilities are just refinements of $worlds_2$. While they are not intrinsically cared about—since two joint truth-possibilities in the same $worlds_2$ will have the same utility—they are cared about derivatively. I may not happen to be concerned with the electrical resistance properties of copper, but those properties certainly have important consequences for the things I do care about such as electrical lighting and heat. Furthermore, because the theory will often restrict what joint truth-possibilities are live, some $worlds_2$ will only have one joint truth-possibility in them. This allows the consequences to not be totally agnostic about the outcome of a wager; secondary system propositions have partial sense in that per the theory, some joint truth-possibilities just are a $worlds_2$. Lastly, because the joint truth-possibilities are refinements of $worlds_2$, they still would permit ethically neutral propositions. This is just due to the fact that one can just appeal to the primary system fragment of the joint truth-possibility to measure the utility of the wager.

One important thing to understand about these joint truth-possibilities is that they are not facts about the world. The name "truth-possibility" is in some sense misleading. Instead, they are about the agent's epistemic or conceptual space—their algebra. Real and fictional propositions alike live in that algebra. And a truth-possibility is just a member of that algebra. So the joint truth-possibilities given by a primary and secondary system can be fictitious even though it has the word "truth" in its name.

Together, this means that wagers on singular, theoretical propositions have joint truth-possibilities between primary and secondary systems as their outcomes. I now have a complete answer to the target question of this essay. Wagers on singular, theoretical propositions are conditionals whose antecedents can be verified by the observational fragment of joint truth-possibilities, which are just sets of $worlds_1$. The consequents of those conditionals are joint truth-possibilities compatible with the theory. From these wagers, one could then compute the probability an agent assigns to the truth of singular, theoretical propositions.

2.6 Conclusion and Further Work

Ramsey's plan to combine "Theories" and a revised "Truth and Probability" into one book made interpreting "Theories" through Ramsey's decision theory an ideal method for helping in part to resolve the abstruseness of "Theories". The central idea guiding Ramsey's decision theory is a dispositional account of belief. Both full beliefs and partial beliefs can be measured through an agent's preferences over gambles. I argued that those gambles are conditionals over propositions in an atomless algebra with outcomes being other propositions given by a partition that is maximal up to utility. Ramsey's philosophy of science, consisting of a primary and secondary system with a dictionary bridging them, can be made to fit with his decision theory. A key first question to be addressed is what are gambles over singular, theoretical propositions. My answer is that wagers can be verified by looking at the primary system fragment of truth-possibilities coherent with the proposition wagered. The outcomes of those wagers are just the partition generated by a theory's vocabulary compatible with the theory.

This leaves several open questions. Perhaps most obviously, I did not answer how theories themselves are wagered. This is an important component of Ramsey's philosophy of science. He discusses disputes and the relations between theories at the end of "Theories" and attempts to provide definitions for equivalent, contradictory, and subset relations over theories (Ramsey, [1929] 1990m, 132). Beliefs in theories like other beliefs should allow wagers to be placed on them.

Finally, an important lacuna still needs to be resolved with respect to singular, theoretical propositions. Wagers are decided on the verification conditions of the propositions that are wagered. But it might turn out that the verification conditions *do not decide* between a proposition and its negation. For example, the joint truth-possibilities given in figure 2.6 allow for $\alpha(0) = 1$ and $\alpha(0) = -1$ in rows seven and eight. But those two rows have the exact same truth values assigned to the primary system propositions. At time zero, one cannot differentiate between these two propositions. So how does one decide on the outcome of a wager? Ramsey's discussion in "Theories" towards the end of the paper suggests the answer has to do with a theory's laws, i.e. about what may still be observed in the future (Ramsey, [1929] 1990m, 133–134). This suggests that his remark elsewhere that his account of theories is a "forecasting" instead of "descriptive" account of theories is important. However, I cannot address that here since it requires an extended discussion of what laws are for Ramsey. I turn to that task in chapters 2 and 3.

Chapter 3

Ramsey's Cognitive Psychology and Philosophy of Logic

3.1 Introduction

So far I have provided a reconstruction of Ramsey's decision theory that is compatible with his philosophical commitments. This has allowed me to show how singular, theoretical propositions have credences. The next step is to explain how laws function in the context of Ramsey's decision theory. This will be necessary to explain ultimately the role of the Ramsey sentence and Ramsey's thoughts on scientific realism. In this chapter, I will start by laying some foundations for an account of Ramsey's views on laws. The chief thing I do here is connect Ramsey's view of laws with his theory of forecasts.

At the end of Ramsey's "General Propositions and Causality", he offers an enigmatic footnote that briefly describes his philosophy of science as a "forecasting theory". What he means by this and by a "forecast" is unclear. However, elsewhere in his unpublished notes, he uses the term sporadically. An examination of those notes reveals the skeleton of a theory of cognition. Ramsey held that all actions are at root driven by the sum total of a person's dispositions or habits. These habits operate in an unconscious process that produces psychological expectations about the realization of desires. When those expectations are frustrated, the violation is registered consciously to the individual as a proposition, and the offending habit is identified. Humans can then regulate and change those habits by the conscious application of logic through deliberation. The applicable logic is Ramsey's decision theory, which aims to make beliefs probabilistically coherent by adopting the laws and chances that signify the habits people might use for guiding behavior. The outcome of this deliberation is to refashion psychological expectations as mathematical expectations on laws and chances. These mathematical expectations are forecasts, and a forecasting theory of science is one that takes scientific theories to provide forecasts.

In the "General Propositions and Causality" paper, Ramsey articulates a new view of universal propositions such as "Arsenic is poisonous" and "All men are mortal" along with a discussion of causal laws. He lists a series of notes at the end of the paper. The first of those notes explicitly describes his philosophy of science as a forecasting theory:

> As opposed to a purely *descriptive* theory of science, mine may be called a *forecasting* theory. To regard a law as a summary of certain facts seems to me inadequate; it is also an attitude of expectation for the future. The difference is clearest in regard to chances; the facts summarized do not preclude an equal chance for a coincidence which would be summarized by and, indeed, lead to a quite different theory (Ramsey, [1929] 1990e, 163).

What Ramsey means by a "forecasting theory" is enigmatic. He does connect laws to expectations about the future, and he also connects chances as well. But beyond that he says nothing nor does he say anything in this paper and other finished writings. More can be said, however, in his notes. Ramsey mentions in a series of notes about existential judgments that forecasts are very important:

> Question. What is the meaning in test of acquaintance?
>
> Suggestion The fundamental proposition is the forecast then the memory (FPRP Existential Judgment, 6).

Here he links forecasts with meaning, acquaintance, and memory. The last part is important: it suggests that forecasts are related to cognition or how humans think. Elsewhere in his notes when he discusses memory, he also discusses cognition generally. This suggests that the key to understanding what Ramsey means by a forecast is to understand how he thinks human cognition works.

Ramsey's theory of cognitive psychology has received little attention in the secondary literature. Sahlin alleges in passing that Ramsey has a view analogous to the representational theory of mind, but he fails to discuss the evidence for that view or to elaborate more fully on it (Sahlin, 1990, 78–80). Dokic and Engel ascribe a functionalist theory of belief to Ramsey, but they do not go into considerable detail or discuss Ramsey's broader theory of cognitive psychology (Dokic and Engel, 2002, 24–25). They do discuss the relationship between beliefs and desires for Ramsey, and they suggest Ramsey would have had to be a kind of coherentist about beliefs and desires in the sense they depend on one another (see section 4.5 in Dokic and Engel, 2002, 62–64). Both Sahlin and Dokic and Engel cite Loar as working out this theory in detail, but Loar mainly identifies Ramsey as thinking some mental states need to play the causal roles of belief and desires without further elaboration on what those roles happen to be (Loar, 1980, 65–68). Additionally, Skorupski (Skorupski, 1980) and Hookway (Way, 1980) only briefly discuss Ramsey's theory of psychology in passing while discussing more modern developments in the philosophy of belief. Later authors like Misak largely ignore a psychological theory Ramsey might with the exception to notice as previous commentators made that Ramsey is seemingly committed to a belief and desire coherentism (Misak, 2016). In short, Ramsey's theory of cognitive psychology is genuinely underdeveloped in the secondary literature.

The key project in this chapter is to develop that theory. This will prove foundational for later chapters on laws, chances, and the Ramsey sentence since it provides a picture of the roles in a person's cognition those items play.

The central component of Ramsey's theory of cognition is the psychological expectation. Psychological expectations are the anticipations of experiences, whether rewarding or punishing. They are both the product of human cognition and also the transmission driving behavior. Cognition produces psychological expectations through an unconscious integration of habits, and cognition is changed by registering violations of those expectations. A violated expectation constitutes the proper propositions. The truth or falsity of those propositions can then be used to deliberate on the habits that generate future expectations.

Habits or dispositions to act are the rules humans use for building psychological expectations. These habits act collectively unconsciously; an expectation is the sum product of every habit. Ramsey analogizes the process through which habits produce expectations to how the automatic telephone dials different households. Like the telephone, this dialing process is associative and largely invisible to people; it is stored in a memory system that is largely inaccessible. Only when those expectations are violated are the habits driving those expectations examinable.

The process of examination occurs consciously. Here violated expectations constitute the proper propositions. These are the propositions of the primary system. They admit to being true or false. When a violation occurs, the mind is able to identify the offending habits, and both the offense and offender are stored in an accessible memory system. Deliberation can then proceed on how to modify those habits. This occurs via the application of logic.

Like Peirce, logic is a means of self-control for Ramsey. It is how the conscious process in deliberation changes the habits that lead to psychological expectations. For Ramsey, the correct logic is his decision theory. Violated expectations are treated as proper propositions while habits are treated as laws and chances. The process of deliberation involves fitting those propositions and habits into a coherent system of credences. This means forcing psychological expectations to act as mathematical expectations.

Psychological expectations that behave as mathematical expectations according to some adopted laws and chances are forecasts. Beliefs in propositions are treated as weighted averages over laws and chances. The weights here are subjective degrees of belief about the trust an agent puts in the laws and chances. When the forecast is purely epistemic, i.e. when the agent only cares about the truth of the forecasted propositions, these expectations are just equal to probabilistic predictions given by a mixture of the laws and chances an agent thinks possible. Thus when Ramsey says his theory of science is a forecasting theory, he means that the point of science is the production of laws and chances and methods for weighing those laws and chances in forecasts. A theory of science is a forecasting theory just in case it is useful for making decisions.

Before I begin, I want to provide a cautionary note about what follows. Ramsey never authored a complete paper discussing his theory of cognition. Most of what follows is a reconstruction from his notes informed by the more mature philosophy in his published papers. So I want to emphasize that this is partially speculative, and I am uncertain whether this account is correct. I have put forward the evidence, and I believe this is the most likely theory of cognition that reflects Ramsey's thoughts in 1928–1930.

Here is how my argument proceeds. First, I argue that Ramsey views human cognition to proceed via unconscious and conscious processes. The meeting point for these processes is psychological expectations. Second, I discuss how the two processes relate to Ramsey's philosophy of logic as self-control. This view of logic Ramsey inherited from Peirce. Finally, I discuss how this leads forecasts to be regimentations of psychological expectations as mathematical expectations. The upshot is that a forecasting theory of science holds science to be a method of decision-making.

3.2 The Cognitive Model

Ramsey has a rudimentary theory of cognition in his notes. The key components of that theory are two processes that work together to produce psychological expectations. The first process is unconscious in the sense that humans are not aware of it, and it is not open to immediate introspection. In contrast, the second process is defined by awareness and the ability of humans to introspect. Awareness is the key dividing line. It is what separates dispositions from acts.

In Ramsey's unpublished book manuscript,[1] he lists the content of the unconscious and conscious processes to be dispositions or acts:

> The most important of these [states of mind] is that between *acts* and *dispositions* [....] When we say he knows he's got to leave or he knows his multiplication table, we are talking of enduring dispositions of his mind, manifested at times in particular acts of knowing, but conceived as existing even when not so manifested, just as a man is called courageous even when not at the moment displaying his courage [....] But we also have other words which refer not to dispositions but to definite dateable (but not necessarily instantaneous) acts of mind. Thinking, as in "I was just thinking that its going to rain," [...], judging, inferring, asserting, perceiving, discovering and learning all refer to acts not to dispositions (Ramsey, 1991b, 98 (early draft)).

[1]Ramsey's book manuscript has multiple drafts per chapter. Here I rely upon two. Both are in the Rescher and Majer collection titled *On Truth*. I will indicate in the citations whether the text is from the earlier or later draft as indicated by Rescher and Majer.

Ramsey divides mental states into acts and dispositions. Starting with the former, he considers dispositions to be claims like knowledge about multiplication tables. These are not claims about the world in the sense when I say I know how to do multiplication that I have in fact multiplied all numbers; instead, my claim about knowing multiplication is a claim that if the right circumstances were presented, I could successfully multiply the presented numbers. In contrast, acts are definitive mental events. Here he considers judgments, assertions, perception, and learning as examples. If I correctly implement a multiplication algorithm in my head, this would be a mental act.

The division between acts and dispositions tracks the split between conscious and unconscious processes. The content of the conscious process is mental acts, while the content of the unconscious process is dispositions. Together these direct and produce behavior.

Two elements of cognition link the conscious and unconscious processes: expectations and memory. I treat expectations first.

3.2.1 Psychological Expectations

Starting with acts, also called "judgments", Ramsey is clear that acts include more than the result of resolutions of doubt. He writes that

> It has been said that judgment is a decision reached from doubt, and presupposes a preliminary process of inquiry and indecision; in ordinary language this may be so, but we shall use the word much more widely so as to include any form of thinking that, whether it be a reasoned conclusion or a guess or a prejudice or a memory or a presentiment or anything else whatever of the same general type. Judgment in our usage presupposes no process of reflexion or weighing of evidence; we may reflect and weigh the evidence before we judge but only too

> often we jump to a conclusion without any such process (Ramsey, 1991b, 46 (later draft)).

No reflection is necessary for a judgment; it is not the abatement of doubt. He uses judgment to include any sort of mental act that is not a disposition. They are mental events.

This means mental images also count as acts. He goes even further, listing other non-linguistic mental representations along with images as judgments:

> Let us take next the case in which he does not say anything to himself but merely has an image of Jones' face. In this case, it still seems to me that this image, just like the words in the last case, would be or express a judgment [....] But suppose he neither said anything to himself nor had an image [in reaction to seeing a man's back], what then? In this case there are, I think, two possibilities: first that he made a judgment of some other kind or in some other way [or second, it could be a disposition] (Ramsey, 1991b, 48 (later draft)).

Acts have propositional content but many of these acts are non-linguistic. In the case of seeing someone's back, the act of thinking can be linguistic (an inner monologue), imagistic (associating the back with the person's face), or neither (some association about who it is). Therefore, propositional content can be had in mental imagery and other forms of intuition that occur as mental events.

What distinguishes acts from dispositions generally is that the act is part of a conscious—aware—process that leads to actions. Ramsey considers the case of seeing someone's back. If there is an explicit event, linguistic or not, that leads to an action and is crucial for that action, then it counts as an act:

> The conclusion we have come to is this: if his seeing the back led either to his

> saying to himself "Hallo, there's Jones" or to his having an image of Jones" face of such a kind or with such accompaniments that it issues in action, then we must say he made a judgment (Ramsey, 1991b, 49 (later draft)).

Mental acts are tied to actions. They have to be somewhere upstream in the process of decision-making. As discussed previously, this means that "accompaniments" can count as acts. Ramsey goes so far as to include even associations: "An immediate (conditioned reflex) response to a stimulus can be in our view a judgment provided it is a response in thought (e.g. words or images) and not in action" (Ramsey, 1991b, 50).[2] I will argue below that their role in issuing actions is through surprise and conscious deliberation. So what makes an act an act is its conscious connection to action. This means that things as diverse as associations and perceptions can count as mental acts.[3]

But how are actions determined? Ramsey provides the same response across his writings. Actions are determined by beliefs in combination with desires: "[A person's actions] result from his desires and the whole system of his beliefs, roughly according to the rule that he performs those actions which, if his beliefs were true, would have the most satisfactory consequences" (Ramsey, 1991b, 45 (later draft)). Here Ramsey's whole system of beliefs includes dispositions.[4] So actions are determined by a combination of previous mental acts and current dispositions and desires. The result is what I call a psychological expectation.

[2]In contrast if it does not, then it is properly characterized as a disposition: "If on the other hand he acted directly on seeing the back without any such intermediary process, then there was no judgment, although we might perhaps say that his response manifested or was due to a belief function" (Ramsey, 1991b, 49–50 (later draft)).

[3]On perceptions, Ramsey divides them into acts and dispositions too:

> The same distinction can be applied to the problem of how far judgment is involved in perception. That a sensation causes us to act, [or leaves a trace which enables us to remember its quality] does not necessarily mean that we judged it to have a certain quality; nor is this involved in its leaving a trace which enables us to remember it afterwards. Whether indeed we could properly say that we perceived that something was so and so, whether we said to ourselves that it was or not, we have according to our definition a judgment (Ramsey, 1991b, 50 (later draft)).

[4]He makes this clear in an earlier draft (Ramsey, 1991b, 100).

These psychological expectations—the production of actions from beliefs and desires—underpin the truth-aptness of mental acts. Ramsey writes that the distinction between acts with propositional reference (propositional content) and dispositions arises from how the former are connected to surprise in action:

> The idea that a piece of conduct has one particular propositional reference only arises when one of the beliefs on which it is based turns out to be mistaken. In general the beliefs on which we act are true, but when just one of them turns out to be false, as for instance when the Union has moved, our attention is fixed on that one and our conduct condemned as erroneous in one particular respect (Ramsey, 1991b, 99–100 (earlier draft)).

Attention focuses on the behavior and in doing so produces a judgment or mental act. The psychological expectation issues the mental act. The conscious mind attends to violations of dispositions—habits—because those habits produce an expectation of satisfaction for the consequences the habit is supposed to lead to. For example, I believe it will not rain today and thus omit my umbrella when walking out the front door. But I then get wet walking to my job. This focuses my attention on the mistaken belief "It will not rain today". My surprise results in a mental act, but that act happens only because my disposition that it will not rain led to an expectation that was violated, i.e. getting wet.

It is crucial to understand that psychological expectations and the acts they produce are intimately tied to the satisfaction of desires. What makes the psychological expectation an expectation is that it anticipates a satisfaction of some desire. When that desire is frustrated, the anticipation is in error, and it reveals the content of the belief as a mental act by showing how the belief is false. This is subtle. Ramsey is pointing out that the way beliefs (considered as propositions) are false is fundamental to individuating them by the propositional content of those beliefs. One cannot merely show when the belief is true; one needs to know when the belief is false. This is a problem that bedevils contemporary, naturalistic accounts of propositional content.[5] His solution is to tie falsity to frustration or disutility in action.

To illustrate how Ramsey's theory works, it would be important to briefly discuss his theory of the truth-conditions of belief in a proposition.[6] Ramsey outlines the basic theory in "Facts and Propositions", and he adheres to this theory in *On Truth*. Basically, the idea is that the content of a proposition is its truth-conditions, and the truth-conditions of a proposition are the causes and effects that follow from the actions taken if the proposition is believed. More specifically, those causes and effects are the causes and effects on utility. Ramsey gives the example of the behavior of a chicken contemplating the proposition of a caterpillar being poisonous:

> In order to proceed further, we must now consider the mental factors in a belief. Their nature will depend on the sense in which we are using the ambiguous term belief: it is, for instance, possible to say that a chicken believes a certain sort of caterpillar to be poisonous, and mean by that merely that it abstains from eating such caterpillars on account of unpleasant experiences connected with them. The mental factors in such a belief would be parts of the chicken's behaviour, which are somehow related to the objective factors, viz. the kind of caterpillar and poisonousness. An exact analysis of this relation would be very difficult, but it might well be held that in regard to this kind of belief the pragmatist view was correct, i.e. that the relation between the chicken's behaviour and the objective factors was that the actions were such as to be useful if, and only if, the caterpillars were actually poisonous. Thus any set of actions for whose utility p is a necessary and sufficient condition might be called

[5]For example, Dretske identifies it as *the* problem for information-theoretic accounts such as his. This is sometimes described as the problem of misrepresentation (Dretske, 1988).

[6]Propositions for Ramsey are not independent entities but really short hands for what he calls the propositional reference of a belief. I will use "proposition" and "belief" interchangeably here.

	The caterpillar is poisonous.	**The caterpillar is edible.**
Eat the caterpillar.	The chicken has an upset stomach.	The chicken is satiated.
Refrain from eating the caterpillar.	The chicken avoids having an upset stomach.	The chicken missed a good meal.

Figure 3.1: A decision matrix for the caterpillar thought experiment. The columns are the proposition or state of the world. The rows are the actions. The cells are the consequence or outcomes of the states and actions. The original rendition of this matrix can be found in Sahlin, 1990, 72.

> a belief that *p*, and so would be true if *p*, i.e. if they are useful (Ramsey, [1927] 1990d, 40).

Ramsey's thought is that the truth-conditions for the chicken's belief that the caterpillar is poisonous are the facts relating the chicken's behavior now and in the past with the objective facts associated with that behavior. This is abstract so consider the decision matrix in figure (3.1) for Ramsey's chicken. The columns of the matrix correspond to the state proposition whose truth-conditions are to be defined, the rows are the actions the chicken might take, and the cells in the table are the consequences of those actions on the states. What Ramsey is saying here is that the cells determine the truth-conditions for the proposition "The caterpillar is poisonous" and its complement. These are the causes and effects alluded to earlier. They are connected with the belief by the actions taken. A belief in "The caterpillar is poisonous" is true if and only if the action the chicken takes leads to the satisfaction of desires. Here, if the chicken believes that proposition the expected desire to be satisfied would be "The chicken avoids having an upset stomach" because the chicken would refrain from eating. And the same applies to the proposition's complement, where if the chicken believes "The caterpillar is edible", it will eat the caterpillar and find itself to be satiated. So the truth-conditions of the proposition are the relations given by the decision matrix between the belief, actions, and consequences of those actions.

Crucially, it is just as important what happens if the chicken mistakenly believes the caterpillar to be poisonous or edible. In that case, the belief could be rendered false because the chicken will have missed a good meal or had an upset stomach—clearly worse outcomes by the chicken's own light. Its desires would have been frustrated. This is why Ramsey mentions the importance of prior experience in the passage: the chicken has the belief that the caterpillar is poisonous when it refrains because it previously ate a caterpillar that gave it an upset stomach or has a model that doing so would lead to an upset stomach. The truth-conditions here include more than just the success conditions of the chicken's action but also the failures the chicken would encounter should it judge poorly. A belief about something cannot be well-formed unless one knows when the belief is false.[7] This is why Ramsey emphasizes that mental acts are formed after violations of expectations. Here in this example, the expectation is given by the matrix of state, action, and outcome. This can be seen in figure (3.2), which gives the utilities of the various actions on propositions. The action is taken in anticipation of the reward given by the outcome. So that would mean that when the chicken believes the caterpillar is edible, it will eat the caterpillar and if it thinks the caterpillar is poisonous, it will refrain from eating the caterpillar. That expectation is violated when the expected outcome does not match the actual outcome; the chicken winds up with an upset stomach or having missed a delicious meal. In the above table, the false conditions are given by the alternative row in the column that is the actual state. Supposing the belief of the chicken is that the caterpillar is poisonous, it takes the action to refrain from eating given by the dark gray row. However, it finds itself having forgone at least one utility because the caterpillar is in fact edible, and so it missed the outcome given in the light gray cell. If the true truth-condition for the belief "the caterpillar is poisonous" is given by the beneficial outcome of avoiding a stomach ache due to the chicken's cautiousness, then the false truth-condition for the belief "the caterpillar is poisonous" is given by poorer outcome of missing a good meal due to refraining when the chicken could have been satiated. In short, the slogan for truth-conditions of belief in a proposition is they are the outcomes given by the action with the highest expected value for that proposition. So both true and false truth-

[7] While it has been pointed out by Sahlin and Dokic and Engel that the truth-conditions are given by the highest utility cells of the decision matrix, I believe they have omitted the importance of the cells where utility is lower (Sahlin, 1990, 72; Dokic and Engel, 2002, 14). These are the cells that fundamentally individuate propositions: they tell agents when those agents are wrong.

	The caterpillar is poisonous.	**The caterpillar is edible.**
eat	$U(upset) = -1$	$U(satiate) = 1$
refrain	$U(avoid) = 0$	$U(miss) = 0$

Figure 3.2: A decision matrix illustrating a proposition's truth-conditions as a relationship between action and utility. Here the truth-condition for the falsity of the belief of "the caterpillar is poisonous" is the outcome of missing a good meal because in refraining to eat, the chicken has an outcome different than avoiding a stomach ache. In essence, the chicken having a false belief about the caterpillar being poisonous suffers the consequence of forgoing a good meal.

conditions are the outcomes driven by the beliefs actions and the utilities over consequences. Falsity plays just as important role here as truth. Thus, Ramsey's theory for truth-conditions more importantly applies to the falsity conditions of propositions: it is baked into his model of how mental acts are produced and cognition proceeds via psychological expectations.

The picture then is that mental acts are differentiated from dispositions by their possession of propositional content. This makes them truth-apt. Their propositional content comes from their role in producing actions and connection to success and frustration in action. The production of those actions comes from a combination of those mental acts in conscious deliberation, dispositions, and desires. This result I call a psychological expectation. Psychological expectations can be violated—people can be surprised or frustrated—and this leads to the further production of mental acts. This suggests that what differentiates mental acts in terms of their conscious deliberation is their connection to how expectations can be violated.

This leaves open two questions: how are dispositions involved in the generation of psychological expectations and what is relevant about the conscious-part to mental acts in their production of expectations? I discuss these in turn.

3.2.2 Unconscious Process

Dispositions for Ramsey operate unconsciously. These are the habits that govern human behavior, and they operate below awareness; most beliefs of this sort are not consciously attended to but only arise in reaction to events. This suggests an unconscious process that governs much of human action. Here I discuss that process and its relationship to the generation of psychological expectations.

The central metaphor for the unconscious process is the automatic telephone. By automatic telephone, Ramsey is referring to the then-new telephone technology that allowed the connection of callers without an operator. These are electro-mechanical devices that proceed via simple rules or programs for connecting callers. In discussing the understanding of a sentence, Ramsey comments that the automatic telephone illustrates part of the cognitive process very well:

> The automatic telephone indeed illustrates some aspects of thought very well; ~~but not the~~ e.g. understanding the words and so the sentence, and if it were not so good it might illustrate failure to understand if you dialed too fast (FPRP Meaning and Experience, 3).

The metaphor exhibits the fact that behavior is the result of habits: "the human mind works essentially according to general rules or habits" (Ramsey, [1926] 1990n, 90). The habits are the particular network switches that can result in observed calls, i.e. the observed mental acts. They appear to be associative in the sense that they connect behaviors. So the central metaphor of the unconscious process is an automatic connection between mental acts.

These dispositions or habits have to be stored and retrieved somehow. Ramsey argues there is "secondary memory" that contains the dispositions, which is normally not directly accessed and hidden:

> There is primary memory but time could be known without it.
>
> Secondary memory is like perception: I can look into the past or not according as I like; but what I see is not chosen by me.
>
> [....]
>
> Most of our mental processes lie below a threshold, which I can always open and let them through; indeed what keeps them out is generally merely crowding of the stage of consciousness by other ideas. (often each just shows its fact but not its whole body). I know what I mean in the sense that I can always when challenged open the door and let an account of it in (FPRP Epistemology, 22).

Habits lie stored in a "secondary memory" that is sealed off from ordinary introspection. Sometimes people gain access to it. This allows people to think about the habits that govern behavior. But most of the time, it is impenetrable to perception and inaccessible.

Furthermore, the use of the name "secondary memory" is meant to separate how habits are stored from how mental acts are stored. I will discuss Ramsey's primary memory more in the next section, but here it is important to keep the two separate. One is accessible to conscious perception while the other is not. This is why Ramsey thinks of them as two separate systems.

Ramsey believes these dispositions in memory to be identifiable with brain states. They are the product of some actual fact about a person's brain, what he calls traces:

> So also in the case of the boy who knows the date of the Conquest, we must suppose his knowledge to depend on some arrangement, 'trace' or 'record' in his mind or brain, which is formed when he learns the date and persists until he forgets it, his forgetting being simply the disappearance of this trace.

> These traces, or in different cases other formations, constitute the positive qualities from which dispositional knowledge and beliefs are derived, but most of us have no idea as to what sort of structures or modifications the traces are, and take them simply as unknown causes which bring it about that if for instance we ask the boy for the date he tells us correctly. So when we are trying to explain as at present what we mean by knowledge, etc. we have no concern with the real nature of these traces but merely with the kind of thoughts or actions which they are supposed to cause. Just as in explaining the meaning of strength, we have only to explain what is meant by supporting a strain without breaking, not what properties of a body they are which enable it to support a strain (Ramsey, 1991b, 44 (later draft)).

Secondary memory is a physical process whose stored habits are some physical state in the world. The dispositions then are dependent, like the strength of a metal, on those physical states and how they work. This is a story for neurophysiology. So dispositions for Ramsey are ultimately fictions of a sort, and their guidance in behavior can be described mechanically in terms of some particular facts about peoples' brains.

The separation between primary, conscious memory and secondary, unconscious memory is also important because Ramsey believes the conscious system operates differently from the unconscious system. Namely, the unconscious process generates behavior holistically.

A person's unconscious process generates behavior from every habit stored in secondary memory. This means that habits cannot be isolated from one another when a psychological expectation is formed. Or in other words, behavior is the result of every disposition, and one cannot say that a particular habit results in a particular behavior:

> [I]t is not possible to take a piece of my conduct and regard it as having a definite propositional reference in the same way as a piece of my thinking has. Take my

> going to Bridge Street; in <doing> this we said I behave as if the Union were there, but also as if the Union had a library, and as if the book I wanted were contained in that library but in no other nearer one from which I could borrow <it>, and as if the library would still be open and so on indefinitely. My conduct is the result or manifestation of my whole system of dispositional beliefs (Ramsey, 1991b, 99 (earlier draft)).

Actions are the result of every belief that is a disposition. They cannot be isolated. Ramsey argues that this follows from the fact that psychological expectations are a product of beliefs plus desires:

> The assertion we make about [a person's] behavior is evidently a very complicated one, for no particular action can be supposed to be determined by this belief alone; his actions result from his desires and the whole system of his beliefs, roughly according to the rule that he performs those actions which, if his beliefs were true, would have the most satisfactory consequences (Ramsey, 1991b, 45 (later draft)).

Since Ramsey assumes that behavior is the result of beliefs and desires, it follows for him that every disposition has to factor into every action, i.e. every expectation. This is a fundamental thesis of Ramsey's, and the thesis dates back to his "Truth and Probability". There he makes the crucial assumption that behavior must be treated fictionally as the product of a person's beliefs and desires. Here that assumption still remains. Interestingly, he has localized this assumption to dispositions; mental acts need not contribute to every behavior. This is crucial because it is another way to separate conscious mental acts from unconscious habits: acts only selectively result in action while habits generate action collectively.

So if habits collectively produce behavior and they are stored in an inaccessible secondary memory, how do people access them?

Ramsey argues that the access comes from how dispositions issue in acts and how they affect our expectations:

> The dispositional beliefs manifest themselves in two ways: firstly by giving rise to corresponding judgments when occasion arises for making them, and secondly by governing our actions, roughly according to the rule that we perform those actions which if our beliefs were true would have the most satisfactory consequences (Ramsey, 1991b, 100 (early draft)).

The case of judgments will be dealt with further down through the use of logic, but the case of behavior is important because it highlights an important connection between the dispositions of the unconscious process and the conscious mental acts they produce: the latter surface when the former are violated.

Ramsey writes that only mental acts—the aware mental states present in the conscious process—have proper propositional reference and thus can be true or false:

> It is clear that in common language both acts and dispositions can be called true or false, and that both have in some sense propositional references. But it seems also clear that the fundamental use of true and reference is that in which they are applied to acts, for whatever is the correct account of dispositions, they must obviously be defined by reference to the acts in which they are manifested (or would be manifested if occasion arose), and the truth or falsity of the disposition arises from that of the acts and not vice versa (Ramsey, 1991b, 98–99 (earlier draft)).

The content of dispositions is derivative of the content of acts. Mental acts are true or false;

dispositions are not. This relates back to Ramsey's earlier comment that dispositions are fictions that stand-in for the unknown brain traces responsible for behavior. It also relates to the fact that dispositions collectively generate behavior; their "meaning" is dependent upon other dispositions and the particular mental acts they produce. So it follows that if they do not refer to a physical process but a fictional abstraction of some process, they have no propositional reference outside of the acts they issue.

In summary, the unconscious process consists of the dispositions or habits that collectively generate behavior. These habits are stored in an inaccessible secondary memory, whose physical implementation is some trace in the brain. Because Ramsey subscribes to the theory that actions and expectations result collectively from beliefs and desires, he thinks that these habits holistically produce behavior. They cannot be isolated from one another; they are not truth-apt.

This tells a particular story about how dispositions aid in the generation of psychological expectations and behavior. Dispositions collectively factor into expectations in a way that particular mental acts may not. So the first question is answered: dispositions produce expectations only together.

3.2.3 Conscious Process

I now need to answer the second question what makes a person's awareness of his mental acts relevant for decision-making. Ramsey argues that mental acts factor at least selectively into expectations. How is this different from dispositions? After all, Ramsey wants to separate a mental act from a disposition by its role in generating a specific psychological expectation. Every disposition is involved with every action and expectation. So what exactly makes acts different from dispositions? The answer is that they play an important role in the deliberation performed by the conscious process.

The conscious process involves the mental acts identified by violations of our expectations. It is these mental acts that Ramsey thinks are the subject of the beliefs that have truth and propositional reference. By propositional reference, Ramsey means what the belief is about:

> Now whether or not it is philosophically correct to say that they [beliefs] have propositions as objects, beliefs undoubtedly have a characteristic which I make bold to call *propositional reference.* A belief is necessarily a belief that something or other is so-and-so, for instance that the earth is flat; and it is this aspect of it, its being "that the earth is flat" that I propose to call its propositional reference (Ramsey, 1991b, 7).

Because dispositions can only be identified through the violation of expectations and the creation of a mental act, the disposition is said to have propositional reference and truth or falsity derivatively. This can be seen in the prior section because dispositions are to be taken as a whole; they cannot be identified in any individual behavior but only become discernible when the mind reflects on them relative to a goal. When goals are frustrated, as when expectations are violated, the mental act that results can be then used in a deliberative process to identify the derivative content of the dispositions that led to the frustrated goal.

The conscious process is inherently a deliberative process. It is important, however, to state that mental acts need not be the result of deliberation. They just need to be involved somewhere in a deliberative process, whether they register the initial violation of expectations or subsequent reasoning over that violation. This makes mental acts the bearers of the primary system propositions. Ramsey lists a number of items he considers to be knowledge. Since knowledge for Ramsey is just a species of mental act, mental acts can then be direct in the sense they do not require argument as in the case of "perception, memory and insight into abstract truths" (Ramsey, 1991b, 59) or they can be indirect in the sense that they do require explicit argument (Ramsey, 1991b, 57).

The mention of memory is important because the conscious process has its own memory through which mental acts are stored. This is the so-called "primary" memory discussed earlier. Those contained in memory are binned in the past, which becomes important for the process of deliberation due to deliberation's connection to action and cause and effect. Importantly, unlike secondary memory, primary memory is consciously accessible and can be easily brought to attention during deliberation. It is its accessibility that makes it the primary memory and repository of mental acts.

With primary memory, deliberation can proceed. Critically, the point of deliberation is recalibration of psychological expectations; these expectations are the determinant for actions. So deliberation is done for the purpose of action.

Deliberation is done in a manner so as to generate laws to act by, which come to form the basis of dispositions. Ramsey writes that "when we deliberate about a possible action, we ask ourselves what will happen if we do this or that" (Ramsey, [1929] 1990e, 154). It is in this process of deliberation that people form the habits based on what propositions they can make true, i.e. what actions can lead directly to mental acts.[8] The conscious process deliberates based on the mental acts stored in memory and given through perception, which enables it to know what is and is not settled. The settled propositions are found in the acts in the memory, those in the past. The not settled propositions are anything not in the memory. By surveying the possible laws that would show how unsettled propositions might follow from the settled ones in memory, behavior can be adjusted through the adoption of those rules that terminate in future desired acts.

[8]This is probably why Ramsey seems to subscribe to the now-named Ramsey Thesis:

> This seems to me the root of the matter; that I cannot affect the past, is a way of saying something quite clearly true about my degree of belief. Again from the situation when we are deliberating seems to me to arise the general difference of cause and effect (Ramsey, 1991b, 158).

He thinks that one's credences about the past should not change based on what action one decides to perform. People are essentially future directed. I will discuss the Ramsey thesis more in a footnote in a future chapter.

The laws adopted following deliberation are initially a mental act—an act to choose to adopt a law—that eventually through practice is formed in secondary memory. Ramsey does not have an explicit theory for how this proceeds, but the outline goes like this. A mental act such as a judgment that a law is correct leads to a conscious sequence initially when that rule is deployed. This is partly what Ramsey means by how acts factor into decision-making; an action has to be conducive to actions by controlling future actions in some respect. By repetition, this deliberative, conscious act can eventually be done unconsciously as a fully formed habit. Slowly over time, those habits are built up and stored in memory. From there, they factor along with other dispositions in forming psychological expectations and actions. So initially laws require deliberate acts, and they eventually are subsumed in secondary memory.

By reflecting on what can and cannot be settled by mental acts, deliberation changes dispositions. In particular, the dispositions that led to a violation of expectations and the resulting mental act can be adjusted based on this reflective process. This is to gain a measure of self-control. Ramsey writes that

> Self-control in general means either
>
> (1) not acting on the temporarily uppermost desire, but stopping to think it out; i.e. pay regard to all desires and see which is really stronger; its value is to eliminate inconsistency in action;
>
> or (2) forming as a result of a decision habits of acting not in response to temporary desire or stimulus but in a definite way adjusted to permanent desire (Ramsey, [1928] 1990j, 99).

The key idea is the formation of habits in (2) and that can only be done through the first process (1). The conscious process receives and stores violated expectations as mental acts. Those mental acts have propositional reference, which allows us to identify the content of the rules that produced the expectation. A person stores those acts in memory, which allows for the difference to arise in awareness between those propositions that are settled and those that are not. This enables a deliberative process that can identify new habits to replace those that led to the violated expectation. If this process is fully general, then it is logic.

3.2.4 Summary

Summarizing Ramsey's model of cognition, he distinguishes between two fundamental types of cognition: mental acts and dispositions. Acts are distinguished from dispositions via their role in conscious deliberation and their specific contribution to individual actions. Collectively, acts and dispositions result in psychological expectations. Dispositions work together to produce every psychological expectation, but acts only contribute to specific expectations. Dispositions are largely inaccessible, except when expectations are violated and actions are frustrated. They dwell in a secondary memory that cannot be introspected. In contrast, acts are registered whenever there is a violation of expectations, and they reside in conscious memory. People can deliberate over those mental acts. They can then use deliberations to adopt new habits, which eventually are transmitted into the unconscious memory.

One way to understand Ramsey's model of cognition is as a primitive version of a two process theory. System one consists of quick-acting, unconscious habits that generate the lion-share of a person's behavior. System two amounts to a slower, conscious executive control of behavior by contemplating how experiences require a change in behavior. Behavior is produced jointly as a function of habits and desires in system one, and system one's outputs are modified by the active involvement of system two. The result of this process is the psychological expectations. Importantly, system two has an attention system that is only activated when those expectations are violated. When attention dwells on a violated expectation, it can use it to intervene in future expectations. This is what imbues those violations with propositional content: their ability to be utilized in deliberation for behavior control.

3.3 Logic as Self-Control

Ramsey's model of cognition is a two-process system where one system produces behavior through associations stored in the brain and the other system modulates behavior by intervening on those associations in response to frustrated desires. This second system of executive control can be more or less successful at modulating behavior. How successful it is depends on what procedure it follows. Some procedures are better than others in the sense that they apply to more cases. The most general case is one where following the procedure for fixing behavior has guarantees. Logic is the most general method or collection of methods. So logic is a method of self-control at the most general.

The goal of this section is to argue that Ramsey believes that logic is the most general method of self-control. The logic he proposes is exactly his decision theory. And the final goal of this decision theory is to regiment psychological expectations as mathematical expectations.

For Ramsey, self-control comes in two varieties. It is either pausing to deliberate or enforcing habits decided on previously:

> Self-control in general means either
>
> (1) not acting on the temporarily uppermost desire, but stopping to think it out; i.e. pay regard to all desires and see which is really strong; its value is to eliminate inconsistency in action;
>
> or (2) forming as a result of a decision of action not in response to temporary

> desire or stimulus but in a definite way adjusted to permanent desire.
>
> The difference is that in (1) we stop to think it out but in (2) we've thought it out before and only stop to do what we had previously decided to do (Ramsey, [1928] 1990j, 99)

Ramsey holds self-control to consist of two parts. The first is the act of deliberation on how to make one's action coherent; the second is to pause in acting so as to follow the plan outlined in the first part. I characterize the first as *finding a regimentation* and the second as *acting out the regimentation*. By regimentation, I mean a series of choices that differ from existing choices. One can think of it as something like what is called a trigger-action plan: when presented with a specific trigger, perform this action instead of what naturally occurs.

Logic applies in both finding a regimentation and acting out a regimentation.

Self-control through deliberation needs a guide for deliberation. The desired guide better work in the sense that it applies across all possible cases one might encounter. Logic aids here because of its generality:

> So also logic enables us
>
> (1) Not to form a judgment on the evidence immediately before us, but to stop and think of all else that we know in any way relevant. It enables us not to be inconsistent, and also to pay regard to very general facts, e.g. all crows I've seen are black, so this one will be—No; colour is in such and such other species a variable quality. Also e.g. not merely to argue from $\phi a \,.\, \phi b \ldots$ to $(x).\phi x$ probable, but to consider the bearing of $a, b \ldots$ are the class I've seen (and visible ones are specially likely or unlikely to be ϕ). This difference between *biassed* and *random* selection (Ramsey, [1928] 1990j, 99).

Ramsey's point is that logic aids in finding all the relevant propositions to ensure one is consistent. This includes both singular propositions and general propositions. This includes not merely inductive inference on the general propositions one believes but also finding the right reference class for observed propositions.

The most important point here is the claim about consistency. By consistency, Ramsey includes deductive consistency and probabilistic coherence. The generality of logic helps here because it aids in going from particular instances to variable hypotheticals. Since deliberation requires adjusting habits, this means one needs to identify problematic general propositions. That can only be done by thinking in general terms, which logic allows one to do. After all, the point of the deliberative process is to settle on new habits, general propositions, to adopt. So generality is required for successful regimentation and logic provides generality.

Once a new set of general propositions are adopted in deliberation, the corresponding habits need to be implemented in behavior. Logic aids here as well:

> (2) To form certain fixed habits of procedure or interpretation only revised at intervals when we think things out. In this it is the same as any general judgment; we should only regard the process as 'logic' when it is very general, not e.g. to expect a woman to be unfaithful, but e.g. to disregard correlation coefficients with a probable error greater than themselves (Ramsey, [1928] 1990j, 99)

Ramsey's point here is that when acting out the regimentation, the act needs to be the same across any successive regimentation. I am following logic in sticking to my agreed upon habits when I have a general procedure—with a corresponding general proposition—I adhere to as I slowly nudge my behavior to incorporate the desired habit. Logic provides the generality necessary to provide guidance for the process of regimentation.

It should be emphasized that this view of logic requires logic to provide *dynamic* guidance of behavior. Beliefs must be continually monitored across time to ensure they stick to the agreed plan of regimentation. So the logic here needs to have generality both for abstracting the rules of that regimentation across its instances and for governing behavior across time. This means that the logic here has to be diachronic.

Contemporaneous notes by Ramsey accentuate his trend to viewing logic as more expansive than deductive logic and even his synchronic decision theory from “Truth Probability”. In the “Weight or Value of Knowledge” manuscript (see Ramsey, 1991a, 285–287), Ramsey proves a theorem that justifies why it is better to be more informed when making a decision as opposed to less informed. As Skyrms discusses, this result and another indicate that Ramsey was aware of the importance of what is now called probability kinematics (Skyrms, 1990, 93–96). It also points to Ramsey’s interest in developing a diachronic logic. The aforementioned need to enforce regimentation across multiple acts would require a theory of logic more substantial than deductive logic.

The upshot is that logic for Ramsey is decision theory. It is a decision theory that can provide the most general tools for both finding a regimentation and also acting out the regimentation.

Logic for Ramsey then must be normative in the sense that it prescribes how to change behavior without describing actual behavior. When deliberating, I use logic to theorize about my habits, my psychological expectation, and the violations of that expectation. So far, I have described deliberation as a descriptive, psychological process. With logic, I consider my deliberation as an approximation to the ideal process given by logic. This requires me to make fictitious, theoretical assumptions about how I in fact decide on action. Logic goes beyond my own actual behavior by focusing on the habits and rules I have buried somewhere in my unconscious process. I postulate fictional propositions that stand in for those habits. After all, how is a habit evaluated for its efficacy in accomplishing my goals? It is not really a proposition, but logic would demand it to be one. These fictions are general propositions, which I discuss in a later chapter. Their fictionality means that when I use decision theory to guide my regimentations, I am not really describing how my actual behavior works.

This makes Ramsey's later views on logic different from his earlier views. And he says as much. His decision theory, which he took to be more descriptive in "Truth and Probability", cannot be viewed that way. It would be meaningless to do so:

> The defect of my paper on probability was that it took partial belief as a psychological phenomenon to be defined and measured by a psychologist. But this sort of psychology goes a very little way and would be quite unacceptable in a developed science. In fact the notion of a belief of degree $\frac{2}{3}$ is useless to an outside observer, except when it is used by the thinker himself who says 'Well, I believe it to an extent $\frac{2}{3}$', 'I have the same degree of belief in it as in $p \vee q$ when I think p, q, r equally likely and know that exactly one of them is true.' now what is the point of this numerical comparison? how is the number used? In a great many cases it is used simply as a basis for getting further numbers of the same sort issuing finally in one so near 0 or 1 that it is taken to be 0 or 1 and the partial belief to be full belief (Ramsey, [1929] 1990i, 95).

Ramsey admits that he had made a mistake in "True and Probability" treating partial belief as a psychological phenomenon. It is not. He argues that credences are meaningless to an observer except maybe to compare to their own degrees of belief and the logical structure of those personal credences. An observed credence only makes sense relative to one's own credences. For example, Jones hearing that Smith believes it will rain to credence two-thirds, can only assess what that means in terms of his (Jones's) own personal credence assignments of two-thirds. Those credences are just the bets Jones's would force himself to take to best optimize for his goals. It is not a descriptive fact of the other person, but a regulative feature of what he would do should he try to be coherent.

The meaning of credences is thus personal in the sense that it only has sense in the context of introspection. That introspection is just the conscious process of adjusting psychological expectations. Ramsey says this is practical decision-making, and the recommendation of logic is the mathematical expectation:

> But sometimes the number [credence] is used itself in making a practical decision. How? I want to say in accordance with the law of mathematical expectation; but I cannot do this, for we could only use that rule if we had measured goods and bads. But perhaps in some sort of way we approximate to it, as we are supposed in economics to maximize an unmeasured utility. The question also arises why just this law of mathematical expectation. The answer to this is that if we use probability to measure utility, as explained in my paper, then consistency requires just this law (Ramsey, [1929] 1990i, 95).

Ramsey claims that the practical use of credences comes through using them to determine behavior "in accordance with the law of mathematical expectation". That is, one should want a decision to behave as a mathematical expectation, a sum of utilities weighted by their probabilities. However, this is impossible since a person will lack a measure of his utilities. The solution is to approximate: some utilities are chosen through preference over a limited selection of worlds. Those utilities are then weighted by the probabilities—a mathematical expectation.

The thought is that instead of considering all possible options, only a limited selection is chosen and a rough preference order is assigned over them. This corresponds to assigning a utility that is within some epsilon of a true utility function. Then an approximate expectation is selected.

Ramsey argues this must be done to ensure consistency (probabilistic coherence). The desirability of consistency is that it prevents Dutch books. And this applies across any decision. So the use of mathematical expectation in practical decision-making is a part of logic.

This then is a normative account of logic. The core prescription that Ramsey thinks logic provides, among other things, is the regimentation of decisions in accordance with mathematical expectation. This will by necessity be an approximation. But it will work well enough when finding a regimentation and when acting on a regimentation. Logic is then a form of self-control that people use to steer action to accomplish goals. Agency is not a description but an ideal people approximate when aiming at their goals.

3.4 Forecasts

I have been reconstructing Ramsey's theory of cognitive psychology and philosophy of logic. These two accounts are complimentary; Ramsey's philosophy of logic is premised on a particular model of human cognition. The goal is to use these two to figure out how Ramsey's philosophy of science is a forecasting theory. The answer is that a forecast is just a regimentation of psychological expectations as mathematical expectations. A forecasting theory of science is a theory that describes how science should augment decision theory for regulating expectations.

There are several pieces in play at this point: a two-process theory of cognition, psychological expectations as the product of that two-process theory, logic as the most general method for self-control, decision theory as logic applied to expectations, and mathematical expectation as the guidance that decision theory provides for directing behavior. These all fit together to recommend a theory of self-control.

Ramsey's core conception of any philosophy is that it must aid in clarifying thought and so guide action.[9] A philosophy of scientific theories must do this. How can any philosophy elucidate thought and so better action? This requires a theory of thought and its connection to action. Ramsey's two-process theory does that: it shows how beliefs as dispositions and as mental acts collaborate along with desires to produce actions. Those actions are psychological expectations about the fulfillment of desires and goals. So for philosophy to better thought and action, it must help in controlling dispositions, mental acts, and psychological expectations.

What does it mean to clarify thought and guide action? Ramsey's psychological theory says it means aiding the conscious process in deliberation and follow-up. However, this aid must be general in the sense that it applies to any possible scenario a human might encounter. A core feature of logic is its association with such absolute generality. This means that logic can be applied as a method of self-control by finding regimentations and acting on those regimentations. Consequently, for philosophy to illuminate thought and action, it must be a form of logic implemented by the conscious process.

The recommendation of logic for thought and action is given by decision theory. Decision theory says a person's behavior should be a mathematical expectation on some probability and utility function. Now this cannot be achieved in practice because a utility function cannot be found via introspection over every possible hypothesis. Instead, people can approximate their utilities by looking at a reduced hypothesis set and simulating their preferences over that set. This results in an approximate mathematical expectation as logic's recommendation for the correct regimentation of psychological expectation. Consequently, a philosophy of science must show how science can help the conscious process regiment psychological expectations as approximate mathematical expectations.

It would be useful to go into a little more depth with this idea. Since logic for Ramsey

[9]He writes clearly in 1929 that "Philosophy must be of some use and we must take it seriously; it must clear our thoughts and so our actions. Or else it is a disposition we have to check, and an inquiry to see that this is so; i.e. the chief proposition of philosophy is that philosophy is nonsense" (Ramsey, [1929] 1990h, 1).

is decision theory, a forecast is the logical analog to the psychological expectation, and a psychological expectation is the result of dispositions and prior mental acts together that produces further mental acts, then the forecast has to be the mathematical expectation that produces acts in decision theory. This fits nicely with the claim that Ramsey makes from "Truth and Probability" that with decision theory I hypothesize that my behavior is the product of my beliefs and desires:

> I propose to take as a basis a general psychological theory, which is now universally discarded, but nevertheless comes, I think, fairly close to the truth in the sort of cases with which we are most concerned. I mean the theory that we act in the way we think most likely to realize the objects of our desires, so that a person's actions are completely determined by his desires and opinions (Ramsey, [1926] 1990n, 69).

Ramsey regiments this psychological theory with the idea that those beliefs and desires produce acts by the mathematical expectation:

> I suggest that we introduce as a law of psychology that his behaviour is governed by what is called the mathematical expectation; that is to say that, if p is a proposition about which he is doubtful, any goods or bads for whose realization p is in his view a necessary and sufficient condition enter into his calculations multiplied by the same fraction, which is called the 'degree of his belief in p' (Ramsey, [1926] 1990n, 70).

Eliminating the talk here that this is a psychological law instead of a logical law, the idea is that I treat my psychological expectations as approximate mathematical expectations in the process of deliberation. That deliberation allows me to isolate my credences in propositions, based on my utilities ascribed to those propositions. Utilities are approximated subjectively through preferences over gambles in a case of limited options. This is not psychological but reflective: I am seeing how hypothetical actions I would take cohere together. Through this reflective process, I use logic to successively regiment my expectations as mathematical expectations, and I can rely upon the same approximations with successive decisions to ensure the original expectation is adhered to. So Ramsey's cognitive psychology makes mathematical expectations the natural recommendation of logic for how to govern psychological expectations.

The end result is a forecast with credences and utilities. In the case where only what is true matters, those utilities can be treated as the indicator function:

$$
\begin{aligned}
\mathbb{E}[P \mid E] &= \Pr(P \mid E)I(P) + (1 - \Pr(P \mid E))I(P^c) \\
&= \Pr(P \mid E)(1) + (1 - \Pr(P \mid E))(0) \\
&= \Pr(P \mid E)
\end{aligned}
$$

where E is the conjunction of any observed propositions, and $I(\cdot)$ is the indicator function that returns 1 if the proposition is true and 0 otherwise. The upshot is that the expectation on this particular utility function returns the probabilities or exact predictions a coherent person would return. In short, logic regiments beliefs through the mathematical expectation.

These regimentations are forecasts. And a philosophy of science is a forecasting theory precisely in the sense that it shows how science factors into the production of forecasts; it is a normative theory for action. So what do scientific theories provide?

Scientific theories provide laws and chances. I will say more on what laws and chances are in a later chapter. But here I want to briefly argue that in Ramsey's theory of cognition, a law or chance is nothing more than a habit. They are the logical rendition of the dispositions in the unconscious system largely responsible for psychological expectations. Ramsey says laws are habits in "General Propositions and Causality" when discussing a particular law "all men are mortal":[10]

> To believe that all men are mortal—what is it? Partly to say so, partly to believe in regard to any x that turns up that if he is a man he is mortal. The general belief consists in
>
> (a) A general enunciation, (b) A habit of singular belief.
>
> These are, of course, connected, the habit resulting from the enunciation according to a psychological law which makes the meaning of 'all' (Ramsey, [1929] 1990e, 148–149).

Laws are habits. So from the above model of cognition, they are the components of the unconscious system that is mostly responsible for psychological expectations. Logic treats these habits as general propositions. Consequently, scientific theories produce general propositions, which are fundamentally habits, and these habits govern behavior through psychological expectations.

The picture provided here is one where a forecasting theory produces forecasts. Forecasts are regimentations of psychological expectations as approximate mathematical expectations. A forecasting theory of x shows precisely how x can produce forecasts. In the case of science, Ramsey's forecasting theory aims to show how scientific theories produce laws, and how those laws are to be used in the production of forecasts.

It remains to be argued how my story fits with the available evidence. I turn to that now.

The principal evidence is the passage I cited at the start of this chapter. In "General Proposi-

[10] Ramsey uses the phrase "law", "variable hypothetical", and "universal proposition" interchangeably throughout "General Propositions".

tions and Causality", Ramsey describes his philosophy of science as a forecasting as opposed to descriptive theory:

> As opposed to a purely *descriptive* theory of science, mine may be called a *forecasting* theory. To regard a law as a summary of certain facts seems to me inadequate; *it is also an attitude of expectation for the future* [emphasis mine]. The difference is clearest in regard to chances; the facts summarized do not preclude an equal chance for a coincidence which would be summarized by and, indeed, lead to a quite different theory (Ramsey, [1929] 1990e, 163).

Two observations should be made. First, Ramsey explicitly talks about laws as "an attitude of expectation for the future". This is very close to what I have proposed where laws are habits instrumental in the formation of psychological expectations. The "expectation" Ramsey uses here has to be psychological expectations. Second, Ramsey contrasts his theory with a "descriptive" theory. He would only do this if his theory is meant to be normative or prescriptive. This fits closely with his view of logic as a prescriptive method of self-control; his philosophy of science is an application of logic toward how science should factor into regimenting decisions. So there is strong evidence here that for Ramsey forecasts are connected with psychological expectations, and forecasting is a prescriptive application of logic to decision-making.

Ramsey uses the phrase "forecast" in two other places. The first occurs in his notes on "Solipsism". He argues against people being automata because he uses his own experience to forecast their behavior:

> I do not believe other people are automata; for I use my experience to forecast their action, and to eliminate experience from this process of inference and recast it in terms of unknown bodily states would be too far fetched. Is X an automaton

> is apt to seem an absurd question? but not as meaningless but simply because the answer is no (unless there is reason to think so). If I made a man I should suppose him to have consciousness by same cause same effect unless there were reason to contrary (Ramsey, 1991a, 68).

Ramsey associates forecast here with a process of inference. This fits well with the idea that forecasts are a type of psychological expectation. Here that expectation must factor in his own conscious experiences. Furthermore, the forecast here applies a principle of same-cause same-effect; a principle that Ramsey's comment indicates should be one that people adopt. So, a forecast is a prescriptive formulation of expectations.

The second use in his notes is the quoted passage from earlier connecting forecasts with psychology:

> Question. What is the meaning in test of acquaintance?
>
> Suggestion The fundamental proposition is the forecast then the memory (FPRP Existential Judgment, 6).

Here Ramsey addresses the meaning of the test of acquaintance. He suggests cryptically that the fundamental proposition happens to be forecasts and then memories. One thing that should be noted immediately is that the question is not the meaning of acquaintance but the test of acquaintance. Ramsey uses a similar phrase in "Theories", when he considers an alternative way of defining theoretical propositions: "the meaning of a proposition about the external world is what we should ordinarily regard as the *criterion* or *test* of its truth" (Ramsey, [1929] 1990m, 122–123). The association of "test" with "criterion" in the meaning of a proposition suggests the same association here. That means that Ramsey is asking what is the meaning of the criterion of acquaintance. This is a prescriptive question: what does it mean for something to count and not count as a criterion or test for being acquainted?

My account provides an answer here: it is how the criterion regiments expectations, i.e. affects the production of forecasts. This is fundamental to how any recommendation of logic should affect behavior; the psychological expectation is fundamental and so the prescription of changing expectations is fundamental. So the note here fits nicely with the theory I have provided.

In summary, forecasts are regimentations of psychological expectations as mathematical expectations. When those regimentations are done with respect to the goal of finding the truth, the forecast becomes a regimentation of beliefs as probabilities. A forecasting theory of philosophy is one that is focused on using logic to show how the philosophical topic can be made general to produce forecasts. In the case of the philosophy of science, this shows how theories and their laws can be used in regimenting behavior as mathematical expectations. Ramsey has provided a fundamentally normative or prescriptive theory of science. This account fits the sporadic uses throughout Ramsey's notes.

3.5 Conclusion

Ramsey mysteriously describes his philosophy of science as a forecasting theory. However, he fails to say what he means by a "forecasting theory" or a "forecast". The limited use of the term in his notes connects it to a model of human cognitive psychology.

That model is at root a two-process system that produces what I call psychological expectations. A psychological expectation is the product of a person's habits, prior mental acts, and desires and the expectation can produce further mental acts when it is violated. The habits that factor into that expectation act collectively in an inaccessible unconscious system. Mental acts are violations of expectations, which means those expectations are properly true or false, and through deliberation they have a localized role in formulating future expectations.

This reflective, conscious system can then generate new habits that allow the person to be more successful at accomplishing his goals. It is this executive control that allows an agent to reason about what methods would best enable deliberation to arrive at an effective new group of habits.

The most successful method of choosing habits comes through logic. The generality of logic allows it to both aid the conscious system in choosing a regimentation of expectations and in sticking to that regimentation through successive actions. Ramsey recommends the logic to apply in both cases is his decision theory, and that decision theory suggests that psychological expectations must be approximations of mathematical expectations. So logic is a means of self-control by adapting psychological expectations to mathematical expectations.

When a psychological expectation is regimented as a mathematical expectation, it becomes a forecast. A forecasting theory is then a philosophy that shows how its subject matter aids in the production of forecasts; it is a branch of logic, and therefore, a prescriptive guide to action. Ramsey's philosophy of science is precisely this because it shows how scientific theories and their laws can be used to regiment psychological expectations as mathematical expectations. What remains to be shown at this point is how exactly that works.

So far in the dissertation, I have reconstructed Ramsey's decision theory to show how singular, theoretical propositions can be wagered, i.e. have credences formed over them. The next step is to say what exactly are laws for Ramsey because the content of a theory is somehow expressed by its laws and laws can occur also in the theory proper. But the problem is that say anything about laws, I have to explain what is the purpose of laws. Ramsey connects laws to forecasts, and I now have an account of forecasts. The next step is to explore in more detail how laws operate for Ramsey. Once laws are understood, I can then proceed to the Ramsey sentence and its relationship with the laws. This will enable me to address the extent to which Ramsey is a scientific realist or anti-realist.

Chapter 4

Ramsey's Laws

4.1 Introduction

So far in the dissertation, I have provided a reconstruction of Ramsey's decision theory compatible with his later philosophical views, described how singular theoretical propositions are wagered in gambles, and provided an account of Ramsey's cognitive psychology that addresses what a forecast is and how Ramsey's philosophy of science should produce forecasts. An important component of that psychology model is the universal propositions and laws due to the laws acting as regimentations of the habits that guide behavior. These are also important in Ramsey's philosophy of science because according to Ramsey they and the consequences of the theory are what "our theory asserts to be true" (Ramsey, [1929] 1990m, 115). Previously, I have punted on addressing what universal propositions and laws are in the context of Ramsey's decision theory. Since they are important, an account of them that fits with Ramsey's theory of cognitive psychology, philosophy of logic, and decision theory must be given. I turn to that now.

In this chapter, my goal is to provide a complete and precise account of laws in Ramsey's philosophy of science. Ramsey provides two accounts of laws that he thinks are distinct, where a law is a universal proposition such as "All men are mortal". The first holds laws to be the axioms and consequences of those axioms in a complete and best system of all propositions that could be known by an omniscient intellect. On this view, laws and with them universal propositions are infinite logical products; they are truth-functions of other propositions. The second holds laws to be rules for judgment in the sense that laws act as inference tickets between premises and conclusions. A pressing question is whether these two accounts of laws are one and the same. Ramsey is seemingly committed to Peirce's conception of truth where propositions are true just in case they are believed at the limit of inquiry. This would imply the two accounts are identical because what is believed by an omniscient intellect should seemingly coincide with whatever rules are logically entailed by the facts at the limit of inquiry. This question can be resolved by showing the two accounts are different. The key insight is that Ramsey came to believe that laws are limiting cases of chance propositions, where chance propositions are logically compatible with every truth-function—finite or infinite—of factual propositions. Chances are systems of credences that agents approximate when making forecasts; this makes laws fail to supervene on factual propositions. So an omniscient intellect at the limit of inquiry could not determine the laws as a deductive fact, which the first account of laws would have them be. Furthermore, Ramsey holds that the correct method of learning chances entails that agents will eventually converge on a unique system of chances with enough observation. This explains Ramsey's seeming commitment to the pragmatist theory of truth, but it avoids assimilating the second account of laws to the first because it is a limited commitment to the methods used in learning chances. Critically, this method of learning chances forces Ramsey to rely upon the Principle of Indifference, which he rejects in earlier work. I suggest removing the use of this principle, and I argue it should be replaced with normal Bayesian conditionalization on the chances. This would make Ramsey's theories more compatible with his overall philosophical program.

This is a long and complicated topic. A natural place to begin is to provide a cursory introduction to Ramsey's two accounts of laws and to examine how Ramsey's apparent commitment to Peirce's conception of truth threatens to collapse those accounts into one and the same.

I start with Ramsey's first account of laws as the axioms in the best system of propositions. Importantly, this account presumes a theory of universal propositions as infinite logical products or conjunctions; laws are just a kind of universal proposition. Ramsey supposes there exists an omniscient intellect—someone who knows every particular fact. Such an intellect would still need to organize its knowledge into a deductive system. Preference would be given to deductive systems that are simpler. In those simpler systems, some of that organized knowledge is universal propositions that serve as axioms or the deductions of other axiomatic universal propositions. These axiomatic universal propositions are the laws along with the universal propositions that follow from them. Thus, laws are infinite logical products that serve as the axioms or consequences of those axioms in the best deductive system of propositions known to an omniscient intellect.

There are three key features of Ramsey's first account of laws as the axioms in the best deductive system of propositions known by an omniscient intellect. First, laws and universal propositions more generally are understood to be infinite logical products or conjunctions. This makes them fundamentally truth-functional; the truth of a universal proposition is just determined by the truth of its arguments. Second, laws are purely factual in the sense that they are determined by the world independent of anyone's beliefs. They are cognitive in the sense that they are about objects and facts in the world. Third, laws fulfill a deductive role in the system in the sense that once all particular propositions are settled, the laws follow as a logical consequence of those propositions. From the omniscient intellect's known particular propositions, it can infer as a deductive fact all universal propositions. The only thing left to do is simply settle what laws are the axioms or the consequences of axioms, which is done based on the deductive simplicity of the overall system.

Ramsey's second account of laws has laws that act as inference tickets. A big change with this account is that universal propositions are no longer infinite logical products; instead, a universal proposition is a rule for inferring one proposition from another. For example, the universal proposition "all men are mortal" is really the rule for when presented with a man, to infer that the man is mortal. Ramsey calls these "rules for judging" or "variable hypotheticals". A law is simply one such rule that has *ceteris paribus* conditions baked into its antecedent. These laws and other variable hypotheticals guide behavior as essentially habits, the inference is nothing more than a habit to associate one proposition with another.

Like the law as axioms in the best system account, there are three key features of Ramsey's second account. First, laws and universal propositions are not really propositional but inference tickets. They fail to be truth-functions of propositions or elementary propositions, and they are something else entirely. Second, laws as rules for judging are non-factual in the sense the world remains undecided on their truth. They are non-cognitive because they fail to depict objects and facts in the world. Third, laws in this account provide an important guiding behavior as inference tickets. They allow people to go from one proposition to another without relying upon the logical connections between those propositions.

An important unresolved riddle from secondary source discussions of Ramsey's theory of universal propositions is the extent to which his rules for judging account is the same as his best systems account. Ramsey declares they are different in "General Propositions and Causality":

> I, therefore, put up a different theory by which causal laws were consequences of those propositions which we should take as axioms if we knew everything and organized it as simply as possible in a deductive system.
>
> What is said above means, of course, a complete rejection of this view (for it is impossible to know everything and organize it in a deductive system) and a

> return to something nearer Braithwaite's (Ramsey, [1929] 1990e, 150).

However, multiple authors have noted that later in "General Propositions and Causality", Ramsey appears committed to Peirce's conception of truth:

> We do, however, believe that the system [of laws] is uniquely determined and that long enough investigation will lead us all to it. This is Peirce's notion of truth as what everyone will believe in the end; it does not apply to the truthful statement of matters of fact, but to the 'true scientific system' (Ramsey, [1929] 1990e, 161).

If this is correct, then despite Ramsey's declarations it would appear that his first account of laws and his second account are one and the same. Should some set of rules for judging be the ones that inquirers are fated to all adopt, then they would form the best system to account for all the facts. There would be no more facts to discover, and so the laws would be just propositions after all. The only difference between them and other propositions is that they could act as the axioms in this fated best system. So the riddle has two parts: are Ramsey's two accounts of laws one and the same, and if not, what is his apparent commitment to Peirce's concept of truth?

To be clear about this riddle, more can be said about what exactly is Peirce's concept of truth in this context and how precisely it would collapse Ramsey's two accounts into one and the same.

Informally, Peirce's account of truth is what is true is what is fated to be believed. A proposition is true just in case it is believed by the community of inquirers after inquiry is finished, where inquiry is the process of science. Sometimes this is glossed as what is true is what is believed at the limit of inquiry, and Ramsey follows this characterization by describing it as "what everyone will believe in the end". One important wrinkle is that what is true is not what will in fact be believed, but what would be believed were science allowed to continue. It is the idea that if enough experiments were permitted, the community would eventually settle on belief in a unique set of propositions. Importantly, this is a definition of the truth of propositions: it specifies the truth of a proposition (the definiens) in terms of a descriptive fact about the epistemology of the community of inquirers (the definiendum).

A natural question is to ask what is the relationship between the Peircian account of truth and Ramsey's omniscient intellect. The immediate answer is they are the same because the omniscient intellect knows what is fated to be believed. The limit of inquiry is just the position that Ramsey's omniscient intellect would be in: it knows the truth value of every proposition and so every experiment inquiry could execute.

Since the omniscient intellect is in the same position as Peirce's hypothetical community of inquirers, Ramsey's commitment to the latter would appear to collapse Ramsey's account of laws to his best system. It does this by assimilating inference tickets to propositions and deciding the correct laws by deductively entailing them from known propositions.

The Peircian account would say that the true laws are the ones adopted by the community of inquirers at the limit of inquiry. Naturally, these would be the laws accepted once every proposition is settled; the true laws are just those scientists converge on once every possible experiment has been run and every fact adjudicated. Importantly, these laws if they are inference tickets would track certain material conditionals because the believed ones would never be violated. Those inference tickets could never issue propositions that have not been settled since by hypothesis all propositions are known. In short, one could define an inference ticket as believed by the community of inquirers if and only if the corresponding conjunction of material conditionals is believed by the community of inquirers. The omniscient intellect knows the same things as the community of inquirers at the limit of inquiry. So these inference tickets will also be equivalent to the corresponding logical products of material conditionals for the omniscient intellect.

Once assimilated to conjunctions of material conditionals, it is easy to see that the correct laws are simply the ones deductively entailed by the settled factual propositions. At that point, one has the view that the inference ticket view would just be equivalent to the axioms of a deductive organization of all knowledge. The two accounts are one and the same.

This changes the features of the laws as rules for judging. Recall that those three features were that laws are not really propositions, laws are undecided by the world, and laws provide guiding behavior for people through non-deductive means. Now, laws are equivalent to infinite conjunctions of material conditionals, which makes them propositions; they are decided by the world in the sense that the particular propositions known by the community of inquirers determine the truth of those conjunctions; they guide behavior strictly through their deductive consequences. That is to say that the three features of the rules for judging account are not just the same features of the best system account.

Of course, Ramsey protests these two accounts are not the same. But he seems committed to Peirce's concept of truth. So the riddle is how are the two accounts not identical and if not, what is the extent of his actual belief in that concept of truth?

My strategy for solving this riddle focuses on why Ramsey changed his mind about his initial account of laws. The key fact I focus on is Ramsey's conclusion that laws are similar to chances. He argues in his later writings that chances are logically compatible with any propositions—including infinite logical products. In fact, he goes even further and argues that laws are just limiting cases of chances. This means that universal propositions must be chances too. Consequently, universal propositions are not truth-functions of ordinary propositions. So the key to understanding the relationship between Ramsey's two accounts of laws is Ramsey's account of chances.

For Ramsey, chances are degrees of belief in a hypothesized system of degrees of belief that people approximate. A chance is a conditional probability function, and chance propositions are fictional propositions about the assignment of chances to values. Since chances are probabilities, they obey the probability axioms. A crucial property of chances for Ramsey is that they obey what David Lewis calls the principal principle: agents defer to the chances when setting their credences, i.e. the probability of a proposition conditional on the chance of that proposition is just the chance probability. So chances are experts that people defer to setting their credences. People approximate the chances by weighting chances by their credences, and forecasts are produced by mixtures of chances. Thus, chances are critically important for the production of forecasts.

A law or universal proposition is just a limiting case of chance. Ramsey calls these variable hypotheticals. The variable hypothetical "if ϕ, then ψ" is really the chance that ψ conditional on ϕ is one. Because they are chances, laws factor into forecasts by weighing the laws with an agent's credences. So laws are only a special case of chance.

One crucial feature of Ramsey's account of chances is that he thinks chances are learned based on simplicity considerations and Fisher's principle of maximum likelihood. The former consideration is less important than the latter. The principle of maximum likelihood states that the correct chance is the one that best predicts observation, i.e. the one with the highest likelihood. Or the probability of a chance is to be given by the ratio between the chance's likelihood and the sum of every chance's likelihoods. Importantly, Fisher's methods have the guarantee (modulo some assumptions) that in the limit of observations, one's estimate of the chances will converge to the "true" chance. It is this that is the source of Ramsey's commitment to Peirce's conception of truth; he thinks that the method of learning chances ensures convergence to a unique chance system. Agents employing the maximum likelihood method can be thought of as fated to converge to the same set of beliefs about the chances. So Ramsey's seeming acceptance of Peirce's concept of truth really falls out of how he thinks chances are learned.

