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INTRODUCTION

Since what seems to be the first treatment in the 1882 [M] as an "algebra", the topic now known as "kinship algebra" has focused on the correct composition of strings of symbols used to form empirical systems of kinship terms, as used in naturally occurring languages. The papers and Comments here, especially [B, R2, WD] have summarized much of the history since that initial treatment. While their principal focus is on a particular set of kinship terminologies which have come to be known as Dravidian, the papers and discussion raise a number of issues on the basic form of, and purpose of, the use of mathematics in developing cultural theory. [B], following the extensive discussions in [T], recognizes the nonassociative context of natural languages and of kinship generally, but refers to associative algebras for kinship, while [R2] seems to simply assume the necessary forms are associative. [B] applies group theory, as have many others summarized in the citations of these papers; [WD] evaluates that use. Finally, the Comments also focus on "careful ethnographic description" [WD].

The present Comment observes that all of those forms, associative as well as nonassociative kinship terminologies, and their group theoretical properties when they exist, are all properties of a common and nonassociative description of natural languages. [S] makes clear on the first page of his basic reference on nonassociative algebras, that when an algebra is called nonassociative this simply means that associativity is not assumed to hold, it does not mean that associativity does not hold. Thus, there is no reason why a nonassociative algebra for natural languages cannot have both associative and nonassociative subsets, and from among the associative such subsets, it is no conflict that some or even many might also be groups.

Similarly, if we focus the entire effort on the act of description, then the power of mathematics to make inferences about what is described is lost. Almost paradoxically, certain parts of the papers here that most extol description, illustrate exactly the value of that power of inference. If the description has found some mathematical property, there is no reason to not study that property, and in doing so, no need to re-do or restate the description. One can make inferences from and about the mathematical property, and test them on other observed or predicted mathematical properties, or on the same or other aspects of the description. One can also use study of the mathematics to predict the forms of description, and then observe if those forms are found. In any of these cases, if a suitable description already exists, one may simply cite it. This is how other theoretical sciences are built and tested, and one sees no reason why theory cannot be constructed for anthropology in a similar way.

While we present this Comment by way of discussing the two original papers here [B, R2], we also think the framework has much more general application in linguistics and other aspects of cultural theory. Few if any natural languages are associative, hence are not moniods, nor semigroups, which however are the usual subject of an algebraic theory of language. We avoid this pitfall. We begin by describing a natural language as a set with a nonassociative partially defined binary operation, subject to a cancellation law, which we call a "partially defined expanded quasigroup" or "peq", on a set called a "dictionary". Using peq's, which are nonassociative, then a relevant grammar of a properly defined natural language on a finite non-empty dictionary induces a non-distributive lattice on that dictionary. A clearly defined relevant grammar of a properly defined natural language induces an ortholattice on its dictionary, which may also define a new class of nonassociative mathematical languages. A subset of a natural language can then be associative or nonassociative, and indeed may be a quasigroup or even a quasigroup which is also a group. Specifically as to the present topic, the Dravidian terminologies are then a good example of a class of isotopic quasigroups, each of which is therefore a grammatical structure of some natural language.

In proposing a nonassociative presentation of natural languages, we generalize the insight of [G&O] that kinship algebras are nonassociative, which led us to review quasigroups as a theory for kinship algebra, which quasigroups in turn are noted here to be subsets of natural languages. We make little use of descriptive terminology of traditional grammar. In particular, we do not discuss phonetics, so if two different dictionary entries might "sound the same", that is not a present concern. Unlike [RJ, PC1, PC2] and very many others in recent linguistic theory, we make no claim relating the structures we describe to "mental" or "neurological" states. While we discuss "languages" and "sentences", we do not present a theory of the logic of propositions [H&G]. However, we follow [HJO] and [C&J] in recognizing that morphology and syntax use similar techniques, and that both are tied to the objects in the dictionary or lexicon of the language. Our terminology is adapted from that of [HJO] but we use especially the term "frame" to refer to a specific defined sequence of components, and not to its "cultural meaning" [HJO: pg. 72]. When we say that some operation is defined, we mean, it reflects the "usage" of the individual(s) who employ that language at some point in time; thus our notion of what is "allowed" or "defined" might be considered as derived from the "experience" as defined in [E]. Specifically, we assume what [HJO] calls "linguistic competence", of knowing how to form grammatically correct sentences by a native speaker; but except in so far as [B, R2 and WD] use the notion of "rules", as we do also in Part II, we do not here study what [HJO] calls "communicative competence" -- the social and cultural contexts in which they may be used. Our terms such as poset, ortholattice and others, follow definitions in [DGG: Chapter 1]. We assume a natural language written from right to left. We denote the relevant non-empty set as D, called a "dictionary"; where $b \in D$ is an <u>entry</u> of D.

PART I: FOUNDATIONS

A quasigroup is a nonempty set D with a fully defined binary operation subject to both a left and a right cancellation law[1], commonly stated [S] as in Definition 1:

Definition 1: A quasigroup $(D, *)$ is a non-empty set D with a binary operation * such that for each $a, b \in D$, there exist unique elements $x, y \in D$ such that:
(1) $a*x = b$; and (2) $y*a = b$.

Definition 2: Let D be a finite non-empty dictionary and let * be a partially defined binary operation on D, such that for $(a, b, x, y) \in D$ if $a*x = b$ and $a*y = b$ are defined, then $x = y$; and if $x*a = b$ and $y*a = b$ are defined then $x = y$. Then:
(1) D is left-expanded, if for $x \in D$ there exist $a,b,c,d \in D$ such that if $a*x = b$ is defined, and $c*x = d$ is defined, and $a \neq c$; such x is called a suffix.
(2) D is right-expanded if for $y \in D$ there exist $a,b,c,d \in D$ such that if $y*a = b$ is defined, and $y*c= d$ is defined, and $a \neq c$; such y is called a prefix.
(3) If D is both left and right expanded then D is doubly expanded.
(4) D is expanded if D is left expanded, or right expanded, or doubly expanded.
(5) We call such object $(D, *)$ a partially defined expanded quasigroup, or "peq".

