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Decision Support

Impact of an “online-to-store” channel on demand allocation, pricing and profitability

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A R T I C L E I N F O

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A B S T R A C T

The growth of e-commerce in the past decade has opened the door to a new and exciting opportunity for retailers to better target different segments of the customer population. In this paper, we develop an analytical framework to study the impact of an “online-to-store” channel on the demand allocations and profitability of a retailer who sells products to customers through multiple distribution channels. This new channel can help the retailer tap new customer segments and generate additional demand, but may also hurt the retailer by cannibalizing existing channels and increasing operating costs. The analytical model allows us to evaluate these fundamental tradeoffs and provide useful managerial insights regarding the specific product and market characteristics that are most conducive for increasing profitability. Our analysis provides some simple conditions under which adding an online-to-store channel would lead to higher profits for products that are only available online. If the product is also available in-store, the analysis becomes more complex. In this case, we performed numerical experiments to generate insights on when the OS channel should be used. Our results imply that the retailer needs to carefully select the set of products to be offered through the online-to-store channel.

1. Introduction

Many retailers with both brick-and-mortar stores and an online presence are now allowing their customers to pick up online orders at brick-and-mortar stores. This allows retailers to leverage their existing physical assets to increase value and convenience for customers while at the same time increase fulfillment flexibility since retailers now offer an additional option in delivering online orders to customers. The online-to-store channel allows retailers to offer a larger variety of products compared to what customers can regularly access through the store channel, due to obvious space constraints. To make the online-to-store channel even more appealing, many retailers also offer dedicated parking spaces and express checkout lanes for online-to-store customers. This practice has gained considerable momentum, as evidenced by the fact that more and more retailers are now adopting this channel.

At the same time, the online-to-store channel will help remedy what a recent Wall Street Journal article (Bustillo & Fowler, 2009) considers as the Achilles heel of online channels: the costs and delays of shipping products to online customers. For products with a low retail price, it is not uncommon for shipping expenses to be higher than the purchase price. To take full advantage of the OS channel’s new found popularity, online retailer Amazon.com has plans to open its first brick-and-mortar store in New York City to allow customers to pick up online orders (Bedford, 2014).

The online-to-store channel combines many of the strengths of the online and store channels, including price reduction and fulfillment flexibility, but it also presents a number of implementation challenges. For example, the retailer needs to cover an additional handling cost in shipping the product from the warehouse to the selected store for customer pickup. Even if a product is available both online and in-store such that the retailer can utilize in-store inventory to satisfy an online purchase under the online-to-store channel, this strategy would still add to the retailer’s cost in filling a customer’s order, as an in-store item is generally more expensive due to the higher labor and storage costs associated with managing in-store inventory. Furthermore, it is unclear as to how this new channel would cannibalize sales from the existing store and online channels.

With these aforementioned implementation issues and associated handling costs, retailers need to carefully consider various factors and market characteristics in selecting the appropriate products to offer under the online-to-store channel. Otherwise, the addition of an online-to-store channel might not necessarily increase the overall profitability of the retailer. For instance, Walmart.com only offers

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its well-known Site-to-Store service for selected product categories and considers a number of important factors including profit margins, bulkiness of items, and the associated shipping costs in selecting these product categories[1]. Hence, these considerations have motivated us to address the interesting question as to which types of products would be most compatible with an online-to-store channel.

Despite the increase in attention given to the online-to-store channel in the retail industry, there is a lack of theoretical research on the topic. Our research aims to fill this gap. In this paper, we provide an analytical framework that captures some essential operating characteristics of a retailer with multiple distribution channels and allows us to evaluate the potential benefits of adding an online-to-store channel to existing distribution channels. Our analysis strives to address the research question of how the addition of an online-to-store channel for a particular product would affect the allocation of customer demand among the multiple distribution channels and the optimal pricing strategy.

Specifically, we first consider a stylized model of a single retailer with two existing distribution channels: a store channel where customers can visit and buy a product at a retail store location, and an online channel where customers can make an online purchase using the store website and have the product shipped to some specified destination. We then evaluate the value of adding a third distribution channel that allows customers to purchase a product online and later pick up the product in a nearby store location.

To capture some key features of the operating environment, we allow customers to differ in two important dimensions. First, we allow customers to differ in their valuation of the product. Second, we allow customers to differ in their inconvenience factors incurred by a store visit. These inconvenience factors can involve the additional time required to visit the store, search for a particular item on the shelves, and expected wait at the checkout counters. We combine these inconvenience factors associated with a store visit for an individual customer into an *inconvenience cost*, which would depend upon the monetary value of time for the individual customer. For instance, a customer living farther away from the store would take longer to drive to the store, resulting in a higher inconvenience cost for a store visit.

We first characterize the demand allocation among the various distribution channels using consumer utility theory. Using these demand characterizations, we then proceed to analyze the impact on the retailer's profit. By comparing the results for any particular product between the two cases with and without the online-to-store channel, we can evaluate the impact of adding an online-to-store channel on the optimal pricing, total demand and total profit of the retailer.

In our model, we allow the online and store prices for the product to be different, as it is common in practice that a retailer could offer discounts on online orders. We analyze various scenarios under which the online price and/or store price can be fixed or optimized. For products that are available online only, we show that the addition of an OS channel would always increase the optimal online price. However, for products that are available both online and in store, we show that the optimal online price can increase or decrease, depending on specific model parameters. To gain additional insights, we performed a comprehensive set of numerical experiments. Our numerical results provide specific operating environments under which the optimal online price would increase due to the addition of the online-to-store channel. Our numerical results further provide specific operating environments under which adding the online-to-store channel would increase the total demand of the retailer.

For products that are available only online, we also provide some simple conditions under which it is profitable for the retailer to offer the product under the online-to-store channel. However, if the product is also available in store, the potential benefits of the online-to-store channel become less clear due to the fact that this online-to-store channel incurs an additional fulfillment cost, due to, e.g., the need to prepare products for express checkout lanes, and it cannibalizes sales in both the store and online channels. This fulfillment cost would include the costs of sourcing the product, shipping the product from the warehouse to the retail store, and managing the product on the store shelves before it is eventually sold to the customer. Indeed, our analysis shows that the retailer's profit would increase only under very specific operating environments. Consequently, the retailer needs to carefully evaluate the underlying operating environment and product characteristics so as to select the appropriate set of products to be offered through the online-to-store channel.

The rest of the paper is organized as follows. We provide a literature review in Section 2. We describe our modeling framework and characterize the demand allocation among the multiple distribution channels in Section 3. In Section 4, we analyze the impact of adding the online-to-store channel for products that are available online only. In Section 5, we extend our analysis for products that are available both online and in-store. Finally, we summarize our results and provide some suggestions for further extensions in Section 6. All proofs are given in the Appendix.

2. Literature review

Multi-channel coordination has received considerable attention in both marketing and operations. One stream of this literature looks at whether or not a manufacturer should introduce a direct channel – either online, physical, or mail-order – to compete with an existing independent retailer that typically sells through the physical channel. For example, Tsay and Agrawal (2004a) consider a model in which a manufacturer can choose among three alternative channel strategies: "direct sales only" with no retailer involved, "retailer only" and "both direct sales and retail sales." They differentiate the direct and retail channels in terms of the amount of sales effort required and show that the direct channel can benefit both channel members if the manufacturer adjusts its wholesale price accordingly. Cattani, Gilland, Heese, and Swaminathan (2006) further consider the case where customers are heterogeneous in their efforts to purchase the product. The authors show that under certain conditions, the manufacturer, the retailer and the customers can all benefit from an equal price strategy. See also Bell, Wang, and Padmanabhan (2003), Chiang, Chhajed, and Hess (2003), Kumar and Ruan (2006), Hendershott and Zhang (2006), and Bernstein, Song, and Zheng (2009) for similar modeling frameworks. All of the above papers involve both vertical (between the manufacturer and the retailer) and horizontal (between multiple sales channels) competition. Tsay and Agrawal (2004b) and Cattani, Gilland, Swaminathan, and Boston (2004) provide some excellent surveys.

Another stream of relevant research deals with horizontal competition among retailers who utilize different channel strategies to sell their products. This stream of research focuses on whether a retailer should introduce a multi-channel strategy, e.g., by opening an online channel in addition to its physical store (or vice versa) to compete with other retailers; e.g., see Bernstein, Song, and Zheng (2008). Agatz, Fleischmann, and van Nunen (2008) provide a recent review in this area. Another area of focus is the price competition between an online retailer and a brick-and-mortar retailer; e.g., see Druehl and Porteus (2006).

Our research contributes to the above two streams of literature in two important aspects. First, we consider two dimensions of customer heterogeneity in our modeling framework with respect to the product valuation and the inconvenience costs associated with a store visit. Cattani et al. (2006) defines customer heterogeneity with respect to the amount of purchasing effort required for a store visit. Our

[1] Private communications.

modeling framework includes an additional dimension of customer heterogeneity in a consumer choice model to capture the competition among the *three distribution channels* (store, online and online-to-store) considered in this paper and characterize demand for each channel. Second, we adopt a centralized model in which one single retailer decides whether to operate an online-to-store channel in addition to its traditional channels. Consequently, the competition involved is among the three different channels of a single decision maker. The centralized model allows us to examine how the introduction of an online-to-store channel would affect the demand allocations and the overall profitability of the retailer.

