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CONSTRUCTION STRESSES IN DWORSHAK DAM

A Report of an Investigation

by

Jerome M. Raphael and Ray W. Clough
Professors of Civil Engineering

to

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INTRODUCTION

One of the most serious problems confronting the designers of massive concrete structures such as large dams is the control of random structural cracking induced by temperature changes as the concrete hydrates and cools. In a number of dams, cracking has been controlled by limiting the dimensions of the pour by a series of contraction joints, and later filling these joints, if necessary, with cement grout. This is a tremendously expensive process, and increased attention has been expended in recent years to alternative construction methods to eliminate this costly detail. With the increased temperature control proposed for the construction of Dworshak Dam, it was desired to investigate the stresses that might be set up in this massive gravity dam if one longitudinal joint were provided, or alternatively if no longitudinal contraction joint were provided.

Because of the successful use of the finite element analysis procedure in determining the distribution of stresses in the cracked section of Norfolk Dam, an investigation was made to see if it would be feasible to make a finite element analysis of the stresses in Dworshak Dam, taking into account the actual construction program for the dam, the individual temperature histories of each lift of concrete, and the time dependent elastic and creep properties of the concrete.

The Norfork Dam analysis had demonstrated the applicability of the finite element method of analysis to large scale problems. However, in the case of Norfork Dam, the concrete was assumed to have a constant modulus of elasticity, and each individual analysis involved a given geometry which was unchanged for that particular problem. In contrast, in the case of Dworshak Dam, a number of important factors changed as the analysis proceeded. The most important problem was that the geometry of the dam changed each time a lift of concrete was placed. Thus, periodically as the analysis proceeded, the size of the problem increased. Also, since the analysis was performed starting from the earliest ages of the concrete, three important factors were time dependent: the thermal history of each independent lift, and the elastic and creep properties of the concrete itself. Each of these four important variables required extension of the analytical procedures used previously before the finite element analysis could be applied to the Dworshak Dam.

The original scope of the investigation included analysis of the incremental construction process and the time dependent creep and elastic properties of the concrete as well as the individual thermal history of each lift. It was intended to determine the maximum stresses in the gravity dam section for two cases: one for a dam without longitudinal joint and one for a dam with a single longitudinal joint at midsection. In the preliminary stages of discussion of this analysis, it was assumed that it was necessary to analyze the complete structure in order to find the maximum stresses. However, as the analysis proceeded, it was found that maximum stresses occurred at the base of the dam, and that extension of the analysis to the complete section of the dam was not necessary.

Following the issuance of a preliminary report in November 1963, a conference was held in Berkeley between members of the Corps of Engineers and the faculty investigators at which the objectives of the investigation were redefined along the following lines.

It was generally agreed that the trend of stresses was sufficiently established in analyzing the first seventeen lifts to terminate the study of the incremental construction procedure at this point. Study of the results of the preliminary analyses disclosed that a number of questions required further elucidation. It was thus decided to extend the study to include the following additional investigations:

1. Effect of length of block between longitudinal contraction joints on thermal stresses.
2. Effect of a delay in the pouring schedule on stresses above and below the cold joint.
3. Effect of modifications to the artificial cooling schedule on the thermal stresses.
4. Effect of inclined foundation surface on thermal stresses.

Investigation of all these points has been conducted within the limitations of funding of the project, and this final report summarizes the results of these investigations.

Following the issue of a second progress report on "Construction Stresses in Dworshak Dam" in July 1964, a conference was held at Walla Walla

Acknowledgments

The investigations reported here were performed at Berkeley under the terms of Contract DA-45-164-CIVENG-63-263 between the Walla Walla District of the U. S. Army Corps of Engineers and the Institute of Engineering Research of the University of California. The U. S. Engineers at Walla Walla were responsible for physical data on the dam and temperatures computed by Civil Engineer James S. Gobble, under the general supervision of Chief Concrete Engineer, Ivan E. Houk, Jr. The stress analyses were performed at Berkeley, under the general direction of the faculty investigators, Professors J. M. Raphael and R. W. Clough, by Graduate Research Engineer, Ian P. King, assisted by Research Assistants, Rolf Hermanrud and Robert S. Dunham. Concrete properties were determined at the University under the direction of Professor David Pirtz and at the U. S. Engineers Concrete Laboratories at the Waterways Experiment Station, Jackson, Mississippi, and at the North Pacific Division Laboratory at Troutdale, Oregon.

METHOD OF ANALYSIS

General Comments

For the purposes of this investigation, the gravity dam structure was assumed to act as a two-dimensional stress system, in which stresses do not vary in the direction of the longitudinal axis, and neither normal nor shear stresses exist on cross sections perpendicular to this axis. The development of normal stress on these planes is assumed to be obviated by the presence of contraction joints parallel to the sections. The structure actually considered in the analysis is a one-inch thick slice of concrete having the outline shown in Fig. 1.

The plane stress problem thus envisaged may be solved readily by the finite element method for any given loading condition, if the concrete is assumed to be linearly elastic and to have properties that do not vary with time. The method has been used in several previous investigations which have been reported in detail (1,2,3,4), thus it will be described here only in general terms. However, the special factors considered in this study for the first time, such as incremental construction, time dependent elastic and creep properties, and local temperature variations, will be described more extensively in this report. Additional details on the purely technical aspects of the investigation may be found in King's doctoral dissertation[5] which also resulted from this project.

The Finite Element Method

The fundamental concept of the finite element method is the subdivision of the actual continuum into a system of individual elements of appropriate shapes which are interconnected only at their corners or nodal points. In order to establish the elastic properties of the individual elements, it is assumed that the deformations which may be developed in each are limited to certain specified forms. On the basis of these assumed displacement functions, the force-displacement relationships for each element may then be found, taking account of its individual geometric and material properties.

In the analysis of a general plane stress problem, such as is considered in this study, it has been found convenient to make use of triangular finite elements, which are easily fitted to arbitrary boundaries, and to assume linearly varying displacement patterns within each element, i.e. a constant state of stress and strain within each. The stiffness properties of each element, representing the forces developed at each nodal point as a result of unit displacements applied successively at each, may then be found by integrating the product of the internal stresses and strains over the volume of the element.

When the individual element stiffnesses have been established in this way, the finite element idealization of the complete plane stress system may then be analyzed by exactly the same methods used in the analysis of normal structural systems. The direct displacement procedure generally has been found convenient for most finite element analyses. By this procedure, the stiffness of the complete assemblage is obtained by adding together the stiffnesses of all the elements contributing to each nodal point. The equations of equilibrium of the joints are then solved simultaneously to determine the joint displacements resulting from the given loads. Finally the stresses in the individual elements are found from their nodal displacements, acting upon the element stiffness properties.

Incremental Construction Procedure

The method of analysis described above is suitable for treatment of any linear elastic plane stress problem for which the geometric and elastic properties do not change with time. It will be recognized, however, that the development of gravitational stresses in a massive structure involves a more complex situation than is envisioned in a typical structural analysis. The gravity stresses develop continuously during construction; it is not proper to assume that the structure is completed and then gravity is applied instantaneously to the entire system.

To account for this effect, it is necessary to carry out the gravity stress analysis in an incremental fashion, corresponding to the actual field construction sequence. As each new lift of the dam is placed, its dead weight contributes to the stressing of the structure already in place. Additional strains due to the new loading are superimposed on the pre-existing state of strain to produce the total state of strain in the portion of the structure awaiting placement of the next lift.

The incremental construction procedure causes no complication in the finite element analysis; it is necessary merely to keep track of the total state of stress and strain in each element by adding the stresses and strains resulting from the placement of each new lift. For the present investigation, the computer program was modified to accomplish this superposition automatically, assuming that each new lift is 60 inches high. The essentially fluid character of the concrete as it is added in each new lift is represented in the analysis by assuming a very low initial value for its elastic modulus. By the time the next lift is placed, however, the concrete has hardened, and it is given a significant modulus in all subsequent analyses.

Creep Behavior of the Concrete

A more complex aspect of the real stress state in a concrete gravity dam, as compared with the ideal elastic behavior assumed in previous studies, results from the creep behavior inherent in portland cement concrete, especially at early ages. This effect may be characterized by time-varying stresses in a system constrained against deformation. The mechanics of creep in concrete is further complicated by the fact that both the modulus of elasticity, relating elastic strain components to stresses, and the creep property itself are not constants, but vary strongly with time.

The effects of both of these time-variation phenomena were accounted for in the same incremental analysis scheme used to represent the incremental construction history. During each interval of time, the material was assumed to have a specified elastic modulus, and to undergo creep stress relaxation at a specified rate. At the end of the increment, these properties were modified as required by the previous stress and strain history of the element, before proceeding with the analysis for the next time increment. It is important to note that the finite element procedure makes possible the selection of appropriate properties for each individual element, independently of all others.

The general analysis procedure may be described for any arbitrary time interval, Δt. At the end of the preceding time increment, the structure is in a known state of deformation, as represented by the displacements of the nodal points from their original positions, and has a known condition of stress within each element. Associated with this internal stress state is a set of nodal point forces--the statically equivalent resultants of the internal stresses. In addition, external loads may be applied directly to each nodal point. In the present study these loads were the dead weight forces, which were taken as one-third of the weight of each element concentrated at each of its nodes. Finally, if the internal stress and external load resultants are not in equilibrium, a set of artificial nodal constraints must be added such as to maintain the existing state of deformation.

The analysis for any given time increment is initiated by a purely elastic evaluation of the deformations which result when the artificial nodal constraints are relaxed. In effect, a set of nodal forces equal but opposite to the constraints is applied, and the resulting elastic contributions to stress and deformation are computed by the normal finite element procedure. The modulus of elasticity used in each element is based on the time elapsed since that element was originally cast in the structure; thus the element properties vary lift by lift.

The total state of deformation is then found by adding the nodal displacements computed in this elastic analysis to those existing at the end of the preceding increment. Similarly the total state of stress is given by the sum of the newly developed stress components and the pre-existing state of stress. After this elastic equilibrium adjustment, which is assumed to take place instantaneously, the nodal points are again fixed against displacement, and the creep mechanism in the concrete takes place in the form of stress-relaxation. In other words, the strains in each element are held constant while its stresses vary according to a specified creep law, as will be discussed in the next chapter. Because the artificial nodal constraints which function during each time interval constitute a violation of equilibrium requirements, it is necessary that they be relaxed at relatively short time intervals to maintain the correct rate of creep.

Temperature changes also may take place in the concrete during each time interval. Thus, the total change of stress in each element is due to temperature effects combined with creep. The temperature stress developed in the fully constrained state which exists in each element during any time interval may be found readily from the product of temperature change in the interval, the thermal coefficient of expansion, and the modulus of elasticity of the element for that time. The nodal force constraints which are relaxed at the beginning of the next interval thus include both thermal and creep effects.

Flexibility of the Foundation

The stresses and deformations in a gravity dam may be affected significantly by deformations of the foundation material upon which it is supported. This factor also may be included in the finite element analysis without difficulty since the material property discontinuity at the interface between concrete and foundation rock may be represented directly in the discrete element idealization.

In previous studies of gravity dams[2] the flexibility of the foundation has been accounted for by including a large foundation zone in the element idealization, as shown in Fig. 2. The great advantage of this technique is that any arbitrary variations of foundation properties, such as seams of softer or harder material, layered rock structures, etc., may be represented realistically. In fact, this is the only practicable way to account for such conditions.

This procedure has a major disadvantage, however, in that the larger part of the computational effort may be devoted to the analysis of the foundation. To make the results independent of artificially assumed boundary conditions in the foundation, it is necessary to include a large foundation zone, generally involving many more finite elements than are in the dam proper. Thus the useful output of the analysis, the stresses and deflections in the superstructure, is only a small part of the total computation.

In the present study a more efficient procedure was developed for taking account of the foundation flexibility effects. The dam is assumed to rest upon an infinite elastic half-plane, as shown in Fig. 3. This procedure is applicable to any case in which the entire foundation zone has homogeneous, isotropic elastic properties. Either plane stress or plane strain conditions may be assumed in the foundation; in this study plane strain behavior was assumed. The dam is assumed to apply forces to the foundation at the nodal points of the base elements. To be consistent with the stress distributions developed within the finite elements, it is assumed that each nodal point force is distributed uniformly over a zone equal in width to the nodal point spacing, as shown in the sketch.

The foundation flexibility is represented in the analysis by a flexibility matrix. Each element of this matrix, f_{ij}, represents the deflection developed at one nodal point "i" due to the application of a unit distributed load at another point "j". Expressions for the deflections at the surface of an infinite elastic half-plane resulting from a distributed load pattern are well known,[6] so it is a simple matter to evaluate these flexibility coefficients. Details of the calculations are presented by King[5] and will not be duplicated here, but two points in the analysis will be noted. First, both vertical and horizontal force components are applied to the foundation by the dam, and both vertical and horizontal displacement components result from the application of each force component, in general. Thus, the order of the base flexibility matrix is twice the number of base nodal points. Second, the total load applied by the dam to the foundation is of no concern in the analysis because the rigid body displacements resulting from the total load cause no stresses in the dam. Only the relative forces acting between the base and the dam, which tend to cause distortion of the dam, are of importance here. Thus, the flexibility coefficients have been calculated on the basis of sets of self-equilibrating interaction forces between the base and the dam. The total base deflection represented by each flexibility coefficient is the result of the simultaneous action of an appropriate set of forces,both vertical and horizontal equilibrium sets being considered.

In order to combine the foundation flexibility effects with the flexibility of the dam itself, it was necessary to express them in the inverse, or stiffness matrix form. After the base flexibility matrix was inverted, the resulting base stiffness matrix was added directly to the stiffness matrix for the corresponding nodal points of the superstructure. The standard finite element analysis procedure was then applied to the combined stiffness matrix to obtain stresses and displacements in the superstructure; no consideration was given to the stresses or deformations in the foundation zone.

A test analysis was made by this procedure to evaluate its reliability in representing foundation flexibility effects. The case considered was a structure consisting of a single lift of concrete supporting a concentrated load at its centerline. In order that the results might be compared with the displacements produced by a concentrated load on the half-space, it was assumed that the modulus of elasticity of the concrete was very low. Vertical displacements at the base of the concrete lift computed by the finite element method are plotted in Fig. 4, together with the theoretical results for the elastic half space. The agreement is seen to be excellent.

Computer Program

The finite element analysis of even a very simple elastic system requires an enormous amount of computational effort, and the method can be considered practical only when used with a powerful digital computer. However, when such facilities are available the analysis of any arbitrary plane stress system subjected to any arbitrary loading may be carried out quite automatically. Programs developed prior to the initiation of the present study were suitable for the treatment of a standard elastic system. In this investigation, it was necessary to extend these programs to account for the time-varying aspects of the problem. In addition, changes were made to increase the capacity of the program and to automate further the input and output operations.

The analysis of both the incremental construction process and the creep relaxation of the concrete are accomplished in this computer program by carrying out a sequence of standard elastic analyses at pre-selected time intervals. In addition, the program is designed to evaluate the creep stress relaxations and thermal stress changes which occur in each element during the specified time intervals, to specify the modulus of elasticity appropriate to each element for each analysis, to recognize the successive addition of new lifts to the structure according to a given construction schedule, and to account for the combined state of stress and strain in each element as a result of the superposition of effects from each time interval.

In developing this new program, a Gauss-Seidel iterative analysis of each elastic state of stress was used, following the idea first used by

Wilson.[4] However, by taking advantage of the symmetry of the matrix defining the stiffness of the structural assemblage, it was possible to increase the capacity of the program from 350 to 600 nodal points. Speed of operation and the rate of convergence were essentially unchanged.

Two other basic improvement were made to speed up and improve accuracy in the overall operation of the program: the addition of a mesh generation routine and of output plotting routines. The mesh generation routine is designed to establish automatically the triangular element subdivision of the dam cross-section and to punch out the x-and y-coordinates of each nodal point on cards. In the past, the hand preparation of these data was quite laborious and was the source of errors which caused considerable waste of computing time.

The mesh generated by this routine is well adapted to the incremental construction process. It is laid out in horizontal lifts, with any specified number of elements in each. (In order to maintain the triangular mesh system, however, the number of nodal points cannot change by more than one in successive lines). Spacing of the nodal points can be made uniform over the whole width or two different spacings may be specified to give a more refined mesh in regions of high stress gradients. A typical automatically generated element mesh is shown in Fig. 5.

The output plotting routines have proved to be even more valuable aids in reducing the time required to carry out a complete analysis. In previous investigations, the plotting of results was the slowest part of the analysis process. The computer produces information at such a prodigious rate that analyses which required only two or three minutes of computer time took several days to plot up in the form of stress contours. This work now is done automatically merely by storing the computer output on magnetic tape and then processing these tapes through a special plotting routine. The output of this routine, which also is stored in magnetic tape, is used to actuate an automatic plotting machine which produces finished stress contours and stress trajectories.

The stress contour plots are constructed from nodal point stress values which are obtained by averaging element stresses as described by Wilson.[4] For plotting purposes, it is assumed that the stresses vary linearly within each element and the contours are located from the nodal values by linear interpolation. Thus, within each element the contours are equally spaced parallel lines, but direction and spacing varies from element to element. A typical machine plotted stress contour graph is shown in Fig. 6.

The stress trajectory plotting routine makes use of the stresses computed within each element rather than the nodal stresses. Each stress trajectory line is constructed by starting at some specified point on an outer boundary of the structure. The line is constructed in the direction of the desired stress quantity within that boundary element. When the trajectory line reaches the boundary of the first element, it proceeds in a new direction corresponding with the stress direction in the second element. This process is then repeated, the direction of the trajectory changing appropriately as each element boundary is crossed. The resulting series of straight line segments closely approximates the true stress trajectory if a reasonably fine element mesh system has been used. A sufficient number of trajectory starting points must be specified along the structure boundaries to provide the desired density in the stress trajectory representation. A machine-plotted set of stress trajectories is shown in Fig. 7.

PROPERTIES OF DWORSHAK DAM

Concrete Properties

The creep phenomenon in concrete is a form of non-linear viscoelastic behavior involving a time-varying relationship between stress and strain. The relationship for concrete is made particularly complex by the strong dependence of the material properties on the age of the specimen, expressed as the length of time since casting.

For a test conducted at constant stress σ, the total strain ϵ may be expressed as follows:

$$\epsilon\ (t) = \epsilon_c(\sigma,t,T) + \frac{\sigma}{E(t)} \tag{1}$$

in which ϵ_c is the creep strain, a function of the state of stress σ, the age at the date of loading T, and the present age t. To this is added the elastic strain which depends upon the current modulus of elasticity E (t).

Results of a test of this type are shown schematically in Fig. 8. It will be noted that the elastic strain reduces continuously as the modulus of elasticity decreases. The creep strain ϵ_c thus increases to compensate for the reduced elastic strain as well as to represent the observed increase in total strain.

A number of different mechanisms and formulas have been proposed in the past to represent the creep behavior of plain concrete.[7,8,9] A thorough study of previous work on this subject was made during the present investigation in an effort to establish the mechanism most suitable for use with the finite element analysis technique. Specific requirements imposed on the mechanism to be employed here were that it be representative of the mass concrete mixes used in gravity dams, and that it depict the stress relaxation behavior envisaged in the incremental analyses procedure. A detailed discussion of this investigation into the creep problem is presented in King's dissertation.[5] Only the specific mechanism that ultimately was selected for this analysis will be described here.

The mathematical formulation of the creep mechanism used in this computer program was proposed originally by McHenry.[8] It may be expressed as follows:

$$\epsilon_c(t) = \sigma \sum_{i=1}^{N} a_i(T) \left\{1-e^{-m_i(t-T)}\right\} \tag{2}$$

McHenry suggested that a sufficient number of terms be included in this series to give satisfactory agreement with available experimental data. In the present study, it was assumed that two terms would suffice. Thus the creep expression becomes:

$$\epsilon_c(t) = \sigma \left\{ a_1(T)(1-e^{-m_1(t-T)}) + a_2(T)(1-e^{-m_2(t-T)})\right\} \tag{3}$$

It is of interest to note that this form of behavior may be represented completely by the rheological model shown in Fig. 9.

The functions $a_1(T)$ and $a_2(T)$ and the constants m_1 and m_2 of Eq. 3 define the creep behavior of the model. They may be evaluated from experimental data obtained in laboratory creep tests to define a mathematical model simulating the observed behavior of the test specimens. Creep tests of the Dworshak Dam concrete which served to define the material properties used in this investigation were carried out by Pirtz.[10] The tests consisted of measurements of creep strain developed under conditions of constant applied stress, and a series of such tests were performed involving a wide range of concrete ages at the time of loading.

From each of these tests, the quantities a_1, a_2, m_1 and m_2 were evaluated using the numerical analysis procedure described by King.[5] The accuracy of the resulting mathematical expression for the creep strain is demonstrated for one case in Fig. 10, which shows the observed and computed creep strains for a specimen loaded at the age of seven days. The agreement clearly is quite satisfactory.

Different values of a_1 and a_2 were obtained for each different age of specimen at the time of loading. The values of these functions determined for ages of loading between zero and 100 days are shown in Fig. 11. The other viscoelastic coefficients involved in Eq. 3 were found to be as follows:

$$m_1 = 0.034$$

$$m_2 = 0.52$$

The modulus of elasticity was found to vary with time according to the following expression.

$$E(t) = (1.0 + 0.663 \log_e t) \times 10^6 \text{ psi} \tag{4}$$

where t is the time since casting, in days. Poisson's ratio for this concrete was taken to be constant at 0.17.