Comment 1: So if such * is fully defined on D, then $(D, *)$ is just a quasigroup, and if $(D, *)$ is a (fully defined) quasigroup and $| D | > 2$, then D is expanded. If $a, b, c \in D$ and $a*b = c$ is defined, we shall refer to the object c (the right hand side of such equation) as a "product".

Definition 3: Let $(D, *)$ be a peq. Let $x \in D$ and let there exist $a, b \in D$ for which $a*x = b$ is defined. Then:
(1) $\mathrm{P}_x := \{ (a, b) \mid x \in D, \ a*x = b$ is defined $\}$, we call such P_x a grammatical component of D determined by x, we call such x the marker of P_x.
(2) $P_x := \{a, b\} \mid (a, b) \in \mathrm{P}_x\} \cup \{x\}$; such P_x is called the support for P_x, and that x generates P_x.
(3) If m is a marker of D, $r,b \in P_m$, $r*m = b$, then r is a root of P_x.

Definition 4: Let D be a dictionary and let P_m be a grammatical component of D, let $P_\mathrm{G} := \{ \mathrm{P}_{mi} \mid \mathrm{P}_{mi} = \mathrm{P}_{mj}, \ m_\mathrm{i} \neq m_\mathrm{j} \}$ be a set of roots of P_m, and $P_{\mathrm{G}m} := \{ m_i \mid P_{mi} \in P_\mathrm{G} \}$ be a set of markers corresponding to that set of roots. Then:
(1) $\mathrm{G} := (P_\mathrm{G}, P_{\mathrm{G}m})$ is a grammatical structure of D;
(2) If G is a grammatical structure of D then $G := P_\mathrm{G} \cup P_{\mathrm{G}m}$ is the support of G;
(3) $G_D := \{\mathrm{G} \mid \mathrm{G}$ is a grammatical structure of $D \}$ is a set of grammatical structures of D;
(4) If G_D is a set of grammatical structures of D, then $\mathrm{k}(G_D) := \bigcup G$, $\mathrm{G} \in G_D$, is the support of G_D.

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[1] We thank Dick Greechie for suggesting use of the cancellation law form applied in Definition 2. All errors are my own.

Comment 2: In each case a support is a subset of D. From Definition 3, (P_x, *) is a peq, and since $P_x \subseteq D$ and both P_x and D are peq's, then P_x is a sub-peq of D. Nothing prevents that b is a product in one context and a root or marker in another. Nothing prevents that m is idempotent so that $m*m = m$ is allowed.

Definition 5: Let X be a non-empty peq; then $r \in X$, $r \neq \varnothing$, is a representative of X. Let X_1, X_2 be finite non-empty peq's, c_1 a representative of X_1, c_2 a representative of X_2, and $c_1c_2 = c_3$ an allowed product, we call such sequence an example.

Convention 1: We shall use "punctuation mark" as an undefined primitive term. We define T_D to be a set of "final markers" of D (commonly written as ".", "?", "!" and possibly others), and I_D to be a set of "intermediate markers" of D (commonly written as ", ";", and others), and assert that $E_D := T_D \cup I_D$, $T_D \cap I_D = \varnothing$ is the set of all "punctuation marks" of D. We assert that T_D and I_D are grammatical components.

Definition 6: Let D be a dictionary, let G_1, G_2 be grammatical structures of D, let $c_1 \in G_1$, $c_2 \in G_2$. Then:

(1) $F := \{ c \mid c_1c_2 = c \text{ is defined} \} := (G_1, G_2)$ is an allowed frame of D;
(2) $F_D := \{ F \mid F \text{ is an allowed frame of } D \}$ is a set of frames of D.
(3) Let F be an allowed frame of D, let $T_D \subseteq D$ be a non-empty set of final markers of D, and let $S_F := \{ s \mid s = ft, f \in F, t \in T_D\} := (F, t)$. Then S_F is a sentence of D, using F.
(4) Let $F \in F_D$, let $S_F = (F, t)$, then $S_F := F \cup \{t\}$ is the support of S.
(5) Let $S_{FD} := \{ S \mid S \text{ is an allowed sentence on } D \}$ be a set of sentences on D.
(6) Let $k(S_{FD}) := \bigcup S_i$, $S_i \in S_{FD}$, be the support of S_{FD}.
(7) A natural language L_D on D is a non-empty set S_{FD} of sentences on D.

Comment 3: The support of L_D is therefore the support $k(S_D)$ of the set S_D of sentences that defines L_D. A sentence (singular) is a class of objects defined by a particular product of a frame and a final marker. If S is a sentence then a representative $s \in S$ is a particular "sentence" in the common language use, and is an example of "the sentence S" in our use. Typically, linguistics uses representatives to define or illustrate frames and sentences. Because frames require grammatical components in particular sequences, they identify a "sentence structure" of a particular language.

Axioms: Let D be a dictionary, let (D, *) be a peq, and let G_D be a set of grammatical structures of D. We adopt two axioms:
Axiom 1: if $a \in D$ there exists $G \in G_D$ such that $a \in G$.
Axiom 2: D may include "punctuation marks".

Comment 4: Axioms 1 and 2 permit to construct a language L_D from the properties of an empirical natural language. Nothing assures that the result of thus constructing sets of grammatical structures G_D of L_D will yield the same sets that carry the traditional names. Our intuition is that if traditional concepts of grammar are correct, that should normally be the result.