There are also a number of papers in the operations management literature that focus on inventory issues in a multi-channel setting under stochastic demand and examine effective mechanisms to manage inventory among multiple channels; e.g., see Boyaci (2005) and Seifert, Thonemann, and Sieke (2006). In our paper, customer demand is assumed to be deterministic and is driven by differences in customer heterogeneity. Here, we do not explicitly consider inventory sharing issues among the multiple distribution channels in our analysis.

3. Modeling framework

Consider a retailer who sells one product to customers through the following three distribution channels: (1) Store channel, where customers can visit a physical store and make a purchase there; (2) Online channel, where customers can order the product online and have the product shipped to a specified address; and (3) Online-to-Store (or simply "OS") channel, where customers can order the product online and then pick up the product at a nearby physical store.

Our model allows for customer heterogeneity with respect to two key features, namely, customer valuation of the product and inconvenience cost of store visits. First, our model allows for customer heterogeneity for the product value, denoted by v, where v is a random variable with support [0, 1]. We assume that the value of a product remains the same regardless of where it is procured[2]. Second, our consumer utility model allows for customer heterogeneity with respect to the inconvenience cost of store visits, which generally depends on the travel distance to the store and time value for money for an individual customer. We model this aspect by assuming that the inconvenience cost of a store visit is equal to w, where w is a random variable with support [0, 1]. We set the upper bound of w at the maximum possible value of v, as no customer will make a store visit to purchase the product if his inconvenience cost to visit the store w exceeds the product value v. Naturally, we assume that the two random variables, v and w, are independent of each other.

To entice customers to use the OS channel, some retailers offer special pickup counters and/or offer convenient reserved parking spaces for their OS customers for pickups. In our model, we introduce a parameter called the inconvenience factor r, with $0 < r \le 1$, to represent the retailer's effort to reduce the inconvenience cost of store visits for OS customers, such that the inconvenience cost of a store visit for OS customer is equal to rw. In particular, $r = 1$ corresponds to the case where the retailer has made no effort in reducing the inconvenience cost of store visits for OS customers.

The retailer can charge different prices for this product, depending on whether the customer purchases the product through the Store channel or orders the product online through either the Online channel or the OS channel. We refer to these two different prices as the store price and online price, denoted by p_s and p_o, respectively. As it is common for a retailer to offer an online discount, we assume that $p_s \ge p_o$ in our model. This result has been substantiated by a seminal paper from Brynjolfsson and Smith (2000) who show that online prices are 9-16 percent lower for many types of products.

We also allow the unit sourcing cost to be different, depending on whether the product is sold through the Store channel, denoted by c_s, or through either the Online channel or the OS channel, denoted by c_o. In general, the retailer would incur a higher cost for products sold through the Store channel due to the extra storage/insurance for keeping the product in the store. Thus, we assume that $c_s \ge c_o$. Without loss of generality, we assume $c_o = 0$ such that c_s simply represents the extra unit sourcing cost for products sold through the Store channel over the Online/OS channels.

The customer pays an additional shipping cost s for a product purchased through the Online channel, but does not incur this cost when the product is purchased through either the Store or OS channel. A purchase through the Online channel could cause additional hassle due to, e.g., delay in receiving the orders and the required effort when the customer needs to return or replace a product ordered online. We can also use the parameter s to capture these inconvenience cost factors associated with an online order. The retailer can meet an order from the OS channel by using existing in-store inventory if available.

If the item is not available in store, the product can be delivered from the warehouse to the store for customer pickup using the next regularly scheduled store delivery, and the retailer does not incur any additional shipping cost. However, we assume that the retailer will incur an extra unit handling cost, denoted by h, for a product sold through the OS channel. For example, a retailer may add a drive-through service for OS customers (Clifford, 2012). Also, the OS channel may introduce additional costs associated with the coordination of online and offline information and logistics. In general, this extra unit handling cost h can depend on the inconvenience factor r. For instance, the retailer can assign a separate counter for only store pickup which helps to reduce the waiting time (and thus, inconvenience cost) for OS customers, but at the expense of a higher operating cost.

The Store, Online and Online-to-Store channels will be abbreviated in our notation by the letters "S", "O" and "OS", respectively. We summarize below the notation used in the paper.

- p_s: unit store price for the product;
- p_o: unit online price for the product;
- c_s: extra unit sourcing cost for products sold through the Store channel;
- s: unit shipping cost incurred by the customer for an online purchase using the Online channel;
- h: unit handling cost incurred by the retailer for a purchase through the OS channel;
- v: product value for a purchase;
- w: customer inconvenience cost for a store visit;
- r: inconvenience factor for OS customers; and
- q_i: demand for the product sold through channel i, i = S, O, and OS.

Under the above model assumptions, we can express the consumer utility functions for the three distribution channels as follows:

$$U_s = v - p_s - w, \qquad U_o = v - p_o - s, \qquad U_{os} = v - p_o - rw. \quad (1)$$

To avoid trivial situations, we make several assumptions in our model: (1) $p_s \le 1$, so as to avoid negative customer utility for any fixed store price in the Store channel; (2) $p_o + s \le 1$, so as to avoid negative channel customer utility for any fixed online price in the Online channel; (3) $r \ge s$, so that the Online channel is not always dominated by the OS channel; and (4) $p_s \le p_o + s$, so that the Store channel is not always dominated by the Online channel. Without loss of generality, we normalize the total market size to one. For analytical tractability, we assume that v follows a uniform distribution on [0, 1] and that w follows a uniform distribution on [0, 1].

[2] For some products such as fashion apparel, online purchase may have an impact on their value to customers. This paper does not cover the analysis for this type of products.

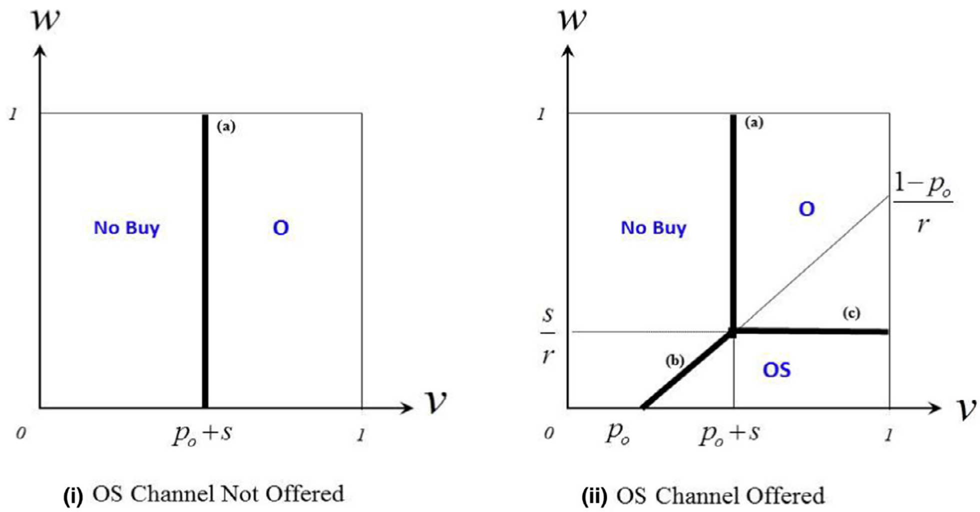

Fig. 1. Purchasing options for a product offered Online only.

4. Products available Online only

We first analyze the scenario where the product is available for purchase only through the Online channel. This is the most typical case since retailers can only stock a limited subset of what is available Online due to constraints on the physical size of retail stores. By utilizing the Online channel, retailers such as Amazon.com have the ability to offer many more niche products compared to what is possible with the Store channel. Although each niche product may have relatively low sales by definition, the sheer number of different types of niches products that can be offered in the Online channel makes this category of product lucrative for retailers; see Brynjolfsson, Hu, and Smith (2006) and Anderson (2009) for detailed discussions.

Under this scenario, customers may order the product online and then pick up the product in a nearby store location once the OS channel becomes available. We can characterize the customers' purchasing behavior in Fig. 1, where (i) denotes the case where the OS channel is not offered, (ii) denotes the case where the OS channel is offered. The purchasing preference for the customers can be analyzed using three indifference curves (a), (b) and (c), where

Line (a): Indifference curve between no-buy and purchase from the Online channel;
Line (b): Indifference curve between no-buy and purchase from the OS channel;
Line (c): Indifference curve of purchase preference between the Online and OS channel.