An additional check on the mathematical formulation of the creep behavior was made by comparing the computer stress relaxation history for a given test condition with the corresponding stress variation observed experimentally. In this series of experiments, which also were conducted by Pirtz[10], the concrete specimens were subjected to a fixed axial strain condition, and the change of stress with time was recorded. The corresponding situation was simulated analytically by assuming the constant stress condition of Eq. 3 to apply during very short time intervals, after each of which the strain was readjusted to its initial value by an instantaneous elastic stress modification. The stress relaxation was thus simulated by a series of steps in the stress history, each readjusting the strain to its applied value.

Because this type of stress relaxation analysis subjects the specimen to a whole sequence of loadings associated with the elastic strain adjustments, it provides a rather complete check of the assumed mathematical formulation of the creep mechanism and of its established coefficients. For each strain adjustment, a different time of loading is involved as well as a different elastic modulus. Thus the comparison of experimentally determined stress relaxation results with analytical results, presented in Fig. 12, demonstrates the satisfactory approximation achieved by this analytical procedure.

It is important to note that this successive strain adjustment approximation of the stress relaxation mechanism is applied in addition to the "time increment" process used in the complete finite element analysis. It was explained earlier how the creep phenomenon in the complete structure was approximated by an incremental analysis procedure, in which the individual finite elements were subjected to stress relaxation during each time interval while the nodal points were constrained against displacement. Now it is seen that the stress relaxation within the elements is computed by an independent time increment process involving considerably shorter time intervals. In general it was found that a one-eighth day time interval was short enough to give a good approximation to the smoothly varying stress relaxation history within the element, while a 2 or $2\frac{1}{2}$ day interval was found to account adequately for the creep mechanism in the complete structure.

It was necessary to consider one additional factor in establishing a creep mechanism suitable for use with the finite element analysis technique. The previous experimental and analytical studies have been concerned only with behavior in a uniaxial stress or strain field, while a complete two-dimensional state of stress is involved in the finite element analysis. To make the transition to the two-dimensional condition, it is assumed that the uniaxial characteristics can be applied to each principal stress component independently. A certain approximation is implied in this assumption because the principal stress directions change slightly as the nodal constraints are relaxed at the beginning of each time increment. Thus the creep due to the newly added stress increment should take place in a slightly different direction. This rotation of the principal stress directions is included in the analysis by treating the creep constants as vectors, transforming them geometrically to values associated with the new stress directions in each time increment.

Temperature History and Construction Schedule

Temperature changes in the concrete represent a major source of danger to the integrity of a gravity dam structure, especially changes which take place during the early ages before the concrete has developed its full strength. The creep mechanism serves to help reduce the full development of thermal stresses, but it is necessary as well to control the temperature variations in the structure if cracking is to be avoided; to this end a number of different temperature control procedures have been employed in the construction of mass concrete structures.

For this investigation, the temperature control procedures and the resulting temperature histories to be expected in each lift were established by the Corps of Engineers. Details of the temperature distributions were evaluated by digital computer, assuming certain cooling cycles to be followed in the cooling pipe system which is imbedded in the concrete. Thermal radiation was assumed to take place only from the upper surface of the structure; the upstream and downstream faces were assumed not to participate in the heat transfer. Although the data supplied by the Corps of Engineers defined temperature variations within the lifts, only the average lift temperature histories were used in the general finite element analysis because each element extended through the full lift height.

The construction schedule which finally was employed in this investigation is shown in Fig. 13. This involves a 4-day sequence in placing each of the first 12 lifts, with increasing times required for the later lifts due to the gradually increasing valley widths. The temperature histories specified for various typical lifts are shown in Fig. 14, based upon a standardized cooling cycle which was adopted during the course of this study. A number of other temperature histories also were considered before the optimum conditions were established. Several of these, represented by the lowest lift temperature histories, are depicted in Fig. 15. The standardized cooling cycle, which yielded the results shown in Fig. 14 is designated as temperature history No. 5 in Fig. 15.

RESULTS OF ANALYSES

Preliminary Studies

The original objective of this investigation, the stress analysis of Dworshak Dam taking full account of the time-varying thermal, elastic and creep properties of the concrete while considering the actual construction history, involves an enormous computational program. Approximately 140 lifts would have to be treated, each involving different variations of material properties. The normal finite element idealization of the 140 lifts would involve over 10,000 elements.

Because the solution of this complete system would require the development of special techniques for handling large numbers of finite elements, it was decided to concentrate the initial analytical effort on the behavior of the lower region of the dam, considering only as many lifts and elements as could be handled directly within the high speed storage capacity of the computer. This initial study would serve to check out all of special features to be built into the program, such as incremental construction and time varying properties, and also would provide valuable information on the expected behavior of the dam during the initial phases of construction.

Results obtained in the preliminary stages of the investigation, however, revealed that the most significant thermal stresses would be developed in the lowest lifts of the dam, and that stresses in the higher lifts would follow a similar pattern but with reduced maxima. On the basis of these observations, the basic objective of the program was modified by mutual agreement.

It was decided that all of the required information could be obtained by analysis of the lowest 70 to 80 feet of the dam and that it would not be necessary to develop an expanded analytical procedure to consider the complete cross-section.

Results of the various preliminary studies which were carried out are discussed in this section; the results of the basic research studies are then described at the end of this chapter.

Nature of Stress Distribution

Although it was not the object of any specific study, the first and most important finding obtained in the preliminary analyses concerned the nature of the stress distribution developed in the lower layers of the dam. The principal stresses were found to be essentially vertical and horizontal, except near the toe and heel of the dam, and the maximum tensile stresses, which are the most critical factors in this project, were found near the center of the lifts. A typical stress distribution is depicted in Fig. 16 which shows the variation of vertical and horizontal normal stresses developed in Lift 1 after 14 days of construction, or two days after the placement of Lift 4. The figure indicates clearly that the edge effects associated with the upstream and downstream faces of the dam are quite local and that the stress at the center-line of the lift is representative of the significant stress conditions for the major portion of the dam. The vertical stresses may be seen to result directly from the dead weight of the material above this lift.

Effect of Finite Element Shape

In previous studies made by the finite element method, it proved desirable to use elements which were nearly equilateral in shape, in order to provide a reasonable approximation of the stress distribution in the system. On this basis it would have been necessary to use well over 100 elements to represent each of the lower lifts of the dam, and the capacity of the computer would have limited the direct solutions to possibly 4 or 5 lifts. For a system such as this in which the stresses in the horizontal direction are nearly constant, however, it is possible to use greatly elongated elements with no loss of accuracy in the central region. Thus, variable length elongated elements as shown in Fig. 17 were used in this analysis, with shorter elements introduced near the faces of the dam to provide a better approximation to the varying stresses in these regions.

The discrepancy in the horizontal stress distribution resulting from the use of this type of irregular coarse mesh system as compared with a uniform fine mesh is shown in Fig. 18. This graph depicts stresses in the lower lift of the dam at three different stages of construction. Although the elongated elements tend to exaggerate the edge effects in the lift, it is evident that they have no effect on the critical stress values at the center of the lift.

Effect of Time Interval

In the description of the method used to treat the creep behavior of the concrete, it was noted that two different processes are carried out by a time increment sequence: the stress relaxation of the concrete within the finite elements, and the creep deformations of the complete structure. In principle, both of these time intervals should be infinitesimal, but in practice if they are made short enough, no further change in the results will be observed from further shortening. Thus it was necessary to establish by trial time intervals short enough to give the desired accuracy but long enough to retain computational efficiency.

The study of the time interval required in the creep relaxation process was carried out with a more complex structure than the lower lifts of the gravity dam. Three different time intervals were considered: one-fourth, one-eighth and one-sixteenth day. Minor principal stress contours obtained in this study are shown in Fig. 19. It is evident that the differences between the one-sixteenth and one-eighth day intervals are insignificant, but the deviation obtained with one-fourth day intervals is more important. On this basis, it was decided to use the one-eighth day stress relaxation interval in all analyses.

The study of the time interval to be used in the creep analysis of the actual dam assemblage was carried out on the lowest lifts of the structure. Various different intervals were tried and it was found that no significant difference resulted from reducing the interval below 2 or $2\frac{1}{2}$ days. Results of such a comparison are presented in Fig. 20, which shows the variation of horizontal stress at the center of the first and third lifts. The differences between the $1\frac{1}{4}$ day and $2\frac{1}{2}$ day analysis intervals clearly are of no consequence.

Effect of Foundation Elasticity

During the early stages of the analytical investigation it was recognized that the flexibility of the foundation rock might have a significant influence on the behavior of the dam. Therefore considerable effort was devoted to developing an analytic procedure which would account for this effect reliably and efficiently. As soon as this procedure was made operational, comparative analyses were carried out to establish the importance of the elastic base. Results of such analyses are presented in Fig. 21 which shows the horizontal stress at the center of the lower lift for three cases. The stresses developed in the lift placed over an elastic foundation are seen to be significantly less than over the rigid foundation, as would be expected. However, it is evident from the third curve on the figure that temperature changes within the foundation rock have little effect on the results. The elastic foundation system was used in all of the regular production analyses after this analytical procedure had been developed; however, some of the early studies on the effects of temperature history were made with the rigid foundation system.

Stress Distribution Near the Base of the Cross Section

In its ultimate version, the finite element computer program was capable of evaluating in a single operation the incremental construction and time-dependent behavior of a 26-lift segment at the base of the dam. Data were prepared for the complete analysis of this 26-lift section, but the analysis was stopped after 73 days of construction, when 17 lifts had been placed.

The computation was terminated at this point because the machine time required to carry out each successive stage was becoming excessive, and because the results indicated that the critical stress area was near the base of the section.

The general nature of the stress distribution results obtained in the analysis carried out 73 days after construction began, for the 17 lift portion of the dam, is shown in Fig. 22 by means of contours of the vertical, horizontal, and shear stresses developed in the section. It will be noted in this figure that the stresses are nearly constant in the horizontal direction except near the upstream and downstream faces. It also is significant that the stresses in the section are almost entirely compressive, except within the upper lifts wherein temperature changes due to hydration of the cement and to artificial cooling are still significant. This fact should be kept in mind when considering the thermal stress effects discussed in the following paragraphs: the tensile stresses which are developed affect only a small portion at the upper part of the cross section, and last for only a short time.

Effect of Temperature History Upon Stresses

One of the most important results to be obtained from this investigation was the comparative evaluation of the several different temperature histories which were proposed for this dam. The temperature histories to be considered, presented in Figure 15, were computed by the Corps of Engineers on the basis of various assumed cooling cycles applied through the cooling pipes.

Results of the most important stress analyses are presented in Figs. 23 to 27. Figs. 23 and 24 show the horizontal stress at successive lift mid-points resulting from temperature histories 1 and 2. Similar results for temperature histories 3 and 4 are presented in Figs. 25 and 26. In all of these analyses, except for temperature history 1, it is clear that the maximum stresses are developed in the lowest lift. The comparison of these lowest lift stresses is shown in Fig. 27, which demonstrate the effectiveness of proper temperature control procedures in reducing tensile stresses in the structure.

The most favorable temperature history considered in these initial analyses was No. 5. Results of a comprehensive investigation using this temperature history are shown in Figs. 28 and 29 in the form of graphs of midsection horizontal stress plotted against time. In Fig. 28, the time axis represents absolute time after the beginning of construction. This figure shows how the stress in each lift is affected by the placement of the subsequent lifts, as the stress history is directly dependent on the construction sequence. The repetitive nature of the stress histories in successive lifts is quite evident in the figure, although the extra constraint provided by the foundation rock is reflected in the higher stress developed in the first lift. The slight variation shown by the stress history of lift 14 is due to the change in placement schedule from a 4 day to a 6 day cycle after lift 12.

The same results are plotted in Fig. 29 on a relative time scale showing time since the placement of each lift, or the age of the concrete in the lift.

This again shows the repetitious nature of the different stress histories, and also the slight deviation found in lift 14. In each lift, these results show an initial horizontal compression due to heating after setting, followed by a sharp reversal of stress into tension as cooling takes place. A short delay occurs when the subsequent lift is placed, but then the stress rise continues until maximum tension is reached at about 14 days. After that the slight continuing drop in temperature is counteracted by the effects of creep so that the tensile stress begins to drop. Subsequent increases of temperature caused further decreases of tensile stress until eventually a compressive stress state is reached.

Further study by the Corps of Engineers of possible cooling cycles led to temperature histories 6 and 7 shown in Fig. 15. The artifical cooling cycles used are defined in Table 1.

Table 1. Program of Artificial Cooling

Temperature Histories	Days of Cooling
No. 5	0-14
No. 6	0-4, 8-12, 16-22
No. 7	0-4, 18-28

Stresses computed at the center of the first lift as a result of applying each of these temperature histories are presented in Fig. 30. A full 35-day construction sequence was considered in each case, although only the first lift stress results are reported. The general conclusion to be drawn from this figure is that the maximum stress is greater the later the date at which cooling is applied. This is due to the reduction in creep at later ages of the concrete.

However, it is not only the stress, but also the strength of the concrete that controls its safety against cracking, and it must be remembered that the strength also increases with time. Assuming the tensile strength to be 10 per cent of the ultimate compressive strength, the estimated development of tensile strength in the Dworshak Dam concrete is indicated in Fig. 31, which is based on data supplied by Pirtz and by the Corps of Engineers. From a design standpoint, the critical factor in the development of thermal stresses is the ratio of the maximum stress at any age to the available strength of the concrete at that age. The variations with time of the ratio of these quantities resulting from temperature histories 5, 6, and 7 is shown in Fig. 32. On this figure it can be seen that both the continuous cooling schedule (5) and the intermittent cooling schedule (6), lead to stress-strength ratios less than 0.5, but that the delayed cooling schedule (7) produces a higher stress-strength ratio as well as the highest stresses. It is clear that either temperature history 5 or 6 is preferable to 7.

Effect of Longitudinal Joints on Stress

Results of all cases studied in the early phases of the investigation showed that horizontal stresses in the lifts were nearly uniform over the entire central portion of the structure, and that reductions in stress could be noted only very near to the upstream or downstream faces as shown in

Figs. 16 and 18. A further study was made of this factor to determine just how effective longitudinal joints might be in reducing tensile stresses. For this purpose, a series of blocks of different lengths were considered, each subjected to the same temperature histories and construction sequences as the actual structure.

Results of this study are shown in Fig. 33 where the maximum stresses developed in each of the first three lifts are plotted against the total block length. This figure shows clearly that only when the block length is less than 50 ft. is there a substantial reduction in the maximum stress. Therefore it may be concluded that longitudinal joints do not provide a practical means of controlling stresses in Dworshak Dam if the proposed temperature controls are used.

Local Stress Distribution Within the Lifts

In all of the analyses discussed above, the calculations were made for finite element idealizations in which the elements were all equal in height to one lift or 60 inches. Therefore, only average stresses within each lift could be computed, and only average temperature variations for the entire lift could be considered. However, because of the localized manner in which the artificial cooling was to be introduced in the lifts, it was recognized that some rather extreme stress variations should be expected within the lifts.

In order to study further these localized stress and creep effects, a detailed finite element idealization was made of a typical interior portion of the lower lifts. Because the cooling pipes were located at five foot spacings staggered at the base of each lift, a $2\frac{1}{2}$ foot wide slice of the lower portion of the structure was considered. To maintain the symmetry of this interior portion of the structure, the slice was assumed to be supported at each side by rollers which permitted vertical movement but prevented lateral movement. Also included in the idealization was an 11 foot deep slice of the foundation rock.

The incremental construction analysis of this idealized structure was carried out for a period of 16 days, during which time the lower four lifts of concrete were placed. The detailed temperature distributions within the lifts, as computed by the Corps of Engineers for Temperature History 5, were employed in the analysis, and the standard creep and modulus variations were considered. An indication of the rather extreme stress variations resulting from these local temperature variations is shown in Fig. 34 which presents the maximum principal tensile stress contours in the first lift six days after it was placed. This is the time at which the maximum values of tensile stress were observed. The extreme stress gradients which developed in the vicinity of the cooling pipes are quite evident in this figure.

The time-history of the stresses at the critical points of the cross section, near the cooling pipes, as well as of the average stress in the lift are presented in Fig. 35 for lifts 1 and 2. The severe stress variations in the concrete near the cooling pipes is quite evident in these figures, as is the four-day phase lag in the placement schedule for lift 2.

The average stress history which is plotted in these graphs was obtained by numerically averaging over each lift the stress values indicated by stress contours plotted at two-day intervals. For comparison purposes, the average stress histories computed in the analysis of the complete lift system are presented by the dashed lines. These are the same results for lifts 1 and 2 which are shown at a reduced time scale in Fig. 28. The agreement between the average stresses given by the fine mesh analysis and the coarse mesh element stresses is quite good. It is not clear why the initial compressive stresses developed in the coarse mesh system are so much smaller in the fine mesh analysis. However, the maximum tensile stresses indicated by both analyses have about the same value and occur at the same time. Thus, it is realistic to think of local stress variations such as are shown in Fig. 35 to be superposed on the lift average stress state obtained in the principal analyses of this investigation and shown in Fig. 22.

Effect of Sloping Foundations on Stresses at the Base of the Dam

Numerical analysis of any modifications to stresses at the base of the dam that might be introduced by a sloping foundation line depends on a corresponding numerical analysis of the effect of this change in geometry on the temperature histories at the base. Such a temperature analysis was not available, but it is still possible to draw valid conclusions on the basis of the stress analyses made thus far. Fig. 36 shows the essential geometric elements of the problem. Each lift placed on the sloping foundation ends in a wedge-shaped element. Since all parts of this element are thinner than a lift placed on a horizontal foundation, it will react more readily to the cooling effects of the foundation below, and the air or cooling pipes above, and hence should reach a somewhat lower maximum temperature. This in turn will result in lower tensile stresses. This conclusion is reinforced when it is remembered that, as shown in Fig. 16, stresses near the ends of a lift are less than those near the center portions of a given lift. Hence no additional precautions need be taken for sloping foundations beyond those taken for any other foundation lift.

CONCLUSIONS

1. Concerning the method of analysis employed in this investigation, the following conclusions may be drawn:
   a. The modified finite element method, developed in the course of this study, provides an effective means of evaluating stresses in gravity dams taking full account of the construction sequence and arbitrary cooling cycles that may be imposed, and including the modifying effects of creep and varying modulus of elasticity.
   b. The mathematical formulation of the creep behavior of the concrete used in Dworshak Dam provides good checks with experimental results, both of creep under constant stress and of the stress relaxation behavior at constant strain.
2. Concerning the application of the method to analysis of large gravity dams, it may be concluded:
   a. The principal stresses in the lowest lifts of the dam are oriented essentially vertically and horizonally. The critical thermal stresses are represented by the horizontal normal stresses at midsection of the lifts.
   b. Because of the uniformity of stress in the central portions of the lifts, it is possible to use greatly elongated finite elements in this region, varying to nearly equilateral elements near the faces.
   c. A 3 hour interval (one-eighth day) is suitable for analysis of the creep relaxation of stresses within the elements; a 2 to $2\frac{1}{2}$ day interval between analyses provides good accuracy in the evaluation of

creep in the complete structure.

d. Foundation elasticity has a significant effect on the stresses developed in the lower lifts of the dam; it may be accounted for effectively by means of a set of foundation stiffness coefficients evaluated for an elastic half-plane.

3. Concerning the behavior of the dam, the following significant observations can be made:

   a. Maximum tensile stresses may be expected in the first lift, with lower stresses resulting in succeeding lifts.

   b. Similar maximum tensile stresses will also be found in the first lift placed after any significant delay in the concrete placement schedule.

   c. Rapid rates of cooling result in higher tensile stresses in these lower lifts than do slower rates of cooling. The temperature histories resulting from cooling programs 5 and 6 (Table 1) provide for the greatest factors of safety against cracking. For these cases the predicted average stresses are at all times less than 50 percent of the expected concrete tensile strengths.

   d. Longitudinal joints do not provide a practical means of reducing still further the already low average tensile stresses predicted for the given cooling cycles; block lengths less than 50 feet are required if the joints are to be effective.

   e. Sloping foundations will not increase the tensile stresses at the base of the dam, and hence will require no special precautions other than those normally taken in base pours.

f. During cooling operations, local stresses in the immediate vicinity of the cooling pipes will be high enough to exceed the uniaxial tensile strength of the concrete. This stress condition lasts only a few days and is followed by a long period of compressive stress during which any microfractures in the still-young concrete should heal.