Ramsey's preference for this method avoids conflating his new view of laws as chances for his old view of laws as axioms and their consequences in the best system. An omniscient intellect may be fated to the same system of chances, but it only finds itself there because of its methods—not from logical entailment. This is subtle. On the old view, the omniscient intellect would deduce the laws from the known facts; on the new view, it only converges to one system of laws because it maximizes its likelihood and not because those laws follow as a matter of logic from the facts. A different method for learning the chances would lead to different results. There is no reduction of the laws to the facts as the old view would have it. So Ramsey's limited commitment keeps the two accounts distinct.

This is bad, however, because it commits Ramsey to the Principle of Indifference. Maximum likelihood estimation interpreted under Bayesian conditionalization is normal updating with a uniform prior over the chances. This means that Ramsey thinks the logically correct prior for ignorance over chances is the uniform prior. But this just is the Principle of Indifference. Ramsey elsewhere rejects that principle for good reason. So he is inconsistent.

The proper revision to Ramsey's account is to reject maximum likelihood estimation for normal Bayesian updating. He would have to abandon the uniform prior, and he would need to learn the chances via methods widely accepted by subjective Bayesians. This rids Ramsey of his inconsistency.

Here is how my argument proceeds. First, I review the secondary literature and document the emergence of the riddle discussed above. Second, I reconstruct Ramsey's first account of laws and the reasons why Ramsey rejected it. Third, I review Ramsey's discussion of his second account of laws and conclude that his theory of chances holds the key to understanding it. Fourth, I develop Ramsey's theory of chances and show how laws are limiting cases of chances. I show how Ramsey believes chances should be learned by using Fisher's principle of maximizing the likelihood. Fifth and finally, I argue that this commits Ramsey to the Principle of Indifference, and I argue for its abandonment.

4.2 A Key Riddle

A key question that has bedeviled the scholarship over Ramsey's view on laws is the relationship between his old view of laws and his new view of laws. As I discussed in section 4.1, those views are the best systems account and the rules for judging account. Cohen has argued that the two accounts are really one and the same (Cohen, 1980). Sahlin has argued they are in fact distinct (Sahlin, 1990). This debate has implications for a more recent analysis of Ramsey's view on laws. Misak has argued *pace* Holton and Price (Holton and Price, 2003) that Ramsey views laws as being cognitive in the sense that they are aimed at the truth. She argues that Ramsey had adopted the pragmatist conception of truth, which makes the laws that happen to be part of the best system to be the ones one should take to be true. Naturally, this agrees with Cohen's view that Ramsey's two accounts of laws are really one and the same, despite the fact that Ramsey himself declares in "General Propositions and Causality" that his new view amounts to "a complete rejection of his old view" (Ramsey, [1929] 1990e, 150). In this section, I aim to show the relationship between these two debates. This sets up the core question I address in this chapter: is Ramsey's account of laws as rules for judging the same as his account where laws are the axioms in a best of systematization all propositions?

4.2.1 Cohen and Sahlin on Ramsey's Laws

The first debate to examine is the one that occurred between Cohen and Sahlin over whether Ramsey's two accounts of laws are one and the same. They both agree that Ramsey denies this, but Cohen argues that Ramsey's commitment to the pragmatist theory of truth means that the best system and rules for judging accounts are identical.

Cohen starts his argument by describing the difficulty Ramsey has in making laws objective given his rules for judging account. The workaround for the seeming subjectivity of laws as rules for judging is the belief that there is some system of laws that is "uniquely determined", and if inquiry is allowed to proceed long enough, it will eventually terminate in that system. Cohen quotes Ramsey from "General Propositions and Causality":

> we do ... believe that the system is uniquely determined and that long enough investigation will lead us all to it. This is Peirce's notion of truth as what everyone will believe in the end; it does not apply to the truthful statement of matters of fact, but to the 'true scientific system' (Cohen, 1980, 215).

Cohen adds that the above quote is not about the system that an inquirer in fact believes but the belief that the inquirer's system will at least, in part, be part of the final system at the end of inquiry.[1] He immediately objects that this view does not work because of several well-known problems: intelligent life may not live long enough for there to be an end to inquiry or it may for various other reasons never settle on a system.

The suggested solution is what Cohen calls "idealised pragmatism" where the limit of inquiry is not what will happen but what would happen if it were to go on. The thought is that because Ramsey is committed to the pragmatist theory of truth, understood counterfactually, he would have to be committed to a theory where believed laws—the rules adopted for judging—just are a bet that those laws exist in the best, simple system found at the ideal

[1]Cohen introduces Ramsey's passage as follows:

> Again, someone might object that Ramsey's view seems to deprive causal laws of the objectivity that we are inclined to attribute to them. But the only facts, on Ramsey's view, are particular occurrences. Like Hume he refused to take causal necessity as a feature of nature. He admits that people may at present find more than one system of truth-functional generalisations that fits the known facts. But, he says
>
> we do ... believe that the system is uniquely determined and that long enough investigation will lead us all to it. This is Peirce's notion of truth as what everyone will believe in the end; it does not apply to the truthful statement of matters of fact, but to the 'true scientific system' (Cohen, 1980, 215).

I will return to the key passage much later in this chapter, but I think it is important to note that Cohen's claim has superficial merit based on the text.

limit of inquiry. Such a system would occur once everything that can be known is known. This is materially no different than the earlier view that believed laws are just what would be found in the best, simplest axiomatization of one's beliefs. A claim that a proposition is a law is a bet that this proposition will be found in the best system.[2] So the second view is just the first view after all by way of the pragmatist theory of truth.

Sahlin disagrees strongly with Cohen's conclusion. After discussing Ramsey's apparent commitment to the pragmatist theory of truth and the problems associated with that view, Sahlin points out that Cohen takes Ramsey's commitment to the pragmatist theory of truth to imply an endorsement of the correct laws being the ones believed by an omniscient agent.[3]. Sahlin argues this view cannot be correct.[4] Sahlin's first point is that he thinks that if Cohen

[2]Cohen summarizes the proposal as follows:

> The obvious way to rescue Ramsey's analysis from the above objections is to identify the true scientific system as the one that everyone *would* believe in the end *if* free scientific enquiry continued long enough. We might call this 'idealised pragmatism'. It has, of course, an appearance of circularity because it seems to use a subjunctive (and probably counterfactual) conditional in order to elucidate the derivability of such conditionals. But in any case no one could be sure that free scientific inquiry had continued long enough unless he already knew every fact, *i.e.* every observable occurrence. And any free scientific inquiry must surely pay due regard to the various criteria of simplicity that deserve to be respected. Accordingly, the true scientific system is now being identified as the one that we should regard as the simplest axiomatisation of our knowledge if we perceived everything. That is to say, Ramsey's second analysis has had to be reformulated in such a way that it is not seriously distinguishable from his first one (Cohen, 1980, 215–216).

[3]Sahlin describes Cohen's argument succinctly as:

> But it has been maintained that from these problems it can be shown that there is no substantial difference between Ramsey's two theories. Jonathan Cohen points that the above problems force Ramsey into idealized pragmatism, which assumes omniscience. The demand for omniscience means that there are no essential differences between the two theories (Sahlin, 1990, 115–116).

The key premise Sahlin notes is the requirement of omniscience.

[4]The argument, in a condensed manner is presented as:

> However, it should be obvious from what has been said above, and even more obvious from reading Ramsey's own texts, that in order to be able to uphold a standpoint like Cohen's, Ramsey must have been either misunderstood or misinterpreted on a number of important points. As regards Ramsey's latter theory [the laws as rules for judging account], for instance, any mention of truth is forbidden. Nor are we trying to set up a true axiomatic system, but we are looking for laws that can help us get through life. If we are omniscient, we can act on the basis of an axiomatic scientific system in such a way that we are always successful; if we are omniscient, our goal is to find a map with which we can find our way into the future; and if this scientific map has been drawn using general propositions, there is no absolute guarantee of

is correct, then Ramsey must have been misunderstood or misinterpreted in multiple ways. He then goes on to make an argument that relies heavily upon metaphor and references to various passages in Ramsey's corpus. He is very unclear. This requires some unpacking so I proceed to do that now.

Sahlin notes that the truth of laws is problematic. This is a reference to Ramsey's claim in "General Propositions and Causality" that "if then it [a universal proposition] is not a conjunction, it is not a proposition at all" (Ramsey, [1929] 1990e, 146). Ramsey does not hold universal propositions to be conjunctions, so they are not propositions, i.e. do not admit of truth or falsity. This automatically would contradict the claim by Cohen that laws are the general propositions in a best system of propositions because there would be no such general propositions. Sahlin's next claim is that the goal for laws is not a "true axiomatic system" but those that are useful in the here and now. This is why Sahlin notes that in the case where one is not omniscient, one shoots for laws that reduce error. This may not coincide with the ideal, axiomatic system found by an omniscient intelligence. That is one may differentiate between rules that minimize error and knowledge that admits no error. The point of universal propositions is the former and not the latter. So Sahlin's argument is that universal propositions cannot be the axioms of the best system arranged by an omniscient intelligence since such a system would have no point for the non-omniscient. Error is impossible for the omniscient whereas minimization of error is the goal for mere mortals. It is the latter case that one finds oneself in when considering laws.

The problem with this argument is that it argues for a distinction when there need be none. If the goal of universal propositions is the minimization of error, then the universal proposition

success. The best we can hope for in the latter case is that our scientific system or map is as free from error as possible—that is, hopefully, it guides us towards success more often than failure. To me there is an obvious difference between a true map, a map that is drawn in perspective, by a more or less reliable procedure, and that is not always successful. There is an obvious difference between a chicken that always knows whether the caterpillar is poisonous or not and the chicken that has a firm and well-supported belief about the matter. There is a difference in degree between knowing and not knowing if this is my last supper (Sahlin, 1990, 116).

that most minimizes error is just the one with zero error. Consider Alice the robot who picks strawberries. She needs to distinguish strawberries from raspberries while picking. She considers multiple programs or rules that take in propositions of her environment and output either the proposition "this is a strawberry" or "this is a raspberry". Her picking training aims to produce a rule that minimizes her picking error, i.e. minimizes the number of identified and picked fruits that happen to be raspberries. In this case, the best rule would simply be the one where she picks no raspberries and only strawberries. What differentiates this rule from the others? It is still a rule, and it still minimizes error. It just minimizes the most error. Apart from the fact this rule happens to be optimum, what differentiates it from the other rules Alice might adopt? There is nothing on Ramsey's account that would differentiate the two.

Another way to put the problem is that nothing in Ramsey's account so far rules out there being optimum rules that inquirers might adopt. Why cannot Ramsey have a best system of rules? If as Sahlin argues, Ramsey happens to think that there are no general facts,[5] then this best system of rules would just coincide with the axioms of a best system. And so Cohen's point would still stand.

The claim that general propositions are not truth-apt is beside the point here. In the ideal system that Cohen alludes to and Ramsey seems committed to based on his appeal to Peirce's limit of inquiry, the rule would coincide just with a conjunction of finitely many facts. So it could in effect be treated as a proper proposition because there would be no more facts forthcoming.

This then leaves a puzzle: if Ramsey asserts that his new view is not the same as his old

[5]Sahlin writes that

> Ramsey did not want to accept general facts. Wittgenstein was correct in stating that the world can be described entirely using particular or atomic facts (Sahlin, 1990, 106).

The upside is that an omniscient intelligence would just have rules that agree with all particular facts described by that system.

view on laws, how do they in fact differ?

4.2.2 Holton, Price, and Misak on Laws

A similar debate has played out in the past two decades over whether there is a significant difference between singular propositions and universal propositions. On one side has been Holton and Price (Holton and Price, 2003), who argue that Ramsey had differentiated between these propositions based on universal propositions applying to more instances than could ever be experienced and those that only apply to what would actually be experienced. They argue that this makes Ramsey incoherent because the same argument could apply to the concepts used to build propositions—making all propositions fiction for Ramsey. Furthermore, they contend that Ramsey's positive account for universal propositions—that they are double dispositions—also applies to singular propositions, meaning there is even less reason to suppose a real difference between general and singular propositions. This furthers the argument that Ramsey is incoherent. Misak (Misak, 2016) has replied that the lack of a distinction between general and singular propositions was precisely Ramsey's point: all beliefs are habits and thus cognitive in the sense that they are aimed at the truth. There is no need to privilege "beliefs of the primary sort" because they are all ultimately evaluated by their usefulness in producing successful, reliable action. This usefulness is captured by what beliefs would be held in the limit of inquiry. She thus argues that Ramsey had internalized the pragmatism attributed to Peirce. The upshot would be that the two accounts of laws are in fact the same. If truth applies equally well to general as to singular propositions and believed general propositions are a bet that they appear in the best system at the end of inquiry, then a general proposition just is an axiom in the best systemization of propositions. Consequently, an examination of this debate would be useful in exploring the puzzle of whether Ramsey's later view of laws is the same as the earlier one.

Starting with Holton and Price, they offer two methods Ramsey uses for distinguishing general from particular propositions. Both, they allege, fail as a distinction.

First, they argue it is not the infinite character of general propositions that Ramsey objects to but their "unsurveyability", which they think is a feature of more than just general propositions. They argue that Ramsey is not objecting to the infinite character of general propositions but something more subtle. They think Ramsey's complaint is due to the fact that knowledge of general propositions outstrips all evidence for those general propositions, which they call knowledge outstripping acquaintance.[6] Their argument rests on a reading of three points they claim Ramsey makes. First, they think Ramsey believes that one cannot even grasp infinite conjunctions, let alone write them out. Second, they allege that Ramsey objects to generalizations because one would not use the full totality of the generalization, and one's understanding of a proposition is constituted only by its use. Third, they quote Ramsey in stating that the degree of certainty in a generalization is really a degree of certainty in a particular case (Holton and Price, 2003, 327–328). They contend that underlying these arguments is the same fundamental objection: Ramsey recognizes that there is a gap between the truth of a universal proposition and the evidence for that universal proposition.[7]

[6] Holton and Price announce their argument thusly:

> We argue that the same considerations concerning infinity come up everywhere, due to the 'infinite' character of our grasp of concepts (though strictly speaking, we contend, the crucial point in both cases is not a matter of infinity, but of something like open-endedness or unsurveyability, which we characterise further below). So the pressure to treat universal generalizations as nonpropositional generalizes to all cases. Thus Ramsey's 'sceptical problem' turns out to be global, and not (as he himself thought) confined to the case of generalizations (Holton and Price, 2003, 326)

[7] Ramsey's supposed objection is written as follows:

> We make two observations about these [Ramsey's] two arguments. The first is that strictly speaking, they depend on a feature of language which is much more modest and ubiquitous than infinity itself. Both arguments apply equally well in a finite domain, so long as that domain is large enough to extend beyond the cases actually 'encountered' by a speaker or group of speakers. At a first pass, *the crucial point is simply that the set of true instances of a generalization normally extends well beyond the set of those instances with which any speaker or group of speakers is or will be acquainted* [emphasis mine]. This notion—let us call it the Extension-Transcends-Acquaintance Principle, or E-TRAP, for short—is usefully refined in various ways, as we shall see. Even as it stands, however, it seems enough to support Ramsey's two arguments. True infinity appears to do no significant work (Holton and Price, 2003, 329).

Take the universal proposition "All men are mortal". I am only familiar with family members and friends that have passed away while the true instances of this generalization extend to many others I have never met and to future generations I will never meet. Likewise, the non-general claim "A grain of sand at Laguna Beach weighs less than one kilogram" (since there are only finitely many grains of sand there) involves true instances of which the vast majority I will never weigh and find to be less than one kilogram. Holton and Price argue that this objection applies just as well to non-general propositions as to general ones. Infinity need not factor in.[8]

Second, they claim that Ramsey's positive account for general propositions applies just as well to particular propositions, which makes it a poor method for demarcating the two. They state that the positive account for general propositions makes them double dispositions. A double disposition is a disposition to activate another disposition. The type of double disposition here is of the sort where one goes from one belief to another and also to be disposed to enunciate specific sentences. The doubleness comes from the fact that all beliefs are dispositions and so to acquire this disposition is to acquire other dispositions.[9] For

The claim is that worries about the infinite are not what drive Ramsey. Instead, it is a property of language's unsurveyability that Ramsey is worried about.

[8]Holton and Price continue to argue that Ramsey has essentially stumbled upon a core problem in the philosophy of language that others such as Goodman (Goodman, 1955) later identified. They argue that a key characteristic of grasping language is to infer new cases from old ones. But no list of old cases—acquainted instances as Holton and Price call it above—uniquely determines the new true cases. This they argue is not native to just general propositions but to all propositions:

> First, it does seem to amount simply to extending to linguistic terms at large the kind of concerns Ramsey has about universal generalizations. In both cases, the concern is precisely that a set of instances (in one case of conjuncts, in the other of true instances of the application of a term) goes beyond what human language users could use or survey. Second, the issue concerned is that at the heart of the so-called rule-following considerations. Indeed, the rule-following considerations seem to expose the real teeth of the E-TRAP. On the one hand, they confront us with the fact that the application of terms to novel cases is essential to language (and hence that in a very strong sense, grasp of meaning necessarily precedes acquaintance with the totality of relevant cases). On the other hand, they out that no finite basis of acquaintance logically determines a unique extrapolation to new cases (and hence that there is no unique way to cantilever ourselves out of the E-TRAP) (Holton and Price, 2003, 330).

This they allege is the core problem that Ramsey faces here, and they think he fails in distinguishing the general from the particular propositions due to this problem.

[9]Holton and Price describe it thusly:

example, the universal proposition "All men are mortal" involves me going from a belief gained by learning that my cousin is human to the belief that my cousin is mortal and the willingness to enunciate so. Stipulating this account is accurate, Holton and Price think it offers no method for distinguishing between universal and singular propositions. There is no principled way to differentiate between universal propositions defined as dispositions and singular propositions. Defining universal propositions as double dispositions does not save them. Furthermore, a bigger worry is that both universal and singular propositions make use of concepts, like "... is mortal", and that ultimately knowledge of a singular proposition requires grasp of a concept, which is itself a disposition akin to the universal judgment. The upshot is that the dispositional nature of concepts prevents Ramsey from dividing singular propositions from universal propositions in the way that he thinks he can do.[10]

Summing up, the view articulated by Holton and Price is that Ramsey cannot differentiate universal propositions from singular propositions based on the former's unsurveyability and dispositional character. They conclude that Ramsey's account is incoherent.

> How should we understand universal generalizations, if not as propositions? Ramsey says that they are 'variable hypotheticals', and that these 'are not judgements but rules for judging "If I meet a Φ, I shall regard it as a Ψ".' [....] In other words, in committing ourselves to a universal generalization we adopt a habit of forming beliefs in a certain way [....] In contemporary jargon, we may say that Ramsey's view is thus that to accept a generalization is to acquire a double disposition—to become disposed to adopt a belief on one sort, whenever one adopts a belief of another sort, and to enunciate a certain sentence (Holton and Price, 2003, 331).

[10]Holton and Price summarize the core of this argument as there being no demarcating line here:

> So what is the distinction between universal beliefs and beliefs of the primary sort? It is hard to see that there is any radical difference in functional terms, by Ramsey's own lights. It might perhaps be suggested that there is a distinction of level: Beliefs of the primary sort are dispositions to act on desires, while universal beliefs are dispositions to act on other beliefs (to act by forming further beliefs). But why should this distinction make all the difference as to what should be counted as a belief? Moreover, even this distinction will not really stand up. Beliefs of the primary sort can themselves be dispositions to form new beliefs on the basis of other beliefs [....] Indeed, dispositionality applies at an even more basic level, that of concepts. Our grasp of a concept surely manifests itself as a disposition—the disposition to apply the term in certain circumstances. Indeed, the complex disposition corresponding to holding a belief ('using a map', in Ramsey's account) depends on simpler dispositions of this kind. Grasp of concepts is like grasp of the map's key. To use a map we need to know what its symbols signify—we need to have adopted a practice which takes us from things in the world to symbols on the map and back again. And these are dispositions. So it seems that here, too, there is no boundary between universal judgements and others (Holton and Price, 2003, 333–334).

Misak argues that Ramsey was aware of this and that the incoherence is only apparent.[11] Ramsey in fact held that there was no fundamental difference between universal and singular propositions.

She summarizes the position ascribed to Ramsey by Holton and Price as an "expressivist" position.[12] Misak believes that Holton and Price's Ramsey has beliefs of the primary sort (singular propositions in the primary system) be properly truth-apt while universal propositions are in some sense not truth-apt. She argues that their Ramsey thinks this because of concerns with infinity.[13] Against this she alleges that Ramsey's view in fact made no distinction. Despite apparent comments that seem to commit Ramsey to such a position,

[11]In what follows, I take Misak's arguments from (Misak, 2016). She mentions in a footnote that she responds to Holton and Price's arguments in more detail with her (Misak, 2017). However, the cited paper only recites arguments from (Misak, 2016) but in less detail. Consequently, I use the longer arguments here.

[12]Misak's summary of Holton and Price lumps it in with philosophical positions she calls "expressivist". This is the idea that some propositions are not truth-apt but really express some other, non-cognitive attitude. She writes:

> Richard Holton and Huw Price have argued [...] that 'General Propositions and Causality' marks a change of mind on Ramsey's part regarding open generalizations. They argue that the change arose because his ideas about infinity had changed, and they place their emphasis on the fact that he seems in this paper to maintain a distinction between generalizations and genuine propositions in the primary language. In 'General Propositions and Causality', that is, it can seem that Ramsey accepts the bifurcation thesis—that some propositions are genuinely descriptive, representational, or truth-apt and so can be accounted for by a semantics centered on reference, while others are not and require a different semantic treatment. These days such a position might be called *expressivist*. Expressivism holds that generalizations (or ethical statements, or modal statements, or statements of whatever other domain is in question) are not descriptive. Despite their 'surface grammar,' they have a different, less objective status, which the expressivist then tells us about. My own view is that it is clear that significant elements in Ramsey's thought cut against the position Holton and Price ascribe to him, and that he was from the mid 1920s moving toward a position that blurred the boundaries between genuinely representational and the merely practical or expressive. That is, he is not offering us a local pragmatism that says that only theoretical statements and other empirical-looking statements that we cannot in fact verify are to be assessed in terms of whether they work. *All beliefs* are habits with which we meet the future, and if they would meet the future well and be in our best theory, then they are true (Misak, 2016, 193–194).

[13]This is inaccurate as Holton and Price's own words make clear. They think infinity is orthogonal to the real issue, which they label as "unsurveyability". See the prior quotes.

she argues his earlier work had him adopting a Peircean view.[14] In addition, she cites several short passages from "General Propositions and Causality" for her view.[15] This leads her to allege that a key development in Ramsey's later thought is that all beliefs are dispositions, blurring the distinction between singular and universal judgments.[16] Misak makes Holton and Price's second observation that all beliefs involve dispositions, making any distinction to be drawn difficult if not impossible. In fact, in one of her footnotes, she writes that while

[14]She writes:

> But we have seen that by 1927, Ramsey was, if you like, exploring an instrumentalism about *all* beliefs, save logical and mathematical ones. He was arguing against the very project that required the instrumentalist move, *if that move applies only to disputed classes of propositions*. He was arguing against the picture in which there is an elementary language of verifiable propositions and then a secondary language composed partly of propositions that are constructed from the elementary language in truth-functional ways, and partly of statements that require another kind of justification, a justification in terms of success or convenience. He was arguing for a pragmatism in which *all* our beliefs, outside logic and pure mathematics, are habits for meeting the future (Misak, 2016, 194–195).

She claims that Ramsey's trajectory of thought had him abandoning the expressivist position in favor of a position where all propositions, such as universal and theoretical ones, are equally likely to be true as propositions of the primary sort. So the evidence here is that Ramsey's prior position before "General Propositions and Causality" had him drifting to the position Misak outlines.

[15]She writes that

> [W]hile it may be the case that Ramsey's view of infinity had changed, and while it may be the case that he retained the Tractarian programme for certain purposes, he was committed to being a pragmatist for *all* judgements, save mathematical ones, at least as far back as the 1926 'Truth and Probability' and 'Mathematical Logic'. And he asserts the general claim at least twice in 'General Propositions and Causality':
>
> since all belief involves habit, so does the criticism of any judgment whatever, and I do not see anything objectionable in this. [Ramsey, [1929] 1990e, 150]
>
> it belong to the essence of any belief that we deduce from it, and act on it in a certain way [Ramsey, [1929] 1990e, 159] (Misak, 2016, 195)

[16]In a passage, she argues as follows:

> One of the main points of 'General Propositions and Causality' is that 'variable hypotheticals involve causality no more and no less than ordinary beliefs'. There is nothing especially problematic about open generalizations and infinity—many predicates can be thought of as ranging over an infinite number of objects. Many ordinary beliefs are in the same boat as generalizations.[46] So when Ramsey says 'Many sentences express cognitive attitudes without being propositions' [...] I read him as demoting the importance of propositions in the primary language, and elevating the importance of all those varied beliefs in the ordinary and theoretical secondary language. Within the realm of cognitive claims, he distinguished mathematical/logical propositions, primary propositions (in the Tractarian sense), generalizations and causal laws, and conditionals (indicative and counterfactual). But they are all cognitive or aimed at the truth (Misak, 2016, 195).

this is Holton and Price's very point, she believes Ramsey had already adopted this stance: "Holton and Price think this is evidence that Ramsey's view [...] is 'unstable' and that they didn't see the rule-following problems. They recommend to him that he abandon the idea of a genuine proposition and adopt global pragmatism. On my reading, Ramsey was already there" (Misak, 2016, 195). Ramsey's account of beliefs as dispositions would, she argues, seem to commit him to the very position that Holton and Price think he should adopt.

It should be noted at this point that the evidence that Misak marshals is very weak. She has several isolated quotes and the argument that this was Ramsey's position in 1927. To bolster her claims, she goes on to provide a bit more.

The first point Misak makes is the familiar one from Holton and Price: universal propositions act as double dispositions to acquire other dispositions. She writes that Ramsey appropriated the Tractarian account for singular propositions.[17] Her reference to the *Tractatus* is a signal that she considers Holton and Price's suggestion that Ramsey might use double dispositions to divide singular from universal propositions. Believing a singular proposition results in a disposition to action; believing a general proposition results in a disposition to believe other propositions. Both are directed towards the future in that they lead either to actions directly or indirectly. But as was noted, Holton and Price dismiss this distinction because they think singular propositions also can count as double dispositions.[18] Misak makes no acknowledgment of this possibility. She does argue that this distinction is only surface level

[17]Misak's argument is to make an explicit connection with the *Tractatus*:

> Ramsey takes the model or map metaphor in the *Tractatus*, on which a picture in the elementary language is a model of reality, and turns it into a tool for getting along in the world: 'A belief of the primary sort is a map of neighboring by which we steer' [Ramsey, [1929] 1990e, 146] Part of his point, I think, is that when you meet the future with a primary, simple, singular belief, you act on the truth of it—beliefs of the primary sort are dispositions to direct action. Other beliefs, such as open generalizations, involve dispositions to acquire other beliefs. When you meet the future with a generalization, you employ a rule that has you acting on autopilot when you encounter an instance to which the generalization applies (Misak, 2016, 196–197).

Her citation here is to a passage where Ramsey discusses the similarities and dissimilarities universal propositions have with conjunctions. I discuss this passage in more detail below.

[18]Holton and Price provide a counterexample: "Beliefs of the primary sort can themselves be dispositions

and that Ramsey is not incoherent here but really does believe that singular and general propositions are fundamentally the same. Her argument rests on the familiar quote used by Cohen committing Ramsey to Peirce's conception of truth. Her thought is that Ramsey's commitment to the Peircian account of truth entails that every proposition that ends up being believed in the best system is taken to be true. The fact that some are double dispositions is no fact to infer that those dispositions are not truth-apt.[19] Misak concludes that since ultimately all beliefs are habits and the habits that are reliably successful will end up in the best system believed at the limit of inquiry, there is no fundamental distinction and Ramsey was are of this.[20]

The relevance of this debate to the question of laws should now be apparent. If there is no fundamental difference between singular and universal propositions, then there should be no difference between laws as rules for judging and laws as the axioms in a best system of propositions. Misak seems to indicate as much when she writes that "Ramsey's version of it

to form new beliefs on the basis of other beliefs: the belief that Martha is dangerous will itself amount to a disposition to form the belief that a dangerous person is approaching given the belief that this is Martha approaching" (Holton and Price, 2003, 334).

[19]Misak writes disjointedly as follows:

> Ramsey is almost right. This is almost Peirce's notion of truth. Ramsey's version of it is that what is true is what can be deduced from our best system. Peirce, however, would have not liked the indicative conditional in Ramsey's passage—we have seen that he thought a true belief is one that *would* be believed *were* we to subject it to as much further testing as would be fruitful.
>
> [....]
>
> Note that all sorts of statements not in Russell's or Wittgenstein's primary language—open generalizations, conditionals, theoretical entities, and on and on—will be part of our best system. We expect that our natural ontological attitude will prevail—that, in the end, our beliefs (e.g. that all sorts of entities exist independently of us) will be left in place (Misak, 2016, 197–198).

[20]Misak concludes that Ramsey had adopted Peirce's answer to the question of what is truth, and this conclusion renders him okay with universal propositions being cognitive in the sense they aim at the truth:

> His [Ramsey's] solution to the problem of general and causal propositions—the problem that maps can't be infinitely extended—is that such propositions are not infinite conjunctions, but automatic rules with which we meet the future. But I have argued that part and parcel of his solution is that there is no special problem about evaluating universal statements—no problem that would call for the answer that they, and only they, are evaluated as *mere* habits with which we meet the future. Rather, this draft paper represents Ramsey's treatment of a few species of belief, showing how they are habits we use to meet the future (Misak, 2016, 198–199).

[Peirce's notion of truth] is that what is true is what can be deduced from our best system" (Misak, 2016, 197). This would mean that Misak would likely side with Cohen in the latter's debate with Sahlin, while Holton and Price would agree that Ramsey's attempts to draw a distinction inline with the one Sahlin wants to draw, though Ramsey is unsuccessful. It should be observed that the key arguments for Cohen and Misak's views rest on Ramsey's commitment to the pragmatist (more specifically, Peirce's) conception of truth as the best system believed in the limit of inquiry. So a crucial component to solving the riddle would be addressing Ramsey's relationship with the pragmatist conception of truth.

4.2.3 Summary

Summarizing, these debates reveal a riddle about how Ramsey's first account of laws and his second account of laws differ. The key issue, as shown by the debates between Cohen and Sahlin and Holton, Price, and Misak, is the extent of Ramsey's commitment to the pragmatist conception of truth. If Ramsey holds that a proposition is true just in case it is in the best system were inquiry to continue, then a universal proposition as a rule for judging would just be the adopted rules in the best system. Since everything would be known, these would just be the axioms of that best system of propositions. In the language of Holton, Price, and Misak's debate, all propositions are dispositions or habits so there would be no fundamental distinction between these laws and other propositions in the best system, apart from how they are deductively organized. The difference between universal and singular propositions would on this account be a matter of that deductive organization. In other words, the rules for judging are just the axioms of the best system.

This is problematic because Ramsey protests things are otherwise. He states unequivocally that the second account is a rejection of the first account:

> What is said above means, of course, a complete rejection of this view [the first

> account] (for it is impossible to know everything and organize it in a deductive system) and a return to something nearer Braithwaite's (Ramsey, [1929] 1990e, 150).

Perhaps something has been missed in these debates. Or perhaps Ramsey is just inconsistent: his first view just is the second view.

It would therefore be good to reexamine what exactly Ramsey's two accounts happen to be. A thorough examination would allow one to have a better handle on the extent to which Ramsey is correct that his old view and new view of laws are not the same and incompatible.

This is important because resolving the riddle would go a long way to meeting the goal of this chapter: an account of universal propositions and laws that fits with Ramsey's philosophy of science. As things stand, it would seem that if Cohen and Misak are right, then Ramsey's account of laws is really the axioms in the best system of laws deduced from the facts known at the limit of inquiry. Universal propositions and laws are cognitive in the sense that they can be said to be true about the world. If Sahlin and Holton and Price are right, Ramsey's account has no substantial claim to the cognitive status of laws at penalty of losing the distinction between universal and ordinary propositions. Either way, an important component of Ramsey's philosophy seems unstable with the proposed accounts of laws; Ramsey cannot really maintain a difference in truth-aptness on both views since on first laws are truth-apt like ordinary propositions while on the second ordinary propositions fail to be truth-apt just like laws. So resolving the riddle and seeing how Ramsey's best system account of laws really differs from his laws as rules for judging account and the extent of his commitment to the Peircian theory of truth would go a long way to providing an account of universal propositions and laws that gels with Ramsey's broader philosophy.

4.3 Best System Account of Laws

In this section, I examine Ramsey's initial conception of laws as the axioms in the best system of propositions were one to know everything. I start by looking at the paper where Ramsey introduces this account and defends it against Braithwaite's alternative theory. Because my goal is to decipher how Ramsey's initial theory of laws differs from his later theory, I end by discussing why Ramsey abandons that initial account.

4.3.1 The Old Account

Ramsey's initial theory of laws occurs in a paper titled "Universals of Law and of Fact" that is dated 1928. It is less of a paper and more a series of notes where Ramsey lists Braithwaite's account of laws, Ramsey's objections to that account, and Ramsey's alternative. I start with Ramsey's positive thesis.

He approaches his positive conception of laws by enumerating the various types of universals. He writes that those universals come in four varieties:

> In order to get nearer a correct solution let us classify universals a little more closely; as we have the following classes:
>
> (1) the ultimate laws of nature
>
> (2) derivative laws of nature, i.e. general propositions deducible from the ultimate laws
>
> (3) what are called laws in a loose sense; i.e. general propositions deducible from the ultimate laws together with various facts of existence assumed to be known by everyone, e.g. bodies fall
>
> (4) universals of fact; but these cannot be sharply distinguished from (3); on a

> determinist view all of them could be deducible from the ultimate laws together with enough facts of existence (Ramsey, [1928] 1990p, 142).

Four types of laws are distinguished. First, there are the fundamental laws of nature, which might be things like the laws of motion in Newtonian mechanics. Second, there are derivative laws. Continuing with the Newtonian example, these would be laws like universal gravitation. Third, there are laws loosely defined. These are dispositions such as heavy bodies fall. And fourth, he lists what he calls "universals of fact". These are taken to be something like conjunctions like everyone in Cambridge voted.

In the immediate passages, he suggests an account that distinguishes between the first two types of laws and the last two by whether they mention specific portions of space-time. Both ultimate and derivative laws of nature supposedly do not mention specific space-time portions while dispositions and universals of facts do. Ramsey dismisses this account on grounds that dispositions and universals of fact may also not mention specific space-time portions.

His positive account comes from the observation that an intuitive difference between ultimate laws and derivative laws of nature on the one hand and dispositions and universals of fact, on the other hand, would persist even if one knew everything. This leads him to conclude that an omniscient agent would still have to systematize their knowledge deductively:

> [E]ven if we knew everything, we should still want to systematize our knowledge as a deductive system, and the general axioms in that system would be the fundamental laws of nature. The choice of axioms is bound to some extent to be arbitrary, but what is less likely to be arbitrary if any simplicity is to be preserved is a body of fundamental generalizations, some to be taken as axioms and others deduced. Some other true generalizations will then only be able to be deduced from these by the help of particular facts of existence. These

> fundamental generalizations will then be our universals of class (1) and (2), the axioms forming class (1) (Ramsey, [1928] 1990p, 143).

The view developed here is familiar to contemporary philosophers of science. Laws of nature are the axioms or immediate consequences of those axioms in the best, simplest system of propositions held by an omniscient agent. These laws of nature are separated from the other sorts of laws by the fact that they do not require particular facts for their deduction. This view was later taken, developed further, and defended by David Lewis in a number of places (Lewis, 1973; Lewis, 1983; Lewis, 1994).

There are several observations to be made of this model. First, the setup is that of an omniscient agent. All particular and universal facts have been settled for this agent, so there is no learning to be done; instead, the task of the knower is systematization. Second, knowledge is to be organized into a deductive system. Ramsey does not say explicitly that the knowledge here is propositional in the sense that it consists of propositions that can be true or false, but the implication from omniscience is that it should be so. Third, some universal propositions are meant to be taken as primitive and others deduced. The primitive ones Ramsey labels as axioms. Fourth, the choice of axioms is arbitrary with the only constraint being "simplicity". Together, the resulting account is one where a law is an axiom or the direct consequence of an axiom in the simplest, deductive system of an omniscient agent.

Of course, Ramsey notes that one is not in fact omniscient. But people still talk about laws. What are these laws relative to the ideal, omniscient agent? Here Ramsey says that people organize their knowledge as if it were found in the mind of an omniscient agent:

> As it is, we do not know everything; but what we do know we tend to organize as a deductive system and call its axioms laws, and we consider how that system would go if we knew a little more and call the further axioms or deductions

> there would then be, law (we think there would be ones of a certain kind but don't know exactly what). We also think how all truth could be organized as a deductive system and call its axioms ultimate laws (Ramsey, [1928] 1990p, 143).

He argues that one in fact organizes one's knowledge deductively. The axioms of that organization are the fundamental laws and their immediate consequences the derivative laws. The relationship between this system and the system of the ideal agent is that one treats the current systematization as if it were the one would organize, were one to know more. Furthermore, it is a bet that once one knows all truths, one would settle on the current axioms and derivative laws as the axioms and derivative laws found in that best system.

There is one more wrinkle to this view. An important question not directly answered is whether the best system and its chosen axioms are facts about the world or counterfactuals that would be obtained for inquirers if they could continue their inquiry. Ramsey's comments indicate there is no difference between the two. He writes immediately following the introduction of his positive account:

> The property of a universal that it *would* be an axiom in a deductive system covering everything is not really hypothetical; the concealed if is only a *spurious* one; what is asserted is simply something about the whole world, namely that the true general propositions are of such forms that they form a system of the required sort with the given propositions in the required place; it is the facts that form the system in virtue of internal relations, not people's beliefs in them in virtue of spatio-temporal ones (Ramsey, [1928] 1990p, 143–144).

His locution here is a bit circuitous. What he says in the first sentence is that his claim appears to be a subjunctive one: if one were to know more, one would organize one's body of knowledge with the believed laws as axioms. But he claims this is not merely a hypothetical because the subjunctive antecedent is spurious; his best systems account is that the world is such that the believed laws are the axioms in the total truths about the world. When someone argues that P is a law, he is not claiming a hypothetical that if he could know all the truths, then P would be an axiom or consequence of an axiom. He is stating that among the in-fact truths about the world, P is an axiom in the best systematization. It is a bet about a fact—not a counterfactual. So the connection between the omniscient agent's and the erring human's laws is not that one happens to be an idealization and the other a fact but that they are really one and the same. The answer to the aforementioned question is that Ramsey's purported view, though couched in counterfactual terms, is really a claim about the actual truths of the world and their best organization.

The upshot from this discussion is that Ramsey's best system account comes offly close to the view Cohen ascribes to him with an important caveat. Recall that Cohen argued that Ramsey seems committed through the pragmatist theory of truth of the correct laws of nature being those found at the limit of inquiry, where the limit of inquiry is an idealized notion. Here the view is close to that where the limit of inquiry is replaced by the omniscient agent. The main difference is that while Cohen thinks Ramsey's final view is purely a counterfactual one, Ramsey ardently says that though seemingly counterfactual, his best systems account is really about the world as it is. One might say that Ramsey is emphasizing the fact that the counterfactual is uniquely determined and thus an objective fact about the world. If that is the case, then the view just is the one Cohen ascribes to him in "General Propositions and Causality".

An important observation is that in this view, natural laws are a species of universal proposition, which is an infinite logical product. Elsewhere, Ramsey enunciates a view of universal propositions as the infinite logical products defined by taking the intersection of all elements in the set defined by propositional functions (Ramsey, [1926] 1990l, 171–172). For example, "all men are mortal" is really the intersection of the elements in the set abstracted by the propositional function “if x is a man, then x is mortal”. This makes universal propositions truth-functions of other propositions, even though they cannot be expressed as conjunctions. While I defer discussion of this aspect of Ramsey’s account to a later chapter, it is important to note that the view expressed here is simpatico with Ramsey’s understanding of universal propositions as infinite logical products. So laws as the axioms or consequences in a best system of propositions are just infinite logical products of every settled proposition.

This means that the view here holds seemingly counterfactual propositions must be truth-functions of ordinary propositions. Counterfactuals are given by the laws, and the laws are the axioms in the best systemization of the facts. Those axioms must be treated as infinite logical products since they are determined by the facts. Consequently, counterfactuals and similar propositions have to be truth-functions on this account.

It is this last point that Ramsey seems to have revisited. I turn to that now, and I connect with why Ramsey changed his mind about the best systems account of laws.

4.3.2 Why Ramsey Changed His Mind

In “General Propositions and Causality”, Ramsey discusses Braithwaite’s account, Ramsey’s objections to Braithwaite’s account, why Ramsey’s previous account of laws is wrong, and how Ramsey’s new positive account avoids the objections against Braithwaite’s theory. I go over Ramsey’s later account of laws as what he calls “variable hypotheticals” or “rules for judging” in the next section. In this section, I want to focus on what many people take to be Ramsey’s objection to his best system account of laws. The key focus is on the following passage:

> I, therefore, put up a different theory by which causal laws were consequences of those propositions which we should take as axioms if we knew everything and

> organized it as simply as possible in a deductive system.
>
> What is said above means, of course, a complete rejection of this view (for it is impossible to know everything and organize it in a deductive system) and a return to something nearer Braithwaite's (Ramsey, [1929] 1990e, 150).

Here Ramsey seems to indicate that the reason he rejects his prior view is that he came to accept that omniscience is impossible. Lewis notes this as does Cohen (Lewis, 1973, 3.3; Cohen, 1980, 214). I claim that this is not in fact the reason Ramsey rejects the best system account of laws. There are several different interpretations of this passage. The interpretation that best fits the text happens to make the passage irrelevant for why Ramsey rejects his old view of laws. Instead, as Sahlin argues, Ramsey rejected the old view because he came to believe that universal propositions are not truth-apt (Sahlin, 1990, 111). This will set the stage for exploring Ramsey's mature account of laws.

The cited passage above is the only discussion Ramsey has of the prior account of laws in "General Propositions and Causality". Ramsey follows it by saying how on his new account "a causal generalization is not, as I then thought, one which is simple, but one we trust (cf. the ages at death of poets' cooks). We may trust it because it is simple, but that is another matter" (Ramsey, [1929] 1990e, 150). And that is it. The available material in his published writings and unpublished notes makes no mention of why he retracted his earlier account of laws. So the evidence here is very thin.

Furthermore, it is very ambiguous. The key sentence starts with "What is said above means, of course, a complete rejection of this view" without stating explicitly what exactly "above" Ramsey is referring to. Ramsey compounds the ambiguity by listing his objection in parentheses with no argument connecting the objection to whatever was previously said. And he does not cleanly state the objection: he does not specify what he means exactly by "impossible" in the objection. This makes it very unclear what Ramsey's argument is.

The result is that there are several interpretations one might make from the above-cited passage.

The first view is that Ramsey does not think as a matter of fact that one could know everything and systematize it in a deductive system. He is objecting to the possibility of any single human knowing everything. However, this objection is very weak because Ramsey in 1928 already admitted it. Instead, he seems to couch omniscience as what ideally could be done: "As it is, we do not know everything; but what we do know we tend to organize as a deductive system and call its axioms laws, and we consider how that system would go if we knew a little more and call the further axioms or deductions there would then be, law (we think there would be ones of a certain kind but don't know exactly what)" (Ramsey, [1928] 1990p, 143). In the prior paper, he was well aware that it is impossible for any individual to know everything. This fact, however, failed to persuade him because he thought it might be ideally still possible.

A second view is that Ramsey came to think it is ideally impossible for someone to know everything. Work needs to be said as to what exactly one might mean by "ideally". Two definitions present themselves. First, Ramsey might think it is physically impossible for such knowledge to be gained in the universe. This is different from the earlier view in that Ramsey might be thinking of some superior, physical intellect to humans knowing everything. The objection would be that there is no such intellect nor could there be any. Second, Ramsey might think it is a metaphysical impossibility to have omniscience. Leaving aside what might be possible in this world, Ramsey might wonder about some God-like intellect knowing everything. His objection would then be that even God could not know everything. Both versions of this view have the advantage that they seem to track pretty closely to the text despite the text's ambiguity; it is pretty straightforward to say that if physical or metaphysical omniscience is impossible, then ideally there is no best system. The disadvantage is that Ramsey gives no arguments as to why he might think it is physically or metaphysically impossible to know everything. Nor is it obvious what argument he might give.

An additional, third view is that Ramsey thinks it is impossible to know what universal propositions would end up as axioms or consequences of those axioms. This objection focuses on the second part of the impossibility claim: that one could not organize it into a deductive system. The idea is that even if it were physically or metaphysically possible to be omniscient, from the standpoint of limited humans one could never know how the propositions known by such an intellect would be organized. So one would not know which universal propositions are fundamental laws of nature or accidental generalizations. The benefit of this view is that it latches onto the heart of the matter for Ramsey in his earlier paper, which was how to distinguish conjunctions from true laws. If it is impossible to distinguish those two in the omniscient intellect, then the account does not serve its primary objective of separating the true from the accidental universals. The cost of this view is that it stretches the text cited above. Ramsey does not say that it is impossible to know the laws when one knows everything—he seems to be stating that it is impossible to know everything. So this view would seem to push the text in ways that are not obvious.

The final and fourth view to consider is that Ramsey is not saying *why* his best systems account is wrong, but is instead arguing that his new view is not the same as his old view.[21] In this section of "General Propositions and Causality", Ramsey has previously provided a positive account of general propositions as rules for judging, and he now wants to differentiate that account from his previous view. He implicitly considers the following worry: could one not say that what distinguishes rules for judging from mere conjunctions is that the former are rules adopted by an omniscient intellect whereas the latter follow from rules not found in such an intellect? The aforementioned passage would say that this is not in fact the case because one adopts general propositions and distinguishes them from mere conjunctions

[21] Thank you to Gerard Rothfus for the discussion as to why this might be the case.

despite the fact that one cannot know everything. This is subtle. The claim Ramsey argues against is that one might identify general propositions as rules for judging through their use in the omniscient intellect. Such a claim would imply that the previous and new accounts of laws are one and the same. They cannot, however, be the same because ignorant human intellects still adopt those rules and distinguish them from conjunctions despite it being impossible for those ignoramuses to be omniscient. The sense of impossibility here is just the mundane fact described in the first view that actual humans are not and will not be omniscient. So the grounds for distinguishing general propositions as rules for judging from conjunctions cannot be their use in an omniscient intellect.

This last view has a big advantage in that it predicts well the structure of Ramsey's argument. Much of the paper that has gone before this provides a positive account for general propositions. After discussing this account, Ramsey abruptly transitions to discussing Braithwaite's account: "This account of causal laws has a certain resemblance to Braithwaite's, and we must compare them closely to see whether it escapes the objections to which his is liable" (Ramsey, [1929] 1990e, 150). Ramsey then summarizes Braithwaite's account and the objections Ramsey leveled against it. It is there that he puts forward the brief summary of his previous account before writing "What is said above means, of course, a complete rejection of this view". It is here that the fourth view has an answer to the question of what the reference for "what is said above". The "what is said above" refers to the larger positive account. Ramsey is saying that the positive account is incompatible with the old view of laws—they are not one and the same because they distinguish general propositions from conjunctions differently. His discussion in the remainder of the paragraph shows continued concern with the question of how to demarcate general propositions for conjunctions. He writes

> We may trust it [a causal generalization] because it is simple, but that is another matter. When I say this I must not be misunderstood; variable hypotheticals are not distinguished from conjunctions by the fact that we believe them, they are

> much more radically different. But the evidence of a variable hypothetical being (often at least) a conjunction, such a conjunction is distinguished from others in that we trust it to guide us in a new instance, i.e. derive from it a variable hypothetical (Ramsey, [1929] 1990e, 150–151)

Ramsey is clearly fixated on distinguishing conjunctions from variable hypotheticals or general propositions. He is arguing that the distinction given by his new account is more radical than the older account. Previously, he distinguished them based on some criteria of simplicity; here is arguing they are to be distinguished based on trust. By trust, he means can be used to forecast future propositions. These do not coincide here—hence his emphasis on them being "much more radically different". The fourth view makes this part of the passage predictable. If the two accounts are different, Ramsey would want to explain in more depth why they are different. The discussion on trust does that.

Furthermore, the fourth view also predicts the brevity of Ramsey's "objection". The "objection" is not really an objection but a point that Ramsey takes to be somewhat obvious: the general propositions believed are believed by ignorant humans, who will never in fact know everything. It is obvious because Ramsey himself previously made the same point in his earlier paper on laws (Ramsey, [1928] 1990p 143). But those general propositions are still "believed", i.e. the rules for judging are adopted, even though omniscience is not in the future. This mundane point does not need to be argued for but Ramsey does need to connect it to why the two accounts of laws are not one and the same. And that is precisely what Ramsey does.

Because the fourth view predicts the text very well without having any obvious difficulties, it is almost certainly the correct account of Ramsey's argument. But if this view is true, then it has the consequence that Ramsey *is not saying why he rejected his best, systems account of laws.* He is doing something slightly different. This then leaves the question of why Ramsey thought the old view had to go.

The immediate candidate and the one defended by Sahlin is that general propositions are not propositions (Sahlin, 1990, 111). Since universal propositions are not conjunctions, they are not propositions. Consequently, they could not be axioms in an organized deductive system of propositions.[22]

The problem with this story is that it is unclear why one could not end up in an idealized omniscient state with every proposition decided and the correct rules just being the ones that agree with all the propositions. Since there are no more facts, every proposition has a truth-value, and the laws of nature are those rules for judging that agree only with those propositions. This would just have the correct laws be those adopted at the Peircian limit of inquiry, and Ramsey seems superficially to be committed to such a view (Ramsey, [1929] 1990e, 161).[23] Sahlin does not have a ready answer to this question, but what he does say seems to suggest something along this view: chances and laws are objective facts that can be settled by other facts, and this implies there is a unique, correct set of chances and laws to be believed.[24] In this context, belief means adopting the corresponding rules for judging. So

[22]Sahlin writes as follows:

> Underlying this change of view on the theoretical status of causal laws is Ramsey's changed attitude to Johnson's and Wittgenstein's analysis of propositions containing the quantifier 'all' and 'some' If these general propositions really are conjunctions and hence express genuine propositions, an axiomatic viewpoint is the obvious one [....] But if these propositions can no longer be identified as conjunctions and thus do not carry truth-value, Ramsey's first theory collapses (Sahlin, 1990, 111–112).

The point is that if they are not propositions, then they cannot be axioms in the simplest, best system. However, this still leaves open the question of whether they can be included in such a system. See below.

[23]I will have a greater discussion of this below.

[24]Sahlin presents the view in his discussion on how Ramsey is really an objective Bayesian:

> One way of looking at Ramsey's theory is to regard the causal laws as limiting our alternatives of action. Two people who both accept the same system of laws ought to be expected to act in similar ways in a great number of choice situations. Viewed in this way, Ramsey's second theory of law and causality forms an interesting superstructure to his theory of probability. We note that Ramsey's theory of probability does not question the agent's beliefs, on the condition that they are coherent. The agent is completely free to assign an event whatever probability he lies. But if Ramsey's theory of law and causality is added on to his probability theory, the situation becomes rather different. What is now a rational assessment of probability is delimited by the laws the agent accepts.

the view being ascribed to Ramsey is that believed general propositions dramatically restrict credences and this leads somehow to objectivity. The natural reason to infer objectivity is that there is a correct class of rules, and these rules would be just the ones believed by an idealized, omniscient agent, i.e. the rules that agree with every proposition decided at the limit of inquiry.

Ramsey's text suggests a reason why one could not end up with a "correct" set of rules in an idealized omniscient state. At the very end of "General Propositions and Causality", Ramsey states that with at least chances, full knowledge of the facts would be *insufficient* for settling the "correct" chance:

> The difficulty comes fundamentally from taking every sentence to be a proposition; when it is seen by considering the position of coincidences that chances are not propositions then it should be clear that laws are not either, quite apart from other reasons (Ramsey, [1929] 1990e, 162).

Here, Ramsey explicitly ties the fact that general propositions are not true propositions with the fact that coincidences exist for chances. What Ramsey is referring to here is that any conjunction of propositions is logically compatible with every chance proposition. Taking the example of an idealized omniscient intellect, a complete description of the facts would not eliminate any chance proposition involving those facts. This is the clue as to why chances are not propositions, i.e. truth-functions of real propositions, and Ramsey suggests the same applies to deterministic general propositions such as laws. So the claim would be that there will be multiple possibly incompatible laws that cohere with a complete description of the universe. That being the case implies that there would be no single, "correct" set of laws

If these laws are of a statistical type—that is, if they contain the notion of chance—the number of rational probability assessments will be severely limited. A combination of theories of this kind takes Ramsey's probability theory still further towards objective Bayesianism (Sahlin, 1990, 112–113).

determined by the facts and held by an omniscient agent at the limit of inquiry because the facts would be insufficient for settling them. This is seen as the radical case of chances, and Ramsey does not hold laws to be fundamentally different from chances.[25]

I want to emphasize that this is the reason why Ramsey rejected his prior account of laws. Chances are not propositions because they are not truth-functions of elementary propositions due to the existence of coincidences. And laws are similar to chances. So laws are not propositions. Thus even a best system of laws known by an omniscient intellect would be radically underdetermined. For this reason, the best system account cannot be right. It is not just that laws are not propositions: it is how they are not propositions. A law is not a summary of the facts; so different laws could be equally compatible with the same summary of facts—even if that summary included all the facts.

Importantly, this means that the more general view of universal propositions as infinite logical products must also be wrong. If laws are similar to chances in that they are compatible with every summary of facts, then even if those summaries are infinite products, they cannot settle the truth of laws. Since laws are just universal propositions about physical facts, those universal propositions cannot be infinite logical products. Their status of universal propositions is what makes them candidates for axioms in the best system. So something must be incorrect with the account of universal propositions as infinite logical products.

Ultimately, Sahlin is right that Ramsey rejects the old view of laws on account of his belief that laws are not propositions. But Sahlin is incorrect in thinking that Ramsey may still view laws as being in some sense "objective". His view on laws, it turns out, is way more radical. Ramsey thinks that no ultimate set of facts can settle the validity of laws. This means that no set of laws would ultimately turn out to be "correct" per the facts. So there would be no logically "true" set of laws found at the limit of inquiry. Consequently, both the view that laws are not propositions and the view that laws are the sort of thing that allows

[25] In fact, he adopts a view that a law just is a special case of a chance. See more below.

them to be compatible with radically different sets of facts is what leads Ramsey to abandon the old view of laws.

While assimilating universal propositions and laws to chances explains why Ramsey would reject his first account of laws, it leaves open the question of why Ramsey came to think universal propositions and laws are like chances. The answer to that question is that Ramsey came to view laws and with them universal propositions as inextricably tied to dispositional propositions and their associated subjunctive conditionals. At the start of "General Propositions and Causality", he explains how universal propositions, which he calls variable hypotheticals, include familiar chestnuts like "All men are mortal" as well as dispositional propositions like "Arsenic is poisonous".[26] He seems to think that these propositions are all one and the same type logically. Importantly, dispositional propositions like "Arsenic is poisonous" are logically compatible with any set of facts involving arsenic and poisoning: such a proposition would be compatible with any world regardless of whether arsenic in fact poisoned anyone in that world. The same fact applies to chances as well; chance propositions like "the chance of the coin landing heads is one" are completely compatible with any sequence of coin tosses. So given their close similarities, an account of chances could be applied to an account of dispositional propositions. And Ramsey thinks dispositional propositions are the same thing as ordinary universal propositions like "All men are mortal". So the link to laws and chances runs through dispositional propositions.

Chronologically, Ramsey seems to have come to this conclusion by thinking about certain problems in the philosophy of physical sciences after having read Norman Campbell's *Physics: The Elements*. He mentions Campbell in three locations in his published works: he cites Campbell in "Truth and Probability", he discusses Campbell briefly in "Causal Qualities", and he criticizes Campbell's views on chance extensively in "Chance". The last is important

[26] Ramsey clearly lumps these two together when he writes: "But right again in radically distinguishing them [finite conjunctions] from the other kind which we may call *variable hypotheticals*: e.g. Arsenic is poisonous: All men are mortal" (Ramsey, [1929] 1990e, 145).

because Campbell thinks that chance is the absence of law; it is fortune's duo to law's fate. As I argue below, Ramsey greatly disagrees with Campbell here by taking laws to be a type of chance. This is not a verbal disagreement because Ramsey includes the propositions Campbell counts as laws under Ramsey's notion of variable hypothetical. So Ramsey is influenced by Campbell's ideas about laws and chances.

One crucial feature of Campbell's ideas about laws is that laws include dispositions. Campbell writes a great deal about physical laws. Importantly, he thinks the fundamental features of laws is a combination of invariability and generality that he calls uniform association.[27] Many propositions assert relations that involve uniform associations in this sense. Campbell includes these like classic physical laws like Ohm's law and Hooke's law as well as dispositional claims like the dissolubility of silver in nitric acid and even existential assertions like "silver exists".[28] While Ramsey's views on laws are clearly different from Campbell, it seems credible that he follows Campbell in including propositions that assert dispositions as laws.

Critically, almost all of Ramsey's writings about Campbell were authored in 1928 and 1929.

[27]Campbell initially defines a law as an assertion of a relation involving uniform association:

> Laws are propositions asserting relations which can be established by experiments or observation. The terms between which the relations are asserted consist largely or entirely of judgments of the material world, immediate or derivative, simple or complex; the relation asserted, if not always the same, have always a common feature which may be described as "uniformity of association." In other words, a law always asserts that A is uniformly associated with B, where A and B are "phenomena," knowledge of which is derived from judgements of the external world (Campbell, 2013, 38–39).

Uniformity of association is later defined as constituting some form of invariability and generality. Invariability is defined as an ability to indefinitely reproduce phenomena: ""Invariability," to my mind, and I think to that of Mill, implies simply the possibility of indefinite repetition; a connection is invariable when, after having observed it on one occasion, I can return later and always observe it again" (Campbell, 2013, 69). Generality is understood to mean the connection asserted by a law can be found in many phenomena: "The observation between which the invariable connection is established must be so general that we are sure that they will occur again; and the more general they are and the more likely they are to occur again, the more important and valuable will be the invariable relation" (Campbell, 2013, 69). He concludes that it is these two features that identify uniform association: "Invariability and generality are then two of the necessary elements which give to laws their importance. It is the combination of these two closely associated elements which I intend by the word "uniformity"" (Campbell, 2013, 70).

[28]Campbell lumps these all together into his discussion of the complexity of laws and how the former dispositional laws are vital to defining what scientists mean by "silver exists". In general, he believes that the interdependence of laws on one another is a universal feature of science—a law holism. See Campbell's

The only exception is a citation in "Truth and Probability". My claim is that by working through his objections to Campbell in 1928, Ramsey came to change his mind about universal propositions and laws. He follows Campbell in assimilating ordinary universal propositions with dispositional propositions, but in disagreeing with Campbell over chances, he found a close similarity between laws and chances. This later proved crucial in his conclusion that universal propositions are not infinite logical products. My account contrasts with the prevalent view among historians like Misak that Ramsey developed his theories of laws by reflecting on Peirce. The problem with that view is that Ramsey largely seems to have internalized Peirce's views from 1924 to 1927—before he changed his mind about laws—and Ramsey fails to discuss Peirce on laws and chances in any depth. In comparison, 1928 and early 1929 seems to have been crucial for Ramsey's change of mind, and it is during this period that he writes most extensively about Campbell and chances. So it stands to reason that this is the most likely vector for Ramsey's shift in views on laws and chances.

Note that this amounts to a partial answer to the central interpretative question of this chapter: how Ramsey's new view of laws differs from his old view of laws? If laws are similar to chances, then a complete specification of the facts will not settle which laws are correct. So whatever the new view of laws happens to be, it cannot be the same as the old view. This goes back to my interpretation of Ramsey's argument from "General Propositions and Causality". Recall that my interpretation holds Ramsey's point to be that the two accounts are not one in the same. They are not one in the same because universal propositions are believed by oblivious humans, who will never know everything. The laws of the new view are supposed to be radically different from the old view. A correct set of laws could not even be settled by an omniscient intellect. This suggests that a crucial feature of the new view of laws that separates them from the old view is that they are not truth-functions of particular propositions.

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discussion in chapter three sections on unrecognized laws, concepts, and defining and non-defining properties (Campbell, 2013, 43–50).

There is still a puzzle here. If laws are radically different from conjunctions and a correct set of laws is not determined by the facts, why does Ramsey still talk about "our system" being uniquely determined? Why does he still seem to be committed to a pragmatist notion of truth? This cannot be answered until I have explored Ramsey's new view of laws.

Summarizing, Ramsey did not reject the old view of laws on account that he held omniscience to be impossible, in whatever sense one understands "impossible". Instead, he held that the mundane fact that no actual human will be omniscient to argue that his new view of laws as rules for judging is not the same as the old view. His real reasons for rejecting the old view stem from a belief that laws are not propositions and a belief that incompatible laws are compatible with the same complete description of the world. The next important step would be to examine the new view of laws that emerge from these twin observations. This is important if I am to resolve the question about the extent of Ramsey's commitment to the Peircian theory of truth. An adequate resolution would go a long way to providing a full account of Ramsey's view on laws and how those laws fit with Ramsey's cognitive psychology and decision theory.

4.4 Laws as Rules for Judging

Recall that so far I have shown that an outstanding problem from the secondary literature is how Ramsey's best system view of laws differs from his view of laws as rules of judging. I have shown in the previous section that Ramsey does not think laws are truth-functions of particular propositions. But I still need to explain what the new account happens to be. In "General Propositions and Causality", Ramsey puts forward an account of universal propositions as rules for judging. He calls these rules for judging "variable hypotheticals" (Ramsey, [1929] 1990e, 145), which he distinguishes from finite conjunctions such as "All the men in Cambridge voted". Ramsey motivates his new account of universal propositions by first discussing how universal propositions are different from and similar to conjunctions. He then provides a positive characterization of his view on universal propositions. I will follow his exposition here. First, I discuss the reasons he gives for why universal propositions are not conjunctions followed by how they are similar to conjunctions. Second, I examine the specific text surrounding the new view of laws. This leads to a central question for this chapter: how does this view on laws connect with Ramsey's decision theory? Laws are intended to be the central pillar of his decision theory and philosophy of science. However, it is unclear how the informal account of laws documented here squares with that decision theory and philosophy of science. Third, the clue I argue for resolving these two key questions is a remark Ramsey makes regarding laws and chances. This sets up the next section, where I discuss chances in depth.

4.4.1 Universal Propositions and Conjunctions

Ramsey considers it important to start his "General Propositions and Causality" by discussing the ways universal propositions are different from and similar to conjunctions. He uses this as stage-setting for his own positive account of universal propositions as rules for judging. So it would be valuable to review what those reasons are in depth. His discussion is surprisingly compact. This will set my own discussion of Ramsey's positive account.

He begins by distinguishing two types of universal propositions. There are those that are conjunctions and those that are not conjunctions. He calls the latter variable hypotheticals, and I adopt his terminology from here. He writes:

> As everyone except us has always said these propositions are of two kinds. First *conjunctions*: e.g. 'Everyone in Cambridge voted'; the variable here is, of course, not people in Cambridge, but a limited region of space varying according to the definiteness of the speaker's idea of 'Cambridge', which is 'this town' or 'the town

> in England called Cambridge' or whatever it may be.
>
> Old-fashioned logicians were right in saying that these are conjunctions, wrong in their analysis of what conjunctions they are. But right again in radically distinguishing them from the other kind which we may call *variable hypotheticals*: e.g. Arsenic is poisonous: All men are mortal (Ramsey, [1929] 1990e, 145).

The idea is that one must distinguish between universal propositions which are limited and those that are unlimited. "Everyone in Cambridge voted" is of the former kind since one could replace the "everyone" with a list of people in a delimited physical space. The proposition would then be a conjunction. In contrast, variable hypotheticals are open-ended in the sense that the "All" in "all men are mortal" could not be replaced with a list of people or "poisonous" could not be replaced with a list of poisonings done by arsenic.

Ramsey then spends about a page trying to say how variable hypotheticals are like conjunctions and not like conjunctions:

> *Why are these not conjunctions*?
>
> Let us put it this way: What have they in common with conjunctions, and in what do they differ from them? Roughly we can say that when we look at them subjectively they differ altogether, but when we look at them objectively, i.e. at the conditions of their truth and falsity, they appear to be the same (Ramsey, [1929] 1990e, 145).

He wants to compare and contrast variable hypotheticals with conjunctions. His first gloss is that when viewed from a particular agent's perspective, they cannot be conjunctions, but when trying to view them from their truth-conditions, they seem like conjunctions. This is opaque at this point but one should keep in mind that Ramsey wants to distinguish how laws are used by an agent from how their supposed truth is evaluated.

His discussion from this point leads with how variable hypotheticals differ from conjunctions and how they are like conjunctions. He presents arguments on both sides. The remainder of this section looks at the first set of arguments for how variable hypotheticals are not like conjunctions and at the second set of arguments for how they are similar to conjunctions.

The next portion of his argument looks at how variable hypotheticals differ from conjunctions. This is the passage that most commentators focus on, and four arguments are typically identified (though two are taken to be much the same). Ramsey lists them as $(a), (b), (c)$ and (d). I will start as with (a):

> $(x).\phi x$ differs from a conjunctions because
>
> (a) It cannot be written out as one (Ramsey, [1929] 1990e, 145).

This argument is typically taken to be the claim that variable hypotheticals cannot be symbolically expressed. It is often connected with a comment Ramsey makes in the following pages that with variable hypotheticals, one is forced to have "a theory of conjunctions which we cannot express for lack of symbolic power" (Ramsey, [1929] 1990e, 146).[29]

Holton and Price quizzically consider this argument to be about an inability to express concepts involving the infinite. They remark that the argument taken in this way is fallacious since obviously, one has the symbolic power to express such concepts. For example, one divided by three might have a decimal expansion that is infinite but one can easily represent it symbolically with $\frac{1}{3}$ (or one-third if one prefers English). The same goes for even more exotic things such as the ratio of a circle's circumference to its diameter. The symbol π seems to work very well there (Holton and Price, 2003, 327). Because such an argument is very bad, Holton and Price think that this is not Ramsey's point, and they consider the real argument to be akin to the one found in (b) and (c) (see above for a discussion concerning "unsurveyability").

[29]I will examine this supposed connection further down. I argue that it is a bad reading.

This view is not right. Taken literally, it seems Ramsey's point here is elementary: one cannot physically write out every instance of a variable hypothetical. If one is to express the formula symbolically, one has to use shorthand. For example, in "Facts and Propositions", Ramsey adopts Wittgenstein's shorthand with the three dots notation: "About these I adopt the view of Mr. Wittgenstein that 'For all x, fx' is to be regarded as equivalent to the logical product of all the values of 'fx', i.e. to the combination fx_1 and fx_2, and fx_3 and ..." (Ramsey, [1927] 1990d, 48–49). It is the fact that one has to use the "..." instead of actually writing out the instances of the variable hypothetical that Ramsey is appealing to here. But one is still able to express the variable hypothetical with shorthand such as "$\forall x fx$" or the addition of the three dots "...". Ramsey's point isn't that one cannot symbolically express variable hypotheticals, but that one cannot express them as conjunctions. This is odd if they are to be considered conjunctions, since any conjunction should, with enough time and space, be able to be expressed using the standard notation.

Ramsey's next two arguments are considered (by him at least) to be largely the same. He presents them back to back, with little commentary:

> (*b*) Its constitution as a conjunction is never used; we never use it in class-thinking except in its application to a finite class, i.e. we use only the applicative rule.
>
> (*c*) [This is the same as (*b*) in another way.] It always goes beyond what we know or want; cf. Mill on 'All men are mortal' and 'The Duke of Wellington is mortal'. It expresses an inference we are at any time prepared to make, not a belief of the primary sort.
>
> A belief of the primary sort is a map of neighbouring space by which we steer. It remains such a map however much we complicate it or fill in details. But if we professedly extend it to infinity, it is no longer a map; we cannot take it in or steer by it. Our journey is over before we need its remoter parts (Ramsey, [1929] 1990e, 145–146)

Ramsey argues that (*b*) and (*c*) are functionally the same argument. That argument's expression in (*b*) is that, unlike any actual conjunction, all of the instances warranted by a variable hypothetical are never used. Instead, one only uses a finite set of instances. For Ramsey, this indicates the interest in the variable hypothetical is its use as a rule. He then makes the point with (*c*) that one's interest never amounts to the entire set of instances but only a part of those instances. He cites Mill here, who remarks in *A System of Logic* that "all inference is from particulars to particulars: General propositions are merely registers of such inferences already made" (Mill, 1843, 216). Ramsey then gives a visual metaphor to accentuate this point by comparing singular propositions to maps. One is only interested in any finite section of a map. Primary beliefs, i.e. beliefs in singular propositions, are like finite maps. In contrast, variable hypotheticals are like an infinite map. Infinite maps, however, would never be used in their entirety since one's journey would always be finite. So the real utility of such infinite maps comes from finite subsections of them.

Two things should be remarked about this argument. First, the problem Ramsey is pointing out concerns the infinite and not "unsurveyability" as Holton and Price hold. Second, the argument has a certain weakness that can only be partially addressed.

Recall that Holton and Price argue that Ramsey's argument here does not depend on the infinite nature of variable hypotheticals. Instead, they claim that Ramsey's point would hold of sufficiently large finite numbers, and this means that Ramsey's point is that some propositions extend beyond the evidence of those propositions (Holton and Price, 2003, 329). For example, "all the grains of sand at Laguna Beach are less than one centimeter in diameter" would go beyond any evidence that I or another ordinary person could collect for its truth, even though there is only a finite amount of such grains. This objection, however, does not make sense. Using some probability theory, statistics, physics, and a bit of experiment, one can provide strong evidence for the sand claim so as to believe it with near certainty. Holton and Price's objection only makes sense if one has a very "strong" notion of evidence.

So propositions involving large finite numbers can be justified on inductive grounds without any difficulty.

Furthermore, Ramsey's point here does not seem to involve evidence at all. He does not mention evidence for variable hypotheticals but discusses their utility. While propositions involving large finitary numbers, such as a large conjunction, may not be used in its entirety for some agents, Ramsey's point is that for other agents they may turn out to be used. For example, Doug the marketing officer may not care whether it is true that his marketing program requires 10^{20} compute cycles but his computer engineer Clara might. Ramsey is making the point that what separates variable hypotheticals from conjunctions is that no possible agent could make use of every instance provided by the variable hypothetical.

This leads to an obvious complaint about Ramsey's argument: it is just unsound. Agents might care about propositions involving the infinite. This is no grounds to distinguish conjunctions from variable hypotheticals. For example, someone playing the game of guess the bigger number might be interested in a theorem that claims for any move an opponent might make, there is a winning strategy. This covers an infinite number of moves. Ramsey's reply to this is that the objection mistakes the rule for the desired outcome: I want to win against this move—not the infinite set of moves. However, it is less clear he has a reply against agents who are interested in the infinite for intellectual or aesthetic reasons. Mathematicians studying the different cardinal infinities might not care about any practical application. If so, then the claim that what separates conjunctions from variable hypotheticals is that one can only care about the totality of instances in the former is just false.

Summarizing, arguments (*b*) and (*c*) are directly concerned with the infinite character of variable hypotheticals. Ramsey's concern does not seem to extend to other types of propositions as Holton and Price suggest. But it does leave Ramsey open to the objection that agents might be interested in the infinite.

The final argument Ramsey gives has received the least amount of attention. Labeling it (d), he writes:

> (d) The relevant degree of certainty is the certainty of the particular case, or of a finite set of particular cases; not of an infinite number which we never use, and of which we couldn't be certain at all (Ramsey, [1929] 1990e, 146).

Like the other arguments, this one is compact. Unlike the other arguments, this one has not received much attention in the secondary literature. It is briefly cited, without discussion, by Holton and Price (Holton and Price, 2003, 328). They imply that the argument supports their thesis that Ramsey's core argument is that the variable hypothetical exceeds its evidence base. This has some plausibility given Ramsey's discussion of certainty here. But this is not Ramsey's conclusion from the argument. Note that Ramsey says the "relevant degree of certainty". This almost surely is a reference to the degree of probability or credence one assigns to the variable hypothetical. Ramsey is not saying that one cannot assign a credence there or that there is a difference between the credence in a variable hypothetical (were it treated as a proposition), but that the correct credence to assign a variable hypothetical is just the credence of a singular proposition or conjunction of propositions. There is no "exceeding the evidence" idea baked in here. So what is Ramsey's point?