Comment 5: Let *a*, *b*, *c* be representatives of grammatical structures A, B and C on a non-empty dictionary *D*. Let *ab* represent a defined frame AB, and (*ab*)*c* represent a defined frame (AB)C. Assume a frame BC is not defined, so a frame represented by *bc* is not defined, and so *a*(*bc*) is not defined, so $(ab)c \neq a(bc)$. And even if *bc* and *a*(*bc*) are defined, nothing requires that $(ab)c = a(bc)$. Thus in general frames are not associative, so sentences are not associative, hence natural languages are not associative, hence are not monoids [I] nor semigroups [P&P], so are not included within the mathematical theory of languages as normally described.

> *Definition 7*: If *D* is a dictionary then a language L_D is <u>properly defined</u> if for $a \in D$ there exists an allowed frame F and an allowed sentence S_F such that $a \in F$.

Comment 6: That is, $k(S_D) = D$ iff every entry in *D* appears in at least one sentence. This condition seems also to be a working axiom of field linguistics to properly define a language.

> *Definition 8*: Let *D* be a non-empty dictionary, let F_D be a finite non-empty set of allowed frames on *D*, let $(F_D^*, \circ)$ be an algebra where $\circ$ is a partially defined symmetric binary operation; if $F \in F_D$ then $F \in F_D^*$; if $F_i, F_j \in F_D^*$ and if $F_i \circ F_j$ is defined then $F_i \circ F_j \in F_D^*$. If $F_i \circ F_j \in F_D^*$ then $F_{ij} := F_i \circ F_j$ is a <u>compound frame</u>. If $S = (F_{ij}, t)$, $t \in T_D$, is an allowed sentence then S is a <u>compound sentence</u>. Then F_i and F_j are each a <u>component</u> of F_{ij}, and $(F_D^*, \circ)$ is a set of <u>frame compositions</u>. If $F_{ij} = F_i \circ F_j$ is a compound frame then $F_{ij} := F_i \cup F_j$.

> *Definition 9*: Let *D* be a dictionary and L_D a language on *D*. Let G_D **:=** { G | G is an allowed grammatical structure on *D* } be a non-empty set of allowed grammatical structures of L_D; let F_D **:=** { F | $G_i, G_j \in G_D$ and $F = (G_i, G_j)$ is a defined frame on *D* } be a finite non-empty set of allowed frames of L_D; let F_D^* be a finite non-empty set of frame compositions on *D*; and let S_{FD} := { S_F | $F \in F_D^*$ and S_F is an allowed sentence on L_D } be a finite non-empty set of allowed sentences on *D*. Then $\boldsymbol{G}_{FD} := F_D^* \cup G_D$ is a <u>finite grammar</u> of L_D, or simply a <u>grammar</u>. If L_D is properly defined using only sentences in S_{FD} then $\boldsymbol{G}_{FD}$ is <u>properly defined on</u> L_D. Then $k(G_D) := \bigcup G$, $G \in G_D$, <u>supports</u> G_D and $k(F_D^*) := \bigcup F$, $F \in F_D^*$, <u>supports</u> F_D^*. If F_D^* is the least set of allowed frames, including compound frames, necessary to define all allowed sentences in S_{FD}, we call the resulting $\boldsymbol{G}_{FD}$ a <u>relevant</u> grammar.

Comment 7: While F_D^* is potentially indefinitely large, in a finite grammar $\boldsymbol{G}_{FD} = F_D^* \cup G_D$ only a finite number of sentences are defined, so only a finite subset of F_D^* are used to form sentences. So we lose nothing by assuming F_D^* is finite, and henceforth do so. From Definition 8, $S_{FD} \subseteq F_D^*$, that is, since a sentence is also a frame, all allowed sentences of a language L_D are also found in a set of frame compositions on L_D, and a grammar contains all defined sentences.

Comment 8: From Definition 8, a set (F_D^*, $\circ$) of frame compositions is a peq, so a grammar $\boldsymbol{G}_{FD}$ is a set of peqs, thus a peq. In general compound frames are not commutative nor associative: a change in sequence of writing a string, or in the arrangements of parenthesis, may result in creating frames that are not allowed. Thus as for peqs, in general a grammar $\boldsymbol{G}_{FD}$ is neither commutative nor associative.

> *Definition 10*: Adopting conditions of Definition 8, let L_D be a natural language on a non-empty dictionary D, T_D a set of final markers of D, and $\boldsymbol{G}_{FD} = F_D^* \cup G_D$ be a grammar on D.
>
> (i) Then for G_i, G_j, $G_w \in G_D$, if
> 1. $G_i \cup G_j \cup \ldots \cup G_w = D$, and
> 2. $G_i \cap G_j = \varnothing$ whenever $i \neq j$, then $\boldsymbol{G}_{FD}$ (resp. L_D) is <u>clearly structured</u>.
>
> (ii) For F_i, F_j, $F_w \in F_D^*$ where F_i, F_j, $F_w \notin S_{FD}$, and F_i, F_j, $F_w \subseteq k(F_D^*)\backslash T_D$,
> 3. $F_i \cup F_j \cup \ldots \cup F_w = D$, and
> 4. $F_i \cap F_j = \varnothing$ whenever $i \neq j$, then $\boldsymbol{G}_{FD}$ (resp. L_D) is <u>clearly framed</u>.
>
> (iii) For F_i, F_j, $F_w \in F_D^*$, where F_i, F_j, $F_w \in S_{FD}$, and F_i, F_j, $F_w \subseteq k(F_D^*)$,
> 5. $F_i \cup F_j \cup \ldots \cup F_w = D$, and
> 6. $F_i \cap F_j = \varnothing$ whenever $i \neq j$, then $\boldsymbol{G}_{FD}$ (resp. L_D) is <u>clearly framed as sentences</u>.
>
> (iv) If $\boldsymbol{G}_{FD}$ (resp. L_D) is clearly structured, clearly framed and clearly framed as sentences, then $\boldsymbol{G}_{FD}$ (resp. L_D) is <u>clearly defined</u>. If $\boldsymbol{G}_{FD}$ (resp. L_D) is not clearly defined then it is <u>ambiguous</u>.