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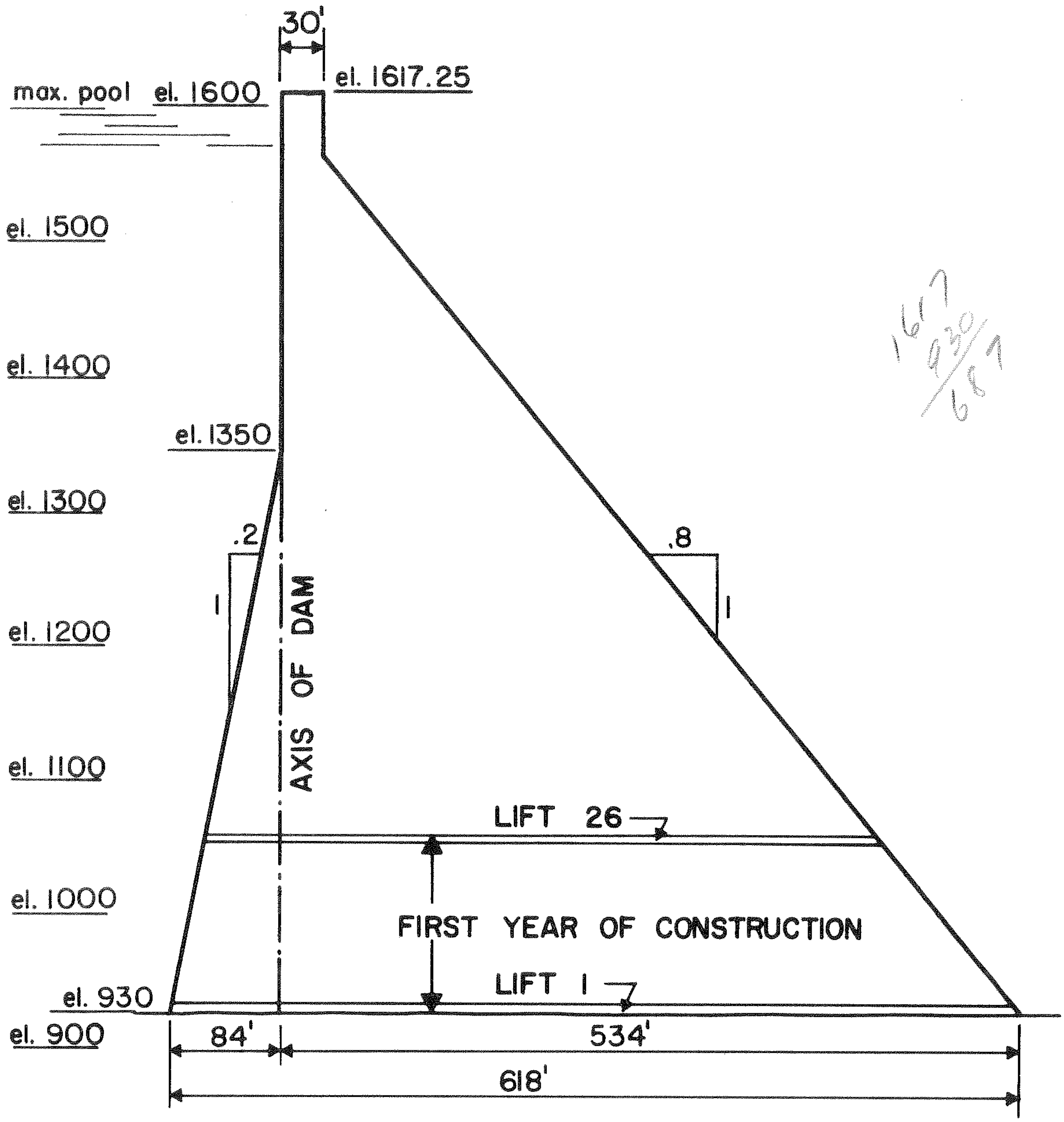