The fact that he is referencing credences should make his points somewhat obvious. Namely, that if one treats a variable hypothetical as an infinite conjunction, then the credence is bounded by any instance of that variable hypothetical, and that the credence of a conjunction will almost certainly differ from the credence of the universal proposition. Or to put it another way, the credence in a conjunction is always less than or equal to its conjuncts, and it is this that people are concerned with—not the credence of an infinite conjunction whose probability must necessarily be lower. Taking the bounding idea and the limiting idea in turn, I can illustrate Ramsey's point.

While Ramsey does not elaborate, his idea prefigures an idea that is later developed in the inductive logic of first-order languages. Namely for any universal sentence:

$$\Pr(\forall x \phi(x)) = \lim_{m \to \infty} \Pr(\bigwedge_{i=0}^{m} \phi(t_i))$$

This follows from what is now called the Gaifman condition (Williamson, 2017, 19). An implication of it is that for any $m \in \mathbb{N}$:

$$\Pr(\forall x \phi(x)) \leq \Pr(\bigwedge_{i=0}^{m} \phi(t_i))$$

The upshot is that as Ramsey suggests, the degree of belief in a variable hypothetical, treated as a proposition, is at most that given by the observed instances.

Now, this point applies to finite conjunctions as well: any sub-conjunction of a given conjunction will bound the credence of the total conjunction. However, when Ramsey says "not of an infinite number which we never use, and of which we couldn't be certain at all", he seems to be referring to a fact that gets expressed in his discussion of how variable hypotheticals resemble conjunctions. That fact is that when considering the truth of variable hypotheticals, we consider them as if their truth-conditions are given by the truth-possibilities of their instances. In other words, we are considering an infinite truth-table. In the context of degrees of certainty, this would be a probability table that gives the probability distribution for all truth-possibilities (see figure 4.1). Now Ramsey's complaint is that this table is fiction since physical beings can obviously only "consider" finitely many probabilities. But if this is the case, then there is no way to marginalize and compute the probability of a variable

P	Q	R	$\dots$	$\Pr(P = p, Q = q, R = r, \dots)$
T	T	T	$\dots$	α
F	T	T	$\dots$	β
T	F	T	$\dots$	γ
$\vdots$	$\vdots$	$\vdots$	$\dots$	$\vdots$

Figure 4.1: A joint probability distribution table involving an infinite number of elementary propositions. α, β, γ represent probability assignments to their rows (conjunctions).

hypothetical directly, since that would require humans to compute an infinite table. Instead, humans would have to only recourse to some finite conjunction of instances from that variable hypothetical. So the bound provided by some finite conjunction in fact becomes the only credence to consider here. This is different from finite conjunctions because one could, hypothetically at least, consider a really large conjunction and compute its credence; one is not restricted to some proper sub-conjunction.[30] So the difference is those variable hypotheticals have their relevant credences given by a proper sub-conjunction, whereas ordinary conjunctions could be gotten by the whole conjunction.

The last point is that Ramsey seems to think that the credence in a universal proposition taken as an infinite conjunction would in fact be different from the credence of the universal proposition. Ramsey states that the relevant usage is the finite number of observations "not of an infinite number which we never use, and of which we couldn't be certain at all" (Ramsey, [1929] 1990e, 146). This last line is an indication that Ramsey is thinking in the context of an infinite sequence of observed trials where the probability assigned to any finite sequence will almost certainly differ from the true probability of the full sequence. The point is familiar to critics of the frequentist theory of probability: the probability of an event such as a coin toss is not given by any actually observed sequence of events but by the limiting relative frequency and the two almost always differ. For example, a frequentist would not claim that the probability of a coin coming up heads is given just by the ratio of observed heads to total

[30] Notice that "hypothetically" does a lot of work here. For sufficiently large numbers of propositions, it is very, very likely physically impossible to compute the marginal distribution.

tosses because it might turn out that one just got lucky; the frequentist claim would instead have to be something about the existence of a limiting relative frequency and its value. Ramsey's point here is that the same reasoning applies to universal propositions understood as infinite conjunctions. The observed conjunctions will have some probability and this probability will almost certainly differ from the probability of the infinite conjunction—in fact, it will have to be at least equal to or lower. In some probability functions, as Carnap later discovered, the two will in fact differ because a universal proposition will be probability zero.[31] So the probability assigned to a universal proposition as an infinite conjunction is useless while it is always the bounding case of the finite conjunction that proves important for deliberation.

Summarizing, Ramsey's four arguments can be reduced to three claims:

1. Variable hypotheticals are unlike conjunctions because they cannot be literally written out as a conjunction.

2. Variable hypotheticals are unlike conjunctions because one is never (so it is claimed) concerned with all the instances of the variable hypothetical whereas one can be concerned with all the conjuncts of a conjunction.

3. Variable hypotheticals are unlike conjunctions because the variable hypotheticals must have their credences be given by a sub-conjunction and this almost always differs from the infinite case.

His strategy is to use these differences to argue that variable hypotheticals are not propositions. Before he can do that, however, he needs to consider why one might consider them propositions. These are the reasons given from an "objective" point of view.

[31]Carnap gets into trouble because his c^* and other inductive methods assign probability by counting cases, which in the limit will assign probability zero. See his initial discussion in Carnap, 1945 and further development in Carnap, 1950 and Carnap, 1952.

There are two arguments for why one might consider a variable hypothetical to be a conjunction. They are that (a) variable hypotheticals entail every finite conjunction and (b) when considering its truth-conditions, one must think of it as a conjunction. I take each in turn.

The first one is just an observation about how variable hypotheticals can churn out every finite conjunction. He writes:

> $(x).\phi(x)$ resembles a conjunction
>
> (a) In that it contains all lesser, i.e., here all finite, conjunctions, and appears as a sort of infinite product (Ramsey, [1929] 1990e, 146).

The observation that makes variable hypotheticals appear as an infinite product is that they contain every finite conjunction. Ramsey does not provide any remark here about why this resemblance is unpersuasive. An argument, however, can be made that this view is misleading. Consider the observation that a property may hold of every finite subset of an infinite set need not hold for the infinite set as a whole. For example, every finite subset of the natural numbers will have a greatest member, but the natural numbers themself will not have a greatest member. Analogously, every implication of a variable hypothetical might be a conjunction does not imply that the variable hypothetical itself is a conjunction. So this argument for resemblance is not very strong.

In the second argument, Ramsey considers how one thinks about the truth-conditions of a variable hypothetical:

> (b) When we ask what would make it true, we inevitably answer that it is true if and only if every x has ϕ; i.e. when we regard it as a proposition capable of the two cases truth and falsity, we are forced to make it a conjunctions, and to

> have a theory of conjunctions which we cannot express for lack of symbolic power (Ramsey, [1929] 1990e, 146).

This argument has been repeatedly misunderstood by commentators. Most philosophers consider the argument to be one that shows how variable hypotheticals are not conjunctions.[32] However, Ramsey is explicitly considering why one might think a variable hypothetical resembles a conjunction. He is not arguing that it is not one at this point. That was done previously. What he points out here is more interesting. Namely treating a variable hypothetical as if it were true forces one to consider its truth-conditions as being equivalent to the truth-conditions of a conjunction. That is, one must consider an infinite truth-table with an infinite number of truth-possibilities and consider the rows where every instance of the variable hypothetical is true. One does this even though one cannot really create such a truth-table.

Another way to put Ramsey's point is that it is not about expressing the variable hypothetical as a conjunction, but instead, it is about considering a variable hypothetical's truth-conditions through the Tractarian device of the truth-table and truth-possibility. When viewed as a truth-table, one sees that the rules for evaluating the "truth" of a variable hypothetical are the same as that given by a conjunction: one checks all the truth-possibilities where its conjuncts are true. This is a very similar method to considering what it would

[32]Consider Holton and Price, who conflate this passage with the earlier point (*a*):

> The first argument Ramsey gives in 'General Propositions and Causality' is the consideration that a universally quantified sentence can be written out, whereas an infinite conjunction cannot. If we treat universally quantified sentences as expressing propositions we will be forced to see them as equivalent to conjunctions which, since they are infinite, 'we cannot express for lack of symbolic power'. But that is no good: 'what we can't say we can't say, and we can't whistle it either'. (1990a, p. 146) (Holton and Price, 2003, 327)

They latch onto the last point about a lack of symbolic power and the reference immediately afterward to Wittgenstein. Sahlin likewise cites this passage in support of Ramsey's argument for why variable hypotheticals are not conjunctions (Sahlin, 1990, 105–106). Misak takes this passage to also give the essence of Ramsey's argument that we lack symbolic power to express variable hypotheticals as conjunctions: "It [a variable hypothetical] cannot be thought of as infinite conjunctions, for we lack the symbolic power or the capacity to express an infinite statement" (Misak, 2016, 190).

take to actually compute the credences of a variable hypothetical using a joint probability distribution table. Ramsey's remark at the end is that one does not in fact do this for the same reason one cannot use the joint distribution probability table. There just is not enough mental or computational power to store the table for any physical being. Hence why when considering these truth conditions, one has to resort to the "..." in the displayed table. It is precisely those "..." that Ramsey is likely referring to when he discusses the "lack of symbolic power" here.[33] It is not that one cannot express an infinite conjunction; it is that one cannot consider such a conjunction's truth-conditions due to an inability to complete the required truth-table.

The upshot of Ramsey's discussion is that the reasons why a variable hypothetical resembles a conjunction are insufficient for rendering it as one. Returning to Ramsey's earlier point, the variable hypothetical subjectively is not a conjunction but objectively is like a conjunction. As discussed in a previous chapter, propositions are not objective entities but subjective violations of psychological expectations. This means the subjective reasons weigh very heavily here. Ramsey closes with the conclusion that variable hypotheticals are not conjunctions: "If then it is not a conjunction, it is not a proposition at all; and then the question arises in what way can it be right wrong" (Ramsey, [1929] 1990e, 146). It is here that Ramsey starts his positive account.

Note that this reasoning about the subjectivity of laws pairs well with Ramsey's remark that laws are like chances. If variable hypotheticals do not supervene on the facts, like chances, then they are more subjective than objective. Ramsey's argument over conjunctions points to this same fact about the importance of the subjectivity of laws. In essence, Ramsey is arguing that because of their subjectivity, laws are not conjunctions. This is close to the

[33] This is a point that Wittgenstein seems to have latched onto if Anscombe is to be believed. She discusses how an infinite truth-table became a real sticking point for Wittgenstein, with the "..." used in that construction something he referred to as "dots of laziness" (Anscombe, 1959, 135). This is likely why Ramsey immediately follows reason (*d*) with a reference to Wittgenstein, who in the Tractatus hoped to show how these "dots of laziness" work despite being unable to say them: "But what we can't say we can't say, and we can't whistle it either" (Ramsey, [1929] 1990e, 146).

argument that because chances are not determined by the facts, they are subjective and fictional.

In summary, Ramsey provides three arguments for why variable hypotheticals differ from conjunctions and two arguments for why they resemble conjunctions. He concludes that since variable hypotheticals cannot in fact be written as conjunctions, since they are not in themselves the ultimate target of interest, and since their assigned credences must be the credences of a proper sub-conjunct, variable hypotheticals viewed subjectively are not conjunctions. This is enough to eliminate them as conjunctions, and because they are not conjunctions, they are not propositions.

4.4.2 Universal Propositions as Rules for Judging

So far, I have documented why Ramsey does not think variable hypotheticals are propositions. He considers the ways variable hypotheticals are like and dislike conjunctions, and he finds that they are not conjunctions because they do not operate as conjunctions when considered subjectively. Because they are not conjunctions, he concludes they are not propositions. Ramsey then moves on to providing the full positive account of variable hypotheticals. His account is very short, and it is couched in an argument over how one can still agree and disagree over attitudes that are not propositions. I argue in this section that there is not much to go on here. In particular, one does not know how Ramsey's proposal fits with his decision theory.

Ramsey's positive account of variable hypotheticals as rules for judging occurs in a section where he has just considered how one might agree and disagree with mental attitudes that are not propositional. He first describes two ways that a proposition might be considered right and wrong. The first is in the case of its truth and falsity. Someone believing a proposition P can be judged right if P is true or can be judged wrong if P is false (Ramsey, [1929] 1990e, 146–147). The second is in the case of unsettled truths. If it is unknown whether P is true, but Jones still believes P, one might hold Jones to be wrong for believing P without oneself believing P or not P. Ramsey takes this kind of rightness and wrongness to apply more widely than just propositions. He enumerates a list of attitudes that might apply to a variable hypothetical (Ramsey, [1929] 1990e, 147–148). It is here that he begins discussing what are variable hypotheticals.

The example Ramsey considers is "all men are mortal". He asks what is it:

> To believe that all men are mortal—what is it? Partly to say so, partly to believe in regard to any x that turns up that if he is a man he is mortal. The general belief consists in
>
> (a) A general enunciation, (b) A habit of singular belief.
>
> These are, of course, connected, the habit resulting from the enunciation according to a psychological law which makes the meaning of 'all' (Ramsey, [1929] 1990e, 148–149).

A variable hypothetical consists of an enunciation of the hypothetical and a habit with singular beliefs. The habit is to infer when an object activates one belief, it activates another. The enunciation and the habit are connected by a psychological law: when one enunciates the variable hypothetical, one also follows the correct habit. In the case of "all men are mortal", the verbal statement of the sentence is underpinned by a willingness to go from beliefs involving men to beliefs about their mortality.

After reviewing the different attitudes one can have to such variable hypotheticals, Ramsey then argues that underlying habit over singular beliefs is one of many that a person uses to navigate the future. He writes that:

> Variable hypotheticals or causal laws form the system with which the speaker

> meets the future; they are not, therefore, subjective in the sense that if you and I enunciate different ones we are each saying something about ourselves which pass by one another like ‘I went to Grantchester’, ‘I didn’t.’ For if we meet the future with different systems we disagree even if the actual future agrees with both so long as it *might* (logically) agree with one but not with the other, i.e. so long as we don’t believe the same things. (Cf. If A is certain, B doubtful, they can still dispute) (Ramsey, [1929] 1990e, 149).

Two things are important here. First, variable hypotheticals together constitute a system used to decide future action. Second, Ramsey emphasizes that variable hypotheticals are not purely subjective. Different variable hypotheticals allow for possible disagreement over future outcomes even when the actual future might agree with both of these variable hypotheticals. Interestingly, the example of disagreement given by Ramsey is a disagreement over partial beliefs. A variable hypothetical that leads to certainty over propositions disagrees with one that leads to doubt over those same propositions.

At this point, Ramsey characterizes variable hypotheticals famously as rules for judging:

> Variable hypotheticals are not judgments but rules for judging ‘If I meet a ϕ, I shall regard it as a ψ’. This cannot be *negated* but it can be *disagreed* with by one who does not adopt it (Ramsey, [1929] 1990e, 149).

He explicitly restates several claims he has made to this point. First, variable hypotheticals are not judgments, i.e. not propositions. Second, he describes them as habits used for making inferences between beliefs in propositions. Third, they are not true or false—cannot be negated—but can be disagreed with. None of these observations are new: he has made each of them previously.

And that is it for the positive account of variable hypotheticals as rules for judging. Ramsey considers nothing mysterious in his account except habit. He writes that any disagreement over a variable hypothetical must itself be a variable hypothetical too, but he finds this to not be circular because “all belief involves habit, so does the criticism of any judgment whatever, and I do not see anything objectionable in this” (Ramsey, [1929] 1990e, 150).

The story is then one where variable hypotheticals are habits that associate singular beliefs. Ramsey’s rules for judging are what have been called by Ryle “inference tickets” (Ryle, 1949, 117). Someone asserting that “Arsenic is poisonous” conveys that they have the habit of inferring from when a person ingests arsenic, they will become sick and die. The habit of making this inference and behavior of asserting the variable hypothetical constitute the entirety of the variable hypothetical “Arsenic is poisonous”. Disagreements over variable hypotheticals reflect different inferences people will make, and the correctness or incorrectness of variable hypotheticals are determined in part by the credences individuals form guided by those variable hypotheticals.

An initial observation is that the positive account given in the text is a very close fit to Ramsey’s theory of cognition discussed in a previous chapter. Recall that Ramsey views cognition to consist of a conscious and an unconscious process. The unconscious process consists of habits or dispositions to behavior. Those habits collectively determine behavior and lead to psychological expectations. Violations of those psychological expectations lead to the activation of the conscious process, which identifies the violation of expectation with a singular proposition. The statements about the unconscious habits very closely match the role variable hypotheticals are supposed to take in Ramsey’s philosophy of science. Applying Ramsey’s philosophy of logic, one might say that they are meant to be habits formed on reflection to regulate future behavior. Variable hypotheticals are the logical rendition of the dispositions that lead to action.

Despite this account, it is unclear what exactly variable hypotheticals are in Ramsey’s logic for decision. The account given here is informal and very vague. While Ramsey says there is nothing mysterious here about variable hypotheticals being in part habits, the story is a bit more complicated.

In the reconstructed decision theory I presented in a previous chapter, there were three types of propositions considered by agents: observational, singular theoretical, and laws. Observational propositions are straightforward. They are just the sub-algebra built out of some privileged partition that constitutes all the propositions the agent ultimately cares about. I provided an account of singular, theoretical propositions, which are also a sub-algebra that has the observational propositions as a coarsening. The main difference between observational and singular theoretical propositions is that credences over the theoretical propositions can only be partially measured because the conditions of the wagers used for measuring those propositions can only rely upon observational propositions. This is supposed to reflect the fictitiousness of those theoretical propositions.

That leaves laws. I noted at the time that laws play an important role in measuring the credences of singular theoretical propositions. But I did not provide an account for laws. Here I have surveyed Ramsey's later view of laws as variable hypotheticals, which are rules for judging. This story needs to be made exact with respect to the decision theory described above. Nothing said so far, however, sheds light on what laws are supposed to be in this decision theory. Showing what they are in the decision theory would help in resolving the chief riddle of this chapter about how the new theory differs from the old theory or the extent of Ramsey's commitment to the Peircian account of truth.

So this is the key question going forward: what are variable hypotheticals in the context of Ramsey's decision theory?

4.4.3 Laws and Chances

A clue for what laws happen to be in Ramsey's decision theory is provided in "General Propositions and Causality". The key passage comes in an extended discussion of the fictitiousness of causal laws.[34] At this point, he is concerned with the apparent objectivity of causal laws. A principal target of this discussion is the view of causal laws requiring a law of causation or necessary connection. Ramsey argues that no such law of causation is needed:[35]

> It is clear that the notion and use of laws presupposes no 'law of causation' to

[34]The discussion starts after Ramsey has shown that he can resolve the problem of unknown laws that afflicted Braithwaite's similar view to his own (Ramsey, [1929] 1990e, 151–153). He starts this section by considering the following question:

> What we have said is, I think, a sufficient outline of the answers to the relevant problems of analysis, but it is apt to leave us muddled and unsatisfied as to what seems the main question—a question not of psychological analysis but of metaphysics which is 'Is causation a reality or a fiction; and, if a fiction, is it useful or misleading, arbitrary or indispensable?' (Ramsey, [1929] 1990e, 153).

He argues that causal laws are ineliminable from human thought because they are "at the root of all praise and blame and much discussion" (Ramsey, [1929] 1990e, 153–154). He then argues that variable hypotheticals are essential for the discussion of what are now called contrary-to-fact counterfactuals such as "if I had eaten cake, I would have gotten a stomach ache" even though I did not eat cake (Ramsey, [1929] 1990e, 154–155). This is where the famous Ramsey test footnote is mentioned. From there, he argues that the use of variable hypotheticals occurs in more than just counterfactuals involving behavior but any sort of hypothetical (Ramsey, [1929] 1990e, 156). He defines causal laws as a particular sort of hypothetical involving the subjunctive mood:

> One class of cases is particularly important, namely those in which, as we say, our 'if' gives us not only a *ratio cognoscendi* but also a *ratio essendi*. In this case which is e.g. the normal one when we say 'If p had happened, q would have happened', $p \supset q$ must follow from a hypothetical $(x) . \phi x \supset \psi x$ and facts $r, pr \supset q$ being an instance of $\phi x \supset \psi x$ and q describing events not earlier than any of those described in pr. A variable hypothetical of this sort we call a causal law (Ramsey, [1929] 1990e, 157).

The formulation here involves what might be called *ceteris paribus* conditions with the proposition r, which are included in the antecedent of the variable hypothetical $\phi x \supset \psi x$.

[35]Ramsey's argument for why this is the case relies upon the claim that one's credences over past propositions are independent of credences over propositions involving one's present action. He frames the asymmetric fact that cause precedes effect as a subjective fact about one's credences:

> What is true is this, that any possible present volition of ours is (for us) irrelevant to any past event. To another (or to ourselves in the future) it can serve as a sign of the past, but to us now what we do affects only the probability of the future.
>
> This seems to me the root of the matter; that I cannot affect the past, is a way of saying something quite clearly true about my degrees of belief. Again from the situation when we are

> the effect that every event has a cause. We have some variable hypotheticals of the form 'If ϕx, then ψx' with ψ later than ϕ, called causal laws: others of the form 'If ϕx, then probability α for ψx'; this is called a chance. We suppose chance to be ultimate if we see no hope of replacing it by law if we knew enough facts. There is no reason to suppose it is not ultimate. *A law is a chance of unity* [emphasis mine]; of course, as is shown in my essay on chance, the chances do not give actual degrees of belief but a simpler system to which the actual ones approximate. So too we do not believe the laws for certain (Ramsey, [1929] 1990e, 159).

Here Ramsey again defines variable hypotheticals. But importantly for the current discussion, he also defines laws relative to chances. He says here that "a law is a chance of unity". This means that laws are just special cases of chances. So knowing what a chance is for Ramsey should inform one what is a law or variable hypothetical.

And Ramsey does not make this same statement in isolation. Elsewhere in "General Propositions" he hints at the strong connection between chances and laws. He writes earlier in the paper when discussing disagreements over counterfactuals that "we each of us have variable hypotheticals (or, in the case of uncertainty, chances) which we apply to any such problem;

> deliberating seem to me to arise the general difference of cause and effect. We are then engaged not in disinterested knowledge or classification (to which this difference is utterly foreign), but on tracing the different consequences of our possible actions, which we naturally do in sequence forward in time, proceeding from cause to effect not from effect to cause. We can produce A or A' which produces B or B' which etc. ...; the probabilities of A, B are mutually dependent, but *we* come to A first from our present volition

The thought is that only propositions about future events should depend on one's present action (volition) while credences over past events are fixed. That is if A is a proposition of the sort "I choose to do x at time t and B is a proposition of the sort "P at time u" where $u < t$ (the events described by B are before the events described by A), then $\Pr(B \mid A) = \Pr(B)$. The intuition is that only the future should be affected by one's future actions and this will appear in one's beliefs. This has been called Ramsey's Thesis. It is a controversial claim and has led to the development of several philosophical theses (see Ahmed, 2014 for a discussion). For an instance of a counterexample, consider the fact that one's present action might be informative about one's past actions. Suppose I am uncertain about what I had for dinner last week at a favored restaurant. Today, I go to that restaurant for dinner and decide on ordering some dish. Given that almost certainly my behavior in the past is related to my behavior in the present, my choice for dinner at and the difference between us is a difference in regard to these" (Ramsey, [1929] 1990e, 155). His final paragraph of the essay I have already quoted suggests a close connection between chances in laws for why laws are not propositions. In his endnotes, Ramsey also lists chances and laws as closely related. I have discussed in a prior chapter how Ramsey's idea of laws as forecasts is best illustrated in the case of chances. This would make a good deal of sense if laws and chances are basically the same.

Due to this clue, a good strategy for seeing how Ramsey's account of laws as rules for judging fits with his decision theory would be to look at his theory of chances. That I turn to now. Doing this would aid in addressing the interpretative puzzle I began this chapter with: how the old account of laws and the new account of laws really differ and whether Ramsey was committed to Peirce's conception of truth. It would also finally address exactly what laws are in Ramsey's philosophy of science.

4.5 Ramsey on Chances

I have argued in this chapter that there is an unresolved riddle from the secondary literature on Ramsey's view of laws. That riddle can be expressed in two questions. First, what is the relationship between Ramsey's account of laws as the axioms in the best system of propositions known by an omniscient agent and Ramsey's account of laws as rules for judging? Second, is Ramsey committed to something like Peirce's concept of truth as the true propositions being those believed at the limit of inquiry? Both depend on one another because if Ramsey has the best system and rules for judging accounts be the same, then he would hold that the true propositions and correct laws are what happen to be believed and held when all facts have been settled and discovered. Likewise, if the true propositions and correct laws are those that happen to be believed at the limit of inquiry, then their truth

present should change my credences in what I had last week.

and correctness are due to their role in the best system of a functionally omniscient agent.

In section 4.3.2, I argued that Ramsey argued that the two accounts are not one and same, and I argued that he rejected the best systems account of propositions because laws are not propositions and are not truth-functions of ordinary propositions. To put this in terms of a more modern slogan, laws do not supervene on the facts.

This partially resolves the riddle, but it is not sufficient to do so because I do not have a full account of Ramsey's account of laws as rules for judging. In section 4.4, I argued that Ramsey's account given in "General Propositions and Causality" is vague and not fully developed. A key outstanding question is how laws, which Ramsey calls variable hypotheticals, fit into his decision theory. This would hopefully resolve the interpretative question over the two accounts of laws because it would give an exact specification for what laws are for Ramsey. I concluded at the end of section 4.4 that finding an answer to this question requires an account of chances for Ramsey. In this section, I will build that account of chances.

Before I begin, it should be worth noting that *there is very little development of Ramsey's account of chances in the secondary literature*. Dokic and Engel and Misak do not even mention Ramsey's views on chances. Blackburn mentions the existence of the account but does not dive into the detail of it (Blackburn, 1980, 179). MacBride and company cite the relevant sources but do not discuss them. Only Sahlin has a more than in-passing discussion of Ramsey's view on chance (Sahlin, 1990, 51–53). Sahlin concludes from his three-page discussion that Ramsey had discovered decades before Carnap the existence of objective probabilities, which Ramsey called chances.[36] Ramsey, Sahlin concludes, is not a subjectivist like De Finetti, but granted the existence of real chances or propensities in the world such as Boltzmann's probabilities (Sahlin, 1990, 53).[37] Furthermore, Sahlin argues

[36]He writes "Carnap distinguishes between probabilities-1 (pr_1 = belief assessments) and probabilities-2 (pr_2 = frequencies). We shall see here that corresponding ideas were already present in Ramsey's texts" (Sahlin, 1990, 51).

[37]He writes:

the existence of chances proves that Ramsey is not a subjective Bayesian but an objective Bayesian: Ramsey believes in there being constraints on priors in addition to them being probabilities, with one of those constraints being subjective probabilities must agree with objective chances (Sahlin, 1990, 53).

While I won't directly show how Sahlin is wrong, an upshot of my account here is that there are serious problems with Sahlin's reading. *Pace* Sahlin, Ramsey's account of chances prefigures important developments years later by subjectivists. This will shed important light on what laws are for Ramsey. And it precisely shows what a rule for judging happens to be in terms of Ramsey's decision theory.

4.5.1 The Account of Chances

The primary document for Ramsey's account of chances is a paper titled "Chance". In there, Ramsey provides an account of chances as systems of degrees of belief that people approximate or defer to when reasoning. There are several components to this account that I discuss in this section. First, chances are not objective in the sense that there are facts such that every possible inquirer can come to agreement on them. Second, chances are defined as degrees of belief. Third, they are not the degrees of belief of any actual agent. Fourth, they are systems of degrees of belief that actual people approximate. Fifth, each system of chances consists of rules for assigning credences called properly the chances of a proposition. These rules can be evaluated as if they are propositions relative to the agent, even though they are

> I believe that Ramsey's concept of chance is closely related to the concept of probability often used in the natural sciences. If we accept certain physical theories, and thus indirectly the experimental evidence for these theories, some probability assessments are more reasonable than others [....] I believe this is what Ramsey means by saying that chances are degrees of belief within a certain system of beliefs and degrees of belief. Ramsey seems to have taken objective probabilities for granted. He believed that Boltzmann's probabilities are a matter for physics, that is, definitely not for logic (Sahlin, 1990, 52–53).

I'll argue below that the picture is way more complicated and that Ramsey ultimately holds such physical chances to be fictions.

not proper propositions. Agents defer to chances in the sense embodied by what David Lewis later calls the principal principle. Sixth, Ramsey believes the supposed objectivity of chances comes from agents having no hope of modifying their laws to account for every proposition. Seventh, laws are limiting cases of chances. Eighth, chances can occur in theories as well as observation, which explains how probabilities function in physical theories. Finally, Ramsey provides a method for learning chances. I discuss points seven and eight in a subsection on laws while the last point is discussed in its own sub-section.

Ramsey begins his discussion of chances by emphatically stating that chances are not objective. By objective, he means that there are no facts that allow for agreement on those facts:

> (1) There are no such things as objective chances in the sense in which some people imagine there are, e.g. N. Campbell, Nisbet.
>
> There is, for instance, no established fact of the form 'In n consecutive throws the number of heads lies between $\frac{n}{2} \pm \epsilon(n)$'. On the contrary we have good reason to believe that any such law would be broken if we took enough instances of it.
>
> Nor is there any fact established empirically about infinite series of throws; this formulation is only adopted to avoid contradiction by experience; and what no experience can contradict, none can confirm, let alone establish.
>
> (N. Campbell makes a simple mistake about this.) (Ramsey, [1928] 1990b, 104).

The view Ramsey is attacking is the view articulated by Campbell and Nisbet, who hold that chances are objective in the sense that the scientific facts are such that everyone can come to agreement, and they are not subjective in the sense they are due to anyone or some group's ignorance of the facts.[38] Ramsey believes, rightly, that such an objective concept

[38] For instance, Campbell differentiates between popular and scientific chance where the former is subjective in the sense that "it refers to the state of mind of the particular person making a statement about it"

of chance requires there to be empirical facts that determine the chances such as limiting frequencies or even infinite throws in the case of coins. He states that such facts just do not exist. For example, an infinite sequence of throws utilizing a coin would lead to that coin eventually disintegrating, rendering such an infinite sequence impossible.

He also points out that the hypothesized facts such as an infinite sequence of coin tosses are specifically constructed to avoid contradiction with experience. Any actual coin flipped will only be flipped finitely many times. This makes it consistent with infinitely many infinite sequences of tosses. Since it is consistent in that way, it cannot contradict nor confirm any objective fact about chances.

With no facts about chances, Ramsey argues that objectivity in Campbell and Nisbet's sense cannot be obtained. Agreement cannot be had on facts that do not exist. So chances must be inherently subjective in the sense that Campbell and Nisbet claim they are not. Consequently, chances are not objective in the sense that agreement can be reached by all inquirers due to there being facts that allow such an agreement.[39]

Second, chances must be degrees of belief (subjective probabilities). But third, they are not the credences of any actual agent:

(Campbell, 2013, 162). Scientific chance, however, is objective:

> On the other hand, scientific chance is objective. It depends not only on the state of the mind of the person who makes a statement about it, but on the state of mind of everyone else who has considered the matter; whether or no some event is governed by chance is a question concerning which universal agreement can be obtained to the same extent as it can be obtained for any law. When it is said to be a matter of chance whether a penny tossed falls heads or tails, I do not merely mean that I do not know which way up it will fall, but that nobody know, and that all the inquiry into the matter which has been made or, so far as can be seen, could be made, is insufficient to enable the fall to be predicted (Campbell, 2013, 162).

Chance is objective in the sense that agreement can be reached after enough inquiry because the facts are such that no deterministic law can be found, and those facts determine what the chance happens to be. So chance is not due to ignorance, in the present or in the future, but due to an empirical investigation of the facts. Campbell later writes this is due to there being random generating processes in the world that reflect the commonly known probability distributions (Campbell, 2013, 207).

[39]This automatically disqualifies Sahlin's account due to his Ramsey believing chances to be objective. Ramsey is adamant that they are not. So Sahlin is clearly wrong here.

> (2) Hence chances must be defined by degrees of belief; but they do not correspond to anyone's actual degrees of belief; the chances of 1,000 heads, and of 999 heads followed by a tail, are equal, but everyone expects the former more than the latter (Ramsey, [1928] 1990b, 104).

Several things are to be noted here. Ramsey does not say that chances are degrees of belief. He says chances are *defined* by degrees of belief. This suggests that chances are treated as something like a proposition: they can be expressed propositionally in terms of degrees of belief. But importantly, they are not defined by some particular agent's degrees of belief. The chance of a coin landing heads on the next toss is not defined as Joe Biden's credence that the coin lands head on the next toss. It is supposed to be non-agentially specific.

Fourth, the chances form a system and actual people approximate the chances, i.e. they defer to the chances for their own credences:

> (3) Chances are degrees of belief within a certain system of beliefs and degrees of belief; not those of any actual person, but in a simplified system to which those of actual people, especially the speaker, in part approximate.
>
> (4) This system of beliefs consists, firstly, of natural laws, which are in it believed for certain, although, of course, people are not really quite certain of them (Ramsey, [1928] 1990b, 104–105).

While Ramsey is not clear here, by system he means a deductive system closed under the probability calculus.[40] Included in the system are certain laws, which are a type of limiting chance. This adds greater evidence to the belief that chances are treated as if they are propositions in the system of chances. Furthermore, Ramsey reiterates that the system is not one about the credences of any one individual but it is one that people attempt to

[40] See below quote.

approximate. By approximate, it appears he means defer: if I know the chances, then I set my credences to those chances. If I'm uncertain about the chances, then I try to approximate what I take the true chance to be. I discuss how this works once I specify the chances exactly below.

Fifth, a system of chances consists of rules for setting credences that Ramsey calls the chances proper:

> (5) Besides these [laws] the system contains various things of this sort: when knowing ψx and nothing else relevant, always expect ϕx with degree of belief p (what is or is not relevant is also specified in the system); which is also written the chance of ϕ given ψ is p (if $p = 1$ it is the same as a law). These chances together with the laws form a deductive system according to the rules of probability, and the actual beliefs of a user of the system should approximate to those deduced from a combination of the system and the particular knowledge of fact possessed by the user, this last being (inexactly) taken as certain (Ramsey, [1928] 1990b, 105).

There is a lot going on in this short paragraph. First, Ramsey provides an exact definition of chance in terms of rules. Second, these rules are deferring rules in the sense that they say how an agent that believes the chance will set their credences. Third, chances are explicitly defined as propositional functions. Fourth, putting these three points together means that chances have to obey what David Lewis calls the Principal Principle. Fifth, the chances must form a deductive system (along with the laws) that obeys the probability calculus. Sixth, the agent can weigh the chances via their credences, leading to an approximation. I go through each point below.

Starting with the first point, Ramsey gives a definition of chances as rules for expecting propositions with fixed credence. Chances are rules for assigning credences: "when knowing

ψx and nothing else relevant, always expect ϕx with degree of belief p". These rules effectively function as something like conditional probabilities. Fixing what is known, the chance assigns a particular credence to some proposition. For example, I might believe in a system of chances about the relationship between rain and the sidewalk being wet. If I know that the sidewalk is wet, that system might contain a rule such that following that rule would assign credence 0.8 to it having rained. In this respect, the chance functions something like conditional probabilities. But note, this allows for chances to also act like marginal probabilities if one conditions on tautology. So if I am curious about the chance of this coin toss landing heads, I might have a rule that says given tautology, assign credence 0.5 to the coin landing heads on the next toss. In this way, Ramsey's definition of chances as rules allows not just for what might be called conditional chances but also marginal chances.

The second point I want to garner from this passage is that these are rules of deference to the chances. That is when someone believes he knows the chance of a proposition, he is committed to his credence of that proposition being just the chance. In this sense, the chance functions as an agent's rules for setting their credences. Now, as I explain below, this does not mean that an agent's credences will agree with the chances because an agent might be uncertain of the chances. But it does mean that agents at least in part defer to the chances for their degrees of belief like how they might defer to the opinion of their doctor for the status of their health.

A third point is that Ramsey explicitly defines chances with a propositional function. When he writes that such rules are "also written the chance of ϕ given ψ is p", he specifies that chances can be treated as if they are propositions with three arguments, ϕ, ψ, and p. This makes a chance proposition an equality between a chance function from propositions to the unit interval and a real number. So letting $Ch : \mathcal{A} \times \mathcal{A} \to [0, 1]$ and $\phi, \psi \in \mathcal{A}$ and $\alpha \in [0, 1]$ where $\mathcal{A}$ is an agent's algebra, a chance can be expressed as if it were the propositions:

$$\text{Ch}(\phi \mid \psi) = \alpha$$

In the case of marginal chances, one can omit the tautology argument and just write the chance as $\text{Ch}(\phi) = \alpha$. Note that even though chances are here treated as if they are propositions, Ramsey's dismissal of them being objective means that they cannot be true propositions like "It rained in Cambridge on Monday". Rather, chances are fictional propositions like theoretical propositions. They exist as propositions in the sense that they are sets of possible worlds from an agent's possibility space. But they are not realistic in the sense that they represent some fact in the world. He says as much later in "Chance" when writing about the use of chances in reasoning. He writes that the apparent use of chance "propositions" in argument contradicts his proposal: "Reasoning which seems incompatible with our solution of the paradox that chance $= \frac{1}{6}$ is inconsistent with this coincidence, which was that ' chance $= \frac{1}{6}$', ' chance $> \frac{1}{6}$', were not propositions and so could not serve as premisses or conclusions of arguments" (Ramsey, [1928] 1990b, 107–108). Thus chances are not true propositions. The likely reason they are not is due to the fact that they are not objective. This happens even though an agent still can treat them as propositions in some fictional sense. So chances are not factual but they can still act as if they were genuine propositions.[41]

Using these three observations, one arrives at a fourth point. That point is that when conditioning on the chances of a proposition, an agent's credence for that proposition should equal the chance. This is just what David Lewis calls the principal principle (Lewis, 1980). It can be expressed using the terminology from above, where A and E are propositions:

[41] Again, this contradicts Sahlin. Sahlin wants there to be objective probabilities. So there would be proper propositions about probabilities that people defer to. Here that is clearly not the case for Ramsey.

$$\Pr(A \,|\, \mathrm{Ch}(A \,|\, E) = x, E) = x \tag{4.1}$$

Because a chance is a rule for setting credences, adopting a chance should lead one to accept the chance credences as one's own credences. That is, one defers to the chance. And because chances can be treated by the agent as a fictional proposition, one should be able to consider their use in conditional probabilities. So conditioning on a chance involving a proposition should lead one's own credence to reflect the chance.

The fifth point to take away from this passage is that the chances are probabilities in the sense they obey the probability axioms. When Ramsey says that the system of chances "form a deductive system according to the rules of probability", he is explicitly stating that the chances must be mathematical probabilities. The rules of probability for Ramsey are what today would be characterized as the Kolmogorov axioms with finite additivity instead of countable additivity.[42] Those axioms can be listed for a chance Ch as follows:

1. $\forall \phi, \psi \in \mathcal{A}, \mathrm{Ch}(\phi \,|\, \psi) \geq 0$

2. $\mathrm{Ch}(\Omega) = 1$

3. $\forall \phi, \psi \in \mathcal{A} s.t. \phi \cap \psi = \emptyset$ and $\phi \cup \psi = \Omega, \mathrm{Ch}(\phi \cup \psi) = \mathrm{Ch}(\phi) + \mathrm{Ch}(\psi)$

4. $\forall \phi, \psi \in \mathcal{A}, \mathrm{Ch}(\phi \cap \psi) = \mathrm{Ch}(\phi \,|\, \psi)\mathrm{Ch}(\psi)$

[42] He lists the probability axioms that his decision theory would have partial beliefs obey in "Truth and Probability" as follows:

(1) Degree of belief in $p+$ degree of belief in $\overline{p} = 1$.

(2) Degree of belief in p given $q+$ degree belief in $\overline{p}$ given $q = 1$.

(3) Degree of belief in (p and q) $=$ degree of belief in $p\times$ degree of belief in q given p.

(4) Degree of belief in (p and q) $+$ degree of belief in (p and $\overline{q}$) $=$ degree of belief in p (Ramsey, [1926] 1990n, 77).

However, because Ramsey wants his chances to apply to the probabilities in one's physical theories, he would likely need chances to obey countable additivity, i.e. for an infinite list of propositions A_i that are disjoint:

$$3'.\ \mathrm{Ch}(\bigcup_{i=0}^{\infty} A_i) = \sum_{i=0}^{\infty} \mathrm{Ch}(A_i)$$

Together with the Kolmogorov axioms (1) and (2), this ensures that chances are probabilities. Importantly, he also includes the laws in this system, which as will be discussed below are defined explicitly as chances assigned probability one.

Lastly, a sixth point is that an agent's credences should approximate the chances by weighing the chances with their own knowledge. Ramsey writes that "the actual beliefs of a user of the system should approximate to those deduced from a combination of the system and the particular knowledge of fact possessed by the user, this last being (inexactly) taken as certain" (Ramsey, [1928] 1990b, 105). He had previously said agents approximate the chances, and here he is a bit clearer. The idea is that an agent does not simply defer to the chances—especially when it is open to what system of chances to adopt—but weighs the chances with their own background knowledge independent of the system of chances. That weighting will be provided by an agent's credences over chances. For example, I might be considering two different systems of chances over the outcomes of coin tosses, where one system holds the chances to be biased heads and the other holds the chances to be fair. But suppose I believe with good credence that the coin is weighted towards heads. Then I

These can be written out as:

$$(1)\ \mathrm{Pr}(P) + \mathrm{Pr}(\neg P) = 1$$
$$(2)\ \mathrm{Pr}(P \mid Q) + \mathrm{Pr}(\neg P \mid Q) = 1$$
$$(3)\ \mathrm{Pr}(P, Q) = \mathrm{Pr}(P)\mathrm{Pr}(Q \mid P)$$
$$(4)\ \mathrm{Pr}(P, Q) + \mathrm{Pr}(P, \neg Q) = \mathrm{Pr}(P)$$

should assign higher credence to the unfair chance than the fair chance, though I may not be certain the biased chance is correct. Then my credences over events will only approximate the unfair chances.

Applying this in the context of Ramsey's decision theory, my expectation of a proposition such as the outcome of this next coin toss, P, to be the weighted sum over chances and E be my evidence that the coin is weighted. Letting the value of the expectation be the indicator function and $\mathrm{Ch}(P \mid E) = b$ be the biased chance and $\mathrm{Ch}(P \mid E) = f$ be the fair chance, my expectation will be:

$$
\begin{aligned}
\mathbb{E}[P \mid E] &= \Pr(P \mid E)I(P) + (1 - \Pr(P \mid E))I(P^c) \\
&= \Pr(P \mid E)(1) + (1 - \Pr(P \mid E))(0) \\
&= \Pr(P \mid E) \\
&= \sum_{c \in \{\mathrm{Ch}(P \mid E)=b, \mathrm{Ch}(P \mid E)=f\}} \Pr(P, c \mid E) \\
&= \sum_{c \in \{\mathrm{Ch}(P \mid E)=b, \mathrm{Ch}(P \mid E)=f\}} \Pr(P \mid c, E)\Pr(c \mid E) \\
&= \Pr(P \mid \mathrm{Ch}(P \mid E) = b, E)\Pr(\mathrm{Ch}(P \mid E) = b \mid E) + \\
&\Pr(P \mid \mathrm{Ch}(P \mid E) = f, E)\Pr(\mathrm{Ch}(P \mid E) = f \mid E)
\end{aligned}
$$

That is, my expectation will be a weighted sum over the chances. Note that conditional probability on the chances will just be an application of the principal principle, leading to my credence there to defer to the candidate chance hypothesis. That credence is then multiplied by the weight my evidence gives for chances. If I assign higher credence to my evidence that a biased chance is correct, my expectation will be closer to that chance. If I

From these you can show the traditional axioms with finite additivity. For example, from the fact that a proposition and its negation are disjoint and (1), one can prove that tautology is assigned probability one.

learn that it was not credible to think the coin was weighted, then I might shift back to the fair chance and more closely approximate that.

It is this sense of approximation that Ramsey seems to have in mind. Chances are approximated in terms of whether one thinks them to be correct. The more one weighs a given chance, the more one approximates it. Knowledge outside the system of chances is then important for how well a given system of chances is approximated.

There is one problem with this account. Namely, the problem of how does one measure the credence in a chance? Ordinary propositions pose no problem. And Ramsey has a story for how the fictional singular, theoretical propositions might be measured (see the first chapter). But chances are altogether different. For starters, the ones I have been considering here are fictional propositions that are in some sense in the primary system: a chance is a function on primary system propositions and bears no mention of any secondary system proposition. So it is unclear how one would measure the chances using the method of theoretical propositions. But perhaps a worse problem is that all chances are compatible with all primary system propositions. This is part of the reason why Ramsey thinks they are not proper propositions. Consequently, there is not a straightforward story for how to measure credences over chances.

I will not address this problem here, except to say that a partial story will be given in a succeeding chapter on the Ramsey sentence.

Returning to the primary discussion of chances, the idea that Ramsey lists in the passage numbered (5) is that he gives an exact definition of chances in terms of rules for setting credences that an agent defers to when they believe the chance. This makes chances fictional propositions in an agent's algebra, similar to how theoretical propositions are fictions. A consequence of this is that Ramsey would require agents to obey Lewis's principal principle, i.e. given the chance of a proposition, the credence in that proposition should equal the chance. Ramsey emphasizes that these chances are probabilities, and he would likely have to require them to be probabilities in the sense they obey the Kolmogorov axioms with countable additivity. Finally, agents approximate the chances by weighing the chances with known propositions.

This leaves a discussion of why Ramsey thinks we think chances are objective, laws as limiting cases of chances, the role of chances in theories, and how to learn chances. I address the first of these here before moving consideration of the other points to their own sections.

Ramsey reasons that the notion of objective chance comes from the lack of laws to account for every proposition:

> (9) What we mean by objective chance is not merely our having in our system a chance $\frac{\phi(x)}{\psi(x)}$, but our having no hope of modifying our system into a pair of laws $\alpha x \,.\, \psi x. \supset_x \,.\phi x : \beta x \,.\, \psi x \supset_x \sim \phi x$ $[\forall x((\alpha(x) \wedge \psi(x)) \supset \phi(x)) \wedge \forall x((\beta(x) \wedge \psi(x)) \supset \neg\phi(x))]$, etc., where $\alpha x, \beta x$ are disjunctions of readily observable properties (previous in time to ϕx). This occurs, as Poincaré points out, when small causes produce large effects.
>
> Chances are in another sense objective, in that everyone agrees about them, as opposed e.g. to odds on horses (Ramsey, [1928] 1990b, 105–106).

He gives two reasons for thinking chances to be objective. The first is what I call the no hope reason. Ramsey's description of it is a bit convoluted. In apparent English, it says one has no hope when there exists some proposition and its complement where there is no pair of laws plus a pair of observable proposition unions that can deduce that proposition and its complement. The lack of such laws leads to their substitution with chances. In common English, it is best illustrated with an example. Consider the case of radioactive decay. I might think the chance of an atom decaying is objective because I cannot deterministically specify when the atom decays and does not decay with a law plus any measurements I take of the atom nor do I ever hope to be able to do so. Since I lack such a law, I formulate a system of chances based on the type of atom and other properties.

The second reason Ramsey gives is that generally, people agree about the chances. This separates them from personal credences because with personal credences disagreement is more likely. What is interesting about this is that Ramsey seems to agree with Campbell that agreement on chances happens. However, it seems that Ramsey does not think this agreement is due to any set of facts, i.e. agreement could occur for other reasons. I return to this below.

The remainder of Ramsey's work on chances, apart from laws as limiting cases of chances, the role of chances in theories, and learning chances, concerns the interpretation of coincidences and comments on causal analysis in statistics. Turning to these last items, the big payoff is to look at the status of laws as chances at credence one.

4.5.2 Laws as Limiting Cases of Chances

Returning to the question of laws and chances, Ramsey states in "Chance" as he does elsewhere that laws are chances where the deferred credence is one. I want to spend some time and consider the implications of this fact for Ramsey's view of laws. I start by reviewing the evidence for the claim. I then discuss four consequences from this view. First, laws are treated by the agent propositionally, even though they are not proper propositions. This is the objective sense of laws that Ramsey comments about at the start of "General Propositions and Causality". Second, the principal principle applies to laws, and this shows that laws govern one's credences in ordinary propositions. Third, laws function like chances in the forecasts of agents, which means that one's credences approximate the laws. Fourth, this shows how Ramsey's account of laws applies to universal propositions and universal instantiation. Fifth, the account of laws here applies to theoretical laws too. Sixth, Ramsey's account of how to learn chances applies just as well to the laws.

In "Chances", the connection between laws and chances is made explicit with note five. Recall that the phrasing used to define chances is as follows:

> [W]hen knowing ψx and nothing else relevant, always expect ϕx with degree of belief p (what is or is not relevant is also specified in the system); which is also written the chance of ϕ given ψ is p (*if* $p = 1$ *it is the same as a law* [emphasis mine]) (Ramsey, [1928] 1990b, 105).

The key line is that when the chance sets credence in ϕ to one, the chance is considered a law. Putting this in terms of the first way Ramsey defines chances, a law can be written as:

> When knowing ψx and nothing else relevant, always expect ϕx.

This is very close to the verbiage of his informal definition of laws as rules for judging (compare "If I meet a ϕ, I shall regard it as a ψ"). The key difference is just in the "nothing else relevant" clause. However, one should think that the same clause applies to Ramsey's own informal definition of a law. His writing clearly indicates he was aware of *ceteris paribus* conditions in the context of causal laws (see footnote 34). When he defines those laws, he lists r as necessary to be conjoined with a variable hypothetical to make a causal inference. Crucially, he lists $p \wedge r$ as being an instance of ϕx in the antecedent of a variable hypothetical $\forall x(\phi x \supset \psi x)$. This indicates that the propositional functions that characterize the conditions of a variable hypothetical include any relevant conditions. Furthermore, the definition of chance allows for chances to have no relevant conditions. So cases of laws without *ceteris paribus* conditions are covered here too. Consequently, the definition of chance is not only very close to Ramsey's definition of a law as a rule for judging: it is the exact same definition.

This coincides with how Ramsey describes the relationship between laws and chances in "General Propositions and Causality". He mentions laws in the context of chances four times there. The first time is in the context of discussing unfulfilled antecedents in subjunctive conditionals. He writes that the difference in disagreement over these conditionals "lies in the fact that we think in general terms. We each of us have variable hypotheticals (or, in the case of uncertainty, chances) which we apply to any such problem; and the difference between us is a difference in regard to these" (Ramsey, [1929] 1990e, 155). The thought is that disagreements over subjunctives amount to disagreements over laws and chances. Importantly, he connects chances to laws in the case of uncertainty. This would mean that the rule is one that applies a credence less than one. The second time Ramsey connects laws and chances occurs in a discussion over the objectivity of causal laws. He mentions in an aside about the rule of effects always following causes that the second law of thermodynamics[43] is discovered after empirical investigation and "what is peculiar is that it seems to result merely from absence of law (i.e. chance), but there might be a law of shuffling" (Ramsey, [1929] 1990e, 157–158). Here the connection is that the second law of thermodynamics has an absence of law or as he describes it, it is chancy. This coincides with the fact that when a rule for judging does not assign credence one to its consequent, it is a chance. The third time is in the quoted clue from earlier where Ramsey says "a law is a chance unity" (Ramsey, [1929] 1990e, 159). And that is it. So on the whole, the evidence supports the claim that Ramsey's view on laws coincides with that of chances: laws are just chances where the deferred credence is unity.

Since I have argued that laws are limiting cases of chances, I need to discuss the consequences of this view.

First, since chances are treated as fictional propositions, it follows that laws are treated as fictional propositions too. Through this fact, one can understand how someone can view a

[43]The entropy in a close system always increases with time.

law objectively. To start, a law is just a chance that assigns probability one. So an example law might be $\mathrm{Ch}(\phi x \mid \psi x) = 1$, where ϕx and ψx are propositional functions (so the law can be thought of as the chance that assigns the instances credence one). This corresponds to a set in the agent's possibility space. One might view it as a contour line where the chance equals one while the other contour lines are where the chance does not equal one. Now, to understand how the law might be thought of objectively, recall that one makes the law resemble a conjunction. It resembles a conjunction in the sense that when one considers its truth or falsity, one must consider the truth of every instance. What this means is that each instance can be thought of as a material conditional with an antecedent and a consequent. An agent's probability function then assigns credence one to every consequent given the antecedent. One might think of all the worlds in the corresponding contour line given by the chance equals one proposition as just those worlds where the agent's probability function acts as described above.

This is the sense in which a law can be viewed objectively. Agent's probabilities are facts about the world just as much as any other particular proposition. So a probability function that assigns credence one to an instance of a law's consequent is a fact of which there is a particular proposition. Those propositions are all subsets of the contour line given by the chance proposition that assigns one to the propositional function given in the chance.

Second, it follows immediately that if a law is a chance, then one's credences must respect the principal principle with respect to that law. This means that one assigns credence one to the consequent of the law. For example, if the law is $\mathrm{Ch}(\phi x \mid \psi x) = 1$ and A is an instance of ψx and B is an instance of ϕx, then

$$\Pr(B \mid \mathrm{Ch}(B \mid A) = 1, A, E) = 1$$

This is just deference to the law, and it matches the formulation of laws as rules for judging. Laws can then be properly viewed as inferences involving real and fictional propositions, where the believed law is reflected in an agent's willingness to infer a proposition when prompted by another. That is, he lets his credence be one in the consequent of the rule when encountering the antecedent.

Third, laws operate in forecasts like chances, where agents approximate the laws by weighing competing laws in a mathematical expectation. The only difference between laws and chances is that when considering only the laws in a forecast, the expectation of a proposition is just the credence in the laws. For example, consider Ramsey's favorite case of the law that "if I eat the cake, I would have a stomachache". I might consider this law's "contradictory", "if I eat the cake, I would not have a stomachache. Let A be the proposition "I eat cake" and B be the proposition "I have a stomachache". And let $\mathrm{Ch}(B \mid A) = 1$ be the first law and $\mathrm{Ch}(B^c \mid A) = 1$ be the second law. Suppose that these are the only two chances I consider possible. And suppose I am uniformly uncertain about them. Then my conditional credence in B given A is an expectation of these two laws:

$$
\begin{aligned}
\mathbb{E}[B \mid A] &= \Pr(B \mid A)I(B) + (1 - \Pr(B \mid A))I(B^c) \\
&= \Pr(B \mid A)(1) + (1 - \Pr(B \mid A))(0) \\
&= \Pr(B \mid A) \\
&= \Pr(B \mid \mathrm{Ch}(B \mid A) = 1, A)\Pr(\mathrm{Ch}(B \mid A) = 1) + \\
&\Pr(B \mid \mathrm{Ch}(B^c \mid A) = 1, A)\Pr(\mathrm{Ch}(B^c \mid A) = 1) \quad (*) \\
&= \Pr(B \mid \mathrm{Ch}(B \mid A) = 1, A)\Pr(\mathrm{Ch}(B \mid A) = 1) + \\
&\Pr(B \mid \mathrm{Ch}(B \mid A) = 0, A)\Pr(\mathrm{Ch}(B \mid A) = 0) \quad (**) \\
&= (1)(0.5) + (0)(0.5) \\
&= 0.5
\end{aligned}
$$

Here, $*$ follows from the fact that I take these two laws to be only the possible chances, so they form a partition that I can then apply the law of total probability. $**$ follows from the fact that assigning chance one to the complement is the same as assigning chance zero to the complement's complement, i.e. B. My credence then just becomes how much I weigh the one law that I get a stomachache, which is here by a half. In English then, my own subjective degrees of belief do not coincide with the law, but my disagreements with my peers are expressed in terms of the differences in our degrees of belief before I eat the cake.

Fourth, these two points are crucial for why this is an account of the universal quantifier. It allows for an elimination rule for universally quantified propositions through the forecasts. Recall that the elimination rule, when characterized deductively, is for any constant a:

$$\forall x \phi(y_1, \ldots, x, \ldots y_n) \Rightarrow \phi(y_1, \ldots, a, \ldots y_n)$$

where ϕ is some propositional function. In short, the elimination rule means that the variable bound by a universal quantifier may be replaced by any constant to produce an instance of the universal proposition. Traditionally, this is a deductive rule; the relationship between the universal proposition and its instance is logical entailment. Ramsey's rule allows something similar through forecasts, but the relationship between the universal proposition and its instance is a property of the agent's probability function. For example, suppose I assign probability one to the universal proposition $\forall x F(x)$, i.e. $\forall x Fx$ is just $\text{Ch}(Fx) = 1$ or the propositions defined by the propositional function Fx have chance one. Then, picking some proposition Fa that is an instance of Fx, my epistemic forecast would just be:

$$\begin{aligned}
\mathbb{E}[Fa] &= \Pr(Fa)I(Fa) + (1 - \Pr(Fa))I(Fa^c) \\
&= \Pr(Fa)(1) + (1 - \Pr(Fa)(0) \\
&= \Pr(Fa) \\
&= \Pr(Fa \,|\, \text{Ch}(Fa) = 1)\Pr(\text{Ch}(Fa) = 1) + \\
&\Pr(Fa \,|\, \text{Ch}(Fa) = 0)\Pr(\text{Ch}(Fa) = 0) \\
&= \Pr(Fa \,|\, \text{Ch}(Fa) = 1)(1) + \Pr(Fa \,|\, \text{Ch}(Fa) = 0)(0) \\
&= \Pr(Fa \,|\, \text{Ch}(Fa) = 1) \\
&= 1
\end{aligned}$$

That is I assign probability one to an instance of the universal proposition $\forall x F(x)$. The deductive rule shows up in my probability function when I assign probability one to any universal proposition. In an important sense, universal elimination applies to universal propositions for Ramsey. So this can be thought of as an account of the universal quantifier.

Fifth, the account of laws as limiting cases of chances applies to theoretical laws as much to observational laws. Theoretical laws are the axioms of a theory; in science, they are laws like Newton's laws of motion. An axiom just specifies that the chance of a theoretical proposition given another is one. These are "higher" rules for judging. Ramsey's view on theories permits this because it allows for theories and those axioms to be theorized by higher level theories; one theory can act as the "primary system" to another theory.[44] So his account of laws here is meant to apply to universal propositions within theories as well as those deduced from theories.

Sixth, laws are learned just as chances are learned. So however Ramsey views chances to be learned, that same learning process should apply to laws. This is important. It would help address the extent of Ramsey's commitment to the pragmatist account of truth as what is believed at the limit of inquiry. In the above account of laws, credences over laws reflect an agent's best guess as to the true laws. Those credences can and should shift over time. The learning process that determines how they shift will then determine, modulo the evidence, the laws settled on in the limit of inquiry. In other words, how laws are learned can determine whether the learning process of those laws ensures convergence of the laws. Some learning processes can lead to convergence. Others will not.

So an important remaining question then is what is Ramsey's account of how to learn the chances. In the next section, I turn to this account and connect it with the puzzling passage Ramsey has at the end of "General Propositions and Causality" which seemingly commits him to the pragmatist account of truth.

4.5.3 Convergence on Chances

I have discussed in detail Ramsey's view on chances and their relationship with laws. A lacuna in that story is how chances and by extension laws are learned. This is important

[44]Ramsey clearly indicates this in his initial description of the primary and secondary system distinction

as I noted in the last subsection because an answer to this story will address the extent to which Ramsey is committed to the pragmatist conception of truth. Here I argue that Ramsey believes the chances to be learned through the method of likelihoods. I then repeat well-known results that such a method is consistent in the sense that it will converge to the "true" chance in the limit. It is in this precise sense that Ramsey accepts the pragmatist conception of truth. The upshot is that while chances will not be settled by the facts (due to the facts not determining the chances), the learning process of likelihood estimation will eventually converge on a single estimation of the chances. This applies just as equally to the omniscient agent as to any actual learner, but it is distinct from Ramsey's earlier account because the old view required the choice of chance to be determined by logic and the facts.

Ramsey has an account for how to learn chances, and his account is presented in the eponymous paper towards the end of the discussion over chances:

> (13) In choosing a system [of chances] we have to compromise between two principles: subject always to the proviso that the system must not contradict any facts we know, we choose (other things being equal) the simplest system, and (other things being equal) we choose the system which gives the highest chances to the facts we have observed. This last is Fisher's 'Principle of Maximum Likelihood', and gives the only method of verifying a system of chances (Ramsey, [1928] 1990b, 107).

Ramsey thinks the two principles one trades off in believing the chances are a simplicity principle and the likelihood principle. He does not elaborate on what he means by 'simplicity', though given the similarity to his view on chances and the best system view of laws, simplicity is something like deductively simple, i.e. the fewest axioms needed to infer the

when he describes the introduction of real numbers into a theory: "If, however, our primary system is already a secondary system from some other theory, real numbers may well occur" (Ramsey, [1929] 1990m, 114).

relevant theorems.[45][46] The other method, the method of likelihoods, has an explicit reference to R. A. Fisher, who invented a method for point estimation when doing statistical modeling (Fisher, 1922). Both Ramsey's citation of the method and his reference elsewhere to two of Fisher's most influential works indicate his familiarity with the method of likelihood estimation.[47] I review this method below.

Let $x_1, \ldots, x_n$ be an n series of observations drawn from random variables $X_1, \ldots, X_n$, M be a probability distribution, and let $\Theta = \{\theta_1, \ldots, \theta_m\}$ be a set of parameters that determine the joint probability distribution of $\Pr(X_1, \ldots, X_n)$. Then the likelihood $\mathcal{L}(x_1, \ldots, x_n, \theta)$ for parameter $\theta \in \Theta$ is just the conditional probability of the observations given θ:

$$\mathcal{L}(x_1, \ldots, x_n, \theta; M) = \Pr(x_1, \ldots, x_n \mid \theta; M)$$

If $X_1, \ldots, X_n$ are independent and identically distributed, then the likelihood is the product of the conditional probabilities of individual observations given θ:

$$\mathcal{L}(x_1, \ldots, x_n, \theta; M) = \prod_{i=1}^{n} \Pr(x_i \mid \theta; M)$$

The principle of maximum likelihood estimation says that the best parameter $\hat{\theta}$ to estimate

[45] This is due to the fact that a system of chances includes laws and is closed under the probability axioms.

[46] Another way to cash out "simplicity" would be with the number of model parameters. Ramsey makes no suggestion of this anywhere else in his writing, but if "simplicity" is understood as model parameter count, then Ramsey is effectively arguing for a prior similar to what is used in the Bayesian Information Criterion. Much of the discussion that follows would still apply because parameter counts vary with description—so objections against uniform priors would apply here. In general, "simplicity" priors have a terrible success rate at addressing the problem of the prior.

[47] Ramsey cites Fisher's "Theory of Statistical Estimation" (Fisher, 1925) and *Statistical Methods for Research Workers* (Fisher, 1992) in a separate note titled "Statistics" (Ramsey, [1928] 1990k, 102).

is the parameter that maximizes the likelihood, i.e. makes the observed data most likely:

$$\hat{\theta} = \underset{\theta \in \Theta}{\arg\max} \mathcal{L}(x_1, \ldots, x_n, \theta; M) = \underset{\theta \in \Theta}{\arg\max} \Pr(x_1, \ldots, x_n \mid \theta; M)$$

Estimating likelihoods results in what is now called a point estimate for each parameter θ and computing the maximum likelihood estimation amounts to finding the most likely parameters θ using just the likelihoods. For example, if it is known that $X_1, \ldots, X_n$ are independent and normally distributed, then one could estimate the parameters μ, σ by computing which set of parameters makes the products of the conditional probabilities for observations $x_1, \ldots, x_n$ the highest.

What this gives is a method for learning the chances. When observing a series of propositions, one lets the credences in those chances be the likelihood the chances assign to the observed propositions. As Ramsey says, one finds the chances that make the observed facts most probable. Credences for all chances are adjusted through their likelihoods. Those likelihoods are then normalized by the probability of the evidence, which is computed from the sum of the likelihoods. If $\mathcal{L}(x_1, \ldots, x_n, \theta_i; M)$ is the likelihood for the $i \in \{1, \ldots, k\}$ parameters, then the new marginal credence for those parameters is:

$$\Pr'(\theta_i) = \frac{\mathcal{L}(x_1, \ldots, x_n, \theta_i; M)}{\sum_{j=1}^{k} \mathcal{L}(x_1, \ldots, x_n, \theta_j; M)} \tag{4.2}$$

This can be treated as a Bayesian update with a uniform prior. That is the likelihoods determine the credences of the chances by treating the chances as uniformly likely. One can then estimate the credence of each individual chance as the ratio between its likelihood

ϕ_1	ϕ_2	Ch_1	Ch_2
1	1	0.36	0.16
1	0	0.24	0.24
0	1	0.24	0.24
0	0	0.16	0.36

Figure 4.2: Two chance hypotheses Ch_1 and Ch_2 over the outcomes of a pair of coin tosses. Intuitively, these correspond to the coin having a bias of 0.6 and 0.4 respectively.

and the sum of all the likelihoods. It is important to note this assumption of uniformity. It applies just in case one considers all observations: this is not meant to be a rule for successive updates. One decides on the current credences of the chances by considering the total evidence. One then computes the credences in the chances.

For example, suppose one is observing two coin tosses ϕ_1 and ϕ_2. Let $\phi_i = 1$ be the proposition "At the i-th toss the coin landed heads" and $\phi_i = 0$ be the proposition "At the i-th toss the coin landed tails". Suppose one partition the possibility space into two chance hypotheses that assign the chances for each sequence of outcomes as found in figure 4.2. The likelihoods here are given by the principal principle, and this corresponds to each row in the aforementioned table. So if one observes two heads, then the likelihood for the first chance will be 0.36 and for the second chance will be 0.16. Likewise, if one head and one tail are observed, then the likelihood of both chances will be the same at 0.24. The most probable chance will then be whichever one is greater. Credences in the chances are then adjusted by equation 4.2. In the case where the coin lands heads on both tosses, the credence in both chances becomes:

$$\mathrm{Pr}'(\mathrm{Ch}(\phi_1 = 1 \cap \phi_2 = 1) = 0.36) = \frac{0.36}{0.36 + 0.16} \approx 0.69$$

$$\Pr'(\text{Ch}(\phi_1 = 1 \cap \phi_2 = 1) = 0.16) = \frac{0.16}{0.36 + 0.16} \approx 0.31$$

If one were to observe further flips, one would have to recalculate the chances using equation 4.2. It is this method that Ramsey seems to have in mind when appealing to Fisher's likelihoods.

To recap, Ramsey suggests that in addition to deductive simplicity, the way chances are learned is through Fisher's likelihood principle. Both the most likely chances as well as credences of chances can be estimated. The most likely chances are found through the maximum likelihood method while credences are learned by estimating likelihoods and then normalizing those likelihoods. Both processes will agree in the sense that the chance that maximizes the likelihood will also be the one assigned the highest credence.

An important property of the method of maximum likelihood estimation is what Fisher calls the criterion of consistency (Fisher, 1922, 316). For Fisher, an estimator can have a number of desirable properties, with consistency being one such property. Roughly, an estimator is consistent if as the size of the data sample grows to infinity, then the estimator converges to the true value of the parameter that it estimates. More precisely, let $X_1, \ldots, X_n$ be a sequence of random variables, let an estimator $\hat{\theta}_n$ be a function from those random variables to some real number, i.e. $\hat{\theta}_n = f(X_1, \ldots, X_n)$, and let θ_* be the true value of the estimated parameter. An estimator $\hat{\theta}_n$ is consistent if for every $\epsilon > 0$, $\Pr(|\hat{\theta}_n - \theta_*| > \epsilon) \rightarrow 0$ as $n \rightarrow \infty$. In other words, there is zero probability that there will be any positive difference between the estimator and the true value being estimated as the sample size increases toward infinity. The upshot is that for any sufficiently large finite sample, one can specify with arbitrary precision the value of the true parameter with a consistent estimator.

One example of a consistent estimator is the maximum likelihood estimator, modulo some important assumptions.[48] Suppose one is estimating the bias of a coin based on samples of coin tosses. Since those samples are independent and identically distributed, the maximum likelihood estimator will be consistent. Increasing the number of throws will result in the estimator returning an estimate of the coin's bias that is closer and closer to the supposed true value. Eventually, that estimate will just be the relative limiting frequency as the number of tosses goes to infinity, i.e. the true parameter.

It is highly likely that Ramsey had read Fisher on consistency and understood its importance. In addition to discussing the method of maximum likelihood estimation, Ramsey cites two of Fisher's key works in a short note titled "Statistics". More concretely, Ramsey seems to have been aware of the accuracy of an estimator based on its sample size. He writes that probable error can be calculated from the sample and that this error is a factor of "the number of instances which we have observed (the weight of our induction) (probable error)" (Ramsey, [1928] 1990k, 102). The implication is that as the sample size increases, the probable error decreases—meaning that Ramsey was aware of Fisher's criterion of consistency and its relevance to maximum likelihood estimation.

This is highly relevant to the convergence and uniqueness of chances. If Ramsey thinks that the method of maximum likelihood estimation is consistent and it applies to chances, then that means he would think agents that use such a method must eventually converge to the true chances. I have already argued that Ramsey does think it applies to chances from his discussion in "Chance". Further evidence comes from a remark in statistics where he throws away Fisher's fiction of infinite population for chances:

> The introduction of an infinite population is a stupid fiction, which cannot be

[48]There are a number of conditions that are sufficient for entailing the consistency of a maximum likelihood estimator. One such condition is identifiability. Let $D(f \| g) = \int f(x) \log f(x) - \log g(x) dx$ be the Kullback-Leibler distance between two probability distribution functions f and g. An estimator $\mathfrak{F}$ with possible parameters Θ and a probability density function f is identifiable if for any $\theta_1, \theta_2 \in \Theta$, if $\theta_1 \neq \theta_2$ then

> defended except by some reference to proceeding to a limit, which destroys its sense. The procedure of calculating parameters by maximum likelihood and probable error can be defined as a process in pure mathematics; its significance is in suggesting a theory or set of chances. Proportion of infinite population should be replaced by chance (Ramsey, [1928] 1990k, 102–103).

Maximum likelihood and probable error are about learning chances. But if likelihood estimation is consistent, then it follows that the chances should eventually converge as more evidence is incorporated into the "true" chance. That is, there will be a unique chance found after inquiry.

Here is the source of Ramsey's remark on inquirers being committed to Peirce's concept of truth. I claim that it is because chances are learned through likelihood estimation that one should expect to find a single true system of chances at the limit of inquiry.

This makes considerable sense of the text at the end of "General Propositions and Causality". Ramsey's whole discussion of Peirce's concept of truth starts in response to a consideration one might have towards being a realist about causation.[49] He gives an example of a society of strawberry abstainers who studiously follow the rule that if they eat strawberries, they will get a stomachache. Because they never try strawberries, they never observe a violation of this rule and in fact can conclude from experience that if they eat strawberries, then they would get a stomachache (because the antecedent of this conditional is always false). From present society, one might conclude they were wrong because one would claim it is a fact that had they eaten them, then they would have not gotten a stomachache. Ramsey, however, states it is not a fact but a rule that one follows. This is the mistaken realist assumption of

$D(f(x;\theta_1)\|f(x;\theta_2)) > 0$. Or in other words, if the parameters differ for an estimator, the generated distributions will be different.

[49] He leads off by writing "The sort of thing that makes one want to take a realistic view of causality is this" (Ramsey, [1929] 1990e, 160–161) and then proceeds into the discussion.

causation. One's conclusion that they are wrong comes from one's present system,[50] a nod to the fact that one is considering a system of laws and chances when modulating behavior into forecasts. This leads Ramsey to consider what happens when two systems of laws and chances agree on all of the facts:

> But their system, you say, fitted all the facts known to them; if two systems both fit the facts, is not the choice capricious? We do, however, believe that the system is uniquely determined and that long enough investigation will lead us all to it. This is Peirce's notion of truth as what everyone will believe in the end; it does not apply to the truthful statement of matters of fact, but to the 'true scientific system' (Ramsey, [1929] 1990e, 161).