Comment 9: If L_D is properly defined and (i)2 holds, then (i)1 follows: since for L_D to be properly defined then each $a \in D$ occurs in at least one frame and therefore in at least one grammatical structure; and if these are disjoint, then each a is in only one grammatical structure, so 1 follows, and L_D is clearly structured. In empirical natural languages, "roots" (which typically in natural languages comprise “morphemes”) often occur in more than one grammatical component; the “role” of markers is to give different "meanings" to the same root. Thus, in general G_D does not partition D, so in general empirical natural languages are ambiguous.

Comment 10: Let D be a finite non-empty dictionary and let $\mathcal{P}(D)$ be a set of subsets of D. If every $X \subseteq \mathcal{P}(D)$ supports at least one grammatical component of a properly defined and clearly structured language L_D, we call such language <u>efficient</u>. Then such L_D is not efficient, unless each grammatical component is an idempotent object $a \in D$, with $r = a$, $m = a$ and $a*a = a$. And apparently also few if any naturally occurring natural languages are efficient for a different reason: some subsets of D may not support any grammatical component.

Comment 11: Let D be a finite non-empty dictionary, and L_D. an allowed language on D. $\mathcal{A}$ set $\mathcal{P}(D)$ of subsets of D is also a Boolean lattice using set inclusion as the partial order. If G, F, S, G_D, F_D*, S_D are allowed on L_D, then the corresponding sets of support G, F, S, $k(G_D)$, $k(F_D^*)$,

$k(S_D) \in \mathcal{P}(D)$, since also G, F, S, $k(G_D)$, $k(F_D^*)$, $k(S_D) \subseteq D$. But not all members of $\mathcal{P}(D)$ will normally be used in a grammar $\boldsymbol{G}_{FD}$ of L_D. Further, while each frame in a set of frame compositions on D is itself a single (possibly compound) frame, a set of two or more frames of a natural language L_D is not a frame. Let F_D^* be a finite non-empty set of (possibly compound) frames of L_D and let $\mathcal{P}(F_D^*)$ be a finite non-empty set of subsets of F_D^*. Then $\mathcal{P}(F_D^*)$ is a Boolean lattice using set inclusion as the partial order, and in particular, is a Boolean lattice of sets of frame compositions. If S_{FD} is a language on D on a set $F_D \subseteq F_D^*$ of frames on D then $S_{FD} \in \mathcal{P}(F_D^*)$. But there may be members of $\mathcal{P}(F_D^*)$ which do not define grammars of L_D.

Comment 12: Let D be a finite non-empty dictionary, let G_1, G_2 be allowed grammatical structures on D, let $\mathrm{F} = (G_1, G_2)$ be an allowed frame on D, and $\mathrm{S_F}$ be an allowed sentence using F. Since $F = G_1 \cup G_2$, then $S_\mathrm{F} = F \cup \{t\} = G_1 \cup G_2 \cup \{t\}$. Since allowed grammatical structures are non-empty sets, and allowed frames are pairs of non-empty sets, we adopt as a convention to denote a "frame" or "grammatical structure" $(\emptyset, \emptyset)$ as "0", as they are not allowed. Using ordinary set inclusion as the partial order, then we have $S_\mathrm{F} \geq (G_1 \cup G_2) \geq G_1 \geq 0$, and also $S_\mathrm{F} \geq \{t\} \geq 0$. (We can write a similar inequality $S_\mathrm{F} \geq (G_1 \cup G_2) \geq G_2 \geq 0$). But we have now constructed a lattice with S_F on the "top", and 0 on the "bottom" and which is isomorphic to the lattice N_5. Therefore, provided $|D \backslash T_D| > 1$, a lattice of sets comprised of dictionary entries supporting grammatical structures, frames and sentences of a natural language L_D is in general neither distributive nor modular.

Comment 13: We now examine a special lattice. Let D be a finite nonempty dictionary, let G be a grammatical structure on D, with support G, and let $R := \{ r \mid r \in G \}$, $M := \{ m \mid m \in G \}$, and $P := \{ p \mid p \in G \}$ partition G. On the "top" is the set G; beneath G are the three disjoint sets R, M, and P of which it is comprised; on the bottom we place 0. Then G is an example of the non-distributive lattice *M3* [G]. So any lattice induced on D by a grammar that induces this special lattice is not distributive. While some empirical natural languages may have some product of a marker and a root equal to the root (for example), in general they do not. So in general a lattice of the supports of a grammar of an empirical natural language will contain this special sublattice, and not be distributive.

Comment 14: Let D be a finite non-empty dictionary, L_D a language on D, F_D^* a set of allowed frames on L_D, G_D allowed grammatical structures on L_D, and $\boldsymbol{G}_{FD} = F_D^* \cup G_D$ a relevant finite grammar on L_D. We construct a lattice $\mathrm{L}(\boldsymbol{G}_{FD})$ of subsets of D, which we shall call the lattice <u>induced</u> on L_D by $\boldsymbol{G}_{FD}$, as follows. Place 0 on the bottom; above 0 place the sets $G \in G_D$ of entries in D supporting members of G; between each G and 0 insert the respective subsets R, M, $P \subseteq D$ as defined in Comment 13; above the row of G's place the objects $F \in F_D^*$, $F \notin S_{FD}$, of entries of D that support frames that are not sentences; above that the sets $F \in F_D^*$, $F \in S_D$, of entries in D that support sentences; place D on top. $\mathrm{L}(\boldsymbol{G}_{FD})$ is a lattice provided $\boldsymbol{G}_{FD}$ is relevant, since $0 \in \mathrm{L}(\boldsymbol{G}_{FD})$ and since if $\boldsymbol{G}_{FD}$ is relevant then for each pair of entries some least upper bound exists; if $\boldsymbol{G}_{FD}$ is not relevant, then some set G and/or F may exist but for which there is no least upper bound with each other element of $\mathrm{L}(\boldsymbol{G}_{FD})$. In general $\mathrm{L}(\boldsymbol{G}_{FD})$ is not distributive, since it contains a sub-lattice per Comment 12; and for that reason and since it will also often contain as a sub-lattice an object as described in Comment 13, $L(\boldsymbol{G_{FD}})$ is not Boolean.