Fig. 1 – TYPICAL CROSS SECTION OF DWORSHAK DAM

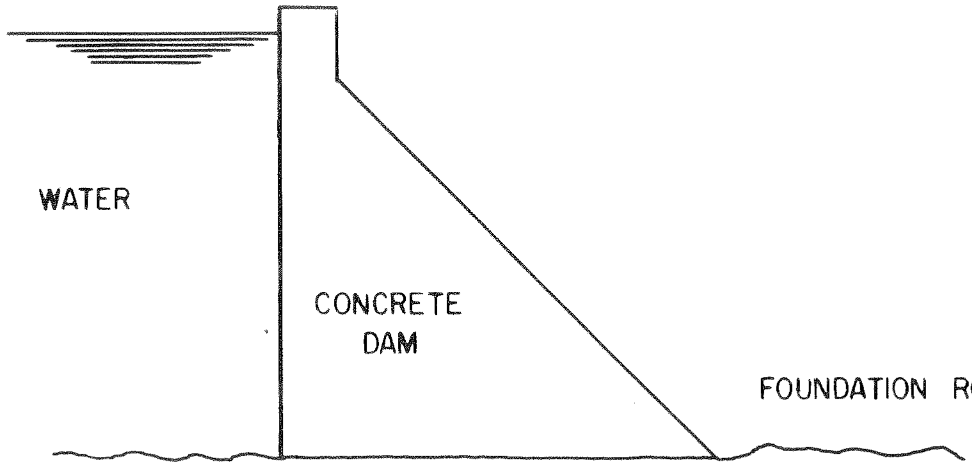

Fig. 2a- DAM ON ELASTIC FOUNDATION

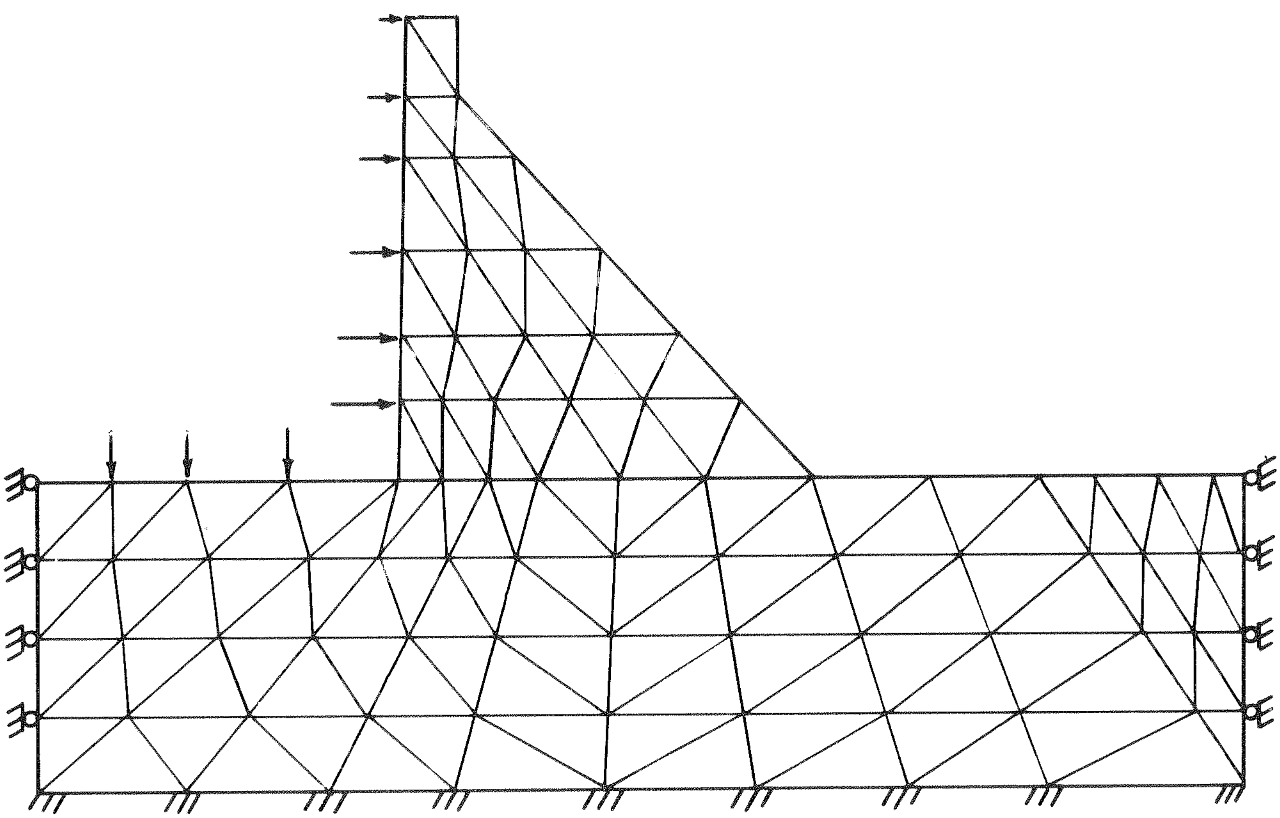

Fig. 2b- FINITE ELEMENT REPRESENTATION OF DAM AND FOUNDATION

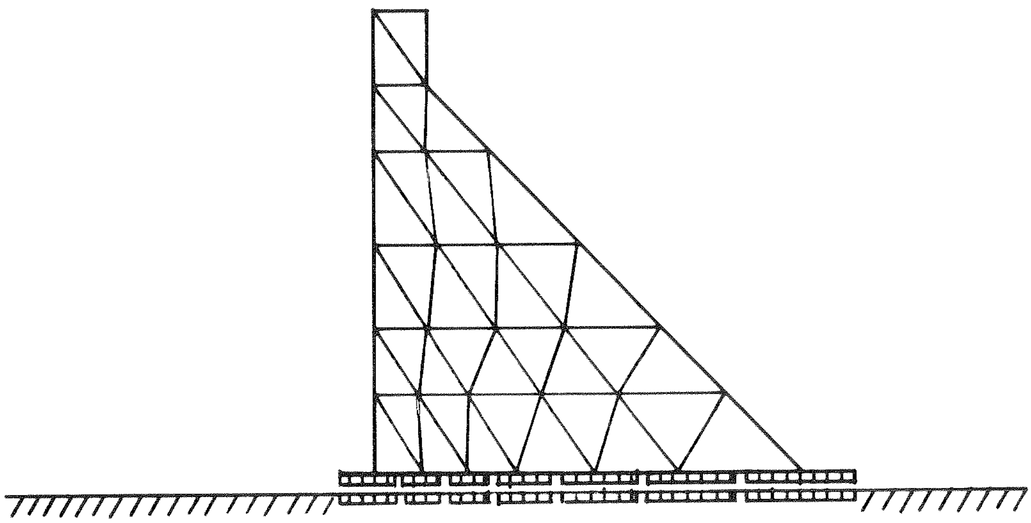

Fig. 3- TRANSMISSION OF FORCES TO ELASTIC HALF PLANE

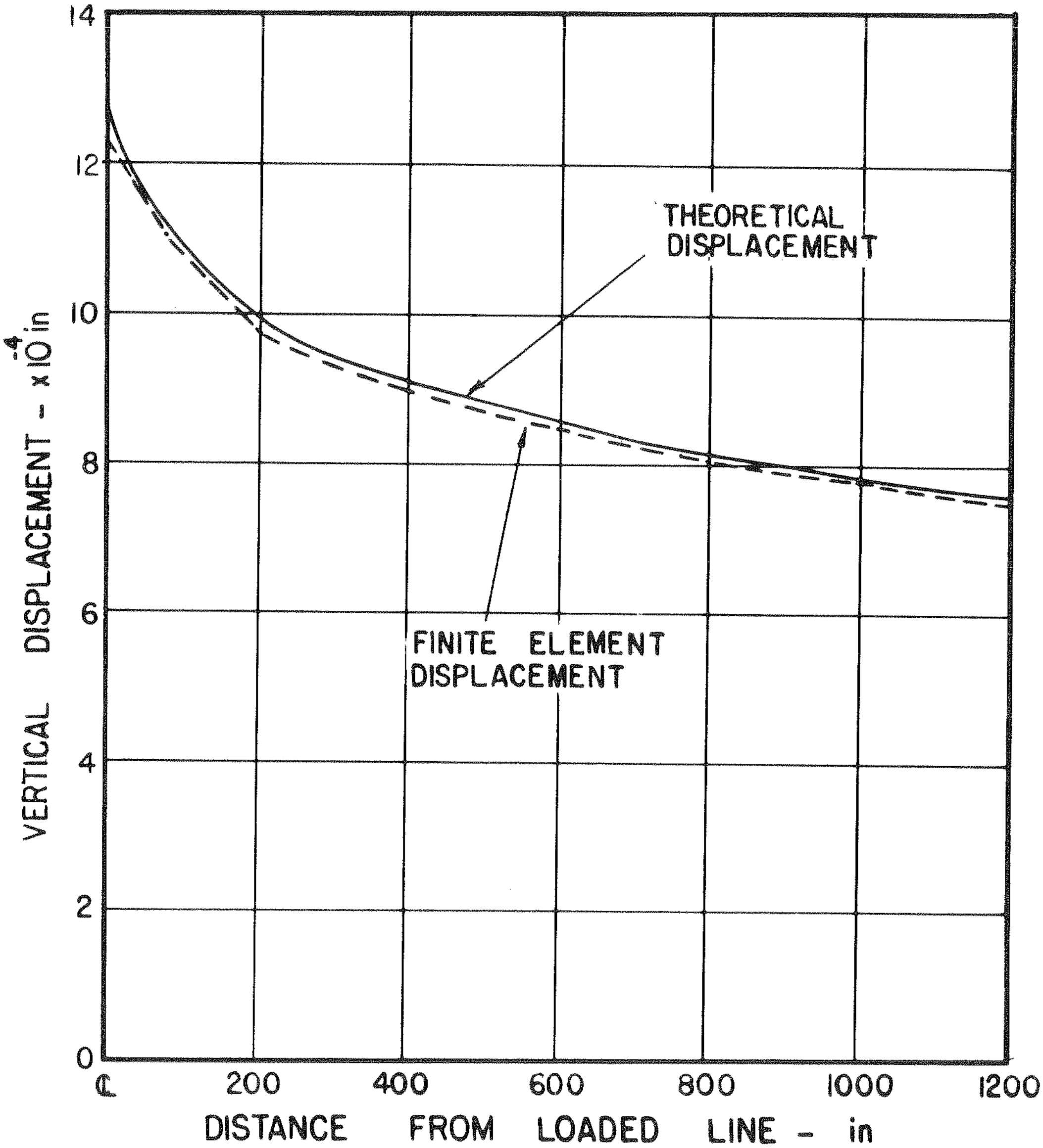

Fig. 4 – COMPARISON OF VERTICAL DISPLACEMENTS IN ELASTIC HALF PLANE

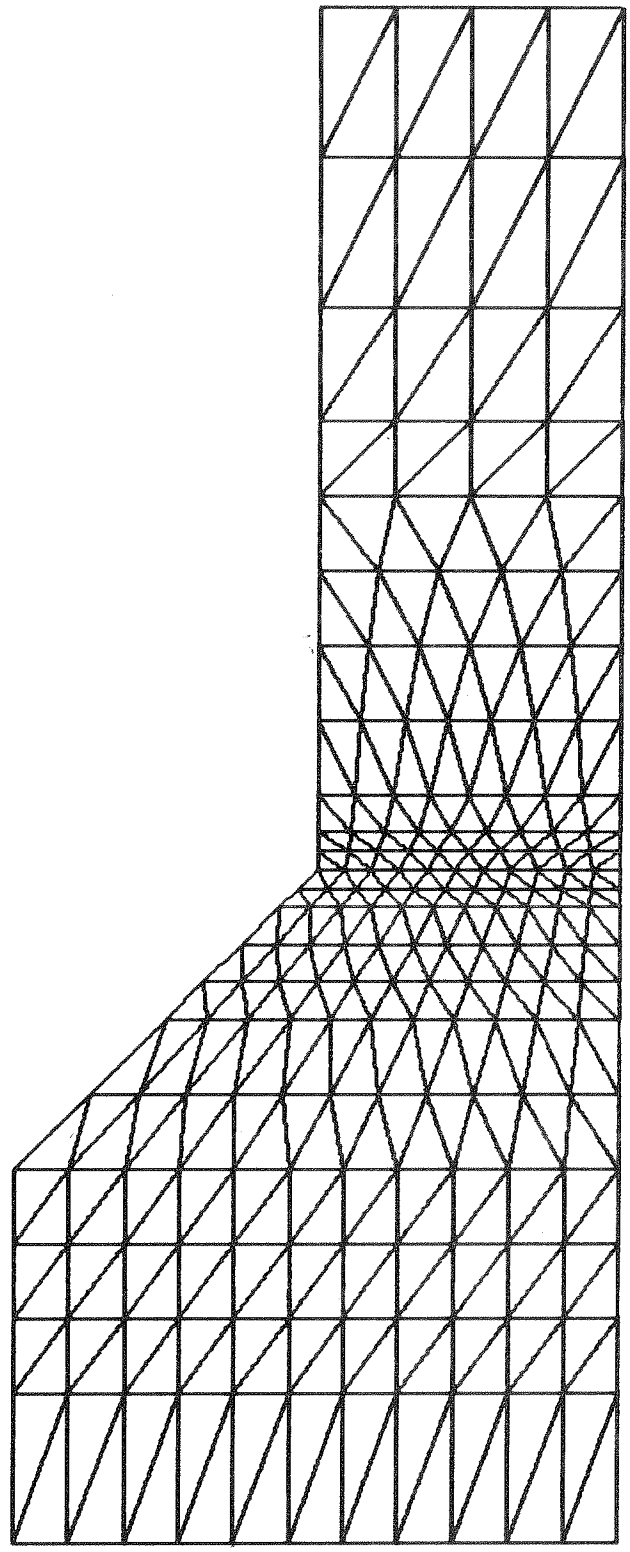

Fig. 5 - TYPICAL MESH GENERATED BY AUTOMATIC PLOTTER

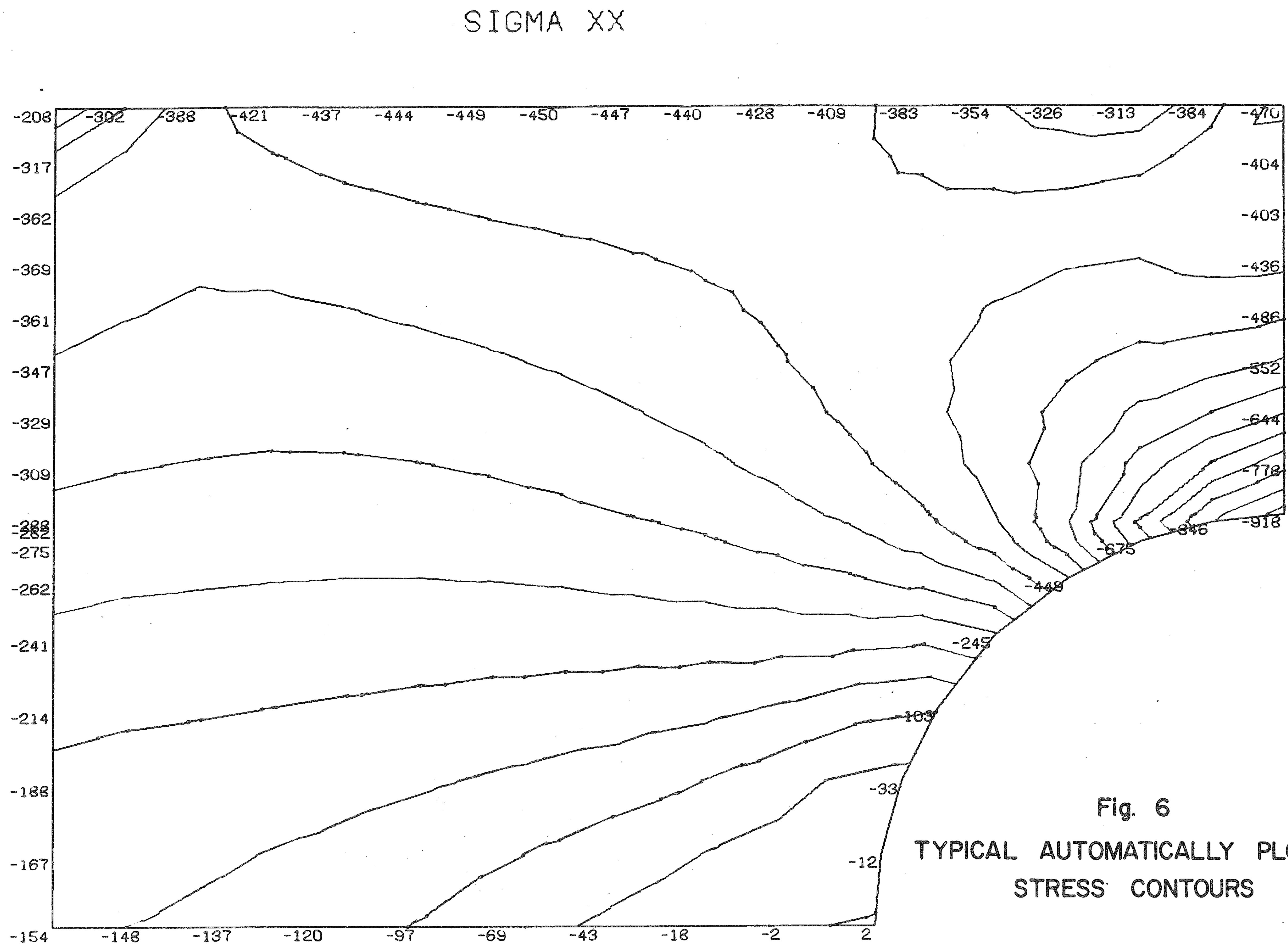

Fig. 6

TYPICAL AUTOMATICALLY PLOTTED STRESS CONTOURS

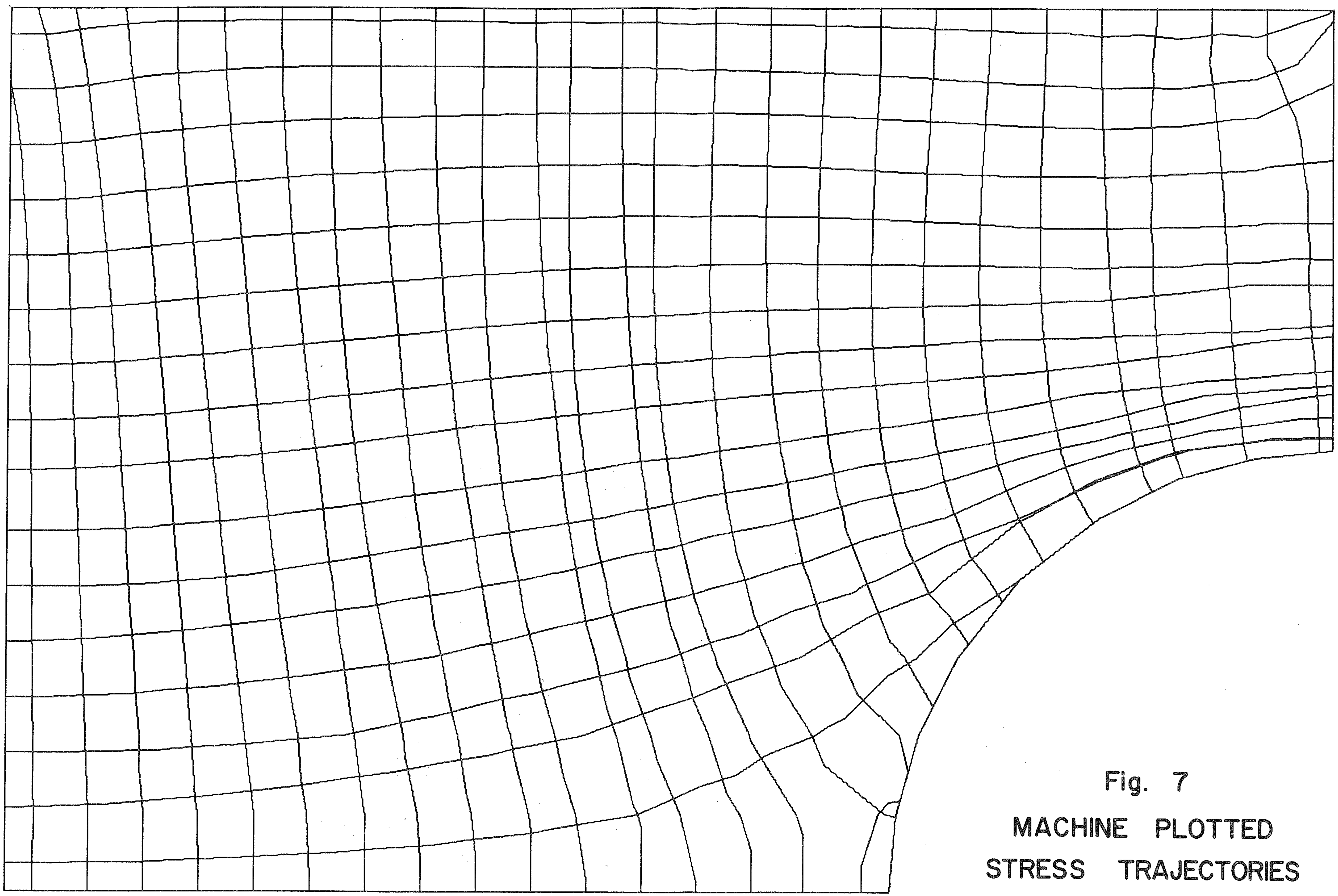

Fig. 7
MACHINE PLOTTED
STRESS TRAJECTORIES