Here Ramsey argues that the choice between two systems of laws and chances is not capricious because one believes "that the system is uniquely determined and that long enough investigation will lead us all to it". Two facts should be observed here. First, the systems here are systems of laws and chances as the previous discussion makes clear. These are not scientific theories (though I discuss how theories relate in a later chapter) but the laws and consequences one uses to regiment psychological expectations as forecasts. Second, he is considering what happens as one learns the system of laws and chances ("long enough investigation will lead us all to it"). Ramsey's claim that investigation will lead one to a unique system of chances and laws is a claim about the learning process: he is stating that the method of scientific inquiry ensures termination in a unique system. Ramsey's immediately following discussion illuminates the importance of learning in for reaching a unique system of laws and chances:

> What was wrong with our friends the strawberry abstainers was that they did

[50] He writes when considering a subjunctive conditional as having sense "if it, or its contradictory, can be

> not experiment. Why should one experiment? To increase the weight of one's probabilities: if q is relevant to p, it is good to find out q before acting in a way involving p. But if q is known it is not worth while; they knew, so they thought, what the issue of the experiment would be and so naturally couldn't bother to do it (Ramsey, [1929] 1990e, 161).

Ramsey explicitly says the problem with the hypothetical society is that they did not try to learn if their hypothesized chances were correct. Learning chances requires adjusting one's credences in the chances. And the method for doing so is via likelihoods. Ramsey is stating that the aforementioned society did not bother to gather data and update their chances via the likelihoods. If they did, then we believe their estimations would converge to the truth.

The upshot is that this passage is an endorsement of the Peircian idea applied to chances. That idea makes sense here because the method for learning the chances is by maximizing the likelihood through gathering observations. Ramsey is a Peircian here because of his account for learning chances; he is not accepting the pragmatist conception of truth for propositions full stop but is applying it here because of how he thinks chances should be learned.

One might ask at this point whether it does turn out that the old view on laws is the same as the new view after all. Ramsey seems to be committed to there being a "true", correct set of laws and chances via the method likelihoods. What is to stop an omniscient intellect to use all the known facts to identify, with credence one, a single system of laws and chances?

The answer is no. The old view of laws held the best system to be a result of an organization of all the facts. Laws are determined by the facts and some consideration of simplicity: an omniscient intellect with all the facts would then systematize those facts into laws. Here the laws are not determined by the facts. Nothing about the facts themselves uniquely

deduced from our system" (Ramsey, [1929] 1990e, 161), indicating one must consider a law in the context of a whole system of laws and chances.

specifies the laws and chances. Instead, an omniscient intellect would have to bring in a separate principle, the rule of maximizing the likelihood, to conclude what laws and chances are correct. It is not from such an intellect's omniscience alone that the correct laws and chances are inferred, but from those facts and features of that intellect's credences that determine the laws and chances.

This is subtle. What differentiates the old view on laws from the new view is the distinction between tautology and probability one and contradiction and probability zero. Assigning probability one to a chance does not imply that chance is a logical consequence of an agent's knowledge just as assigning probability zero to a chance does not imply that chance to be a logical impossibility. An omniscient intellect could, even if it used the method of maximizing the likelihoods, be wrong about the chances. Consistency does not entail correctness. For Fisher it does. But that is because he is a frequentist. Ramsey is a personalist about probability. The probability function given in the definition of consistency is *someone's* credences. Naturally, this would be the agent who is considering the method of likelihoods for estimating the chances. So for that agent, he knows his method (modulo some assumptions) will lead him to the "true" chance. He believes his system will converge to the "truth." And so does anyone else who adopts the method of likelihoods, including the omniscient agent. What differentiates this omniscient agent from the one in the old view of laws is that she does not know the true chance because of deductive entailment between the chances and the facts; she knows it because her method of learning the chances provides her confidence in the "true" answer.

I now have a solution to the other part of the riddle that I began this chapter with. The first part of that riddle was to what extent Ramsey's two accounts of laws are one and the same. I argued previously that they cannot be because in the laws as rules for judging, Ramsey held laws to not be propositions due to them not being truth-functions of elementary propositions. This left the second part of the riddle. That second part was the extent Ramsey is committed to the pragmatist conception of truth. Here I have concluded that his commitment to this Peircian ideal comes from his method of learning chances: Fisher's method of maximizing the likelihoods. Because that method under certain assumptions is consistent, i.e. the agent believes he will converge to the "true" chance, it implies that there will be a unique system of laws and chances at the limit of inquiry. Importantly, this does not make the view of laws as rules for judging collapse to the first view because the belief in a unique system follows from the method of maximizing the likelihoods—not from the deductive structure of any elementary propositions. In short, Ramsey's commitment to Peirce's conception of truth is not a profound epistemological and metaphysical thesis but falls out from how Ramsey thinks laws and chances are learned in science. Ramsey derives Peirce's thesis as a special case due to the mathematical technicalities of his new view on laws.

4.6 Ramsey and the Principle of Indifference

My goal in this chapter has been to provide a resolution to a riddle that has plagued the secondary literature. Along the way, I developed a precise version of laws as rules for judging through Ramsey's account of chances. This account fits with a revised decision theory because chances are just propositions in an agent's algebra. What remains to be seen is the philosophical merits of that account. I argue here that Ramsey's theory for how laws and chances are learned is fundamentally in tension with his stated views on probability. Namely, learning chances through maximizing likelihoods commits Ramsey to the Principle of Indifference (PI). But Ramsey unequivocally disavows that principle in "Truth and Probability", and his whole personalist theory of probability is aimed at eliminating the need for PI. While it is unclear whether Ramsey is aware of this conflict, I hypothesize that he might have adopted Fisher's method for learning chances as an answer to the problem of what he calls the logic of truth. Regardless, I argue that the solution to the problem is to abandon learning laws and chances through likelihoods alone. Instead, Ramsey should just conditionalize on the laws and chances.

To see why Ramsey is committed to PI when learning chances, it would be useful to review what maximizing the likelihood is in a Bayesian context. Namely, equation 4.2 is really Bayes's Theorem with a uniform prior. Let $\Theta = \{\theta_1, \ldots, \theta_m\}$ be a partition on an agent's sample space corresponding to the different chances they might consider (so each $\theta_i \in \mathcal{A}$ for $i \in \{1, \ldots, k\}$). Suppose $\Pr(\theta_i) = \Pr(\theta_j) = \frac{1}{k}$ for all $i, j \in \{1, \ldots, k\}$, i.e. one assigns uniform probability to each member of the θ partition. Then solve for $\Pr(\theta_i \,|\, x_1, \ldots, x_n; M)$ for $i \in \{1, \ldots, k\}$:

$$\begin{aligned}
\Pr(\theta_i \,|\, x_1, \ldots, x_n; M) &= \frac{\Pr(x_1, \ldots, x_n \,|\, \theta_i; M)\Pr(\theta_i; M)}{\sum\limits_{j=1}^{k} \Pr(x_1, \ldots, x_n \,|\, \theta_j; M)\Pr(\theta_j; M)} \\
&= \frac{\Pr(x_1, \ldots, x_n \,|\, \theta_i; M)\left(\frac{1}{k}\right)}{\sum\limits_{j=1}^{m} \Pr(x_1, \ldots, x_n \,|\, \theta_j; M)\left(\frac{1}{k}\right)} \\
&= \frac{\Pr(x_1, \ldots, x_n \,|\, \theta_i; M)\left(\frac{1}{k}\right)}{\left(\frac{1}{k}\right)\sum\limits_{j=1}^{k} \Pr(x_1, \ldots, x_n \,|\, \theta_j; M)} \\
&= \frac{\Pr(x_1, \ldots, x_n \,|\, \theta_i; M)}{\sum\limits_{j=1}^{k} \Pr(x_1, \ldots, x_n \,|\, \theta_j; M)} \\
&= \frac{\mathcal{L}(x_1, \ldots, x_n, \theta_i; M)}{\sum\limits_{j=1}^{k} \mathcal{L}(x_1, \ldots, x_n, \theta_j; M)} \\
&= \Pr'(\theta_i; M)
\end{aligned}$$

This only works if the prior is uniform. So adapting one's credences by the likelihoods is just conditionalization with a uniform prior. This makes Fisher's maximum likelihood estimator just computing the maximum *a posteriori* probability for some member of Θ when there is a uniform prior over $\theta \in \Theta$:

$$\hat{\theta} = \underset{\theta \in \Theta}{\arg\max} \Pr(\theta \,|\, x_1, \ldots, x_n; M)$$

So likelihood-based learning is Bayesian, but only when one happens to have a uniform prior over the hypotheses.[51]

Ramsey's uniform prior is PI applied to chances. According to him, the proper method for learning the chances is to start with a uniform prior over the chances and then update via Bayes's theorem. That is, he states that PI offers a solution to the question of what prior credence one should have over the laws and chances.

This is bad. PI has some well-known problems that Ramsey knew about. Keynes, in his *A Treatise on Probability*, recounts a number of now well-known objections to PI (Keynes, 1921).[52] Ramsey had read Keynes thoroughly. He knew those objections because he mentions them in passing, and he concurs with Keynes in saying that "To be able to turn the Principle of Indifference out of formal logic is a great advantage; for it is fairly clearly impossible to lay down purely logical conditions for its validity, as is attempted by Mr Keynes" (Ramsey, [1926] 1990n, 85). He remarks that it is a genuinely good feature of his theory of probability as subjective degrees of belief that it eliminates the need for PI.[53] It is striking then that

[51]A natural question that might occur to the reader is that Fisher would have known this fact, and he would have discussed it in his publications on likelihood estimation so shouldn't Ramsey have been aware? The short answer is that Fisher denied Bayesian inference—the method of inverse probabilities— as a valid form of statistical inference. Fisher was a committed frequentist. In fact, he held such an intense antagonism to Bayesian methods that when Karl Pearson pointed out that one could understand likelihood estimation as a variant of inverse probability estimation, Fisher publicly slandered Pearson and ended their friendship (see chapter four of Clayton, 2021).

[52]See Keynes chapter four (Keynes, 1921, 44–70). For a full discussion of the philosophical problems, a good modern starting point is van Fraassen (Fraassen, 1989), and for a review of the history of PI, Zabell is good (Zabell, 2005.

[53]Ramsey brags about this in "Truth and Probability" by writing:

despite banishing PI through the front door, Ramsey lets it in through the back door with his recommended learning rule for chances. Either he is inconsistent or he happened to change his mind about PI.

It is unclear if Ramsey knew that learning the chances through maximizing the likelihoods committed him to PI. If he did, it would be a stunning reversal. I am speculating here, but if he did change his mind, it is likely because of a problem that bedeviled him at the end of "Truth and Probability". There in section five, Ramsey considers whether there is more to logic than consistency, i.e. whether formal logic is all there is to the "logic of truth" or "human logic". He explicitly considers induction as a part of this "logic of truth", and the ideas he gives are largely derivative of Peirce (he admits this freely in a footnote) (see the discussion on Ramsey, [1926] 1990n, 86–90). Zabell comments that these ideas are unsatisfactory, and Ramsey himself seems to have thought so given several writings in his *Nachlass* (Zabell, 2005, 133). Perhaps what happened is that Ramsey in his final year settled on his account of chances and Fisher's likelihood methods as "the logic of truth". If so, he would have been saddled with PI's many problems—problems that are largely now deemed unsolvable.

Philosophically, the right move is to abandon PI and with it Fisher's likelihood principle as the proper learning method for chances. Doing so would make Ramsey more of a subjective Bayesian, where nearly any prior can be had on the chances. Convergence results proved since Ramsey's death would then allow one to recover some of the desired Peircian ideal, where the priors wash out in the end and agents can come to agreement as long as they have non-dogmatic priors. It is this view that would ultimately be more in keeping with the spirit of Ramsey's groundbreaking subjective theory of probability. And it would make him closer

> Secondly, the Principle of Indifference can now be altogether dispensed with; we do not regard it as belonging to formal logic to say what should be a man's expectation of drawing a white or a black ball from an urn; his original expectations may within the limits of consistency be any he likes; all we have to point out is that if he has certain expectations he is bound in consistency to have certain others (Ramsey, [1926] 1990n, 85).

Ramsey here states what is now taken to be the canonical answer to the problem of the priors: (almost) any prior will do.

to his near contemporary De Finetti.

4.7 Conclusion

This chapter's goal is to identify an account of universal propositions and laws that is compatible with Ramsey's theory of cognitive psychology, philosophy of science, and decision theory. Such an account is necessary to complement the theory of singular theoretical propositions proposed in chapter one and to explain how scientific theories are useful in action. Theories for Ramsey provide forecasts. Forecasts appeal to laws to regiment psychological expectations. So a story of theories needs to include a story of what are laws and how they operate in forecasts.

I began the chapter with an important riddle obstructing a coherent account of laws: Ramsey has two accounts of laws and thanks to his seeming commitment to Peirce's conception of truth, those accounts appear to be one and the same—even though Ramsey declares they are very different. Any account of Ramsey's laws would have to resolve this riddle somehow. It would have to resolve the riddle while also fitting laws with Ramsey's philosophy of science and decision theory.

I have argued in this chapter that Ramsey's old account of laws as the best system and his new account of laws as rules for judging are not one and the same. They are not identical because the latter view makes laws and chances not supervene on the facts. I have also argued that Ramsey's commitment to the pragmatist conception of truth as what is believed in the limit of inquiry is a narrow commitment born of how he thinks laws and chances are learned. Laws are a species of chance, and chances are rules for setting credences. Both laws and chances are learned through maximizing their likelihoods. This commits Ramsey to the Principle of Indifference, which likely makes him inconsistent. To avoid that principle's problems, I suggested that Ramsey abandon it, and rely upon normal Bayesian conditionalization.

This sets up the remainder of the dissertation to focus on the famed Ramsey sentence and Ramsey's views on scientific realism. With an account of both singular theoretical propositions and laws, I can now relate those fictional entities to the Ramsey sentence. This will require two discussions. The first is to situate the Ramsey sentence in relation to laws, forecasts, and Ramsey's decision theory. That will be the discussion in the next chapter. The second is to explain how existentially quantified propositions work now that I have a basic account of universally quantified propositions. That will be the following chapter. Once the Ramsey sentence and its quantifier are explained, I can finally turn to the big question surrounding Ramsey's philosophy of science: the extent of his commitment to scientific realism.

Chapter 5

The Ramsey Sentence

5.1 Introduction

In "Theories", Frank Ramsey introduces a formal sentence that has since earned his name. Subsequent philosophers have attributed great significance to the Ramsey sentence because they view it as a bridge between observational and theoretical languages that addresses the problem of scientific realism. Ramsey, however, only discusses the sentence in passing. He ignores the question of realism in "Theories" entirely. The query Ramsey does consider in "Theories" is whether the fictional propositions of a physical theory can be reduced to purely observational propositions in the style given by Carnap's *Aufbau*. Ramsey concludes that this can be done but at the cost of making the theory useless for decisions. Elsewhere Ramsey argues that a theory's meaning is given by its laws—cognitive rules agents use to regiment their expectations. Theories are ultimately fictions used in guiding expectations to allow for forecasts. This suggests a connection between the utility of scientific theories and observation's inability to depict how agents make decisions; it suggests a connection between the anti-reductionism of Ramsey's project and his anti-realism of scientific theories.

My fundamental goal in this chapter is to address this connection, the problem it leads to, and how the Ramsey sentence fits into a solution for that problem. Ramsey's problem is explaining the function and utility of scientific theories with a beggared observation language. The Ramsey sentence solves this problem, allowing deliberation and communication with others, by the use of latent structures in the agent's conceptual algebra that mirror the cognitive rules used in the agent's decision-making.

The difficulty of capturing Ramsey's understanding of the Ramsey sentence comes from Ramsey's very brief use of it in "Theories" and in his notes. The sentence appears in "Theories about two-thirds through. Immediately before introducing the sentence, Ramsey considers the option of replacing theoretical propositions with explicit definitions in a Carnapian observation language. He concludes that while such an option is available, it makes the theory useless and difficult to grow. At this point, he asks the question:

> Taking it then that explicit definitions are not necessary, how are we to explain the functioning of our theory without them?
>
> Clearly in such a theory judgment is involved, and the judgments in question could be given by the laws and consequences, the theory being simply a language in which they are clothed, and which we can use without working out the laws and consequences.
>
> The best way to write our theory seems to be this $(\exists \alpha, \beta, \gamma)$ dictionary . axioms.
>
> Here it is evident that α, β, γ are to be taken purely extensionally (Ramsey, [1929] 1990m, 130–131).

The penultimate sentence introduces the Ramsey sentence. Later sentences consider the theoretical and practical consequences with respect to the meaning and communication of the Ramsey sentence. The total discussion of the Ramsey sentence includes the quotes above and about a page and a half discussion. And that is it.

Ramsey's remaining corpus only mentions the Ramsey sentence sporadically. The relevant published articles are "Causal Qualities" and "General Propositions and Causality". In both, the Ramsey sentence is obliquely referenced. Ramsey's other use of the Ramsey sentence appears in a handful of incomplete notes, and there his discussion is fragmentary and unclear. In summary, inside of "Theories" Ramsey's use of the Ramsey sentence is little, and outside of "Theories", it is almost completely absent.

The paucity of primary source material on the Ramsey sentence undergirds an expansive zoo of often incompatible interpretations about that sentence. Braithwaite (Braithwaite, 1953) and Lewis (Lewis, 1972) believe that the deductive relations in the Ramsey sentence show how the theoretical terms function while avoiding the reference of those terms. Psillos (Psillos, 2000) argues the Ramsey sentence expresses a theory's content through the sentence's existential quantifiers. Those quantifiers prevent a theory from being a mere summary of observation because they make a commitment to the theory's multiple realization. Carnap (Psillos, 2004) claims Ramsey thought the Ramsey sentence successfully eliminates "bothersome" theoretical terms while still preserving the observable laws and consequences scientists care about. Ramsey shows how to avoid metaphysical questions like what is the reference of theoretical terms. Demopoulos (Demopoulos, 2011) envisions Ramsey's Ramsey sentence as a successful execution of Russell's maxim of scientific philosophy: when possible, logical constructions should substitute for inferred entities. Finally, Majer (Majer, 1989) understands the Ramsey sentence as an application of intuitionistic philosophy of science. Each of these interpretations is radically different and sometimes incompatible with one another.

Every interpretation may be rated on how well it satisfies certain desiderata. Three criteria stand out as a good starting point for a satisfactory understanding of the Ramsey sentence. First, an account of the Ramsey sentence needs to connect it to decision-making. A core tenet of Ramsey's philosophy is that "philosophy must be of some use and we must take it seriously; it must clear our thoughts and so our actions" (Ramsey, [1929] 1990h, 1). The "use" here is the "use" involved in guiding decisions and expectations. A philosophical account of theories with the Ramsey sentence at its center must show how theories are relevant for crafting decisions. Second, Ramsey is adamant that theories are fictions. So an interpretation of the Ramsey sentence must be anti-realist in the sense that it considers theories as fictions. Third, Ramsey understands the quantified functions in the Ramsey sentence "extensionally". What he means exactly by that is unclear at this point. But because he immediately declares them to be "extensional", an interpretation of the Ramsey sentence should have the functions be "extensional" as Ramsey understands it. My interpretation is built to solve these three criteria first and foremost.

My interpretation of the Ramsey sentence has the following important features. First, *existentially quantified propositions are anti-realistic in the sense they act as descriptions for other propositions.* Second, *the Ramsey sentence corresponds with important structures in an agent's algebra given by the theory's theoretical functions.* Third, *those structures represent a reification of the cognitive rules an agent uses for producing forecasts.* Fourth, *these structures allow for deliberation to proceed across time even though the propositions an agent considers might be different.* Fifth, *the Ramsey sentence's induced structures allows communication between agents with different algebras.*

The resulting interpretation has the Ramsey sentence be a description of the axioms and dictionary of a scientific theory. A theory's axioms and dictionary are universally quantified sentences where the axioms limit the range of a theory's functions and the dictionary specifies the behavior of observational propositions in terms of the theoretical functions. A description of a proposition is a finite disjunction of that proposition's instances. For example, if the available names in a language are a, b, c, then a description of Fa is the disjunction $Fa \vee Fb \vee Fc$. Similarly, if the possible functions in a language are F, G, H, K, then the description of a theory's axioms and dictionary $\forall xF \wedge \forall xO \equiv G$ would be $(\forall xF \wedge \forall xO \equiv G) \vee (\forall xH \wedge \forall xO \equiv K)$. Importantly, descriptions are finite because they are only warranted in the case when there is a witness for the description. This means that the description has no meaning independent of its witness. So the Ramsey sentence as a description is anti-realistic in the precise sense that it has no meaning apart from the theory's axioms and dictionary.

A Ramsey sentence's theoretical functions are extensional because each instance of a function induces a partition on an agent's possibility space with a special set of properties. Ramsey understands the extension of a logical function to be a set of propositions. Propositions here are understood as being sets of epistemic possibilities in the agent's possibility space. In the case of the Ramsey sentence, the described functions induce a partition on that possibility space whose elements are propositions. The axioms and dictionary rule out certain elements of those partitions as being incompatible with the theory. Consequently, the extension of a particular instance of a theoretical function from the Ramsey sentence happens to be the remnants of the partition compatible with the axioms and dictionary. The extension of the theoretical functions is the set of partition residues that agree with an instance of the axioms and dictionary. So the Ramsey sentence's functions understood extensionally amount to a modeling of the theory in the agent's algebra as one among many partitions that are compatible with some instance of the theory's axioms and dictionary.

This is relevant for decision-making. According to Ramsey, the laws and chances of a theory are what make it pertinent to decision-making. They act as the rules that guide deliberation; laws and chances are treated as "experts" the agent uses to regiment expectations. Since a theory says nothing more than what is given by its laws and chances, it is equivalent to those laws and chances. Every induced partition given by the extension of the Ramsey sentence reflects important behaviors of the agent's conditional credences given the laws and chances. Those show up in the form of the sets of possibilities the agent takes to be live in his algebra. Combined with marginal credences over individual theoretical propositions, the theoretical propositions allow for the formation of expectations over observational propositions. This is to say, in modern terminology, the theory provides latent variables an agent can use to make predictions of observational propositions. So the induced partitions given by the Ramsey sentence express half of the agent's plan for self-control: they reify the agent's rules for regimenting expectations. Thus, the laws and chances an agent uses in deliberation appear in his algebra as the structural relations between theoretical and observational propositions. Each member of the description of a theory's axioms and dictionary duplicates this behavior. So the Ramsey sentence is directly relevant for decision-making because it summarizes the decision rules the agent utilizes in managing expectations.

My account of the Ramsey sentence explains the functioning and utility of scientific theories relative to an impoverished observation language. It allows the generation of laws from the theory, which are instrumental for decision-making. But most importantly, it allows for an agent to deliberate across time and to communicate with other agents. The rules an agent follows for regimenting his expectations are captured by the agreeing structures in the propositions induced by the Ramsey sentence. When agents learn new facts, they can safely ignore worries about how their updates might apply to the same fictional theoretical propositions; every possible theoretical partition they consider will objectify the same rules. Those rules drive the decision-making process. So decisions will be coherent even though strictly speaking, the fictional theoretical propositions might differ from update to update. Deliberation can proceed because any partition induced by the Ramsey sentence applies the same rules for calculating credences.

This also allows for communication between individuals. What matters in communication is that people obey the same rules for registering their credences. They may not—and will almost surely not—consider the same propositions. But if they believe the same Ramsey sentence, their propositions will terminate in the same rules for estimating credences. And that is what ultimately leads to action. So these different propositions will actually have the same meaning. Consequently, communication follows from the Ramsey sentence.

My plan for demonstrating how this account follows from the text and how it is the best reconstruction of Ramsey's thoughts is as follows. First, I review the main views of the Ramsey sentence philosophers have developed. Second, I look at every usage Ramsey has of the sentence. Together these sections will allow for the construction of multiple criteria that my account of the Ramsey sentence must satisfy. Third, I construct an example of a theory involving chances to better see how the Ramsey sentence works in the case of chances. Fourth, I discuss this example in detail and construct my own view of the Ramsey sentence using Ramsey's decision theory. Fifth, I argue that this constructed account satisfies every criterion enunciated.

5.2 Review

The Ramsey sentence has received significant attention in the philosophy of science since Braithwaite introduced the concept to philosophers in Braithwaite, 1953. I review the different views of the Ramsey sentence here and document what is flawed about each view.

Braithwaite's account is first. In his book *Scientific Explanation*, Braithwaite explains an early version of what I call the functionalist account of the Ramsey sentence. By showing the deductive relationships between theoretical and observational terms, the Ramsey sentence documents the functions theoretical terms perform in the theory's deductive structure. But the sentence ignores the reference of its theoretical terms because the sentence's existential quantifier and bound variables replace those terms. So the Ramsey sentence avoids reference to theoretical terms yet it preserves the deductive form of the theory.[1]

[1]Braithwaite writes in response to the question of what is the status of theoretical concepts:

> One way of answering this question which is in essence the answer given by Ramsey is to say that the status of a theoretical concept (e.g. an electron) is given by the following proposition which specifies the status of an electron within the deductive system of contemporary physics: There is a property E (called "being an electron") which is such that certain higher-level propositions about this property E are true, and from these higher-level propositions there follow certain lowest-level propositions which are empirically testable. Nothing is asserted about the 'nature' of this property E; all that is asserted is that the property E exists, i.e. that there are instances E, namely, electrons (Braithwaite, 1953, 79).

There are three problems with Braithwaite's account. First, the text hinders Braithwaite's interpretation. Ramsey introduces his eponymous sentence with a remark that the judgments of a theory are its laws and consequences, and the theory is just clothing for those judgments one can use without working them out. Then he gives the sentence and remarks that the secondary system functions are to be taken extensionally and that any addition to the theory must occur within the scope of the quantifier. He fails to discuss the relevance of the Ramsey sentence's deductive structure for the theoretical terms; instead, he argues the form of a theory is important for how one understands a theory's meaning and for how one agrees or disagrees with a theory. Both have little to do with the reference of a theory's terms. Second, the Ramsey sentence is a facile way of avoiding asserting anything about the "nature" of the theoretical properties. As Hempel dryly notes, an existential sentence is just an accounting trick for asserting the theoretical concept; it still references theoretical entities without explicitly naming them.[2] Third, Ramsey is some sort of anti-realist about theories. He writes in "Causal Qualities" that "The truth is that we deal with our primary system as part of a fictitious secondary system. Here we have a fictitious quality, and we can also have fictitious individuals" (Ramsey, [1929] 1990a, 137). He would not assert that theoretical properties exist because he thinks theoretical properties are fictions.

Other functionalist accounts of the Ramsey sentence inherit the problems from Braithwaite's interpretation.

David Lewis adopts a similar view to Braithwaite. In "Psychophysical and theoretical identifications", Lewis suggests that the Ramsey sentence identifies of the causal roles of our

[2]He writes:

> But this means that the Ramsey-sentence associated with an interpreted theory T' avoids reference to hypothetical entities only in letter—replacing Latin constants by Greek variables—rather than in spirit. For it still asserts the existence of certain entities of the kind postulated by T', without guaranteeing any more than does T' that those entities are observable or at least fully characterizable in terms of observables. Hence, Ramsey-sentences provide no satisfactory way of avoiding theoretical concepts (Hempel, 1958, 81).

theoretical terms. The conjunction of the axioms and the dictionary in the Ramsey sentence provides the causal roles of theoretical terms.[3] Like Braithwaite, Lewis believes the causal roles specify that theoretical terms have real meaning and the existential quantifier really asserts that they exist. Unlike Braithwaite, Lewis makes the Ramsey sentence a first-order sentence: the terms are no longer the properties but individuals quantified over by the theory.

But each problem plaguing Braithwaite's view also infects Lewis's view with a further problem. First, Ramsey fails to discuss how the Ramsey sentence documents the causal roles of theoretical entities within the theory. Second, Lewis's "realist" interpretation has the Ramsey sentence assert a lot about theoretical entities. Third, Lewis's realism clashes with Ramsey's anti-realism on theories. Ramsey explicitly says theories are not judgments but the "clothes" for such judgments. Finally, Lewis introduces a new problem: the Ramsey sentence is now a first-order sentence. This is wrong because Ramsey never uses his sentence in that way; his brief discussion is always with it as a second-order sentence.

Psillos takes the Ramsey sentence to give a different perspective on scientific realism. He thinks the Ramsey sentence expresses a theory's content, but the theory's content is not the theory's laws and consequences. He argues that existentially quantifying the theoretical vocabulary allows for the fictitious propositions of the theoretical language to be truth-

[3] He writes:

> Suppose we have a new theory, T, introducing the new terms $t_1, \ldots, t_n$. These are our T-terms. (Let them be names.) Every other term in our vocabulary, therefore, is an O-term. The theory T is presented in a sentence called the postulate of T. Assume this is a single sentence, perhaps a long conjunction. It says of the entities—states, magnitudes, species, or whatever—named by the T-terms that they occupy certain *causal roles*; that they stand in specified causal (and other) relations to entities named by T-terms, and to one another (Lewis, 1972, 253).

He then gives the Ramsey sentence as quantifying out the T-terms with an existential quantifier. This view is essentially Carnap's. But unlike Carnap, Lewis takes the existential quantifier literally:

> If I am right, T-terms are eliminable—we can always replace them by their definientia. Of course, this is not to say that theories are fictions, or that theories are uninterpreted formal abacuses, or that theoretical entities are unreal. Quite the opposite! Because we understand the O-terms, and we can define the T-terms from them, theories are fully meaningful; we have reason to think a good theory true; and if a theory is true, then whatever exists according to the theory really *does* exist.

apt. The sentences of a theory are really open formulas where the exact denotation of theoretical vocabulary is not given. By binding them with the existential quantifier, they become true judgments. This is important because it allows theories to carry ontological commitments.[4] Consequently, the existential quantifier prevents theories from being only summaries of observational propositions. Theories have surplus content. Psillos concludes the existential quantifier in the Ramsey sentence allows the theoretical terms to be multiply realized, and the existential quantifier shows that theories have a certain meaning holism that allows them to change over time.[5]

While Psillos's points can be justified from Ramsey's text, they can be justified *without an appeal to a realist's interpretation of the existential quantifier.*

A realist interprets the existentially quantified sentences to be about objects in the world. Consider the existential claim that for every integer, there exists an additive inverse, i.e.

[4]He writes:

> Against the backdrop of Schlick's approach, we can now see Ramsey's insight clearly. We need not divorce the theory from its content, nor restrict it to whatever can be said within the primary system, provided that we treat a theory as an existential judgement. Like Schlick, Ramsey does treat the propositional functions of the secondary system as variables. But, in opposition to Schlick, he thinks that advocating an empirical theory carries with it a claim of realisation (and not just an if-then claim): there are entities which satisfy the theory. This is captured by the existential quantifiers with which the theory is prefixed. They turn the axiom-system from a set of open formulas into a set of sentences. Being a set of sentences, the resulting construction is truth-valuable. It carries the commitment that not all statements such as 'α, β, γ stand to the elements of the primary system in the relations specified by the dictionary and the axioms' are false. But of course, this ineliminable general commitment does not imply any specific commitment to the values of α, β, γ (Psillos, 2004, 71).

[5]Summarizing what he takes Ramsey's insights to be, Psillos writes:

> First, a theory need *not* be seen as a summary of what can be said in the primary system. Second, theories, *qua* hypothetico-deductive structures, have excess content over their primary systems, and this excess content is seen when the theory is formulated as expressing an existential judgement. Third, a theory need *not* use names in order to refer to anything (in the secondary system). Existentially bound variables can do this job perfectly well. Fourth, a theory need *not* be a definite description to be a) truth-valuable, b) ontically committing, and c) useful. So uniqueness of realisation (or satisfaction) is not necessary for the above. Fifth, if we take a theory as a *dynamic* entity (something that can be improved upon, refined, modified, changed, enlarged), we are better off if we see it as a *growing existential sentence* (Psillos, 2004, 72).

$\forall n \in \mathbb{Z}, \exists m \in \mathbb{Z},\ n + m = 0$. A Platonist realistically interprets the existential to say there are actual abstracta, additive inverses, in the world. Non-Platonists, however, interpret the existential fictionally: they deny the existence of abstracta, but some might believe this sentence to be true because he can articulate an algorithm for constructing integers that satisfy the additive inverse property.

Non-realistic interpretations of the quantifier can justify Psillos's observations. First, a non-realist may view theories to be more than summaries of observation. Consider again the example of the proposition every integer has an additive inverse. Non-realists of the type described previously view theories of the integers to have surplus content because they specify algorithms for constructing the integers. Second, a non-realist of the existential quantifier can avoid the use of names. He ignores specific naturals in constructing the integers. Third, non-realists would allow for multiple realizations of the quantified theoretical terms. Fourth, non-realists could have theories to be dynamic because they could amend the algorithms that construct theoretical entities over time.

So did Ramsey have a realist interpretation of the quantifier? The answer is negative. Psillos appeals to Ramsey's discussion from the 1926 essay "Mathematical Logic" for his realist interpretation of the quantifier. But Ramsey had by 1929 abandoned the view of logic and mathematics given in that essay for something closer to Weyl's (see Majer, 1989 for a discussion), Hilbert, and Wittgenstein. Furthermore, Ramsey makes several comments in "Theories" and associated work that writing scientific theories are not judgments. He says in "Theories" that "the totality of laws and consequences will be the eliminant when $\alpha, \beta, \gamma, ..$, etc., are eliminated from the dictionary and axioms, and it is this totality of laws and consequences which our theory asserts to be true" (Ramsey, [1929] 1990m, 115). If more than just the laws and consequences were asserted by a theory, then why does Ramsey omit to say so? He also writes that the existential quantifier prefaces the meaning of the theoretical propositions it binds like "once upon a time". Why use a clearly fictitious phrase if the quantifier is supposed to be about stuff in the real world? Lastly, the Ramsey sentence is the proper form of a scientific theory; it does not transform a theory into a true judgment. But theories are fictions, as he says in "Causal Qualities". Consequently, he must interpret the existential quantifier non-realistically in the same way as "once upon a time" makes clear what follows is fictional.

I have argued that the functionalist interpretations of Braithwaite and Lewis fail for various reasons, and I have argued that the more robust realist interpretation given by Stillos fails because it is unlikely that Ramsey had a realist interpretation of the existential quantifier. This leaves Carnap's and Demopoulos's view that the Ramsey sentence is a device for eliminating theoretical terms.

The first person to articulate this view is Carnap. Carnap argues that Ramsey believes scientists only care about a theory's observational consequences. Theoretical terms are "bothersome" and should be eliminated. Ramsey devised the Ramsey sentence to jettison a scientific theory's theoretical terms while preserving the theories observational consequences.[6] This allows Ramsey to sidestep questions about the existence of theoretical terms. Carnap proposes that Ramsey's insight is that the meaning of the theoretical vocabulary is captured by the Ramsey sentence. Because he avoids reference to theoretical vocabulary, he ignores thorny metaphysical questions surrounding scientific theories. However, theories are not meaningless. Observational confirmation means the theory is true since the content of the theory is

[6]He summarizes Ramsey's view of the Ramsey sentence in a talk at the Pacific APA in 1959:

> Now Ramsey showed that this existential sentence—which we call now the Ramsey-sentence—is O-equivalent to the theory TC. And he made the following practical proposal. He said: The theoretical terms are rather bothersome, because we cannot specify explicitly and completely what we mean by them. If we could find a way of getting rid of them and still doing everything that we want to do in physics with the original theory, which contains these terms, that would be fine. And he proposes this existential sentence. You see, in the existential sentence the T-terms no longer occur. They are replaced by variables, and the variables are bound by existential quantifiers, therefore that sentence is in the language L'_O, in the extended observation language. And he said: let's just forget about the old formulation TC about the T-terms; let's just take this existential sentence, and from it we get all the observational consequences which we want to have, namely, all those which we can derive from the original theory (Psillos, 2000, 163).

expressed by the Ramsey sentence.[7] Carnap attributes to Ramsey an instrumentalist view of theories that coincides with his view of meaning (Carnap and Gardner, 1966, 255).

The view attributed to Ramsey is anti-realist and reductionist. It is anti-realist because it attributes the truth of a theoretical proposition to the confirmation of its observable consequences. Carnap's Ramsey ignores whether theoretical terms denote anything separate from observation. It is reductionist because Carnap's Ramsey wants to eliminate talk of theoretical terms, and the Ramsey sentence replaces that talk with bound variables. Carnap attributes to Ramsey the explicit philosophical goal of soothing the scientist from the "bothersome" theoretical terms. Ramsey's panacea for metaphysics is the Ramsey sentence.

Carnap's Ramsey is an incorrect reading because Ramsey opposes the broader project of eliminating theoretical vocabulary. A review of his primary argument in "Theories" shows this. The chief discussion in "Theories" is around the desirability of explicit definitions for theoretical vocabulary:

> 2. Can we reproduce the structure of our theory by means of explicit definitions within the primary system?

[7] Carnap promulgated this view of the Ramsey sentence to a generation of philosophers in an influential textbook (Carnap and Gardner, 1966; Carnap, 1974). In that textbook, Carnap recognizes Hempel's insight that the Ramsey sentence does not solve the problem of asserting the existence of theoretical entities. He then writes how Ramsey allows one to avoid even asking existential questions about theoretical entities:

> The important fact is that we can now avoid all the troublesome metaphysical questions that plague the original formulation of theories and can introduce a simplification into the formulation of theories. Before, we had theoretical terms, such as "electron", of dubious "reality" because they were so far removed from the observable world. Whatever partial empirical meaning could be given to these terms could be given only by the indirect procedure of stating a system of theoretical postulates and connecting those postulates with empirical observations by means of correspondence rules. In Ramsey's way of talking about the external world, a term such as "electron" vanishes. This does not in any way imply that electrons vanish, or, more precisely, that whatever it is in the external world that is symbolized by the word "electron" vanishes. The Ramsey sentence continues to assert, through its existential quantifiers, that there is something in the external world that has all those properties that physicists assign to the electron. It does not question the existence—the "reality"—of this something. It merely proposes a different way of talking about that something. The troublesome question it avoids is not, "Do electrons exist?" but, "What is the exact *meaning* of the term 'electron'?" (Carnap and Gardner, 1966, 252).

> [This question is important because Russell, Whitehead, Nicod and Carnap all seem to suppose that we can and must do this.] (Ramsey, [1929] 1990m, 120)

He argues across nine pages (a third of the paper) that while such definitions are always possible, multiple definitions will be possible and any definition will be disjunctive. He then asks the question "Is this *necessary* for the legitimate use of the theory?" (Ramsey, [1929] 1990m, 129). He concludes that no, it is not necessary:

> To this the answer seems clear that it cannot be necessary, or a theory would be no use at all. Rather than gives all these definitions it would be simpler to leave the facts, laws and consequences in the language of the primary system. Also the arbitrariness of the definitions makes it impossible for them to be adequate to the theory as something in process of growth (Ramsey, [1929] 1990m, 130).

He argues that such a reduced theory will be too complicated to use, and he argues their arbitrariness will prevent the theory from growing over time[8] This argument demonstrates Ramsey's broader opposition to eliminating theoretical vocabulary. He admits in the argument that explicit definitions are always possible, but he still insists eliminating theories with these definitions is inappropriate. Theories are useful—they should not be replaced with their observable consequences. So Ramsey's project is to show how theories are not eliminable.

Carnap's Ramsey has the goal of eliminating "bothersome" theoretical terms for observable consequences. A theory generates the singular, observable propositions scientists care about. The Ramsey sentence shows how theories generate those observable propositions without the metaphysical baggage through the elimination of theoretical terms. But Ramsey wants to

[8] The theory could not grow because each new axiom added to the theory will force change in the definitions and therefore the meaning of theoretical propositions.

preserve the theoretical vocabulary. So the point of the Ramsey sentence cannot be to eliminate the theoretical vocabulary.

Demopoulos's interpretation of the Ramsey sentence suffers from the same problem as Carnap's. According to Demopoulos, Ramsey continues Russell's project from *Our Knowledge of the External World*. Ramsey develops a sophisticated application of Russell's supreme maxim of scientific philosophy: "Wherever possible, logical constructions are to be substituted for inferred entities" (Russell, [1914] 1951, 115). Russell aims to show how the logical structure of a theory secures the "reasoning" of the theory, i.e. the observable consequences and hypothetical propositions while avoiding dubious hypothetical entities. The Ramsey sentence obtains the "reasoning" of the theory through its deductive structure, and it eliminates theoretical terms without explicit definition's twin problems of arbitrary and complicated definitions.[9]

The problem with Demopoulos's story is that Ramsey explicitly desires to conserve theoretical terms. It is odd if he claims that reductionism by explicit definitions fails (though

[9]Demopoulos writes:

> For him [Ramsey], the notion simply gave precise expression to the fact that the derivation of propositions belonging to what he called the 'primary system' (the L_O-sentences in our terminology) does not depend on our assigning any meaning to the 'secondary' or theoretical terms beyond the isolation of their logical category. As Ramsey wrote:
>
> We can say, therefore, that the incompleteness [that results when, in the secondary propositions, non-logical constants are replaced by variables] affects our disputes but not our reasoning. (Ramsey [1929], p. 232)
>
> Ramsey's point is that our disputes, insofar as they involve questions of truth and mutual compatibility, depend on the completeness of our propositions, and completeness is precisely what is set to one side when we pass from secondary propositions to the propositional functions which replace them under ramsification. But our reasoning with secondary propositions does not require their completeness. By 'our reasoning' Ramsey clearly intended to include both the derivation of consequences of the theory—especially the derivation of L_O-consequences—and the use of the theory to reason hypothetically to conclusions which, although not consequences of the theory, are conclusions that would not be forthcoming without its help. So far as our reasoning with the theory in any of these senses is concerned, the secondary vocabulary is entirely eliminable in favor of variables, provided evident consistency conditions are respected in their use. Moreover, the eliminability of the secondary vocabulary in the reconstruction of our reasoning does not require its *reduction by explicit definitions* to the primary vocabulary (Demopoulos, 2011, 181–182).

it admittedly does not) and then says "oh by the way, there is another method of elimination that works perfectly well." Ramsey certainly overlooks saying so. Furthermore, the textual evidence that Demopoulos cites for his view hinders his view. Consequently, it is a stretch to say that Ramsey's point is about the elimination of the theoretical terms for their observational consequences.[10]

In summary, the core problem with both Carnap's and Demopoulos's interpretations is that they attribute to Ramsey the goal of eliminating theoretical terms for their observational consequences. This has the obvious difficulty that Ramsey's primary argument in "Theories" is against the elimination project. Ramsey thinks that if scientists want to eliminate the theoretical for the observational, then they would not have bothered with constructing a theory to begin with (Ramsey, [1929] 1990m, 130).

The final major interpretation to discuss is Majer's (Majer, 1989). I set this discussion aside for a later chapter dedicated to the existential quantifier. For the moment, it is sufficient to note his interpretation also has some major problems. Primarily, it is ill-motivated: Majer thinks that Ramsey is interested in justifying laws of the sort where a universal quantifier

[10]The claim about the "reasoning" of the theory does not fit with what Ramsey says surrounding the words "reasoning". Ramsey emphasizes that when deciding on the adequacy of two incompatible theoretical propositions, one should include both propositions under the scope of the same quantifier:

> So far, however, as *reasoning* is concerned, that the values of these functions are not complete propositions makes no difference, provided we interpret all logical combination as taking place within the scope of a single prefix $(\exists \alpha, \beta, \gamma)$; e.g.,
>
> $\overline{\beta(n,3).\overline{\beta}(n,3)}$ must be $(\exists \beta) : \overline{\beta(n,3).\overline{\beta}(n,3)}$, not $\overline{(\exists \beta)\beta(n,3).(\exists \beta)\overline{\beta}(n,3)}$.
>
> For we can reason about the characters in a story just as well as if they were really identified, provided we don't take part of what we say as about one story, part about another (Ramsey, [1929] 1990m, 132).

Ramsey only discusses how the claims of different theoretical propositions are supposed to be understood. He neglects theoretical propositions' consequences. If he is considering the different consequences of theoretical propositions, then those propositions would be treated as two different theories with two different quantifiers. But since a single quantifier ranges over those different propositions, the consequences of the theory remain the same between both propositions. The theory of "there once was a hobbit Bilbo, who went to the Lonely Mountain or did not go to the Lonely Mountain" has only a single set of empirical consequences whereas the theory of "there once was a hobbit Bilbo, who went to the Lonely mountain" has different consequences from the theory "there once was a hobbit Bilbo, who did not go to the Lonely mountain". This is hypothetical reasoning. It does not concern the derivation of consequences from the theory.

is followed by an existential. But this does not explicitly show up anywhere in Ramsey's writings on science.

The problems with different interpretations of the Ramsey sentence illustrate several properties that a satisfactory view must have. First, it needs to provide a compelling story for what the Ramsey sentence is supposed to do in Ramsey's broader philosophy. What is it for? I have argued in a previous chapter that Ramsey's philosophy of science should be understood in the context of Ramsey's decision theory. How does the Ramsey sentence relate to decision-making? Call this the **decision-making criterion**. Second, an account of the Ramsey sentence should satisfy several correct observations authors have made about it: it signifies in some way the surplus content of the theory. Call this the **surplus content criterion**. Third, Ramsey's fictionalism about theories dictates that the existential quantifier be interpreted in the anti-realist fashion I have described. That is the quantifier cannot be taken to provide judgments about stuff in the world. One then needs an illustration of how that should work with a given interpretation of the quantifier. Call this the **anti-realism criterion**.

5.3 Ramsey on the Ramsey Sentence

The different accounts of the Ramsey sentence have various problems. Before I produce an account that corrects those problems, it would be useful to lay out what Ramsey himself says about the Ramsey sentence. This will add to the list of criteria from the previous section.

5.3.1 Ramsey's Use of the Ramsey Sentence

Ramsey's introduction of the Ramsey sentence follows his attack on explicit definitions. He argues that while it is possible to provide eliminative definitions of the theoretical system, it should be avoided because

> [A] theory would be no use at all. Rather than give all these definitions it would be simpler to leave the facts, laws and consequences in the language in the language of the primary. Also the arbitrariness of the definitions makes it impossible for them to be adequate to the theory as something in process of growth (Ramsey, [1929] 1990m, 130).

Ramsey makes two points. First, theories without theoretical terms are useless. He means by useless that a useless theory makes no practical difference in making decisions. Earlier in the paper, Ramsey explicitly ties the utility of the theory to its laws and consequences in the observation language. A theory provides laws, which as I have argued are crucial to decision-making for Ramsey. Explicitly defining a theory with those laws makes the theory superfluous because one already has the laws and consequences used in decision-making. Second, these arbitrary and complex explicit definitions hinder the theory's ability to grow and change. Because the explicit definitions would change every time the theory changes, the theory's meaning would shift. Thus the theory would remain constant.

After having dismissed reductionism, Ramsey asks how scientific theories work: "Taking it then that explicit definitions are not necessary, how are we to explain the functioning of our theory without them?" (Ramsey, [1929] 1990m, 130). It is here that he introduces the Ramsey sentence:

> Clearly in such a theory judgment is involved, and the judgments in question could be given by the laws and consequences, the theory being simply a language in which they are clothed, and which we can use without working out the laws and consequences.
>
> The best way to write our theory seems to be this $(\exists\alpha, \beta, \gamma)$: dictionary . axioms.

> The dictionary being in the form of equivalences.
>
> Here it is evident that α, β, γ are to be taken purely extensionally. Their extensions may be filled with intensions or not, but this is irrelevant to what can be deduced in the primary system (Ramsey, [1929] 1990m, 131)

There are several things to note from this passage. With the "theories as clothes passage", Ramsey seems to either endorse instrumentalism or contrast instrumentalism with the Ramsey sentence. Additionally, Ramsey remarks that the theoretical terms are meant to be taken extensionally. It is unclear what he means by "extensionally" here.

Starting with the "theories as clothes passage", Ramsey says that a theory involves judgment and that judgment "*could* [emphasis mine] be given by the laws and consequences". The word "could" has resulted in some controversy. While one might take the view to be expressed here as Ramsey's, Psillos argues that the "could" is meant to be taken contrastively with the scientific realism given by the Ramsey sentence (Psillos, 2004, 69–70). This is unlikely. As other authors have gleaned from Ramsey's remarks in his notes, Ramsey viewed theories instrumentally (Demopoulos, 2011, 190–191). Furthermore, the remarks in the "clothes" passage match very closely what Ramsey says unambiguously earlier in the paper: "The totality of laws and consequences will be eliminant when α, β, γ.., etc., are eliminated from the dictionary and axioms, and it is this totality of laws and consequences which our theory asserts to be true" (Ramsey, [1929] 1990m, 115). The assertion—the judgment—given by one's theories are just the laws and consequences deduced from the theory. So the "could" in the above passage is not meant to be an alternative to Ramsey's view: it just is what he takes theories to be doing.

The upshot is that one needs to show how an anti-realist interpretation of the Ramsey sentence allows the deduction of the laws and singular judgments. However the Ramsey sentence is interpreted to not be about objects in the world, it needs to show how laws and consequences about the world can be deduced from it. Call this the **law and consequences criterion**.

Ramsey states that the α, β, γ are meant to be taken extensionally. What he means by "extensionally" is that the functions α, β, γ are meant to be taken as sets. This is how he defines "extensionally" in "Foundations of Mathematics":

> Here, of course, we are using 'extension' in its logical sense, in which the extension of a predicate is a class, that of a relation a class of ordered couples; so that in calling mathematics extensional we mean that it deals not with predicates but with classes, not with relations in the ordinary sense but possible correlations, or "relations in extension" as Mr Russell calls them (Ramsey, [1926] 1990l, 177).

Predicates understood extensionally are classes of individuals, binary relations are classes of ordered couples, and so on. By a class, Ramsey means a type of set:

> I do not use the word 'class' to imply a principle of classification, as the word naturally suggests, but by a 'class' I mean any set of things of the same logical type. Such a set, it seems to me, may or may not be definable either by enumeration or as the extension of a predicate. If it is not so definable we cannot mention it by itself, but only deal with it by implication in propositions about all classes or some classes. The same is true of relations in extension, by which I do not merely mean the extensions of actual relations, but any set of ordered couples (Ramsey, [1926] 1990l, 178).

A class is a set defined by some logical type. The logical types here are individuals, sets of individuals, sets of sets of individuals, and so on. Individual classes may be defined by some predicate or non-predicate such as a function. The point is that however they are defined, the extension happens to be a set of a particular type.

This is relevant to "Theories" because the only individuals in the "domain" of Ramsey's toy example that α, β, γ could be true of are integers and pairs of integers. It is unlikely the logical types of α, β, γ are defined by the properties of their members. Any interpretation of the Ramsey sentence must identify these integers, what exactly is the logical type of α, β, γ, and how those integers fit with this logical type. In short, an account of the Ramsey sentence needs to say what exactly the class is here, what is the logical type, and how the aforementioned sentence connects with the extension. Call this the **extension criterion**.

So far, I have documented the context where Ramsey initially uses the Ramsey sentence. After introducing it, Ramsey moves to two applications of the sentence. The first deals with theory growth and the meaning of individual theoretical propositions. The second concerns how one entertains theoretical propositions and how one entertains disputes with theoretical propositions.

Ramsey directly connects theory growth with the Ramsey sentence:

> Any additions to the theory, whether in the form of new axioms or particular assertions like $\alpha(0, 3)$, are to be made within the scope of the original α, β, γ. They are not, therefore, strictly propositions themselves just as the different sentences in a story beginning 'Once upon a time' have not complete meanings and so are not propositions by themselves (Ramsey, [1929] 1990m, 131).

Two things should be noted here. First, he considers the existential quantifier to be equivalent to the "Once upon a time" introduction from fairy tales. This is additional evidence for the anti-realism criterion. Second, he explicitly ties the introduction of new theoretical propositions to them being introduced within the scope of a theory's quantifier. Thus, individual theoretical propositions can only be understood with respect to the whole theory. Ramsey makes this meaning holism explicit:

> This makes both a theoretical and practical difference:
>
> (a) When we ask for the meaning of e.g. $\alpha(0,3)$ it can only be given when we know to what stock of 'propositions' of the *first and second* systems $\alpha(0,3)$ is to be added. Then the meaning is the difference in the first system between $(\exists\alpha,\beta,\gamma)$: stock $.\alpha(0,3)$, and $(\exists\alpha,\beta,\gamma)$. stock. (We include propositions of the primary system in our stock although these do not contain α,β,γ.)
>
> This account makes $\alpha(0,3)$ mean something like what we called above $\tau\{\alpha(0,3)\}$, but it is really the difference between $\tau\{\alpha(0,3)+$ stock$\}$ and τ(stock) (Ramsey, [1929] 1990m, 131).

He concludes theory growth is understood as introducing new theoretical propositions under the scope of a theory's quantifiers implies the meaning of a theoretical proposition is the difference between the theory with and without that proposition. A theoretical proposition's meaning is a shift in the theory's verification conditions. The τ Ramsey references here is the τ discussed in a prior chapter. Recall that the τ of a secondary system proposition is the set of primary system truth-possibilities that if false would falsify said secondary proposition. So Ramsey's example shows how the meaning of a theoretical proposition comes from the difference that proposition makes to the verification conditions of the theory.

So the Ramsey sentence is supposed to be connected with the verification conditions of a theory. Any account of the Ramsey sentence needs to factor this in. Call that requirement the **verification conditions criterion**.

Ramsey also uses the Ramsey sentence when considering the truth of theoretical propositions. He discusses the practical difference the Ramsey sentence makes in understanding a theory:

> (b) In practice, if we ask ourselves the question "Is $\alpha(0,3)$ true?", we have to adopt an attitude rather different from that which we should adopt to a genuine

proposition.

> For we do not add $\alpha(0,3)$ to our stock whenever we think we could truthfully do so, i.e., whenever we suppose $(\exists\alpha,\beta,\gamma)$: stock . $\alpha(0,3)$ to be true. $(\exists\alpha,\beta,\gamma)$: stock . $\overline{\alpha}(0,3)$ might also be true. We have to think what else we might be going to add to our stock, or hoping to add, and consider whether $\alpha(0,3)$ would be certain to suit any further additions better than $\overline{\alpha}(0,3)$. E.g. in our little theory either $\beta(n,3)$ or $\overline{\beta}(n,3)$ could always be added to any stock which includes $\overline{\alpha}(n,3). \vee . \overline{A}(n).\overline{B}(n)$. But we do not add either, because hope from the observed instances to find a law and then to fill in the unobserved ones according to that law, not at random beforehand (Ramsey, [1929] 1990m, 131–132).

When I consider the truth of a theoretical proposition, I entertain the effect of adding other theoretical propositions to my theory's Ramsey sentence. For example, "Bilbo had a gold-plated pocket watch" is true is not the same as saying "Once upon a time, the Hobbit and Lord of the Rings story and Bilbo had a gold-plated pocket watch" is true. I need to think whether it makes sense in Middle Earth for there to be gold-plated pocket watches and how a hobbit such as Bilbo might have one. Thus, for a theoretical proposition to be true, it must fit with the broader theory.

When I think about the fit of a theoretical proposition within a theory, I am reasoning hypothetically. This type of reasoning is different from the reasoning that occurs when two people dispute a theoretical proposition. Ramsey explicitly contrasts these two cases:

> So far, however, as *reasoning* is concerned, that the values of these functions are not complete propositions makes no difference, provided we interpret all logical combination as taking place within the scope of a single prefix $(\exists\alpha,\beta,\gamma)$; e.g., $\overline{\beta(n,3).\overline{\beta}(n,3)}$ $[\beta(n,3) \vee \overline{\beta}(n,3)]$ must be $(\exists\beta)$: $\overline{\beta(n,3).\overline{\beta}(n,3)}$, not $\overline{(\exists\beta)\beta(n,3).(\exists\beta)\overline{\beta}(n,3)}$ $[\overline{(\exists\beta)\beta(n,3)} \vee \overline{(\exists\beta)\overline{\beta(n,3)}}]$.

> For we can reason about the characters in a story just as well as if they were really identified, provided we don't take part of what we say as about one story, part about another.
>
> We can say, therefore, that he incompleteness of the 'propositions' of the secondary system affects our *disputes* but not our *reasoning* (Ramsey, [1929] 1990m, 132).

When doing hypothetical reasoning, I must consider any hypothetical additions under the scope of the same quantifier. I need to ensure my hypothetical additions fit with my theory. When I reason about the consequences of Bilbo going to the Lonely Mountain, I have to ensure my Bilbo is the same Bilbo in the *Hobbit*. I replace the name "Bilbo" with a bound variable under the existential quantifier of my *Hobbit* Ramsey sentence. A dispute over rival theories, however, involves two quantifiers because the two theories have different Ramsey sentences. The "incompleteness" of theories—theoretical propositions must occur under the scope of an existential quantifier—makes disputes different from hypothetical reasoning.

The Ramsey sentence account of theories has important consequences for the communication of theories. In Ramsey's example of a dispute, each disputant quantifies out over their respective theories. Disputes over theories involve disputes over their Ramsey sentences; any communication must go through the Ramsey sentence. Ramsey references how the "incompleteness" of theoretical propositions affects disputes but not the reasoning. By "incompleteness" he means that the propositions have no meaning. Their verification conditions provide their meaning, but those verification conditions can only come once theoretical propositions mesh within a theory. A theory's Ramsey sentence demonstrates this meshing. Consequently, people with different theories will then mean very different things since their Ramsey sentences will be different.

An account of the Ramsey sentence needs to explain how the Ramsey sentence facilitates communication and affects disagreement. How do people who believe the same Ramsey sentence believe the same theory? How do they in fact disagree when they do not believe the same Ramsey sentence? Answering these questions would satisfy the **communication criterion**.

And that is it for Ramsey's use of the Ramsey sentence. The remainder of the paper "Theories" is notable for its absence of the Ramsey sentence. Instead, Ramsey moves on to discuss what it means for theories to be incompatible and when they are compatible. None of this discussion appeals to the Ramsey sentence. It is justifiable to say that despite the importance that philosophers of science have attached to the Ramsey sentence, Ramsey himself largely ignores it.

5.3.2 Summary and Strategy

I have discussed a number of different criteria that any account of the Ramsey sentence must satisfy. Some of those criteria were given in the previous section where I looked at the main interpretations given of the Ramsey sentence and found them wanting. In this section I looked at Ramsey's entire use of the Ramsey sentence in "Theories", and I added several more criteria from that examination. A recap would be useful.

The first criterion I argued for is the **decision-making criterion**. Ramsey's larger philosophical project included his decision theory. One should know how the Ramsey sentence fits with that decision theory. Second, an account of the Ramsey sentence needs to connect with an account of the surplus content of a theory. This is the **surplus content criterion**. Third, an account of the Ramsey sentence needs to explain how the existential quantifier does not quantify over stuff in the world separate from the agent. I need an anti-realist explanation of the existential quantifier and Ramsey sentence, which I called the **anti-realism criterion**. Fourth, an account of the Ramsey sentence needs to show how laws and consequences are deduced from the Ramsey sentence. I call this the **law and consequences criterion**. Fifth, the Ramsey sentence is supposed to be understood extensionally, where its extension is a class of objects of the same logical type. Any account of the Ramsey sentence needs to specify what the class is, the logical type, and how the sentence connects with the extension. This is the **extension criterion**. Sixth, Ramsey explicitly tied the Ramsey sentence of a theory together with its verification conditions. I have provided in a previous chapter an account of those verification conditions. So an explanation of the Ramsey sentence needs to relate a theory's verification conditions with its Ramsey sentence. I call this the **verification conditions criterion**. Finally, I argue that the Ramsey sentence is relevant to how people communicate theories. A story of the Ramsey sentence needs a compelling account of how that communication works with the Ramsey sentence. This is the **communication criterion**.

My goal for the remainder of the chapter is to craft an interpretation of the Ramsey sentence that satisfies all these criteria. The strategy is to appeal to an insight Ramsey gives about his own philosophy of science. I have argued previously how Ramsey thinks his philosophy of science is a forecasting theory, and what that happens to be exactly. Most importantly, Ramsey thinks his philosophy of science is best illustrated with chances:

> As opposed to a purely *descriptive* theory of science, mind may be called a *forecasting* theory. To regard a law as a summary of certain facts seems to me inadequate; it is also an attitude of expectation for the future. The difference is clearest in regard to chances; the facts summarized do not preclude an equal chance for a coincidence which would be summarized by and, indeed, lead to a quite different theory (Ramsey, [1929] 1990e, 163).

The allusion to chances is important. It illustrates characteristics of Ramsey's approach that a deterministic example does not. For that reason, I need to document how the Ramsey sentence works in the case of chances.

5.4 Working Through An Example and Its Consequences

To illustrate a number of important points about how Ramsey must have understood his eponymous sentence, it would be fruitful to have an example. Every example of the Ramsey sentence places it in the context of a deterministic, deductive system. Ramsey thinks that important features of his philosophy of science are illuminated as concerns chances. For that reason, the example I construct deals with chances. Ramsey writes extensively about chances in his published and unpublished papers. I have previously argued that laws are a variety of chance at unity. Since the Ramsey sentence is meant to deduce laws, it must then be relevant for the deduction of chances. I want to see how that works.

5.4.1 The Example

Consider the simplest case of flipping coins. The primary system consists of propositions that indicate the outcome of a coin toss. In the primary system, there are chances that one defers to setting one's credences. The secondary system consists of two things: a generating chance distribution and parameters over those chance distributions. Propositions in the secondary system are equalities between those parameters and their values. The axioms limit the range of values these parameters take and the dictionary specifies how the chance distribution sets the values of the primary system propositions.

Going into more detail, suppose that one fixes that all observations start at time 1. Let the primary system propositions be $\phi(n)$ such that $\phi(n)$ takes the values 1 or 0. So $\phi(n) = 1$ means "at time n, the coin landed heads" and $\phi(n) = 0$ means "at time n, the coin landed tails." Let the secondary system propositions consist of functions $\beta(n)$ that take either the value 0.6 or 0.4 and $\gamma(n)$ that takes a real value on the unit interval $[0, 1]$. These specify the bias of the coin and the drawing of a random sample. Next, I define the probability distribution function of flipping a coin in a sequence of trials. Let $f_\Phi : \{1, \dots\} \times \{1, \dots\} \times [0, 1] \to [0, 1]$ be defined with respect to the random variable Φ, which is the sum of successes and failures, i.e. $\Phi(n) = \sum \phi(i)$ and k being the total number of trials (so $n = k$ for $\Phi(n)$):

$$f_\Phi(x; k, p) = \Pr(\Phi(k) = x) = \binom{k}{x} p^x (1-p)^{k-x}$$

Next specify the axioms that $\beta(n)$ only takes the values 0.6 and 0.4, i.e. $\forall n, \beta(n) = 0.6 \vee \beta(n) = 0.4$, that β is fixed $\forall n, m, \beta(n) = \beta(m)$, and $\gamma(n)$ takes some value $[0, 1]$, $\forall n, \exists m \in [0, 1] \gamma(n) = m$. Finally, the dictionary requires that one sample from the chance distribution to set the value of $\phi(n)$. This requires the inverse cumulative distribution function. First, define the cumulative distribution function through f_Φ:

$$F_\Phi(x; k, p) = \sum_{i=0}^{\lfloor x \rfloor} f_\Phi(i; k, p)$$

And then define the inverse as just the greatest lower bound of a value of F_Φ that passes some threshold q:

$$F_\Phi^{-1}(q; k, p) = \inf\{x \in \mathbb{R} : q \leq F_\Phi(x)\} = \inf\{x \in \mathbb{R} : q \leq \sum_{i=0}^{\lfloor x \rfloor} \binom{k}{i} p^i (1-p)^{k-i}\}$$

We can then specify the values of $\phi(n)$ as follows:

Proposition	English
$\phi(n) = 1$	At time n, the coin landed heads.
$\phi(n) = 0$	At time n, the coin landed tails.
$\beta(n) = 0.6$	A time n, the bias of the coin is 0.6.
$\beta(n) = 0.4$	At time n, the bias of the coin is 0.4.
$\gamma(n) = x$	At time n, the random number was x.

(a) Vocabulary

Axioms:

1. $\forall n, \beta(n) = 0.6 \vee \beta(n) = 0.4$
2. $\forall n, m, \beta(n) = \beta(m)$
3. $\forall n, \exists m \in [0, 1], \gamma(n) = m$

Dictionary:

1. $\phi(n) = F_{\Phi}^{-1}(\gamma(n); 1, \beta(n))$

(b) Axioms and Dictionary

Figure 5.1: The primary and secondary system of the chapter.

$$\phi(n) = F_{\Phi}^{-1}(\gamma(n); 1, \beta(n))$$

This says that the value of a coin flip is the result of randomly sampling a Bernoulli distribution given some parameter $\beta(n)$. Note that $\gamma(n)$ fulfills the role of a random sample from that Bernoulli distribution. So what this simple theory says is that each coin flip is the result of sampling from a chance process given mathematically by a Bernoulli distribution.

The next step is to specify how chances are extracted from this secondary system.

Since f_{Φ} is defined with respect to the number of successes and failures in a sequence of coin tosses, I define the chance measure with respect to all possible conjunctions of $\phi(n)$ for fixed n. In other words, the chance measure's domain is the algebra generated by a partition X_n induced on possibility space through the primary system at a fixed time n. The partition X_n is a sample space for the chance measure, and individual primary system propositions are unions and complements of the cells in X_n. For example, if $n = 3$, then the partition X_3 has eight members, and propositions such as $\phi(1) = 1$ are the union of all members in X_3 where $\phi(1) = 1$ is true, i.e. the three toss sequences where the coin landed heads in the first toss. I specify $Y(\psi(m)) = \{x \in X_n : x \subseteq \psi(m)\}$ as the set of cells that are subsets of a given proposition ψ. Next, I define the function $g : X_n \to \mathbb{N}$ to be the function that counts the number of successes in a given partition cell. The chance function $\text{Ch} : \mathcal{A}_{O_n} \to [0, 1]$ where $\mathcal{A}_{O_n}$ is the subset of an agent's algebra consisting just of primary system propositions up through time n is defined as follows:

$$\text{Ch}(\psi(n)) = \sum_{y \in Y(\psi(n))} \beta(n)^{g(y)}(1 - \beta(n))^{n-g(y)}$$

The chance of a proposition is the sum of the individual probabilities of its elements, i.e. the sequences that compose it. Since these sequences are disjoint, the chance measure will satisfy finite additivity. By definition, each element of X_n is greater than or equal to zero and so propositions formed from those elements must be at least zero. The chance of X_n will equal one because its elements sum to one through the generating distribution. So the chance measure is a probability measure. Furthermore, the chances of conjunctions of propositions are independent because they are equal to sequences of Bernoulli trials.

With this chance measure, I can define credence in an individual coin toss (or any finite conjunction of propositions about coin tosses) through the law of total probability. Recall that the law of total probability implies that the marginal credence in any given proposition is a summing out through a conditional probability factorization. In this case, if I let $\psi(n)$ be any truth-function of ϕ, I have:

$$\begin{aligned}\text{Pr}(\psi(n)) &= \sum_c \text{Pr}(\psi(n), \text{Ch}(\psi(n)) = c) \\ &= \sum_c \text{Pr}(\psi(n) \,|\, \text{Ch}(\psi(n)) = c)\text{Pr}(\text{Ch}(n) = c)\end{aligned}$$

The first part of this product within the sum is the law that credence should defer to chance, and the second part is the prior credence over chances. As I have argued previously, Ramsey thinks that chances of propositions obey the principal principle.[11] This is just the following property:

$$\Pr(\psi(n) \,|\, \mathrm{Ch}(\psi(n)) = x, \Gamma) = x$$

where Γ is any set of propositions relevant to $\psi(n)$ and the chance. In this case, each value of the first part of the above product will just be c for each $\mathrm{Ch}(\psi(n)) = c$. Thus credence in $\psi(n)$ is a weighted sum of the chances by one's credences in the chances. Letting the value of $\psi(n)$ be the indicator function, the expectation of $\psi(n)$ is a weighted sum of chances. Consequently, this chance law yields a forecast as discussed in a prior chapter.

Now the Ramsey sentence can be applied to this toy theory. The secondary system here consists of two functions, β, γ, and the axioms and dictionary. Quantifying out, I have the following Ramsey sentence for the theory:

$$\begin{aligned} &\exists\beta\exists\gamma[\forall n(\beta(n) = 0.6 \vee \beta(n) = 0.4) \wedge \forall n\forall m(\beta(n) = \beta(m)) \wedge \\ &\qquad \forall n\exists m \in [0,1](\gamma(n) = m) \wedge \forall n(\phi(n) = F_{\Phi}^{-1}(\gamma(n); 1, \beta(n)))] \end{aligned} \tag{5.1}$$

The first three conjuncts under the quantifiers are the axioms and the last conjunct is the dictionary.

[11] He writes in "Chance" that "Besides these the systems contains various things of this sort: when knowing ψx and nothing else relevant, always expect ϕx with degree of belief p (what is or is not relevant is also specified in the system); which is also written the chance of ϕ given ψ is p (if $p = 1$ it is the same as a law)" (Ramsey, [1928] 1990b, 105). This can be compactly expressed in David Lewis's notation in what follows (Lewis, 1980).

What is this sentence saying? A couple of observations are helpful here.

5.4.2 Chances and the Scope of the Quantifier

Chance propositions appear to be in the primary system. But the chance measure is defined through chance distributions. Any chance proposition falls under the scope of the quantifiers given by chance's theory because the chance distribution is parameterized by theoretical functions. Thus chance propositions exceed the primary system in a similar way to the secondary system. This is an important observation.

Starting with the chance measures themselves, they are in the primary system in the sense that their arguments are primary system propositions, and they yield real numbers along the unit interval. Those real numbers reflect an agent's conditional credences in a proposition given the chance of the proposition. So the chances count as laws in the primary system. Viewed from the perspective of an agent's possibility space, one might think of the chance proposition as providing a contour map of possibility space within each cell of the partition given by the primary system. Each line in that map corresponds to some value the chance measure takes on.

But a closer look shows these laws are derivative of the secondary system because they are derived from the secondary system's functions. Recall that the chance of a primary system proposition is:

$$\mathrm{Ch}(\psi(n)) = \sum_{y \in Y(\psi(n))} \beta(n)^{g(y)}(1-\beta(n))^{n-g(y)}$$

This makes chances a *function* of the secondary system propositions β and γ. There are free parameters here. These chances not reducible to any arbitrary truth-function of primary system propositions because each chance proposition is compatible with each possible sequence of coin tosses. Consequently, the theoretical system is not a truth-function of primary system propositions too. As Ramsey states, the chances are not summaries of facts.

Another way to put this point is that a chance measure only makes sense if there is some distribution that generates it. Some parameters will characterize that distribution, even if the distribution is a complicated mixture of Gaussians. In statistics, this would count as a model for the chances. The Ramsey sentence reflects that the chances are a product of that model.

There are two upshots here.

First, it helps solve a problem I discussed in an earlier chapter: how do credences over laws work? Here the answer is straightforward. Since a chance measure is just a function of singular theoretical propositions β and γ, the credences over those theoretical propositions act as the credences over chances. Consider the following example. If the chance of landing two heads in two tosses is 0.36, my credence over that chance is really my credence over $\beta(n) = 0.6$ given the theory. So there is no need for an account of credences over chances.

Second, chances make sense only under the scope of the quantifiers given by the Ramsey sentence. A dispute over chances would then equate to a dispute over theories. This is despite the chances apparently dwelling in the primary system.

These observations apply to laws as well as chances. Consider Ramsey's toy model. Ramsey generates a law for the opening and closing of someone's eyes. If $\chi(n)$ specifies the feeling of that person's eyes opening or closing, then the law is:

$$\forall n, m |\sum_{r=n}^{m} \chi(r)| \leq 1$$

This law is derivable from Ramsey's secondary system axioms and dictionary. Pick for arbitrary n and m the sum, substitute in the definition ($\chi(n) = \gamma(n) - \gamma(n-1)$), and expand. The following cancellation behavior is observed:

$$\begin{aligned}
\sum_{r=n}^{m} \chi(r) &= \sum_{r=n}^{m} \gamma(r) - \gamma(r-1) \\
&= \gamma(m) - \gamma(m-1) + \gamma(m-1) - \gamma(m-2) + \gamma(m-2) - \gamma(m-3) \\
&+ \cdots - \gamma(n) + \gamma(n) - \gamma(n-1) \\
&= \gamma(m) - \gamma(n-1)
\end{aligned}$$

Now consider the four cases where $\gamma(m) = 1$ and $\gamma(n-1) = 1$, $\gamma(m) = 1$ and $\gamma(n-1) = 0$, $\gamma(m) = 0$ and $\gamma(n-1) = 1$, and $\gamma(m) = 0$ and $\gamma(n-1) = 0$. It is easy to show that the absolute value of the sum has to be less than or equal to one. Importantly, though the law fails to mention γ, this result only makes sense in the context where $\chi(n)$ is defined by $\gamma(n)$ in the way given by the theory. Thus, the deterministic case is exactly the same as the chance case.