When the OS channel is not offered, customers can only purchase the product through the Online channel. From Fig. 1(i) and the assumption of a uniform distribution for v, it is clear that the demand for the Online channel is given by

$$q_o = 1 - p_o - s. \tag{2}$$

Accordingly, the retailer's profit function can be written as

$$\Pi = p_o q_o = p_o(1 - p_o - s). \tag{3}$$

When the OS channel is offered, the retailer can now generate demand from both the Online and OS channels. In particular, we can characterize the demand functions by analyzing the area of the corresponding regions in Fig. 1(ii).

$$\tilde{q}_o = (1 - p_o - s)\left[1 - \frac{s}{r}\right] \tag{4}$$

$$\tilde{q}_{os} = \frac{s}{2r}(2 - 2p_o - s). \tag{5}$$

Hence, the total realized demand for the retailer is given by

$$\tilde{q}_o + \tilde{q}_{os} = (1 - p_o - s) + \frac{s^2}{2r}, \tag{6}$$

and the retailer's total profit is given by

$$\tilde{\Pi} = p_o\tilde{q}_o + (p_o - h)\tilde{q}_{os} = p_o(1 - p_o - s) + p_o\frac{s^2}{2r} - \frac{hs}{2r}(2 - 2p_o - s). \tag{7}$$

4.1. Effects of the OS channel under fixed online pricing

We will now consider the case where the OS channel is offered and the online price is fixed as before. In this case, we can easily compare the corresponding demand and profit functions derived above to obtain the following result:

Proposition 1. *Under fixed online pricing, adding an OS channel has the following effects.*

(i) *(Demand) The retailer's total demand increases by $\frac{s^2}{2r}$; its demand from the Online channel decreases by $\frac{s(1-p_o-s)}{r}$; and its demand from the OS channel is given by $\frac{s}{2r}(2 - 2p_o - s)$.*
(ii) *(Profit) The retailer's total profit increases if and only if*

$$h \le \bar{h} = \frac{p_o s}{2 - 2p_o - s}. \tag{8}$$

Furthermore, the threshold value $\bar{h}$ increases in p_o and s, and is independent of r.

Proposition 1(i) shows the impact on the demand allocations when the OS channel is offered. Essentially, some customers from the Online channel will switch over to the OS channel to avoid the shipping cost s, thereby reducing the demand from the Online channel. In addition, we can see that the OS channel is able to generate new demand which was not there before. As a result, the retailer's total demand increases.

Proposition 1(ii) shows that the retailer would benefit from adding an OS channel only when the extra unit handling cost incurred by the retailer for products sold through the OS channel is lower than some threshold level $\bar{h}$. The result further shows that a higher online price p_o or a higher shipping cost s will make it more likely for the retailer to benefit from the OS channel. A higher value of p_o can benefit the retailer in two ways. First, a higher p_o increases the profit due to the extra new demand generated by the OS channel. Second, a higher p_o reduces the number of existing customers in the Online channel switching to the OS channel, which benefits the retailer as he needs to pay an extra handling cost of h for any purchase through the OS channel. Similarly, a higher value of s increases the potential of generating extra demand, which also benefits the retailer.

Finally, it is interesting to note that the threshold value $\bar{h}$ is independent of the inconvenience factor r. We can explain this as follows. On one hand, the extra revenue generated by offering the OS channel is the product of the unit selling price and the extra demand created, i.e., $\frac{p_o s^2}{2r}$. On the other hand, the extra cost incurred by addition of the OS channel is the product of the unit inventory handling cost at the store and the demand from the OS channel, i.e., $\frac{hs}{2r}(2-2p_o-s)$. Since both the extra revenue and cost are dependent on r only through a term $\frac{1}{r}$, whether the addition of the OS channel would benefit or hurt the retailer is independent of r. However, the magnitude of the corresponding profit gain or loss will be amplified by a lower inconvenience factor r, i.e, the magnitude decreases in r. Observe that if Eq. (8) does not hold, the retailer's profit $\tilde{\Pi}$ given in (7) would increase in r. This interesting result implies that in this case, providing more convenience to customers through the OS channel (i.e., reducing the inconvenience factor r) could negatively affect the retailer's profit if a product with high extra holding cost h (such as large appliances) is offered through the OS channel.

4.2. Effects of the OS channel under optimal online pricing

Now consider the general case where the retailer can adjust the online price p_o both before and after the addition of an OS channel so as to maximize his profit. For the model without the OS channel, the retailer's profit function given in (3) is concave in p_o, and is maximized at $p_o^* = \frac{1-s}{2}$. Hence, the optimal realized demand is given by $q_o^* = \frac{1-s}{2}$, and the retailer's optimal profit is equal to $\Pi^* = \frac{(1-s)^2}{4}$.

For the model with the OS channel, it is straightforward to show that the retailer's profit function given in (7) is concave in p_o and achieves its maximum at

$$\tilde{p}_o{}^* = \min\left(1-s, \frac{2r(1-s)+s^2+2hs}{4r}\right). \tag{9}$$

Note that $\tilde{p}_o{}^* = \frac{2r(1-s)+s^2+2hs}{4r}$ only when $0 \le h \le \frac{2r(1-s)-s^2}{2s}$. Otherwise, $\tilde{p}_o^* = 1-s$ which results in positive demand for the OS channel but zero demand for the Online channel. The retailer's optimal profit can be obtained by substituting this optimal online price into the retailer's profit function (7).

Comparing the optimal online prices and retailer's optimal profits between the models with and without an OS channel, we can obtain the following proposition regarding the effects of the OS channel:

Proposition 2. *Under optimal online pricing, adding an OS channel would:*

(i) *increase the retailer's optimal online price, i.e.,* $p_o^* \le \tilde{p}_o^*$;
(ii) *reduce the demand for the Online channel;*
(iii) *increase the retailer's total demand if* $0 \le h \le \min\left(\frac{s}{2}, \frac{2r(1-s)-s^2}{2s}\right)$, *or if* $h \ge \frac{2r(1-s)-s^2}{2s}$ *and* $s \le r \le \frac{s^2}{1-s}$; *and*
(iv) *increase the retailer's profit if* $0 \le h \le \min\left(\frac{2r-s^2-2\sqrt{r^2+rs^3-2rs^2}}{2s}, \frac{2r(1-s)-s^2}{2s}\right)$, *or if* $\frac{2r(1-s)-s^2}{2s} \le h \le \frac{(1-s)(2s^2-r(1-s))}{2s^2}$ *and* $s \le r \le \frac{s^2(2-s)}{1-s^2}$.

The above proposition provides a number of interesting insights. First, Proposition 2(i) shows that the retailer should charge a higher online price after adding the OS channel. The reason is as follows. When the OS channel is added, extra demand in the amount of $\frac{s^2}{2r}$ will be created by those customers who had previously opted not to purchase when the online price is fixed; see Proposition 1(i). Consequently, the demand function for the Online channel becomes less price-elastic at each price point, which gives the retailer the incentive to increase the online price. In the meantime, the extra demand generated by the OS channel also introduces some additional inventory handling cost associated with this channel. Hence, the retailer needs to increase its revenue to cover some of this cost. This increase in online price, coupled with the fact that some customers would switch from the Online channel to the OS channel, results in a decrease in the number of customers using the Online channel; see Proposition 2(ii).

Proposition 2(iii) shows that adding an OS channel will increase the total demand for the retailer under some mild conditions. Recall from Proposition 1(i) that, at a given online price, adding the OS channel increases the total demand by an amount of $\frac{s^2}{2r}$. However, any subsequent increase in the online price due to the OS channel (see Proposition 2(i)) will decrease the total demand. It is the effect of these two forces that determine the net change in the total demand. When the handling cost h is small (i.e., when $h \le \min\left(\frac{s}{2}, \frac{2r(1-s)-s^2}{2s}\right)$), the retailer can still generate more demand by adding the OS channel since the decrease in total demand, due to an increase in the optimal online price in this case, is relatively small. On the other hand, under a high value of h, the optimal online price in the model with the OS channel is forced to take its maximum possible value $1-s$, which leads to zero demand for the Online channel in this case. Observe that the demand from the OS channel in this case is given by $\tilde{q}_{os} = \frac{s^2}{2r}$ and the demand from the Online channel before the addition of the OS channel is given by $q_o^* = \frac{1-s}{2}$. Hence, adding an OS channel can increase the total demand only when $\tilde{q}_{os} \ge q_o^*$, or equivalently, $r \le \frac{s^2}{1-s}$.

Proposition 2(iv) provides the condition under which the addition of the OS channel will increase the retailer's profit. Specifically, it shows that the OS channel would increase the retailer's profit if the unit handling cost h is below some threshold value, which depends on the relative values of s and r. This result seems intuitive as the additional handling cost is borne by the retailer, and therefore must be compensated by either higher prices or increased demand, both of which depend on the values of s and r. When the unit handling cost h is on the high side, but not too high, following the effect of the OS channel on the total demand described above, the Online channel becomes degenerated (with no demand from this channel), and hence, an increase in the total profit, due to the addition of the OS channel, can only occur when $s \le r \le \frac{s^2(2-s)}{1-s^2}$.