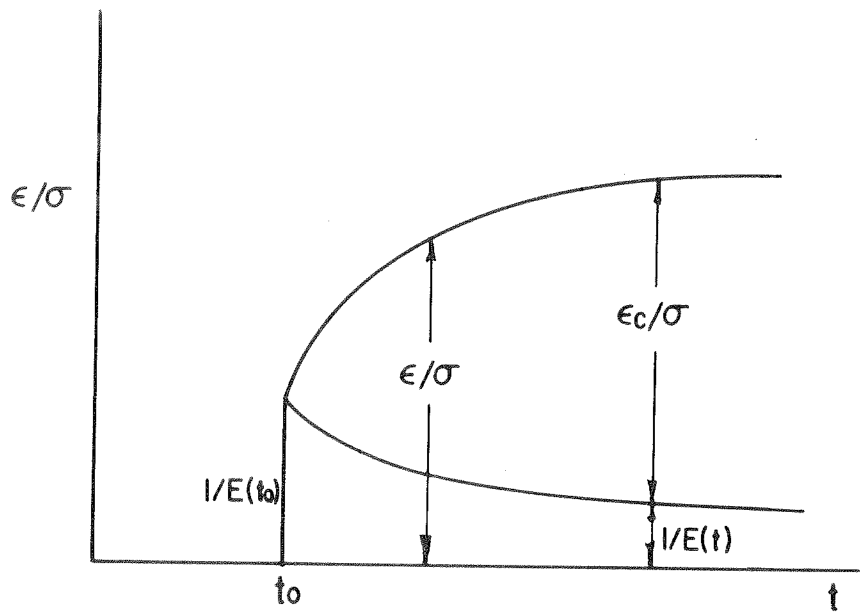

Fig. 8 - TYPICAL CREEP TEST RESULTS

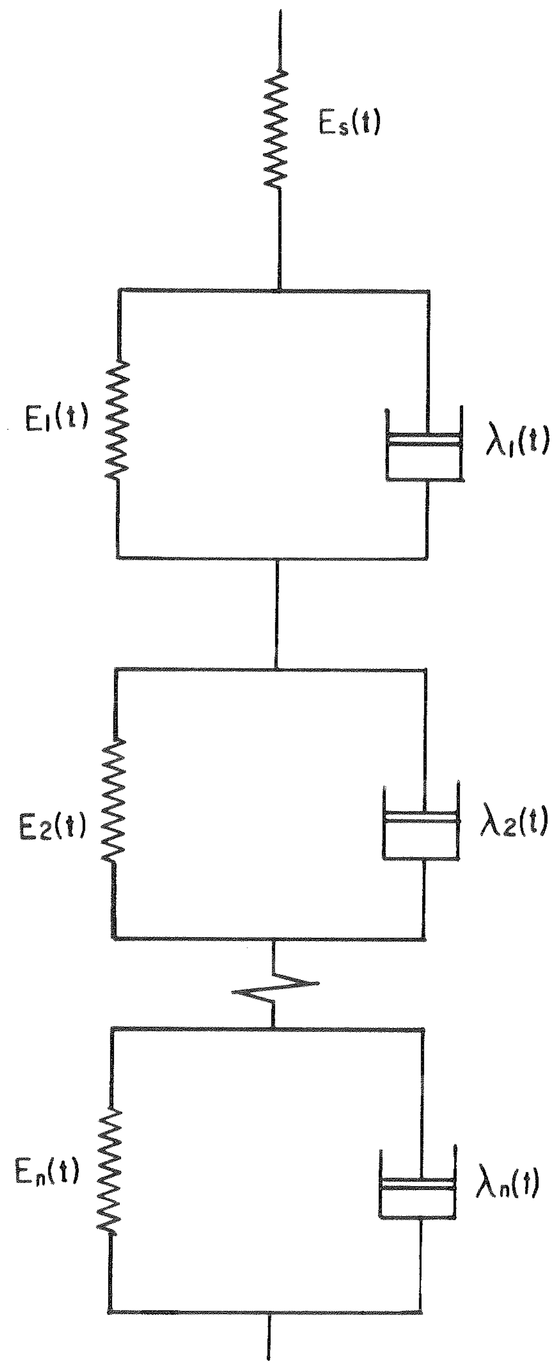

Fig. 9 - RHEOLOGICAL MODEL OF McHENRY'S CREEP EQUATION

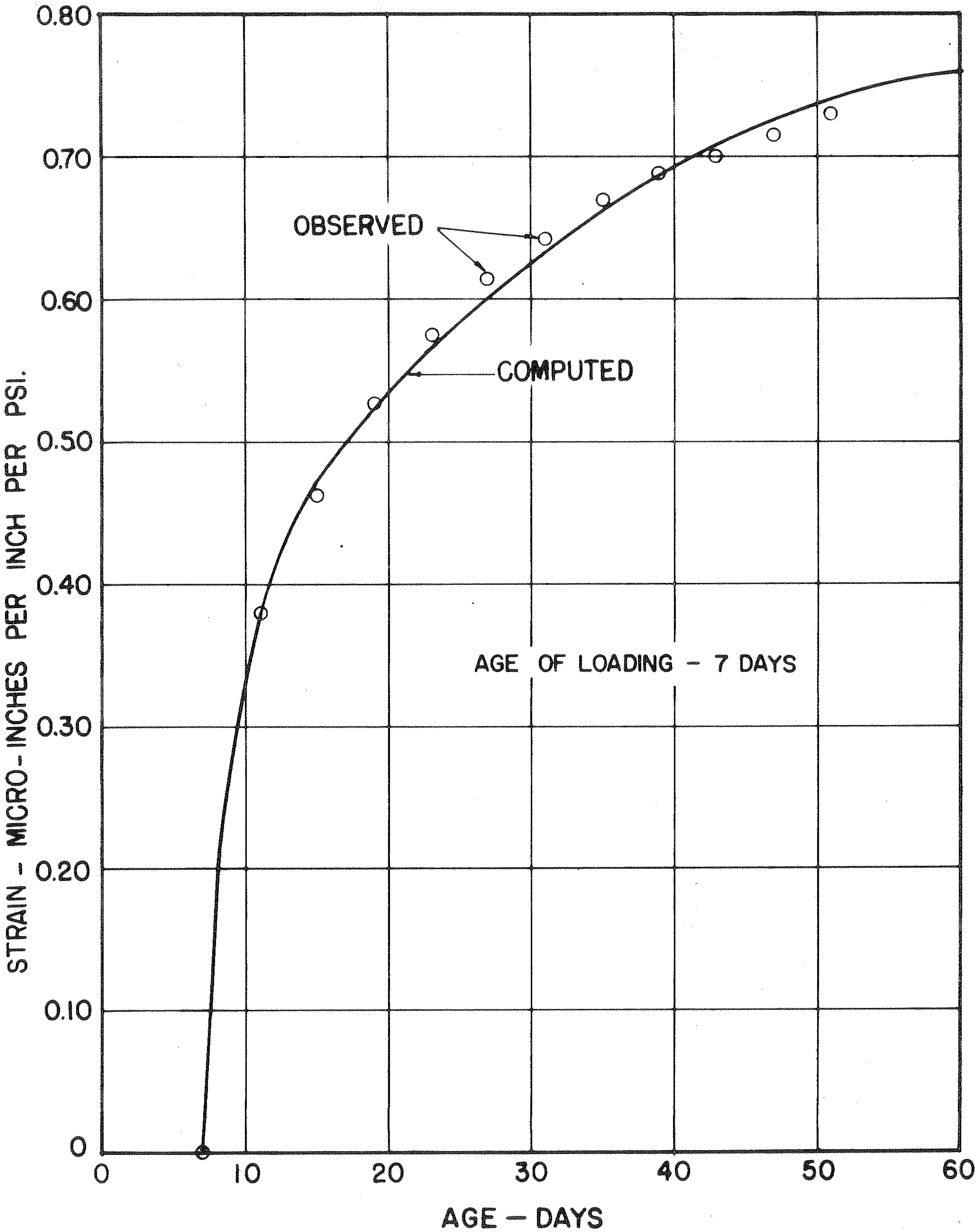

Fig. 10 — CREEP OF DWORSHAK DAM CONCRETE

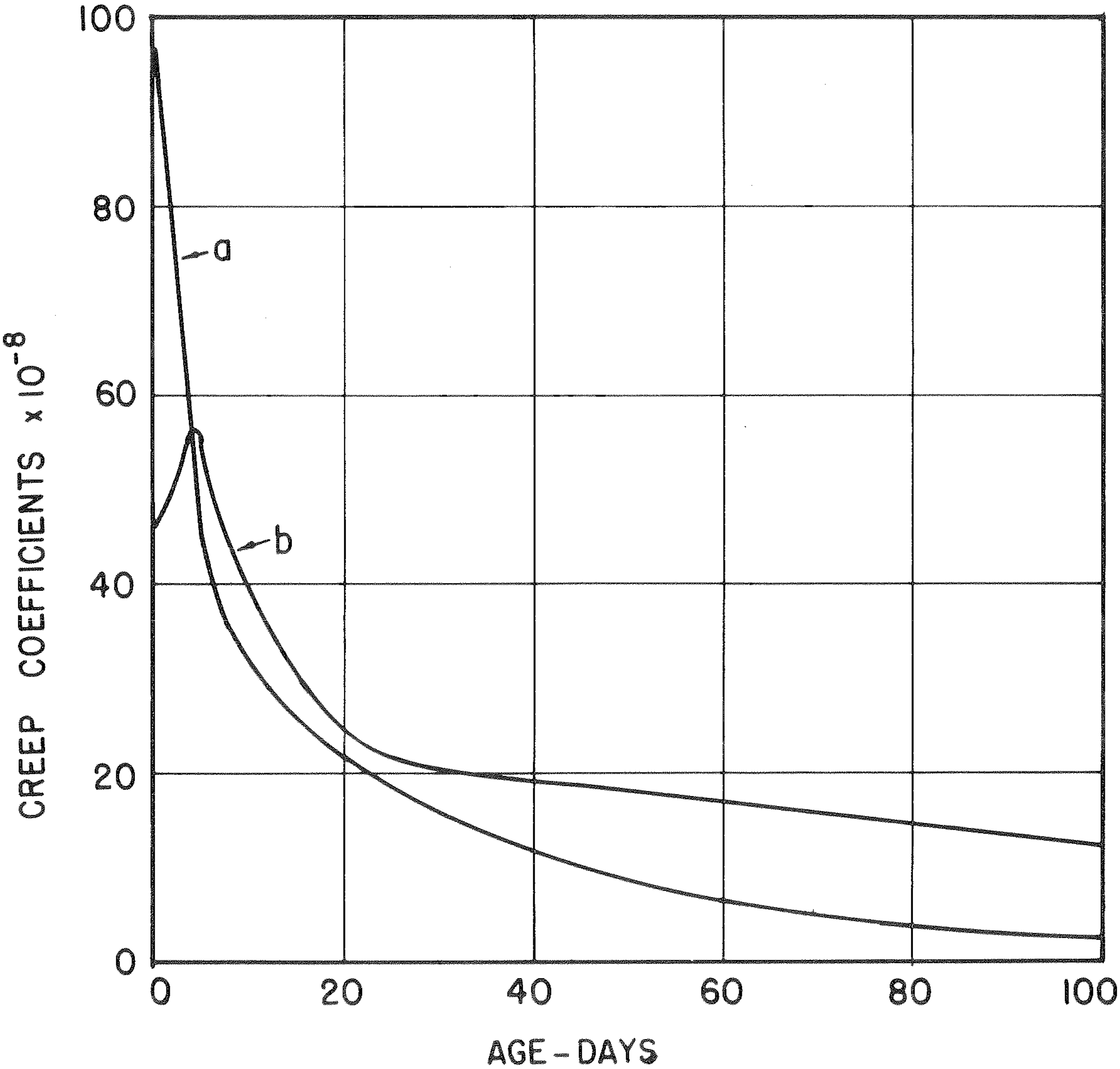

Fig. 11 - VARIATION OF CREEP COEFFICIENTS WITH AGE

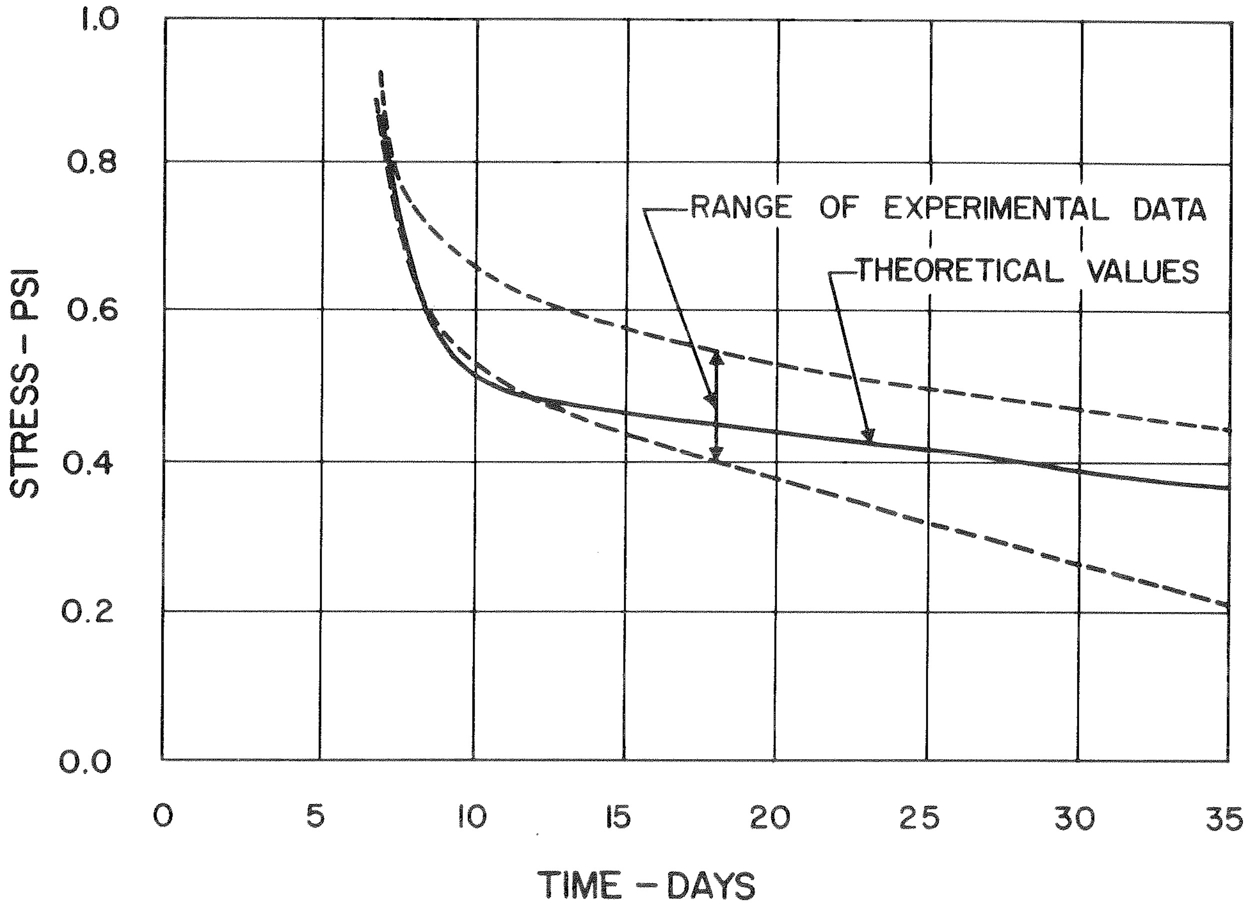

Fig. 12 - STRESS RELAXATION OF DWORSHAK DAM CONCRETE

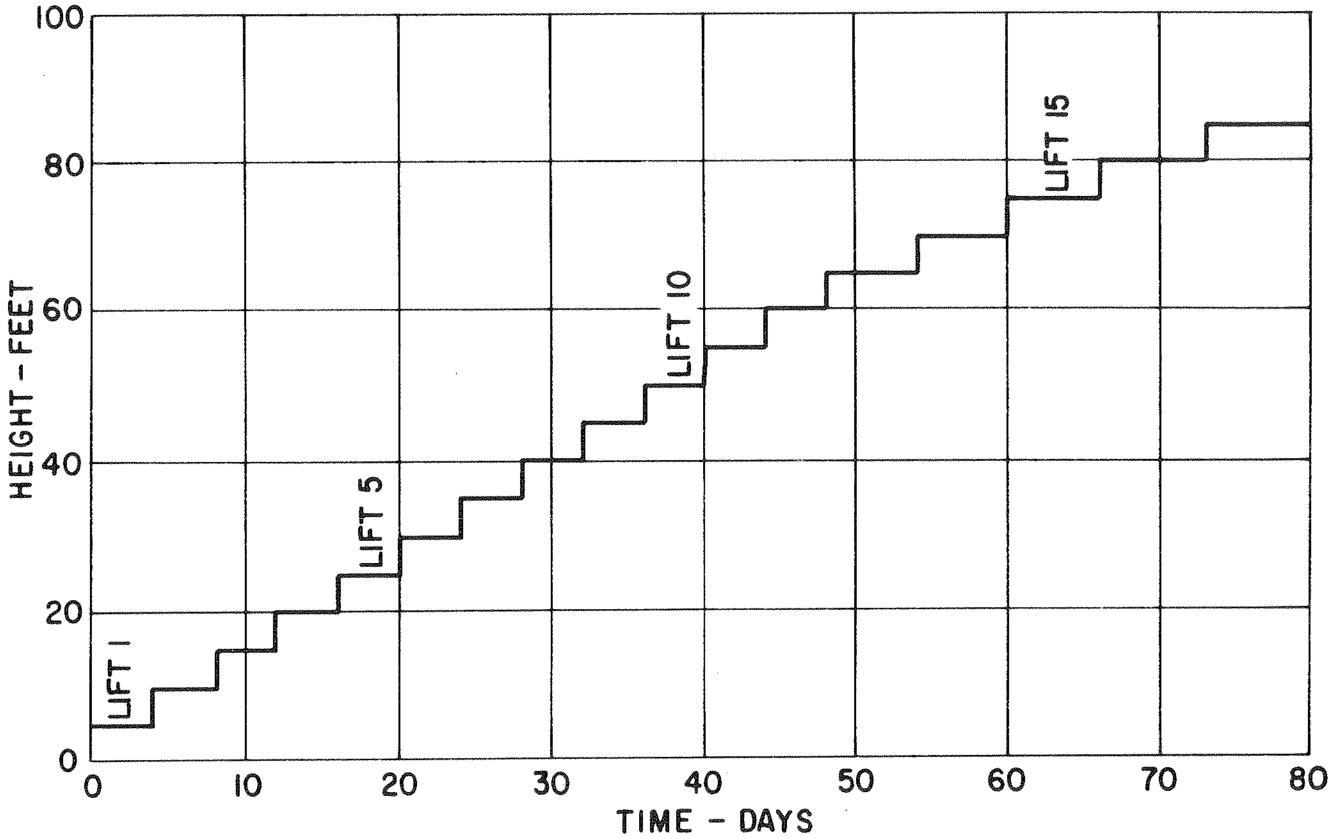

Fig. 13 - ASSUMED CONSTRUCTION SCHEDULE

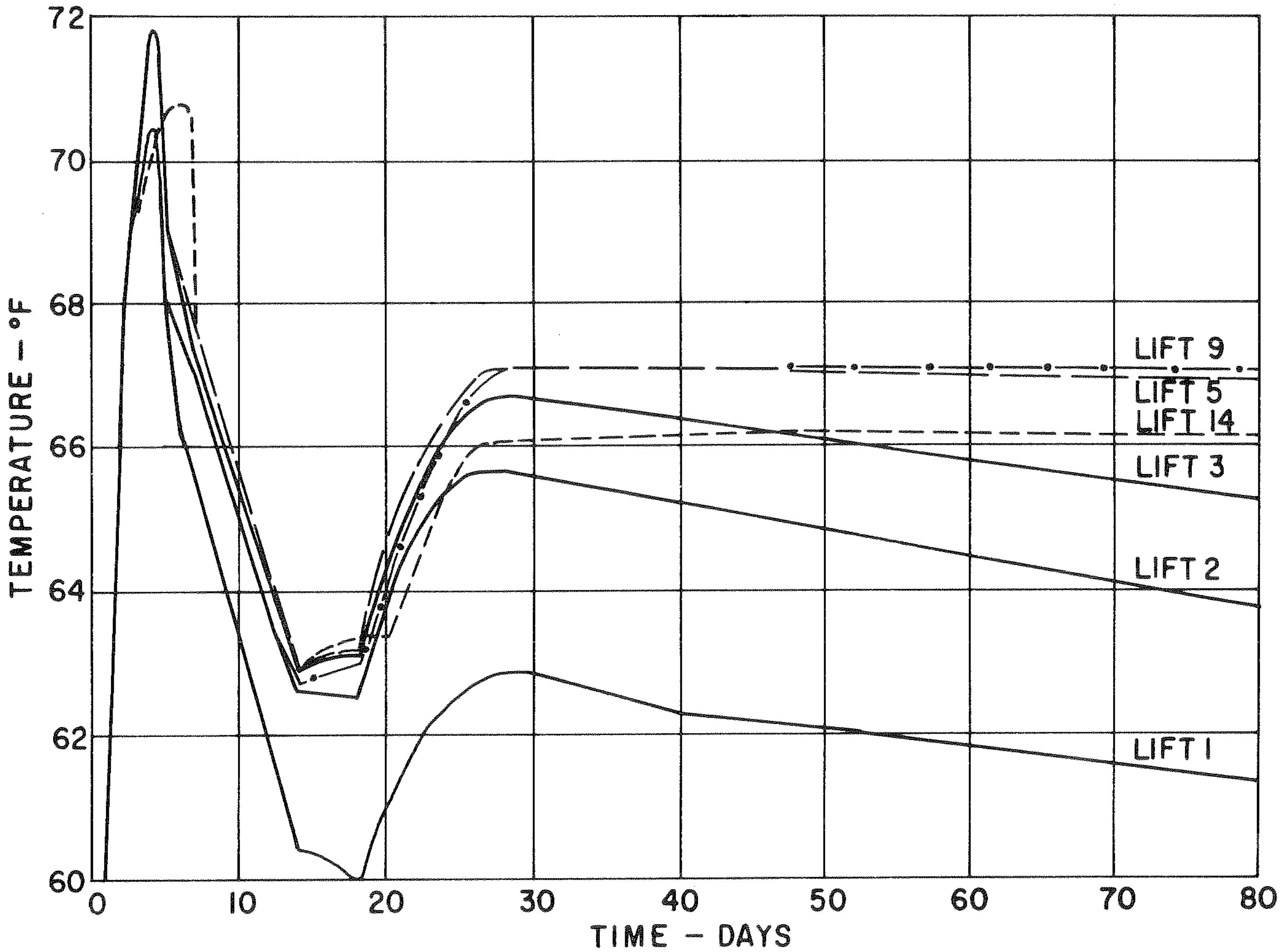

Fig. 14 – TEMPERATURE HISTORIES IN TYPICAL LIFTS

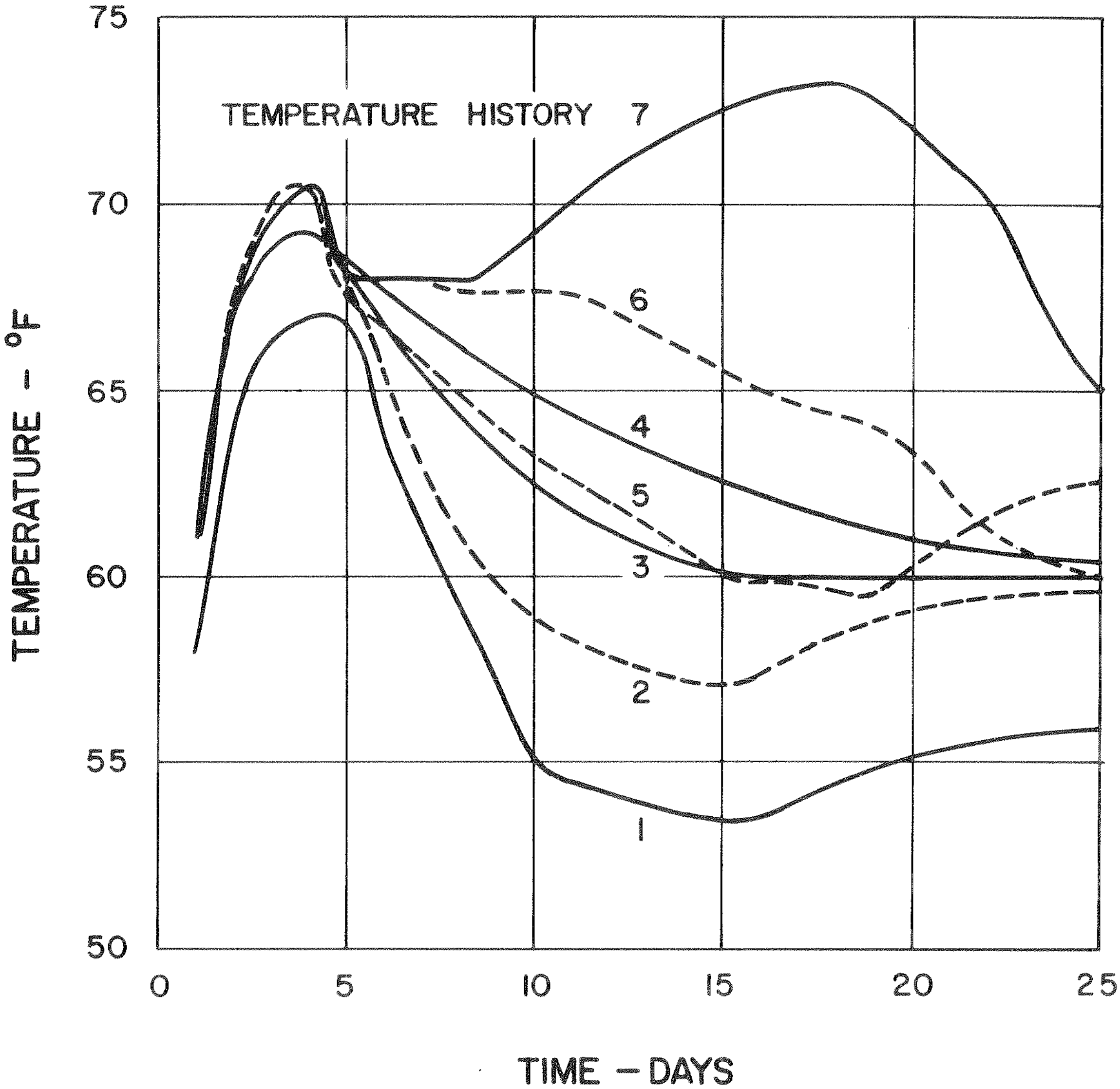

Fig. 15 - TEMPERATURE HISTORIES IN LOWEST LIFT

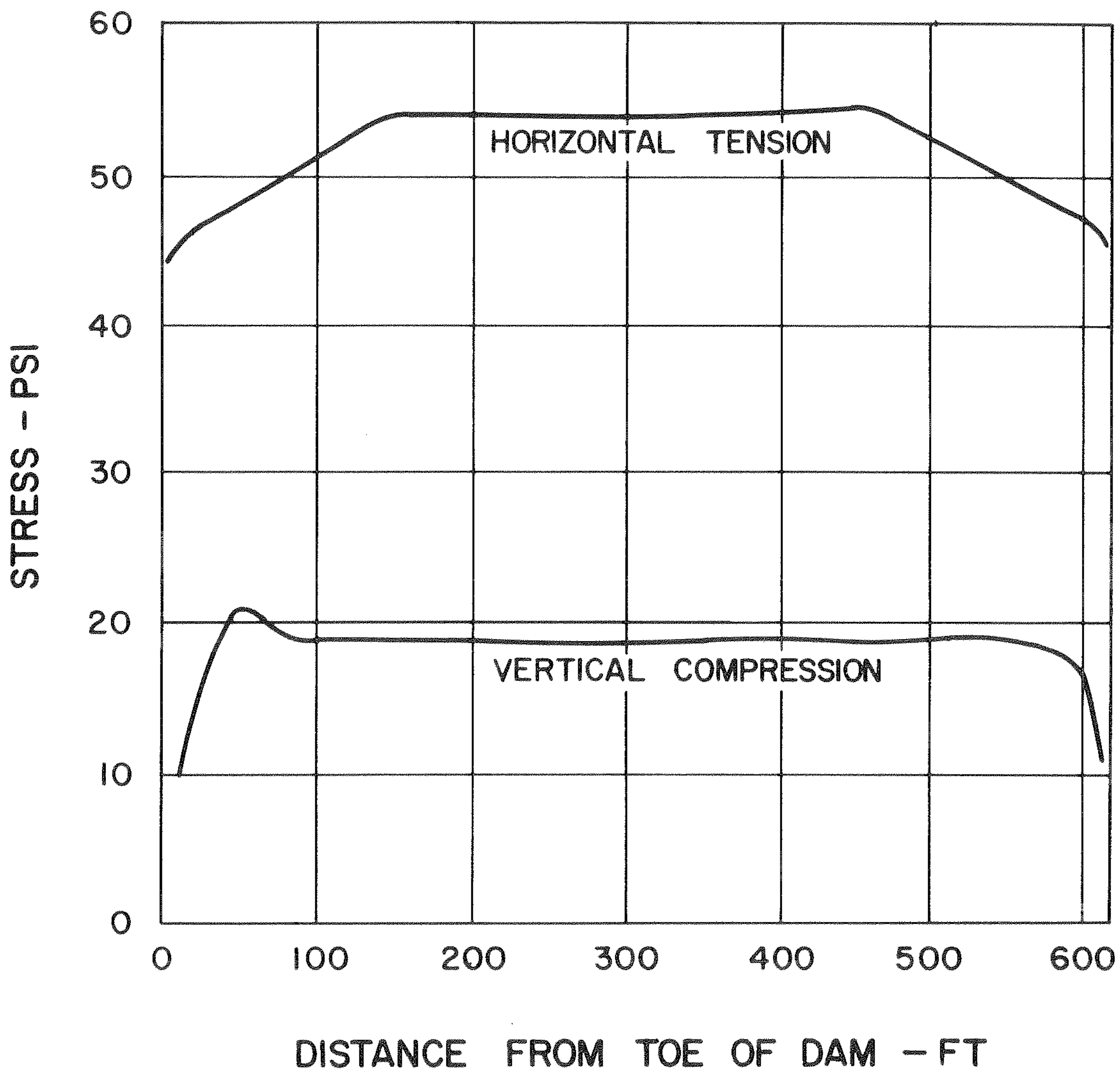