Summing up, chances are "in" the primary system in the sense they do not refer to any secondary system proposition, but they are really an extension of the secondary system because the secondary system propositions define them.

5.4.3 Chances as Fictions and Theories as Fictions

Chances are fictions. Ramsey is adamant about this: "There are no such things as objective chances in the sense in which some people imagine there are" (Ramsey, [1928] 1990b, 104). As discussed in a previous chapter, he views chances as credences in a best-simplified system of credences that people approximate. This example shows how chances are fictions. And by extension, it also shows how theories are fictions too.

Chances are fictions due to them being more than summaries of primary system propositions. As I discussed earlier, the chance propositions are not truth-functions of primary system propositions because each chance proposition is logically compatible with every truth-function of primary system propositions. But the only propositions properly true here are the primary system propositions. Therefore, the chance propositions have no truth-value, and they are fictions.

The whole secondary system must also be a fiction. Chance propositions are equivalent to the theoretical system of chance distributions. By extension, those theoretical propositions are also fictions. Random samples and biases of coins are not truth-apt: they do not say anything about objects in the world because they are compatible with every possible primary system proposition. I could get a million heads to land in a row and that would still be compatible with the bias of the coin being only 0.4 heads and with the sample being random.

What about the case of deterministic theories and laws? Here there appears to be a logical incompatibility between chance and observation. For example, if in my toy example $\beta(n) = 1$, then it would appear that only $\phi(n) = 1$ can appear for all n. In other words, a coin biased heads must according to this theory result in a limiting relative frequency of all heads; any observation of a tails should immediately falsify it by force of logic. This is strictly false for my example. Consider the case when $\gamma(n) = 0$ and $\beta(n) = 1$. The dictionary says the value of $\phi(n)$ is equal to the inverse cumulative distribution of a Bernoulli trial. This is the infimum of the reals, the greatest lower bound, that has the Bernoulli trial be greater than or equal to $\gamma(n) = 0$. That is obviously 0 since the CDF of a Bernoulli distribution (or a one-trial binomial) on 0 (meaning 0 successes) is equal to 0—this is just equal to $\gamma(n)$ so it must be the greatest lower bound. So even in the deterministic case for the chance example I outline here, one can have a chance zero event happen! Given what Ramsey has said about the presence of coincidences and laws just as limiting cases of chances, this same fact will hold even for deterministic laws. So deterministic laws are logically compatible *with every possible truth-function of primary system propositions*.

The upshot from this discussion is that the Ramsey sentence when viewed as being equivalent to its laws must also be fictional. If one equivalent set of "propositions" are not truth-apt, so must be the other set. Chances illustrate this vividly.

5.4.4 The Theory Determines Probabilities

The last thing to observe is the connection between the theory and a person's probabilities. What is special about chances for Ramsey is that they are in fact rules for determining credences. This can be seen in Ramsey's definition of chance being essentially the principal principle:

$$\Pr(\psi(n) \mid \mathrm{Ch}(\psi(n)) = x, \Gamma) = x$$

Using this definition of chance, an agent can weigh the chances and develop marginal credences in the chancy propositions. This weighing results in a forecast when taking the indicator function to be the value of the chancy proposition.

Critically, because the chance measure is defined through the chance distribution, a person's credences are really a function of the theory. Substituting the appropriate values in the prior equation, I have:

$$\Pr(\psi(n) \,|\, \sum_{y \in Y(\psi(n))} \beta(n)^{g(y)}(1-\beta(n))^{n-g(y)}, \Gamma) = \sum_{y \in Y(\psi(n))} \beta(n)^{g(y)}(1-\beta(n))^{n-g(y)}$$

The theory yields conditional credences. Credences over the theoretical propositions yield credences over primary system propositions, i.e. to forecasts. One can dispense with the chances and speak just in terms of the theory.

Thus any truth-possibility of the theory logically entails a person's conditional credences will be. There is a direct connection between theory and credences. Quantifying out that theoretical vocabulary means that the Ramsey sentence has a direct effect on credences too. It fixes the conditional credences on the quantified theoretical terms and through those conditional credences the marginal probabilities of individual propositions in the primary system.

This is important both for the process of coordination and for the process of deliberation. Knowing how the theory determines probabilities entails knowing how the theory determines behavior because probabilities are just certain dispositions to act. And it is relevant for deliberation since deliberation on credences for primary system propositions can now be replaced with deliberation over secondary system propositions. The theory greatly simplifies this due to the fact that credences over large sequences of coin flips can easily be found through a weighted average of the underlying chances, which are given by the propositions fixed by the theory.

5.4.5 Summary

I suggested an adequate account of the Ramsey sentence would be best tackled by way of an example involving chances. I have provided an example that consists of a primary system of sequences of coin flips, and a secondary system of biases, random sampling, and a distribution to generate those coin tosses. The secondary system induces a chance measure on the primary system, much like how the deterministic example Ramsey uses in "Theories" induces a set of laws. There are several observations: the chance measure is really code for the theory, the chances and the theory are clearly fictitious, and the theory plays an important role in determining probabilities, which is useful for coordination and deliberation. I build on these observations in the next section.

5.5 Decision and Extension in Chances

I have argued that a productive method for understanding Ramsey's philosophy of science is to interpret it through his decision theory. That method has led to a better understanding of how credences in singular, theoretical propositions work. I apply that same method with the aid of the example just sketched to interpret the Ramsey sentence. My goal is to satisfy the three hardest criteria outlined earlier in this chapter: the decision-making criterion, the anti-realism criterion, and the extension criterion. Afterward, I will solve the remaining criteria.

5.5.1 Viewing the Example in Possibility Space

The first criterion is to connect the Ramsey sentence with Ramsey's broader philosophy. As an aid to satisfying that criterion, I interpret the example I sketched in the previous section through Ramsey's decision theory. A good starting task is to figure out the structure of the agent's algebra in the example and to figure out how the chance measure works on that algebra.

In the example, the vocabulary of the primary system consists of propositions $\phi(n)$, which correspond to the outcomes of coin tosses. The secondary system has propositions $\beta(n)$ and $\gamma(n)$ that represent the bias of the coin and the outcome of a random sample. When viewed through Ramsey's decision theory, the outcomes of the coin toss form a coarse partition with 2^{n+1} cells (due to there being four propositions at time n). The partitions given by the secondary system are more complicated. Since both β and γ assume real values on the unit interval, their partitions consist of every real-valued assignment they assume. These form crisscrossing contour maps in each cell of a partition given in the primary system. Since the axioms rule out any values for $\beta(n)$ asides from 0.6 and 0.4, I ignore almost all of the contours given by β, and I treat it as dividing a given partition by one further step. Treat $\beta(n)$ first and then partition each cell given by the ϕ and β partition with the contour lines of γ. The resulting presentation can be seen in figure 5.2 ($\beta(n)$ need not continuously divide possibility space).

I can now prune cells with the axioms and the dictionary. It is a consequence of the theory that $\phi(n)$ has to equal 1 or 0 but not both. So the corresponding coarse partition cells can be deleted. Looking at the remainder, the relationship between γ, β, and ϕ is fixed by the inverse cumulative distribution function. What it says is that the greatest lower-bound of $\phi(n)$'s values must be the value of $\phi(n)$ when the cumulative distribution function of a Bernoulli distribution parameterized by $\beta(n)$ crosses the threshold set by $\gamma(n)$. To see what this means in terms of the diagram, focus on the remaining coarse cell where $\phi(n) = 1$ and $\phi(n) \neq 0$. Locate the contour line corresponding to $\gamma(n) = 0.6$. In the $\beta(n) = 0.6$ side of the coarse-partition cell, everything past that γ line will be eliminated since the greatest lower-bound x on $F_{\Phi}(x)$ is 0 (meaning $\phi(n) \neq 1$). Similarly, in the $\beta(n) = 0.4$ side of the cell,

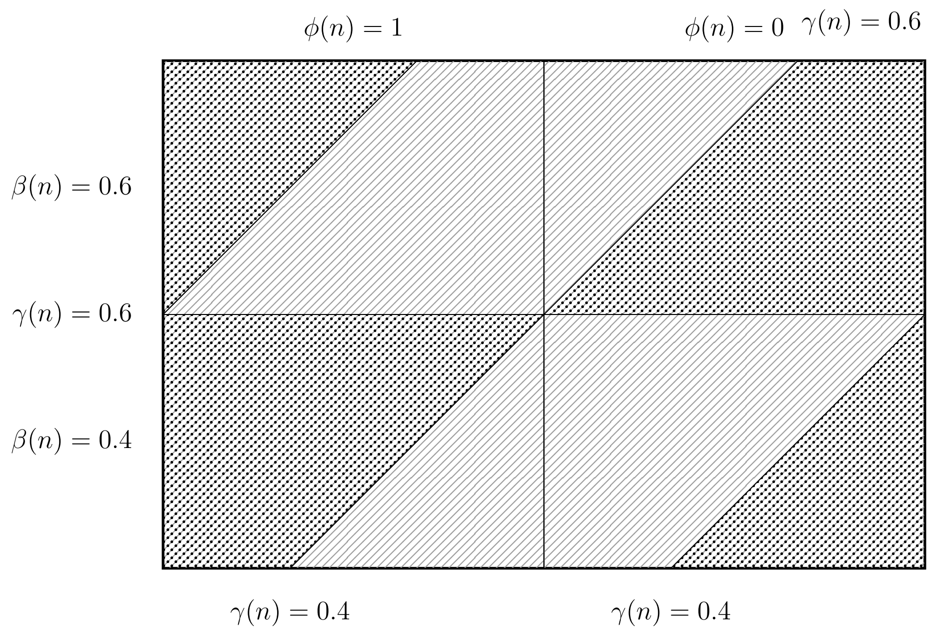

Figure 5.2: A diagram of an agent's possibility space after accounting for theoretical functions γ and β. I ignore the propositions $\phi(n) \neq 1$, $\phi(n) \neq 0$, and all other values of $\beta(n)$ since they are eliminated by the axioms and dictionary. The lightly shaded lines indicate live propositions of $\phi(n), \beta(n), \gamma(n)$. At the threshold of $\gamma(n)$, the crosshatch indicates eliminated propositions.

everything past the $\gamma(n) = 0.4$ contour line will be eliminated. The same result applies in the other coarse-partition cell, except the results are flipped. When $\beta(n) = 0.6$, everything before $\gamma(n) = 0.6$ line is eliminated while everything after it is live. And when $\beta(n) = 0.4$, everything after $\gamma(n) = 0.4$ is live while everything before is eliminated.

In the above example, I only considered the simple case where $n = 1$. When moving to more complicated sets of propositions, the live sections of the coarse partitions will form more complex regions depending on the respective contour lines for each $\gamma(n)$. For example, in the coarse cell for $\phi(1) = 1 \wedge \phi(2) = 1$ there will be two layers of contour lines corresponding to $\gamma(1)$ and $\gamma(2)$. The threshold will be the region given before $\gamma(1) = 0.6$ and $\gamma(2) = 0.6$ in the part of the cell for $\beta(n) = 0.6$ (remember, each $\beta(n)$ is identical to every other). And in the part of the cell where $\beta(n) = 0.4$, the live region will be those before $\gamma(1) = 0.4$ and $\gamma(2) = 0.4$. The same holds for similar propositions and so on.

Interestingly, one can compute the chance for a proposition from the live space in the cells that compose that proposition. Fix the cells of the primary system partition. Then divide the cells by the bias, and apply the contour maps corresponding to each $\gamma(n)$. Let the total "volume" within each bias subcell be one.[12] Then the "volume" of the space found to be live per the contours will just be the chance of the primary system truth-possibility given the bias of the relevant subcell. For example, in the case of the single coin toss, the chance of $\phi(n) = 1$ conditional on the bias being 0.6 will be 0.6 due to the region below $\gamma(n) = 0.6$ being live within the $\beta(n) = 0.6$ subcell. So the chances of specific sequences of coin tosses can be thought of as being specific volumes in possibility space.

If I apply the definition of the chances, the chances of any proposition turn out to be the sum of the volumes in each primary system truth-possibility for the relevant bias. Different chance measures then correspond to the different volumes found in the subcells of each bias. So the chance measure can be rightfully viewed as a possibility aggregator.

Two things should be noted.

First, this is possible because of the $\gamma(n)$ function, which acts like a probability measure. It is strictly not a measure: it is a partition of possibilities, i.e. each $\gamma(n) = x$ is a proposition. But because the partition has members along the unit interval, each proposition can act like the propositions that are the assignment of credence functions. Having that the real unit interval many propositions might seem strange, but Ramsey states this is perfectly allowable in a secondary system: "If, however, our primary system is already a secondary system from some other theory, real numbers may well occur" for the values of functions like ϕ (Ramsey, [1929] 1990m, 114). The implication is that secondary systems allow for at least real-value many propositions. This would be expected to occur in any physical theory with continuous space or time. The same applies to any chance theory like here with γ and β.

[12] We effectively treat at this point the agent's algebra as a measure algebra.

Second, these volumes are not the chance propositions themselves. The chance measures assuming a certain value are treated as their own proposition: $\mathrm{Ch}(\psi(n)) = x$ is a proposition "in the primary system" in the sense that I may view it on possibility space as providing contours on the primary system truth-possibilities. The worlds in these contours need not be the same worlds that constitute the volumes described above. What the volumes illustrate is an important equivalence between computed volumes and assignments of the chance measures. Because the chance measure is defined through the biases and outcomes of tosses are due to the dictionary functions of those biases and γ, the chance measure will assume a certain value if and only if the volume computed as described assumes that same value. The volumes represent the chances, but they are not the same set.

Since the agent obeys the principal principle, those volumes represent conditional probabilities. The agent has carved their possibility space up in a way to mimic some of their credences. Note that this is not the entirety of their credences: they still need a prior probability distinct from the description of volume given above. But the reflection of some conditional credences in possibility space in this example is suggestive. Similar facts are likely to obtain in other theories.

There is evidence that a difference in agents' algebra volumes is what Ramsey has in mind when discussing the differences between agents who adopt contrary laws. He writes in the "General Propositions and Causality" that when arguing over subjunctive conditionals with unfulfilled conditions, people are appealing to different laws or chances:

> The meaning of these assertions about unfulfilled conditions, and the fact that whether the conditions are fulfilled or not makes no different to the difference between us, the common basis, as we may say, of the dispute lies in the fact that we think in general terms. We each of us have variable hypotheticals (or, in the case of uncertainty, chances) which we apply to any such problem; and the

> difference between us is a difference in regard to these (Ramsey, [1929] 1990e, 155).

The difference between people who disagree on a subjunctive conditional is expressed in terms of a difference in laws or chances. The conflict of opinions before the antecedent condition is true or false is one of the conditional credences: "Before the event we do differ from him in a quite clear way: it is not that he believes p, we $\overline{p}$; but he has a different degree of belief in q given p from ours; and we can obviously try to convert him to our view" (Ramsey, [1929] 1990e, 155). So the differences in chances before the event leads to a difference in conditional probabilities. Those laws and chances are equivalent to certain theories. The theoretical propositions will have volumes that represent the different conditional probabilities given by the chances. So a dispute over chances is really a dispute over the volumes given by the chance's theory.

This is an important property of theories generally. They shape possibility space in a way that is equivalent to having certain conditional degrees of belief. One might say that theories project credences into the world by having possibility space and the algebra have volumes that correspond with conditional credences. The agent is effectively "objectifying" their credences. If I can fix what the extension of the Ramsey sentence is with regard to these volumes, then I can make a direct connection to Ramsey's decision theory.

In summary, I have documented what the chance setup from section 5.4.1 looks like in terms of an agent's possibility space. The chances of a proposition can be viewed as volumes of possibility space in the example discussed above. Effectively, the live elements of the algebra provided by the chance's theory are equivalent to the agent's own conditional credences in the chances. My next goal is to provide a connection from this with the Ramsey sentence.

5.5.2 Extension as Partitions

Recall that immediately after introducing the Ramsey sentence, Ramsey says the functions the Ramsey sentence quantifies over are meant to be taken extensionally. He defines extension elsewhere in his writings to be a class of objects of the same logical type. What is the class that is the extension of a Ramsey sentence's functions, and how does the extension relate to the Ramsey sentence? I argued in the previous section from the example that there was an important connection between the elements of an agent's algebra and that agent's conditional credences. I combine that observation with a look at what the Ramsey sentence could be in terms of that possibility space and algebra.

The first observation is that the Ramsey sentence given in equation 5.1 is a second-order quantification over the propositional functions β and γ. These functions map to propositions in the agent's algebra. A particular function for β on each n fixes a partition with respect to possibility space. Aggregating all the time instants, the specific function $\beta(n)$ is just a set of propositions that are exclusive and exhaustive, i.e. a partition. The same applies to $\gamma(n)$. So both $\beta(n)$ and $\gamma(n)$ when pinned down to specific functions yield partitions. Because the theory does not quantify out the primary system and has the dictionary in it, the addition of $\beta(n)$ and $\gamma(n)$ must result in finer partitions than provided by the primary system.

A second observation is that this finer partition is compatible with the axioms and the dictionary. When considering the propositions mapped to by $\gamma(n)$ and $\beta(n)$, they have to be propositions compatible with the theory's axioms and dictionary. So certain cells in the finer partition will be eliminated. For example, all cells apart from those with $\beta(n) = 0.6$ and $\beta(n) = 0.4$ are not included in the partition given by a specific $\gamma(n)$ and $\beta(n)$. Call the set of elements from a given partition that is compatible with the theory of the compatible partition.

I claim that this compatible partition is the extension of those specific functions $\beta(n)$ and

$\gamma(n)$. What else could it be? Ramsey states that the extension of a predicate is just a set of objects of the same logical type. There are two equivalent ways to understand $\beta(n)$ and $\gamma(n)$. The first is in the mathematical form where they are functions from integers to real numbers. The second is the logical form that $\beta(n)$ and $\gamma(n)$ are propositional functions that map time instants to propositions. The mathematical form differs from the logical form by having the extension be sets of ordered pairs consisting of integers and reals while the logical form has them be ordered pairs of integers and propositions. Ramsey, however, says that both forms are about the same stuff. Thus I will examine the logical form since it is easy to apply in the decision theory. This leaves three candidates for the logical type of the theoretical functions' extension: pairs of time instants and propositions, pairs of time instants, and propositions. It cannot be pairs of time instants and propositions because in other cases with propositional functions, Ramsey does not describe the extension as pairs (see Ramsey, [1926] 1990l, 178 cited earlier). It also cannot be pairs of time instants. If it were, then $\beta(n)$ and $\gamma(n)$ would be the same set since they apply to all-time instants. Consequently, it has to be propositions. Propositions are just members of an agent's algebra; they are sets of worlds. So $\beta(n)$ and $\gamma(n)$ have as their extension sets of propositions. Those propositions form a partition. Since they are defined relative to a primary system, they are a finer partition than that provided by the language of observation. Eliminating the elements of this finer partition that are incompatible with the axioms and the dictionary, they form a compatible partition. Therefore, the logical type's objects are propositions of the type that form a compatible finer partition on possibility space finer than the primary system. So the extension of specific functions $\beta(n)$ and $\gamma(n)$ are sets of propositions that form a compatible partition.

Now I have to specify what is the extension of β and γ. They occur under the scope of a second-order quantifier. So their extension should be the set of specific propositional functions $\beta(n)$ and $\gamma(n)$. Again, what else could it be? It cannot be propositions because that would be a type below. Nor can it be time instants for type reasons. Finally, their extension cannot be sets of time instants because that would not differentiate between individual $\beta(n)$ and $\gamma(n)$. So it has to be the specific propositional functions $\beta(n)$ and $\gamma(n)$. Those functions understood extensionally are compatible partitions. Therefore, the extension of β and γ should be sets of compatible partitions.

So far I have considered what β and γ might be in the example given here. The same considerations apply to Ramsey's own toy example from "Theories". There he has three secondary system functions, α, β, γ, which correspond to places, colors of places, and eyes being open or shut. Individual $\alpha(n), \beta(n), \gamma(n)$ are theory compatible finer partitions. Consequently, α, β, γ are sets of compatible partitions. So the extension of α, β, γ in Ramsey's example is just the different ways to carve up possibility space that results in a compatible finer partition than the primary system.

I can now connect this to the Ramsey sentence. The extension of a Ramsey sentence's theoretical functions yields a set of compatible partitions from the axioms and dictionary. The existential quantifier is related to this set because it specifies the relationship between the theoretical functions and the axioms and dictionary. It helps show that the extension of the theoretical functions only includes those partitions that happen to be compatible with the theory's axioms and the dictionary. Modulo a theory of the extensional quantifier, I can show how the extension of the theoretical functions might be relevant to the belief in a theory's Ramsey sentence.

Summing up, an important question that needed to be addressed was what is the extension of the functions used in the Ramsey sentence. In my particular example, the extension of those functions is the set of compatible partitions induced by β and γ. Each $\beta(n)$ and $\gamma(n)$ has to be the elements of a partition compatible with the theory. Since the functions are quantified with a second-order quantifier, their extension is a set of those $\beta(n)$ and $\gamma(n)$. Ramsey's requirement that these functions be understood extensionally ensures the extension of β and γ is a set of compatible partitions. This applies not just to my own example but also to Ramsey's example. Finally, this account of extension is connected to the Ramsey sentence. The Ramsey sentence restricts the extension of those theoretical functions to just the partitions that are compatible with the axioms and the dictionary.

5.5.3 Agreement Between Credences and Chances

An important question about my example is whether every compatible partition induced by the theoretical functions has the same volumes. Recall that those volumes correspond to the proposition considered live within each bias cell. Viewing them as volumes is to effectively treat the $\gamma(n)$ propositions as a measure on the bias cell. For each n, $\gamma(n)$ yields the total volume of the bias cell held to be live. The chance relative to a specific bias of a specific primary system proposition is the sum of the volumes found in the proposition's bias subcells. Now each compatible partition given by the theoretical functions of the Ramsey sentence will have these volumes. The question is are the volumes leading to the same measure?

The answer has to be yes. Since a given primary system proposition has a specific volume x if and only if the chance measure deduced from the theory assigns x to that proposition, it follows that if compatible partitions lead to the same chances then they have the same volumes. And those partitions lead to the same chances just as they lead to the same laws. So they will assign the same volumes.

This should not be surprising because of Ramsey's comment that "the theory" is "simply a language in which they [the laws and consequences] are clothed" (Ramsey, [1929] 1990m, 131). What is surprising is that each element of the extension of the theoretical functions from the Ramsey sentence share an important structure in these volumes. Membership in the extension of the theoretical functions is properly characterized as those compatible partitions that have the same volumes. In other words, the Ramsey sentence identifies sets of propositions that have the requisite structure to produce chances.

Furthermore, those partitions all capture the same interesting fact: agents mirror their conditional credences in the structure of their theoretical and observational propositions. Recall the third observation from the example that stated a theory determines some conditional probabilities. Each member of the set of compatible partitions given by the extension of a Ramsey sentence's theoretical terms produces the same conditional credences. Someone who believes in the theory believes in the Ramsey sentence. So his conditional credences will be duplicated in the structure of his algebra. An agent's belief in a theory reifies his own credences.

The upshot in terms of the decision theory is that theories can be understood as credences and credences as theories. If one knows that an agent believes in a theory, then one knows their conditional credences will be a certain way and from those their actions. Conversely, if from some actions one infers conditional credences, then one can infer that an agent's algebra and possibility space is structured a certain way. This is important. It means that an agent's conditional credences allow access to properties of his algebra.

To illustrate, consider the fact that conditional bets can represent conditional preferences and through them, conditional credences. I can define my conditional credences as follows. I must be indifferent between two bets. The first bet is some truth-possibility of the primary system Φ if the chance of the coin landing heads on the next toss is 0.6 and the primary system truth-possibility Ψ otherwise. The second bet is the sequence of following conditional prospects: a third truth-possibility of the primary system Θ if the coin lands heads on the next toss and the chance of the coin landing heads on the next toss is 0.6, a fourth truth-possibility Δ if the coin lands tails on the next toss and the chance of the coin landing heads on the next toss is 0.6, and Ψ if the chance of the coin landing heads on the next being 0.4. Then the conditional credence can be measured as the ratio of the difference between Φ and Δ to the difference between Θ and Δ (Ramsey, [1926] 1990n, 76).[13] Using Ramsey's method

[13]I wish to solve for $\Pr(\phi(n) = 1 \,|\, \mathrm{Ch}(\phi(n) = 1))$. Setting both gambles equal to one another, I have:

of measuring utilities from preferences, I can find the value of each of the above-mentioned truth-possibilities, and, from them, I can find the conditional credence. If that credence happens to be 0.6 and I apply the same rule across other chances, then I know that my credences respect the principal principle. So I am deferring to the chances. I know from my theory that these chance propositions are equivalent to a structure of binary beliefs that have the appropriate chance volume. Therefore, I must be believing in the Ramsey sentence of this theory: there is some compatible partition that has the right volumes in it. I have read off from my preferences facts about the structure of my algebra.

But this is not enough. Some reflection reveals that *all chances satisfy this property.* Whether the agent has adopted those chances as rules for inference is not shown by this method (whether they believe the chance propositions). It is necessary that an agent's chances obey the principal principle. However, it is not sufficient to show that those are in fact the correct chances.

To get those, additional properties of an agent's credences would need to be established. Ramsey, however, mentions no additional story for the required properties. A needed property for this example would be exchangeability. A credence function is exchangeable just in case that function assigns the same credences to different permutations of random vari-

---

$$
\begin{aligned}
\Pr(\mathrm{Ch}(\phi(n)=1))\overline{\Phi} + (1-\Pr(\mathrm{Ch}(\phi(n)=1)))\overline{\Psi} &= \Pr(\phi(n)=1, \mathrm{Ch}(\phi(n)=1))\overline{\Theta} \\
&\quad + \Pr(\phi(n)\neq 1, \mathrm{Ch}(\phi(n)=1))\overline{\Delta} + (1-\Pr(\mathrm{Ch}(\phi(n)=1)))\overline{\Psi} \\
\Pr(\mathrm{Ch}(\phi(n)=1))\overline{\Phi} &= \Pr(\phi(n)=1, \mathrm{Ch}(\phi(n)=1))\overline{\Theta} + \Pr(\phi(n)\neq 1, \mathrm{Ch}(\phi(n)=1))\overline{\Delta} \\
\overline{\Phi} &= \frac{\Pr(\phi(n)=1, \mathrm{Ch}(\phi(n)=1))\overline{\Theta}}{\Pr(\mathrm{Ch}(\phi(n)=1))} + \frac{\Pr(\phi(n)\neq 1, \mathrm{Ch}(\phi(n)=1))\overline{\Delta}}{\Pr(\mathrm{Ch}(\phi(n)=1))} \\
\overline{\Phi} &= \Pr(\phi(n)=1 \,|\, \mathrm{Ch}(\phi(n)=1))\overline{\Theta} + (1-\Pr(\phi(n)=1 \,|\, \mathrm{Ch}(\phi(n)=1)))\overline{\Delta} \\
\overline{\Phi} &= \Pr(\phi(n)=1 \,|\, \mathrm{Ch}(\phi(n)=1))\overline{\Theta} + \overline{\Delta} - \Pr(\phi(n)=1 \,|\, \mathrm{Ch}(\phi(n)=1)))\overline{\Delta} \\
\overline{\Phi} - \overline{\Delta} &= \Pr(\phi(n)=1 \,|\, \mathrm{Ch}(\phi(n)=1))\overline{\Theta} - \Pr(\phi(n)=1 \,|\, \mathrm{Ch}(\phi(n)=1)))\overline{\Delta} \\
\overline{\Phi} - \overline{\Delta} &= \Pr(\phi(n)=1 \,|\, \mathrm{Ch}(\phi(n)=1))(\overline{\Theta} - \overline{\Delta}) \\
\frac{\overline{\Phi} - \overline{\Delta}}{\overline{\Theta} - \overline{\Delta}} &= \Pr(\phi(n)=1 \,|\, \mathrm{Ch}(\phi(n)=1))
\end{aligned}
$$

where the overlines are the respective utilities.

ables. De Finetti shows that this property is sufficient for representing an agent as having a credence function that is a mixture of latent Bernoulli random variables and chance distributions. This is not present here. As Zabell notes, however, Ramsey comes tantalizingly close to discovering the importance of exchangeability in identifying chances (Zabell, 2005, 133–134). In an unpublished note titled "Rule of Succession", Ramsey gives the definition of exchangeability and uses it to prove a more general form of Laplace's rule of succession:

> n things are A what is the chance of $n+1^{th}$ being A.
>
> Suppose chance a priori of μ out of $n+1$ being A is $\phi(\mu)$. *all permutations equally probable.* [emphasis mine]
>
> req((uire))d chance is
>
> $$\frac{(n+1)\phi(n+1)}{(n+1)\phi(n+1)+\phi(n)} = \frac{n+1}{n+1+\frac{\phi(n)}{\phi(n+1)}}$$
>
> If $\phi(n) = \phi(n+1)$ [the above] $= \frac{n+1}{n+2}$ (Ramsey, 1991a, 279).

The important point is that Ramsey both has exchangeability and recognizes its importance to chance, but he fails to take the additional step of connecting it with a latent chance variable and distribution. Instead, he returns to the problematic uniform prior. Perhaps had Ramsey lived longer, the connection would have been made. Regardless, for the Ramsey sentence to be useful for identifying facts about an agent's algebra, Ramsey needs the symmetry properties of credences that De Finetti and others have proposed for eliminating chances (see Skyrms, 1984 for a full discussion).

What is different here, however, is that Ramsey would not use those symmetry properties and representation theories for the elimination of chances, but for inferring the believed chances from an agent's credences. Once those chances are found, then the equivalent Ramsey sentence can be used to read off properties of an agent's algebra. So the idea is to go the opposite direction of De Finetti and others: instead of a reduction, one can use the properties of credences to infer chances and from the chances infer the believed theory.

Speaking non-historically, one might wonder what the theoretical representation of the chances adds to the chances themselves. Remember, chances are viewed as fictional propositions too. The toy example I gave has chance propositions $\mathrm{Ch}(\cdot)$. Why not stop with those propositions? My response is that the chances just are theoretical propositions. Recall the first observation: specifying a chance means specifying a chance distribution. This makes the derivation of the chance falls under the scope of the Ramsey sentence quantifier. So to say that an agent believes in a chance proposition is also just to say that they believe in a generating chance process with parameters. The picture I have sketched here is that one can give any account of what those parameters are in terms of the structure of the agent's algebra. Parameters like the bias in my example can be exactly described as a compatible partition with the right structure.

This is relevant to Ramsey's view of logic as self-control. I have to consider preference over options when introspecting how I should modify my behavior. To figure out how I should connect my action up with the model where I believe in certain propositions, I have to know how to go from actions to propositions and propositions to actions. If I fix what my conditional credences are going to be in the future, I can have myself act in a way that control's my behavior so that I will effectively believe certain propositions. And likewise, when considering what my conditional credences and thus preferences should be, I need to go from propositions to credences. I need to specify how my conditional credences work. Chances allow me to do that. But entertaining chances is just to entertain the free parameters that specify those chances. So I construct my conditional credences by theorizing about how the chances operate.

Summarizing then, there is an important connection between the Ramsey sentence viewed in terms of his decision theory and in terms of its extension. The Ramsey sentence specifies a set of compatible partitions that are in agreement with respect to certain structures they have. This leads to them generating the same chances and laws. This set is just the extension of the theoretical terms of the Ramsey sentence. It allows one to read off structural facts of an agent's algebra from their conditional credences. This is important for Ramsey's view of logic as a method of self-control. To regulate one's future behavior, one needs to know what that behavior is about and infer from what it is about to what the correct actions should be.

From these points, I have addressed two of the criteria I laid out at the start of this section. The decision-making criterion has been addressed in part by viewing the Ramsey sentence in the context of an agent's algebra and possibility space, and it has been addressed in part by the Ramsey sentence's utility in reading off some structure of an agent's algebra. The extension criterion has been handled due to the extension of the Ramsey sentence's theoretical terms being a set of compatible partitions that are in agreement with respect to the volumes discussed here. That leaves the anti-realism criterion.

5.5.4 Anti-Realism and Summary

I satisfy the anti-realism criteria in the next chapter by viewing existentially quantified sentences as descriptions of propositions. For Ramsey, an existentially quantified sentence describes a proper proposition, and existential sentences are worthless without the appropriate witness. Those witnesses must contain either individuals that are named or functions that have been correctly defined. If the latter happens to be the case, then an introduced existential quantifier is justified by a universal quantifier. The paradigmatic case for this is the dictionary of a theory. Each definition in the dictionary is a universally quantified sentence that warrants the introduction of an existential over the defining theoretical functions.

A second-order existential quantifier can then be introduced if a proper dictionary and requisite theoretical function have been constructed. The dictionary (and with it the axioms), thus justifies the Ramsey sentence. Since multiple constructed functions might be properly defined, one can have dictionaries that are structurally similar but strictly speaking different. This means that the Ramsey sentence quantifier is really a description of a disjunction of axioms and dictionaries that each induce a compatible partition on an agent's possibility space. I argue for this view at length in the following chapter, but for now, I conclude that the anti-realism criterion is satisfied.

This is very important. I claim there is now an account of the Ramsey sentence that matches the decision-making, extension, and anti-realism criteria. Both decision-making and extension criteria are satisfied due to the extension of the Ramsey sentence theoretical functions being a set of compatible partitions that are in agreement with respect to certain volumes in them. These volumes correspond with the conditional credences on the chances an agent would have if they had those partitions. The anti-realism criterion comes in because the existential sentence is warranted due to the universal propositions that are the axioms and dictionary taken jointly. Each agent has constructed a specific $\gamma(n)$ and $\beta(n)$ function for their dictionary. Agents abstract the axioms and dictionary to produce a disjunction of axioms and dictionaries for a class of compatible partitions that have the right volume properties. The existentially quantified Ramsey sentence is not truth-apt, but it is a description of the appropriate dictionary and axioms constructed by the agents. Such a sentence does not justify anything: it is merely a summary of the important bits of what the agent has already done when they believed the theory. Thus the anti-realism criterion is satisfied.

In this section, I provided an account of the Ramsey sentence that satisfied the decision-making, extension, and anti-realism criteria. The Ramsey sentence is a description of the axioms and dictionary that have defined theoretical functions. It is used to pick out a class of compatible partitions induced by axioms, dictionaries, and constructed functions that agree with volumes corresponding to the agent's conditional credences in the chances. In short, the Ramsey sentence is a summary of how an agent reifies their conditional credences.

Each of the aforementioned criteria is satisfied. First, the decision-making criterion is satisfied because this account makes a direct connection between the Ramsey sentence and an agent's credences. Second, the extension criterion is satisfied because the extension of the theoretical functions for the Ramsey sentence is just a set of compatible partitions. Third, the anti-realism criterion is satisfied because the Ramsey sentence is itself not taken to be a proper proposition but is warranted based on the universal proposition of the axioms and the dictionary with appropriate constructed functions. It is instead used to signify a disjunction of axioms, dictionaries, and constructed functions with the right properties whose extensions are the corresponding compatible partitions.

The next task is to see if the offered account of the Ramsey sentence can satisfy the other remaining criteria. Those remaining criteria are the surplus content, law and consequences, verification conditions, and communication criteria. I address this in the next section.

5.6 Laws, Verification, Content, and Communication of Scientific Theories

I have an account of the Ramsey sentence. The Ramsey sentence is a description of the axioms, dictionary, and constructed functions that induces a compatible partition X whose volumes represent conditional credences in the chances. The extension of the Ramsey is the set of all compatible partitions in agreement with respect to the volumes in X. I argue this account satisfies three of the seven criteria needed for any account of the Ramsey sentence. In this section, I conclude that it satisfies the remaining five criteria. I address each of these criteria in order from what I take to be the easiest to the hardest: the laws and consequences, the verification, the surplus content, and the communication criteria.

5.6.1 The Laws and Consequences Criterion

The first criterion is the laws and consequences criteria. Recall that this criterion says that an account of the Ramsey sentence must be anti-realist and show how one can deduce the primary system laws and consequences from the theory. The problem is that an anti-realist interpretation of the Ramsey sentence is not about objects in the world, but it still produces laws and consequences that are about the world. I need to show how that can be the case with the anti-realist interpretation of the sentence I gave earlier.

The interpretation I am defending is anti-realist in the sense that the Ramsey sentence is a description of the theory's axioms and dictionary, which are universal propositions. The quantified terms are the constructed functions in the dictionary. Those axioms and dictionary induce a particular compatible partition on the agent's possibility space with the property that certain volumes in that partition mirror the chances. The induced volumes report the chances, laws, and consequences deduced by the axioms and dictionary. So the axioms and the dictionary do all the work. Thus, the Ramsey sentence is superfluous for producing the laws and chances.

This last point must be emphasized. The anti-realism of the Ramsey sentence is that the sentence itself means nothing; it acts as a signifier of the universal propositions, i.e. the axioms and dictionary, the agent has used to shape their beliefs. Those universal propositions are what warrant the laws and consequences. Believing in the Ramsey sentence means that the axioms and dictionary are already believed. So there is no need to appeal to the Ramsey sentence for justifying laws and chances.

To see how this works, consider the chance example. The definition of individual coin tosses automatically justifies the chance propositions since it uses the same generating distribution and parameters $\gamma(n)$ and $\beta(n)$. The chances are just the sum of the probabilities given by the chance distribution conditional on $\beta(n)$ for each n length sequence of coin tosses. Because the theory defines each individual coin toss as the outcome of one trial using that chance distribution, it follows that the chances should hold so long as the chance bias is the bias given by the theory, which here is either 0.6 or 0.4.

The same applies to Ramsey's own toy example. If the axioms, dictionary, and constructed functions α, β, γ justify the Ramsey sentence, they also justify the laws and consequences of the theory. Believing in his toy example Ramsey sentence is just to believe in specific α, β, γ functions plus the axioms and dictionary. So one would then also believe in the laws and consequences.

Summing up, the laws and consequences criterion is easily satisfied by the anti-realist interpretation given here. Because the Ramsey sentence is only believed if some specific axioms and dictionary is believed, the chances, laws, and consequences of the theory are deducible trivially.

5.6.2 The Verification Criterion

The second criterion to be fulfilled is the verification conditions criterion. Recall that it states that the Ramsey sentence is supposed to be connected to the verification conditions in a way that leads to meaning holism. Ramsey argues that the meaning of a theoretical proposition is the difference between that proposition added to a theory under the scope of the existential quantifier and the theory without the proposition. He connects this to the theory's verification conditions. The meaning of a theoretical proposition is the difference between the necessary verification conditions of the theory plus the proposition and the necessary verification conditions of the theory alone. My job is to show how my proposed account allows this to happen.

It is important to review verification conditions for Ramsey. In "Theories", Ramsey discusses two sets of verification conditions: the sufficient and necessary conditions. The sufficient verification conditions of a theoretical proposition are the primary system truth-possibilities that entail that proposition, i.e. would confirm it. Ramsey views these as the union of truth-possibilities that are incompatible with the negation of the theoretical proposition and the axioms and dictionary. The necessary verification conditions of a theoretical proposition are the primary system truth-possibilities that are entailed by that proposition and the axioms and the dictionary. He writes that these are the union of truth-possibilities that are compatible with the theoretical proposition and the axioms and dictionary. A slogan for both sets of conditions is that sufficient conditions confirm and necessary conditions can falsify.

Verification conditions of a theoretical proposition can be the same even when theoretical propositions are not identical. Both sets of verification conditions can be viewed in terms of the subset relation for elements of the finer theoretical partition within the coarser observational partition. So two theoretical propositions with the same verification conditions would have their component theoretical truth-possibilities bear the same subset relations to observational truth-possibilities. Two theoretical propositions in the same coarse partition might be different propositions because they do not share the same $worlds_1$. But they would have the same verification conditions. Consequently, both propositions would have the same meaning since any addition or subtraction will result in the same verification conditions.

My interpretation of the Ramsey has the extension of the Ramsey sentence theoretical functions be the set of compatible partitions that are in agreement with respect to certain volumes. These volumes are nothing more than the finer, theoretical partition cells that the theory takes as possible. Any union of these cells forms some proposition p. The verification conditions of p are the unions of the coarser, observational cells that p is a subset of.

Importantly, preserving the same volumes across different compatible partitions would also preserve the same theoretical cells and their subset relations to the coarser observational cells. This is due to the volumes fixing these theoretical cells within each observational cell.

For example, the γ and β functions might subdivide the ϕ cells in the chance example differently between each partition given by the Ramsey sentence. However, because they have volumes that agree, there will be equivalent live γ and β truth-possibilities in each ϕ truth-possibility. They won't "cross" the lines of each cell in the partition nor will they be absent when they should be present and present when they should be absent in the corresponding primary system cell. The result is that the same subset of relations will hold between the secondary and primary system propositions.

The upshot is that agreement between volumes is enough to ensure the same verification conditions for theoretical propositions. In addition to providing information about conditional credences, the Ramsey sentence also specifies the verification conditions of theoretical propositions. This should not be surprising because the Ramsey sentence is a description of the axioms and dictionary such that it captures the structure of the induced compatible partition. The verification conditions are just parts of that structure.

So the account of the Ramsey sentence offered here does offer a connection between the Ramsey sentence and the verification conditions. But does it lead to meaning holism?

The answer is yes. Since the meaning of a theoretical proposition is in part through its verification conditions and the verification conditions are defined through the axioms and dictionary of the theory, then the meaning of a theoretical proposition is *always* dependent on the axioms and the dictionary (with the dictionary being particularly important). The interpretation of the Ramsey sentence here is that it is a description of the axioms, dictionary, and constructed functions an agent has built. So if the Ramsey sentence is believed, then so is the axioms and dictionary that warranted it. And so any introduced theoretical proposition will have its meaning determined by the axioms and dictionary.

Using the chance example again, any introduced theoretical proposition will have to be done with respect to the axioms and dictionary. This means it might as well be introduced within the scope of the existential quantifier. For example, $\gamma(1) = 0.5$ would have its meaning fixed by the dictionary, where in the worlds that the bias is 0.6 the coin would have landed heads, and in the worlds that the bias is 0.4 the coin would have landed tails at time 1. Its conditions for confirmation and falsification would be predicated on what the bias is and on how the chance distribution relates bias and γ to the outcome of the coin toss. The meaning of $\gamma(1) = 0.5$ would then be fixed by the theory as a whole. Consequently, it would be treated as if it were introduced only under the scope of the existential quantifier.

To summarize, the interpretation of the Ramsey sentence given here does satisfy the verification conditions criterion. It satisfies it by directly tying the anti-realism of the Ramsey sentence to the axioms and dictionary. The extension of the sentence's theoretical terms is in effect just the class of compatible partitions with the same verification conditions for the theoretical propositions. And the direct connection to the axioms and dictionary ensures that any theoretical term will have its meaning determined by the theory as a whole.

5.6.3 The Surplus Content Criterion

The third criteria that need to be fulfilled are the surplus content criterion. The first of these states an important observation about theories for Ramsey: they are not reducible to observation without seriously compromising their usefulness. Theories have excess "content" above observation. The Ramsey sentence is supposed to somehow signify that surplus content.

I have argued previously that the surplus content of a theory can be observed by the fact that the verification conditions cannot settle the truth of a theoretical proposition. This is due to the theoretical truth-possibilities forming a finer partition than the observational partition. More to the point, if there happens to be any cell (truth-possibility) of the observational system that has more than one subcell (truth-possibility) of the theoretical system within it, then at least one theoretical proposition will not be equivalent to its verification conditions. In other words, theories are not truth-functions of observation. This is exactly what Ramsey means by the extra content of a theory.

There is something missing from this story. While it is fine to say exactly what causes the surplus content, it does provide a philosophical story about that cause. The hope is that the Ramsey sentence might provide some insight.

What has to be done then is to show how the interpretation of the Ramsey sentence I have given leads to theories not being a truth-function of observation. Furthermore, it would be great if that same story could explain philosophically what the surplus content happens to be.

Starting with the anti-realism of the Ramsey sentence, it merely signifies that the agent believes axioms and dictionary with constructed functions that induce a finer partition than the coarser partition provided by the primary system. So the Ramsey sentence already suggests the theory is not a truth-function of observation. But it alone would not ensure it. Importantly, for the secondary system to fail as a truth-function of the primary system, there must be a primary system truth-possibility that has more than one secondary system truth-possibility live in it. It is possible for the believed theory to only have one truth-possibility per primary system truth-possibility. For example, I could define a theory of coin tosses so that the outcome of each toss was predetermined by the hand that flips the coin. If the coin is flipped by my right hand, it lands heads, and if the coin is flipped by my left hand, it lands tails. In this case, I have a theory that is a truth-function of observation since the truth of every secondary system proposition would be determined by the outcome of each toss.

There is no surplus content in this theory. So just believing in a theory's axioms, dictionary, and constructed functions is not sufficient for that theory to have suplus content.

A remark elsewhere in "Theories" suggests that Ramsey believes every scientific theory to not be like the aforementioned coin case. Instead, every theory must have surplus content. He writes when one tries to invert the dictionary for an explicit definition of the secondary system in terms of the primary system, one finds this to be impossible:

> We conclude, therefore, that there is neither in this case nor in general any simple way of inverting the dictionary so as to get either a unique or an obviously preeminent solution which will also satisfy the axioms, the reason for this lying partly in difficulties of detail in the solution of the equations, partly in the fact that the secondary system has a higher multiplicity, i.e. more degrees of freedom, than the primary. In our case the primary system contains three one valued functions, the secondary virtually five $[\beta(n,1), \beta(n,2), \beta(n,3), \alpha(n), \gamma(n)]$ each taking 2 or 3 values, and such an increase of multiplicity is, I think, a *universal* characteristic of useful theory (Ramsey, [1929] 1990m, 122).

Taking multiplicity here to be the number of truth-possibilities, Ramsey's point is that in his constructed example the number of joint truth-possibilities between primary and secondary system is greater than the number of primary system truth-possibilities. So there must be at least one primary system truth-possibility that has at least two secondary system truth-possibilities in it. Consequently, the secondary system is not a truth-function of the primary system. The important remark here is the last clause where Ramsey states that the secondary system's increased multiplicity is a universal feature of useful theories. He is claiming, without much argument, that all useful scientific theories have greater multiplicity than observation and hence surplus content over observation. For example, knowing that my right hand or left hand flipped a coin adds *nothing* that I already did not know with the outcome of the coin toss. I could eliminate the whole secondary system of right and left-handed coin tosses. My knowledge of the coin toss outcomes already summarizes that. So this particular theory would not be very useful according to Ramsey.

The fact that a given axiom, dictionary, and constructed function has surplus content is presumed by Ramsey if the theory happens to be useful. The connection between the Ramsey sentence and surplus content then is a derivative one: the Ramsey sentence only entails surplus content of the theory if the justifying axioms, dictionary, and constructed functions happen to have surplus content. Presumably, the theory being believed is a useful one; otherwise, why would it be believed? Since the theory is useful, then the Ramsey sentence describes it as having surplus content derivatively. That means the extension of the Ramsey sentence's theoretical functions is of special importance. Each element of that extension will entail that theoretical propositions are not truth-functions of observational propositions.

More specifically, the extension of the theoretical functions in the Ramsey sentence has each compatible partition agree with respect to volumes within them. Since the theory is presumably useful, the truth-possibilities corresponding to those volumes are not truth-functions of observation. And this should be so because *the volumes are the objectification of the agent's conditional credences.* Observation here has no term for the agent. Facts about what the agent believes and the forecasts the agent will make are not possible within the primary system. The secondary system in objectifying the conditional credences through the corresponding volumes can represent the chances and conditional credences the agent will have. And chances and conditional credences are if nothing but surplus content. So the surplus content here is not mysterious: it is a representation of the rules the agent is following.

This is important. The surplus content of the theory, as represented by the Ramsey sentence, is just a representation of the agent and his decision-making. It is precisely this that is missing from observation. By objectifying the agent's credences through agreeing volumes, the Ramsey sentence locates the agent and his decision-making. Namely, it identifies how the agent makes decisions via the compatible partition. The volumes in that partition correspond to the conditional wagers the agent will and will not take. So the Ramsey sentence shows how the agent might represent himself in a manner that avoids the poverty of the observation language.

So this is the story of how surplus content works with the Ramsey sentence. Ramsey presumes that any believed theory happens to be a useful one. This entails the theory will have greater multiplicity and so surplus content. The Ramsey sentence describes the believed theory, i.e. axioms, dictionary, and constructed functions. So it has its theoretical terms' extension be a set of compatible partitions with agreeing volumes whose corresponding truth-possibilities are not truth-functions of observational propositions. This makes the surplus content of the theory just the theory representing the agent's belief in the chances and conditional credences. Furthermore, the surplus content of the theory is also what allows the theory to solve the problem bedeviling the observation language: locating the agent and his decisions. It objectifies the agent through the agreeing volumes.

The conclusion is that my account can satisfy the surplus content criterion. It satisfies the surplus content criterion by showing that if the believed theory is useful, which Ramsey presumes is the case, then the Ramsey sentence will derivatively have surplus content.

5.6.4 The Communication Criterion

The fourth and final criterion that needs to be satisfied is the communication criterion. Recall an account of the Ramsey sentence needs to explain how the Ramsey sentence facilitates communication and affects disagreement. Two questions need to be answered: how do people who believe the same Ramsey sentence believe the same theory, and how do they in fact disagree when they do not believe the same theory? My answer to these questions will proceed by first addressing how my account of the Ramsey sentence affects communication inter-self. How can deliberation across time proceed over the same theory? This requires recounting briefly Ramsey's views on logic as self-control. The strategy is to target a special case of the problem of communication: communicating with one's future self. From there, I can extend the account to communicating with other agents.

Underlying Ramsey's philosophy of logic is a model of cognition. According to this model of cognition, behavior is driven by two processes: an unconscious, rule-driven process and a conscious, reflective process. Actual decision-making is conducted according to habits or rules in the unconscious process. These habits and rules are stored in a memory that is susceptible to change, allowing for the modulation of those rules. That modulation occurs through the conscious process. Habits in the unconscious process act together to produce psychological expectations. When those expectations are violated, they enter into the conscious process as mental acts. Mental acts are treated as propositions. Deliberation then begins to identify the relevant offending habit, which is treated as a chance (in the deterministic case laws). When identified, the habit can be amended along with the violation in conscious memory, which over time ports the habit into unconscious memory. Cognition then allows for the regulation of behavior.

Logic enters as a generalization of this regulation of behavior. Logic for Ramsey is self-control. Mental acts are treated as approximating propositions. Habits are treated as chances, and in the limit, as laws. Psychological expectations and with them, actions are represented as forecasts, which are mathematical expectations with credences and utilities. Someone does this by measuring their credences and utilities through preference over outcomes. Ensuring those preferences are coherent ensures their behavior is in accordance with the mathematical expectation. Logic allows for those actions to be regimented so as to avoid books that ensure a failure to satisfy goals, and it allows the best decisions possible to be made. Insofar as one amends one's habits in accordance with forecasts, then one is rational. Self-control can then be achieved and the odds of achieving one's goals are maximized.

Here is a key issue that has not been resolved in that story: how does one ensure that the regimentation is constant across successive deliberations?

Suppose Jones is engaged in gambling over coin tosses. He had a habit that led him to expect not to see ten heads in ten flips. This expectation is violated when he sees ten heads in a row, and he has the mental act that can be expressed as "the coin in ten tosses landed heads on all of them". He now deliberates and treats his action as the result of the mathematical expectation of the chances that the coin would land heads. He revises his credences according to Bayes. Note that his belief in the chances is equivalent to a belief in a theory about a generating chance process where the coin has a bias. In effect, his credences in the chances are credences in the bias of the coin. After storing his revised credences in his memory, they are translated into a change of habit in his unconscious memory. Now he observes more coin tosses. Again he has a violation of expectations and has to deliberate and revise his credences. What ensures that the hypotheses over the biases are in fact the same hypotheses for Jones? Each act of deliberation is in essence dealing with a different fiction: Jones has to reconsider what his various preferences would be over his algebra. Jones does not have direct access to that algebra, but instead only has his preference for gambles and conditional gambles. But he needs to track what his credences were in the chances and what the chances and affiliated biases happened to be. How does he do this?

The issue here is not exactly extracting the agent's algebra from their preferences. Ramsey assumes one can do that through the provided mental acts. One can deliberate over some privileged propositions and outcomes. The issue is instead about the theory of treating one's actions as the result of one's credences and utilities over the chances. Those chances are fictions—propositions one has to invent—or equivalently fictions of a theory with a chance distribution and parameters. When deliberating over the privileged propositions and outcomes, one needs to ensure that the represented chances and theory are in some sense the same as those previously deliberated over. One needs to effectively measure the same thing when deliberating over time to ensure ones credences are updated over the same propositions.

A solution to this problem is instead of trying to keep the chance and theory propositions identical, an agent fixes how those chances and theoretical propositions impact his credences.

Here credences are simply understood as ultimately preferences over gambles. Gambles over singular theoretical propositions are understood as being conditional wagers. The conditions of those wagers are the theoretical proposition's verification conditions, and the outcomes of the wager are the joint truth-possibilities of theory and observation. Importantly, gambles over singular, theoretical propositions can be used to find an agent's credences over observational propositions involving chances. Credences over chances are just credences over singular, theoretical propositions. For example, the credence of the next coin toss landing heads given the chance of it landing heads can be found by finding the credence of the next coin toss landing heads given the coin's bias is so and so. To find an agent's credence in the next coin toss landing heads, that agent can multiply that conditional credence by the marginal credence that the coin's bias is so and so. Both of these credences are wagers. The first is a conditional wager in an observational proposition given a singular theoretical proposition. The second is a wager in a singular, theoretical proposition.

A potential problem is the identity of singular, theoretical propositions here across deliberation. How is the proposition at time i the same as the proposition at time $i+1$? Here is a proposal: ignore the identity of the theoretical proposition and focus on the observational propositions. A person wants to ensure coherence on those propositions because those propositions are ultimately the ones that matter, i.e. they can lead to a violation of expectations. Recall the marginal credences in observation can be factored by the law of total probability into the product of the conditional credences given the theoretical propositions and the marginals over the theoretical propositions. If an agent continues to maintain the same conditional credences of observation given theoretical propositions over time and to maintain the same marginal credences, he can effectively treat the learning process here as a change in the marginal credences of the theoretical propositions. This will ensure the correct update in the credences of the observational proposition. But note this can be the case even though strictly speaking, the theoretical proposition considered at a later time may not be the "same", i.e. the exact same set of $worlds_1$, as it was at an earlier time. So long as the conditional and marginal credences are the same, the new theoretical proposition can be treated as effectively the same one from earlier.

To illustrate, again consider the chance example where I want to establish my credence that "at time n, the coin toss lands heads", i.e. $\phi(n) = 1$.[14] Before the first toss, I make a forecast using the expectation and the law of total probability:

$$
\begin{aligned}
\mathbb{E}[\phi(n) = 1] &= \Pr(\phi(n) = 1)I(\phi(n) = 1) + (1 - \Pr(\phi(n) = 1))I(\phi(n) \neq 1) \\
&= \Pr(\phi(n) = 1)(1) + (1 - \Pr(\phi(n) = 1))(0) \\
&= \Pr(\phi(n) = 1) \\
&= \sum_b \Pr(\phi(n) = 1 \,|\, \beta(n) = b)\Pr(\beta(n) = b)
\end{aligned}
$$

Now I observe the toss and collect my evidence E. I want to update $\phi(n) = 1$. So I am in effect doing the conditional expectation. Except this time, I factorize according to β':

[14] I simplify here and am treating a proposition something like the next coin toss lands heads.

$$
\begin{aligned}
\mathbb{E}[\phi(n) = 1 \,|\, E] &= \Pr(\phi(n) = 1 \,|\, E) I(\phi(n) = 1) + (1 - \Pr(\phi(n) = 1 \,|\, E)) I(\phi(n) \neq 1) \\
&= \Pr(\phi(n) = 1 \,|\, E)(1) + (1 - \Pr(\phi(n) = 1 \,|\, E))(0) \\
&= \Pr(\phi(n) = 1 \,|\, E) \\
&= \sum_b \Pr(\phi(n) = 1 \,|\, \beta'(n) = b, E) \Pr(\beta'(n) = b \,|\, E)
\end{aligned}
$$

The important thing here is that $\Pr(\phi(n) = 1 \,|\, \beta(n) = b) = \Pr(\phi(n) = 1 \,|\, \beta'(n) = b, E)$ and $\Pr(\beta(n) = b) = \Pr(\beta'(n) = b)$. If that is the case, then the update should be as if I am updating over the previous proposition β. Furthermore, the update will be coherent for $\phi(n) = 1$. And this is what ultimately matters.

In the context of thinking of credences as ultimately preferences over wagers, what this means is that I have deliberated with the new proposition assuming a certain structural role in my preferences as the previous one. There is constancy in preference structure across time.

So the key here is ensuring those conditional credences and marginal credences remain the same across similar theoretical propositions. For the marginal credences, an agent can just declare it, and he treats the new theoretical proposition as if it were the old one. The conditional credences are more complicated. Since these are supposed to mirror the chances, they have to reflect deeper structures in the algebra.

Those deeper structures are just what is given by the volumes of the appropriate Ramsey sentence. Recall that the extension of the Ramsey sentence's theoretical terms is just the set of compatible partitions whose volumes are in agreement with one another. This means that someone who believes a given Ramsey sentence will have the corresponding conditional credences of observational propositions given the theoretical propositions. It fixes those conditional credences in the way desired. And those conditional credences also duplicate the conditional credences of observation given the chances.

The upshot is that the Ramsey sentence provides an important component for communicating with a person's future self. In believing the Ramsey sentence, he can reconstruct from it theoretical propositions that have the right relationships with observational propositions. This ensures conditional credences remain the same. So on Ramsey's model of cognition, one fiction is sufficiently identical to another fiction to allow for coherent deliberation and learning across time with respect to the propositions that matter.

If the same story is true of an individual deliberating, then it should also apply to groups of people deliberating together. People are almost certainly going to be regimenting their behavior according to different propositions. What matters, however, is ultimately their credences—their preferences over gambles. If credences over their common observational propositions can be maintained, then they can effectively communicate the same meaning of their theoretical propositions. As Ramsey writes in companion notes to "Theories", the meaning of theoretical propositions are these observational propositions: "The essence of a theory is that we make our assertions in a form containing a lot of parameters, which have to be eliminated in order to get our real meaning" (FPRP Theories, 9). So in communicating, what is important is the same as what is important in deliberation, i.e. communicating one's credences vary in the primary system functions.

The upshot is that the Ramsey sentence allows for communication about theories. When asserting a Ramsey sentence, a person is allowing for a whole zoo of algebras with compatible partitions that still preserve the correct volumes. Those volumes reflect the fact that two adherents of the theory have set their conditional credences in the same way. They in effect defer to the same chances or believe the same laws. They can do this despite the fact that strictly speaking, they are not even discussing the same propositions. But because their algebra has the right structure, they agree on the meaning of the theory through having isomorphic conditional credences.

This addresses the first question given by the communication criterion. That question was how do people who believe the same Ramsey sentence believe the same theory? The answer is that they have agreeing volumes in their algebra that reflect their conditional credences in the theoretical propositions. They in effect have similar preferences over certain gambles. This ensures that their credences over observation have important similarities.

What about the second question? That question was how do two people in fact disagree when they do not believe the same theory? The answer is that they will differ in their conditional credences on the chances. If I believe the coin flips are generated by a binomial process while Jones believes it is Poisson, then the two of us will have very different conditional credences in observational propositions given the chances. This is just due to the third observation from my toy example that the theory determines the probabilities.

Note that the difference in the conditional credences given the chances here are more than just differences in conditional credences. I and Jones may have both observed the coin toss but still differ in the chances. This means that these disagreements reflect deep disagreements over the laws, where laws are more than summaries of fact. As Ramsey writes in "General Propositions" this amounts to disagreements over subjunctive conditionals, i.e. what would have happened:

> Before the even we do differ from him in a quite clear way: it is not that he believe p, we $\overline{p}$; but he has a different degree of belief in q given p from ours; and we can obviously try to convert him to our view. [Here is the famous Ramsey test footnote.] But after the event we both know that he did not eat the cake and that he was not ill; the difference between us is that he thinks that if he had eaten it he would have been ill, whereas we think we would not. But this is *prima facie* not a difference of degrees of belief in any proposition, for we both agree as to all the facts (Ramsey, [1929] 1990e, 155).

The disagreement in a theory then reflects a difference in how one applies the chances. This is reflected in how one's algebra has different volumes in them. It is as Ramsey puts it, a matter of difference between the "variable hypotheticals" or rules for one's credences that people differ:

> The meaning of these assertions about unfulfilled conditions, and the fact that whether the conditions are fulfilled or not makes no difference to the difference between us, the common basis, as we may say, of the dispute lies in the fact that we think in general terms. We each of us have variable hypotheticals (or, in the case of uncertainty, chances) which we apply to any such problem; and the difference between us is a difference in regard to these (Ramsey, [1929] 1990e, 155).

So disagreement in theories are disagreements in the laws that those theories give one. That is reflected by the Ramsey sentence in terms of the different volumes ascribed to each observational proposition. Jones and I have objectified our credences in different ways. And it is those objectifications that we end up at loggerheads over.

So I have answered both questions required for addressing the communication criteria. The Ramsey sentence shows how one can communicate about the same theory and disagree over theories through its theoretical term's extension being the compatible partitions who agree with respect to certain volumes.

5.6.5 Summary

In this section, my goal was to show how the proposed account of the Ramsey sentence satisfied the four remaining criteria. Those criteria for an adequate account of the Ramsey sentence were the laws and consequences, verification conditions, surplus content, and communication criteria. The first is trivially answered thanks to the anti-realism of the Ramsey sentence being a description of the axioms, dictionary, and constructed functions an agent believes. The second is answered likewise thanks to Ramsey's anti-realism. By tying the Ramsey sentence to the axioms and dictionary, the verification conditions of theoretical propositions will depend on those axioms and dictionary. Consequently, the meaning of individual theoretical propositions depends on the theory as a whole. The third criterion is satisfied by the fact that Ramsey presumes any believed Ramsey sentence will be useful, guaranteeing that it will derivatively have surplus content. Furthermore, the surplus content is really just the objectification of a person's credences. Finally, the fourth criterion is satisfied due to the extension of the Ramsey sentence theoretical functions. That extension allows agents to grasp each other's theoretical meaning because it captures their conditional credences. Disagreement then becomes disagreement over those same conditional credences.

5.7 Conclusion

In this chapter, I argued for a new interpretation of the Ramsey sentence that is maximally faithful to Ramsey's philosophy and decision theory. I reviewed every interpretation of the Ramsey sentence, and I argued that these interpretations have major problems. From these problems, I enunciated a set of criteria that an account of the Ramsey sentence needs to satisfy to approximate Ramsey's own views. To better understand those views, I did a line-by-line examination of every use Ramsey had of the eponymous sentence. These led to additional criteria to augment the previously enumerated ones. Since Ramsey thought his philosophy of science is best exemplified by the case of chances, I constructed a chancy theory. This toy model was used to begin building my own view of the Ramsey sentence. That view holds the Ramsey sentence to be a description of the axioms and dictionary of an agent's theory, where the extension of the quantified theoretical functions is a set of compatible partitions that are in agreement with respect to certain volumes. I argued that this view satisfies every criterion I outlined previously. Except for one.

The remaining criterion that still needs to be addressed is the anti-realism criterion. I offered a pass, deferring to a future chapter to address that criterion. My plan for the next chapter is to solve it and define exactly what "a description of the axioms and dictionary of an agent's theory" happens to be.

The upshot of my discussion is that Ramsey understood his eponymous sentence as an answer to the problem of decision-making with an impoverished observation language. An observation language is just one where only proper, non-general propositions have truth-values. One example is the phenomenal language of Carnap's *Aufbau*. Ramsey argues that this language is useless for decision-making, and augmenting it with general propositions is insufficient for representing how an agent might make decisions. The Ramsey sentence addresses the poverty of the underlying observation language by reifying an agent's decision-making rules, the laws and chances of the theory, through fictional theoretical propositions. These fictional theoretical propositions allow an agent to deliberate across time and allow an agent to communicate with other agents about their decisions.

Once the anti-realism criterion is satisfied, this will allow me to address the final part of this dissertation: the extent of Ramsey's commitment to scientific realism. That will address a long-standing debate scholars have had over the extent to which Ramsey thinks scientific theories are descriptive and factive.

Chapter 6

The Existential Quantifier

6.1 Introduction

In the previous chapter, I articulated a view of the Ramsey sentence that I claim is anti-realist in the sense that existentially quantified propositions are not about objects in the world apart from the agent. I did not, however, demonstrate why exactly my view is anti-realist in the claimed sense. My goal here is to argue for an anti-realist interpretation of existential propositions that supports my earlier claim. To that end it is important to consider Ramsey's old view of the quantifiers and why he changed his mind. Ramsey's old view held existential propositions to be the infinite union of the proposition's instances; this is sometimes glossed as an infinite disjunction. By 1929, Ramsey had abandoned this view for something closer to Weyl's interpretation of the existential propositions. Existential propositions are no longer proper propositions—they are not truth-apt—but they are descriptions of a witnessing proposition. Witnesses can be either proper, singular propositions, or they can be non-truth-apt universal propositions. In either case, a description is just a finite disjunction over available names or relations. Ramsey believes that this alternative account dispenses with his previous account because it shows how infinite sets are fictions, and thus it eliminates the need for infinite unions. This is relevant to the Ramsey sentence since the Ramsey sentence is the existential generalization of a universal proposition. A non-truth-apt description of a non-truth-apt rule for judging results in a fictional collection of objects just as a mathematical rule results in a fictional infinite collection of ghostly objects such as numbers. So the Ramsey sentence is deeply anti-realistic in the precise sense it is doubly not a proper proposition whose fictional reference is really a reification of the rules for judging contained in the theory.

I want to begin by emphasizing there is an important mystery that needs to be addressed before building an anti-realist theory of existential propositions. Writing earlier in his career in 1926, Ramsey juxtaposes his view of existential propositions explicitly with the anti-realist views of the quantifiers found in Weyl and Hilbert. He writes that the difference in his philosophy of mathematics from their philosophy of mathematics comes down to the quantifiers, which he rejects:

> We must begin with what appears to be the crucial question, the meaning of general and existential propositions, about which Hilbert and Weyl take substantially the same view. Weyl says that an existential proposition is not a judgment, but an abstract of a judgment, and that a general proposition is a sort of cheque which can be cashed for a real judgment when an instance of it occurs (Ramsey, [1926] 1990f, 233).

Ramsey then provides a list of objections against Weyl and Hilbert's account that focus on how they eliminate much of higher mathematics and beliefs about ordinary objects. What is mysterious is that there years later Ramsey writes in notes from 1929 that "A proposition containing it [an existential quantifier] is a description (abstract) of a proposition rather than a proposition itself, and the proposition described must be able to be given" (Ramsey, [1929] 1991c, 197). He uses explicitly Weyl's language and then gives a view that has substantial similarities to Weyl's—the same account that Ramsey pilloried a few years earlier. This suggests the following questions: if Ramsey did change his mind, what shifted his view; and how does the new view avoid the objections levied against its cousin?

The key shift came in Ramsey's theory of the universal proposition. In a previous chapter, I argued that Ramsey came to believe that universal propositions are really a species of chance. Consequently, universal propositions are logically compatible with every singular proposition; they are not truth-functions of singular propositions.[1] This makes them not truth-apt. What this means is that many of the jobs the existential quantifier plays in human cognitive economy can be completely performed by universal propositions as chances. For similar reasons, existential propositions inferred from singular propositions have little utility. Ramsey concluded that existential propositions provide no epistemic value apart from the singular or universal propositions that warrant their introduction. So the existential quantifier can be dispensed with entirely.

This fact can be seen in how Ramsey treats infinity as it is used in mathematics. An important motivation Ramsey has in his earlier criticism of Weyl and Hilbert is that existential propositions play important roles in thought about infinite sets and higher mathematics. However, the new view on universal propositions shows that those infinite sets can be interpreted as fictions used to handle universal propositions. So an important motivation for interpreting existential propositions as infinite unions is lost.

It would be a mistake, however, to consider Ramsey an eliminativist about the infinite and existential propositions. This is what most clearly separates Ramsey's new views on universal and existential propositions from the view he attributes to Weyl and Hilbert. Even though

[1] By logically entailed I mean the usual sense of deductive entailment. If one construes propositions as sets in an algebra, then one proposition A logically (deductively) entails proposition B just in case A is a subset of B. One can still meaningfully talk about how Ramsey's variable hypotheticals are universal propositions because they obey something like universal elimination in an agent's forecasts.

existential propositions might be eliminable, they should not because they are convenient. Their underlying meaning is tied up with their witness. And in the case of truly general existential propositions, that witness will be a universal proposition. Universal propositions play an essential role in human cognition. So they should not be eliminated—nor should the existential propositions generalized from them.

The upshot is that Ramsey is a fictionalist about general propositions, but he views them as load-bearing beams in the house of human decision-making. So his objections against Weyl and Hilbert fail to wreck his own views.

This is relevant to the Ramsey sentence because it means that the anti-realism criterion I articulated in the previous chapter can be satisfied. That criterion states that the Ramsey sentence as an existential proposition is not about objects in the world apart from the agent. If my argument about Ramsey is correct, then the Ramsey sentence's witness is the theory's axioms and dictionary. Those are universal propositions. They dictate the behavior of an agent's credences when making forecasts. So the Ramsey sentence is really about the rules an agent uses for deliberating over their credences; it is not about objects in the world, but it is about how the agent's beliefs work.

Here is how my argument proceeds. First, I review Ramsey's old view of the existential quantifiers, I review his objections to Weyl and Hilbert, and I review why Ramsey's new view of universal propositions avoids those objections. Second, I review Ramsey's notes on the existential quantifier, and I articulate an account of existential propositions compatible with those notes. I discuss how Ramsey's shift on universal propositions results in a revision of his thoughts about the infinite. I then connect this to how he changes his mind about quantified propositions while avoiding his previous objections to Weyl and Hilbert. Third, I argue that the proposed account of existential propositions satisfies the anti-realism criterion. Finally, I conclude by discussing my comparing account with Majer's similar account. I argue that Majer's account has important shortcomings that my account avoids.

6.2 Ramsey's Old View of the Quantifiers and of Weyl

In a previous chapter, I have put forward an account of universal propositions, what Ramsey calls variable hypotheticals, that is anti-realist in the sense that those propositions are not truth-apt. The view I develop has universal sentences be a species of chance. This makes them logically compatible with any description of the world—rendering Ramsey's old view of laws as the axioms in the best system of propositions inadequate.

Here I want to focus on Ramsey's view of the existential quantifier. I will argue that Ramsey's shift on the existential quantifier is intimately connected with his shift on the universal quantifier. But before I do, I need to discuss his old view of the existential quantifier.

In this section, I examine the view of the quantifiers that Ramsey adopts prior to 1929, which he attributes to Wittgenstein from the *Tractatus*.[2] I then look at Ramsey's argument against Weyl in "Mathematical Logic." I conclude with a review of why Ramsey's mind changed about how to understand universal propositions.

6.2.1 Ramsey's Old View of the Quantifiers

Ramsey endorses a view of the quantifiers in "Foundations of Mathematics" that he attributes to Wittgenstein (Ramsey, [1926] 1990l). He later defends this view in "Mathematical Logic" and "Facts and Propositions" (Ramsey, [1927] 1990d). A succinct summary of this account is that universal and existential quantified sentences are infinite truth-functions. While one might gloss universal sentences as infinite conjunctions and existential sentences as infinite disjunctions, this is not the actual position Ramsey endorses. Critically, Ramsey states they are not conjunctions and disjunctions because their arguments cannot be enumerated. However, this does not prevent them from being truth-functions. Proper truth-functions can still

[2]It is highly probable that this view is not Wittgenstein's view in the *Tractatus*. See Rogers and Wehmeier

be defined through set-theoretic abstraction using propositional functions and infinite intersections and unions. So general propositions are truth-functions, and they can be analogized to infinite conjunctions and disjunctions.