Comment 15: For any grammar $\boldsymbol{G_{FD}}$ in which there exists $t \in T_D$ such that t occurs in two or more distinct sentences of $\boldsymbol{G_{FD}}$ (which seems to be the case for terminal markers of most empirically occurring natural languages) then each such sentence S has relative complement $S^{\sim}$ in D such that $\{t\} = S \cap S^{\sim} \neq 0$, so $L(\boldsymbol{G_{FD}})$ is not a bounded involution poset (thus not an orthoposet nor ortholattice.)

Comment 16: However, if D is a finite non-empty dictionary, L_D is properly defined on D, and $\boldsymbol{G_{FD}}$ is a relevant clearly defined grammar of L_D, then $L(\boldsymbol{G_{FD}})$ is a bounded involution poset, an orthoposet and ortholattice. This occurs since first, each object in $L(\boldsymbol{G_{FD}})$ other than D or 0 is the support of a defined sentence, frame, or grammatical structure. Then following notation of Comment 12 we have that $L(\boldsymbol{G_{FD}})$ is constructed as a collection of partial orders $D \geq S_F = (F \cup \{t\}) = (G_1 \cup G_2 \cup \{t\}) \geq (G_1 \cup G_2) \geq G_1 \geq 0$. Because $\boldsymbol{G_{FD}}$ is clearly defined, each object in each "layer" of such partial orders of $L(\boldsymbol{G_{FD}})$ is the cell of a partition of D and thus each such cell is uniquely relatively complemented as a subset of D. In the ascending direction each object is a union of some set of cells in the partition in the layer beneath it. Then clearly also, $0 = D^{\sim} \leq (G_1 \cup G_2 \cup \{t\})^{\sim} \leq (G_1 \cup G_2)^{\sim} \leq G_1^{\sim} \leq 0^{\sim} = D$. So the orthoposet condition holds everywhere, and thus $L(\boldsymbol{G_{FD}})$ is also an ortholattice. Thus a clearly defined relevant grammar on a properly defined language on a finite non-empty peq, while apparently not describing empirical natural languages, forms a non-distributive orthoposet. This also appears to be a new class of nonassociative formal languages.

Conclusion to Part I: The topic known as mathematical linguistics is based on semigroups; but as an associative fully defined binary operation on a set semigroups do not describe natural languages. In general natural languages are not associative, are partially defined, and when defined are not commutative. Thus we construct nonassociative non-commutative objects called peqs: partially defined expanded quasigroups. Quasigroups are a well known topic, but the notion of a partially defined quasigroup appears to be new here.

PART II: TERMINOLOGIES AND CONSISTENT CULTURES

We next recognize a special subset of a natural language L_D on a finite non-empty dictionary D.

> *Definition 11*: Let $G = (P_G, P_{Gm})$ be a grammatical structure on D. If every object in P_{Gm} is doubly expanded with every object in P_S, then G is a <u>complete terminology</u>.

Comment 17: Definition 11 is equivalent to stating that the operation * is fully defined on the set $G = P_G \cup P_{Gm}$, which is to say, that $(G, *)$ is a quasigroup. So a complete terminology is also a quasigroup (and vice versa). Thus in a complete terminology $P_G = P_{Gm}$, so citing definitions of Comment 13, $R = M = P = P_G = P_{Gm} = G$. Then if D is a finite non-empty dictionary, $G \neq \varnothing$ and G is a complete terminology of D with support $G \subseteq D$, then G is a grammatical component of D, and (in contrast to the special lattice of Comment 13) $D \geq G \geq R \geq 0$ is a chain; note that any of R, M, P, P_G, P_{Gm} or G can be substituted for R in that chain. Now notice that because a complete terminology G is a quasigroup $(G,*)$ therefore $(G,*)$ is a set of permutations. Because quasigroups may also be groups, then at least some complete terminologies may also be groups. Let $g(\mathcal{P}(D)) :=$ { G | D is a finite non-empty dictionary, G is a complete terminology of D, the support of G is $G \in \mathcal{P}(D)$ and $(G, *)$ is a group }. Then $g(\mathcal{P}(D)) \subseteq \mathcal{P}(D)$ is a set of complete terminologies (quasigroups) that also form groups.

Comment 18: In [R, R2] it is required that in a kinship algebra, if Q is a set of kinterms, then for each "pair" of kin terms, $x, y \in Q$, if $xy \in Q$ then $yx \in Q$; such pairs of sequences are called "reciprocals"; and also $xy = yx = 1$. But this simply restates Definition 1: the condition if $a, b \in Q$, $ab \in Q$, assures existence of reciprocals since it also implies $ba \in Q$ and therefore also, and $ab = ba = 1$ assures existence of a cancellation law. Thus these conditions imply that a kinship algebra is a quasigroup, which is a complete terminology and a grammatical structure of a natural language. In fact, in [R2] not only are kinship terminologies quasigroups, they are peqs: [R2] identifies a kinship algebra as a set of kin terms with binary operation which allows that "a product … does not yield a kin term", hence the operation is partially defined relative to the set Q. Therefore, following [R2] a kinship algebra is a set with a partially defined binary operation subject to a cancellation law: a peq. [R2] also seems to assume such algebras are associative; in fact, apparently many are. Those properties together are part of why many kinship algebras also form groups. In study of quasigroups [KKP, SVA, S] sets of permutations are arranged in tables, called Latin Squares or "Cayley Tables" [Go]. Then empirical examples of complete terminologies, hence quasigroups, are found in "kinship algebras" using a finite set of symbols and a rule of concatenation of those symbols [G1, G2, G&O, H, R, W], which may be organized as Cayley Tables [K, G1, G2], and which often also form groups [G1, G2, K, W].