Fig. 16 - DISTRIBUTION OF VERTICAL AND HORIZONTAL STRESSES IN LIFT 1 AT 14 DAYS

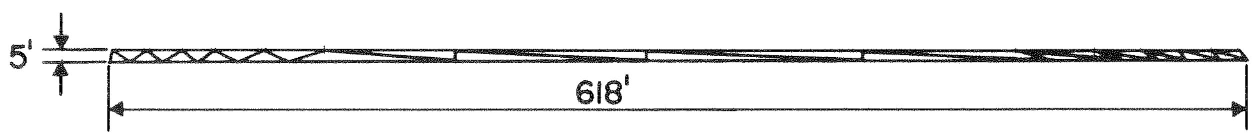

Fig. 17 - MESH LAYOUT FOR TYPICAL LAYER

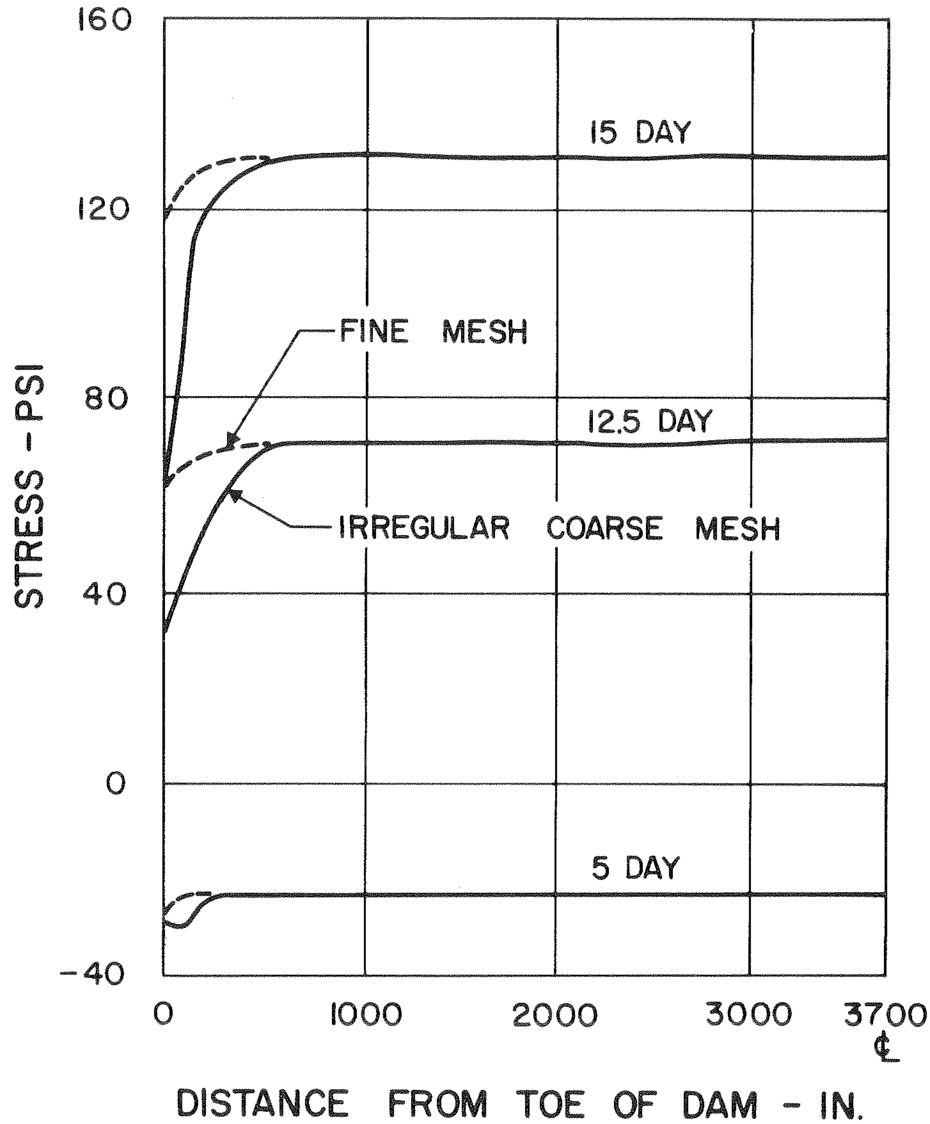

Fig. 18 - EFFECT OF IRREGULAR COARSE MESH

CONTOURS OF SIGMA MIN FOR DIFFERENT RELAXATION INTERVALS

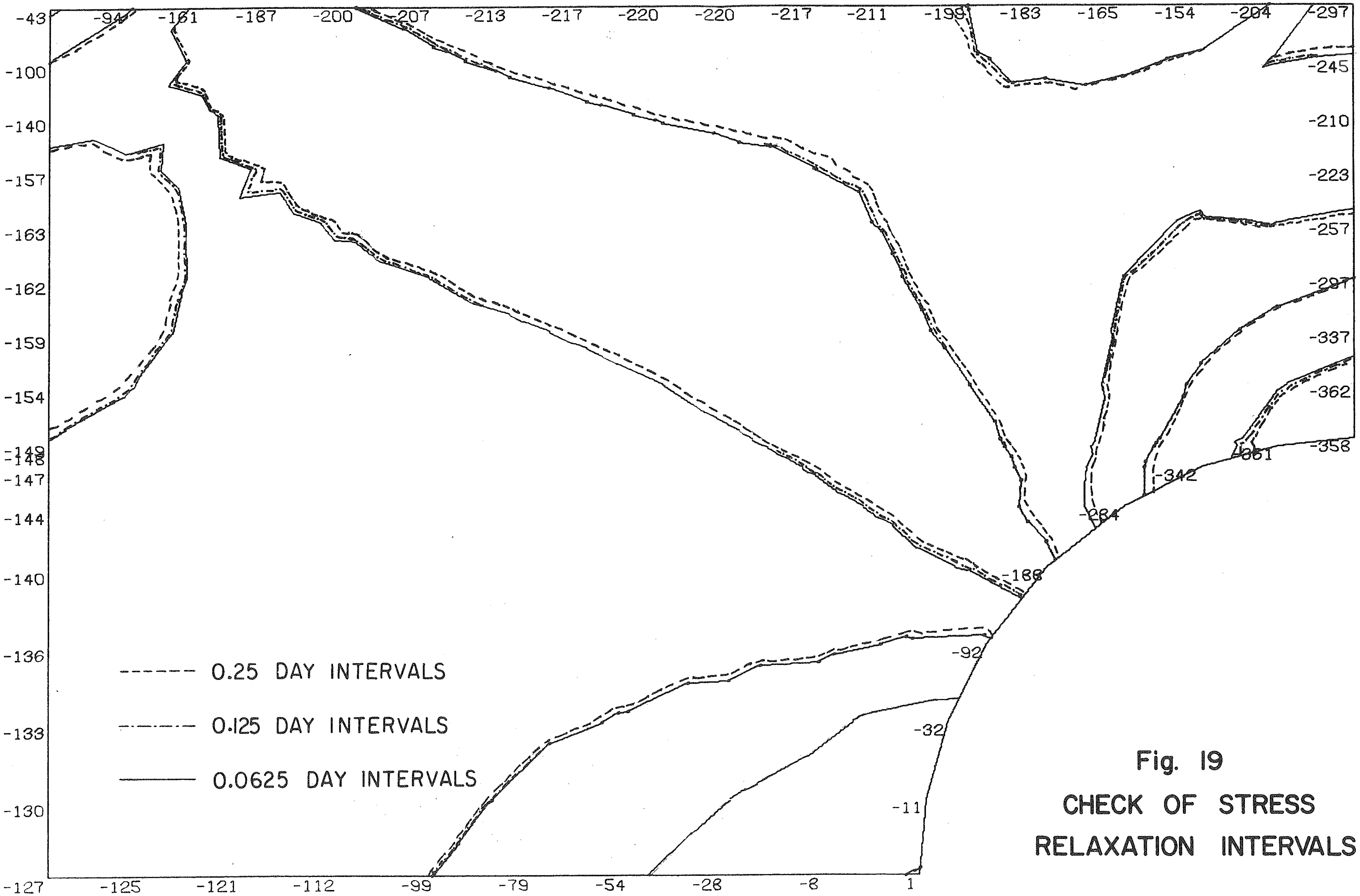

Fig. 19
CHECK OF STRESS
RELAXATION INTERVALS

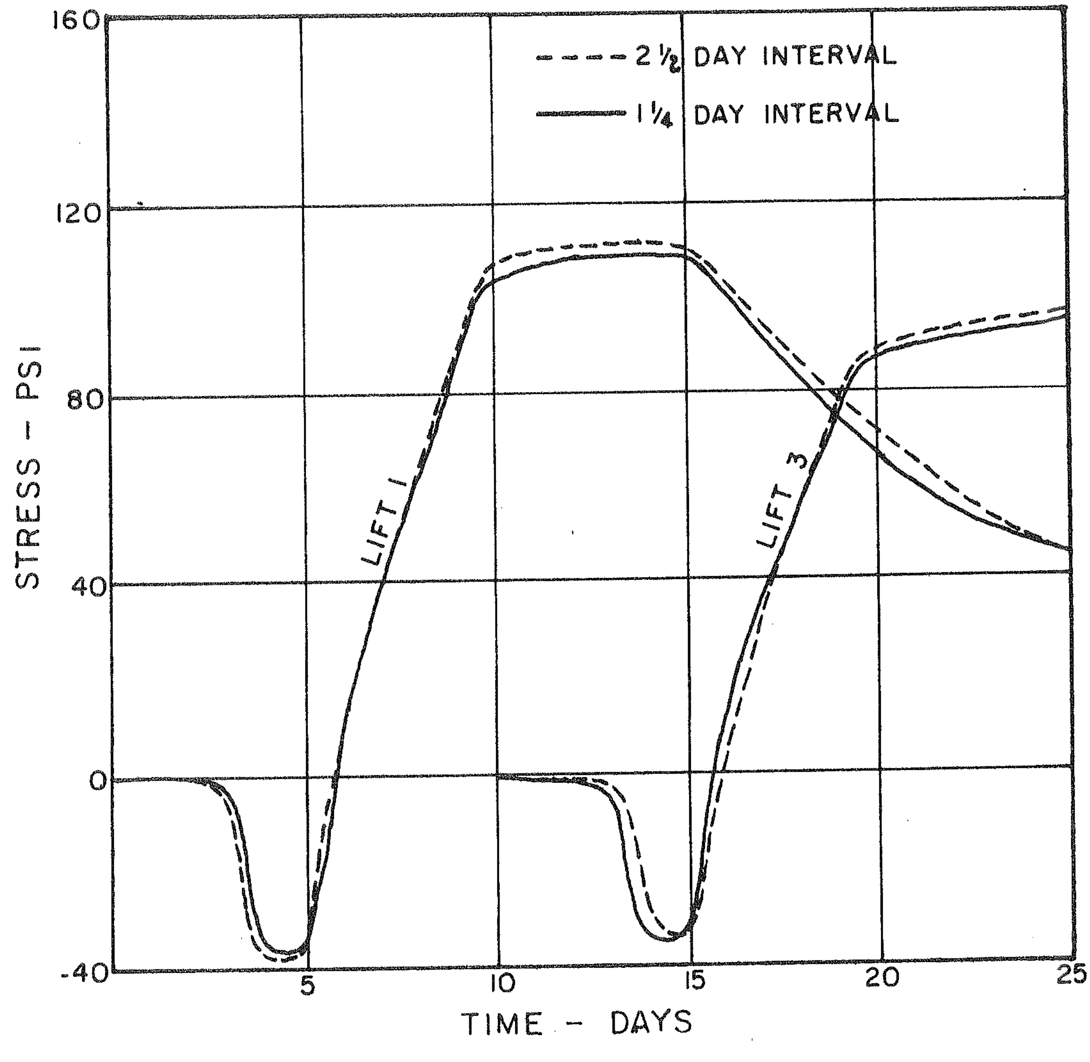

Fig. 20 CHECK OF ANALYSIS INTERVALS

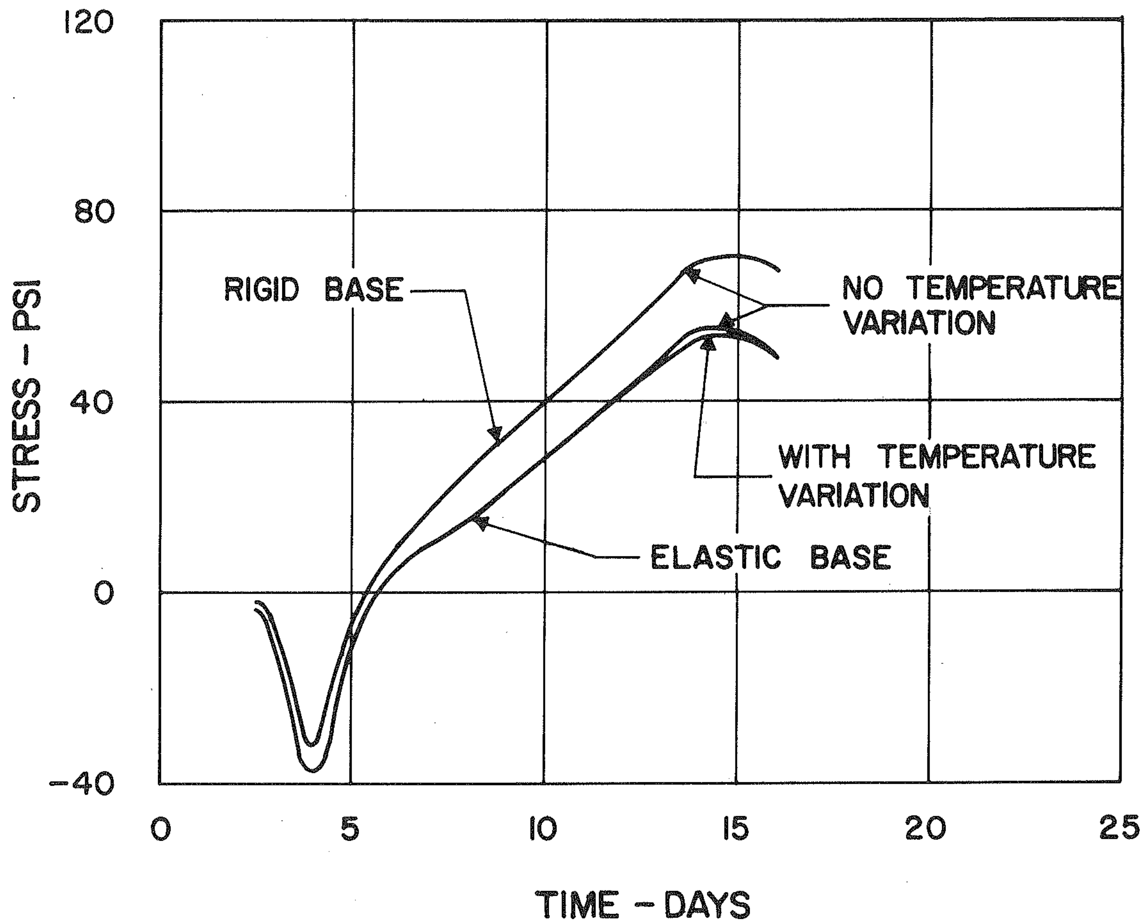

Fig. 21 - EFFECT OF FOUNDATION FLEXIBILITY ON HORIZONTAL STRESS

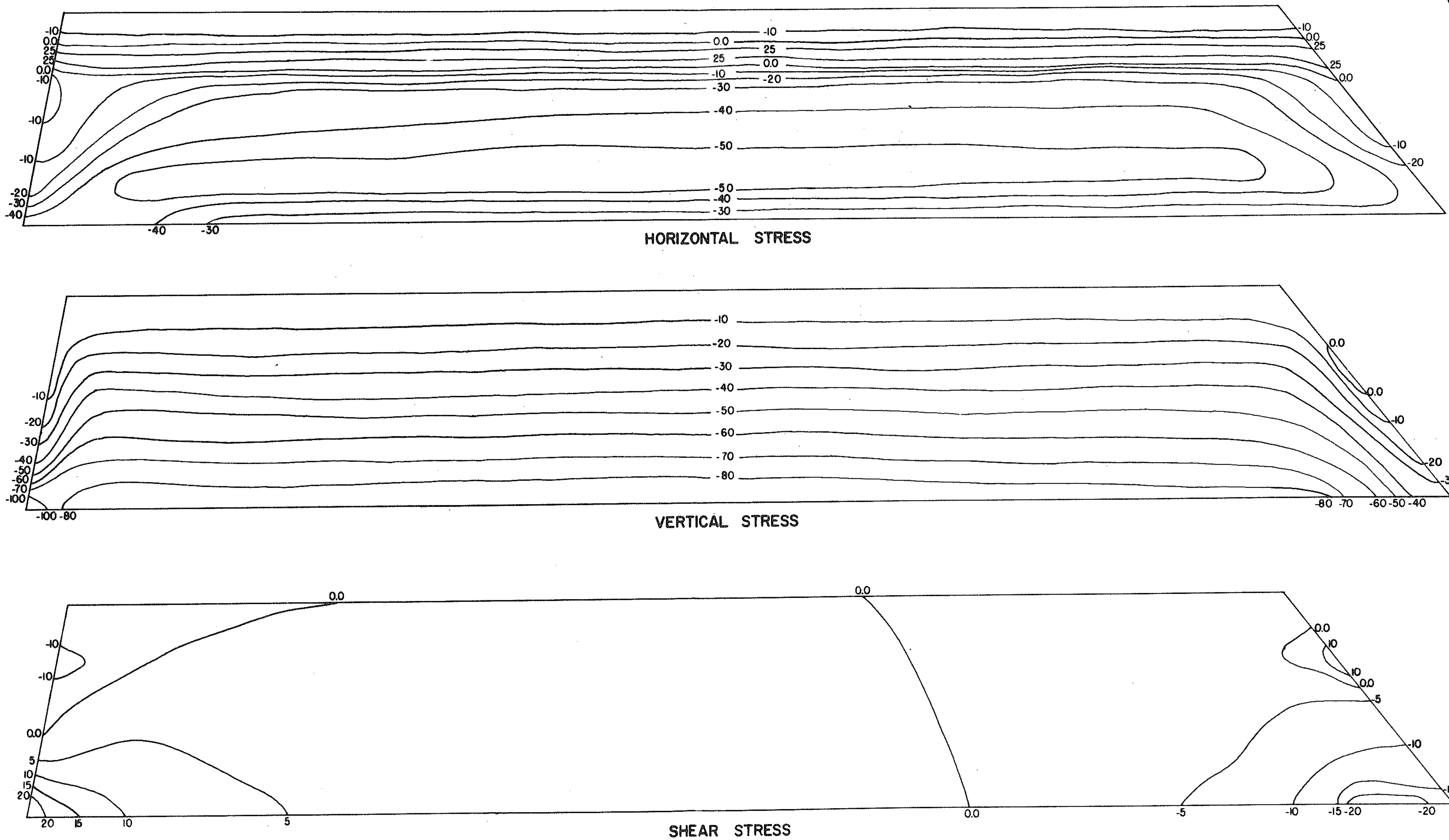

Fig. 22

DISTRIBUTION OF STRESS THROUGH 17-LIFT SECTION

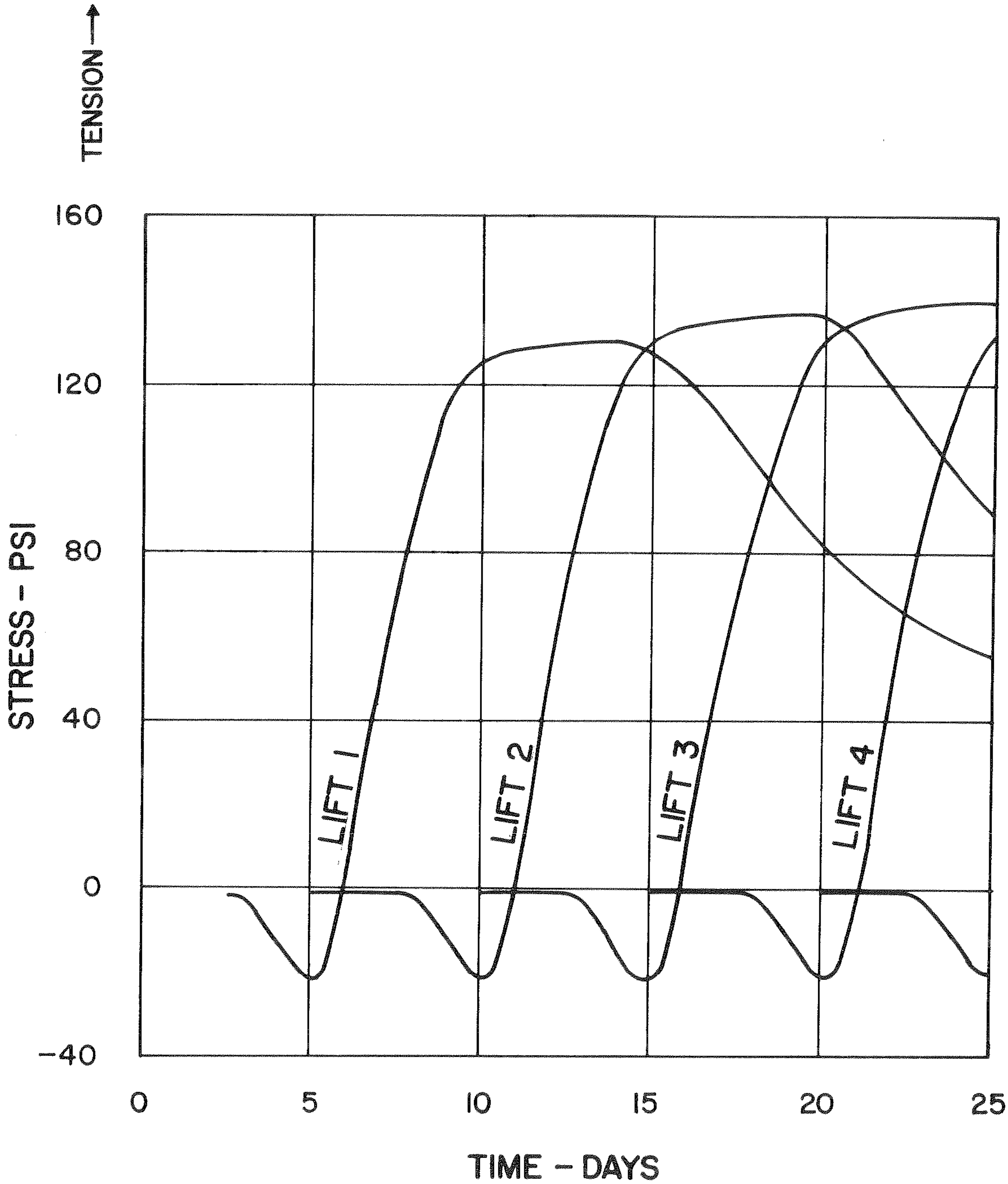