In "Foundations of Mathematics", Ramsey appeals to Wittgenstein's theory of propositions when he introduces his view of the quantifiers. Ramsey's goal is an account of logic and mathematics that is general and tautologous, and he thinks that Wittgenstein's theory of propositions can yield such an account. He argues it is mistaken to equate logic and mathematics with just general propositions:

> It is really obvious that not all such propositions [general propositions] are propositions of mathematics or symbolic logic. Take for example 'Any two things differ in at least thirty ways'; this is a completely general proposition, it could be expressed as an implication involving only logical constants and variables, and it may well be true. But as a mathematical or logical truth no one could regard it; it is utterly different from such a proposition as 'Any two things together with any other two things make four things,' which is a logical and not merely an empirical truth. According to our philosophy we may differ in calling the one a contingent, the other a necessary propositions, or the one a genuine proposition, the other a mere tautology; but we must all agree that there is some essential difference between the two, and that a definition of mathematical propositions must include not merely their complete generality but some further property as well (Ramsey, [1926] 1990l, 167).

Ramsey wishes to distinguish between contingent or genuine propositions and necessary or tautologous propositions. It is the latter that he thinks are the best candidates for mathematical propositions. He argues that mathematical propositions are those whose form

for a full discussion (Rogers and Wehmeier, 2012).

is that of a tautology, and whose content is a full generality:

> It is obvious that a definition of this characteristic [tautological] is essential for a clear foundation of our subject, since the idea to be defined is one of the essential sides of mathematical propositions—their content and their form. Their content must be completely generalized and their form tautological.
>
> The formalists neglected the content altogether and made mathematics meaningless, the logicians neglected the form and made mathematics consist of true generalizations; only by taking accounts of both sides and regarding it as composed of tautologous generalizations can we obtain an adequate theory (Ramsey, [1926] 1990l, 168).

Russell identifies the content of mathematical propositions with their generality, but he fails to explain how they are tautologous. The formalists provide the form of mathematical propositions, yet omit a demonstration of their generality. With Wittgenstein's theory of propositions, Ramsey hopes to provide an explanation that captures the content and form of mathematical or logical propositions.

Wittgenstein's theory of truth-functions is crucially important for Ramsey's project. Truth-functions are defined through truth-possibilities. Recall a truth-possibility of some set of propositions is an intersection of those propositions or their complements, and the set of truth-possibilities for some propositions is the set of every possible intersection of those propositions or their complements.[3] Two propositions are the same truth-function of some truth-possibilities if they both agree with those truth-possibilities:

> A proposition which expresses agreement and disagreement with the truth-possibilities of $p, q, \ldots$ (which need not be atomic) is called a truth-function of the

[3]When viewed sententially instead of propositionally, these are conjunctions of sentences or their nega-

> arguments $p, q, \ldots$. Or more accurately, P is said to be the same truth-function of $p, q, \ldots$ as R is of $r, s, \ldots$ if P expresses agreement with the truth-possibilities of $p, q, \ldots$ corresponding by the substitution of p for r, q for $s, \ldots$ to the truth-possibilities of $r, s, \ldots$ with which R expresses agreement (Ramsey, [1926] 1990l, 170).

The thought is familiar to students of propositional logic who construct truth-tables. First, consider two sets of propositions that need not be atomic. Then consider all the different combinations of truth-values those propositions may take on. These are the truth-possibilities with respect to their prospective sets. Then two propositions ϕ and ψ are the same truth-function with respect to their truth-possibilities if they have the same pattern of agreement and disagreement with respect to their truth-possibilities. Two propositions are the same truth-function if they have the same truth-table with respect to their truth-values.

It is here that Ramsey takes Wittgenstein to have made an important observation. Truth-functions need not be finite:

> Mr Wittgenstein has perceived that, if we accept this account of truth-functions as expressing agreement and disagreement with truth-possibilities, there is no reason why the arguments to a truth-function should not be infinite in number.[1]
>
> [1] [Footnote] Thus the logical sum of a set of propositions is the proposition that one at least of the set is true, and it is immaterial whether the set is finite or infinite. On the other hand, an infinite algebraic sum is not really a sum at all, but a *limit*, and so cannot be treated as a sum except subject to certain restrictions (Ramsey, [1926] 1990l, 170).

The innovation of an infinite truth-function Ramsey immediately applies to what he calls logical sums and logical products. He immediately answers a possible objection that one could not enumerate the arguments for these truth-functions:

> As no previous writer has considered truth-functions as capable of more than a finite number of arguments, this is a most important innovation. Of course if the arguments are infinite in number they cannot be all be enumerated and written down separately; but there is no need for us to enumerate them if we can determine them in any other way, as we can by using propositional functions (Ramsey, [1926] 1990l, 170–171).

I cannot provide an infinite list of arguments for a truth-function. His workaround is that I need not require such an enumeration because I can specify the arguments through other means. Propositional functions provide the other means.

Ramsey defines a propositional function through substitution and from propositional functions, universal and existential propositions. He writes that

> A propositional function is an expression of the form '$f\hat{x}$', which is such that it expresses a proposition when any symbol (of a certain appropriate logical type depending on f) is substituted for '$\hat{x}$' [....] We can use propositional functions to collect together the range of propositions which are all the values of the function for all possible values of x. Thus '$\hat{x}$ is a man' collects together all the propositions 'a is a man', 'b is man', etc (Ramsey, [1926] 1990l, 171).

Propositional functions take as arguments individuals and output propositions.[4] Here Ramsey expresses this with substitution. A propositional function yields a proposition when the unbound variable is substituted for a name of the proper type. He argues that these

tions.

[4]If the function is higher order, it might take higher ordered types as arguments like functions.

functions can define a set of propositions. The natural target is then to define an infinite collection of propositions given by the universal and existential quantifiers:

> Having now by means of a propositional function defined a set of propositions, we can, by using an appropriate notation, assert the logical sum or product of this set. Thus, by writing '$(x).fx$' we assert the logical product of all propositions of the form 'fx'; by writing '$(\exists x).fx$' we assert their logical sum. Thus '$(x).x$ is a man' would mean 'Everything is a man'; '$(\exists x).x$ is a man', 'There is something which is a man'. In the first case we allow only the possibility that all the propositions of the form 'x is man' are true; in the second we exclude only the possibility that all the propositions of the form 'x is a man' is false.
>
> Thus general propositions containing 'all' and 'some' are found to be truth-functions, for which the arguments are not enumerated but given in another way (Ramsey, [1926] 1990l, 171).

The thought is that if propositions are truth-functions, quantified propositions are truth-functions with infinite arguments. Those infinite arguments are described—not enumerated—by the propositional functions the quantifiers bind. Thus, general propositions can be understood as truth-functions.

It is worth emphasizing that the view here is not that universal propositions and existential propositions are infinite conjunctions and disjunctions respectively. Ramsey explicitly states that the arguments for these truth-functions cannot be enumerated; they are not like conjunctions and disjunctions in the sense that they can be written out and checked. Instead, the content of general propositions can only be described set theoretically. By abstracting the set of propositions that are instances of some propositional function, Ramsey can provide a well-defined set and then take an intersection or union of its elements. This intersection or union being possibly infinite is no problem; it does not need to be written out because it can be described in the appropriate way. While this can be thought of as analogous to an infinite conjunction or disjunction, this is just a way of speaking. So Ramsey's account is slightly different from the view of the quantified propositions as infinite conjunctions and disjunctions.

He attacks Hilbert and Weyl's views in "Mathematical Logic" in 1926, and he defends his truth-functional view. Importantly, he explicitly connects his logical products and sums with conjunctions and disjunctions:

> How then are we to explain general and existential propositions? I do not think we can do better than accept the view which has been put forward by Wittgenstein as a consequence of his theory of propositions in general.
>
> [....]
>
> Mr Wittgenstein holds that all propositions express agreement and disagreement with truth-possibilities of atomic propositions, or, as we say, are truth-functions of atomic propositions; although often the atomic propositions in question are not enumerated, but determined as all values of a certain propositional function. Thus the propositional function 'x is red' determines a collection of propositions which are its values, and we can assert that all or at least one of these values are true by saying 'For all x, x is red' and 'There is an x such that x is red' respectively. That is to say, if we could enumerate the values of x as $a, b \dots z$, 'For all x, x is red' would be equivalent to the propositions 'a is red and b is red and $\dots$ and z is red' (Ramsey, [1926] 1990f, 237).

Ramsey makes the analogy between universal propositions and conjunctions. In cases where the values of a propositional function can be finitely enumerated, then the universal proposition is a finite conjunction. Likewise, an existential proposition would be a finite disjunction.

The important upshot is that with "Mathematical Logic" Ramsey solidifies the position he develops in "Foundations of Mathematics".

The last place Ramsey defends his old view of the quantifiers is in the 1927 paper "Facts and Propositions". There he repeats the accepted view and comes closest to equating quantified sentences with infinite conjunctions and disjunctions:

> About these [general propositions] I adopt the view of Mr Wittgenstein that 'For all x, fx' is to be regarded as equivalent to the logical product of all the values of 'fx', i.e. to the combination fx_1 and fx_2 and fx_3 and $\ldots$, and that 'There is an x such that fx' is similarly their logical sum. In connection with such symbols we can distinguish first the element of generality, which comes in specifying the truth-arguments, which are not, as before, enumerated, but determined as all values of a certain proposition function; and secondly the truth-function element which is the logical product in the first case and the logical sum in the second (Ramsey, [1927] 1990d, 48–49).

The view here is the same as it was before. Propositional functions specify the range of values, i.e. propositions, whose intersections and unions constitute the universal and existential propositions respectively. While Ramsey does not equate these with infinite conjunctions and disjunctions, these propositions can be still thought of as ordinary truth-functions. Though their truth-arguments are not enumerable, general propositions are in spirit not any different from singular propositions. Again, the view is the same as that given in "Foundations of Mathematics".

In summary, Ramsey's old view is that the quantifiers are infinite truth-functions defined by the right propositional functions. While these can be analogized with conjunctions and disjunctions, Ramsey admits that this is just talk; the arguments cannot be enumerated.

But the inability to enumerate the truth-arguments is no problem because Ramsey believes one can still properly define an infinite truth-function through propositional functions.

6.2.2 Ramsey on Weyl in 1926

In a previous chapter, I argued that Ramsey's view on laws shifted, and with them so too did his views on universal propositions. The old view of universal propositions as logical products was abandoned for a view of laws as "rules for judging." Multiple authors have noted the similarity in phraseology with Weyl's terminology. They have concluded that Ramsey adopts Weyl's view of the quantifiers wholesale. In the case of universal propositions, this is not quite right; Weyl had no conception of chance or theory of decision-making that could he could assimilate with a theory of quantifiers. So there is an important difference between Weyl and Ramsey on that count. But Ramsey's view of universal propositions still has the fundamental similarity with Weyl's that makes those propositions be rules. This makes Weyl's theory of the existential quantifier and Ramsey's reaction to both it and Weyl's theory of the universal quantifier relevant to Ramsey's final views on existential propositions.

Ramsey records his reaction to Weyl's theories in two places. The first is in the 1926 paper "Mathematical Logic" where he defends his account of universal and existential propositions against Weyl and Hilbert's heterodoxy; the second is in a collection of later notes concerning finitist and intuitionist mathematics. I review Ramsey's initial reaction here because Ramsey lays out a number of objections. It is important to understand why Ramsey initially dismissed Weyl's views so that I can better understand how his theory of the quantifiers changed.

The goal of "Mathematical Logic" is to defend to a broader audience logicism. To that end, Ramsey focuses on a substantial difference between the logicist and finitists such as Hilbert and Weyl. That difference is those of the quantifiers: "We must begin with what appears to be the crucial question, the meaning of general and existential propositions, about which Hilbert and Weyl take substantially the same view" (Ramsey, [1926] 1990f, 233). He immediately explains both views, starting with Weyl:

> Weyl says that an existential proposition is not a judgment, but an abstract of a judgment, and that a general proposition is a sort of cheque which can be cashed for a real judgment when an instance of it occurs.
>
> Hilbert, less metaphorically, says that they are ideal propositions, and fulfill the same function in logic as ideal elements in various branches of mathematics. He explains their origin in this sort of way; a genuine finite proposition such as 'There is a prime between 50 and 100', we write 'There is a prime which is greater than 50 and less than 100', which appears to contain a part, '51 is a prime, or 52 is a prime, etc., *ad inf.*,' and so be an infinite logical sum, which, like an infinite algebraic sum, is first of all meaningless, and can only be given a secondary meaning subject to certain conditions of convergence. But the introduction of these meaningless forms so simplifies the rules of inference that it is convenient to retain them, regarding them as ideal, for which a consistency theorem must be proved (Ramsey, [1926] 1990f, 233).

Ramsey reiterates Weyl's terminology without much discussion, and he then proceeds to discuss Hilbert's view in more detail. The idea for Hilbert is that quantified sentences act like infinite algebraic products and sums that unless strict conditions are met, have no meaning whatsoever. For example, there is no sum of the sequence defined by $1, -1, 1, -1, \ldots$. That is, the following has no value

$$\sum_{i=0}^{\infty}(-1)^{i+1}$$

due to the fact that its limit does not exist. So the view Ramsey attributes to Hilbert and by extension to Weyl is that logical products and sums are idealizations so long as consistency proofs can be given.[5]

Ramsey then enumerates several problems with the Weyl and Hilbert account of quantifiers.

First, Ramsey thinks that such a view eliminates much of mathematics, and thus does not offer room for any uses of those idealizations. He writes that

> [I]t is hard to see what use these ideal can be supposed to be; for mathematics proper appears to be reduced to elementary arithmetic, not even algebra being admitted, for the essence of algebra is to make general assertions. Now any statement of elementary arithmetic can be easily tested or proved without using higher mathematics, which if it be supposed to exist solely for the sake of simple arithmetic seems entirely pointless (Ramsey, [1926] 1990f, 233–234).

The argument is that the Weyl and Hilbert view equates mathematics with elementary arithmetic, which obviates the need for ideals because higher mathematics such as analysis is eliminated. Since any statement of arithmetic is easily verified, there is no need for the exotic idealizations given by universal and existential propositions. Call this the **elimination objection**.

Second, Ramsey thinks that eliminating general propositions with ideals presupposes the possibility of general knowledge:

> Secondly, it is hard to see how the notion of an ideals can fail to presuppose the possibility of general knowledge. For the justification of ideals lies in the fact that *all* propositions not containing ideals which can be proved by means of them are

[5] I omit here whether this is an accurate characterization of both Weyl and Hilbert's views.

> true. And so Hilbert's metamathematics, which is agreed to be genuine truth, is bound to consist of general propositions about all possible mathematical proofs, which, though each proof is a finite construct, may well be infinite in number. And if, as Weyl says, an existential proposition is a paper attesting the existence of a treasure of knowledge but not saying where it is, I cannot see how we explain the utility of such a paper, except by presupposing its recipient capable of the existential knowledge that there is a treasure somewhere (Ramsey, [1926] 1990f, 234).

Ramsey claims that the method of ideals, which is supposed to eliminate general propositions, in fact, presupposes those general propositions. So they cannot really be used to banish general propositions. In the case of Hilbert, Ramsey argues that the ideals presuppose an infinite application of them. For Weyl, Ramsey thinks that one cannot make any sense of a treasure map without knowing that it is a treasure map. The objection more concretely is that the utility of existential propositions can only be had with an understanding of their generality—not merely from the witness that allows their introduction. I know that someone is a student of UCI if I know that "someone" need not apply to merely an individual, but could apply across a whole domain of students, which might possibly be infinite. Call this the **presupposition objection**.

Third, Ramsey thinks Hilbert and Weyl's accounts fail to make sense of generality outside of mathematics:

> Moreover, even if Hilbert's account could be accepted so long as we confine our attention to mathematics, I do not see how it could be made plausible with regard to knowledge in general. Thus, if I tell you 'I keep a dog', you appear to obtain knowledge of a fact; trivial, but still knowledge. But 'I keep a dog' must be put into logical symbolism as 'There is something which is a dog and kept by me';

> so that the knowledge is knowledge of an existential proposition, covering the possibly infinite range of 'things' (Ramsey, [1926] 1990f, 234).

Ramsey claims that ordinary knowledge claims conceal generalizations. These generalizations are not mathematical but deal with ordinary objects like people and dogs. He thinks that Hilbert and Weyl's proposals have no way of dealing with these ordinary objects. Call this the **ordinary object objection**.

Finally, Ramsey thinks that even arithmetic has generality, which neither Hilbert nor Weyl's proposals address:

> Lastly, even the apparently individual facts of simple arithmetic seem to me to be really general. For what are these numbers, that they are about? According to Hilbert marks on paper constructed out of the marks 1 and +. But this account seems to me inadequate, because if I said 'I have two dogs', that would also tell you something; you would understand the word 'two', and the whole sentence could be rendered something like 'There are x and y, which are my dogs and are not identical with one another'. This statement appears to involve the idea of existence, and not to be about marks on paper; so that I do not see that it can be seriously held that a cardinal number which answers the question 'How many?' is merely a mark on paper.

The claim is that arithmetical propositions are general propositions. Propositions involving cardinal numbers can be understood as existential generalizations. For example, "I met two friends for lunch" could be written in first-order logic as:

$$\exists x \exists y (R(a,x) \wedge R(a,y) \wedge x \neq y \wedge \forall z (R(a,z) \supset (z = x \vee z = y)))$$

where $R(x, y)$ is understood as "x met y for lunch" and a is me. Ramsey then goes on to demonstrate that finite sums can be written similarly. Since any claim involving cardinal numbers can be written using an existential quantifier, Ramsey argues that arithmetic presupposes some concept of generality. But, so Ramsey argues, Hilbert and Weyl have an account of generality that takes arithmetic for granted. So Hilbert and Weyl ignore really interesting cases of generality. Call this the **arithmetic objection**.

Summarizing, Ramsey disputes Hilbert and Weyl's account of the quantifiers. Hilbert and Weyl's theories reduce mathematics to elementary arithmetic; their accounts seem to presuppose some notion of generality; their view fails to cover ordinary objects; and Hilbert and Weyl cannot account for non-mathematical propositions covertly involving cardinal numbers. Ramsey concludes that his preferred account suffers from none of these objections. A few years later, he changes his mind. It is why he changes his mind that I turn to now.

6.2.3 Ramsey's Objections to Universal Propositions

Most scholars agree that Ramsey changed his mind about the quantifiers. This is most obvious with the universal quantifier. The reason why he changed his mind about the universal quantifier is somewhat disputed; explanations range from concerns about the infinite (Misak, 2016), about the unverifiability of universal propositions (Holton and Price, 2003), about the existence of general facts (Sahlin, 1990), and about properly defined sets (Majer, 1989). I argue that these explanations are either inaccurate or incomplete. Ramsey changed his mind on the interpretation of universally quantified propositions because he concluded that universal propositions are chances. Chances are logically compatible with every non-chancy proposition. This means that a conjunction—whether finite or infinite—is not equivalent to any chance proposition. So the old view of universal propositions as logical products is incorrect.

Ramsey's new view is very close to Weyl's account that Ramsey criticizes in "Mathematical Logic". Do Ramsey's objections against Weyl apply to Ramsey's new account of universal propositions? I address this question now.

Recall that the ordinary and arithmetic objections held that important propositions, such as those in ordinary language, secretly had existential quantifiers and were thus general propositions after all. Similarly, the presupposition objection was that Weyl's account presupposed knowledge of general propositions when trying to eliminate them. So these three objections could basically be summed up with the slogan: general propositions are not eliminable. And the elimination objection could be simply stated: nor should one try to eliminate them. The cost would be too high.

By itself, Ramsey's conclusion that universal propositions are chance propositions still faces the objections leveled against Weyl and Hilbert. If I suppose that because chances are not real propositions, they should be eliminated, then Ramsey's objections still sting. Arithmetical and ordinary claims are full of universal generalizations. These would have to be abandoned. Likewise, much of my knowledge seems to presuppose universal propositions, and I should conclude it was really ignorance or nonsense gussied up as knowledge. Finally, most of higher mathematics would have to go—rendering me with little beyond counting. At least, this would have to go if I conclude that universal propositions are chances and chances should be eliminated.[6]

Ramsey, however, does not accept the premise that chances should be eliminated. When he considers the possibility of eliminating them in "General Propositions and Causality", he declines:

[6]It is an open question whether I should view mathematics and ordinary propositions as full of chances. The latter seems to be the case: most of my concepts are affordances or dispositions that admit subjunctives and other chance-like propositions. But mathematics might still be thought to only involve "proper" propositions. Of course, a non-Platonist might scoff that these are true, proper propositions at all. Some other approaches, such as treating them as secret conditionals, would have to bring them back under the umbrella of ordinary propositions.

> We can begin by asking whether these variable hypotheticals play an essential part in our thought; we might, for instance think that they could simply be eliminated and replaced by the primary propositions which serve as evidence for them [....] This view can be supported by observing that the ultimate purpose of thought is to guide our action, and that on any occasion our action depends only on beliefs or degrees of belief in singular propositions. And since it would be possible to organize our singular beliefs without using variable intermediaries, we are tempted to conclude that they are purely superfluous.
>
> But this would, I think, be wrong; apart from their value in simplifying our thought, they form an essential part of our mind. That we think explicitly in general terms is at the root of all praise and blame and much discussion. We cannot blame a man except by considering what would have happened if he had acted otherwise, and this kind of unfulfilled conditional cannot be interpreted as a material implication, but depends essentially on variable hypotheticals (Ramsey, [1929] 1990e, 153–154).

He acknowledges that only proper propositions are needed to guide action. However, he thinks humans should still make decisions according to universal propositions and chances. They are useful because they greatly simplify thought, but they are also essential elements of human cognition. This goes back to Ramsey's theory of cognitive psychology. Decisions are made according to habits, and universal propositions and chances are just regimented habits. So Ramsey's point is that it is part of how humans actually cognize that decisions will be made with variable hypotheticals. To avoid doing so is impossible—for humanity at least.[7] This fact shows up in how humans obsess about counterfactuals when evaluating one another's decisions. And Ramsey concludes that this fact drives much discussion about

[7]One might ask whether this would be the case for any agent. This gets back to Kant's question that predates Ramsey about the necessary preconditions of experience for various intellects. Human intellects might require the use of variable hypotheticals. However, others may not.

responsibility and blame. So Ramsey thinks variable hypotheticals and chances should be kept in the human decision-making toolkit.

By not accepting a need to eliminate universal propositions, Ramsey can avoid the objections leveled against Weyl and Hilbert's views. What it opens up for him is that an anti-realism about the universal quantifier *does not imply the elimination of universal propositions*.[8]

Instead, it offers a novel and rich philosophy of mathematics. Consider the ordinary object and arithmetic objections. Universal propositions involving those dry, medium-sized goods are automatically explained in terms of Ramsey's theory of chances and laws. Arithmetical universal propositions when they appear are likewise understood: they depend on the functions that warrant their assertion, and functions are just rules as chances are rules. The same applies to the presupposition and elimination objections. Keeping variable hypotheticals around allows them to act as presuppositions in most of ordinary knowledge. And the elimination objection is just directly denied; variable hypotheticals should not be eliminated and with them much of higher mathematics. So universal propositions can be kept around in a slightly different guise.

This covers the case of universal propositions. It fails to address existential propositions, and those propositions must be accounted for because many of Ramsey's concerns with Weyl and Hilbert's views concern existential generalizations. Propositions like "there are two dogs in the meadow" must be preserved in some manner. The same applies to elementary arithmetic and the higher reaches of mathematics. So even though I can claim that Ramsey's objections are no problem for his view of universal propositions as chances, they still leave open the question of whether there is a view of existential propositions that can meet those objections.

I need an anti-realist account of existential propositions to square with these objections. Once that account is on the table, I can then address two remaining questions: first, why did

[8]It should be noted that this really does not apply to Hilbert's account as Ramsey claims.

Ramsey change his mind about existential propositions, and second, how does the account satisfy the anti-realism criteria of the previous chapter? I turn to an account of existential propositions now.

6.3 Ramsey on the Existential Quantifier

Ramsey rejects his old view of the universal quantifier by 1929 for an account of universal propositions as rules for judging. It is less clear whether he renounces his old view of existential propositions as logical sums (unions) of the instances given by a propositional function. Symmetry considerations would suggest he did; if he abandons the position that universal sentences are proper propositions, then the equivalence between existential and universal sentences would imply existential sentences are not proper propositions either. But it is unclear what an existential proposition would be if universal propositions are rules for judging.

There is some debate over this in the secondary literature. Psillos argues that Ramsey maintained the old view of existential propositions (Psillos, 2004, 71), which allowed him to contemplate theories as proper propositions (Psillos, 2004, 73). Majer, however, suggests that Ramsey adopted Weyl's view on existential quantifiers to solve a particular problem about iterated quantified sentences (Majer, 1989).[9] So it is not obvious that symmetry considerations should push Ramsey to adopt a new interpretation of the existential quantifier.

What does militate that he changed his mind is the anti-realism criterion I developed in a previous chapter. Ramsey thinks theories are fictions. He also seems to indicate a change of mind about the existential quantifier in his notes. And when he does discuss the existential quantifier in "Theories", he treats it like the preface "Once upon a time ..." used in fairy tales.

[9] I discuss Majer's views in a later section for this chapter.

So Psillos is almost certainly wrong about Ramsey retaining the old view about existential quantifiers.

The task then for this section is to 1) provide an anti-realist account of existential propositions and 2) explain why Ramsey changed his mind. Majer has pointed out that some unpublished notes of Ramsey about finitist mathematics are relevant here. In this section, I examine those notes—along with other unpublished notes—to reconstruct a view of the existential quantifier that is very close to Weyl's view. This will turn out to be different from Majer's reading in important ways, which I discuss in the following section. I show that Ramsey's change of mind about existential propositions is symmetric with his change of mind about universal propositions. His views about universal propositions really drove his views about quantified propositions generally.

6.3.1 Ramsey's Reading of Weyl

There is very little in Ramsey's published work that discusses existential judgments. Apart from "Theories", there is the fragment "Causal Qualities" and some references in "General Propositions and Causality". In each of these pieces, Ramsey omits an explicit discussion of existential propositions.

More can be found in Ramsey's unfinished notes. Three documents stand out. One is an untitled piece that contains erratic notes directly about the existential quantifier. The other two contain explicit discussions of Weyl and Skolem's understanding of the existential quantifier. These are the two pieces that authors like Majer use to argue that Ramsey had adopted Weyl's interpretation of the existential quantifier. The first is a document titled "Principles of Finitist Mathematics", and the second is a document titled "The Formal Structure of Intuitionistic Mathematics". The latter is a series of lecture notes and other writings affiliated with Brouwer, where Ramsey proposes a logic without the introduction rule for universal propositions and the elimination rule for the existential quantifier. It is unclear whether Ramsey himself endorses this logic or is documenting a logic compatible with Brouwer's intuitionism. The former document seems to track Ramsey's own views. The beginning section is most relevant for the view to be argued for, so I consider only that part of the paper.

Ramsey begins by claiming the existential quantifier should be replaced, and he uses terminology associated with Weyl:

> (1) Ex must be able to be dispensed with. cf. Skolem.
>
> A proposition containing it is a description (abstract) of a proposition rather than a proposition itself, and the proposition described must be able to be given. If it is $(x)(Ey)$, y stands for a function. The proposition described has fx for y (Ramsey, [1929] 1991c, 197).[10]

Two things are worth noting. First, Ramsey has an anti-realist view when he says any proposition with existential quantification is a "description (abstract)" instead of a proposition. This is cashed out directly in what follows. He gives the example of dropping the existential quantifier and replacing that quantifier's bound variable with a function whose arguments are the variables of a universal quantifier whose scope the function falls in. This elimination rule is now known as Skolemization. Consequently, the reference to Skolem is appropriate because Skolem demonstrated this procedure with his construction of primitive recursive arithmetic minus apparent logical variables (unbounded quantifiers).[11] Second, the phrase

[10] I have included the additions found in Ramsey, 1991a due to the incomplete nature of the notes.

[11] Skolem writes in the "Foundations of Elementary Arithmetic" that

> Now what I wish to show in the present work is the following: *If we consider the general theorems of arithmetic to be functional assertions and take the recursive mode of thought as a basis, then that science can be founded in a rigorous way without use of Russell and Whitehead's notions "always" and "sometimes".* This can also be expressed as follows: A logical foundation can be provided for arithmetic without the use of apparent logical variables. To be sure, it will often be advantageous to introduce apparent variables; but we shall require that these variables

"description (abstract)" is not accidental. It is a nearly exact phrase that Weyl uses in his "On the New Foundational Crisis in Mathematics", where Weyl writes that

> An *existential statement*—say, "there exists an even number"—is not at all a judgment *in the strict sense, which claims a state of affairs [behauptet einen Sachverhalt]*. Existential states of affairs are empty invention of logicians. "2 is an even number": This is an actual judgment expressing a state of affairs; "there is an even number" is merely a *judgment abstract [Urteilsabstrakt]* gained from this judgment (Weyl, [1921] 1997, 97).

The phrase "judgment abstract" along with the argument that existential statements are not judgments match Ramsey's phraseology and ideas. An existential proposition only has "meaning"[12], i.e. correctness conditions, if an instance has been found. In other words, an existential proposition needs a witness. Weyl compares the existential sentence to a treasure map: it only has significance if there is an X that marks the spot—an original proposition that generates the existential sentence. In mathematics, this original proposition would be the successful construction of a function. Existential sentences signify the successful construction of a proof (Weyl, [1921] 1997, 98). This closely matches Ramsey's idea that an existential proposition is a description of a proposition that must be deliverable. That deliverable proposition can be given by a function. So Ramsey's remark closely tracks Weyl's own view.

Following his declaration of the dispensability of existential sentences, Ramsey argues that these sentences should be kept around because they are convenient. So rules for introducing existential quantifiers need to be proposed:

> range over only finite domains, and by means of recursive definitions we shall then always be able to avoid the use of such variables" (Skolem, [1923] 1967, 304).

This is very close to Ramsey's own formulation except narrower. Ramsey seemed to apply a similar lesson to all of mathematics.

[12] I use the word "meaning" here very loosely. They lack truth-conditions and so sense, i.e. meaning.

> (2) But though it *can* be dispensed with, this would be very inconvenient and it is better to lay down rules governing its use in the above sense.
>
> (3) In the first place we must explain when and how it can be introduced.
>
> From e.g. $2+2=4$ we may proceed to $(Ex)2+x=4$ meaning as it were we had (and could have again a proposition of the form) $2+x=4$ (namely $2+2=4$).
>
> This has the simple formula $\phi(a)$ to $(Ex)\phi(x)$, (ϕ a propositional function, f a numerical function), and so from $(x)x+2=2+x$ we would have $(Ey)(x)x+y=y+x$ (Ramsey, [1929] 1991c, 197–198).

He proposes the standard existential introduction rule here. However, he is quickly unsatisfied with the rule because it leads to the wrong rule for introducing existential quantifiers under the scope of a universal quantifier:

> (4) But difficulty arises when it is a question of getting an (Ey) within an (x).
>
> e.g. how do we get to $(x,y)\{x>y(Ez)x=y+z\}$ [$\forall x,y(x>y\supset\exists z(x=y+z))$]?
>
> [N.B. definition of $x>y$ is $x<1$ is false $x<y+1 :=: x<y \vee x=y$]
>
> This raises the general difficulty of introducing descriptive functions. On the old view the descriptive function was defined by means of E, on the new E is defined or derived from the descriptive function (Ramsey, [1929] 1991c, 198).

The issue is that the previous rule would only allow one to infer $\exists y\forall x\phi(x,y)$ from $\forall x\phi(x,b)$, but he needs a rule for $\forall x\exists y\phi(x,y)$. He ties this to the problem of the relationship between descriptive functions and existential quantifiers. Since existential quantifiers are dispensable—they are abstracts for a witnessing proposition, then descriptive functions can no longer be defined through them. Instead, descriptive functions must define existential propositions. Taking Ramsey's example, I want to know whether there is a difference between two positive integers. Answering this question requires the introduction of an existential quantifier over a descriptive function "the difference of x and y". This necessitates an adequate definition for "the difference of x and y", which as Ramsey notes cannot utilize an existential quantifier. So the existential quantifier will in part be defined by the properly constructed descriptive functions.[13]

Ramsey then considers Skolem's solution to introducing existential quantifiers under the scope of universal quantifiers and some associated problems with that solution:

> (5) Skolem's method is to limit the scope of z in the above to the numbers 1 to x, and defined $E_1^x z$ by recursion.
>
> (6) The disadvantage of this method is that it does not really pave the way to an introduction of E in the way we want to use it without explicit indication of scope. We should still have to turn the scope into an apparent variable with a new E.
>
> (7) Secondly Skolem defines $x - y = z$ to mean $x + y = z$.
>
> (8) The disadvantage of this is that it doesn't in the least make clear under what conditions $x - y$ is a one valued descriptive function, and so does not make any preparation for the use of a variable descriptive function which is essential in analysis (Ramsey, [1929] 1991c, 198).

Ramsey outlines Skolem's solution from the latter's article on primitive recursive arithmetic. The trick Skolem employs there is to use a bounded quantifier over a recursively defined

[13]This fits Majer's concern. Majer believes that Ramsey wants to justify laws of the form $\forall x \exists y R$ such as "all children have a father". However, Ramsey's follow on remarks about descriptive functions indicate that what's at issue here is much broader. The new account of existential propositions has to result in a drastically changed treatment of descriptive functions. Descriptive functions are the masonry of language: they are used everywhere in constructing anything more than basic sentences in all subjects. So a proposed logic must have a way to treat them. For Russell, this is done through the use of incomplete symbols and the definite description operator. He takes the existential quantifier to be more basic than Ramsey, and as

formula. Ramsey's complaints are that this is not what is desired because an existential quantifier should be an unbounded quantifier[14], and that it does not define $x - y$ correctly. The alternative Ramsey considers is to adopt Weyl's recommendation and require the introduction of an unbounded quantifier through the specification of the correct definition for $x - y$: "(9) the proper method seems to be Weyl's (MZ vol. 10 1921 p. 60)" (Ramsey, [1929] 1991c, 198).

It is at this point that Ramsey provides a rough characterization of what counts as an adequate definition of a descriptive function. He describes the function Weyl defines for the difference between two positive integers:

> He defines $x - y$ as meaningless if $x \leq y$
>
> $y + 1 - y$ to be 1
>
> $x + 1 - y$ to be $(x - y) + 1$
>
> $x - y$ is thus defined uniquely for $x > y$ and we have
>
> $(x, y)(x > y \rightarrow x = y + (x - y))$
>
> $x - y$ taken as $f(x, y)$ may be replaced by Ez so we have
>
> $(x, y)(x > y(Ez)x = y + z)[(x, y)(x > y \rightarrow (Ez)x = y + z)]$

Ramsey mentions, defines the descriptive functions through them. But if existential propositions are not proper propositions, then one cannot define descriptive functions through them.

[14] Skolem mentions this problem in his postscript (Skolem, [1923] 1967, 333).

(Ramsey, 1991a, 198–199).

Ramsey references a proposed function that Weyl does not explicitly construct.[15] He then claims that the offered function is sufficient to entail the right universal proposition. I want to make a couple of observations that I will expand on in the following section. First, a definition of the descriptive function has to specify when that function is meaningless. In this case, because the numbers are positive integers (not including zero), negative numbers do not exist so $x-y$ is undefined if x is less than or equal to y. The upshot is that constructed functions have to specify when they are meaningful. Second, the descriptive function appeals to another well-defined function. Here that is the successor function (taking $x+1$ to just be $S(x)$). Such a function must also be well-defined (in the case of the successor, it will be just taken as a primitive). Consequently, constructed functions have to be defined through other well-defined functions or else be taken as primitive. Third, the definition in terms of another well-defined function must be proper in some sense. What is interesting about Ramsey's definition and his criticism of Skolem is that the definition he provides is recursive: it appeals to a base and recurrent case for constructing the necessary function. I will have more to say about this later, but it is important to observe at this point that not only should the function be either primitive or appeal to another more primitive function, but it also has to appeal to the other function in the correct manner. Fourth, a defined function requires functionality or uniqueness. Descriptive functions are functions. The definition has to provide a unique output for every input. Here every difference is uniquely defined for $x > y$. This is the "the" in "the difference between x and y". Ramsey solves the uniqueness

[15]Weyl does not in fact mention this in the passage that Ramsey cites. The section Ramsey seems to be referring to is a passage for properly introducing a subtraction function between natural numbers:

> Finally, we also wish to admit those cases where the function is not defined for all possible argument values; in this case we shall speak of a *scattered [zerstreut] sequence*. This is a law that from each number generates a number or nothing. For example, $n-5$ is generated from n by the law that the numbers from 1 to 5 do not generate anything, whereas all the other generate a particular number according to the rule: $6-5=1; n'-5=(n-5)'$ (Weyl, [1921] 1997, 102).

Here the n' is the successor function. The key claim Ramsey is referring to is "each number generates a number or nothing" which specifies the form of the rule Ramsey is interested in.

claim of definite descriptions through appeal to the underlying descriptive function. The definition thus has to respect that.

In summary, the constraints on a constructed function for Ramsey include meaningfulness, defined through proper functions, have a proper definition, and functionality. First, when constructing a function, the construction must specify when propositions involving the function are meaningful. Second, the constructed function must be defined through other well-defined functions or be in the same category as the successor function. Third, the definition must be proper in some important sense. Fourth, the construction must ensure that the function is a function, i.e. it always has a unique output for every input. Such a constructed function will provide a good definition of a descriptive function.

Ramsey argues that one can then use these well-defined descriptive functions to introduce existential quantifiers under the scope of a universal quantifier. So long as that universal quantifier applies toward a properly defined function, the existential can be introduced. He generalizes this to the rule:

> (10) The formula for this process is
>
> $(x, y)\{\phi(x, y) \rightarrow \psi(x, y, f(x, y))\}$
>
> to $(x, y)\{\phi(x, y) \rightarrow (Ez)\psi(x, y, z)\}$.
>
> (11) We can bring it under the simpler form
>
> $(x, y)\psi\{x, y, f(x, y)\}$
>
> to $(x, y)(Ez)\psi(x, y, z)$
>
> If we agree that $x - y$ shall have some definite meaning, say 1, when $x \leq y$ but only use it when $x > y$ (Ramsey, [1929] 1991c, 199).

The idea is something like reverse Skolemization. Instead of eliminating the existential quantifier for an arbitrary function, Ramsey says Weyl's method is to only introduce the existential when such a function has been constructed. The important difference between Skolem and Weyl here is that the function $f(x, y)$ is unbounded due to it being under the scope of the unbounded (x, y), while Skolem kept it limited. That is Weyl's trick and effectively what Ramsey thinks to be the correct move. The existential quantifier effectively captures the unboundedness of the universal quantifier it is introduced under. The universal quantifier justifies the introduction of an unbounded existential quantifier.

After this point, Ramsey continues on to elaborate the proposed logic. For my purposes here, I will ignore the remainder of the document.

Four things should be noted. First, Ramsey thinks that existential quantifiers can be eliminated but they are kept around as a convenience. Second, the rules that govern their use cannot be naive introduction rules because of the difficulty of introducing them under the scope of a universal quantifier. Third, the existential quantifier for its proper introduction under a universal quantifier requires the specification of the correct kind of function. Fourth, that constructed function enables the introduced existential quantifier to inherit the unboundedness of the surrounding universal quantifier. These facts together illustrate the importance of the universal quantifier and constructed functions to the meaning of the existential quantifier. Both enable the "true", unbounded existential generalization.

But what are these constructed functions? Where do they come from, and what are they ontologically in Ramsey's philosophy of mathematics and philosophy of science? One answer is that they are sets of ordered pairs as ordinarily viewed in set theory. However, this is unsatisfactory because sets and ordered pairs are not ontological primitives for Ramsey: there are no sets and ordered pairs in the world. Ramsey has abandoned his Platonism from "Foundations of Mathematics" (see Sahlin's introduction and chapter six in Sahlin, 1990). So some other characterization needs to be given for these constructed functions. I turn to that now.

6.3.2 Ramsey's Constructed Functions

I need to say more precisely what are the constructed functions and what is their relationship with the universal quantifier. The key thing I argue here is that constructed functions are really just well-defined constellations of particular and universal propositions. The latter are just "rules" or "habits" however that is cashed out. The upshot is that existential propositions when unbounded are really abstracts of some collection of variable hypotheticals. Constructed functions are eliminable for talk of habits.

Recall that the four criteria constructed functions must satisfy are 1) the definition of the constructed function must specify when it is "meaningful", 2) the definition must appeal to other well-defined constructed functions, whether primitive or well-built, 3) the definition must be proper in some sense, and 4) the definition must provide a function, i.e. a relation that specifies a unique output for every input.

Each of these criteria is fairly straightforward, except for criterion three. "Proper" or "well-defined" needs to be specified. Ramsey is concerned with some sense of "well-defined" because he criticizes Skolem's definition as improper in some sense.[16] Part of his praise of Weyl's method for introducing existential quantifiers is that Weyl provides the correct sort of function to allow the introduction of an existential under the scope of a universal. A closer examination of Ramsey's definition explains what "well-defined" might mean in criterion three.

An essential feature of the definition Ramsey ascribes to Weyl is that it is recursive. Ramsey provides both a base case and a recurrent case for the behavior of the function. The base case is that adding a number to one minus that number yields one. The recurrent case is that adding one number to one and then subtracting another is just the same as subtracting the latter from the former:

[16] His critique is that Skolem provides the wrong definition and fails to identify when $x - y$ is one-valued

Base Case: $\forall y(y + 1 - y = 1)$

Recurrent Case: $\forall x \forall y(x + 1 - y = (x - y) + 1)$

Ramsey adds the condition that for any x, y, these cases hold if $x > y$. Importantly, the structure of this definition allows the following law:

$$\forall x \forall y(x > y \supset x = y + (x - y))$$

This law allows the introduction of the existential quantifier to replace $(x-y)$—an existential quantifier that would then be under the scope of $\forall x \forall y$. Without this structure, no proof can be given.

To see how, recall the facts that if $x > y$ then there is some n such that $x = y + n$ and that $x + y = x + z$ implies $y = z$. Then one can show the desired law by supposing that $x > y$:

or makes it of use for analysis.

$$
\begin{aligned}
x + x &= x + x \\
&= y + n + x \\
&= y + \sum_{i=1}^{n} 1 + x \\
&= y + \sum_{i=1}^{n} (y + 1 - y) + x \\
&= y + \sum_{i=1}^{n} y + \sum_{i=1}^{n} (1 - y) + x \\
&= y + \sum_{i=1}^{n} y + \sum_{i=1}^{n-1} (1 - y) + x + 1 - y \\
&= y + \sum_{i=1}^{n} y + \sum_{i=1}^{n-1} (1 - y) + (x - y) + 1 \\
&= y + \sum_{i=1}^{n} y + \sum_{i=1}^{n-1} (1 - y) + (x - y) + y + 1 - y \\
&= y + \sum_{i=1}^{n} y + \sum_{i=1}^{n} (1 - y) + (x - y) + y \\
&= y + (x - y) + y + \sum_{i=1}^{n} (y + 1 - y) \\
&= y + (x - y) + y + \sum_{i=1}^{n} 1 \\
&= y + (x - y) + y + n \\
&= x + y + (x - y) \\
&\Rightarrow x = y + (x - y)
\end{aligned}
$$

So the desired law follows straightforwardly from Ramsey's definition of $x - y$. This means that the definition of $x - y$, with crucially its base and recurrent cases, is sufficient in its totality to directly prove the required law necessary for introducing the existential quantifier under the scope of the universal.

It is worth pausing and reflecting on how it is the universal propositions given in the base and recurrent cases for $x - y$ that yield the entire content of the desired law. *The function is merely a notation for a series of universal propositions in a recursive definition that provides the content of the function.* An existential claim about the function $x - y$ is only warranted if the witness that is these laws is had in hand. Those universal propositions do all the work. Here then is an explication of what "well-defined" or "proper" might mean in the third criterion for specifying constructed functions: the definition of the function must appeal only to either particular propositions or universal propositions in defining the function.

One remaining question is whether the definition must be recursive. Ramsey's example is recursive. So it stands to reason that the way particular and universal propositions are used in defining a function must be in a recursive format. I do not know if this is true. Ramsey's example suggests it. But a natural worry is that such a restriction eliminates many non-recursive definitions like the construction of the reals in Dedekind cuts. While he does not need these definitions in the kind of scientific theories he discusses,[17] most of his other philosophical projects like his decision theory require irrational numbers and more.[18] If those are admitted, one might wonder what restriction the "well-defined" criterion puts on the sort of functions that are constructed. It might be too thin.

If Ramsey's suggestion here is that the particular propositions and universal propositions in the definition must recursively define the appropriate function, then he faces a series of choices. He can recover most of mathematics if he admits otherwise non-recursive functions like Dedekind cuts as part of the primitive rules or habits people come equipped with. This seems ad hoc. Another possibility is that he was okay with eliminating most of the mathematics. This means that Ramsey is guilty of the elimination objection, a weird outcome given his earlier professed desire to save mathematics from the Bolshevism of Brouwer and

[17] The constructed functions he relies upon in "Theories" all involve just the integers.

[18] Ramsey indicates in "Theories" that reals might be needed in some scientific theories. He writes that "If, however, our primary system is already a secondary system from some other theory, real numbers may well occur" (Ramsey, [1929] 1990m, 114).

kind. Finally, Ramsey may have genuinely thought that every function could be sufficiently described or tested with recursive functions. The idea is that while perhaps not every function is recursive, the entirety of the relevant behavior for human concerns of non-recursive functions can be captured by other functions that are recursive. This last hypothesis has two appealing features: Ramsey was trying to solve the decision problem in his last year or so of life, which means that he thought some recursive characterization of mathematics is possible, and Ramsey seems to have been influenced by Wittgenstein who had something like the view that recursion was sufficient for characterizing mathematics.

I characterize this as the "two misfortunes argument" for why Ramsey might have held that properly constructed functions must be recursive. Two things make this plausible. First, Ramsey spent the last two years of his life trying to develop a solution to the decision problem. His paper "On a Problem of Formal Logic" shows that some fragment of first-order logic is decidable, which he proves with the help of what is now called Ramsey's theorem. Perhaps inspired by this success, Ramsey thought the decision problem solvable; his misfortune then is that he died only a few years before the problem he devoted considerable mathematical energy towards turned out to be unsolvable. As Mellor succinctly describes the situation, "Ramsey's enduring fame in mathematics, which was his job, rests on a theorem he didn't need, proved in the course of trying to do something we now know can't be done!" (Mellor, 1983, 12). Second, Ramsey spent the last year of his life engaged in a series of extensive discussions with Wittgenstein. These discussions coincided with his shift in views on the philosophy of logic and Ramsey's renewed interest in Brouwer and Weyl. Importantly, Wittgenstein held a view at the time that many objects built in non-constructive manners, like the reals, could be still properly characterized by recursive laws (see Rodych, 2018 sections 2.4 and 2.5). It is probable these ideas influenced Ramsey; his misfortune here is that Wittgenstein is just wrong about the mathematical details. Thus if Ramsey did think that "well-defined" functions must involve recursive definitions, it is likely he thought this because of the twin misfortunes of early death and Wittgenstein's company.

Summarizing, Ramsey seems to have thought that constructed functions must involve particular propositions and universal propositions to enable them to entail laws whose universal quantifiers allow the introduction of an existential in their scope. His example suggests he thought those particular and universal propositions must form a recursive definition. However, this makes Ramsey's proposal mathematically untenable without some serious compromise. I think he may have made this step because he suffered from twin misfortunes that led him to these ideas.

So the answer then is that ultimately rules and habits underpin existential quantifiers in the unbounded case through the role those rules and habits play in articulating placeholders people call functions. One final question then is what are rules or habits? Ramsey never defines these things except by perhaps explaining them in terms of brain traces. I do not have a good answer. The best candidate if the claim that functions must be recursively defined is true is that rules or habits are ultimately mechanical procedures or algorithms. However, this is largely speculation on my part since Ramsey never says what these things are except to gesture at their obviousness.

There is now enough information to now construct a picture of how Ramsey understands existential propositions. Existential propositions are descriptions of some witnessing propositions. What is a description? Ramsey's comments elsewhere make clear that a description is something like a disjunction of finite terms: "I verily believe that at last I *see* that they [existential propositions] are disjunctions of a finite number of terms" (FPRP Existential Judgment, 1). Such a disjunction must include the witnessing proposition. Where those witnesses are singular propositions, the existential proposition is a disjunction with that singular proposition as one member of the disjunct. For example, if Fa is the witness and the available names are a, b, c, then the existential proposition $\exists Fx$ as a description is $Fa \vee Fb \vee Fc$. This makes introducing an existential proposition something like introducing a disjunction. However, it should be emphasized that the witness is what does the work here;

the introduced disjunction is only true because the witness is true. For example, $Fa \vee Fb \vee Fc$ is warranted only due to Fa having already been warranted. So an existential proposition is really nothing without its witness.

What about witnesses that are not singular propositions where the existential quantifier can be thought of as unbounded, i.e. cases like $\forall x \exists y \phi(x, y)$? Here things become interesting. The case Ramsey explicitly considers in connection to Skolem in Weyl are witnesses given by universal propositions and constructed functions. Recall that universal propositions are not proper propositions but really chances at unity; they are "rules for judging" used as cheques for drafting proper propositions. The obvious question is: when can I existentially generalize the drafted propositions? Ramsey's apparent answer is when there is a constructed function that enables the draft to act as a witness for the cheque. But a constructed function is just a bundle of particular and universal propositions that entail the cheque law. Those propositions, sometimes described as a definition, do all the work; they let the cheque law draft when needed and provide the necessary liquidity, i.e. an unbounded witness, for the unbounded existential quantifier. For example, the cheque law "every man has a father" relies upon a definition of fatherhood that appeals to other laws and some base case for how fatherhood is related between generations. That definition with its laws allows proper, unbounded existential quantification in the cheque law—they underpin the inference ticket that if x is a man, then x has a father. So ultimately the witness here is the dictionary, which can be captured by a second-order quantifier over the relations in the dictionary.[19] Importantly, an upshot of Ramsey's rules for the introduction of the existential quantifier is that only existential propositions involving the universal propositions in a dictionary count as unbounded. So this means that the second-order existential quantifier is tightly connected

[19] Astute observers will note that this basically is the Ramsey sentence. The view here is also very close to the one defended by Majer (Majer, 1989). I discuss below how this view is similar to and how it differs from Majer's account of the Ramsey sentence. In short, Majer correctly identifies many features of Ramsey's account but misunderstands the epistemic role of the Ramsey sentence: Majer thinks the Ramsey sentence justifies laws of the form $\forall x \exists y F$. I will say at this point that if my account is right, the Ramsey sentence justifies *nothing* because it is a description and can be dispensed with.

with unbounded existential quantification at the first-order level.

It is worth reiterating at this point that the "meaning" or correctness conditions of an existential proposition is driven entirely by its witness. In the case of singular propositions, this straightforwardly makes existential propositions nothing but a receipt for a proposition that one can always produce. However, within the scope of universal propositions, existential propositions become more interesting. They are receipts for rules and functions constructed from those rules. This allows the existential proposition to range over a non-finite domain of values. This has ramifications for Ramsey's views on the infinite.

6.3.3 Existential Propositions and the Infinite

There has been some discussion of whether Ramsey is a finitist. Holton and Price make this claim based on some remarks from Ramsey's notes (Holton and Price, 2003, 332–333). Sahlin cites Ramsey from "General Propositions and Causality" to infer he became a finitist (Sahlin, 1990, 176–177). However, no one has suggested a close connection between Ramsey's apparent finitism and his view on the existential quantifier. I want to argue for that here. This is important because it explains why Ramsey changed his mind about existential propositions.

Ramsey considers existential propositions to be descriptions of other propositions. My previous discussion showed that with proper propositions, this makes an existential proposition really a finite disjunction that includes that proper proposition. Existential propositions can also describe universal propositions, which are really not proper propositions but chances at unity. It is this latter case that connects Ramsey's view of the existential quantifier with his views on the infinite.

The key observation is that existential generalization within the scope of universal propositions can be a substitute for an infinite collection. Ramsey notes that Skolem's solution to introducing existential quantifiers under the scope of universals only results in bounded quantification; however, a true existential should be unbounded—range over a possibly infinite set—and unbounded existentials follow from Weyl's procedure, which Ramsey adopts. Because such an existential is unbounded, the equivalent statement of the second-order existential quantification about the property *yields an effectively infinite set*. For example, consider the successor rule "for every natural number, there exists a unique successor". This can be translated into first-order logic as:

$$\forall x(Nx \supset \exists y(Syx \land \forall z(Szx \supset z = x))) \tag{6.1}$$

where Nx stands for "x is a natural number" and where Syx stands for "y is the successor of x". The existential quantifier here is warranted because there exists a well-defined function $s(x)$ that produces a successor. That is to say the following justifies the introduction of the existential quantifier:

$$\forall x(Nx \supset (Ss(x)x \land \forall z(Szx \supset z = s(x)))) \tag{6.2}$$

Importantly, equation (6.1) is equivalent with a second-order generalization:

$$\exists \phi(\forall x(Nx \supset \phi x)) \tag{6.3}$$

where ϕ is a propositional function that can be used to define the set of all unique successors. Recall that propositional functions at a base level go from individuals to propositions. Through abstraction, one can then define via equivalence classes over individual numbers the set of successors where one and only one member of ϕ's range is true of each successor. This defined set is effectively infinite because both the rules that characterize successor, as well as the law where the existential quantifier is introduced, are unbounded: it applies indefinitely to any "object" one might encounter.[20] So existentially propositions whose witnesses are drafted from universal propositions can be used to describe infinite sets.

Infinite collections such as the set of unique successors, however, are entirely fictitious. These collections are the existential generalizations of universal propositions. Those universal propositions *are not truth-apt*—they are chances or rules for setting credences; they are compatible with every factual proposition, and so they are not truth-functions of real propositions. An infinite collection is really just a rule. It is a property of an agent's beliefs; it is a reification of an agent's habits. Consequently, this sort of infinite is really a fiction that guides action.[21]

The account of the infinite here stands in stark contrast to Ramsey's earlier view. It is a rejection of the high Platonism that characterizes "Foundations of Mathematics". But it is not a form of intuitionism or Hilbert's formalism. Ramsey's rules are habits that generate actions, and they are not phenomenal proofs like in Brouwer or Hilbert. Furthermore, Ramsey's view is not a species of strong finitism—as I argue below, Ramsey maintains the importance and utility of the infinite in mathematics. So the characterizations of it given by

[20]Crucially, the abstraction here would not be naive abstraction. Two things push against the abstraction being naive. First, the antecedent condition in the law and in the definition of the constructed function restrict the individuals that the propositional function ϕx has in its domain. Second, the reliance on another well-defined constructed function (here successor)—itself is given by some laws—restricts the domain of the individuals further.

[21]It is unclear whether Ramsey would conclude that there are no infinities. The type gestured to here might be characterized as the mathematical sort. But there might be infinite collections in the real world. I do not think Ramsey would foreclose that possibility; he would only insist that is an empirical fact to be discovered. It is not something humans should presuppose.

Holton, Price, and Sahlin are inaccurate.

As I mentioned previously, Ramsey's view of the quantifiers seems to have been influenced by and have some interesting overlap with Wittgenstein's views from 1929–1933. In 1929, the two were in constant collaboration, and parts of the *Philosophical Remarks* almost surely originated in conversations with Ramsey. Both are fictionalists about infinite sets and both hold that the laws of mathematics are what people mistake for the infinite (see Rodych, 2018 section 2.2). But Ramsey differs from Wittgenstein in still permitting unbounded quantification.[22] Furthermore, Ramsey has a richer view of mathematics as more than just syntax; he thinks of mathematical laws as habits just like every other chance, which makes mathematics another part of how humans make decisions. Wittgenstein restricts mathematical laws to rules in a particular game or activity—there is no integration with the process of decision-making. The upshot is that Ramsey's philosophy of mathematics is novel and unique from his time period.

So is Ramsey a finitist? The answer is complicated. Ultimately, Ramsey is a fictionalist about the infinite sets in mathematics; those sets are artifacts of how existential propositions can generalize universal propositions, and universal propositions are not proper propositions. So Ramsey is a finitist of the sort that thinks there are no infinite collections: "So too there may be an infinite totality, but what seem to be propositions about it are again variable hypotheticals and 'infinite collection' is really nonsense" (Ramsey, [1929] 1990e, 160). However, he thinks universal propositions *are essential to human cognition*, and thus so are the fictions of infinite sets. Fictions may lack sense, but they may still be important in cognition for non-epistemic reasons. Mathematical infinities cannot be dispensed with; they are too useful for humans. So Ramsey is not a finitist in the sense of eliminating the mathematics of the infinite. He does not want to be thrown out of Cantor's paradise.

In summary, Ramsey's view of the existential quantifier leads him to be a fictionalist about

[22]Wittgenstein only allows the restricted quantification that Ramsey criticizes Skolem for using. See

infinite collections while maintaining a method to understand those collections and tie them to human cognitions. He rejects abandoning the infinite in mathematics. Furthermore, this picture of how to interpret the infinite can explain why Ramsey changed his mind about existential quantifiers.

6.3.4 Ramsey's Change of Mind

I can now address why Ramsey changed his mind about existential quantifiers. In short, he altered his view about existentials because he had changed his mind about universal propositions. His anti-realism about existential propositions flows from his anti-realism of universal propositions.

Ramsey came to view universal propositions as chances, which made them no longer truth-functions of ordinary, singular propositions. It forced a change in how he thought about the infinite; he found that he could characterize supposed infinite collections through universal propositions. For example, the set of natural numbers can be best defined through the rules that consist of Peano's Axioms. So the rules are sufficient for the specification of these supposed infinite sets—including for every proper proposition that applies to the members of that supposed set.

This change in view about universal propositions suggests the following reasons why existential propositions may no longer be considered infinite logical sums. While Ramsey does not state these reasons explicitly, the contour of how he likely may have thought can be seen.

The key consideration is the role of constructed functions and the defining variable hypotheticals. Those variable hypothetical license laws that provide propositional functions that can define an infinite set. One can then effectively reduce that infinite set to just those variable hypotheticals in the constructed function's definitions. After all, they provide all the inferences needed for the members of that nonsensical set. So then why is an existential generalization as an infinite logical sum necessary? The rules provide *everything I would ever need about that set.* An infinite logical sum about that set's members is useless once I have the rules. So I might as well dispense with such a sum.

This is the key move. Logical sums provide nothing the rules in the constructed function's definition already do not yield. So an existential proposition interpreted as a logical sum reveals that the existential proposition is really contentless; it adds nothing its witness does not provide. Yes, when pressed to view its truth-conditions, I might treat it like some infinite disjunction. But I know that since I have the universal propositions in hand that this is merely supposed book-keeping: the universals already warrant some instance of that supposed disjunction. The work is already done.

The fate of existential propositions in Ramsey's thinking can be thought of as analogous to the fate of chances in De Finetti's thinking. De Finetti's theorem shows that a probability function that is exchangeable acts as if there are chances; but no chance needs to be posited since exchangeability is satisfactory for the behavior of that probability function. Likewise, universal propositions can account for every proposition that existential propositions supposedly provide. Universal propositions through definitions function like the exchangeability of probabilities for existentials as chances. One is sufficient for the elimination of the other.

Summarizing, Ramsey's mind changed about existential propositions as logical sums because he figured out an independent way to capture the mathematical and logical facts those sums seemed to provide. Since universal propositions in definitions and individual, singular propositions do all the work for existentials, there is no need to treat existentials as truth-functional. They too are not proper propositions.

Rodych, 2018 sections 2.2 and 2.3.

6.3.5 Ramsey's Objections

Ramsey's later account of existential quantifiers has much in common with Weyl's account. It is an open question if it suffers from the same problems Ramsey discussed in "Mathematical Logic". Earlier, I enumerated the objections there: the elimination, presupposition, ordinary object, and arithmetic objections. I argue here that Ramsey's new account avoids those objections for the same reasons his new account of universal propositions avoids those objections.

I review the aforementioned objections first. The elimination objection against Weyl and Hilbert's accounts of the quantifiers is that those accounts would eliminate everything but elementary arithmetic from mathematics; the presupposition objection holds that the utility of an existential proposition comes from the generality presupposed in the proposition and not merely from the witness; the ordinary object objection states that ordinary objects conceal general propositions that cannot be eliminated; and the arithmetic objection claims any proposition involving cardinality conceals generality. What is interesting about these objections is that they apply more strongly against existential propositions as opposed to universal propositions. In particular, the presupposition objection is most directed at existential propositions. So while I argued previously that they do not afflict Ramsey's new account of universal propositions, they may have more sting with existential propositions.

The new view of existential propositions invests their entire correctness conditions in the witnessing proposition. An existential proposition is a finite disjunction of some propositions that contain the witness. There are two cases to consider. The first is when the witness is a singular proposition, and the second is when the witness is the bundle of universal propositions in a function's definition. I check each case, in turn, to see how that existential proposition fares against the listed objections.

Starting with an existential proposition whose witness is a singular proposition, the primary objection to worry about is the presupposition objection. An existential proposition whose witness is a singular proposition is exactly the case that worries Ramsey. If I am a student at UCI and from that I infer there is a student at UCI, is that premise proposition sufficient to understand its conclusion? I can consider a domain of students; does that domain need to be infinite? With students, the answer has to be no because I in fact can only consider finitely many individuals. This is not a general proposition, but it really is a disjunction of finite many individuals that could be students. This insight Ramsey leverages in differentiating variables hypotheticals from conjunctions when he writes that propositions like "Everyone in Cambridge voted" are really conjunctions (Ramsey, [1929] 1990e, 145). So there is no real generality to presuppose here.[23]

The other objections do no damage to existentials as descriptions of singular, propositions. These cases are all really finite disjunctions. They apply easily to ordinary objects as well as to elementary arithmetic statements as Ramsey understands them in "Mathematical Logic". "Two dogs barked at the cars" becomes a conjunction of finite disjunctions plus a general proposition. So the case of existential propositions whose witness is a singular proposition is not problematic.

Moving onto existential propositions that have a constructed function's definitions as a witness, each objection can be dismissed in the same manner they were dismissed with universal propositions. Existentials nor their witnessing universal propositions should be eliminated due to their convenience and essential role in human thought. Ramsey just wants to argue that the definition that witnesses for the existential is what is doing the work mathematically and decision-theoretically. In the case of presuppositions, his account documents how an existential proposition presupposes generality with the witnessing universal proposition. Ordinary objects that involve variable hypotheticals, like "arsenic is poisonous", still retain

[23] One might wonder about mathematical singular propositions like "there is an even number". However, this clearly falls in the same case as singular, theoretical propositions where one is really considering some mathematical theory.

those variable hypotheticals when they are proclaimed by an existential proposition. And arithmetic is completely recovered in the singular proposition case and in the case where one wants to be more general. So every objection is met for the same reason they are met with universal propositions because in this case, the witnessing universal proposition is okay.

In summary, Ramsey's proposed account of existential quantifiers avoids the objections he levies at Weyl. It avoids them because Ramsey's account of universal propositions is different from Weyl's, and Ramsey's view of universal propositions is seen as non-eliminable for human thought. Weyl, as Ramsey reads him, places universal propositions as an auxiliary tool with limited applicability. Ramsey does not go that route. Since existential propositions inherit their generality from universal propositions, their generality is safe. So Ramsey can avoid the problems he thought plagued Weyl and Hilbert.

6.4 The Anti-Realism Criterion

I have an account of existential propositions. My goal is to show how this account can be applied to the Ramsey sentence and to demonstrate how the goal satisfies the anti-realism criterion. Recall the anti-realism criterion holds that the existential quantifier must fail to act as it is traditionally understood through quantifying over objects in the world separate from the agent. So I must show how this account of the existential quantifier and existential propositions avoids being about stuff in the world separate from the agent.

Here is a brief plan. I briefly review the chance example developed in a prior chapter. Then I use it to illustrate some important points about the Ramsey sentence. This will show how the existential quantifier in the Ramsey sentence is anti-realistic.

It would be useful to review the chance example. In that example, the observation language consists of individual coin flips. Those coin toss outcomes are reflected in the proposition

$\phi(n)$, which when equal to 1 means the coin at time n landed heads, and when equal to 0 means it landed tails. The theoretical language has a vocabulary of $\beta(n)$ and $\gamma(n)$, which represent the value of the coin's bias and the outcome of a random sample. These functions have their values restricted by a set of axioms. And they are connected to the outcomes of the coin tosses by a dictionary that uses the inverse cumulative distribution function of a chance distribution.

An important observation about the proposed account of existential propositions is that unbounded existential propositions only occur under the scope of a universal quantifier. This captures when an existential proposition generalizes a universal proposition. Examining this model, is there any constructed function applied in a formula that involves an existential quantifier under the scope of a universal quantifier? The answer is in the dictionary and axioms. This is also precisely when the new theoretical functions have their behavior specified. Each definition in the dictionary specifies the primary system propositions as a function of time and the axioms specify the theoretical functions also as a function of time. For example, with the chance example the definition of $\phi(n)$ is:

$$\phi(n) = F_{\Phi}^{-1}(\gamma(n); 1, \beta(n))$$

$\phi(n)$ is a function of the inverse cumulative distribution function on $\gamma(n)$ and $\beta(n)$ arguments. While this is called a dictionary, note that it is also a law: the n is implicitly universally quantified. The functions $\gamma(n)$ and $\beta(n)$ fit the description of being functions on n. Together with the axioms, these can be used to provide proper definitions. The axioms are restrictions on the values that γ and β can assume. They are taken to be properly defined since β is just the function that outputs either 0.6 or 0.4 on any input, and γ always returns a unique real on the unit interval for any input. Together with the dictionary, they then provide a characterization of the theoretical functions in terms of the observational functions. Taking Ψ to be the dictionary equality plus the axioms, I can go from:

$$\forall n \Psi(\gamma(n), 1, \beta(n))$$

to the law:

$$\forall n \exists m \exists p \Psi(m, 1, p) \tag{6.4}$$

This fits what Ramsey says about introducing the existential quantifier. The constructed functions here are just γ and β. Consequently, the dictionary warrants the introduction of an unbounded existential quantifier.

The same applies to Ramsey's own toy example. Consider his $\chi(n)$, which specifies whether one feel's one's eyes opening, closing, or doing nothing. Then letting Φ be $\chi(n) = \gamma(n) - \gamma(n-1) \wedge \gamma(n) = 0 \vee \gamma(n) = 1$ (recall γ in this context says whether one's eyes are open or closed) one goes from:

$$\forall n \Phi(\gamma(n), \gamma(n-1))$$

to the law:

$$\forall n \exists m \exists p \Phi(m, p) \tag{6.5}$$

which again fits what Ramsey says about introducing the existential quantifier. So here the constructed function γ plays the same role as the constructed functions γ and β from the chance example. Again, the axioms are just a restriction on the values of γ and Ramsey's other theoretical functions. Consequently, just as the chance toy example has the dictionary warrant the introduction of an unbounded existential quantifier, the dictionary in Ramsey's toy example does the same.

The presence of the existential proposition under the scope of a universal quantifier signifies the existential generalization of the variable hypothetical expressed by the dictionary and axioms. In the case of the chance example, this means that equation (6.4) allows:

$$\exists \Psi' \forall n \Psi'(n, 1)$$

and similarly Ramsey's example becomes:

$$\exists \Phi' \forall n \Phi'(n)$$

These are the Ramsey sentences for their perspective theories (or fragment in the case of Ramsey's example). Note that the Ramsey sentence here fails to justify the dictionary and axioms: the conjunction of the dictionary and axioms is the witness for the Ramsey sentence.

So the Ramsey sentence is merely a description of the rules given by the dictionary.

What is the point of the Ramsey sentence if it does not justify the dictionary definitions? The anti-realism here about the existential quantifier is that it is merely a description or abstract of real propositions. The "real" propositions are the universal propositions that are the axioms and the dictionary.[24] So the question could be reformulated as Why would one want a description or abstract of the axioms and the dictionary?

A clue for answering this question comes in Ramsey's comment that the theoretical functions of the Ramsey sentence are meant to be taken extensionally. Recall that the extension of those functions is just the set of partitions that happen to have volume structures in agreement with one another. In the case of the chance example, these partitions are induced by the constructed functions γ and β plus the dictionary and the axioms. Since each constructed function leads to a different dictionary, one might say that the partitions each correspond to different dictionaries on possibility space. If Abe constructs a different $\gamma(n)$ from Betty's $\gamma(n)$ (for example, Abe's might be $\gamma(1) = 0.3$ while Betty's might be $\gamma(1) = 0.5$), then technically they have different dictionaries due to the functions being different. Corresponding to each of these different functions will be different compatible partitions. But importantly, those compatible partitions will have agreement with the volumes in Abe and Betty's algebras.

To connect this back to the anti-realism of the Ramsey sentence, note that a description of a proposition *can describe multiple propositions*. For example, "someone is a university student" could describe the proposition that I am a university student at present, that any of my graduate student colleagues are university students, or that any arbitrary student at my school or at any school is a student, and so on. This might be the reason why Ramsey writes elsewhere that "I verily believe that at last I *see* that they [existential judgments]

[24]Astute observers will note that these are not real propositions either since universal propositions are not truth-apt too. Hence Ramsey's comment at the end of "General Propositions and Causality" that "Of course the theoretical system is all like a variable hypothetical in being there just to be deduced from; and a law in the theoretical system is at two removes from deduction" (Ramsey, [1929] 1990e, 162).

are disjunctions of a finite number of terms" (FPRP Existential Judgment, 1). Existential propositions are eliminable for disjunctions of proper propositions that include at least one proposition that warrants the existential. If I apply this observation plus the known facts about the extension of the Ramsey sentence theoretical functions, then I have the conclusion that the Ramsey sentence is really a disjunction of different dictionaries plus axioms. Each dictionary corresponds to the different constructed functions $\gamma(n)$ and $\beta(n)$ found within the scope of the universal quantifiers. The respective dictionary then induces the particular compatible partition given by the theoretical functions.

The upshot is that the Ramsey sentence describes the different rules the agent might use to configure his algebra in a way that respects the volumes given in every similar algebra. Its utility is that it maps out the different various ways an agent might objectify his rules for finding credences. This is extremely anti-realistic: no proposition about the world (apart from the agent perhaps) is quantified over. The Ramsey sentence merely describes how the agent configures his algebra to reflect the various variable hypotheticals that the agent in fact employs.

In summary, I have an account now that satisfies the anti-realism criterion. The Ramsey sentence is not a proper proposition but a description of one of the various axioms and dictionaries the agent may have built. That dictionary induces the right compatible partition on possibility space that is a member of the extension of the theoretical functions. It shows the different objectifications that all agree on those properties an agent may adopt. The Ramsey sentence is not about the world but about the agent. Thus, this account satisfies the anti-realism criterion.

6.5 Majer's Account of the Existential Quantifier

I have an account of existential propositions that is anti-realistic, and I have shown how that account fits with the Ramsey sentence. An important question that might linger is how my anti-realist account squares with the other anti-realist account in the secondary literature: Majer's intuitionistic interpretation (Majer, 1989). Majer's view is notable because it is the only approach that explicitly adopts an anti-realist interpretation of the existential quantifier. Thus it avoids many of the pitfalls that beleaguered views such as Braithwaite's and Psillos's.

In this section, I discuss Majer's view in depth. I compare it with my account, and I argue that it goes subtly wrong in the role it gives existential propositions. The problem with Majer's account is that it treats existential propositions as justificatory for laws.

What separates Majer's view from others is that he interprets Ramsey's existential quantifier as Weyl's existential quantifier. Majer thinks that Ramsey adopted an intuitionistic view of mathematics. And with it came a radically different view of the quantifiers. Namely, the existential quantifier cannot be negated because it only has an introduction rule: $f(a) \rightarrow \exists x f(x)$ (Majer, 1989, 245–246). There is no elimination rule, which does not allow the quantifier equivalence to hold.

Majer's reasons for thinking that Ramsey had converted to a form of intuitionism comes from the manuscript "Principles of Finitist Mathematics". I examined that manuscript previously. Majer is convinced by a single sentence in this manuscript that Ramsey adopted a "moderate" form of intuitionism that admits the law of excluded middle for propositional logic but denies the quantifier equivalences.[25]

[25] Majer writes:

> Because this [Ramsey's conversion] is by no means obvious from Ramsey's published writings—although I suggested it some years ago—let me quote from an unpublished manuscript entitled "Principles of Finitist Mathematics"" contained in the Frank Ramsey Collection in Pittsburgh. There Ramsey says: "The proper method seems to be Weyl's," referring to the solution of a problem that became crucial for Ramsey's conception of theories. In fact Ramsey's conception

This has implications for Ramsey's philosophy of science, which Majer correctly describes as non-reductionistic. Majer writes that Ramsey views scientific theories like laws, in that they are both theories and laws are not proper propositions. Theories are like laws in that they are not truth-apt. But theories are not identical to laws.[26] What differentiates them can be found in how certain laws are to be justified. Majer considers the case of the existential quantifier under the scope of a universal quantifier. He thinks these judgments are particularly important in scientific reasoning, but they are difficult to justify because of the presence of the universal quantifier.[27] For example, how does one justify the assertion that every child has a father? Since universal propositions are not judgments, they cannot be negated. So one cannot justify this law by fixing an arbitrary child and negating the law that no one is its father. The limitations of Ramsey's interpretation of the quantifier prevent this. Majer claims Weyl found a workaround for this problem, and Weyl's workaround is at

> of theories and theoretical terms simply emerged as a generalization of Weyl's procedure (Majer, 1989, 244).