Notational Citation: To next discuss properties of internal consistency of cultures, we need some terminology for properties other than kinship algebras. We adopt the notation of [B1, B2], to which the reader is referred for more detail. In brief we assume a finite non-empty set $\mathbb{P}$ whose members are called <u>individuals</u>. If $d \in \mathbb{P}$ the there exists a discrete "generation at time t" such that $d \in \mathbb{G}^t \subseteq \mathbb{P}$. We defined three binary relations D, B, and M on $\mathbb{P}$, satisfying these four axioms: (1) D is totally non-symmetric and transitive; (2) M is symmetric; (3) if bDc and there exists no $d \in \mathbb{P}$, $d \neq b,c$ for which bDd and dDc, then we write cPb, and then require B = { (b,c) | there exists $d \in \mathbb{P}$ with both dPb and dPc }; and (4) $\#bM \leq 2$, where $bM :=$ { $c \in \mathbb{P}$ | $(b,c) \in M$ } and a <u>rule</u> $R \in \mathbb{R}$, $\mathbb{R} \neq \emptyset$, is a statement concerning the relationships between the D, B, and/or M, which does not violate those four axioms. Since a viable rule also implies a sequence over time we can also describe the same idea as a history, and denote a history (that is, the rules which generate it) with a notation α. A set $\mathbf{G} =$ {$\mathbb{G}^t$ | $t \in \mathcal{T}$} is called a <u>descent sequence</u> of $\mathbb{S}$ in case, for all $\mathbb{G}^t \in \mathbf{G}$, each cell B occurs in only one generation, each subset M occurs in only one generation, and when $\mathbb{G}^t \in \mathbf{G}$, $b \in \mathbb{G}^t$, and cPb, then $c \in \mathbb{G}^{t-1}$ (that is, the set $\mathbb{G}^{t+1}$ contains all of, and only, the immediate descendants of individuals in $\mathbb{G}^t$). Given a generation $\mathbb{G}^t$, $a,b,c,..,k \in \mathbb{G}^t$, $a \neq b \neq c \neq \ldots \neq k$, a <u>regular structure</u> is a closed cycle of alternating M and B relations, such as aBb, b*M*c, c*B*d, … k*M*a. If there are j instances of M in such cycle, then the cycle is of type Mj. An ordered list counting the numbers of regular structures present in a particular $\mathbb{G}^t$ is a configuration C:= (m_1, …, m_j, …) where m_j is the number of regular structures of type Mj in C. Thus a configuration consisting only of 2 of the M2 structures would be written (0,0,2,0, …). We let $\mathbb{C} = \{ C_i \mid i = 1,2, ..., n \}$ be a finite non-empty set of n distinct configurations C_i. We here consider only finite sets $\mathbb{C}$. In general, if C_i and C_j are configurations then $C_i + C_j$ is also a configuration, though $C_i + C_j \in \mathbb{C}$ is not required (since $\mathbb{C}$ is finite). All marriages in a regular structure are assumed to be reproducing. So if C is a configuration, then $\mu_C = \Sigma(jm_j)$, or simply "μ" when C is understood, is the number of reproducing marriages in C; β is the number of cells induced by *B* (sibships) in C; and $\gamma = 2\mu$ is the total population included into regular structures of the generation $\mathbb{G}^t$ on which C_i is formed. We denote the set of smallest "self-reproducing" configurations ("minimal structures") of a history **α** by $\mathbf{M}_\alpha$. Often a history **α** has one minimal structure; so if $C \in \mathbf{M}_\alpha$ and $|\mathbf{M}_\alpha| = 1$, we write simply $\mathbf{M}_\alpha = C$, not $\mathbf{M}_\alpha = \{C\}$. If C is a minimal structure of **α**, then $\Sigma(jc_j) = s$ is the structural number of **α**.

Definition 12: We have required that if c*P*b and if $b \in \mathbb{G}^t \subseteq \mathbb{P}$ then $c \in \mathbb{G}^{t-1} \subseteq \mathbb{P}$. If $\mathcal{G} = \{\mathbb{G}^t \mid t \in \mathcal{T}\}$ is a descent sequence of $\mathbb{P}$, $\mathbb{G}^k, \mathbb{G}^j \in \mathcal{G}$, j, k $\in \mathcal{T}$ then the integer g = |j - k| is the generation interval between $\mathbb{G}^k$ and $\mathbb{G}^j$. If $\mathbb{G}^k, \mathbb{G}^j \in \mathcal{G}$ $b \in \mathbb{G}^k$ and $c \in \mathbb{G}^j$ then we also say that g = |j – k| is the generation interval between b and c. We shall write that when c*P* b and d *P*c then dP^2b, and in general if there is a transitive sequence of g such pairs c*P*b , d*P*c, … n*P*m we write mP^gb. Then b*B*c iff ∃(d)(dP^1b and dP^1c), so we set bB^gc iff ∃(d)(dP^gb and dP^gc), and say that b is a g^{th} order relative of c, and say b is a g^{th} order descendant of d and d is a g^{th} order ancestor of b. We define bB^0c iff b=c.

Comment 19: So if c*P*b then g = |t – (t-1)| = 1. And if c*P*b and d*P*c then the generation interval between b and d is 2. Note that "b is a g^{th} order relative of c" is symmetric.