5. Products available both Online and In-Store

We next analyze the scenario where the product is available both online and in-store. In this scenario, we assume that there are two basic types of customers. Type I customers will consider all available channels when making a purchase, while Type II customers will only make a purchase in store, regardless of whether the Online or OS channels are available. For instance, Type 2 customers could correspond to those who do not have Internet access or simply those who are old-fashioned and prefer to not shop online. To simplify our analysis, we further assume that a fixed proportion α of all customers is of Type 1 and the remaining proportion $(1-\alpha)$ is of Type 2.

For any fixed store price, the addition of an OS channel has no effect on Type 2 customers. Thus, we first focus our analysis on Type 1 customers where the OS channel would compete for these customers from both the Online and Store channels. We can characterize the customer purchasing behavior in Fig. 2, where (i) denotes the case when the OS channel is not offered, whereas (ii) denotes the case when the OS channel is offered. The purchasing preference for the customers can be analyzed using the corresponding difference curves, from which we can derive the demand for each channel.

When the OS channel is not offered, Type 1 customers have three options: no purchase, purchase from the Store channel, or purchase from the Online channel. The tradeoffs are captured in Fig. 2(i). Under the assumption of uniform distributions for v and w, we can characterize the demand function for each channel by finding the area of

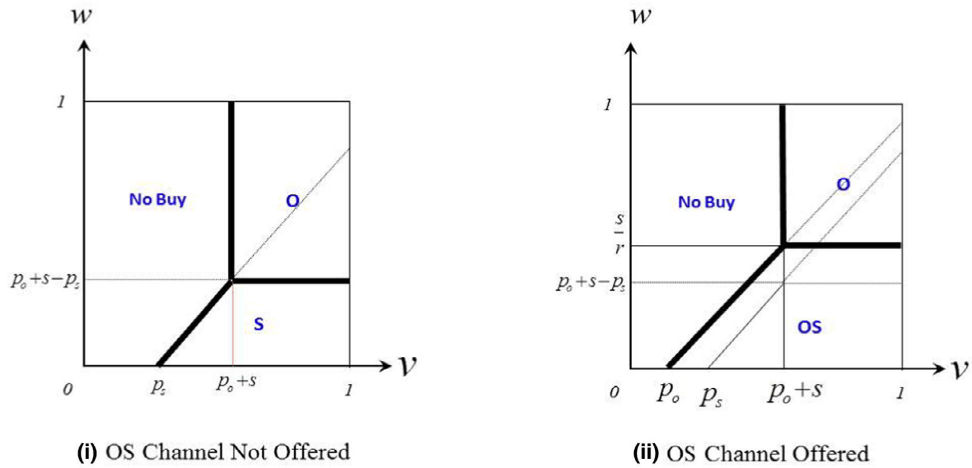

Fig. 2. Purchasing options for a product offered Online and In-Store.

the corresponding region in Fig 2(i). Specifically,

$$q_s = \left(1 - \frac{p_o + s + p_s}{2}\right)(p_o + s - p_s) \tag{10}$$

$$q_o = (1 + p_s - p_o - s)(1 - p_o - s). \tag{11}$$

Hence, the total demand generated through the Store and Online channels is given by

$$q_s + q_o = (1 - p_o - s) + \frac{(p_o + s - p_s)^2}{2}. \tag{12}$$

Accordingly, the retailer's total profit can be written as

$$\begin{aligned}\Pi &= (p_s - c_s)q_s + p_o q_o \\ &= (p_s - c_s)\left(1 - \frac{p_o + s + p_s}{2}\right)(p_o + s - p_s) \\ &\quad + p_o(1 + p_s - p_o - s)(1 - p_o - s).\end{aligned} \tag{13}$$

When all three channels are offered, it is clear from Fig. 2(ii) that all Type 1 customers who previously chose the Store channel will now switch to the OS channel due to a possible lower online price and smaller inconvenience cost. Consequently, Type 1 customers will now only consider the Online or OS channels in making a purchase, and the demand functions for these two channels are simply given by the results in the case where the product is available online only. Specifically, the demand function for the Online channel $\tilde{q}_o$ is given by (4), and the demand function for the OS channel $\tilde{q}_{os}$ is given by (5). Note that the demand for the Store channel is equal to 0. Then, the total demand for the retailer is equal $(\tilde{q}_o + \tilde{q}_{os})$, which is given by (6). Also, the retailer's total profit is given by

$$\begin{aligned}\tilde{\Pi} &= p_o\tilde{q}_o + (p_o - h)\tilde{q}_{os} = p_o(1 - p_o - s) \\ &\quad + p_o\frac{s^2}{2r} - \frac{hs}{2r}(2 - 2p_o - s).\end{aligned} \tag{14}$$

5.1. Effects of the OS channel under fixed online and store Pricing

Consider the case where the retailer cannot change either the online or store price after the OS channel is offered. In this case, all Store customers will switch to the OS channel, some Online customers will switch to the OS channel, and the OS channel will attract some customers who previously opted not to buy; see Fig. 2. In particular, we can derive the following results regarding the effect of adding an OS channel to the existing Store and Online channels:

Proposition 3. *Under fixed online and store product pricing, adding an OS channel has the following effects:*

(i) *(Demand) The retailer's total demand increases by* $\frac{s[s(1-r)+2r(p_s-p_o)]-r(p_s-p_o)^2}{2r}$*; its demand from the Online channel decreases by* $\frac{1-p_o-s}{r}[s(1-r)+r(p_s-p_o)]$*; and its demand from the OS channel exceeds the demand from the Store channel by* $\frac{1}{2r}[(1-r)(2s-s^2)-2s(1-r)p_o-r(p_s^2-p_o^2)+2r(p_s-p_o)]$.

(ii) *(Profit) The retailer's total profit increases if and only if*

$$h \le \frac{2p_o(1-p_o-s)(r(p_o+s-p_s)-s)+sp_o(2-2p_o-s)-r(p_s-c_s)(2-p_o-s-p_s)(p_o+s-p_s)}{(2-2p_o-s)s}.$$

Proposition 3(ii) shows that the retailer's profit would increase due to the addition of an OS channel only when the additional handling cost h is sufficiently low. A lower value of h makes the OS channel more likely to be beneficial since it leads to a lower cost of fulfilling orders from the OS channel.

5.2. Effects of the OS channel under optimal online pricing

Consider next the case where the store price is fixed, but the retailer can adjust the online price both before and after the addition of an OS channel so as to maximize his profit. For a retailer offering only the Store and Online channels, we can derive the optimal online price from the total profit function given by (13).

For a retailer offering all three distribution channels, all Type 1 customers who previously chose the Store channel will now switch to the OS channel. As a result, the optimal online price is essentially the same as that provided in Eq. (9), except that the online price is also required to be no more than the store price. We summarize the optimal online prices in the following proposition:

Proposition 4.

(i) *For a retailer offering only the Store and Online channels, the optimal online price is given by*

$$p_o^* = \begin{cases} p_s, & \text{if} \quad p_s \le \frac{(1-s)(1-s-c_s)}{2-2s-c_s}; \\ \frac{1}{6}\left[3p_s - c_s + 4(1-s) - \beta\right], & \text{if} \quad \frac{(1-s)(1-s-c_s)}{2-2s-c_s} \le p_s \le \frac{1+2s-c_s}{2+s-c_s}; \\ p_s - s, & \text{otherwise}, \end{cases} \tag{15}$$

where $\beta = \sqrt{[3p_s - c_s + 4(1-s)]^2 - 12(1-s)(2p_s - c_s + (1-s))}$.

Table 1
Changes in optimal online price, total demand, and total profit with the addition of the OS channel ($p_s = 0.6$ and $c_s = 0.1$)

	$r = 0.5$				$r = 0.7$				$r = 0.9$			
s	0.0	0.1	0.2	0.3	0.0	0.1	0.2	0.3	0.0	0.1	0.2	0.3
	Change in optimal online price ($\tilde{p}_o^* - p_o^*$)											
$h = 0.00$	−0.1000	−0.0450	0.0037	0.0225	−0.1000	−0.0465	−0.0020	0.0096	−0.1000	−0.0473	−0.0052	0.0025
$h = 0.10$	−0.1000	−0.0350	0.0237	0.0525	−0.1000	−0.0393	0.0123	0.0311	−0.1000	−0.0417	0.0059	0.0192
$h = 0.20$	−0.1000	−0.0250	0.0437	0.0825	−0.1000	−0.0322	0.0265	0.0525	−0.1000	−0.0362	0.0170	0.0358
$h = 0.30$	−0.1000	−0.0150	0.0637	0.1125	−0.1000	−0.0250	0.0408	0.0739	−0.1000	−0.0306	0.0281	0.0525
	Change in total demand ($\tilde{q}^* - q^*$)											
$h = 0.00$	0.1000	0.0550	0.0362	0.0649	0.1000	0.0536	0.0305	0.0520	0.1000	0.0528	0.0273	0.0449
$h = 0.10$	0.1000	0.0450	0.0162	0.0349	0.1000	0.0465	0.0162	0.0306	0.1000	0.0473	0.0162	0.0282
$h = 0.20$	0.1000	0.0350	−0.0038	0.0049	0.1000	0.0393	0.0019	0.0092	0.1000	0.0417	0.0051	0.0115
$h = 0.30$	0.1000	0.0250	−0.0238	−0.0251	0.1000	0.0322	−0.0124	−0.0123	0.1000	0.0362	−0.0060	−0.0051
	Change in total profit ($\tilde{\Pi}^* - \Pi^*$)											
$h = 0.00$	0.0100	0.0070	0.0161	0.0297	0.0100	0.0057	0.0113	0.0197	0.0100	0.0050	0.0087	0.0143
$h = 0.10$	0.0100	−0.0028	−0.0027	0.0033	0.0100	−0.0013	−0.0024	0.0001	0.0100	−0.0005	−0.0021	−0.0013
$h = 0.20$	0.0100	−0.0124	−0.0207	−0.0213	0.0100	−0.0082	−0.0156	−0.0186	0.0100	−0.0059	−0.0125	−0.0163
$h = 0.30$	0.0100	−0.0218	−0.0379	−0.0441	0.0100	−0.0151	−0.0285	−0.0363	0.0100	−0.0013	−0.0228	−0.0307