The judgment is that when Ramsey says the "proper method seems to be Weyl's", Ramsey is referring to an account of the universal and existential quantifiers.

[26]Majer writes that

> The secondary system—like the variables hypothesis $\forall x f(x)$ of implication (1) [$\forall x f(x) \rightarrow f(a)$]—cannot be judged as true in itself, however it justifies, like the variable hypothesis in formula (1), the deduction of an infinite variety of primary propositions $\{f(a)\}$ called laws and consequences. This hypothetical-deductive feature puts theories and general sentences on a par: both are there just to be deduced from. To be sure, the similarity between the secondary systems and general sentences is not a complete one; otherwise it would turn that theories are *identical* with general sentences in the variable-hypothetical sense of formula (1)—but they are not! In addition to the common hypothetical-deductive feature there is an important difference, which distinguishes theories from general sentences (Majer, 1989, 246).

Theories and laws are very similar in that they are not judgments. However, they are dissimilar in how they are not judgments.

[27]Majer argues that we one cannot properly justify such laws because they do not follow easily from other universal propositions:

> But what happens when we come across a general existential sentence in which the order of the quantifiers is *reversed*, such as: "For every number m there exists a number n such that they stand in the relation $R'(m,n)$"? How is such a sentence justified? This can question can be reformulated as: From which general sentence, from which "variable hypothetical" is this general-existential sentence abstracted? And here we face a problem of utmost seriousness, because the answer in general is: *We don't know*! (Majer, 1989, 247)

He concludes that something must be found to justify such a sentence.

the core of Ramsey's conception of the Ramsey sentence. The Ramsey sentence justifies such laws. The idea is that theories are used to justify sentences of the form $\forall x \exists y F$. They do this by constructing laws that allow the creation of particular judgments. Those particular judgments can then justify the introduction of existential quantifiers under the scope of said laws. What is required is the existence of the constructing law. And that is exactly what the Ramsey sentence signifies.[28] For example, the existence of Mendelian genetics—a constructed function that produces offspring from parents—justifies the law that every child has a father. According to Majer, the Ramsey sentence is merely accounting of these constructed functions.[29]

The upshot is that adoption of Weyl's method leads to a view of the Ramsey sentence where the sentence signifies the construction of theoretical functions. The existentially quantified

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[28]Majer describes it at length thusly:

> First, we construct a *law* (*Gesetz*), that is, a function $\phi(m)$ which generates out of every number m a new number $n = \phi(m)$ such that the general sentence $\forall m R(m, \phi(m))$ is justified. Then, in a second step, we further *abstract* from this general-law sentence (as I will call this peculiar sentence) the existential sentence: There *exists a law* $\phi(m)$ such that fore very given number m the relation R between m and $\phi(m)$ holds:
>
> (4) $\exists \phi(m) \forall m : R(m, \phi(m))$.
>
> Now we see how our original sentence has to be justified; it is justified precisely by the way we are able to *construct* a law $\phi(m)$ between m and n such that the general-law sentence: $\forall m : R(m, \phi(m))$ can be maintained; and this is the case if and only if every singular judgment $R(a, \phi(a))$ deducible from the general-law sentence turns out to be true. Once such a law $\phi(m)$ is constructed, we can abstract from every singular sentence, deducible from the general-law sentence $\forall m : R(m, \phi(m))$, and existential sentence of the form $\exists R(a, n)$. Hence, what Weyl really proposes is no more and no less than this: The proper meaning of a general-existential sentence of the form $\forall m \exists n : R(m, n)$ is expressed by the *existence of an appropriately constructed law* $\phi(m)$ such that the general-law sentence $\forall m : R(m, \phi(m))$ is justified (Majer, 1989, 248).

The purpose of the Ramsey sentence, and its epistemology significance is the introduction of iterated quantified formulas like $\forall x \exists y F$. A quantified formula $\exists F \forall x F$ justifies $\forall x \exists y F$.

[29]He concludes that Ramsey adopted Weyl's purported solution wholesale:

> Ramsey not only adopted Weyl's proposal entirely but made it the core of his conception of theories. This emerges when he explains the role of theoretical terms in "Theories." "The best way to write our theory seems to be this $(E\alpha, \beta, \gamma)$ dictionary $\cdot$ axioms." If this is not immediately apparent, this is mainly due to the fact that the sentence just quoted only expressed the second and almost trivial part of Weyl's proposal: the abstraction of the second order existential sentence from the general-law sentence; whereas the sentence omits the first and most important part of Weyl's proposal: the construction of a law or a theoretical function. It should, however, be crystal clear from what was said before, that according to Weyl the

sentence can then justify sentences of the form $\forall x \exists y F$ where the existential quantifier falls under the scope of the universal. So this is the meaning and purpose of the Ramsey sentence.

Summarizing this complicated idea, Majer believes that Ramsey adopted Weyl's interpretation of the existential quantifier. The existential quantifier is defined with only its introduction rule. This eliminates the quantifier equivalences. Theories allow one to deduce laws and consequences. Only they can fulfill this role, however, because the restriction on the existential quantifier prevents the deduction of laws of the form $\forall x \exists y F$ from general propositions alone. Instead, one constructs functions that one can introduce an existential quantifier over. This allows the deduction of the aforementioned laws. The form of the theory captures this existential generalization as a Ramsey sentence.

How does Majer's interpretation fit with my own? And are there any notable problems with it? I take the second question first.

The first problem is that Ramsey's interpretation of the existential quantifier is a little different from Weyl's interpretation. While Ramsey does adopt the unbounded introduction rule, he differs from Weyl in how he treats the meaning of the unbounded existential quantifier. For both Ramsey and Weyl, the existential proposition is nothing without its witness. So the meaning of the existential proposition depends very much on what other proposition vouches for it. In Weyl's case, the witness for an unbounded proposition is the mathematical law that warrants its introduction. Mathematical laws are given in constructive proofs. There is an apparent similarity with Ramsey because a theory's dictionary and axioms are constructed. However, here the similarity ends due to the fact that laws or variable hypotheticals are chances. They are not constructive proofs but really degrees of belief that agents

second step, the *abstraction* of the existence of theoretical functions α, β, and γ only makes sense if the first step, the *construction* of the appropriate functions α, β, and γ has already been established! Otherwise, the sentence "There exist theoretical functions α, β, γ satisfying the dictionary and axioms," which is now called the "Ramsey-sentence", would tell us *nothing*, it would be a mere verbal promise without any assurance that it indeed explains or justifies the facts (Majer, 1989, 248–249).

approximate; these are different things entirely. Since the meaning of existential propositions depends on their witness, any existential proposition found under the scope of a universal proposition is really validated by some variable hypotheticals, which for Ramsey will be very different in content from Weyl. As I have documented in the previous section, this explains in part why the Ramsey sentence can be thought of as a set of different algebras with the same agreeing volumes. It is why Ramsey can say that "I verily believe that at last I *see* that they [existential judgments] are disjunctions of a finite number of terms" (FPRP Existential Judgment, 1). Thus, Ramsey's existential propositions differ from Weyl's because Ramsey has a different view on universal propositions.

A second and bigger problem is that Ramsey never explicitly mentions the importance of an existential proposition under the scope of a universal proposition *when discussing scientific theories*. This is crucial to Majer's theory because he wants the Ramsey sentence to justify such a formula. Nowhere in "Theories" or any other associated writings with "Theories", does Ramsey mention the case of a law with the form $\forall x \exists y F$ or connect the Ramsey sentence to justification. Except for the passage I discussed in "Principles of Mathematics", Ramsey does not mention the importance of this rule and associated formula anywhere else. If this were the motivation for the Ramsey sentence, as Majer's interpretation would logically entail, then the absence of any discussion of this motivation is damning. Majer does not address this point. So even if Ramsey had adopted a view of the existential quantifier similar to Weyl's, another justification for that view should be had.

It is with respect to this bigger problem that there is a considerable distance between my interpretation of the existential quantifier and Majer's. Recall that the view I put forward has the dictionary and axioms justify the Ramsey sentence. This is the opposite of what Majer claims to be the direction of justification. Remember his argument is that universal propositions of the form $\forall x \exists y R(x, y)$ need to be justified. They are justified by the construction of functions $f(x)$. One can then infer from the second order existential quantified $\exists f(x)\forall R(x, f(x))$ sentence the required law. Majer is partially incorrect. Ramsey does not appeal (nor Weyl for that matter[30]) to the second-order quantifier for justifying the introduction of the existential quantifier. While he does appeal to a constructed function, that function is really understood as a combination of particular propositions and variable hypotheticals. Those particular propositions and variable hypotheticals are what warrant the actual introduction of the existential under the scope of the universal quantifier: they substitute for the term given by the constructed function. The second-order quantifier does no justificatory duty here. Instead, the solution obviously appeals to the use of a law: it is not $\exists f(x)\forall xR(x, f(x))$ but really the laws in the definition for $f(x)$ that drives the required sentence of $\forall x\exists yR(x, y)$. After all, the second-order existential quantifier is nothing without its witnesses—the laws that form the function $f(x)$. The second-order existential quantifier just signifies that it is in fact some laws that are doing the work.

The problem is even more profound in the case of the Ramsey sentence proper. Majer's

[30]This is very clear from Weyl's own discussion of the subject. He writes that:

> By contrast, an instruction concerning judgment abstracts is nothing at all if it is not backed up by an instruction regarding the proper judgments from which it has been obtained as an abstract. Example: For every number m there is a number n such that the relation $R(m, n)$ holds between them. We must in truth be dealing here with an abstract from a judgment instruction. Which judgment instruction? Apparently the following one. Let ϕ be a certain law that from every number m generates a number $\phi(m)$. Let us assume that the general judgment instruction $R(m, \phi(m))$ is justified. Then we can draw from it the abstract: There is a law ϕ, such that for every number m the relation R holds between m and $\phi(m)$. This is the way in which the above statement must be meaningfully interpreted. If we now encounter any number, say, 7, then the law ϕ will produce a particular number, say, $\phi(7) = 19$, and we are entitled to say: The relation R holds between 7 and 19, and thus we also have a justification for the judgment abstract: There is a number n that stands to 7 in the relation $R(7, n)$. The "existential" [*da "es gibt"*] must hence include the "universal" [*das "jeder"*], and not vice versa, if we formulate the statement according to the way in which they are drawn as abstracts from self-sufficient statements (Weyl, [1921] 1997, 99–100).

Weyl's point here is not that the existential judgment "there exists a law ..." justifies the universal judgment: his point is the opposite. He claims that if the law is justified—if the universal judgment $\forall xR(x, \phi(x))$ is justified—then so is the existential judgment $\forall x\exists yR(x, y)$. Note that this makes philosophical sense for Weyl. Universal judgments contain an infinitude of actual, real judgments whereas existential judgments contain *nothing*—they are just place signifiers. It makes no sense then for an existential judgment to justify a universal. Hence why his final remark that "the "existential" [*da "es gibt"*] must hence include the "universal" [*das "jeder"*], and not vice versa, if we formulate the statement according to the way in which they are drawn as abstracts from self-sufficient statements" (Weyl, [1921] 1997, 100).

argument is grossly incorrect if one considers the earlier observation that it is the universal quantifier over the dictionary definitions and the axioms that justifies the introduction of the existential within its scope. Recall that for Ramsey, the problem of introducing the existential in these contexts was how to get in an unbounded existential quantifier. Skolem's solution does not work because it limits the values that the existential can range over. Weyl's solution does work because it uses the unbounded universal quantifier in the definition of the constructed functions to warrant the law that yields the existential's introduction. So *the axioms and dictionary justify the Ramsey sentence—not the other way around.*

Majer might reply that the constructed functions in the Ramsey sentence justify the Ramsey sentence and from that sentence, judgments of the form $\forall x \exists y F$ are justified. But a constructed function only can do that work if the underlying variable hypotheticals have been built; $f(x)$ is only well-formed when given the proper definition with requisite laws. That leads to the underlying laws or axioms and dictionary in the case of theories. They do the witnessing for the iterated quantifier formula and the Ramsey sentence. So Majer's emphasis on the constructed function cannot do the work he wants.

In summary, Majer offers an anti-realist view of the existential quantifier that gets some things right and others wrong. Namely, it correctly identifies Ramsey's anti-realism about the existential quantifier as in the vicinity of Weyl's. But it mistakes Ramsey's actual view for Weyl's view by confusing Ramsey's interpretation of the universal proposition for Weyl's. More fundamentally, Majer mistakes the direction of justification for the Ramsey sentence. He thinks the Ramsey sentence justifies laws of the form $\forall x \exists y F$ when Ramsey actually holds that the underlying law justifies the Ramsey sentence. So Majer's position is close but wrong.

6.6 Conclusion

I set out in this chapter to determine Ramsey's views of the existential quantifier. This was necessary to satisfy the final criterion for any account of the Ramsey sentence, the anti-realism criterion. That criterion held that a probable story needs to be given that has the existential quantifier not be about objects in the world. I satisfied that criterion in this chapter. First, I discussed Ramsey's theory of the existential quantifier prior to 1929, and why he had at that time rejected Weyl's version. Second, I discussed the view of the existential quantifier Ramsey adopted inspired in part by Weyl's work. This was connected to a shift in Ramsey's view of the infinite, and ultimately, a rejection of his older view on the quantifier. Third, I argued that Ramsey's new view satisfies the anti-realism criterion when applied to the Ramsey sentence. Finally, I considered Majer's similar but flawed view that Ramsey had adopted Weyl's philosophy of mathematics wholesale.

Chapter 7

Ramsey's Anti-realism

7.1 Introduction

Ramsey argues that philosophy must be of some use for clarifying thought and action or else it is a disposition that should be checked (Ramsey, [1929] 1990h, 1). In a not-so-veiled reply to Wittgenstein, he concludes that when philosophy is nonsense it should be accepted as nonsense and not important nonsense! One philosophical question that teeters into nonsense is the debate over realism and the sciences. Are scientific theories approximately true and do scientific terms refer to actual entities in the world? A Tractarian response would hold this question to be nonsense because the propositions with sense are only those given by science and a philosophy that answers this question necessarily is composed of utterances that pretend to not be scientific propositions. Later empiricists like Carnap try to save the question by distinguishing between internal and external questions, where the realism question viewed internally obviously is answered with a yes, and viewed externally the question becomes a pragmatic one about what linguistic framework scientists should adopt. Carnap understands the Ramsey sentence to play a role in the empiricist answer since it allows one to avoid metaphysical disputes over the reference of terms. I have argued that Carnap's understanding of the Ramsey sentence is not Ramsey's. So given Ramsey's philosophy of science, what is his view on the realism question? Is there an answer or is the question nonsense? The realism question is meaningful for Ramsey. His whole distinction between the primary and secondary system hinges upon a distinction between truth-apt propositions that have meaning independently of other propositions and fictional laws, chances, and theoretical 'propositions' that only have meaning as a collective whole. Knowing this distinction is useful for clarifying thought and action because it helps locate the person as an agent in the world instead of as a separate perceiver of the world. A major limitation of Ramsey's decision theory is that it restricts the ultimate set of utility-bearing propositions to those without gambles and deliberation. This set of propositions unique up to utility constitutes the primary system; as a result, the primary system cannot represent the agent's own deliberation and gambles. So fictional laws, chances, and theories must be introduced to capture the extra degrees of freedom necessary to account for the agent acting in a world. Ramsey is an anti-realist about scientific theories by way of being realistic about agency.

The spirit of Ramsey's philosophical program is the pragmatic spirit that philosophy must be useful for action. Belief is ultimately a disposition to act, and when adopting a belief fails to lead to an appropriate change in actions were circumstances to permit, the lack of change in action is a warning sign that the belief is nonsense. So any philosophical belief—about ethics, aesthetics, or science—that makes no difference to action is meaningless. A claim about the approximate truth of scientific theories or the reference to theoretical terms is sensical just in case the claim has consequences for action. Ramsey affirms positively that a mistaken realism can lead to confusion about the role of the supposedly true proposition in cognition when he writes that to view laws as a fact invites such a confused position:

> Variable hypotheticals have formal analogies to other propositions which makes us take them sometimes as facts about universals, sometimes as infinite conjunc-

> tions. The analogies are misleading, difficult though they are to escape, and emotionally satisfactory as they prove to different types of mind. Both these forms of 'realism' must be rejected by the realistic spirit (Ramsey, [1929] 1990e, 160).

Ramsey argues that realisms that take variable hypotheticals to either be facts about supposed universals or infinite conjunctions are mistaken though emotionally satisfying. They are mistaken because they project facts about human cognition in the world. This makes them remote from their important role in deliberation and action; the realistic spirit here is the spirit of a pragmatic philosophy that ultimately connects belief to action. What role does a Platonic universal or an infinite product or sum have for action here and in the future? Only the rule—the expert used for setting credences—has actual utility in action, and these realisms blind inquirers to that fact. Self-knowledge about how one's own cognition works is perhaps the most important belief for action. Addressing the question of realism thus has vital consequences for action because it leads to an appropriate model of how agency really works.

It is prudent to address the realism of scientific theories because the laws and chances that guide action are produced by those theories. Ramsey is no Tractarian here. Throughout his writings on the philosophy of science, he talks about the difference between truth-apt primary system propositions and fictional secondary system propositions. This distinction between the real and fictional applies to the quantifiers as well. Both universal and existential propositions are considered fictional and not truth-apt.

The meaning of propositions consists of their sense or truth-conditions. A proposition P's truth-conditions are simply the consequences an agent would expect to receive when successfully acting on belief in P and the consequences the agent would not expect to receive when frustrated in acting on belief in P. Those consequences are ultimately the truth-possibilities the agent intrinsically cares about, and those truth-possibilities are the truth-possibilities of the primary system. So the meaning of any proposition—including general and theoretical propositions—must be cashed out in terms of the primary system.

There is a strong connection between the meaning of general propositions and theories because of the Ramsey sentence. That sentence is a second-order, existentially quantified sentence, and it is the proper way to write scientific theories. I argued in a previous chapter that existential sentences are descriptions of propositions. So at one level, they are not truth-apt, and at another level, they have meaning due to the witnessing proposition.

But in the case of theories, the witnessing proposition for the Ramsey sentence is a universal proposition. The existential quantifiers bind the relations of the theory's dictionary and axioms along with any other theoretical proposition. Importantly, the dictionary and axioms are universal propositions. But universal propositions are not proper propositions but rules for judging. So scientific theories are not truth-apt at a deeper level: they are descriptions of laws.

By being bound by an existential quantifier, the propositions of a theory have their meaning tied to the dictionary and axioms and so their meaning is tied to the meaning of the theory as a whole. To give the meaning of a theoretical proposition, I must relate it to other theoretical propositions and to observation. This is unlike primary or observational propositions whose meaning is independent of other propositions. So what separates primary and secondary systems is that primary system propositions have sense separately from other primary system propositions.

The reason why primary system propositions have meaning independent of one another is their fundamental utility for guiding action. Primary or observational propositions are the propositions that an agent ultimately cares about. These are the propositions that pay rent for agents; that is what makes them truth-apt. In contrast, gambles are not truth-apt.

These represent the choices or acts the agent could take, but crucially they cannot be valued intrinsically, i.e. belong to the set of outcomes the agent ultimately cares about, because this would break Ramsey's measurement procedure for utilities and partial beliefs. So their value has to be calculated in terms of the gamble's conditions and consequences. Importantly, Ramsey proposes that the utility of a gamble—and consequently of a rational choice—is the utility of the gamble's consequences weighted by the marginal probability of the gamble's conditions. This differs from propositions in the primary system, whose utility should be calculated in terms of the utility of their truth-possibilities weighted by the probabilities of those truth-possibilities conditional on the proposition, because the gambles are causal propositions. Since gambles are not valued intrinsically, and they are causal propositions whose value is calculated differently from ordinary propositions, their truth-conditions must be other gambles. But this results in the truth-conditions of gambles depending on the agent's subjective beliefs about propositions other than the gamble's consequences; it makes the truth-conditions "incomplete" in the sense that they require information about what would happen with the gamble than what will. This fact about gamble truth-conditions applies also to any chance or law due to chances and laws effectively being betting preferences over certain wagers and likewise to theoretical propositions. So ultimately every outcome of a gamble must bottom out in an intrinsic set of non-chancy propositions, the primary system.

An important problem with this primary system is its inability to represent how people make decisions. Unless an agent obtains utility from deliberation and thinking about the decisions they will take, those propositions will not be in the primary system. But Ramsey's decision theory rules this out; an implicit assumption is that gambling is not valued for its own sake because it would break how utilities are measured. This is a cardinal feature of Ramsey's consequentialism. Making decisions is instrumental to the true goals. So the goal propositions—the propositions whose beliefs are true or false—must not be about the gambles and deliberations of agents themselves.

The secondary system addresses the weakness of the primary system by reifying the conditional credences of agents. Agents can represent themselves and their decision-making through fictional propositions. An agent projects themself into the world; this enables Ramsey to avoid a solipsism he attributes to Carnap. It also yields a direct explanation for what the surplus content of a theory happens to be: the agent themselves and their rules for cognition.

Here is how my argument proceeds. First, I discuss the two levels of anti-realism of Ramsey's view of theories. There is one level given by his attitude about the existential quantifier, and there is another level given by the dictionary and axioms of theories. Second, I articulate the difference between the primary and secondary systems in detail. Finally, I discuss how this difference points to a foundational issue in decision theory Ramsey tries to address about the representation of the agent in decision-making.

7.2 What Sort of Anti-Realism

The Ramsey sentence is a description of the axioms, dictionary, and constructed functions an agent accepts, which induces a compatible partition that has volumes that mirror the agent's conditional credences. The extension of the Ramsey sentence functions are all such compatible partitions with agreeable volumes. This makes the Ramsey sentence a projection of the laws and chances adopted by an agent into their algebra.

A question that might remain for the reader is the question that everyone who has ever been interested in the Ramsey sentence asks: does it support a philosophy of science that is realist or anti-realist? And if anti-realist, how is it anti-realist? Another way to put this second question is what separates the primary from the secondary system. If the secondary system is fictional, what divides the fictional propositions from the legitimate, non-fictional propositions?

My goal in this section is to address these questions about realism and anti-realism. I argue that the interpretation of the Ramsey sentence here implies a kind of anti-realism. That anti-realism, however, is an anti-realism about laws. The Ramsey sentence is really a special type of universal proposition. And Ramsey holds universal propositions to not be truth-apt. But this asks a deeper question: if the anti-realism of the Ramsey sentence is an anti-realism of universal propositions, what is the anti-realism of universal propositions?

7.2.1 Anti-Realism at One Level

The first job is to figure out what sort of anti-realism the Ramsey sentence gives to Ramsey's philosophy of science. It is a species of anti-realism because of Ramsey's prolific comments that theories are fictions and not true propositions. For example, in "Causal Qualities", a companion piece to "Theories", Ramsey writes:

> The truth is that we deal with our primary system as part of a fictitious secondary system. Here we have a fictitious quality, and we can also have fictitious individuals. This is all made clear in my account of theories (Ramsey, [1929] 1990a, 137)

Ramsey states clearly that the secondary system propositions are fictional and not true propositions. Similar language is used in the paper "Theories" too. There he states that the laws and consequences of a theory are what happen to be asserted by the theory,[1] the entire content of the theory can be given by the primary system,[2], the theory is just "clothing" for

[1]He writes that "the totality of laws and consequences will be the eliminant when α, β, γ.., etc., are eliminated from the dictionary and axioms, and it is this totality of laws and consequences which our theory asserts to be true" (Ramsey, [1929] 1990m, 115).

[2]He says with his first question "1. Can we say anything in the language of this theory that we could not

the laws and consequences,[3] and the theory's quantifiers function like "once upon a time" from the fairy tales.[4]. So theories are fictions for Ramsey, and he is clearly some sort of anti-realist.

Importantly, Ramsey connects his anti-realism with the Ramsey sentence. He introduces the Ramsey sentence shortly after stating that theories clothe the true judgments of laws and consequences. So what sort of anti-realism is provided by the Ramsey sentence?

At first pass, there is an anti-realism that stems from the anti-realism of the existential quantifier. For Ramsey, the existential quantifier is not a true proposition but a description of a proposition. In the context of the Ramsey sentence, the existential quantifier is a description of the axioms and dictionary of the theory. So at one level, theories are fictions for Ramsey because their expression as Ramsey sentences makes them descriptions of the theory's axioms and dictionary.

A description of a proposition is just a receipt for the witnessing proposition. More precisely it can be seen as a finite disjunction that has the witness as one of its disjuncts. However, the disjunction is really treatable as a pseudo-proposition because the disjunction is only warranted due to the existence of the witness. That is, the witness performs the entire cognitive labor, and no elimination rule is given for this finite disjunction since elimination is unnecessary thanks to the witness. So the existential proposition treated as an actual proposition is just filler: it is nothing without its witness and so should not be treated as truth-apt.

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say without it? Obviously not; for we can easily eliminate the functions of the second system and so say in the primary system all that the theory gives us" (Ramsey, [1929] 1990m, 119).

[3]In answering the question about the function of theories, Ramsey writes "Clearly in such a theory judgment is involved, and the judgments in question could be given by the laws and consequences, the theory being simply a language in which they are clothed, and which we can use without working out the laws and consequences" (Ramsey, [1929] 1990m, 131).

[4]He writes conclusively that theoretical propositions are not true propositions "Any additions to the theory, whether in the form of new axioms or particular assertions like $\alpha(0,3)$, are to be made within the scope of the original α, β, γ. They are not, therefore, strictly propositions by themselves just as the different sentences in a story beginning 'Once upon a time' have not complete meanings and so are not propositions by themselves" (Ramsey, [1929] 1990m, 131).

This is a very shallow anti-realism. "There is a philosopher" maybe just a description of a proposition and thus not truth-apt, but it still describes a real proposition, namely "Socrates is a philosopher". So at one level, the anti-realism given by the existential quantifier is pretty thin: one is simply denying that there is falsification conditions for that proposition. Ramsey does not think the existential quantifier has elimination rules. This means that it cannot be properly negated, which implies there is no way to render the proposition false. I cannot falsify "there is a philosopher" because I would have to check every person who has ever lived or ever will live, which would be an infinite conjunction that Ramsey thinks is nonsense (Ramsey, [1929] 1990e, 144–145).

The upshot is that Ramsey's anti-realism about scientific theories comes from the witness for the Ramsey sentence. Of course, existential propositions fail to have truth values—his philosophy of logic makes them fictional. But this fictionality is surface-level because if the witness is truth-apt, then one might say there is some real factive heft to the existential judgment. So what is the witness for the Ramsey sentence?

7.2.2 Anti-Realism At Another Level

A closer inspection of the witness to the Ramsey sentence reveals that the sentence has a deeper anti-realism. A Ramsey sentence's witness is the theory's axioms and dictionary. Importantly, the axioms and the dictionary are *universal* propositions. This means that the Ramsey sentence is a description of certain laws and chances. Ramsey holds these propositions to also not be truth-apt. A universal proposition is actually a rule for judging, which in the case of chances is a rule for assigning credences. It allows for the production of singular propositions in the primary system through the dictionary. But itself it is not a proper proposition, which Ramsey believes cannot be negated like the aforementioned existential propositions (Ramsey, [1929] 1990e, 149). So the Ramsey sentence is a description of rules for judging—a description of non-real propositions.

The upshot is that the anti-realism of the Ramsey sentence makes theories like universal propositions. They are not proper propositions but descriptions of rules that one constructs for inferring the proper propositions. And Ramsey says this in "General Propositions and Causality". He remarks in an end note that "Of course the theoretical system is all like a variable hypothetical [universal proposition] in being there just to be deduced from" (Ramsey, [1929] 1990e, 162). He means this literally; a theory just is a rule for judging. The difference between a theory and a law is that a theory is a description of a law involving constructed functions.

This has the convenient consequence that strictly speaking, theories have no real propositions but are really a mechanism for producing real propositions. Since a theory is a description of a law involving constructed functions, the dividing line between the theory and observation is that the theory never has proper singular propositions. To produce a proposition it must always proceed from the rules given by the dictionary and axioms.

A key difficulty for this view, however, is that not every sentence in the scope of the Ramsey sentence need be a law. In my chance example, I might add $\gamma(1) = 0.5$ in addition to the dictionary and axioms. This is not forbidden for Ramsey, and he even discusses this as a proper thing to do to find singular propositions in the primary system:

> So far we have only shown the genesis of *laws*; *consequences* arise when we add to the axioms a proposition e.g. a particular value of n, from which we can deduce propositions in the primary system not of the form $(n)\ldots$ These we call the *consequences* (Ramsey, [1929] 1990m, 119).

Singular, theoretical propositions may be added to the axioms of the theory for singular, observational propositions as consequences. Ramsey himself does this later in "Theories" when considering the meaning of theoretical propositions. So it would appear that there should be some theoretical propositions that are not universal.

The difficulty is resolved by observing that Ramsey always forces these singular, theoretical propositions to occur under the scope of the existential quantifier. This means that they are tied to the other theoretical propositions including the axioms and the dictionary. This is the reason for his extended discussion surrounding meaning holism in theories, which I discuss in the following section. Meaning holism restricts the independence of singular, theoretical propositions. They have to be yoked to universal propositions like the axioms and the dictionary. The upshot is that for *any theoretical proposition to have meaning, it must be conjoined with the other theoretical propositions including universal propositions like the dictionary.*

So the conclusion is that the anti-realism of the Ramsey sentence has theories be descriptions of non-truth-apt universal propositions. It is an anti-realism of laws and chances. Those laws and chances are not truth-apt because they are really rules for judging and setting credences. And since all theoretical propositions have to either be derived from these rules or conjoined with these rules, all theoretical propositions are really fictitious in the same sense that laws and chances are fictitious.

7.3 The Primary and Secondary System

I have argued that theoretical propositions have a radical meaning dependence on the whole theory. I now need to explain why this is the case. That requires first saying what a theory's "meaning" happens to be, how its meaning dependence differs from the primary system, and why theories are meaning-dependent. I conclude by discussing how this anti-realism is different from the observable versus non-observable anti-realism prevalent in empiricisms from Ramsey's day.

7.3.1 Theory Meaning Dependence

Ramsey's anti-realism of scientific theories is an anti-realism of laws and chances. Every theoretical proposition depends, in part, for its meaning on the theory's dictionary, which is a universal proposition. A universal proposition is a rule for judging—an expert that agents defer to for setting their credences. They are not truth-apt. So the meaning of any theoretical proposition depends on a non-truth-apt rule.

I want to pause here and say exactly what Ramsey understands here by "the meaning of a proposition" where the "proposition" can be a primary or secondary system proposition. The meaning of a proposition is its sense; meaning is the truth-conditions for the proposition. Recall that for Ramsey the truth-conditions of a proposition P are the consequences that would be realized were action successful or frustrated when acting on belief in P. Note that consequences are picked based on their utility; what makes a difference in the truth-conditions—what fundamentally individuates one consequence from another—is its desirability. In terms of Ramsey's decision theory, this is captured by $worlds_2$. Importantly, those $worlds_2$ just are the truth-possibilities of the primary system since the primary system describes the world as it is desirable. This leads to an intermediate conclusion that the sum total of the meaning of a proposition is captured by the primary system. Secondary system propositions, however, have as their consequences when acting truth-possibilities that describe the world at a finer grain than $worlds_2$. They still have meaning in the sense that their truth-possibilities are refinements of the primary system, but they add additional details that do not drive action except derivatively. So any meaning ascribed to theoretical propositions comes solely from the observational propositions that would have to be true were the theoretical propositions "true".

Ramsey's discussion in "Theories" supports this idea. When writing about whether theoretical propositions are meaningless, he indicates that a theory's meaning comes from its primary system consequences:

> People sometimes ask whether a 'proposition' of the secondary system has any meaning. We can interpret this as the question whether a theory in which this proposition was denied would be equivalent to one in which it was affirmed. This depends of course on what else the theory is supposed to contain; for instance, in our example $\beta(n, 3)$ is meaningless coupled with $\overline{\alpha}(n, 3) \vee \overline{\gamma}(n)$. But not so coupled it is not meaningless, since it would then exclude my seeing red under certain circumstances, whereas $\overline{\beta}(n, 3)$ would exclude my seeing blue under these circumstances. It is possible that these circumstances should arise, and therefore that the theories are not equivalent. In realistic language we say it could be observed, or rather might observed (since 'could' implies a dependence on our will, which is frequently the case but *irrelevant*), but not that it will be observed (Ramsey, [1929] 1990m, 133–134).

Ramsey's discussion makes it clear that the meaning of a proposition is in terms of its primary system consequences. The example he gives has the proposition $\beta(n, 3)$ (place 3 is blue) yield no consequences when paired with $\overline{\alpha}(n, 3) \vee \overline{\gamma}(n)$ (either I am not at place 3 or my eyes are closed) because I would not be able to see blue after the right movements. If not coupled, it would have meaning because I could be at place 3 and see blue and if the proposition were false I would not see blue. In either case, the meaning of $\beta(n, 3)$ is in terms of the primary system consequences—the primary system truth-possibilities or $worlds_2$.

In summary, the meaning of a proposition in the following discussion should be understood as the primary system truth-possibilities that would be realized if the action were successful or frustrated when acting on that proposition.

Returning to the question of meaning dependence, the upshot of the connection between theoretical propositions and the axioms and dictionary is that no theoretical proposition has meaning independent from the theory as a whole, even though the theory is a fiction. This is so important because it is the first consequence Ramsey draws from the Ramsey sentence (Ramsey, [1929] 1990m, 131). It is one thing for a proposition to have its meaning dependent on other propositions; it is another for the meaning of a proposition to depend on non-propositions. For example, a disjunction $P \vee Q$ depends for its meaning on its disjuncts P and Q. But those disjuncts have independent truth-values so the disjunction is treatable as a truth-function of its arguments. Theories are different. With theories, it is like P depends for its meaning on $P \vee Q$ where Q is a rule, and with it, the disjunction is not truth-apt. There is a radical meaning holism where every theoretical proposition depends on every other theoretical proposition for meaning and on the theory's rules for meaning. It is this last part about the dependence on rules for meaning that makes this meaning holism radical. Since each theoretical proposition depends on the dictionary and the axioms, they are all connected, and there is no easy separation of a theoretical proposition's meaning from its peers.

This is such an important consequence of Ramsey's view of theories that he discusses it immediately after introducing the Ramsey sentence. The whole passage is worth quoting in full:

> Any additions to the theory, whether in the form of new axioms or particular assertions like $\alpha(0, 3)$, are to be made within the scope of the original α, β, γ. They are not, therefore, strictly propositions by themselves just as different sentences in a story beginning 'Once upon a time' have not complete meanings and so are not propositions by themselves.
>
> This makes both a theoretical and a practical difference:
>
> (a) When we ask for the meaning of e.g. $\alpha(0, 3)$ it can be only given when we

> know to what stock of 'propositions' of the *first and second* systems $\alpha(0,3)$ is to be added. Then the meaning is the difference in the first system between $(\exists\alpha,\beta,\gamma) : \text{stock} \,.\, \alpha(0,3)$, and $(\exists\alpha,\beta,\gamma) \,.\, \text{stock}$. (We include propositions of the primary system in our stock though these do not contain α,β,γ).
>
> This account makes $\alpha(0,3)$ mean something like what we called above $\tau\{\alpha(0,3)\}$, but it is really the difference between $\tau\{\alpha(0,3)+\text{stock}\}$ and $\tau(\text{stock})$ (Ramsey, [1929] 1990m, 131).

The first point Ramsey makes is the relevance of the existentially quantified theoretical parameters. These witness any singular theoretical propositions in addition to the axioms and dictionary of the theory. So Ramsey is making the point that the meanings of any additional theoretical proposition are tied to the other theoretical propositions, and importantly, the universal propositions that are the axioms and the dictionary. His second point is that this leads to a theoretical difference where fixing the meaning of any new theoretical proposition, singular or otherwise, requires one to compare the difference it makes between the Ramsey sentence of an existing stock of observational and theoretical propositions and that Ramsey sentence with its stock plus the new proposition. The addition of observational propositions is required because of the dictionary. The upshot is that the meaning of theoretical propositions is dependent on the whole theory.

Ramsey illustrates this radical meaning holism with the case of mass in physics. He writes that mass depends for its meaning on hypotheticals that can never be examined:

> In dealing with the motion of bodies we introduce the notion of mass, a quality which we do not observe but which we use to account for motion. We can only 'define' it hypothetically, which is not really intelligible when you think it out. E.g. 'It had a mass 3 = If we had fired at it a given body (mass 1) at 3 times its velocity which coalesced with it the resulting body would have been at rest'

> is an unfulfilled conditional intelligible only as a consequence of a law, namely a law of mechanics stated in terms of mass" (Ramsey, [1929] 1990a, 137).

Ramsey argues that the use of unfulfilled conditionals dependent on the theory of mechanics makes unintelligible the whole idea of an independent proposition "It had a mass of 3". The thought is that the dependence singular propositions about mass have on laws—rules compatible with every possible observable proposition—makes it impossible to nail specific truth-conditions independent of other theoretical propositions about mass. One cannot cash out the meaning of theoretical propositions as truth-functions of other propositions because of their dependence on non-truth-functional laws and chances.

This stands in contrast with the primary system. Observation propositions can have meaning independent of other observation propositions. Recall here meaning is understood as the sense or truth-conditions of a proposition; the truth conditions of some primary system propositions can vary independently of the truth conditions of some other primary system propositions. For example, what makes "Jones perceives red" true or false does not depend on what makes "Mount Kilimanjaro is an active volcano" true or false. The same is false for the secondary system. Since every theoretical proposition depends on the rule that is the dictionary, the "truth-conditions" (really their verification conditions) of each theoretical proposition are connected. In contrast, primary system propositions are independent of at least some other primary system proposition.[5] So some degree of meaning independence is an important feature of the primary system.

The distinction between propositions whose meaning is radically holistic and those that have some degree of meaning independence is the distinction between the primary and secondary systems. It is the difference between propositions whose meaning is totally reliant upon

[5]This is likely not true of every primary system proposition. Ramsey is not so radical as to adopt the priority of sense thesis that every primary system proposition has its meaning independent of every other primary system proposition. All Ramsey needs is that for every primary system proposition, there is at least one other primary system proposition whose sense is independent of it.

every other proposition and those that are not so reliant. This is what separates the primary and the secondary system. The primary system can have singular propositions independent of one another and the laws and chances; the secondary system cannot. Secondary system propositions are ultimately those that are wedded together through the laws under a different guise. Observation tracks particular facts. Theory follows rules that are constructed to infer particular facts. This is the dividing line between observation and the theoretical.

Still, one might ask: why is meaning independent for the observational? Why should the primary system propositions be truth-functional? After all, Ramsey states in passing that "all belief involves habit" (Ramsey, [1929] 1990e, 150).[6] I address this next.

7.3.2 Gambles and Incomplete Truth-conditions

Another way to put the question I am confronted with is why is there a separation between non-supervenient variable hypotheticals and truthful singular propositions. Answering this version of the question brings in an important technical feature of Ramsey's decision theory: Ramsey's use of gambles to measure probabilities and the difference between the utility of a gamble and other types of propositions. Jeffrey first identified this feature when discussing the difference between the utility of ordinary propositions and Ramsey's proposal for the utility of gambles. Namely, the feature is that a gamble's expected value includes more of the agent's beliefs than the expected value of an ordinary proposition; gambles include the agent's beliefs about what would happen instead of the agent's beliefs about what logically follows from the belief. What I will argue is that this fact dictates that gambles—and by extension all variable hypotheticals—have "incomplete" meanings, i.e. truth-conditions. By "incomplete", I mean that the truth-conditions of a gamble are neither primary system propositions nor intrinsically valuable gambles, but must be other gambles. They cannot

[6] Misak uses this line to argue there is no real distinction between singular and general propositions (see Misak, 2016 section 6.6). She argues Ramsey is not an expressivist about general and theoretical propositions.

be just primary system propositions or truth-possibilities because the utility of the gamble is different from the utility of such a proposition, and they cannot be intrinsically valued gambles because this breaks Ramsey's decision theory. This only leaves derivatively valuable gambles, which are not truly objective in the sense they depend on more of the agent's subjective beliefs than they should because of how the utility of a gamble is calculated.

Ramsey measures utilities and probabilities via gambles. A gamble is a causal proposition in the precise sense that it specifies outcomes as the effects of antecedents; by a causal proposition, I mean that the different sides of the gamble are really subjunctive claims about what would happen were that side's antecedents realized. As Jeffrey notes, when offering an agent a gamble one is convincing the agent that one can make the outcomes be realized conditional on the truth of the gamble's antecedents.[7] Importantly, these gambles are necessary for Ramsey to measure utilities and probabilities. The calibration of an agent's utility-scale requires the use of gambles conditional on some number of ethically neutral propositions. This means that to use Ramsey's decision theory descriptively or prescriptively,

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To say this stretches the content of the text is an understatement.

[7] Jeffrey identifies the causal nature of gambles in his discussion of Ramsey's decision theory. He writes:

> In general, a gamble of form
>
> A if C, B if not
>
> exists if there is a causal relationship between C, A, and B, in virtue of which A will happen if C does, and B will happen if C does not. If I offer to bet you a dollar at even money that C will happen, and you accept the bet, we have set up a causal relationship between C, A and B, where A is the proposition that
>
> you pay me \$ 1 at the time we learn whether C is true or false,
>
> B is the proposition that
>
> I pay you \$ 1 at the time we learn whether C is true or false.
>
> The basis of the causal relationship lies in our good faith and in our ability to produce the required cash at the time in question. It is a relationship that we brought into being of our own free will. Nonetheless, it is a genuine causal relationship: as genuine as the relationship between the proposition that the gas tank is empty and the proposition that the car does not go (Jeffrey, 1990, 156–157).

Gambles are thus not ordinary material conditionals but something more.

one needs gambles—causal propositions. It is the core, foundational feature of Ramsey's approach to measuring utilities and probabilities.[8]

This makes a gamble a type of law. Recall that laws for Ramsey include the usual suspects like "All men are mortal" but also dispositional claims like "Arsenic is poisonous." The latter means that laws or what Ramsey calls variable hypotheticals include subjunctive conditionals like "if I were to eat the cake, then I would get a stomach ache." Despite their seeming appearance as singular propositions, these conditionals are general in the sense that they describe what propositions would follow from other propositions in a variety of conditions; I could eat cake today and get a stomach ache or eat it tomorrow and get a stomach ache and

[8]Jeffrey identifies it as the foundational building block for Ramsey's entire decision theory. He writes

> To avoid confusion that may result for alternative interpretations of the word "if," let us introduce a special symbol
> (10-7)
> $$[\ ,\ ,]$$
> for the operation which, applied to three propositions
> $$X, Y, Z$$
> yields the proposition
> $$[X, Y, Z]$$
> which asserts that a gamble on Y is in effect, with outcomes X (if Y happens) and Z (if Y fails). Then (10-3) [A if C, B if not] is to be interpreted as $[A, C, B]$.
> The symbol (10-7) expresses the key operation of Ramsey's theory. His rule for computing estimated desirabilities of gambles is
> (10-8)
> $$des[X, Y, Z] = prob Y\, des XY + prob \overline{Y}\, des \overline{Y} Z$$
> The condition that Y be ethically neutral relative to X and to Z is simply that both XY and $X\overline{Y}$ be ranked with X and that both ZY and $Z\overline{Y}$ be ranked with Z. Also, if Y is ethically neutral relative to X and to Z, and X and Z are not ranked together, Ramsey's condition for Y to have probability 1/2 is that the gambles $[X, Y, Z]$ and $[Z, Y, X]$ be ranked together (Jeffrey, 1990, 158).

Jeffrey associates the gamble (or wager or bet or conditional prospect) as the key operation for Ramsey's decision theory. It is what makes everything work, as Jeffrey explains, since ethically neutral propositions are used to calibrate utility scales.

so on. A gamble just is a subjunctive conditional, e.g. "if I were to draw an ace, then I would get a heifer; otherwise, I would get a goat." Likewise, they specify what would happen across a variety of conditions; if I were to rerun the wager over and over, I would receive a heifer when drawing an ace. So to accept or reject a gamble is to accept or reject a subjunctive conditional, and thus to accept or reject a gamble is to accept or reject a particular law.[9]

Importantly, Ramsey uses a particular rule for the expected utility of a gamble. The value of a gamble is the expectation where the weights are the unconditional probabilities of the gamble's conditions. For example, if the gamble Γ is α if P; β otherwise, then its value U is:

$$U(\Gamma) = \Pr(P)U(P \cap \alpha) + (1 - \Pr(P))U(P^c \cap \beta) \tag{7.1}$$

One question that might be asked is how does the utility of a gamble compare to the utility of a non-gamble proposition? The answer is not so straightforward because Ramsey's original decision theory only considers the value of acts (gambles) and outcomes (worlds). He never specifies what the utility of an ordinary proposition might be. However, it would be prudent to consider the value of normal propositions since as I discuss below, they are the only other candidate apart from other gambles for being the truth-conditions of gambles.[10] But what would their value be on the reconstructions of Ramsey's decision theory I have offered? The natural and immediate candidate is the standard formula first introduced by Jeffrey.

[9]The reader might note that this creates a circularity: laws are supposed to be defined in terms of how an agent's conditional credences work, which are defined through a constellation of conditional gambles, yet here I have just said that gambles are really laws. So laws are really defined through other laws. This is a circularity, but it is not fatal; Ramsey holds credences to not being descriptive but a prescriptive fiction one uses to modulate psychological expectations. And credences are nothing but dispositions to accept or reject certain wagers. Logic is a fiction—a useful fiction for regulating behavior—but a fiction nevertheless.

[10]Depending on whether one allows for every gamble the existence of outcomes, $worlds_2$, that are of equal in value to that gamble, one could then substitute in those $worlds_2$ for other gambles that act as outcomes. However, this has a very undesirable property: it would make the truth-conditions of a gamble dependent on some outcomes that *could have nothing to do with the gamble's wagered proposition and outcomes.* I will consequently ignore this as a live option for this reason, apart from the additional fact that reconstructing Ramsey's decision theory also allows for the abandonment of the axiom that would make this an option.

That is, the reconstructed decision theory for Ramsey allows for the evidential calculation of a non-act proposition's utility.[11] The utility of P (a singular proposition in the primary system and so excluding gambles) is just the weighting of the utility of outcomes $o \in O$ by the conditional probability of the outcomes on the truth of P:

[11] This is examining the utility of propositions not treated as acts or subjects of choice. When considering a proposition as an act, like choosing between eating cake or eating ice cream, Ramsey has the expected value of the act computed by the formula for a wager in equation (7.1). That is to say that Ramsey is a predecessor of causal decision theorists in that he thinks the value of choices is not the same as the value of newsworthiness or evidence for a proposition. However, Ramsey also considers the possibility that the value of propositions and their use in action may be captured by an alternative rule. He writes that:

> We can begin by asking whether these variable hypotheticals play an essential part in our thought; we might, for instance, think that they could simply be eliminated and replaced by the primary propositions which serve as evidence for them. This is, I think, the view of Mill, who argued that instead of saying 'All men die, therefore the Duke of Wellington will', we could say 'Such-and-such men have died, therefore the Duke will'. This view can be supported by observing that the ultimate purpose of thought is to guide our action, and that on any occasion our action depends only on beliefs or degrees of belief in singular propositions. And since it would be possible to organize our singular beliefs without using variable intermediaries, we are tempted to conclude that they are purely superfluous (Ramsey, [1929] 1990e, 153).

Ramsey considers the possibility that the value of singular propositions may be weighted by just the agent's probabilities (including conditional probabilities) without appeal to any variable hypotheticals. This he seems to admit is sufficient for act propositions too when he writes "on any occasion our action depends only on beliefs or degrees of belief in singular propositions". Note how close this is to evaluating the utility of a proposition just in terms of its outcomes weighted by conditional probabilities, what is now called Jeffrey's ratio rule. Ramsey is thinking of a decision rule closer to Evidential Decision Theory (EDT) than Causal Decision Theory (CDT). But while Ramsey admits this makes sense when acting only on beliefs in singular propositions, he disagrees with its correctness as a decision rule. Instead, he suggests that the right decision rule is to appeal to variable hypotheticals:

> But this would, I think, be wrong; apart from their value in simplifying our thought, they form an essential part of our mind. That we think explicitly in general terms is at the root of all praise and blame and much discussion. We cannot blame a man except by considering what would have happened if he had acted otherwise, and this kind of unfulfilled conditional cannot be interpreted as a material implication, but depends essentially on variable hypotheticals (Ramsey, [1929] 1990e, 153–154).

Ramsey's proposal is that the correct decision rule treats act propositions as essentially gambles; to do P or to do otherwise is a bet captured in the variable hypothetical that is the gamble. He thinks two reasons push towards a decision rule based on variable hypotheticals: how they simplify thought and how they are relevant in blameworthiness and responsibility. The simplicity idea has come up in some discussions around the descendant of Ramsey's recommended decision rule, CDT. The thought in some discussions is that the simplification here is a computational simplification; by adopting Ramsey's proposed decision rule, one cuts out keeping track of all the primary system propositions that might provide evidence for a particular act. The blameworthiness and responsibility idea has been adopted by some defenders of CDT (see Halpern, 2016). This proposal ties causation's relevance to decision-making through human moral cognition.

So the picture then is that for Ramsey, act propositions should really have their expected value given by his formula for gambles. And this is because all acts are just an instance of a gamble. Non-gambles, like ordinary primary system propositions, would have their utility calculated by something like Jeffrey's ratio rule. This is because those propositions are not considered acts.

$$U(P) = \sum_{o \in O} \Pr(o \mid P) U(o) \tag{7.2}$$

where the outcomes here are the set of $worlds_2$. Note that $worlds_2$ work as outcomes because P and P^c are a coarsening of the partition given by $worlds_2$. If the gamble was treated like an ordinary proposition, like a disjunction as Jeffrey proposes, its value should really be:

$$U(\Gamma) = \sum_{o \in O} \Pr(o \mid \Gamma) U(o) \tag{7.3}$$

$$= \Pr(\alpha \mid P) U(P \cap \alpha) + \Pr(\beta \mid P^c) U(P^c \cap \beta) \tag{7.4}$$

Equation (7.1) is not the same as equation (7.4). Ramsey has proposed an alternative way to measure the value of a gamble—it just is a different formula.[12] In Ramsey's decision theory, all gambles are structured this way because they involve recourse to some ultimate set of outcomes. Gambles supervene on those conditions and outcomes for their probabilities and values.

Why does Ramsey consider the value of gambles to be derivative of their outcomes? The problem is that if he had they would change the agent's utilities in the act of measuring them. Borrowing an example from Jeffrey (Jeffrey, 1990, 157), suppose someone offers me the following gamble involving a fair coin flip: heads there is a nuclear war next week; tails the weather is sunny next week. Now as a causal rule, the person offering the gamble is telling me that conditional on this coin coming up heads that person (or something else about the world) *will cause a thermonuclear war* and likewise for the coin landing tails. But my beliefs

[12] I will say more about why this is the case below.

about coins, wars, and the weather in no way included the possibility that someone could causally relate to them. The gamble, if I believe it to act on it, would effectively change my best model of the way the world would work and my attendant beliefs about the world. And so my beliefs in a war tomorrow and the weather would change in the act of trying to measure them. As Jeffrey writes, if I have preferences over gambles that treats them as causal rules makes them illicit devices for measuring my utilities:

> Not so for Ramsey's causal operation: to ask the agent to locate the gamble $[X, Y, Z]$ in his preference ranking when X, Y, and Z are the propositions (X) there will be a thermonuclear war next week, (Y) this coin will land head up when I toss it, and (Z) there will be fine weather next week, is not to invite him to take pains in the interest of clarity and self-knowledge. To the extent that he can bring himself to consider the gamble seriously, he must entertain alarming and bizarre hypotheses about the person who is offering the gamble: hypotheses that he can only entertain by altering his sober judgments about the causes of war and weather, and thereby altering the very probability assignments which the method purports to measure (Jeffrey, 1990, 159–160).

In short, measuring preferences with causal rules changes those preferences. This is why Ramsey appeals to the somewhat bizarre thought experiment in "Truth and Probability" for measuring a person's preferences over gambles by convincing that person that one is God. God fulfills the role of avoiding problematic alterations of an agent's beliefs when measuring them. A fiction needs to be believed to perform the magic trick necessary for measurement. So Ramsey has to attribute the value of the gambles to be an expectation over their condition proposition and outcomes.

For these gambles, and so for any causal proposition, to have truth values they must have well-defined truth-conditions. Ramsey's theory of truth-conditions for a proposition can be summarized as the causes and effects of beliefs in that proposition, where those causes and effects are understood as the usefulness in action of belief in the proposition. Recall that he writes in "Facts and Propositions" that:

> Thus any set of actions for whose utility p is a necessary and sufficient condition might be called a belief that p, and so would be true if p, i.e. if they are useful.[1]
>
> [1] It is useful to believe aRb would mean that it is useful to do things which are useful if, and only if, aRb; which is evidently equivalent to aRb (Ramsey, [1927] 1990d, 40).

Ramsey's thought is that the belief in p is a relation between the utility of the proposition or state p and the actions an agent would take were p. The belief that p would be true just in case the action the agent takes leads to the agent's desired outcome; and the belief that p would be false just in case the action the agent takes leads to an undesired outcome. Thus, the truth-conditions are the facts found by a relationship between belief, action, and utility, i.e. by a decision matrix. One such example can be found in figure (7.1), which comes from an earlier chapter. Here the chicken's belief that the caterpillar is poisonous is given by the action the chicken takes, to refrain from eating, and their expected outcomes conditional on the belief being true, to avoid an upset stomach. The belief is true just in case the desired outcome of avoiding an upset stomach is realized, and the belief is false just in case the undesired outcome of missing a good meal is realized; the truth-conditions of a proposition just are the outcomes in the row of the decision matrix corresponding to the action the agent would take were they to believe the proposition. Importantly, this includes misrepresenting a proposition, i.e. when the proposition is false. Such a misrepresentation leads to an unexpected outcome and in this example, to a lower-than-expected utility, since being satiated is preferable to missing a good meal. Consequently, Ramsey can individuate propositions by an appeal to the expected behavior of the believer and the utility of the outcomes.

	The caterpillar is poisonous.	**The caterpillar is edible.**
Eat the caterpillar.	The chicken has an upset stomach.	The chicken is satiated.
Refrain from eating the caterpillar.	The chicken avoids having an upset stomach.	The chicken missed a good meal.

Figure 7.1: A decision matrix for the caterpillar thought experiment. The columns are the proposition or state of the world. The rows are the actions. The cells are the consequence or outcomes of the states and actions. Here the truth-conditions of a belief in a particular proposition are given by the row of the action the belief induces. So the first column's truth-conditions are given by the second row, and conversely, the second column's truth-conditions are given by the first row. The original rendition of this matrix can be found in Sahlin, 1990, 72.

For gambles to have meaning, they must have truth-conditions. Here I am treating a gamble as the antecedent in another gamble where I accept or reject that gamble. For example, I can either accept or reject the gamble "I receive a heifer if I draw an ace; I receive a goat otherwise." The meaning of a gamble, its truth-conditions, would just be the consequences that follow from accepting it and it paying off or not paying off. Importantly, for the payoffs to be selected they must have a utility, but for Ramsey, the utility of a gamble's payoffs is not the value of any ordinary proposition but the alternative formula he gives in equation (7.1). What proposition would have this utility? This has ramifications for understanding what are the exact truth-conditions in believing a gamble. It means that those truth-conditions are "incomplete" in the sense that they depend on more than an agent's best guess about the consequences of the truth-condition. They do not correspond exactly to primary system propositions and truth-possibilities nor do they correspond to other intrinsically-valuable gambles.

One might think that perhaps the truth-conditions of a gamble can be given by a disjunction of its conditions and consequences. One hypothetical way of doing this can be seen in figure (7.2).[13] Here the consequences of accepting or rejecting a gamble and its complement can be thought of as a disjunction or negation of disjunction. An immediate problem is that it is unclear how exactly the consequences of accepting the negation of a gamble track the

[13]This idea goes back to Jeffrey, who proposes it as an alternative to Ramsey's formulation of gambles. So instead of treating gambles as organic, non-decomposable propositions, Jeffrey suggests gambles should be understood as disjunctions. He then notes that this will not work for Ramsey because the desirability of such

consequences of the original gamble. Is it really true that the falsity of receiving a heifer if an ace is drawn and a goat otherwise is just the fact that either I will not receive a heifer or not draw an ace and that either I will not receive a goat or draw an ace? I could after all still happen to receive a heifer from my farmer grandfather and draw an ace at poker tonight! A similar problem might appear for equating the consequences of accepting or rejecting the gamble with some proposition involving the gamble's conditions and outcomes. These gambles are chancy propositions—they do not seem to be equivalent to any actual possible proposition. This is precisely Ramsey's point in "General Propositions and Causality" when he discusses a similar gamble about eating or not eating cake (Ramsey, [1929] 1990e, 154–155). It speaks to a larger problem about rendering the outcomes of causal propositions as Boolean operations over ordinary propositions while pretending to claim the desirability of a gamble is given by its expectation (see Sneed, 1966 for a full discussion). Note that this problem is immediate if one compares the expected utility of the gamble as Ramsey calculates it with the expected utility of the Boolean formula in figure (7.2). The value of those propositions does not correspond with the values of the gamble. So ordinary propositions cannot straightforwardly be the truth-conditions of gambles.

How about using the utility of other gambles as in figure (7.3)? Here Ramsey presents a formula for calculating the utility of the gamble. However, as I discuss below this has an undesirable consequence that makes the truth-conditions for the gamble depend on the agent's best estimate of the truth of other propositions. To avoid this, one would have to value gambles intrinsically. What if gambles are allowed to have intrinsic value like other propositions? The problem is that Ramsey cannot measure my partial beliefs without changing those beliefs for the reasons Jeffrey discusses. Gambles are supposed to be neutral measuring devices and that is facilitated by the expedient of treating their value derivatively. Change that and now gambles can change my beliefs when I am offered them. So Ramsey's decision

a disjunction is not equal to the expected value of gambles as Ramsey defines it (Jeffrey, 1990, 157–158). Note that here, I am appropriating his solution for thinking about the consequences of wagers on gambles themselves.

	A if P; B if not P	**not (A if P; B if not P)**
Accept the gamble.	$(P \wedge A) \vee (\neg P \wedge B)$	$(\neg P \vee \neg A) \wedge (P \vee \neg B)$
Reject the gamble.	$(\neg P \vee \neg A) \wedge (P \vee \neg B)$	$(P \wedge A) \vee (\neg P \wedge B)$

Figure 7.2: A decision matrix for wagering over a gamble A if P; B if not P. The consequences of accepting the gamble or its negation just a disjunction or negated disjunction.

	A if P; B if not P	**not (A if P; B if not P)**
Accept the gamble.	C if Q; D if not Q	E if R; F if not R
Reject the gamble.	G if T; H if not T	I if U; J if not U

Figure 7.3: A decision matrix for wagering over a gamble A if P; B if not P. The consequences of accepting the gamble or its negation are some other gambles.

theory enforces a hard constraint here: I have to treat the utility of gambles derivatively from proper propositions. That leaves the last option, which is that gambles only have value derivatively. How does that work?

Ramsey's proposal that gambles have utility only derivatively through his expectation formula makes problems. Namely, it makes the utility of the prospective consequences include beliefs that are broader than the consequences of the gamble. This means that subjective beliefs about propositions that do not logically entail the gamble's consequences *can affect an agent's subjective assessment of the utility of the gamble*; an agent's utility is now dependent on more than just the truth of the gamble's outcomes but any proposition that logically entails the gamble's antecedent. In short, the agent considers not just what will happen with the gamble but what could happen. For example, my utility over the gamble I have a heifer if an ace is drawn and a goat otherwise depends not just on my belief that I will have a heifer with an ace or I will have a goat with some other card, but whether I win the lottery with no heifer and draw an ace or I am elected President goatless and draw a king. All the consequences incompatible with my winnings in the gamble yet possible with drawing some card are factored into my beliefs and so the utility I ascribe to the gamble. This is why Ramsey has the utility of the gamble depend not on the probability I assign to its consequences but on just the probability of its conditions. Consequently, my utility and so the truth-conditions for a gamble depend on more than just what will happen but *what would happen by my best lights.*

This is subtle. The problem isn't that I can have varying utilities about outcomes. After all, I can change my mind about whether I prefer a heifer to a goat or vice versa over time. This would involve by equation (7.2) a shift in my beliefs about the outcomes of owning a heifer or a goat. What is unusual here is that I am factoring my beliefs that should be *irrelevant* to the expected utility of the gamble; I am weighting my credences in $worlds_2$ that are adjacent to the outcome of the gamble in the sense they include the truth of the gamble's antecedent. This is a direct consequence of weighting the probability of the gamble's outcomes by the probability of P instead of α conditional on P or β conditional on P^c in equation (7.1). But this means that treating gambles as truth-conditions includes more of my subjective beliefs in those truth conditions than they should: I am including my beliefs about all the consequences of the gamble's antecedent conditions.[14] I have included much more of my model about the world than ordinary propositions!

A consequence of this is that the truth-conditions of a gamble can shift on a change in my beliefs about propositions orthogonal to the consequences of those truth-conditions. The truth-conditions of primary system propositions can change as the utility of those truth-conditions shifts. However, these shifts only occur due to a change in the probabilities an agent assigns to the consequences of those truth-conditions. In gambles (and as I will argue, with laws generally and secondary system propositions), those truth-conditions can change even though I may think the utility of the gamble's outcomes remains fixed. This makes the truth-conditions of gambles incomplete in the sense that they are not solely determined by the consequences of those truth-conditions. More has to be brought in—namely the agent's

[14]To see this, just note that taking all o_i in $worlds_2$ where $o_i \cap P \neq$, one can define

$$\Pr(P) = \Pr(P \cap o_1) + \Pr(P \cap o_2) + \cdots + \Pr(P \cap o_i) + \dots)$$

larger beliefs about the world.

This is important. *A gamble's truth-conditions dependence on an agent's beliefs about more than the truth-conditions' consequences is the root of laws' and theories' meaning dependence*. It is this incompleteness of truth-conditions that drives the separation between primary and secondary systems.

A gamble's truth-conditions are incomplete because they depend on more subjective factors than ordinary primary system propositions. This follows from the fact that gambles have derivative value as given by Ramsey's formula for calculating their expected value. That formula plus their subjunctive nature prevents them from having their utility in terms of some truth-function of primary system propositions; and the formula cannot be abandoned to yield gambles with intrinsic value because it allows Ramsey to measure an agent's degrees of belief without changing those beliefs. So gambles have underspecified truth-conditions in the sense that those conditions depend on more than an agent's best guess about their consequences for their validity.

There are consequences here for chance and law propositions. Since those propositions are understood as a systematic series of bets, they have incomplete truth-conditions too. A law or chance is just a rule a person follows in their betting preferences, i.e. in what gambles they are willing to accept or reject. This makes the laws' and chances' truth-conditions dependent on the "truth" of gambles. While this may seem circular since gambles are themselves just variable hypotheticals, it presents no real difficulty except to the ardent reductionist. The case of laws and gambles is just like the case of causal propositions and subjunctive conditionals—a definition of one resides in the other and vice versa. The upshot is that variable hypotheticals have a meaning holism. Laws and chances as gambles depend for their truth on other laws and chances, though ultimately those truth-conditions are derivative of the primary system—the primary system propositions provide the ultimate value or payoff for those laws and chances. Through the expectation formula, the correctness of laws and chances rides on the truth of the primary system propositions.

I want to emphasize that *this is exactly the same problem singular theoretical propositions face*. A singular theoretical proposition has as its consequence a theoretical truth-possibility. That consequence only has value insofar as its observational coarsening has value. However, one thing that was unclear when I first introduced this idea was the connection between the observational and theoretical joint truth-possibility's utility and the $world_2$'s utility. Here it is possible to say more precisely that the value of a theoretical truth-possibility is a function of some $worlds_2$' utilities along with the probability assigned to the truth of the theory just as the utility of a gamble is a function of some $worlds_2$ or gambles whose values are derived from $worlds_2$ plus the probability of the wagered proposition. And this should be the case because ultimately the content or meaning of a theory is given by its laws and consequences. But then that means that the truth-conditions of a theory are dependent in part on the agent's best guess about the theory and its relation to other known facts. So the meaning of a theoretical proposition—singular or general—depends on the rest of the theory. And the theory's meaning comes ultimately from its laws and consequences.

The upshot is that the independence of primary system propositions and their fundamental separation from chances and laws is due to a technical feature of Ramsey's decision theory. He needs to measure preferences over propositions without affecting those preferences in the process. This places constraints on any solution to the truth-conditions of gambles. The measurement problem excludes gambles from having intrinsic value. Along with the problem of coincidences, Ramsey's theory for the utility of gambles prevents gambles from having truth-functions of ordinary propositions as their truth-conditions. So gambles must have other gambles, valued derivatively per Ramsey's expected value formula, as their truth-conditions. But these truth-conditions are incomplete in the sense that their validity depends on more than just their consequences but additional subjective factors. Since chances and laws are just configurations of gambles, they too must have incomplete truth-conditions and the same applies to theoretical propositions. This results in meaning dependence for laws and theories.

In summary, Ramsey splits the primary from the secondary system propositions due to their differences in meaning dependence. Secondary system propositions have a radical meaning holism because they are tied to a universal proposition, the theory's dictionary. In contrast, primary system propositions have meaning independent of at least some other primary system propositions. This difference tracks the broader distinction between singular propositions and non-supervenient laws and chances. That distinction is fundamentally due to the fact that Ramsey must treat the value of gambles, a form of causal propositions, as completely dependent on some non-causal proposition components. If he did not, then gambles would affect the preferences they are used to measure. This means that gambles cannot have intrinsic utility, and so they have incomplete truth-conditions. Thus they must ultimately be fictions. In short, Ramsey's anti-realism follows from a meaning dependence born out of technicalities in his decision theory that results in gambles—and with them laws, chances, and theories—having truth-conditions that depend on an agent's subjective model of the world.

7.3.3 A Different Anti-Realism

The anti-realism I have sketched here is very different from the anti-realism of the empiricist common in the decades before Ramsey wrote. That empiricism holds the distinction between the real propositions found in observation and the fictional propositions found in theories is a distinction between what can be directly observed or perceived and that which is hypothesized. Genuine propositions are things like lab reports about the values of instruments and other reports via perception; fake propositions involve entities that could not be directly perceived like the weights of atoms.

This is not the anti-realism that Ramsey offers for two reasons.

First, Ramsey makes no mention of a privileged set of observations connected with perception. While Ramsey's examples make use of this, the real distinction is between propositions intrinsically cared about and those that are used to make forecasts about the intrinsically cared propositions. Real propositions are those that an agent actually values and not merely as an instrument. The latter have truth-conditions but their truth-conditions depend in part on the agent's own subjective degrees of belief, while the former have no such dependence. This results in a meaning dependence between the fictional propositions just not found in the actual propositions. That meaning dependence and its rationale has nothing to do with a divide between perceivable and non-perceivable entities. This issue is orthogonal to the one that Ramsey really cares about: on what propositions do people's beliefs really ride?

Second, Ramsey's anti-realism *is relative*. Namely, what people intrinsically care about can change over time. This can make propositions that were formerly fictional real. The cases Ramsey gives in his notes are heat, bacteria, and genes:

> Of course, causal, fictitious, or 'occult' qualities may cease to be so as science progresses. E.g. heat, the fictitious cause of certain phenomena of expansion (and sensations, but these could be disregarded and heat considered simply so far as it comes into mechanics), is discovered to consist in the motion of small particles.
>
> So perhaps with bacteria and Mendelian characters or genes (Ramsey, [1929] 1990a, 138–139).

People might come to care intrinsically about the temperature, independent of the sensations of hot or cold, as families that fuss over the thermostat can attest. Likewise, mysophobiacs just really care about avoiding bacteria above and beyond any other consequences and similarly so might more health-conscious people. Finally, as the history of the twentieth century attests, people can become very concerned with genes for their own sake. So these things elapse from mere instruments used in making forecasts to propositions cared about for their own sake. They are part of an enlarged primary system. So Ramsey's anti-realism allows a change of status over time for theoretical propositions. Fictions become fact as their truth-conditions shift to being intrinsically cared about.

The anti-realism here is thus unique to Ramsey and not present in his contemporaries. The distinction between observable and non-observable plays practically no role here. Instead, the distinction is based on a meaning dependence due ultimately to the status of the truth-conditions of theoretical propositions.

7.4 Resolving a Core Problem

Ramsey's primary system fails to represent the agent in deliberation and action. The truth-apt propositions are just those whose truth is determined by the subjective factors of the consequences of those propositions and an objective factor, the outcomes of the action picked. Consequently, Ramsey throws out of the primary system the natural propositions for representing agent's actions and deliberation: gambles. Gambles cannot have non-causal propositions as truth-conditions because of the existence of coincidences. Nor can they have intrinsic value because of the problem of measuring credences. So their truth-conditions must involve the utility of other gambles and so the agent's credences about more than the gamble's consequences, i.e. what the agent thinks would happen with the gamble instead of what will. So the primary system is shorn of gambles along with theoretical propositions, leaving it unable to represent the agent in deliberation and action.

A secondary system fills in for the primary system's weakness. By using fictional theoretical propositions, the agent can reify the rules they follow for producing judgments in the structure of those theoretical propositions. Theories serve as fictional projections onto the world about the agent's own decision-making. While they still lack truth-conditions, their verification conditions function as a method for constraining how the agent might deliberate in the future. The value of the theory comes in the laws and chances and consequences, which are all tools reflecting how the agent deliberates and acts. So the theory is useful in guiding action because it is useful in helping the agent to think about themselves.

This last point about the true utility of a theory points to a deep element of Ramsey's philosophy. In short, Ramsey thinks that humans should be viewed as part of and in the world. Methven characterizes this as the "realistic spirit" (Methven, 2014), a reference to the following line from "General Propositions and Causality":

> Variable hypotheticals have formal analogies to other propositions which makes us take them sometimes as facts about universals, sometimes as infinite conjunctions. The analogies are misleading, difficult though they are to escape, and emotionally satisfactory as they prove to different types of mind. Both these forms of 'realism' must be rejected by the realistic spirit (Ramsey, [1929] 1990e, 160).

Ramsey's core complaint about realists for laws and chances is that they refuse to incorporate these characteristic features of human cognition into the world properly. Instead of putting the essence of the variable hypothetical—its use as a rule—front and center, these realisms treat rules as real propositions and end with monstrosities such as universals and infinite conjunctions. The idea is not germane to just "General Propositions and Causality". When interrogating the question of whether meaning is causal, Ramsey obliquely criticizes Wittgenstein for the same reasoning: "We cannot really picture the world as disconnected selves; the selves we know are in the world" (Ramsey, 1991a, 51).[15] Similar thoughts are expressed years earlier in "Epilogue", a paper presented to the Cambridge Apostles in 1925, where Ramsey attacks the style of philosophy that looks out into the world without placing us into it:

> My picture of the world is drawn in perspective, and not like a model to scale. The foreground is occupied by human beings and the stars are all as small as threepenny bits. I don't really believe in astronomy, except as a complicated description of part of the course of human and possibly animal sensation. I apply my perspective not merely to space but also to time. In time the world will cool and everything will die; but that is a long time off still, and its present value at compound discount is almost nothing. Nor is the present less valuable because the future will be blank. Humanity, which fills the foreground of my picture, I find interesting and on the whole admirable (Ramsey, 1990c, 249).