Definition 13: Let $\mathbb{P}$ be a finite non-empty set $\mathbb{P}$, b, c, d$\in\mathbb{P}$. Let *R*, *S* be binary relations on $\mathbb{P}$. Let ¬ be a binary operation such that ¬(cB^gb) iff c ∉ bB^g. Let ∧ be a symmetrical binary operation such that *R*∧*S* iff b*R*c and b*S*c. Let ∨ be a symmetrical binary operation such that *R*∨*S* iff b*R*c or b*S*c. Let **KBR** be a finite non-empty set such that:
(1) If ρ:= (for non-negative integer g, c*M*b only if cB^gb) then ρ ∈ **KBR;**
(2) If ρ:= (for non-negative integer g, c*M*b only if ¬(cB^gb)) then ρ ∈ **KBR;**
(3) if ρ, σ∈**KBR** then ρ∧σ∈**KBR**;
(4) if ρ, σ∈**KBR** then ρ∨σ∈**KBR**;
(5) nothing else is a member of **KBR;**
then if ρ∈**KBR** then ρ is a kin based marriage rule.

Comment 20: **KBR** uses a simplified version of the notation of [G&O]. Members of **KBR** are rules, since they define when an *M*-relation can form and do not violate the applicable four axioms. The operations ∧ and ∨ are each idempotent, commutative and associative, and the absorption laws also hold. Therefore if we construct only rules consisting of statements of type

(1), (2) and (3) then we have a meet semilattice which we describe as $\mathbf{KBR}_\wedge$. If we allow only statements of the form (1), (2) and (4) then we have a join semilattice which we denote as $\mathbf{KBR}_\vee$. And therefore (**KBR**, $\wedge$, $\vee$, $\neg$, 0) is a lattice.

Comment 21: We give examples of rules in **KBR**.

(1) Let α_0 := (c*M*b only if cB^0b). Then c*M*b only if c = b. Since *D* is onto the sets M (see [2]) a set $\mathbb{P}$ subject to α_0 is a set of "self-mating" or "self-reproducing" organisms, such as single cell organisms. $\mathbf{M}_{\alpha 0}$ = M0.

(2) Let α_1 := (c*M*b only if $\neg$(cB^0b)). Then c*M*b only if c $\neq$ b, so a set $\mathbb{P}$ subject to α_0 is a set of "bisexual" organisms in the sense that descent can only be assigned to a "marriage" of two distinct individuals. $\mathbf{M}_{\alpha 1}$ = M1.

(3) Let α_2 := $\alpha_1 \wedge$ (c*M*b only if cB^1b). If c*M*b then b and c are required to have "committed incest", since cB^1b. $\mathbf{M}_{\alpha 2}$ = M1 as well.

(4) Let α_3 := $\alpha_1 \wedge$ (c*M*b only if $\neg$(cB^1b). α_3 requires to "avoid incest". $\mathbf{M}_{\alpha 3}$ = M2.

(5) Let α_4 := $\alpha_1 \wedge \alpha_3 \wedge$ (c*M*b only if cB^2b). α_4 is marriage to a "first cousin". $\mathbf{M}_{\alpha 4}$ = M2.

(6) Let α_5 := $\alpha_1 \wedge \alpha_3 \wedge$ (c*M*b only if $\neg$(cB^2b)). α_5 prohibits marriage among second degree or closer (lower degree) relatives, often called "first cousin prohibition". $\mathbf{M}_{\alpha 5}$ = 2(M2). Only 2(M2) is a fixed point under α_5 on a descent sequence whose generation size is always 2s. However α_5 also allows 2(M2) to create both 2(M2) and M4, but allows M4 only to create 2(M2). So we can denote a history $\kappa = \alpha_5\alpha_5 = \alpha_5^2$, in which κ is an automorphism on $\mathbb{C}$ with fixed points {2(M2), M4}) that occur in a two generation cycle of application of α_5.

(7) Let α_6 := $\alpha_1 \wedge \alpha_3 \wedge \alpha_5 \wedge$ (c*M*b only if cB^3b). α_6 prohibits marriage among second degree or "closer" relatives, but requires marriage with a third degree relative, or "second cousin". $\mathbf{M}_{\alpha 6}$ = 2(M2) as well.

Definition 14: (1) let $\mathbb{C}$ be a set of configurations, let C,D$\in\mathbb{C}$, let C be the configuration defined on a generation $\mathbb{G}$, and let b,c$\in\mathbb{G}$. Then $h : \mathbb{C} \rightarrow \mathbb{C}$ is a homomorphism (isomorphism when one-one and onto) of configurations iff both h(b*B*c)=h(b)*B*h(c) and h(b*M*c)=h(b)*M*h(c). (2) let *X* and *Y* be non-empty sets, let x,y$\in X$, and let f: $X \rightarrow Y$ be a function. Then ker f : = {(x,y) $\in X^2$: f(x) = f(y)}. If *S* and *T* are algebras and f: $S\rightarrow T$ is a homomorphism, then ker f is a congruence.

Comment 22: Let $\mathbb{C}$ be a finite non-empty set of configurations, let **KBR** be a set as defined in Definition F, and define *Min* : **KBR** $\rightarrow \mathbb{C}$ as *Min* (α) = { C : C$\in\mathbb{C}$, α $\in$**KBR**, C is a minimal structure of α } so *Min* (α) = $\mathbf{M}_\alpha$. That is, if $A \subseteq$ **KBR** is a non-empty set of rules then ker *Min* identifies the equivalence classes of rules in *A* that share the same sets of minimal structures. For example from Comment 21, ker *Min* maps { α_5, α_6 } $\rightarrow$ 2(M2), { α_3, α_4 } $\rightarrow$ M2, { α_1, α_2 } $\rightarrow$ M1, { α_0 } $\rightarrow$ M0.