Fig. 23 – MIDPOINT HORIZONTAL STRESS DUE TO TEMPERATURE HISTORY I

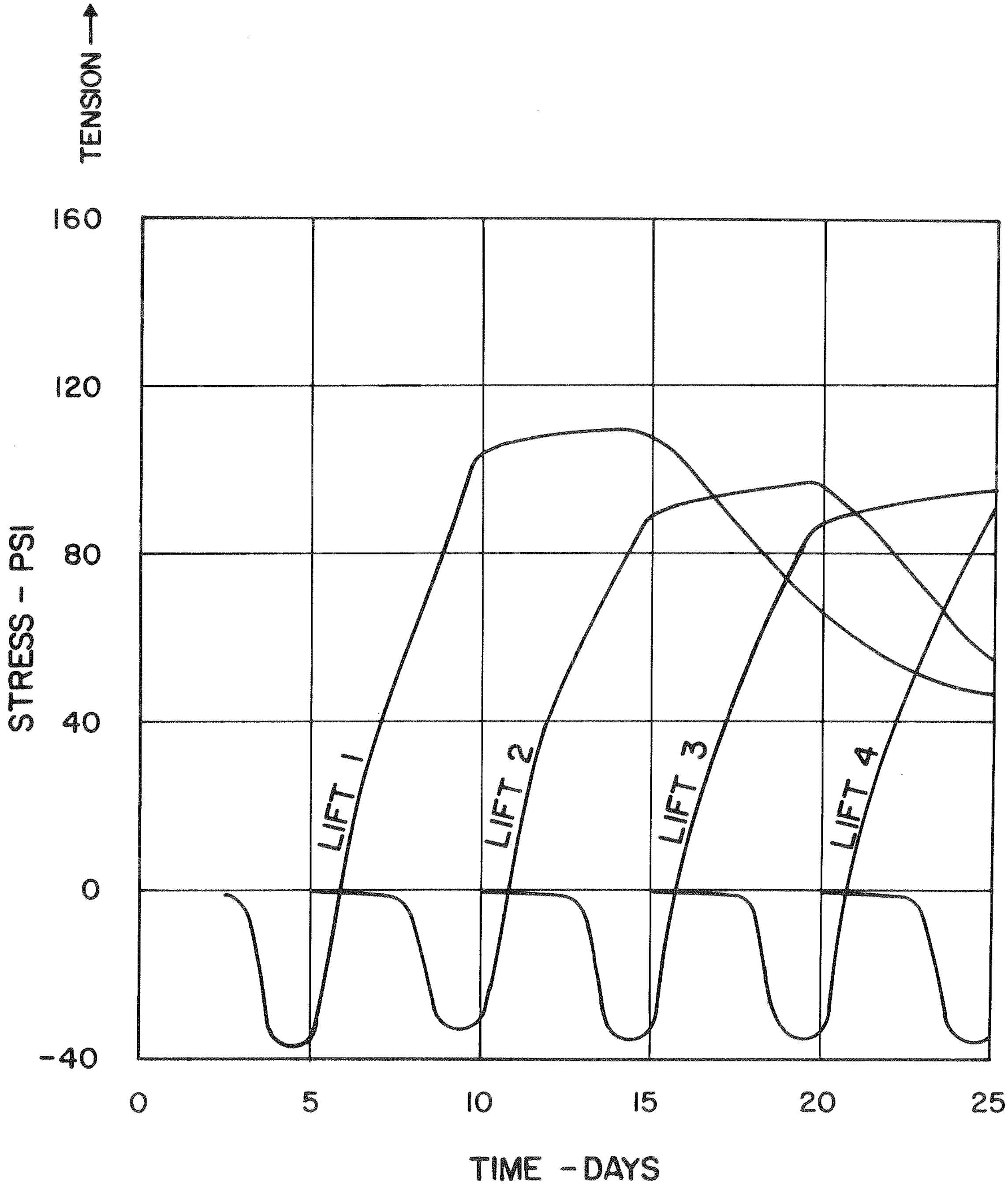

Fig. 24 – MIDPOINT HORIZONTAL STRESS DUE TO TEMPERATURE HISTORY 2

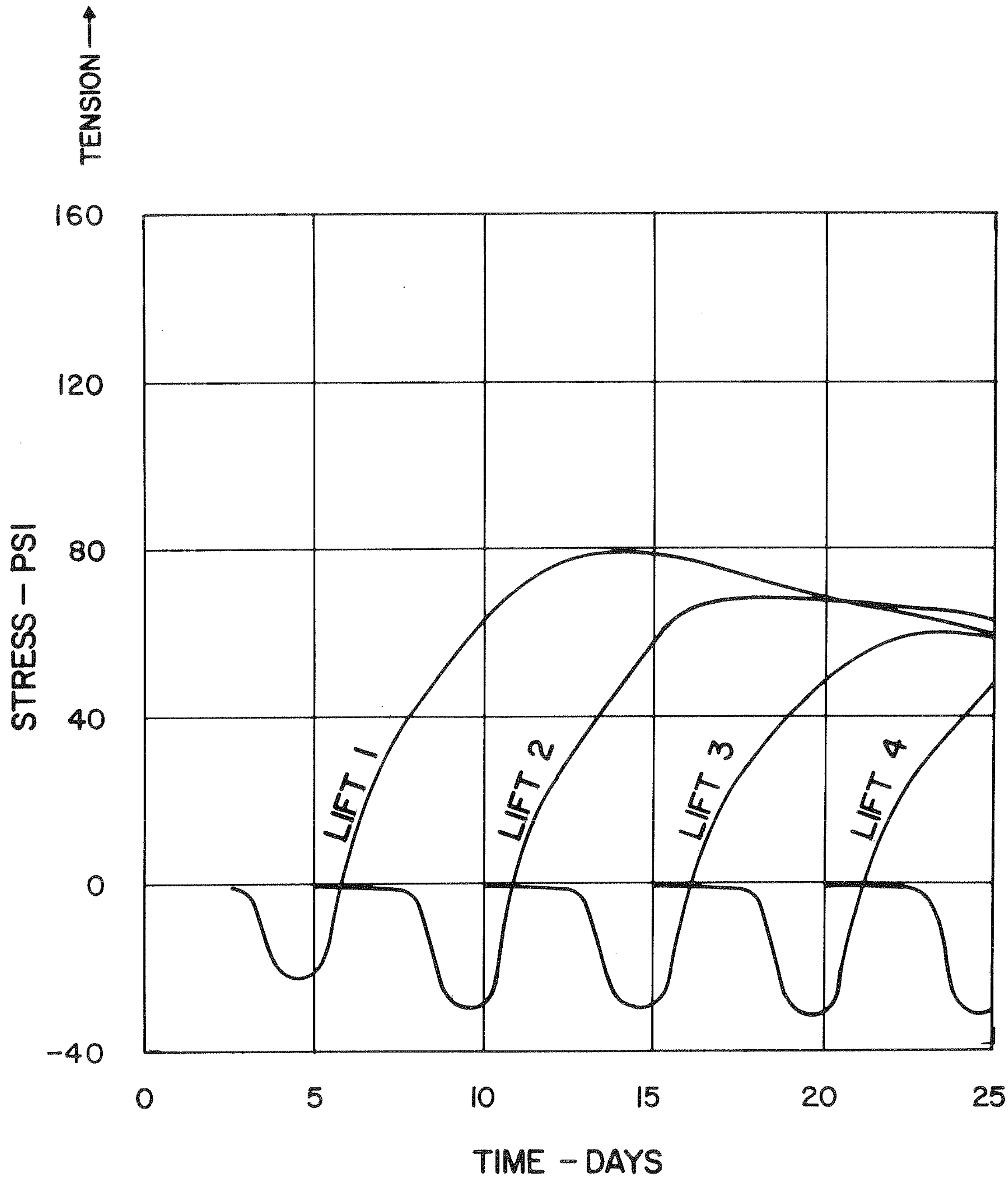

Fig. 25 — MIDPOINT HORIZONTAL STRESS DUE TO TEMPERATURE HISTORY 3

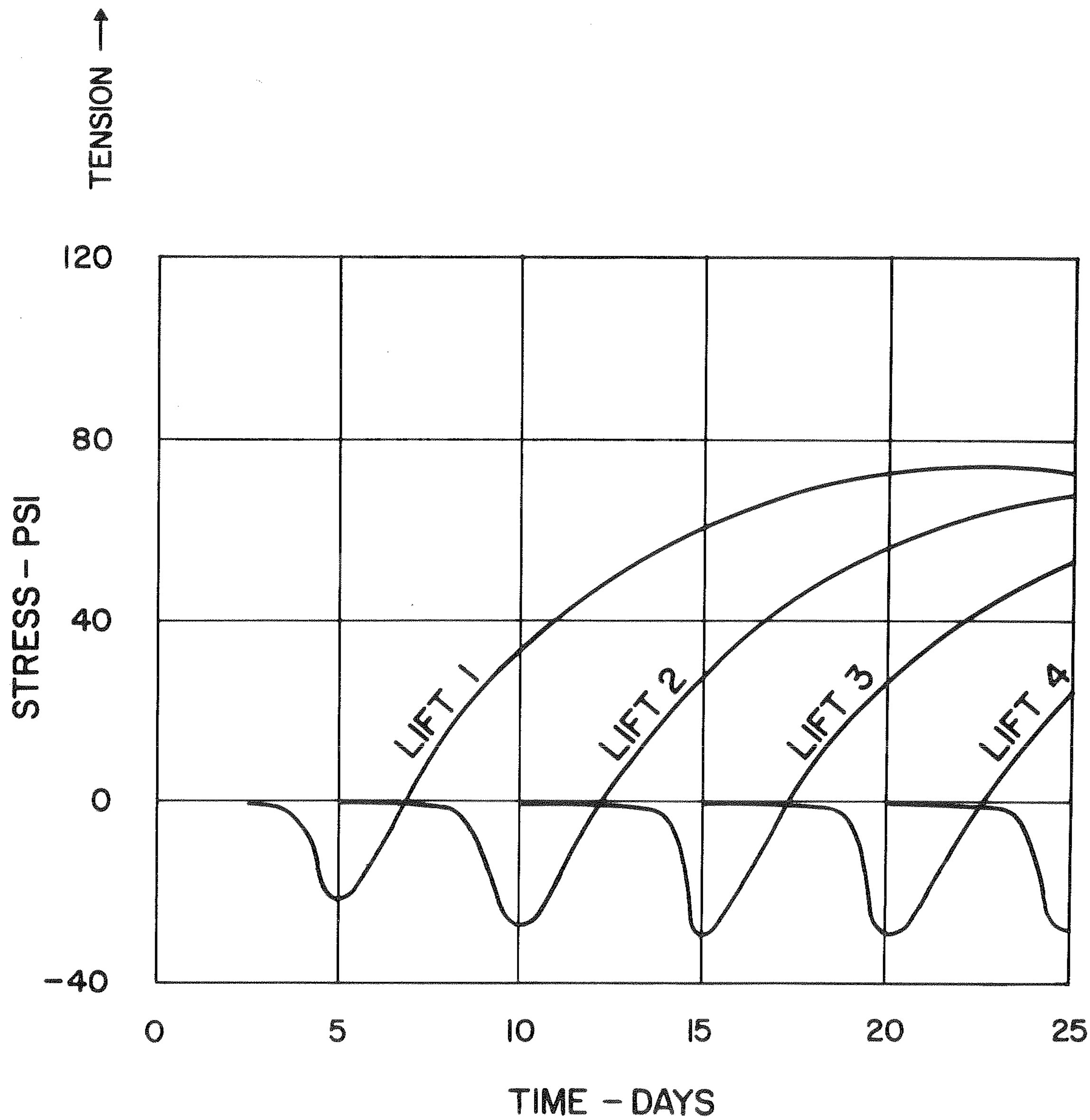

Fig. 26 — MIDPOINT HORIZONTAL STRESS DUE TO TEMPERATURE HISTORY 4

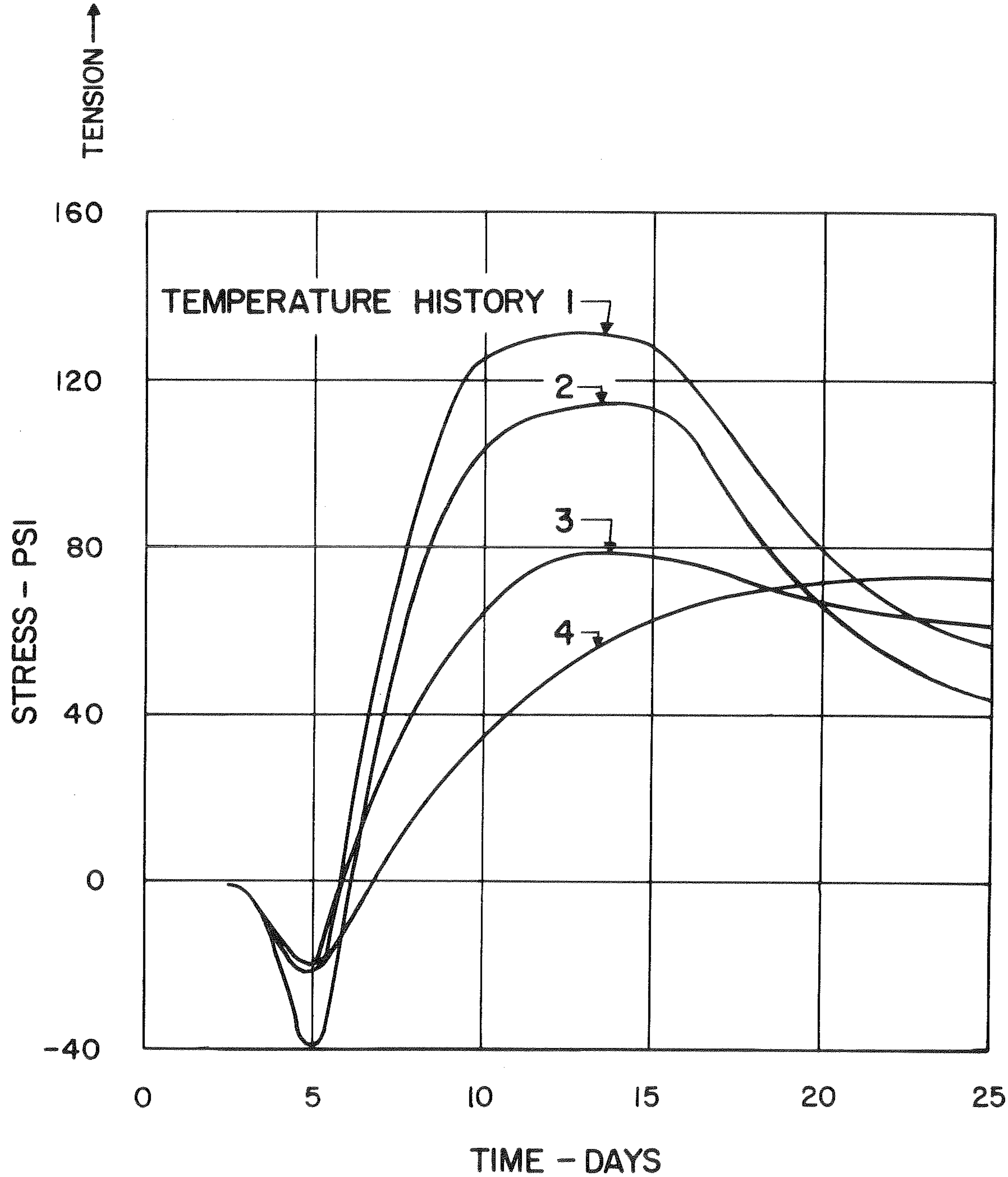

Fig. 27 - COMPARISON OF MIDPOINT STRESSES DUE TO TEMPERATURE HISTORIES 1,2,3,4

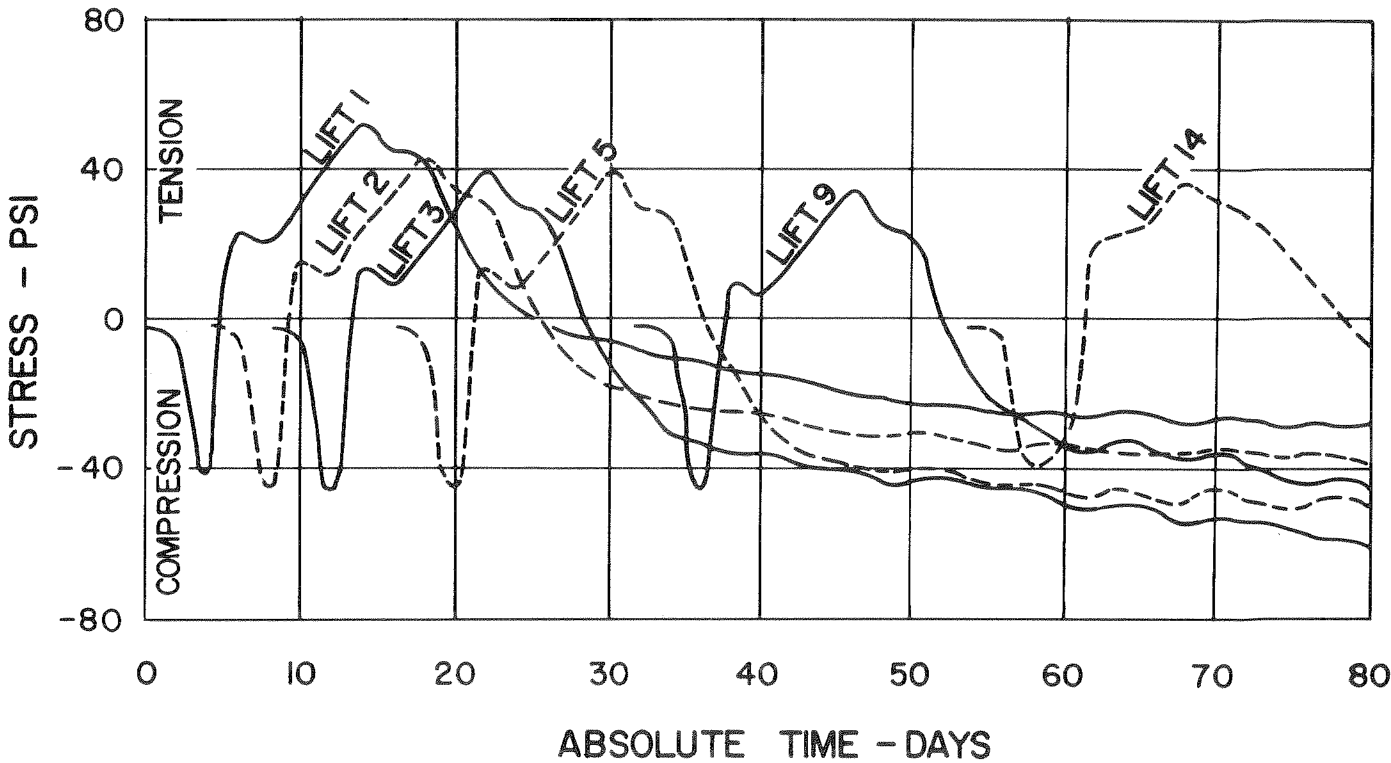

Fig. 28 - HORIZONTAL STRESSES DUE TO TEMPERATURE HISTORY 5

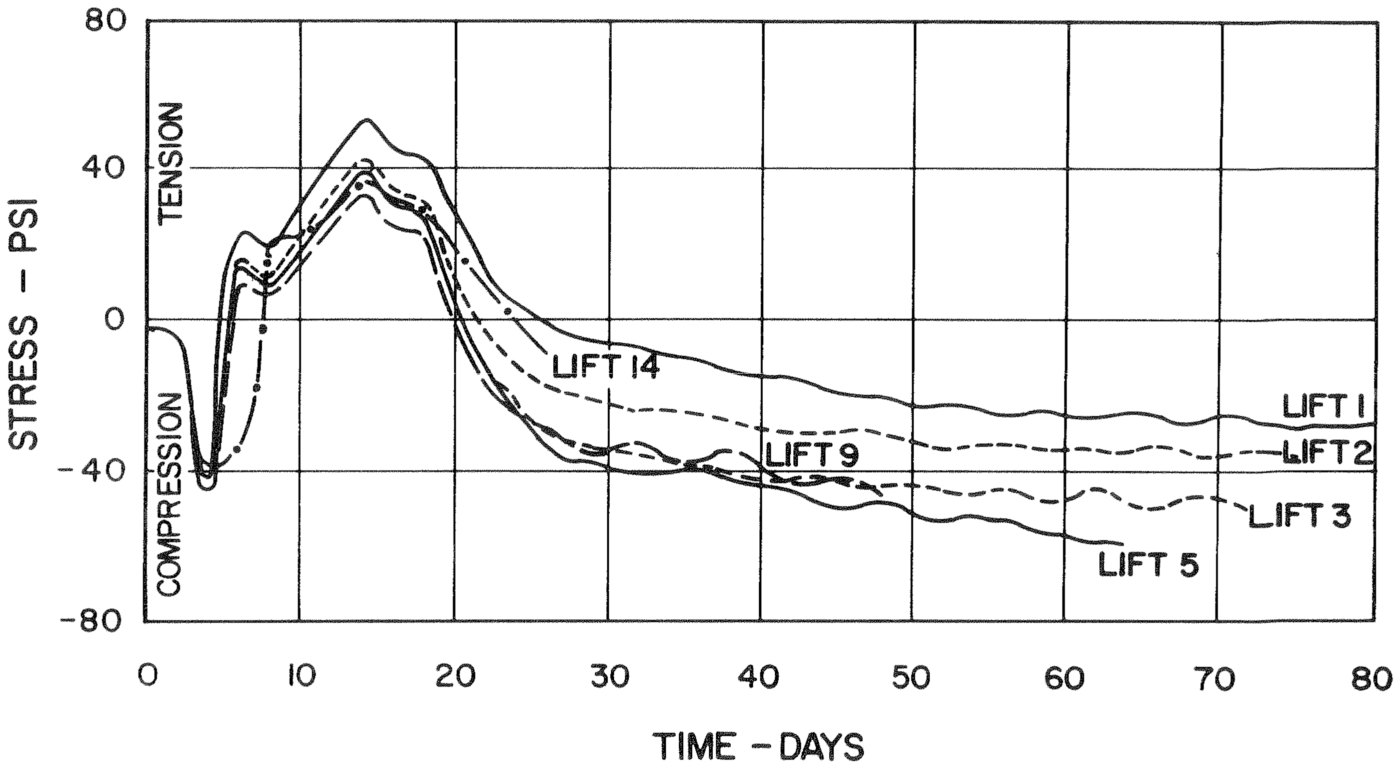

Fig. 29 – HORIZONTAL STRESSES IN VARIOUS LIFTS AS A FUNCTION OF AGE OF CONCRETE FOR TEMPERATURE HISTORY 5