Ramsey, perhaps cheekily, mocks the despairing philosophy he attributes to Russell and Wittgenstein. They view the world as in picture; Ramsey views the world as foregrounded with human observers. The critique here is that Russell's despair is driven by an inappropriate model of the world with no one in it. In contrast, Ramsey's philosophy places humans immediately in action. A person in the world should always be looking for how he fits into the world as he understands it. Ramsey's account of laws and chances as expressions of properties of one's credences in the propositions one believes puts the agent front and center in the world. Laws and chances are not real; they are projections of one's own attitudes onto the world. Remember that "real" here for Ramsey is that they have sense as proper propositions in the primary system. This is due to his theory of truth. But laws and chances

[15]He then follows on to attack Wittgenstein explicitly: "What we can't do we can't do and it's no good trying. Philosophy comes from not understanding the logic of our language; but the logic of our language is not what Wittgenstein thought. The pictures we make to ourselves are not pictures of facts" (Ramsey, 1991a, 50).

can still account for a person in the world in the sense that they allow a person to take meaningful actions when considering them. In that sense, one can account for oneself as someone who is part of the world looking out into it.

Ramsey's appeal to the realistic spirit and his complaint that Russell or Wittgenstein's philosophy is at odds with that realistic spirit happens to be duplicated in Ramsey's attack on Carnap's project in the *Aufbau*. In drafts to "Philosophy", Ramsey attacks the solipsism incipient in Carnap's project[16] and points out that Carnap's project is useless in the precise sense that it avoids putting the person in the world and so is useless for guiding action:

> But this is clear that the definitions are to give at least our future meaning, and not merely to give any pretty way of obtaining a certain structure. They must give not merely the Logisches Wert [logical value] but also the Erkenntnis Wert [cognition value] in Carnap's terms. That is why his book is so misguided. (The structure could be obtained formally with no relation to the facts in question, a reductio ad absurdum) (Ramsey, 1991a, 43).

Ramsey argues that Carnap's constructive method has no space for learning and affecting future meaning because it has to always reconstruct the agent from the autopsychological. Definitions, as Ramsey says in the previous paragraph, must guide future meaning—influence expectations and through expectations action.[17] But a definition that just obtains some

[16]The initial drafts of "Philosophy" make it very clear that Ramsey views Carnap as edging very close to solipsism. He writes in a note titled "Can we put the problem of philosophy thus?":

> The most idealistic philosophy accepts only the given in the sense of present experience, memory, and expectation.
>
> This is solipsism of the present moment. It seems to me untenable, because in order to describe the present I should never make such elaborate constructions (Ramsey, 1991a, 34–45).

And he then goes on to describe Carnap's workarounds for this problem. Likewise, in a long note titled "Refutation of Solipsism" he targets his "proof" against the solipsism he thinks Carnap is trapped in: "Solipsism in the ordinary sense in which as e.g. in Carnap the primary world consists of my experiences past present and future will not do" (Ramsey, 1991a, 66).

[17]Ramsey says that "I do not think it is necessary to say with Moore that the definitions explain what we

pretty structure about the given cannot help in guiding future meaning because that will always require a reconstruction of the meaning in the future. This is essentially Ramsey's critique of the *Aufbau* in "Theories" when he writes "That is to say, if we proceed by explicit definition we cannot add to our theory without changing the definitions, and so the meaning of the whole" (Ramsey, [1929] 1990m, 130); definitions through pretty structures cannot guide action because when new experiences are had a new structure must be built, changing the meaning of what was meant originally and so their influence on future action. Carnap's proposal is thus purely formal without any bearing on someone living in a world and guiding his expectations in accordance with what he knows and learns about that world. So Carnap's project, like Russell's and Wittgenstein's, is at odds with the realistic spirit of foregrounding humans and their cognitive quirks in the world.

Understanding theories and their laws and chances as workarounds for the primary system's limitations helps address what exactly is a theory's surplus content (see chapter five on the Ramsey sentence)? Recall that Ramsey presumes that all theories have greater multiplicity than observation. Why should one assume that? The answer here is that the primary system is not rich enough to account for the observer. To capture oneself in the world, one always needs to go further and think of oneself as in the world through a fictional secondary system. That secondary system must allow for the possibility of one being an observer, distinct from the observation. So it must always outstrip what is provided by the primary system.

The view then is that in foregrounding the agent through the laws and chances, Ramsey is trying to solve a still unresolved problem: how to have an account of the logic of decision where the agent is part of the world that the agent acts in. Ramsey's solution is philosophical, not technical. Some of the propositions that decisions are made with respect to really be propositions about the agent themself. Laws and chances and theories are really about figuring out where I stand in the world and how I am a part of that world. In this sense,

have hitherto meant by our propositions, but rather that they show how we intend to use them in future" (Ramsey, 1991a, 42–43).

one might say that Ramsey is really the most dyed-in-the-wool realist.

7.5 Conclusion

Various philosophers have long tussled over the extent to which Ramsey is a scientific realist or a scientific anti-realist. As a realist, Ramsey would believe in the approximate truth of scientific theories; while as an anti-realist, he would believe those theories to be some sort of fictional instrument. This question might even be meaningless for Ramsey in the sense empiricists such as Carnap think the problem is a pseudo-problem.

I have argued that Ramsey did think there is a meaningful question here about the realism of scientific theories. In so far as a question has implications for clarifying thought and so action, it has some content. The answer Ramsey gives to the question then is a decisive rejection of scientific realism.

A scientific theory is fictional in two senses. First, as an existential proposition, it is a description of another witnessing proposition. This type of anti-realism, however, is shallow because the witness could be a real proposition. Second, the witness for scientific theories is a universal proposition given by the dictionary and axioms. The dictionary and axioms, as universal propositions, are not truth-apt. And since every theoretical proposition depends for its meaning on the theory's axioms and dictionary and so every other theoretical proposition, it will depend on fictional propositions. So Ramsey's anti-realism about theories is at its root an anti-realism about laws and chances.

This anti-realism explains the separation between the primary and secondary system. Fictional propositions in the secondary system depend on their meaning in the whole secondary system via the dictionary; in contrast, the primary system propositions can be independent in their meaning from other primary propositions. So a radical holism separates the two systems. This difference in holism stems from a difference in how to articulate the meaning of propositions. The meaning of a belief for Ramsey comes from the causes and effects that would follow given the actions the believer would take. These are the truth-conditions of propositions. Because causes and effects for Ramsey must be the consequences that an agent ultimately cares about, this rules out gambles having proper meaning, and so with them, laws and chances. So Ramsey's separation of propositions into primary and secondary systems stems from his theory of meaning, belief as a disposition to act, and technical features of his decision theory.

A consequence of Ramsey's view on the content of the primary system is that the primary system fails to represent the agent's deliberation and action. But importantly, Ramsey wants his philosophy to be able to represent humans as being in the world. His solution is that theories and their laws and chances are really reifications in people's algebra of propositions of the cognitive rules that they employ when making decisions. This allows in a weak sense of "real", a realistic account of agency as in the world. It is precisely this accounting that constitutes the surplus content of scientific theories.

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Appendix A

Appendix

This appendix provides a careful, in-depth recapitulation of Ramsey's toy example from the paper "Theories". The appendix is broken into three parts. The first provides a formal characterization and discussion of Ramsey's toy model. The second discusses in more detail the arguments considered earlier in this paper that Ramsey thought about the truth-possibilities as interpreted sets of possible worlds. I consider the same arguments for what they could be I discussed earlier in the paper. The third and final part provides a series of Python functions that can be run to implement Ramsey's toy model. The whole appendix is found in a Google Colab Notebook that can be found on my website.

In "Theories", Ramsey's first question is whether a scientific theory is a language. Ramsey writes, however, that he will not answer that question directly. Instead, he says that he will "describe a theory simply as a language for discussing the facts the theory is said to explain" so that "if we knew what sort of language it would be if it were one at all, we might be further towards discovering if it is one" (Ramsey, [1929] 1990m, 122). He never directly answers the question of whether scientific theories are languages. I argue that there is an answer that can be inferred from his discussion of an example he uses in the paper. He believes that if theories are languages, they are languages that describe what is possible, i.e. what is in one's space of possibilities. This means theories are more than just languages: they are about possibilities that provide laws for the observed facts.

Here is how my argument will go. I first discuss in brief outline the example Ramsey uses in "Theories" to illustrate his points. I argue that this toy model makes sense in the context of what Ramsey calls the truth-possibilities of sets of propositions. I argue that the truth-possibilities of some propositions are just sets of possibilities where those propositions are true or false. This would only work in the context if the truth-possibilities of theories shared the same underlying space of possibilities as the facts to be explained. I then use this fact to build a working program of Ramsey's toy model that illustrates how truth-possibilities behave according to a theory.

A.1 Ramsey's Toy Model

Ramsey's toy model is about an individual who experiences colors, feeling their eyes opening and shutting, and stepping forward and backward. It comes in two parts. The first part is what Ramsey calls the primary system. It consists of functions representing those immediate experiences of colors, eye movements, and body movements. The second part is what Ramsey calls the secondary system. This consists of functions that represent a map with three places that form a ring, the color experiences at those places, and whether the individual's eyes are open or closed. In Ramsey's discussion, the primary system corresponds with what philosophers call the observation language and the secondary system corresponds with what is now called the theoretical language. Both primary and secondary system are presented either in a logical form with propositional functions or a mathematical form as a system of equations. Below, I go through each system in more detail.

A.1.1 Primary System

The primary system has three functions in its mathematical form and six propositional functions in its logical form. The domain of both functions are the integers. Those integers represent time instants. Each mathematical function represents a particular type of experience. ϕ represents color experience, χ represents the individual feeling their eyes opening and shutting between two neighboring times, and ψ represents movement forward and backward. The values of these functions and their associated logical form is given in figure 1

Mathematical Form	Logical Form	English Translation
$\phi(n) = 1$	$A(n)$	I see blue at n.
$\phi(n) = -1$	$B(n)$	I see red at n.
$\phi(n) = 0$	$\neg A(n) \wedge \neg B(n)$	I see nothing at n.
$\chi(n) = 1$	$C(n)$	Between $n-1$ and n I feel my eyes open.
$\chi(n) = -1$	$D(n)$	Between $n-1$ and n I feel my eyes shut.
$\chi(n) = 0$	$\neg C(n) \wedge \neg D(n)$	Between $n-1$ and n I feel my eyes neither open nor shut.
$\psi(n) = 1$	$E(n)$	I move forward a step at n.
$\psi(n) = -1$	$F(n)$	I move backward a step at n.
$\psi(n) = 0$	$\neg E(n) \wedge \neg F(n)$	I do not move at n

The primary system is about a few things. In English, it is about whether an individual sees blue, red, or nothing at specific time instant. It also represents the individual perception of their eyes opening, closing, or doing neither between two neighboring time instants. And it depicts the individual moving forward a step, backward a step, or not moving at a time instant. Every argument to the mathematical functions and their propositional function counterparts are time instants. In short, the primary system provides an order relative to time of some of an individuals experiences. Next Ramsey introduces a secondary system for deducing general propositions and consequences in the primary system.

A.1.2 Secondary System

Ramsey's example includes a secondary system with three new functions in both mathematical and logical form. Their presentation differs slightly between the two forms. Like their primary system counterparts, they take integers as arguments. Unlike the primary system, those integers need not represent time instants but represent an ordering between propositions. I will explain this shortly. But first, the secondary system functions can be seen in figure 2.

Mathematical Form	Logical Form	English Translation
$\alpha(n) = 1$	$\alpha(n, 1)$	At time n I am at place 1.
$\alpha(n) = 2$	$\alpha(n, 2)$	At time n I am at place 2.
$\alpha(n) = 3$	$\alpha(n, 3)$	At time n I am at place 3.
$\beta(n, 1) = 1$	$\beta(n, 1)$	At time n place 1 is blue.
$\beta(n, 1) = -1$	$\neg\beta(n, 1)$	At time n place 1 is not blue.
$\beta(n, 2) = 1$	$\beta(n, 2)$	At time n place 2 is blue.
$\beta(n, 2) = -1$	$\neg\beta(n, 2)$	At time n place 2 is not blue.
$\beta(n, 3) = 1$	$\beta(n, 3)$	At time n place 3 is blue.
$\beta(n, 3) = -1$	$\neg\beta(n, 3)$	At time n place 3 is not blue.
$\gamma(n) = 1$	$\gamma(n)$	At time n my eyes are open.
$\gamma(n) = 0$	$\neg\gamma(n)$	At time n my eyes are closed.

The twin functions α and β need some explanation. α represents being at specific places at a time instant. Ramsey presents α in its logical form as $\alpha(n, m)$. He then states that m

can take only the three values 1, 2, and 3. In the mathematical form, this second argument is dropped entirely. Instead, $\alpha(n)$ can assume those same three values. What this means is that Ramsey encoded three separate propositional functions under the guise of one with $\alpha(n, m)$. This shows up in figure 2. Ramsey could have chosen different functions, such as α, η, ζ, on one argument to achieve the same effect. The same story holds for β–even in its mathematical form. β represents a specific place being blue at a time instant. For that reason, Ramsey has $\beta(n, m)$ also take two arguments for both forms. And like α, the second argument, m, can only assume three values. So like α, Ramsey could have presented the different βs on their second argument as separate functions. The reason why he did not is likely due to compactly representing the axioms and definitions. Nevertheless, the fact that m can only take three values indicates that α and β really correspond to different propositional functions in their logical form.

The last function, γ, is more straightforward. It represents the individual's eyes being open or closed. This is different from the primary system's χ, which represents the feeling of eyes being opened or closed. γ depicts those eyes continuing to be open or closed.

And that is all for the functions. The remaining part of the secondary system are the axioms.

Ramsey provides two different versions of his toy model's axioms: a logical form and a mathematical form. The number of axioms is different between both versions. In the logical form, there are four axioms:

Axioms (Logical Form)

1. $\forall n, m, m'((\alpha(n, m) \wedge \alpha(n, m')) \supset m = m')$
2. $\forall n \exists m \alpha(n, m)$
3. $\forall n \beta(n, 1)$
4. $\forall n(\beta(n, 2) \equiv \neg\beta(n + 1, 2))$

Axiom 1 says that the individual can only be at one place at a time. Axiom 2 states that the individual is at a place at all times. Axiom 3 asserts that place 1 is always blue. And axiom 4 says that place 2 alternates between being blue and not blue at neighboring time steps. Importantly, the axioms collectively permit which propositions in the secondary system are true. Equivalently, the mathematical form of the axioms allow what values the secondary system functions might assume. There are five axioms in that form:

Axioms (Mathematical Form)

1. $\forall n(\alpha(n) = 1 \lor \alpha(n) = 2 \lor \alpha(n) = 3)$
2. $\forall n(\beta(n, 1) = 1)$
3. $\forall n(\beta(n, 2) \neq \beta(n + 1, 2))$
4. $\forall n, m(\beta(n, m) = 1 \lor \beta(n, m) = -1)$
5. $\forall n(\gamma(n) = 0 \lor \gamma(n) = 1)$

Axioms 2 and 3 in the mathematical form match axioms 3 and 4 from the logical form. But axioms 1, 4, and 5 do not have exact equivalents in the logical form. Axiom 1 entails the logical form's axioms 1 and 2 along with the following axiom:

$$f(m) = \begin{cases} 2 & m = 1 \\ 3 & m = 2 \\ 1 & m = 3 \end{cases}$$

Combined with the definitions, f permits m in $\alpha(n, m)$ to assume only three values. Together with axioms 1 and 2 from the logical form, this makes α effectively three propositional functions at each time step: $\alpha(n, 1), \alpha(n, 2)$, and $\alpha(n, 3)$. And that is what axiom 1 in the mathematical form states. Lastly, axioms 4 and 5 of the mathematical form limit the mathematical versions of β and γ to only have as many values as there are as many propositional functions β and γ in the logical form. And that is it for the axioms.

A.1.3 Definitions

Ramsey also presents a dictionary of his toy model. The dictionary connects the primary and secondary system. Like the other parts, it comes in two forms: a mathematical and logical form. In its mathematical form, the functions of the primary system are ϕ, χ, and ψ. Ramsey provides the following equations for them:

Definitions (Mathematical Form)

1. $\phi(n) = \gamma(n) \times \beta(n, \alpha(n))$
2. $\chi(n) = \gamma(n) - \gamma(n-1)$
3. $\psi(n) = (\alpha(n) - \alpha(n-1)) \mod 3$

The first definition has ϕ be a function of every secondary system function. Recall that ϕ is the function that encodes the experiences of seeing red, blue, or nothing. In other words, the individual seeing a color is a function of that individual having their eyes open and being at a place that might be blue. The second definition has χ be a function of γ. Because χ represents the individual feeling their eyes open or close and γ represents those eyes being open or closed, γ's value should determine χ. And the third definition has ψ be a function of α in modular arithmetic. Since ψ depicts forward and backward steps and by stipulation there are only three places in the example that form a ring, the movement between those places is represented by α under arithmetic modular three.

In the logical form, the definitions compute the value of the atomic propositional functions

A, B, C, D, E, and F. This tracks their mathematical counterparts as was seen in figure 1. Consequently, the number of definitions is twice as long:

Definitions (Logical Form)

1. $A(n) = \exists m(\alpha(n, m) \wedge \beta(n, m) \wedge \gamma(n))$
2. $B(n) = \exists m(\alpha(n, m) \wedge \neg\beta(n, m) \wedge \gamma(n))$
3. $C(n) = \neg\gamma(n - 1) \wedge \gamma(n)$
4. $D(n) = \gamma(n - 1) \wedge \neg\gamma(n)$
5. $E(n) = \exists m(\alpha(n - 1, m) \wedge \alpha(n, f(m)))$
6. $F(n) = \exists m(\alpha(n - 1, f(m)) \wedge \alpha(n, m))$

These definitions can be read as equivalences between the left-hand side and right-hand side of the equalities. This means that $A(n)$ is equivalent to the individual being at a place at n that is blue with their eyes open. Likewise, $B(n)$ is equivalent with that individual being at a place at n that is not blue with their eyes open. The two definitions for C and D are even more straightforward. $C(n)$ is equivalent with the individual having had their eyes closed at $n - 1$ and having their eyes open at n. $D(n)$ is equivalent with reverse. Finally, E and F make use of the function f defined earlier. $E(n)$ is equivalent with there being a place that the individual was at previously $(n - 1)$ and the individual being at a place that is after the previous place. $F(n)$ reverses the order of the right-hand side. Its definition says that there is a place the individual is at presently and previously the individual was at a place after the present one.

A.1.4 Conclusion

Summing up, Ramsey's toy model treats as primary or observational the phenomenal experiences of colors and eye and foot movements. His toy model considers as secondary or theoretical a map correlating those experiences with places. The two systems are connected by a dictionary that defines the primary in terms of the secondary.

A key question to address is how the two systems operate together. What bridges them? Or more directly, what allows the dictionary to connect primary and secondary systems if they are conceived as two different languages? I argue in the next section that they are connected by a common possibility space. That means that the two systems share the same possibilities with their propositions.

A.2 Possibility Space

In this section, I argue that the primary and secondary system of Ramsey's toy model share the same possibility space. A possibility space is just a collection of possibilities or worlds. Propositions in both systems are then just sets of those possibilities. I argue that Ramsey's conception of truth-possibilities implies an underlying possibility space. Here is how my argument will proceed. I first document how truth-possibilities are used by Ramsey in "Theories". I then argue that these truth-possibilities contain possibilities or the possible worlds of "Truth and Probability". I conclude that this leads naturally to the interpretation of Ramsey's toy model as describing movements over sets of truth-possibilities.

A.2.1 Truth-Possibilities

The concepts of truth-possibility appears in the middle of "Theories" when Ramsey is discussing verification conditions. Ramsey writes about verification conditions because he wants to see if it is possible to explicitly define propositions of the secondary system propositions in terms of those propositions' primary system verification conditions. He provides two definitions of verification conditions. The first is in terms of logical consequence. The second in terms of truth-possibilities:

> We can elucidate the connection of $\sigma(p)$ and $\tau(p)$ as follows. Consider all truth-possibilities of atomic propositions in the primary system which are compatible with the dictionary and axioms.
>
> Denote such a truth-possibility by r, the dictionary and axioms by a. Then $\sigma(p)$ is the disjunction of every r such that
>
> $r \wedge \neg p \wedge a$ is a contradiction
>
> $\tau(p)$ the disjunction of every r such that
>
> $r \wedge p \wedge a$ is not a contradiction
>
> (Ramsey, [1929] 1990m, 123).

Ramsey introduces the truth-possibilities of atomic propositions in the primary system. By a truth-possibility of a set of propositions, Ramsey means the possibilities where those propositions are either true or false. The term first appears in Ramsey's review of the *Tractatus* where he discusses Wittgenstein's concept of the sense of a proposition. Ramsey summarizes Wittgenstein's view as being that propositions express their sense by the agreeing or disagreeing with the truth-possibilities of elementary propositions. He writes that "with regard to n elementary propositions there are 2^n possibilities of their truth and falsehood, which are called the truth-possibilities of the elementary propositions; similarly there are 2^n possibilities of existence and non-existence of the corresponding atomic facts" (Ramsey, 1923, 470). So the truth-possibility of a set of propositions is just as its name states: the possibilities where those propositions are true or false. Put diagrammatically, they are the rows in a truth-table. Note that this means that the number of truth-possibilities of a set is relative to the number of propositions in its set. So the number of truth-possibilities Ramsey discusses in the above passage will be 2^n for n atomic propositions in the primary system.

With truth-possibilities on the table, I can now explain what Ramsey means by a sufficient and necessary verification condition. A *sufficient* verification condition of a secondary system proposition P is the set of truth-possibilities incompatible with the negation of P and the axioms and dictionary. Intuitively, these are the truth-possibilities of the primary system ruled out by P being false. For example, consider the truncated table of truth-possibilities at $n = 0$ in figure 3:

TP #	$\phi(0) = 1$	$\phi(0) = -1$
1	1	1
2	1	0
3	0	1
4	0	0

Recall that $\phi(0) = 1$ says I see blue at time 0 and $\phi(0) = -1$ says I see red at time 0. Some of the sufficient conditions for $\alpha(0) \neq 1$ are going to be truth-possibilities one and three because they are incompatible with $\alpha(0, 1)$ and axiom 3. Of course, the actual truth possibility table will be considerably bigger than what I present here because there will be sixty-four such possibilities. But the idea is the same.

Consider now the *necessary* verification conditions of secondary system propositions P. One such condition will be a truth-possibility that is compatible with P and the axioms and dictionary. Take figure 3. Here let P be $\alpha(0) = 1$. Then truth-possibilities two and four are such conditions since they are compatible with $\alpha(0) = 1$ and the axioms and dictionary. Again, because the set of truth-possibilities formed from all atomic propositions in the primary system will be much bigger, there will be considerably more. Nevertheless,a necessary verification condition of P will be just one such set of worlds that are truth-possibilities in the primary system that are compatible with P.

Ramsey's definition of verification conditions in terms of truth-possibilities suggests the following question: could one consider the truth-possibilities of both primary and secondary systems? If there are truth-possibilities for the atomic propositions in the primary system, then there are also truth-possibilities for the atomic propositions in the secondary system. What if one considered the joint truth-possibilities for atomic propositions in both systems? Then, one would have something like figure 4, which considers the truth-possibilities over only $\{\phi(0) = 1, \phi(0) = -1, \alpha(0) = 1\}$:

TP #	$\phi(0) = 1$	$\phi(0) = -1$	$\alpha(0) = 1$
1	1	1	1
2	1	1	0
3	1	0	1
4	1	0	0
5	0	1	1
6	0	1	0
7	0	0	1
8	0	0	0

Recall that axiom 3 of the logical form says that place 1 is always blue. It rules out truth-possibilities one and five because the individual could not be at place 1 at time 0 and see red at time 0. So the table would be stripped of these due to them being eliminated. The result would be figure 5:

TP #	$\phi(0) = 1$	$\phi(0) = -1$	$\alpha(0) = 1$
2	1	1	0
3	1	0	1
4	1	0	0
6	0	1	0
7	0	0	1
8	0	0	0

The result is a truncated truth-table. I bring this up because the verification conditions Ramsey defines can be viewed as sets of rows from these truncated tables. When Ramsey defines those verification conditions in terms of truth-possibilities from atomic propositions in the primary system, the various compatibility conditions he lays down amount to selecting for certain truth-possibilities from tables like figure 5 (expanded of course with the full allotment of atomic propositions in both systems). For example, $\phi(0) = 1 \wedge \phi(0) \neq -1$ is compatible with $\alpha(0) = 1$ and the axioms and dictionary because there is a truth-possibility in figure 5 where $\phi(0) = 1, \phi(0) \neq -1$ and $\alpha(0) = 1$ (number three). And since every proposition in the secondary system is either atomic or a truth-function of atomic propositions from the secondary system, Ramsey's alternative proposal for verification conditions of propositions is basically a truth-table method. One forms the truth-possibilities from atomic propositions of both systems. Then one prunes those truth-possibilities that are incompatible with the axioms and dictionary. The leftovers amount to what could be the verification conditions for any secondary system proposition.

What this means is that Ramsey is likely thinking about his toy model as operating over sets of truth-possibilities. They are truth-possibilities of atomic propositions in both systems. The axioms and dictionary eliminate certain truth-possibilities. And the verification conditions can be thought of as the primary system fragment of these larger truth-possibilities.

This provides an alluring and integrated picture of what is happening in Ramsey's toy model. I use this to provide working code for the toy model. But what allows Ramsey to do this? Are not primary and secondary systems different languages? Do they share anything in common? To answer that question, I need to address what exactly truth-possibilities represent.

A.2.2 What exactly are truth-possibilities

I need to address why Ramsey would be able to have truth-possibilities formed from the atomic propositions of his primary and secondary systems. This means I need to address what truth-possibilities represent. Ramsey himself does not say explicitly what they represent in "Theories". Instead, I argue that the available evidence best supports they represent possibilities. There are several other options. I consider each in turn and argue they do not fit Ramsey's writings well.

The most obvious option is that truth-possibilities represent the existence or non-existence of facts. Ramsey introduces the term from his review of the *Tractatus*. So perhaps he kept how they functioned from his understanding of the *Tractatus*. This is not very likely. First, Ramsey expresses puzzlement at Wittgenstein's account of facts and the picture theory in his review of the *Tractatus* (Ramsey, 1923, 466). Second, in his paper "Universals", Ramsey explicitly rejects the distinction between objects and forms that is critical to the Tractarian account. So it would be odd for truth-possibilities to represent Tractarian facts when Ramsey thinks there are none.

The next hypothesis is that Ramsey has no account here for truth-possibilities. He never figured out what they represent. Instead, he only meant them to correspond with just the rows of the truth-tables. Support for this comes from the fact that in *On Truth* he never provides a complete account of the content of beliefs or what he calls propositional reference. A brief idea of how this goes is sketched in "Facts and Propositions". The content of a belief is given by its the causes and effects (broadly construed). These causes and effects include the bets or actions that an individual might make with such a belief. And that is it. There are several problems with this hypothesis. The first is that Ramsey does have a concept of possibility or possible world available to him. In "Truth and Probability", Ramsey introduces possible worlds or "possible courses of the world", which are the "different possible totalities of events between which our subject chooses–the ultimate organic unities" (Ramsey, [1926] 1990n, 72–73). These exhaustive options are ultimately how Ramsey constructs wagers in his decision theory. The choice of such an expansive option on a wager amounts to all the causes and effects of that wager. Since the content of a belief is just its causes and effects, it stands to reason that the worlds common to all wagers accepted on that belief just are its content. So the lack of a detailed account about the content of beliefs is perfectly compatible with how Ramsey had understood worlds from his earlier writings. Furthermore, there is evidence that Ramsey had intended continuity between what is said in "Truth and Probability", "Facts and Propositions", and his later work. Ramsey did not publish "Truth and Probability" because he intended to work the essay into a larger book along with the manuscript now called *On Truth* and his later papers (see Misak, 2020). Possible worlds and truth-possibilities were meant to live in the same system. And since truth-possibilities are just propositions and thus have propositional reference, they can be the content of beliefs. So they have causes and effects, i.e. possible worlds. Despite this story not being worked out fully, there is a story here. So the hypothesis that Ramsey has no account for truth-possibilities looks unlikely.

Another hypothesis is that Ramsey had an alternative but it is through languages and not possibilities. The idea is that truth-possibilities are syntactic: they are just sets of consistent sentences. The evidence for this comes from Ramsey not mentioning possible worlds at all in "Theories". There are several problems for this hypothesis. First, it ignores Ramsey's other work that states that the truth-bearers are *not sentences* (Ramsey, 1991b, 7). In "Theories", Ramsey explicitly talks about propositions bearing truth. So he does not interpret "proposition" as sentence. Instead, propositions—construed as propositional reference—are the contents of beliefs. Second, Ramsey's work is before the distinction between syntax and semantics was drawn. "Theories" was very likely written in 1929—a full year before the completeness and soundness theorems were proved for propositional and first order logics. While Ramsey's work does mention possible worlds, he always appears to view formal systems as being interpreted, i.e. the objects of beliefs. So truth-possibilities have to be related to beliefs and are not sentences. Together, these facts push the credence on this hypothesis low.

An additional hypothesis is that Ramsey was just not consistent. The work in "Truth and Probability" concernsitself with partial belief and probability. "Theories" deals in full belief and deductive logic. Worlds only apply to partial beliefs. So the two are just very different approaches to knowledge. This view cannot be right. It would extraordinarily surprising if Ramsey did not realize that deductive and inductive logic are perfectly compatible. And again, Ramsey intended "Theories" to be part of the same book as "Truth and Probability". It would be stunning if he thought they were fundamentally different and inconsistent. So this third hypothesis is not very likely.

Finally, this just leaves the truth-possibilities of propositions as being just as they are defined: they are the sets of possibilities where those propositions are true or false. Possibilities here are the possible worlds of "Truth and Probability". The evidence for this comes from Ramsey's definition, his introduction of worlds in "Truth and Probability", his intention to create a book centered around "Truth and Probability", and for him to be consistent, his account of partial belief would have to apply to theoretical propositions. So truth-possibilities are just sets of possibilities. To be clear, this *does not mean that Ramsey intended his possibilities to be metaphysical.* Ramsey's turn to pragmatism intended the content of beliefs to be in terms of their causes and effects. Those causes and effects were what wagers the person who holds the belief would be willing to make. Since wagers are defined over worlds, the causes and effects of a belief are connected with those worlds. Thus, it appears that

Ramsey intended worlds to be epistemic in the sense that they concern beliefs and bets.

Recapping, the most likely hypothesis is that Ramsey took truth-possibilities to represent sets of possibilities. The other hypotheses do not fit well with the available evidence. I now turn to argue that Ramsey's primary and secondary system share the same possibility space.

A.2.3 Shared Possibility Space

I argued earlier that Ramsey's discussion of truth-possibilities and verification conditions suggest that his toy model can be viewed as operating over the truth-possibilities of both primary and secondary system propositions. I have also argued that truth-possibilities are sets of possibilities or possible worlds. I claim now that the primary and secondary system share the same set of possibilities when crafting their truth-possibilities.

My argument is straightforward: there could be no joint truth-possibilities between the two systems nor could the dictionary connect propositions in one with propositions in the other unless they shared the same set of possibilities, i.e. the same possibility space. If the primary and secondary systems operated from different possibility spaces, the joint truth-possibilities formed from their atomic propositions would be empty. Furthermore, because the definitions in the dictionary compute the value of primary system atomic propositions from secondary system propositions, the truth-possibilities of the secondary system must be connected with their primary counterparts. These two facts plus the idea that truth-possibilities are just sets of possibilities means that there is an underlying possibility or conceptual space for primary and secondary systems.

The upshot is that secondary and primary systems are related by how the former divides up and eliminates truth-possibilities of the latter. The truth-table given figure 4 amounts to a finer partition of the truth-possibilities given in the truth-table found in figure 3. The secondary system axioms and the dictionary further eliminate certain truth-possibilities from the primary system. This is just to say that the secondary system in Ramsey's toy model provides through the joint truth-possibilities a finer partition of the possibility space than the primary system alone. And it eliminates areas in that possibility space by restricting what truth-possibilities are allowed.

Since the truth-possibilities of some propositions are just the sets of possibilities that make those propositions true or false, both the logical form and mathematical form truth-possibilities refer to the same sets of possibilities from Ramsey's conceptual space. Both representations are about the same thing. This can help address the question that Ramsey leads the paper with: "whether a theory is only language"? (Ramsey, [1929] 1990m, 112). Recall that instead of answering this question directly, he addresses the question of if it were a language, what sort of language would it be. The answer here is that if it were a language, it would be a language that describes an individual's possibility space. Whether the language is completely mathematical or given the gloss of a formal theory, it describes fundamentally how groups of possibilities behave. The upshot is that theories are not just languages: what makes the theory work, what allows people to make predictions, is how they affect possibility space. This changes the bets–the actions–people will take. Ramsey's evolving pragmatism applies just as much to his philosophy of science as it does elsewhere in his philosophy.

A.2.4 Conclusion

I have argued that theories are more than just languages for Ramsey: they represent an underlying possibility space. I now move to show how this shared possibility space and the joint truth-possibilities can be visualized in a working model.

A.3 Building Ramsey's Toy Example

In this section, I build the fundamentals of a working program of Ramsey's toy example using Python and the module numpy. I can do this because of the data structure created by joint truth-possibilities and the fact that this data structure is possible due to a shared possibility space underlying primary and secondary systems. Here is how I will proceed. First, I describe the main data structure representing truth-possibilities. Second, I program functions that build out the complete set of truth-possibilities of both the primary and secondary systems in the toy model. Third, I construct the axioms and dictionary of the secondary system. This will be important for deriving the laws in the following section.

A.3.1 Core Data Structure of Truth-possibilities

Since truth-possibilities of some propositions are just the possibilities where those propositions are true or false, they can be thought of as a set of assignments of truth-values to each of those propositions. This naturally lends itself to be programmed as an array of boolean values.

To illustrate, consider the primary system from Ramsey's toy model. In its logical form, he has six propositional functions, $A(n), B(n), C(n), D(n), E(n),$ and $F(n)$. Let these correspond to indices $0, 1, 2, 3, 4,$ and 5 respectively for a given time step n. Then an example (of a fragment of a) truth-possibility at $n = 0$ would be

```
[True, False, False, False, True, False]
```

which is the same as saying in the logical form $A(0) \wedge \neg B(0) \wedge \neg C(0) \wedge \neg D(0) \wedge E(0) \wedge \neg F(0)$ or in the mathematical form $\phi(0) = 1 \wedge \phi(0) \neq -1 \wedge \chi(0) \neq 1 \wedge \chi(0) \neq -1 \wedge \psi(0) = 1 \wedge \psi(0) \neq -1$. That is the first (zeroth) index corresponds to the value of $A(0)$, the second of (1st) index to $B(0)$, and so on. Of course, this is a truncated truth-possibility because one would have to include also the truth-possibilities given by $1, 2$, and so on up to some time. To make things legible, I will assume the convention of indexing truth-possibilities relative to a time step, but it should be understood that the actual truth-possibilities involve combining these arrays from different time steps.

The secondary system behaves similarly. In its logical form, it has seven propositional functions, $\alpha(n,1), \alpha(n,2), \alpha(n,3), \beta(0,1), \beta(0,2), \beta(0,3),$ and $\gamma(0)$. Again, index them from 0 to 6. An example at $n = 0$ would be

```
[True, False, False, False, True, True, True]
```

which says in its logical form that $\alpha(0,1) \wedge \neg\alpha(0,2) \wedge \neg\alpha(0,3) \wedge \neg\beta(0,1) \wedge \beta(0,2) \wedge \beta(0,3) \wedge \gamma(0)$ or $\alpha(0) = 1 \wedge \alpha(0) \neq 2 \wedge \alpha(0) \neq 3 \wedge \beta(0,1) = -1 \wedge \beta(0,2) = 1 \wedge \beta(0,3) = 1 \wedge \gamma(0) = 1$. The first (zeroth) index says that $\alpha(0,1)$ is true, the second (first) index that $\alpha(0,2)$ is false, and so on.

Finally, both truth-possibility formats can be combined in the manner documented in figure 5. Here, I put the propositions in the primary system first and the secondary system propositions second. Combined that results in an array with thirteen boolean values (six from the primary and seven from the secondary). The combined output from the two prior examples would look like this:

```
[True, False, False, False, True, False, True, False, False, False, True,
True, True]
```

which corresponds to the assertions from the previous two truth-possibilities. Indices 0 through 5 denote the primary system propositions while indices 6 through 12 denote the secondary system propositions. For example, $\alpha(0,2)$ is false because index 7 is false. I will use this combined truth-possibility because it allows for keeping track of the relationship between primary and secondary system truth-possibilities. This will be especially useful with understanding the dictionary.

I have sketched what the basic data structure looks like. Now I build it. The first thing is to import the necessary functions and methods that will make manipulating the data structure easier.

[ ]:
```
from mpl_toolkits import mplot3d
import numpy as np
import matplotlib.pyplot as plt
%matplotlib inline
%matplotlib notebook
```

Next, I add the functions for building the un-edited truth-possibilities. These will make the initial truth-tables for Ramsey's toy model.

[ ]:
```
def build_worlds_func(set_worlds, indx, world, maximum):
    """
    Used to build individual sets of truth-possibilities.


    :param set_worlds: the final set of worlds to be built
    :param world: the current world being built
    :param maximum: the maximum size of the worlds to be built
    :return Void: Fills out a set.
    """
    if len(world) >= maximum:
        set_worlds[indx] = world
    else:
        world_1 = world.copy()
```

```
        world_1.append(True)
        world_2 = world.copy()
        world_2.append(False)
        build_worlds_func(set_worlds, 2*indx, world_1, maximum)
        build_worlds_func(set_worlds, (2*indx)+1, world_2, maximum)

def expand_worlds_func(n):
    """
    Builds a single set of truth-possibilities.

    :param n: the number of atomic propositions
    :return list: Returns a list of sets of truth-possibilities.
    """
    worlds = [[] for j in range(2**n)]
    build_worlds_func(worlds,0,[],n)
    return worlds
```

Axioms and Dictionary

Now I add the axioms and the dictionary. The axioms for the secondary system are given below. I use the logical form of the axioms for easier implementation.

[ ]:
```
def axiom1_func(w):
    """
    The axiom disallows propositions 6, 7, and 8 from being true in the␣
 ↪same world.  A violation happens if
    any of these two are true.
```

```
    :param w: Possible world w.
    :return Boolean: Returns true if the axiom is violated.
    """
    return (w[6] and w[7]) or (w[6] and w[8]) or (w[7] and w[8])

def axiom2_func(w):
    """
    The axioms states that one of the propositions 6, 7, or 8 must be␣
 ↪true in every world.

    :param w: Possible world w.
    :return Boolean: Returns true if the axiom is violated.
    """
    return not(w[6] or w[7] or w[8])

def axiom3_func(w):
    """
    The axiom states that proposition 9 is always true.

    :param w: Possible world w.
    :return Boolean: Returns true if the axiom is violated.
    """
    return not(w[9])

def axiom4_func(W, indx):
```

```
    """
    Checks to see whether the index world agrees in its truth values with␣
↪every world in the next step over
    on propositions 10.

    :param W: Sets of possible worlds.
    :param indx: The world to be checked if it is possible.
    :return Boolean: Returns true if the axiom is violated.
    """

    W_prime = W.copy()
    W_prime[indx[0]] = [W[indx[0]][indx[1]]]

    #Check the forward case
    if indx[0] < (len(W_prime) - 1):
        w = np.asarray(W_prime[indx[0]])
        wp = np.asarray(W_prime[indx[0]+1])
        if (np.all(w[:,10] == True) and np.all(wp[:,10] == True)) or (np.
↪all(w[:,10] == False) and \
                                                                          ␣
↪np.all(wp[:,10] == False)):
            return True

    #Check the backward case
    if indx[0] > 0:
        w = np.asarray(W_prime[indx[0]])
```

```
        wp = np.asarray(W_prime[indx[0]-1])
        if (np.all(w[:,10] == True) and np.all(wp[:,10] == True)) or (np.
↪all(w[:,10] == False) and \
                                                                          ␣
↪np.all(wp[:,10] == False)):
            return True
        else:
            return False
    else:
        return False
```

Now I add the dictionary. Again, I use the logical form:

```
[ ]: def prop0_func(w):
    """
    Flags a violation if the world has A(n) as true while there is not a␣
↪place that you are at that is also blue
    with your eyes open.

    :param w: A possible world.
    :return Boolean: Returns true if the world violates the definition␣
↪for proposition 0.
    """
    if w[0] == True:
        for i in range(6, 9, 1):
            if w[i] == True and w[i+3] == True and w[12] == True:
                return False
```

```
        return True
    else:
        for i in range(6, 9, 1):
            if w[i] == True and w[i+3] == True and w[12] == True:
                return True
        return False


def prop1_func(w):
    """
    Same as prop0_func except checks to see whether it is not blue when␣
 ↪you open your eyes.

    :param w: A possible world.
    :return Boolean: Returns true if the world violates the definition␣
 ↪for proposition 1.
    """
    if w[1] == True:
        for i in range(6, 9, 1):
            if w[i] == True and w[i+3] == False and w[12] == True:
                return False
        return True
    else:
        for i in range(6, 9, 1):
            if w[i] == True and w[i+3] == False and w[12] == True:
                return True
        return False
```

```python
def prop2_func(W, indx):
    """
    Returns a violation if the indexed world is true at prop 2 and true␣
 ↪at prop 12 but has no world in the set
    before it where prop 12 is false.

    :param W: A history of possible worlds.
    :param indx: The index of the possible world to be tested.
    :return Boolean: Returns true if the world violates the definition␣
 ↪for proposition 2.
    """
    W_prime = W.copy()
    W_prime[indx[0]] = [W_prime[indx[0]][indx[1]]]
    w = np.asarray(W_prime[indx[0]])

    if np.all(w[:,2] == True):
        if np.all(w[:,12] == False):
            return True
        elif indx[0] > 0 and np.all(np.asarray(W_prime[indx[0]-1])[:,12]␣
 ↪== True):
            return True
        else:
            return False
    else:
        if np.all(w[:,12] == False):
```

```
            return False
        elif indx[0] > 0 and np.all(np.asarray(W_prime[indx[0]-1])[:,12]␣
 ↪== True):
            return False
        elif indx[0] == 0:
            return False
        else:
            return True


def prop3_func(W, indx):
    """
    Returns a violation if the indexed world is true at prop 3 and true␣
 ↪at prop 12 or the previous world
    is false at 12.

    :param W: A history of possible worlds.
    :param indx: The index of the possible world to be tested.
    :return Boolean: Returns true if the world violates the definition␣
 ↪for proposition 3.
    """
    W_prime = W.copy()
    W_prime[indx[0]] = [W_prime[indx[0]][indx[1]]]
    w = np.asarray(W_prime[indx[0]])

    if np.all(w[:,3] == True):
        if np.all(w[:,12] == True):
```

```
            return True
        elif indx[0] > 0 and np.all(np.asarray(W_prime[indx[0]-1])[:,12]␣
↪== False):
            return True
        else:
            return False
    else:
        if np.all(w[:,12] == True):
            return False
        elif indx[0] > 0 and np.all(np.asarray(W_prime[indx[0]-1])[:,12]␣
↪== False):
            return False
        elif indx[0] == 0:
            return False
        else:
            return True


def prop4_func(W, indx):
    """
    Check to see if proposition 4 is true and the index is greater than 0.
↪  If so, it defines f_i on a loop
    between 6 and 8.  Then goes through that loops and checks to see if␣
↪the previous set of worlds has a world
    where proposition i (6,7,8) - 1 is true.  If so, there is no␣
↪violation.
```

```
    :param W: A history of possible worlds.
    :param indx: The index of the possible world to be tested.
    :return Boolean: Return true if the world violates the definition for␣
↪proposition 4.
    """
    W_prime = W.copy()
    W_prime[indx[0]] = [W_prime[indx[0]][indx[1]]]
    w = np.asarray(W_prime[indx[0]])

    if np.all(w[:,4] == True):
        # Check if the forward movement proposition is true and consonant␣
↪with where one has been.
        for i in range(6, 9, 1):
            if i < 8:
                f_i = i+1
            else:
                f_i = 6

            if indx[0] > 0 and np.any(np.asarray(W_prime[indx[0]-1])[:,i]␣
↪== True) and \
            np.any(w[:,f_i] == True):
                return False
            elif indx[0] == 0 and np.any(w[:,f_i]) == True and np.all(w[:
↪,5] == False):
                return False
        return True
```

```
    else:
        # Check if the forward movement proposition is false and␣
↪consonant where one has been.
        for i in range(6, 9, 1):
            if i < 8:
                f_i = i+1
            else:
                f_i = 6

            if indx[0] > 0 and np.any(np.asarray(W_prime[indx[0]-1])[:,i]␣
↪== True) and \
            np.any(w[:,f_i] == True):
                return True
            elif indx[0] == 0 and np.any(w[:,f_i] == True) and np.all(w[:
↪,5] == False):
                return True
        return False

def prop5_func(W, indx):
    """
    Check to see if proposition 4 is true and the index is greater than 0.
↪  If so, it defines f_i on a loop
    between 6 and 8.  Then goes through that loops and checks to see if␣
↪the previous set of worlds has a world
    where proposition i (6,7,8) - 1 is true.  If so, there is no␣
↪violation.
```

```
    :param W: A history of possible worlds.
    :param indx: The index of the possible world to be tested.
    :return Boolean: Return true if the world violates the definition for␣
↪proposition 4.
    """
    W_prime = W.copy()
    W_prime[indx[0]] = [W_prime[indx[0]][indx[1]]]
    w = np.asarray(W_prime[indx[0]])

    if np.all(w[:,5] == True):
        for i in range(6, 9, 1):
            if i < 8:
                f_i = i+1
            else:
                f_i = 6

            if indx[0] > 0 and np.any(np.asarray(W_prime[indx[0]-1])[:
↪,f_i] == True) and \
            np.any(w[:,i] == True):
                return False
            elif indx[0] == 0 and np.any(w[:,i] == True) and np.all(w[:
↪,4] == False):
                return False
        return True
    else:
```

```
        for i in range(6, 9, 1):
            if i < 8:
                f_i = i+1
            else:
                f_i = 6

            if indx[0] > 0 and np.any(np.asarray(W_prime[indx[0]-1])[:
↪,f_i] == True) and \
            np.any(w[:,i] == True):
                return True
            elif indx[0] == 0 and np.any(w[:,i] == True) and np.all(w[:
↪,4] == False):
                return True
        return False
```

The next step is to add the functions that implement the axioms and dictionary across truth-possibilities:

[ ]:
```
def possible_sec_func(history, indx):
    """
    :param history: A history of truth-possibilities.
    :param indx: Index of the truth-possibility to be tested.
    :return Boolean: Returns true if the truth-possibility is possible.
    """
    # Check the axioms and then the definitions
    w = history[indx[0]][indx[1]]
```

```
    if axiom1_func(w) or axiom2_func(w) or axiom3_func(w) or␣
↪axiom4_func(history, indx):
        return False
    elif prop0_func(w) or prop1_func(w) or prop2_func(history, indx) or␣
↪prop3_func(history, indx) or \
    prop4_func(history, indx) or prop5_func(history, indx):
        return False
    else:
        return True


def screen_second_func(history):
    """
    Check to see whether a history of truth-possibilities is possible.
    It checks every truth-possibility at that history's frontier for␣
↪possibility.

    :param history:  A history of sets of truth-possibilities.
    :return list:  Returns a pruned history of sets of␣
↪truth-possibilities.
    """
    frontier_indx = len(history)-1
    W = history[:frontier_indx]
    safe_frontier = []
    for i in range(len(history[frontier_indx])):
        if possible_sec_func(history, (frontier_indx,i)):
            safe_frontier.append(history[frontier_indx][i])
```

```
    W.append(safe_frontier)

    return W

def DLS_worlds_func(history, limit, n, screen_func):
    """
    Starter limited depth-first search.  The core function called below␣
↪is recurse_DLS_worlds_func, which
    duplicates some of this machinery but returns a singleton intsead of␣
↪a set of worlds.  While I could
    use one function here, separating the two out mechanically allows for␣
↪a much faster search process.

    :param history: list of sets of truth-possibilities
    :param limit: the maximum depth to be searched
    :param n: the number of propositions
    :screen_func: the screening function for invalidating␣
↪truth-possibilities
    :return list: returns the history with a safe frontier
    """
    # Construct and perform initial screen of the frontier
    history_prime = history.copy()
    history_prime.append(expand_worlds_func(n))
    history_prime = screen_func(history_prime)

    # Construct the safe frontier
```

```
    safe_frontier = []
    for w in history_prime[len(history_prime)-1]:
        alt_hist = history_prime.copy()
        alt_hist[len(alt_hist)-1] = [w]
        alt_hist.append(expand_worlds_func(n))
        alt_hist = screen_func(alt_hist)
        if recurse_DLS_worlds_func(alt_hist, limit, n, screen_func):
            safe_frontier.append(w)
    history_prime[len(history_prime)-1] = safe_frontier
    return history_prime


def recurse_DLS_worlds_func(history, limit, n, screen_func):
    """
    Standard limited depth-first search.  Failure condition is the empty␣
 ↪list.  Success condition is either
    a non-empty list at the limit or a successful singleton found.

    :param history: list of sets of truth-possibilities
    :param limit: the maximum depth to be searched
    :param n: the number of propositions
    :screen_func: the screening function for invalidating␣
 ↪truth-possibilities
    :return list: returns a singleton world.
    """
    init_frontier = history[len(history)-1]
    if not(init_frontier):
```

```
        return init_frontier
    elif limit <= 0:
        return init_frontier
    else:
        single_frontier = []
        for w in init_frontier:
            alt_hist = history.copy()
            alt_hist[len(alt_hist)-1] = [w]
            alt_hist.append(expand_worlds_func(n))
            alt_hist = screen_func(alt_hist)
            if recurse_DLS_worlds_func(alt_hist, limit-1, n, screen_func):
                single_frontier.append(w)
                return single_frontier
        return single_frontier


def construct_worldline_func(worldlines, hist, limit, n, screen_func):
    if limit == 0:
        return hist
    else:
        hist_prime = DLS_worlds_func(hist, limit+3, n, screen_func)
        for w in hist_prime[len(hist_prime)-1]:
            alt_hist = hist_prime.copy()
            alt_hist[len(alt_hist)-1] = [w]
            if limit > 1:
                worldlines = worldlines + construct_worldline_func([],␣
 ↪alt_hist, limit-1, n, screen_func)
```

```
            else:
                worldlines.append(construct_worldline_func(worldlines,␣
↪alt_hist, limit-1, n, screen_func))
        return worldlines
```

A.3.2 Visualization Tools

With those parts in place, we need visualization tools for seeing how the toy model operates. These will help with visualizing the worlds.

```
[ ]: def print_history(history):
         """Prints the first 3 worlds from each time step."""
         hist_np = np.asarray(history)
         for i in range(hist_np.shape[1]):
             print("At time step " + str(i) + " three truth-possibilities are:
↪")
             print(hist_np[:3,i,:,:])


     def convert_hist2numpy_func(history):
         """
         :param history: A list of lists of possible worlds.
         :return array: Returns an array with filled in missing values as␣
↪zeros.
         """

         # Build zeros of the maximum number of possible worlds contained in␣
↪the history
```

```
    data = np.zeros((len(history),len(max(history)),2))

    # Fill in the data
    for i,ws in enumerate(history):
        for j,w in enumerate(ws):
            data[i,j,0] = int(w[0]) + 2*int(w[1])
            data[i,j,1] = int(w[4]) + 2*int(w[5])

    return data

def convert_histories_func(set_hist):
    data = np.zeros((len(set_hist),len(set_hist[0]),1,2))
    for i,h in enumerate(set_hist):
        data[i] = convert_hist2numpy_func(h)
    return data

def plot_3d_hist_func(histories):
    zdata = np.zeros((histories.shape[1],histories.shape[2]))
    for i in range(zdata.shape[0]):
        zdata[i,:] = i

    fig = plt.figure(figsize=(8,6))
    ax = plt.axes(projection='3d')
    ax.set_xticks([0.0,1.0,2.0])
    ax.set_xticklabels(['Nothing','Blue','Red'])
    ax.set_xlabel('Colors')
```

```python
ax.set_yticks([0.0,1.0,2.0])
ax.set_yticklabels(['Nothing','Forward','Backward'])
ax.set_ylabel('Movement')
ax.set_zticks(zdata[:,0].tolist())
ax.set_zlabel('Time Step')

for h in histories:
    xdata = h[:,:,0]
    ydata = h[:,:,1]
    ax.scatter3D(xdata,ydata,zdata)

    ax.plot(xdata.flatten(),ydata.flatten(),zdata.flatten())

plt.show()
```

A.4 An Example

To illustrate how this would work, consider the sequence where I am at place 1 with my eyes open. This state corresponds to the following truth-possibility:

`[False, True, True, False, True, False, False, True, False, True, False, True, True]`

`[False, True, False, False, True, False, False, False, True, True, True, False, True]`

`[True, False, False, False, True, False, True, False, False, True, False,`

```
False, True]
```

This says that I was at place two, saw red, and moved forward. Then I was at place three, saw red, and moved forward. And then I was at place one after moving forward and saw blue. I now want to ask what possibilities remain over the next two time steps:

```
[ ]: root1 = [[[False, True, True, False, True, False, False, True, False,␣
      ↪True, False, True, True]],
             [[False, True, False, False, True, False, False, False, True,␣
      ↪True, True, False, True]],
             [[True, False, False, False, True, False, True, False, False,␣
      ↪True, False, False, True]]]


     future1 = construct_worldline_func([], root1, 2, 13, screen_second_func)
```

An example of what this history looks like can be viewed below, where I print out the first three truth-possibilities from each time step.

```
[ ]: print_history(future1)
```

```
At time step 0 three truth-possibilities are:
[[[False  True  True False  True False False  True False  True False
    True  True]]


 [[False  True  True False  True False False  True False  True False
    True  True]]


 [[False  True  True False  True False False  True False  True False
    True  True]]]
```

```
At time step 1 three truth-possibilities are:
[[[False  True False False  True False False False  True  True  True
   False  True]]

 [[False  True False False  True False False False  True  True  True
   False  True]]

 [[False  True False False  True False False False  True  True  True
   False  True]]]
At time step 2 three truth-possibilities are:
[[[ True False False False  True False  True False False  True False
   False  True]]

 [[ True False False False  True False  True False False  True False
   False  True]]

 [[ True False False False  True False  True False False  True False
   False  True]]]
At time step 3 three truth-possibilities are:
[[[ True False False False  True False False  True False  True  True
    True  True]]

 [[ True False False False  True False False  True False  True  True
    True  True]]

 [[ True False False False  True False False  True False  True  True
    True  True]]]
```

```
At time step 4 three truth-possibilities are:
[[[ True False False False  True False False False  True  True False
    True  True]]

 [[ True False False False False  True  True False False  True False
    True  True]]

 [[ True False False False False  True  True False False  True False
   False  True]]]
```

We see here that we start to have certain regularities appear in time steps 3 and 4. For example, we see that when the sixth Boolean value is `True`, the first Boolean value is `True` for time step three. This says that taking a step forward will ensure we see blue. Why? We can observe that we are at place 2 (the eighth Boolean value is `True`). And three time steps before, we were at place 2 and we observed red. Since place 2 always alternates blue and red, that means that by time step 3, place 2 will be blue. And we happened to have moved three steps forward and one step backward so we have ended up back at place 2.

This same lesson applies generally to the pattern of colors and movements. We can focus on the behavior of the first, second, fifth, and six Boolean values in the truth-possibilities. These correspond with seeing blue, red, or nothing and moving forward, backward, or staying still. If we plot those behaviors in time, we have the following collection of total truth-possibilities:

[ ]:
```
data = convert_histories_func(future1)


plot_3d_hist_func(data)
```

Each line in the above graph documents a particular truth-possibility across time. An examination reveals that we cannot see red at time step three unless we move backward.

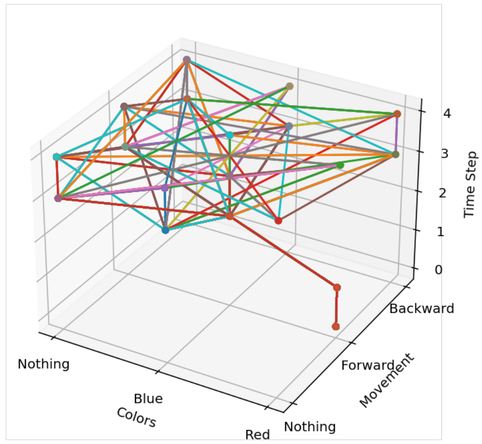

Figure A.1: A complete plot of the worlds given by the axioms and dictionary of the theory.

These patterns are what Ramsey calls laws in his toy model. The laws describe these regularities among the truth-possibilities. I now turn to introduce them and provide code for them to see if they match what the secondary system produces.

A.5 Laws

There are five fundamental laws in the primary system that Ramsey deduces with the aid of his secondary system. A law for Ramsey in the toy model is a universal proposition. Those laws, like the rest of his toy model, come in a logical and a mathematical form. I present both forms here.

A.5.1 Law 1

The first law essentially rules out certain combinations of propositions being true together in the primary system:

Mathematical Form

$$\phi(n) = -1 \vee 0 \vee 1, \quad \chi(n) = -1 \vee 0 \vee 1, \quad \psi(n) = -1 \vee 0 \vee 1$$

Logical Form

$$\forall n((\neg A(n) \vee \neg B(n)) \wedge (\neg C(n) \vee \neg D(n)) \wedge (\neg E(n) \vee \neg F(n)))$$

This rules out seeing blue and red at the same time, opening and closing one's eyes at the same time, and taking a step forward and backward at the same time. It's code is:

[ ]:
```
def lawOne(w):
    return (w[0] and w[1]) or (w[2] and w[3]) or (w[4] and w[5])
```

A.5.2 Law 2

The second law says that if one opened or closed one's eyes and then did the same again later, one must have done the other action in between:

Mathematical Form

$$\forall n, m(|\sum_{r=n}^{m} \chi(r)| \leq 1$$

Logical Form

$$\forall n_1, n_2((n_1 > n_2 \wedge C(n_1) \wedge C(n_2)) \supset \exists n_3(n_1 > n_3 > n_2 \wedge D(n_3))$$

$$\forall n_1, n_2((n_1 > n_2 \wedge D(n_1) \wedge D(n_2)) \supset \exists n_3(n_1 > n_3 > n_2 \wedge C(n_3))$$

The code for it is also below:

[ ]:
```
def lawTwo(W, indx):
    """
    Checks to see whether a possible world keeps your eyes closed or open␣
 ↪between two time steps.

    :param W: Set of possible worlds
    :param indx: The index world being tested
    :return Boolean: Returns true if the world violates law 2 or false if␣
 ↪it does not.
    """
    W_prime = W.copy()
    W_prime[indx[0]] = [W[indx[0]][indx[1]]]
    w = np.asarray(W_prime[indx[0]])

    for i in range(len(W_prime)):
        if indx[0] > i and np.all(w[:,2] == True) and np.all(np.
 ↪asarray(W[i])[:,2] == True):
            for k in range(i,indx[0],1):
                if k > i and np.any(np.asarray(W[k])[:,3] == True):
                        return False
            return True
        elif indx[0] > i and np.all(w[:,3] == True) and np.all(np.
 ↪asarray(W[i])[:,3] == True):
            for k in range(i,indx[0],1):
                if k > i and np.any(np.asarray(W[k])[:,2] == True):
```

```
                return False
            return True
    return False
```

A.5.3 Laws 3 and 4

The third and fourth laws complement one another. The third law says that if your eyes have been open, then you have to see either red or blue:

Mathematical Form

$$\forall n(\exists m(\sum_{r=m}^{n}\chi(r) = 1) \supset \phi(n) \neq 0$$

Logical Form

$$\forall n(\forall v(\exists n_1(C(n_1) \wedge n_1 \leq n \wedge n \geq v > n_1) \supset \neg D(v)) \supset (A(n) \vee B(n))$$

The fourth law says something very similar. It states that if your eyes have been shut, then you will see neither red nor blue:

Mathematical Form

$$\forall n(\exists m(\sum_{r=m}^{n}\chi(r) = -1) \supset \phi(n) = 0$$

Logical Form

$$\forall n(\forall v(\exists n_1(D(n_1) \wedge n_1 \leq n \wedge n \geq v > n_1) \supset \neg C(v)) \supset (\neg A(n) \wedge \neg B(n))$$

The code for both laws is given below:

```
[ ]: def lawThree(W, indx):
        """
        Checks to see whether the indexed world is a world where it asserts␣
     ↪that you see nothing without having opened
        and closed your eyes before the indexed world.

        :param W: Possible Worlds
        :param indx: The index world being tested
        :return boolean: Returns true if the law is violated, false otherwise.
        """

        # Create a set of worlds where the index world is the only world at␣
     ↪its time slice.
        # This W_prime will be the set of worlds the rest of
        W_prime = W.copy()
        W_prime[indx[0]] = [W[indx[0]][indx[1]]]

        if W_prime[indx[0]][0][0] == True or W_prime[indx[0]][0][1] == True:
            return False
        else:
            # Find a n1 <= indx[0] such that I eyes openened at n1 and I did␣
     ↪not shut my eyes after it.
            for n1 in range(indx[0]+1):
                if np.all(np.asarray(W_prime[n1])[:,2] == True):
                    violation = True
```

```
                if n1 < indx[0]:
                    # Checks all the vs > n1 to see if the eyes were shut.
                    for v in range(n1,indx[0]+1,1):
                        if np.any(np.asarray(W_prime[v])[:,3] == True):
                            violation = False
                            break
                if violation == True:
                    break
            else:
                violation = False
        return violation


def lawFour(W, indx):
    """
    Checks to see whether the indexed world is a world where it asserts␣
 ↪that you see red or blue but your eyes have
    been closed previously without opening.

    :param W: Possible worlds.
    :param indx: The index of the world being tested.
    :return boolean: Returns true if the law is violated, false otherwise.
    """

    # Create a set of worlds where the index world is the only world at␣
 ↪its time slice.
    # This W_prime will be the set of worlds the rest of
```

```
    W_prime = W.copy()
    W_prime[indx[0]] = [W[indx[0]][indx[1]]]

    if W_prime[indx[0]][0][0] == False and W_prime[indx[0]][0][1] ==␣
↪False:
        return False
    else:
        # Find a n1 <= indx[0] such that I closed my eyes at n1 and I did␣
↪not open my eyes after it.
        for n1 in range(indx[0]+1):
            if np.all(np.asarray(W_prime[n1])[:,3] == True):
                violation = True
                if n1 < indx[0]:
                    # Checks all the vs > n1 to see if the eyes were shut.
                    for v in range(n1,indx[0]+1,1):
                        if np.any(np.asarray(W_prime[v])[:,2] == True):
                            violation = False
                            break
                if violation == True:
                    break
            else:
                violation = False
        return violation
```

A.5.4 Law 5

The fifth law is the most complicated. What it says is that if one has had a pattern of movement such that one has not seen red during those movements, then one will see either nothing or red. Ramsey envisions this law as essentially keeping a counter. The law tracks how many steps forward or backward one has taken modulo three. If one has taken the right number of steps and not seen red, the law ensures red will be seen. The mathematical and logical forms are given below:

Mathematical Form

$$\forall n \exists m (\forall n' (\sum_{r=n}^{n'} \psi(r) \equiv m \mod 3 \supset \phi(n') \neq -1) \wedge \forall n', n'' ((\sum_{r=n}^{n'} \psi(r) \equiv \sum_{r=n}^{n''} \psi(r) \equiv$$

$$m - 1 \mod 3 \wedge n' \equiv n'' + 1 \mod 2) \supset (\phi(n') \times \phi(n'') = 0 \vee -1$$

Logical Form

$$\forall n \exists m (m = 0 \vee 1 \vee 2 \wedge \forall v (m(v, n) \supset \neg B(v)) \wedge \forall v_1, v_2 (((m-1)(v_1, n) \wedge$$

$$(m-1)(v_2, n) \wedge v_1 \not\equiv v_2 \mod 2) \supset ((\neg A(v_1) \vee \neg A(v_2)) \wedge ((\neg B(v_1) \vee \neg B(v_2)))$$

where the functions $m(n_1, n_2)$ are defined as the proposition asserting that the number computed by counting the number of times one has stepped forward from n_1 to n_2 minus the number of times one has stepped backward is equivalent with m modulo three. In code, this is complicated but it can be seen below:

[ ]:
```
def lawFive(W, indx):
    """
```

```
    This function is complicated but it basically defines what you will␣
↪see depending on how many steps you have
    taken forward or backward (whether a world's index 4 or 5 are True).

    The return value indicates whether the law is violated by the␣
↪possible world given at the index.

    :param W, indx:
    :return Boolean:
    """

    # Create a set of worlds where the index world is the only world at␣
↪its time slice.
    # This W_prime will be the set of worlds the rest of
    W_prime = W.copy()
    W_prime[indx[0]] = [W[indx[0]][indx[1]]]

    def mSub(m):
        if m < 1:
            return 2
        else:
            return m - 1

    def congruent(a, b, n):
        if a % n == b % n:
            return True
```

```
        else:
            return False

    def Nabstraction(con1, con2):
        # Returns the difference of the size of two sets defined by two␣
↪conditions.
        Nc1 = 0
        Nc2 = 0

        for i in range(len(W_prime)):
            if con1(i):
                Nc1 += 1
            if con2(i):
                Nc2 += 1

        return Nc1 - Nc2

    def RamseyCondition(m, n1, n2):
        # Satisfies the first condition of Ramsey's conditional.
        if not(m == 0 or m == 1 or m == 2):
            return True
        else:
            func1 = lambda x: x > n1 and x <= n2 and np.all(np.
↪asarray(W_prime[x])[:,4] == True)
            func2 = lambda x: x > n1 and x <= n2 and np.all(np.
↪asarray(W_prime[x])[:,5] == True)
```

```
        if congruent(Nabstraction(func1, func2), m, 3):
            return True
        else:
            return False

def clause1(m, n):
    """
    Checks to see whether the world satisfies the Ramsey condition␣
↪and it red.  If so, flags it as potentially
    an impossible world.

    What it says is that if a counterexample to clause 1 can be␣
↪found, then return False.  Otherwise, return
    True.

    :param m: The right hand modulo term.
    :param n: The upper bound of search on the Ramsey condition.
    :return Boolean:
    """
    for v in range(n):
        if RamseyCondition(m, v, n) and np.all(np.
↪asarray(W_prime[v])[:,1] == True):
            return False
    return True
```

```
    def clause2(m, n):
        """
        Checks to see whether there are two worlds that are spaced apart␣
↪from one another modulo 3 and not
        congruent modulo 2 and that happen to either both be blue or both␣
↪be red.

        What it says is that if a counterexample to clause 2 can be␣
↪found, return False.  Otherwise, return True.

        :param m:
        :param n:
        :return Boolean:
        """
        for v1 in range(n):
            for v2 in range(n):
                if RamseyCondition(mSub(m),v1,n) and␣
↪RamseyCondition(mSub(m),v2,n) and not(congruent(v1,v2,2)) and \
                        ((np.all(np.asarray(W_prime[v1])[:,0] == True)␣
↪and \
                          np.all(np.asarray(W_prime[v2])[:,0] == True))␣
↪or \
                         (np.all(np.asarray(W_prime[v1])[:,1] == True)␣
↪and \
                        np.all(np.asarray(W_prime[v2])[:,1] == True))):
                    return False
```

```
        return True

    # The test is run here. We take our index and then check if the␣
↪modified possible world structure (W_prime)
    # values of m satisfies either clauses.  If it does, then the␣
↪possible world is illegitimate.
    for m in range(3):
        if clause1(m, indx[0]) and clause2(m, indx[0]):
            return False
    return True
```

[ ]:
```
def possible_prim_func(history, indx):
    """
    Checks to see whether a given world at an index violates any of␣
↪Ramsey's laws.

    :param history:  A history of sets of possible worlds.
    :param indx:  A 2-tuple that specifies what world is being tested in␣
↪the history.
    :return Boolean:  Returns True if the world is possible; False␣
↪otherwise.
    """
    return not(lawOne(history[indx[0]][indx[1]]) or lawTwo(history, indx)␣
↪or lawThree(history, indx) or
               lawFour(history, indx) or lawFive(history, indx))


def screen_prim_func(history):
```

```
    """
    Check to see whether a history of worlds is possible.  It checks␣
↪every world at that history's frontier
    for possibility.

    :param history:  A history of sets of possible worlds.
    :return list:  Returns a pruned history of sets of possible worlds.
    """
    frontier_indx = len(history)-1
    W = history[:frontier_indx]
    safe_frontier = []
    for i in range(len(history[frontier_indx])):
        if possible_prim_func(history, (frontier_indx,i)):
            safe_frontier.append(history[frontier_indx][i])
    W.append(safe_frontier)

    return W
```

We now can see whether the laws match up with Ramsey's full system. We keep the first six entries from the previous truth-possibilities and have:

```
[False, True, True, False, True, False]

[False, True, False, False, True, False]

[True, False, False, False, True, False]
```

We run the same functions and observe that:

[ ]:
```
root2 = [[[False, True, True, False, True, False]], [[False, True, False,␣
↪False, True, False]],
        [[True, False, False, False, True, False]]]


future2 = construct_worldline_func([], root2, 2, 6, screen_prim_func)
```

Checking the history we see overlap with the first three functions and what can be observed in the first three entries of each truth-possibility:

[ ]:
```
print_history(future2)
```

```
At time step 0 three truth-possibilities are:
[[[False  True  True False  True False]]


 [[False  True  True False  True False]]


 [[False  True  True False  True False]]]
At time step 1 three truth-possibilities are:
[[[False  True False False  True False]]


 [[False  True False False  True False]]


 [[False  True False False  True False]]]
At time step 2 three truth-possibilities are:
[[[ True False False False  True False]]


 [[ True False False False  True False]]
```

```
 [[ True False False False  True False]]]
At time step 3 three truth-possibilities are:
[[[ True False False False  True False]]


 [[ True False False False  True False]]


 [[ True False False False  True False]]]
At time step 4 three truth-possibilities are:
[[[ True False False False  True False]]


 [[ True False False False False  True]]


 [[False  True False False  True False]]]
```

And we can view the same history in the three dimensional plot:

[ ]:
```
datap = convert_histories_func(future2)


plot_3d_hist_func(datap)
```

The pattern matches what we saw from the secondary system exactly, meaning the laws are consequences of Ramsey's secondary system. We see that we can only see red at time step 3 if we move backward. The other observations are similar. This means that with regard to the observables of color and movement, our secondary system is an exact match to our primary system. This means that the secondary system coded here has verification conditions that match the possibilities given in the primary system. But if we look at the total number of truth-possibilities at a given time step, say time step 3, we will see a discrepancy:

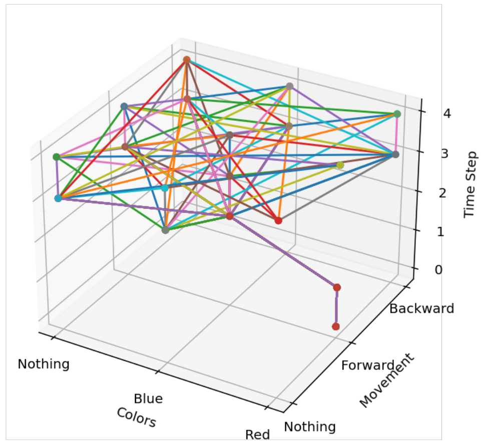

Figure A.2: A plot of the worlds after application of the laws.

```
[ ]: print("The total number of truth-possibilities at time step 3 for the␣
      ↪joint system are:")
     print(np.asarray(future1)[:,3,:,:].shape)
     print("The total number of truth-possibilities at time step 3 for the␣
      ↪primary system are:")
     print(np.asarray(future2)[:,3,:,:].shape)
```

```
The total number of truth-possibilities at time step 3 for the joint system␣
 ↪are:
(144, 1, 13)
The total number of truth-possibilities at time step 3 for the primary␣
 ↪system
are:
(55, 1, 6)
```

We have 144 truth-possibilities in the join system but only 55 in the primary system. This means that the content of the secondary system outstrips the primary system. There are just more truth-possibilities. The upshot is that the secondary system produces a finer-grained partition of possibility space than the primary system–even when they rule out the same possibilities (the laws here coincide with the same observable truth-possibilities).

A.6 Conclusion

I hope to have provided the reader a thorough overview of Ramsey's toy model. Along the way I argued that Ramsey has an underlying possibility space. Theories are more than just languages. They represent possibilities. Through the artifice of truth-possibilities, I then showed how the joint truth-possibilities of the primary and secondary systems in Ramsey's toy model behave. I showed how there is an overlap between the observable consequences of the secondary system and the laws governing those observables in the primary system. Furthermore, I showed that this overlap exists despite the joint system having more truth-possibilities than just the primary system alone.