(ii) *For a retailer offering all three channels, the optimal online price is given by*

$$\tilde{p}_o^* = \min\left(p_s, 1-s, \frac{2r(1-s)+s^2+2hs}{4r}\right). \quad (16)$$

Proposition 4 illustrates an important fact that the optimal online price with only Store and Online channels depends heavily on the fixed store price p_s, the shipping cost for online orders s, and the extra unit sourcing cost for products sold through the Store channel c_s, while the optimal online price with all three channels depends heavily on s, the inconvenience factor r, and the unit handling cost for purchase through the OS channel h. Moreover, when the OS channel is offered, the optimal online price $\tilde{p}_o^*$ increases in the unit handling cost h and decreases in the inconvenience factor associated with store visiting r.

Using the results given in Proposition 4, we can examine how the addition of the OS channel affects the optimal online price and the corresponding channel demands and profit. First of all, the addition of the OS channel could lead to either a higher or lower optimal online price, depending on the specific model parameters. To illustrate this fact, consider the parameter set with $c_s = 0,\ s = 0.2$ and $p_s = 0.6$. One can easily verify that for the model with only Store and Online channels, $\frac{(1-s)(1-s-c_s)}{2-2s-c_s} \le p_s \le \frac{1+2s-c_s}{2+s-c_s}$, and hence $p_o^* = 0.4319$. For the model with all three channels and the parameter set with $s = 0.2,\ p_s = 0.6,\ r = 0.4$ and $h \in [0, 0.1]$, it is easy to verify that $p_s \ge \frac{2r(1-s)+s^2+2hs}{4r}$, and hence $\tilde{p}_o{}^* = 0.425 + 0.25h$. In this example, we have $p_o^* \ge \tilde{p}_o^*$ when $0 \le h \le 0.0276$, and $p_o^* < \tilde{p}_o^*$ when $0.0276 < h \le 0.1$.

To provide additional insights, we have conducted a set of numerical experiments to evaluate the effect of the OS channel. We shall focus our discussion on the comparison of the optimal online prices, the resulting total demands and optimal profits between the model with only Store and Online channels and the model with all three channels. We use the notation q^* and $\tilde{q}^*$ to denote the resulting total demands, and Π^* and $\tilde{\Pi}^*$ to denote the optimal profit in these two models, respectively.

Table 1 summarizes one set of numerical results to illustrate the main observations. For this set of results, we set $p_s = 0.6$ and $c_s = 0.1$, with different values of r, s and h. The first section in Table 1 suggests that the optimal online prices are generally higher with the addition of the OS channel (i.e., $\tilde{p}_o^* - p_o^* > 0$) for higher values of s. Apparently, the retailer cannot afford to offer a higher online price in the model with only Store and Online channels due to the high shipping cost. With the OS channel, some customers can now avoid the high shipping cost, which supports a high online price.

The second section in Table 1 suggests that the total demand are generally higher with the addition of the OS channel (i.e., $\tilde{q}^* - q^* > 0$), except in few cases in which s and h are high. This observation illustrates the complex interactions among the various factors captured in our model. While a higher shipping cost s would entice more customers to adopt the OS channel, the retailer would need to increase the optimal online price more significantly to compensate for the extra handling cost h incurred for a purchase through the OS channel. When h is sufficiently high, this increase in the optimal online price could result in a decrease in the total demand in such cases.

Finally, the third section in Table 1 further suggests that the total profit would increase when the value of h is small. This effect on the total profit is also consistent with that in the case when the online price is fixed in Proposition 3(ii). We also note that in the cases where the addition of the OS channel increases the total profit, the magnitude of this profit increase is generally larger as the inconvenience factor r decreases.

We summarize our main observations of our numerical results below:

Observation 1. Under optimal online pricing, adding an OS channel would:

(i) increase the optimal online price when the shipping cost s is high;
(ii) increase the total demand except when both s and the extra handling cost h are high;
(iii) increase the total profit only when either s or h is low.

For the numerical results given in Table 1, we assume that the unit extra handling cost h is independent of the inconvenience factor r. In practice, it is likely that the values of h and r are correlated. For example, some stores have separate pickup counters for OS customers, which reduces the wait/inconvenience cost of store pickup, but requires a higher handling cost. To allow for this dependence, we have also conducted another set of numerical experiments by assuming that $h = a(1-r)$, where the parameter $a \in (0, 1)$ measures the cost of convenience improvement for OS customers to do store pickup, i.e., a higher value of a corresponds to a more costly convenience improvement. In practice, even before the OS channel is introduced, some retailers may already have a pick-up counter for customers to pick up their in-store purchases of bulky items. In this case, one may interpret that parameter a for these retailers is relatively small when they add the OS channel. The results are summarized in Table 2.

The results in Table 2 show that the qualitative insights given in Observation 1 remain valid when h and r are dependent. In particular, the optimal online price would increase with the addition of the OS channel only when the shipping cost s is relatively high, e.g., $s \ge 0.1$

Table 2
Changes in optimal online price, total demand and total profit with the addition of the OS channel ($p_s = 0.6$, $c_s = 0.1$ and $h = a(1-r)$)

	$r = 0.5$				$r = 0.7$				$r = 0.9$			
s	0.0	0.1	0.2	0.3	0.0	0.1	0.2	0.3	0.0	0.1	0.2	0.3
	Change in optimal online price ($\tilde{p}_o^* - p_o^*$)											
$a = 0.1$	−0.1000	−0.0400	0.0137	0.0375	−0.1000	−0.0443	0.0023	0.0161	−0.1000	−0.0467	−0.0041	0.0042
$a = 0.2$	−0.1000	−0.0350	0.0237	0.0525	−0.1000	−0.0422	0.0065	0.0225	−0.1000	−0.0462	−0.0030	0.0058
$a = 0.3$	−0.1000	−0.0300	0.0337	0.0675	−0.1000	−0.0400	0.0108	0.0289	−0.1000	−0.0456	−0.0019	0.0075
$a = 0.4$	−0.1000	−0.0250	0.0437	0.0825	−0.1000	−0.0379	0.0151	0.0354	−0.1000	−0.0450	−0.0008	0.0092
	Change in total demand ($\tilde{q}^* - q^*$)											
$a = 0.1$	0.1000	0.0500	0.0262	0.0499	0.1000	0.0515	0.0262	0.0456	0.1000	0.0523	0.0262	0.0432
$a = 0.2$	0.1000	0.0450	0.0162	0.0349	0.1000	0.0499	0.0219	0.0392	0.1000	0.0517	0.0251	0.0415
$a = 0.3$	0.1000	0.0400	0.0062	0.0199	0.1000	0.0472	0.0176	0.0327	0.1000	0.0512	0.0240	0.0399
$a = 0.4$	0.1000	0.0350	−0.0038	0.0049	0.1000	0.0450	0.0133	0.0263	0.1000	0.0506	0.0299	0.0382
	Change in total profit ($\tilde{\Pi}^* - \Pi^*$)											
$a = 0.1$	0.0100	0.0021	0.0066	0.0163	0.0100	0.0036	0.0072	0.0137	0.0100	0.0045	0.0076	0.0127
$a = 0.2$	0.0100	−0.0028	−0.0027	0.0033	0.0100	0.0015	0.0031	0.0078	0.0100	0.0039	0.0065	0.0111
$a = 0.3$	0.0100	−0.0076	−0.0118	−0.0092	0.0100	−0.0006	−0.0010	0.0020	0.0100	0.0034	0.0054	0.0096
$a = 0.4$	0.0100	−0.0124	−0.0207	−0.0213	0.0100	−0.0027	−0.0051	−0.0037	0.0100	0.0028	0.0044	0.0080

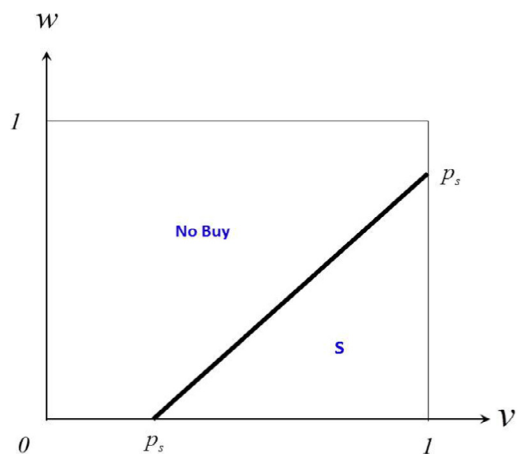

Fig. 3. Purchasing options for Type 2 customers.

in these numerical examples. Also, the total profit increases with the addition of the OS channel when s is low, the value of parameter a is small, or the inconvenience factor r is high. (Note that a small value of a or a high value of r generally corresponds to a low value of h.) Apparently, this observation implies that adding the OS channel is more likely to benefit retailers who already have pickup counters for some in-store purchases.