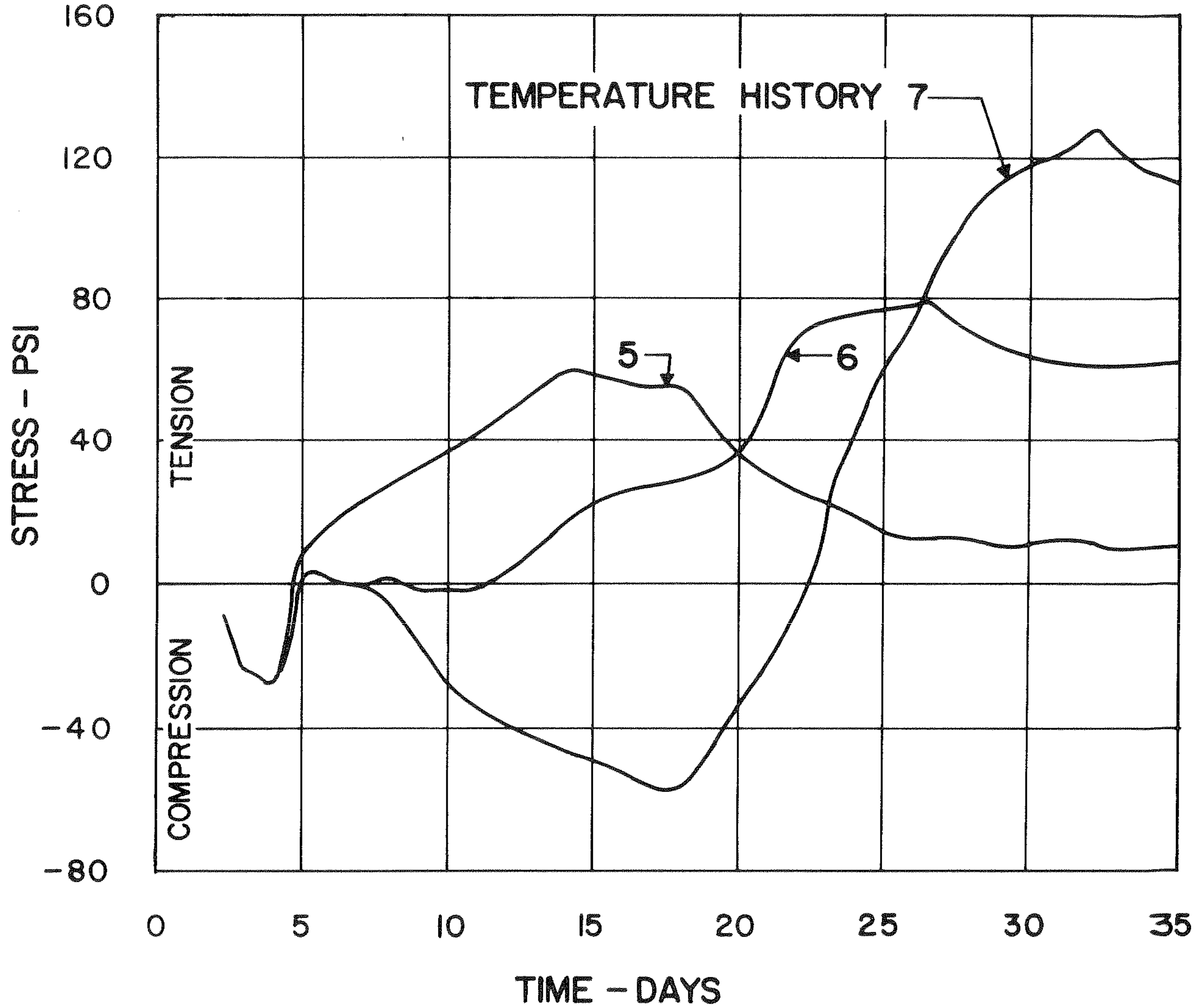

Fig. 30 - COMPARISON OF MIDPOINT STRESSES DUE TO TEMPERATURE HISTORIES 5,6,7

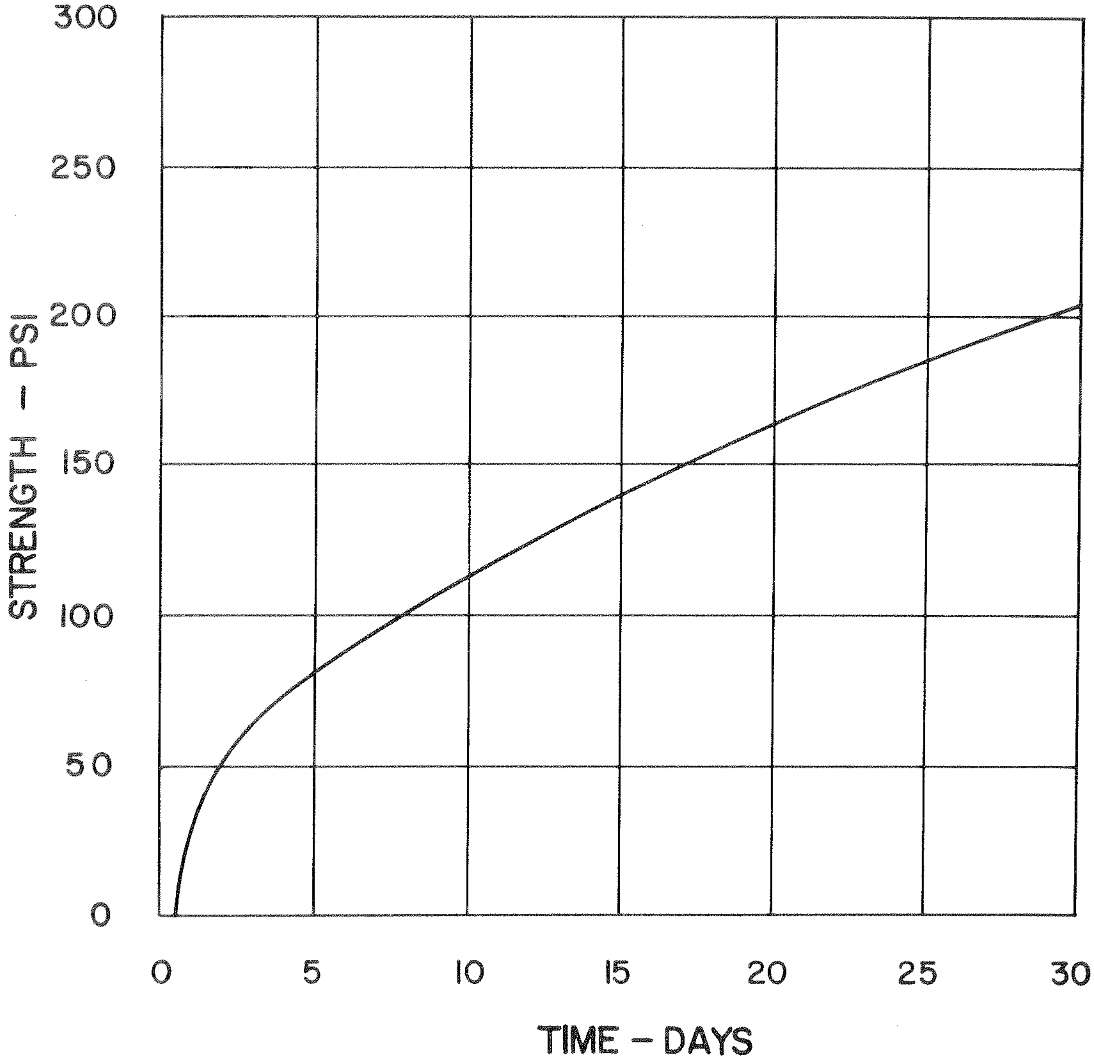

Fig. 31 - ESTIMATED TENSILE STRENGTH

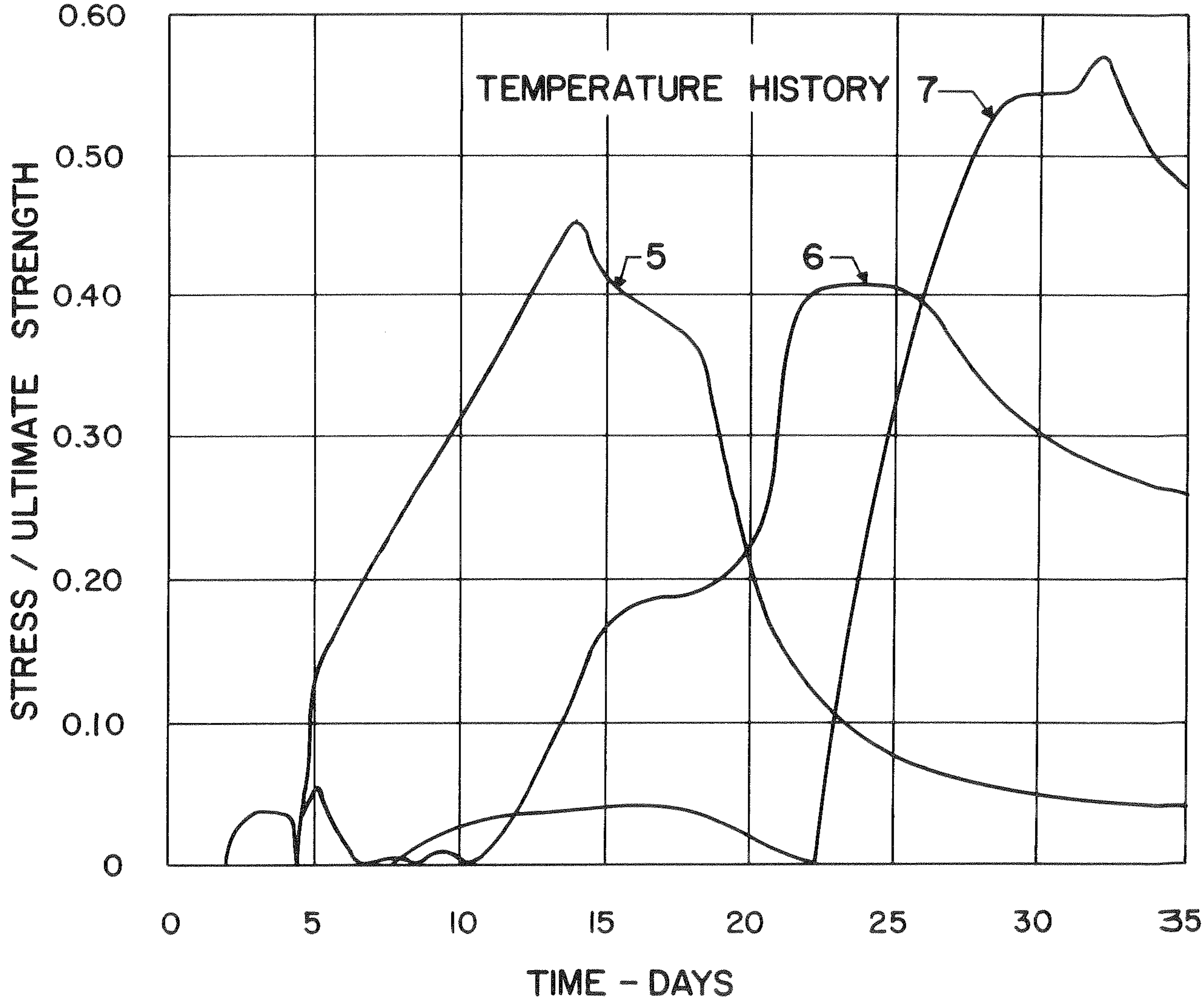

Fig. 32 - EFFECT OF COOLING SCHEDULE ON STRESS TO STRENGTH RATIO

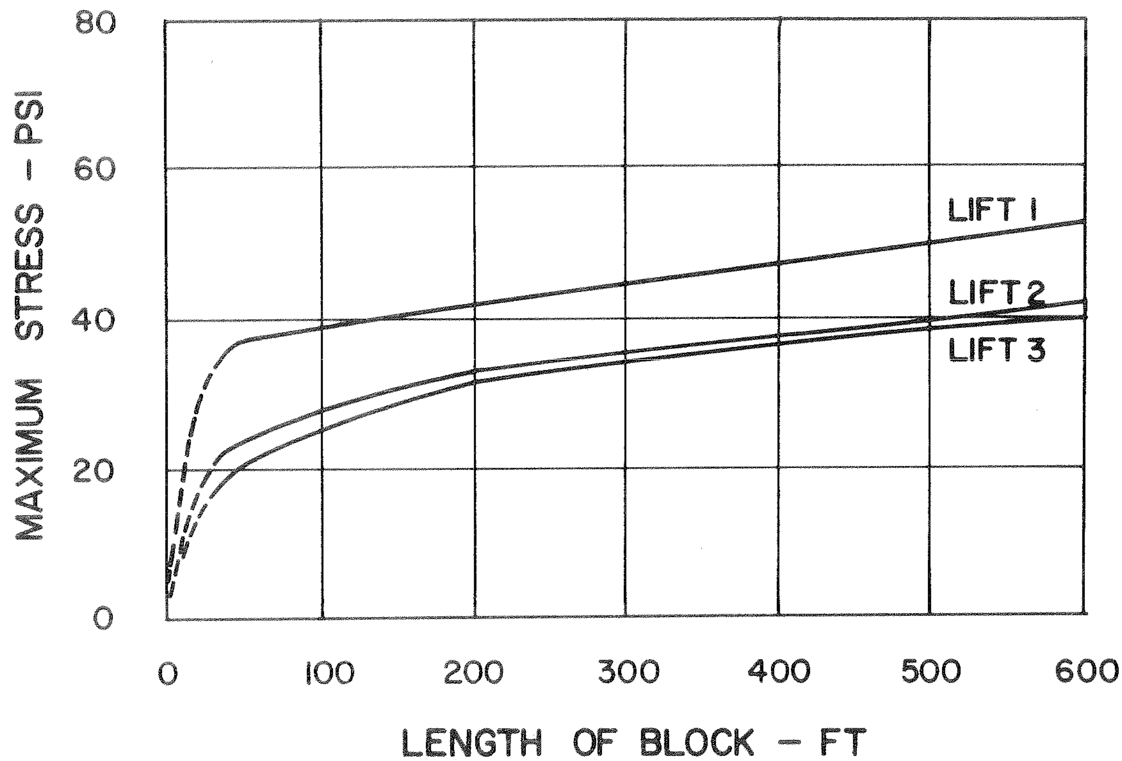

Fig. 33 - EFFECT OF BLOCK LENGTH ON HORIZONTAL STRESS

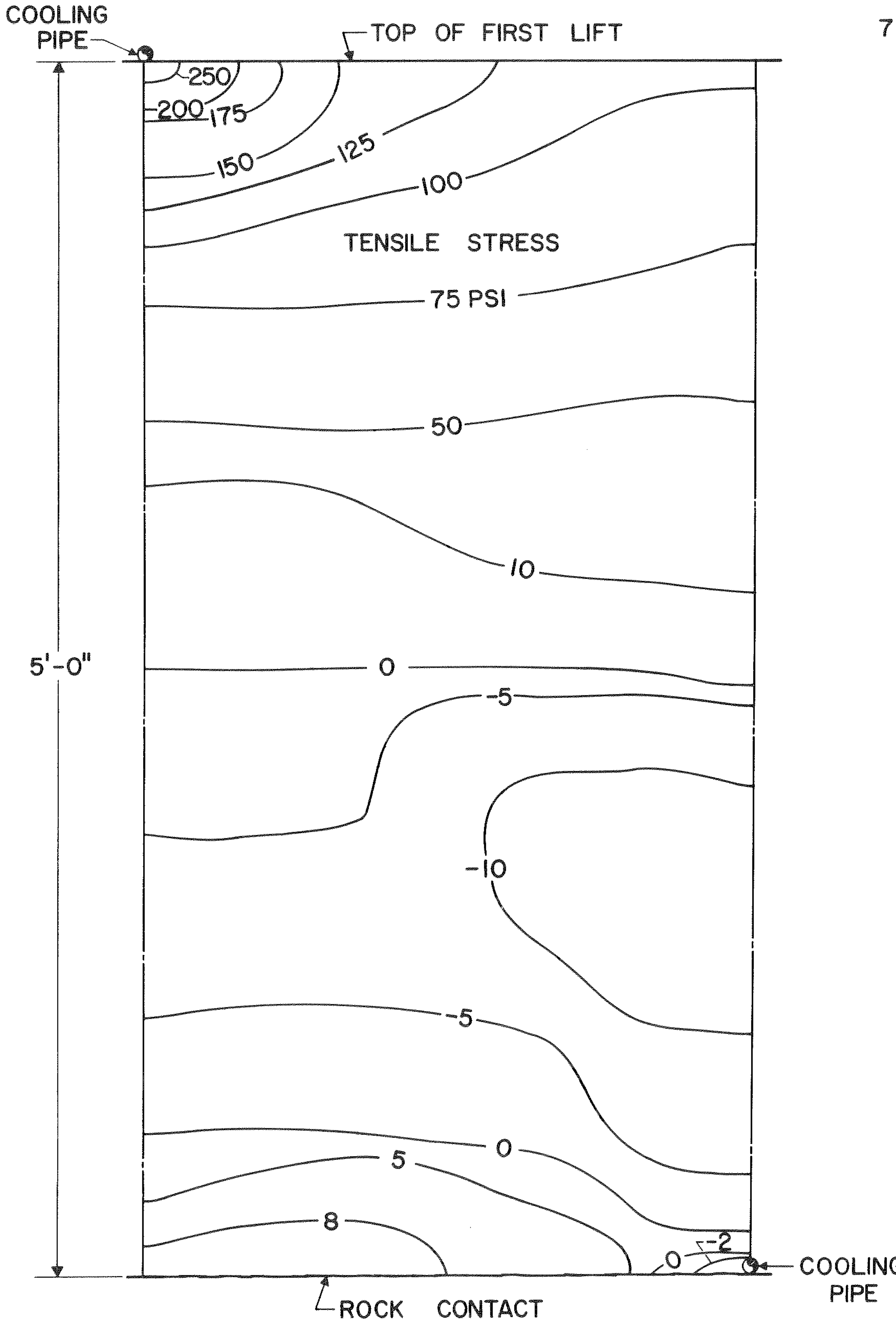

Fig. 34 – STRESS DISTRIBUTION IN LOWEST LIFT AT TIME OF MAXIMUM TENSION

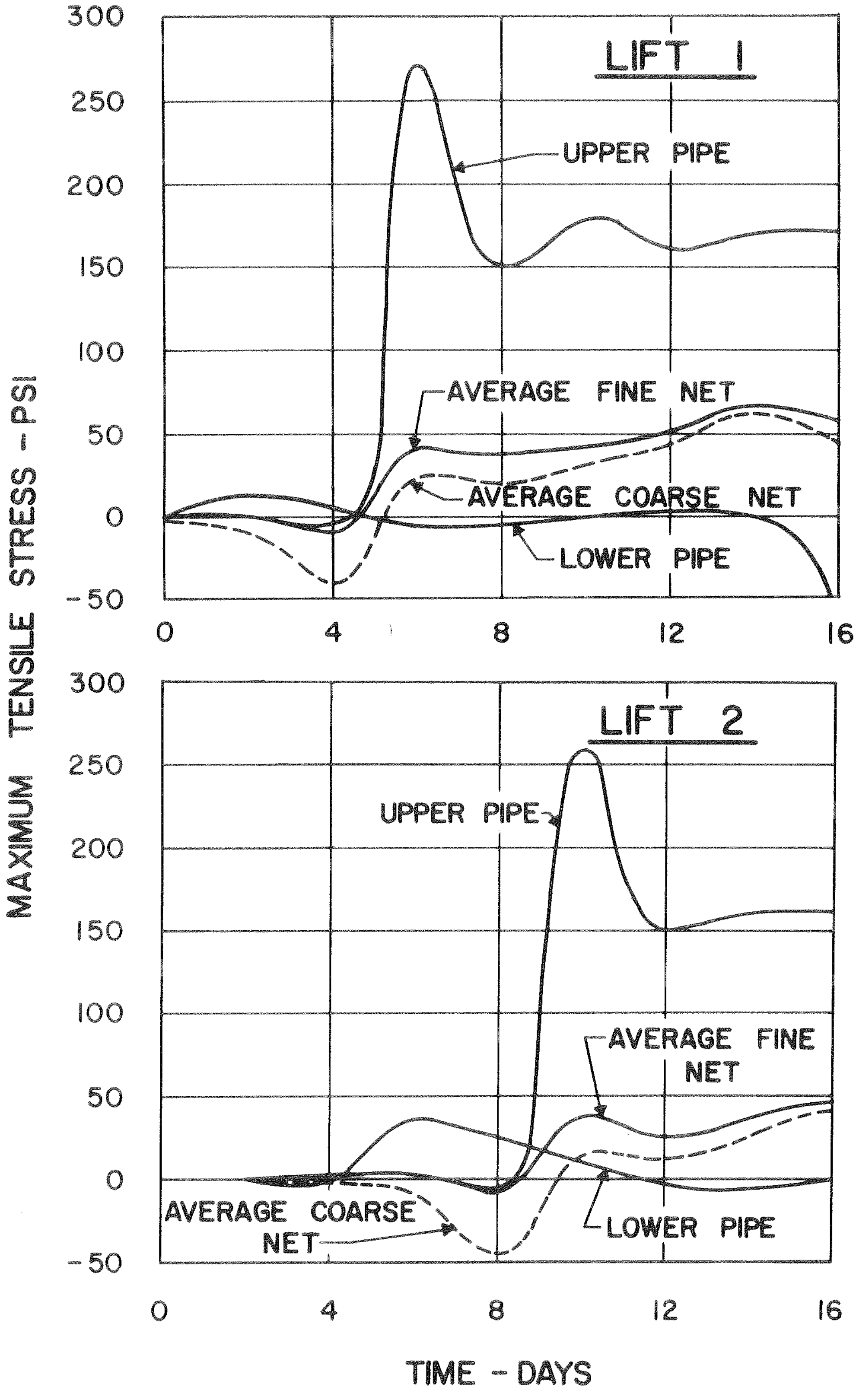

Fig. 35 - STRESS HISTORY FOR SELECTED POINTS IN FIRST TWO LIFTS

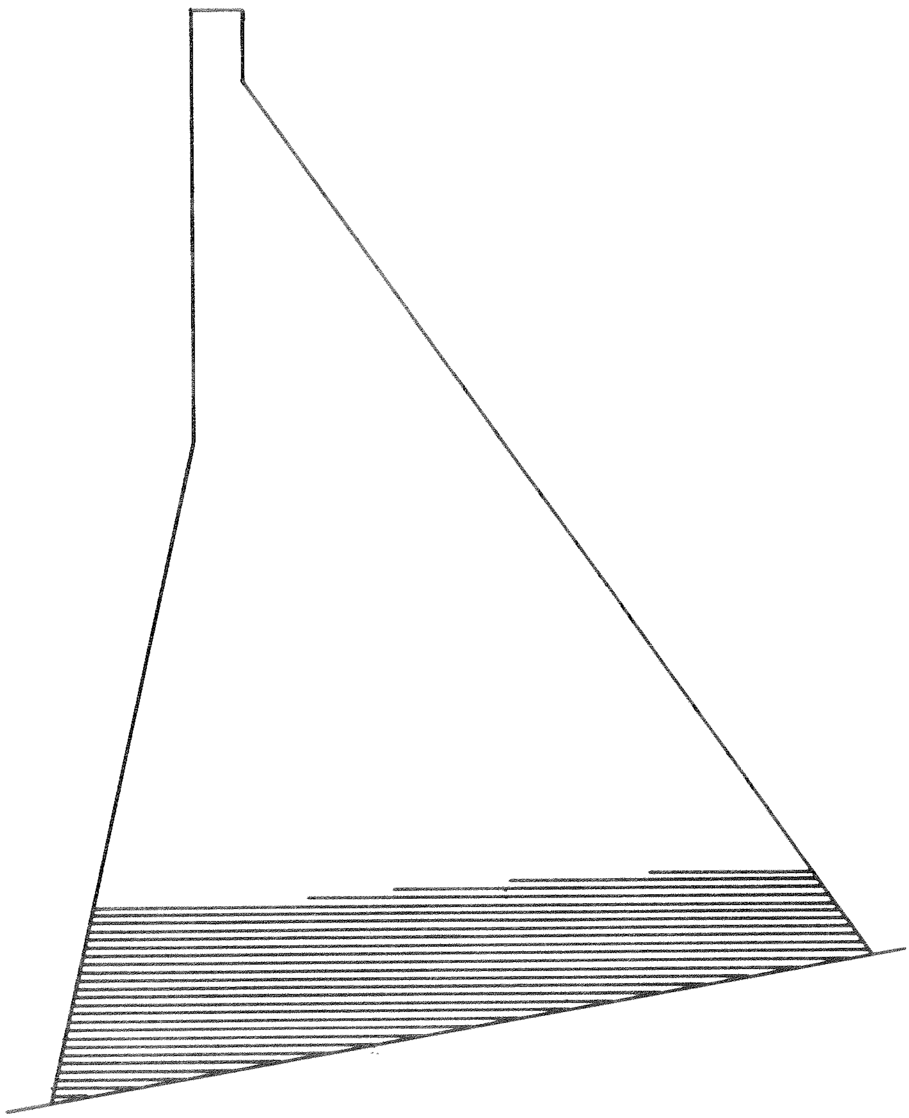

Fig. 36 - DAM WITH SLOPING FOUNDATION