Furthermore, it is interesting to note that the change in the optimal online price due to the addition of the OS channel always increases in a. In other words, a more costly convenience improvement (higher value of a) would lead to a larger difference in the optimal online price $(\tilde{p}_o^* - p_o^*)$ if $(\tilde{p}_o^* - p_o^*) > 0$, but a smaller difference in $(\tilde{p}_o^* - p_o^*)$ if $(\tilde{p}_o^* - p_o^*) < 0$. In contrast, the changes in the total demand and total profit due to the addition of the OS channel always decrease in a. This supports the intuition that a more costly convenience improvement would make the OS channel less attractive to the retailer.

5.3. Effects of the OS channel under optimal online and store pricing

Consider the most general case where the retailer can adjust both the online and store prices before and after the addition of an OS channel so as to maximize his profit. Since any change in the store price would also affect the demand of Type 2 customers who only purchase from the Store channel, we now need to also include the demand from Type 2 customers in the analysis here. For Type 2 customers, the customer utility is given by $v - p_s - w$, and the indifference curve between no-buy and purchase from the Store channel is given by the solid line in Fig. 3. Thus, the demand of Type 2 customers for the Store channel is equal to $\frac{(1-p_s)^2}{2}$, and the retailer's profit from Type 2 customers is given by $(p_s - c_s)\frac{(1-p_s)^2}{2}$.

First, consider the model without the OS channel. In this case, the retailer's total profit from both Type 1 and Type 2 customers can be expressed as

$$\Pi^{nos} = \alpha\Pi + (1-\alpha)(p_s - c_s)\frac{(1-p_s)^2}{2} \quad (17)$$

where $\Pi = (p_s - c_s)\left(1 - \frac{p_o+s+p_s}{2}\right)(p_o + s - p_s) + p_o(1 + p_s - p_o - s)(1 - p_o - s)$, given in (13). Let p_s^* denote the optimal store price in this model.

Next, consider the case with the OS channel. In this case, the retailer's total profit from both Type 1 and Type 2 customers can be expressed as

$$\Pi^{os} = \alpha\tilde{\Pi} + (1-\alpha)(p_s - c_s)\frac{(1-p_s)^2}{2} \quad (18)$$

where $\tilde{\Pi} = p_o(1 - p_o - s) + p_o\frac{s^2}{2r} - \frac{hs}{2r}(2 - 2p_o - s)$, given in (14). Let $\tilde{p}_s^*$ denote the optimal store price in this model.

We can then compare the respective optimal store and online prices that would maximize the total profit given in (17) and (18) to evaluate the effects of adding an OS channel when the retailer can adjust both the store and online prices so as to maximize his profit. Unfortunately, it is analytically intractable to derive these optimal store and online prices. Therefore, we have conducted a numerical study to gain a better understanding of the effect of the OS channel on the optimal store and online prices, the total demand and the total profit.

Tables 3 and 4 present the results of a set of numerical experiments to illustrate some basic observations in our numerical study. In this set of numerical experiments, we set $c_s = 0.1$, with different values of r, s and h. To examine the effect of how the existence of the two types of customers could affect the resulting solutions, we considered two different values of α: $\alpha = 1$ and $\alpha = 0.5$. The $\alpha = 1$ case corresponds to the scenario where all all customers would consider all channels offered by the retailer, while the $\alpha = 0.5$ case corresponds to the scenario that half of the customers would only buy in the store. The results are given in Tables 3 and 4, respectively.

Based on our numerical study, we summarize our main observations below:

Observation 2. Under optimal store and online pricing, adding an OS channel would:

(i) increase the optimal online price;
(ii) decrease the optimal store price except when the handling cost h is high and the inconvenience factor r is low, or when all customers would consider all available channels, i.e., $\alpha = 1$;

Table 3
Changes in optimal online and store prices, total demand and total profit with the addition of the OS channel ($\alpha = 1$ and $c_s = 0.1$)

	$r = 0.5$				$r = 0.7$				$r = 0.9$			
s	0.0	0.1	0.2	0.3	0.0	0.1	0.2	0.3	0.0	0.1	0.2	0.3
	Change in optimal online price ($\tilde{p}_o^* - p_o^*$)											
$h = 0.00$	0.0000	0.0050	0.0183	0.0396	0.0000	0.0036	0.0125	0.0268	0.0000	0.0028	0.0094	0.0196
$h = 0.10$	0.0000	0.0150	0.0383	0.0696	0.0000	0.0107	0.0268	0.0482	0.0000	0.0083	0.0205	0.0363
$h = 0.20$	0.0000	0.0250	0.0583	0.0996	0.0000	0.0179	0.0411	0.0696	0.0000	0.0139	0.0316	0.0530
$h = 0.30$	0.0000	0.0350	0.0783	0.1296	0.0000	0.0250	0.0554	0.0911	0.0000	0.0195	0.0427	0.0696
	Change in optimal store price ($\tilde{p}_s^* - p_s^*$)											
$h = 0.00$	0.4999	0.4500	0.4902	0.5189	0.4999	0.4500	0.4902	0.5189	0.4999	0.4500	0.4902	0.5189
$h = 0.10$	0.4999	0.4500	0.4902	0.5189	0.4999	0.4500	0.4902	0.5189	0.4999	0.4500	0.4902	0.5189
$h = 0.20$	0.4999	0.4500	0.4902	0.5189	0.4999	0.4500	0.4902	0.5189	0.4999	0.4500	0.4902	0.5189
$h = 0.30$	0.4999	0.4500	0.4902	0.5189	0.4999	0.4500	0.4902	0.5189	0.4999	0.4500	0.4902	0.5189
	Change in total demand ($\tilde{q}^* - q^*$)											
$h = 0.00$	0.0000	0.0050	0.0175	0.0352	0.0000	0.0036	0.0118	0.0223	0.0000	0.0028	0.0086	0.0152
$h = 0.10$	0.0000	−0.0050	−0.0025	−0.0052	0.0000	−0.0036	−0.0025	0.0009	0.0000	−0.0028	−0.0025	−0.0015
$h = 0.20$	0.0000	−0.0150	−0.0225	−0.0248	0.0000	−0.0107	−0.0168	−0.0206	0.0000	−0.0083	−0.0136	−0.0182
$h = 0.30$	0.0000	−0.0250	−0.0425	−0.0548	0.0000	−0.0179	−0.0311	−0.0420	0.0000	−0.0139	−0.0247	−0.0348
	Change in total profit ($\tilde{\Pi}^* - \Pi^*$)											
$h = 0.00$	0.0000	0.0045	0.0144	0.0262	0.0000	0.0032	0.0096	0.0162	0.0000	0.0025	0.0070	0.0108
$h = 0.10$	0.0000	−0.0053	−0.0044	−0.0002	0.0000	−0.0038	−0.0041	−0.0034	0.0000	−0.0030	−0.0038	−0.0047
$h = 0.20$	0.0000	−0.0149	−0.0224	−0.0248	0.0000	−0.0108	−0.0173	−0.0220	0.0000	−0.0084	−0.0142	−0.0197
$h = 0.30$	0.0000	−0.0243	−0.0396	−0.0476	0.0000	−0.0176	−0.0302	−0.0398	0.0000	−0.0318	−0.0245	−0.0342

Table 4
Changes in optimal online and store prices, total demand and total profit with the addition of the OS channel ($\alpha = 0.5$ and $c_s = 0.1$)

	$r = 0.5$				$r = 0.7$				$r = 0.9$			
s	0.0	0.1	0.2	0.3	0.0	0.1	0.2	0.3	0.0	0.1	0.2	0.3
	Change in optimal online price ($\tilde{p}_o^* - p_o^*$)											
$h = 0.00$	0.0000	0.0076	0.0244	0.0476	0.0000	0.0066	0.0205	0.0348	0.0000	0.0060	0.0182	0.0276
$h = 0.10$	0.0000	0.0148	0.0385	0.0700	0.0000	0.0117	0.0305	0.0551	0.0000	0.0100	0.0260	0.0443
$h = 0.20$	0.0000	0.0221	0.0528	0.0913	0.0000	0.0169	0.0406	0.0700	0.0000	0.0140	0.0388	0.0584
$h = 0.30$	0.0000	0.0294	0.0674	0.1131	0.0000	0.0211	0.0508	0.0852	0.0000	0.0180	0.0417	0.0700
	Change in optimal store price ($\tilde{p}_s^* - p_s^*$)											
$h = 0.00$	0.0000	−0.0179	−0.0316	−0.0356	0.0000	−0.0189	−0.0355	−0.0356	0.0000	−0.0195	−0.0378	−0.0356
$h = 0.10$	0.0000	−0.0107	−0.0175	−0.0182	0.0000	−0.0137	−0.0255	−0.0332	0.0000	−0.0155	−0.0300	−0.0356
$h = 0.20$	0.0000	−0.0034	−0.0032	0.0031	0.0000	−0.0086	−0.0154	−0.0182	0.0000	−0.0115	−0.0222	−0.0299
$h = 0.30$	0.0000	0.0039	0.0114	0.0249	0.0000	−0.0034	−0.0052	−0.0031	0.0000	−0.0074	−0.0143	−0.0182
	Change in total demand ($\tilde{q}^* - q^*$)											
$h = 0.00$	0.0000	0.0048	0.0116	0.0204	0.0000	0.0041	0.0090	0.0139	0.0000	0.0038	0.0076	0.0104
$h = 0.10$	0.0000	−0.0009	0.0005	0.0040	0.0000	0.0001	0.0011	0.0030	0.0000	0.0007	0.0015	0.0020
$h = 0.20$	0.0000	−0.0065	−0.0107	−0.0127	0.0000	−0.0039	−0.0068	−0.0088	0.0000	−0.0025	−0.0047	−0.0067
$h = 0.30$	0.0000	−0.0122	−0.0220	−0.0296	0.0000	−0.0079	−0.0148	−0.0208	0.0000	−0.0056	−0.0109	−0.0160
	Change in total profit ($\tilde{\Pi}^* - \Pi^*$)											
$h = 0.00$	0.0000	0.0035	0.0081	0.0137	0.0000	0.0028	0.0058	0.0087	0.0000	0.0025	0.0045	0.0060
$h = 0.10$	0.0000	−0.0016	−0.0014	0.0004	0.0000	−0.0008	−0.0011	−0.0011	0.0000	−0.0003	−0.0009	−0.0018
$h = 0.20$	0.0000	−0.0066	−0.0107	−0.0122	0.0000	−0.0044	−0.0079	−0.0105	0.0000	−0.0032	−0.0063	−0.0093
$h = 0.30$	0.0000	−0.0115	−0.0197	−0.0243	0.0000	−0.0080	−0.0146	−0.0196	0.0000	−0.0060	−0.0115	−0.0166

(iii) increase the total demand when h is low;
(iv) increase the total profit only when h is low.

First, it is interesting to observe that the optimal online price always increases due to the addition of the OS channel. Furthermore, this increase in the optimal online price is always larger when either the value of h or s is higher, or when the value of r is lower.

As for the optimal store price, we note that when all customers would consider all channels offered by the retailer for a purchase (i.e., $\alpha = 1$), the store price is actually irrelevant when the OS channel is offered as the Store channel will be completely dominated by the OS channel. As a result, the store price would not have any impact on the total demand or the total profit. In this case (as in Table 3), the optimal store price with the OS channel will be simply set to the highest possible value to allow for maximum flexibility for the retailer to set his optimal online price, as we assume that the online price cannot exceed the store price.

For $\alpha < 1$, the retailer would generally lower the optimal store price with the addition of an OS channel due to the fact that the availability of an additional channel increases more internal competition and causes the retailer to lower its store price to keep customers in the Store Channel. This also implies that the difference between the optimal store and online prices will be reduced due to the addition of the OS channel. However, the observation that the OS channel would lower the optimal store price is not valid when h is high and r is low (see Table 4). When h is high, the retailer needs to significantly increase the online price $\tilde{p}_o^*$ when the OS channel is added in order to compensate for the additional high cost of h. However, note that the consumer utility function for the OS channel is given by $(1 - p_o - rw)$, so it requires that $p_o \leq 1 - rw$ for a customer to use the OS channel. Hence, the value of r needs to be small to allow the retailer to significantly increase its online price. As a high $\tilde{p}_o^*$ would lead to a high $\tilde{p}_s^*$, adding the OS channel could result in a higher optimal store price only when h is sufficiently high and r is sufficiently low.

Our results also suggest that the addition of an OS channel would increase the total demand only when the value of h is low. Furthermore, for the cases where the addition of an OS channel can increase the total demand, this total demand increase becomes larger when the value of s increases, or when the values of h or r decrease.

Finally, our numerical results suggest that adding an OS channel is beneficial to the retailer only when the extra handling cost h is low. This effect is robust across various scenarios in all numerical experiments, regardless of whether the underlying store and/or online prices are fixed or optimized, or whether the extra handling cost h is independent or negatively correlated with the inconvenience factor r.

6. Conclusion and future research

Many retailers have adopted a multi-channel approach to better target different customer segments in order to increase profitability. A number of major retailers have recently begun to offer an "online-to-store" channel to allow for more flexibility and convenience for their shoppers with the objective of increasing overall demand. However, adding such a channel could cause cannibalization among the existing channels. At the same time, this new channel can add to the operating cost, as the retailer might incur extra handling costs to have the products available at the store for customer pickup. As such, retailers need to carefully evaluate these tradeoffs so as to select the appropriate products to be offered through this new channel.

In this paper, we develop an analytical modeling framework to study the impact of an online-to-store channel on demand allocations and profitability of a retailer who sells products to customers using multiple distribution channels. Our modeling framework allows for an analysis of the resulting tradeoffs and provides useful insights for understanding the specific product characteristics and operating environments under which the retailer would benefit from the addition of this new channel.

We first characterize the demand allocation among the various distribution channels using a consumer utility model. Using these demand characterizations, we analyze the impact of adding an online-to-store channel on the retailer's profitability. For products that are only available online, we provide some simple conditions under which it is profitable for the retailer to offer the online-to-store channel. However, if the product is also available in-store, the analysis becomes much complicated, and we have performed a numerical study to illustrate under which conditions the retailer is better off.

Our modeling framework provides a simplified approach for understanding the impact of the online-to-store channel, which can be extended in several directions. First, our analysis shows that an online-to-store channel would increase the total number of customers visiting the store through the Store channel or the OS channel, which can provide additional benefits to the retailer. For example, a higher store traffic can increase cross-selling opportunities in the store, and thus makes the online-to-store channel more appealing for the retailer. Also, this new channel provides added convenience for customers, which could help to increase customer loyalty and benefit the retailer in the long run. Our current modeling framework does not address this potential long-term impact on the retailer. On the other hand, we do not explicitly capture the associated cost of reducing the inconvenience cost of store visits for customers using the online-to-store service. Extending our modeling framework to include some of these considerations would help to capture more realism in our analysis.

Second, our model does not allow for customers to differ in their acceptance of an online purchase for certain types of products such as fashion apparel, where customers might assign a lower valuation of an online purchase than of an in-store purchase; see Bushong, King, Camerer, and Rangel (2010) and Chiang et al. (2003). For example, many customers prefer to purchase items such as clothes and shoes in stores in order to be able to touch and feel the product and determine its fit. We can extend our model to allow for this differentiation between different types of products from clothes and shoes to more standardized products such as books.

Also, many papers in the existing literature studying multi-distribution channels deal with the horizontal and/or vertical competition among multiple players in the market. One motivation for a retailer to adopt the online-to-store channel is to provide another service differentiator against its competitors. Thus, an interesting extension would be to introduce competition among multiple retailers in a game-theoretic framework to study how retailers may use this additional channel to compete in the market.

Appendix

Proof of Proposition 1. (i) The increase in the retailer's total demand can be simply derived from the difference between the total demand in the model with the OS channel as given in (6) and that in the model without the OS channel as given in (2). The decrease in the corresponding Online demand is simply given by the difference between (4) and (2). The demand from its OS channel is simply equal to $\tilde{q}_{os}$ as given by (5).

(ii) By comparing the retailer's total profit in the model with the OS channel as given in (7) and that in the model without the OS channel as given in (3), we have

$$\tilde{\Pi} - \Pi \geq 0 \quad \text{if } h \leq \bar{h} \equiv \frac{p_o s}{2 - 2p_o - s}.$$

Note that the numerator in the expression of $\bar{h}$ is positive and increases in both s and p_o, while the denominator is also positive but decreases in both s and p_o. Hence, $\bar{h}$ increases in both s and p_o. Clearly, $\bar{h}$ is independent of r. □

Proof of Proposition 2. (i) It is straightforward to show that

$$\tilde{p}_o^* = \min\left[1 - s, \frac{2r(1-s) + s^2 + 2hs}{4r}\right] \geq \frac{(1-s)}{2} = p_o^*.$$

(ii) For any fixed online price, it is obvious that adding an OS channel can only reduce the Online customers due to customer switching from the Online channel to the OS channel. Also, since adding the OS channel will increase the optimal online price as proved in part (i), it will further reduce the number of Online customers.

(iii) From the analysis in the beginning of Section 4.2, it is known that the optimal demand in the model without an OS channel is $q_o^* = \frac{1-s}{2}$. In the model with an OS channel, the optimal online price is given in Eq. (9). To remove the min operator in this expression, we consider two cases.

(1) If $0 \leq h \leq \frac{2r(1-s)-s^2}{2s}$, then $\tilde{p}_o^* = \frac{2r(1-s)+s^2+2hs}{4r}$. Substituting this value into the total demand function in Eq. (6) and simplifying it leads to the optimal total demand

$$\tilde{q}_o^* + \tilde{q}_{os}^* = \frac{(1-s)}{2} + \frac{s(s-2h)}{4r}.$$

Clearly, adding an OS channel would increase the retailer's total demand if and only if $s \geq 2h$, or equivalently, $h \leq \frac{s}{2}$. Taking into consideration the feasible region in this case, we have that the OS channel would increase the total demand if

$$0 \leq h \leq \min\left(\frac{s}{2}, \frac{2r(1-s) - s^2}{2s}\right).$$

(2) If $h \geq \frac{2r(1-s)-s^2}{2s}$, which could occur, e.g., when $r \leq \frac{s^2}{2(1-s)}$, we have $\tilde{p}_o^* = 1 - s$ and the resulting total optimal demand is

$$\tilde{q}_o^* + \tilde{q}_{os}^* = \frac{s^2}{2r} \geq \frac{1-s}{2} = q_o^* \quad \text{if } s \leq r \leq \frac{s^2}{1-s}.$$

(iv) Similarly, the analysis in the beginning of Section 4.2 indicates that the retailer's optimal profit in the model without an OS channel is $\Pi^* = \frac{(1-s)^2}{4}$. In the model with an OS channel, we again consider the two cases discussed in item (iii) above.

(1) If $0 \le h \le \frac{2r(1-s)-s^2}{2s}$, we substitute $\tilde{p}_o^* = \frac{2r(1-s)+s^2+2hs}{4r}$ and the corresponding $\tilde{q}_o^*$ and $\tilde{q}_{os}^*$ into the retailer's profit function given in Eq. (7) and simplifying it yields

$$\tilde{\Pi}^* = \frac{s^2h^2}{4r^2} + \frac{(4s^3-8sr)h}{16r^2} + \frac{(2r-2sr+s^2)^2}{16r^2}.$$

The difference in the retailer's profit due to the OS channel can be written as

$$\tilde{\Pi}^* - \Pi^* = \frac{s^2h^2}{4r^2} + \frac{(4s^3-8sr)h}{16r^2} + \frac{s^2(4r-4rs+s^2)}{16r^2}.$$

The difference is a quadratic and convex function in the inventory handling cost h. Hence,

$$\tilde{\Pi}^* - \Pi^* \ge 0 \text{ if } h \le h_1 \equiv \frac{2r-s^2-2\sqrt{r^2+rs^3-2rs^2}}{2s} \text{ or if}$$
$$h \ge h_2 \equiv \frac{2r-s^2+2\sqrt{r^2+rs^3-2rs^2}}{2s}.$$

Given the feasible region of this case, it is never possible to have $h \ge h_2$. Hence, the addition of an OS channel would increase the retailer's optimal profit if

$$0 \le h \le \min\left(h_1 \equiv \frac{2r-s^2-2\sqrt{r^2+rs^3-2rs^2}}{2s}, \frac{2r(1-s)-s^2}{2s}\right).$$

(2) If $h \ge \frac{2r(1-s)-s^2}{2s}$, we substitute $\tilde{p}_o^* = 1-s$ and the corresponding $\tilde{q}_o^*$ and $\tilde{q}_{os}^*$ into the retailer's profit function and simplifying it yields $\tilde{\Pi}^* = \frac{(1-s-h)s^2}{2r}$. It is greater than or equal to $\Pi^* = \frac{(1-s)^2}{2}$ if $h \le \frac{(1-s)(2s^2-r(1-s))}{2s^2}$. Combining the feasible region of this case, we conclude that the addition of an OS channel would increase the retailer's optimal profit if

$$\frac{2r(1-s)-s^2}{2s} \le h \le \frac{(1-s)(2s^2-r(1-s))}{2s^2},$$

which implies that $s \le r \le \frac{s^2(2-s)}{1-s^2}$. □

Proof of Proposition 3. (i) The increase in the retailer's total demand can be derived from the difference between the total demand in the model with the OS channel as given in Eq. (6) and that in the model without the OS channel as given in (12). The decrease in the corresponding Online demand is given by the difference between (4) and (11). The difference between its demand from the OS channel and the demand from the Store channel before the addition of the OS channel is simply given by the difference between (5) and (10).

(ii) By comparing the retailer's total profit in the model with the OS channel as given in (14) and that in the model without the OS channel as given in (13), we have $\tilde{\Pi}^* - \Pi^*$ is linearly decreasing in h and hence it is positive if and only if

$$h \le \frac{2p_o(1-p_o-s)(r(p_o+s-p_s)-s)+sp_o(2-2p_o-s)-r(p_s-c_s)(2-p_o-s-p_s)(p_o+s-p_s)}{(2-2p_o-s)s}. \quad \square$$

Proof of Proposition 4. (i) For a retailer who only offers the Store and Online channels, his profit function is given in (13). We can substitute (10) and (11) into (13) and obtain

$$\Pi = (p_s-c_s)\left(1-\frac{p_o+s+p_s}{2}\right)(p_o+s-p_s) + p_o(1+p_s-p_o-s)(1-p_o-s).$$

We differentiate Π above with respect to p_o and obtain

$$\frac{\partial\Pi}{\partial p_o} = 3p_o^2 - [3p_s-c_s+4(1-s)]p_o + (1-s)(1+2p_s-c_s-s). \tag{A.1}$$

It is then straightforward to show that $\frac{\partial\Pi}{\partial p_o}$ is convex in p_o.

Note that any feasible online price must lie between p_s-s and p_s $(\le 1-s)$. Also, $\frac{\partial\Pi}{\partial p_o} = -p_s(1-s) < 0$ when $p_o = 1-s$. This implies that Π is unimodal in $p_o \in [p_s-s, p_s]$. So there are three possible scenarios as follows.

(1) Suppose that $\frac{\partial\Pi}{\partial p_o} \ge 0$ when $p_o = p_s$. This also implies that $\frac{\partial\Pi}{\partial p_o} \ge 0$ when $p_o = p_s - s$, and indeed, $\frac{\partial\Pi}{\partial p_o} \ge 0$ for the whole feasible range of p_o. So, Π is increasing in p_o. Hence, $p_o^* = p_s$. It is straightforward to confirm that:

$$\frac{\partial\Pi}{\partial p_o} \ge 0 \textit{ at } p_o = p_s \textit{ when } p_s \le \frac{(1-s)(1-s-c_s)}{2-2s-c_s}.$$

(2) Suppose that $\frac{\partial\Pi}{\partial p_o} \le 0$ when $p_o = p_s - s$. This also implies that $\frac{\partial\Pi}{\partial p_o} \le 0$ when $p_o = p_s$, and indeed, $\frac{\partial\Pi}{\partial p_o} \le 0$ for the whole feasible range of p_o and Π is decreasing in p_o. Hence, $p_o^* = p_s - s$. It is also straightforward to confirm that:

$$\frac{\partial\Pi}{\partial p_o} \le 0 \textit{ at } p_o = p_s - s \text{ when } p_s \ge \frac{1+2s-c_s}{2+s-c_s}.$$

(3) Suppose that $\frac{\partial\Pi}{\partial p_o} \ge 0$ when $p_o = p_s - s$ and $\frac{\partial\Pi}{\partial p_o} \le 0$ when $p_o = p_s$. This is the case where Π first increases and then decreases in p_o. Hence, the optimal online price p_o^* can be found by solving the first-order condition $\frac{\partial\Pi}{\partial p_o} = 0$. In particular, it is the smaller root of the equation, i.e.,

$$p_o^* = \frac{1}{6}\left[3p_s - c_s + 4(1-s) - \beta\right], \tag{A.2}$$

where $\beta = \sqrt{[3p_s-c_s+4(1-s)]^2 - 12[(1-s)(2p_s-c_s+(1-s))]}$. The condition for this scenario is the complement of the conditions in scenarios (1) and (2). That is,

$$\frac{(1-s)(1-s-c_s)}{2-2s-c_s} \le p_s \le \frac{1+2s-c_s}{2+s-c_s}.$$

(ii) For a retailer offering all three channels, Type 1 customers will only choose between the Online and OS channels in making a purchase. Thus, the analysis is the same as for the case where the products are only available online, except that it is also required that $p_o \le p_s$. The result follows by combining this additional restriction and the equation given in (9) in the beginning of Section 4.2. □

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