Comment 23: Following Comment 22, we can extend a congruence "back" to sets of kinship terminologies (that is, to complete terminologies of a natural language; that is, to quasigroup subset of a peq) which map to the sets of rules. Let **Kin** be a set of quasi-groups, each of which generates a kinship terminology. That is, let **Kin** be a set of complete terminologies. Let ***G*** be a set of finite permutations. Let g : **Kin** $\rightarrow$ ***G***. Then ker g(**Kin**) classifies kinship algebras (quasigroups) by permutations, if any, to which they are homomorphic. If we let f : **KBR** $\rightarrow$ (ker g(**Kin**)) then ker f(**MBR**) identifies rules in **KBR** to kinship terminologies, also according to the permutations. When the permutations are also groups [G1, G2] then we have also applied [N, Theorem 4.3] "Every lattice can be embedded into the lattice of subgroups of a group". In the notation of Comment 17, we have found a set ker f(**MBR**) = $g(\mathcal{P}(D)) \subseteq \mathcal{P}(D)$ of complete terminologies (quasigroups) that also form groups. Thus we have a congruence between marriage rules and permutations, and a classification of marriage rules and kinship terminologies of similar orders. Isotopic quasigroups (which have the same "structure" but use distinct sets), hence isomorphic kinship terminologies expressed using languages with different systems of representation whether notational or phonological, will be similarly classified by this procedure. It is natural to think of the existence of mappings which have non-empty kernels as identifying <u>internally consistent cultures</u>.[2] We know at least that for viable rules, such kernels are non-empty, therefore consistent cultures exist. In fact the entire topic of "Dravidian kinship terminologies" is itself an example of a set of classes of isotopic quasigroups. The examples are isotopic since the subject of Dravidian terminologies discusses languages which might have very different phonologies and other properties, yet share similar quasigroup or peq structures as kinship algebras. [A] is a very good example of an ethnographic analysis of Dravidian which uses the minimal structure of a pure system of a rule, illustrating the use of the identity transform on a minimal structure, and a mapping of a kinship terminology onto a descent sequence of that pure system, showing permutations of the kinship labels onto successive presentations of the minimal structure.

Comment 24: Let α be a viable rule with minimal structure C $\in$ $\mathbf{M}_\alpha$ and structural number s. Let G be a grammatical structure which is also a kinship terminology (a complete terminology of a natural language) placed in 1-1 correspondence with the individuals in C, called a <u>labeling</u> of that configuration. Then | G | = 2s terms since C has 2s individuals. Thus if we label the minimal structure of a viable rule with the kin terms of a quasigroup of order 2s, then each permutation of the terms defines a permutation of the minimal structure, such as by one of the symmetries of a dihedral group [G1, G2]. If we consider isomorphic configurations with different orientations of labelings as distinct, then in a pure system, the orbits are among the differently labeled copies of the minimal structure. Ethnography very often uses these properties to draw illustrations of the use of kin terms and application of marriage rules. A very good example of this is provided by [R2] whose Figure 9, though only illustrating one generation, is exactly an example of rule α_4 on its M2 minimal structure; this this figure thus also illustrates that the particular kinship terminology in association with that rule and that configuration form an internally consistent culture. Examples of such illustrations, used for exactly such purposes, are pervasive in ethnography. While not all cultures can be represented by configurations that assume discrete generations quite evidently a very large class of cultures do so.

[2] [WD] would apparently prefer a weaker concept than homomorphism here; simply a many-one map.

Comment 25: Following Comment 24, let *D* be a finite non-empty dictionary and let L_D be a natural language on *D*. Let G be a set of kin terms assigned to each relation form $cB^g b$, let *G* be the support of G so that $G \subseteq D$, and let $M \in D$ be an entry of *D* that expresses when the relation *M* holds. Then the forms of Definition 13 are frames, indeed are compound frames. If we require that each of the forms of statement used in Definition 13(1) through 13(4) must end with a terminal marker such as ".", as is done in the examples of Comment 21, then each defines a frame which is also a sentence. Thus modified, Definition 13 defines a language on the set $\{ G \cup M \} \subseteq D$ since it identifies a set of sentences on that set, and also { G ⋃ **KBR** } is a grammar on $\{ G \cup M \}$. We further invite comparison of the concept of grammatical structure, from Definition 4, to the description by [B] of classes of "contracted words", which therefore appear to be examples of grammatical structures. Thus we can formalize a commonly if not universally held intuition that study of kinship terminologies is a sub-topic of linguistics, which if one adds a terminal marker to his notation, appears to be how [B] views his subject.

Comment 26: A commonly found definition of a quasigroup is to write the unique solutions to Definition 1 equations (1) and (2) as $x = a \backslash b$ and $y = b / a$. Here, '\' and '/' denote, respectively, binary operations of left and right division which are taken as primitive. Then:

> *Definition 1A*: A <u>quasigroup</u> $(D, *, \backslash, /)$ is a type (2, 2, 2) algebra satisfying the identities: $y = x * (x \backslash y)$; $y = x \backslash (x * y)$; $y = (y / x) * x$; $y = (y * x) / x$.

If $(D, *)$ is a quasigroup according to Definition 1, then $(D, *, \backslash, /)$ is an equivalent quasigroup under Definition 1A [S]. If the operations in the first form are partially defined, then the second form is also only partially defined. Note that if in this second definition we take as the elements *x* and *y* some particular pair composed of a marker *m* and a root *r*, then the second definition identifies unique "divisions" and thus shows how to "add or remove the markers". That is, the logic of "divisions" in Definition 1A seems to be a model for how field linguists go about "decoding" an observed language, in order to then construct a grammar, or structured terminology, using definitions of the first sort.

CONCLUSION:

We have defined a mathematical object called here a "partially defined expanded quasigroup", to describe natural languages, which may have as subsets objects with the known properties of kinship algebras. While the topic often called "mathematical linguistics" uses semigroups, hence does not describe natural languages, peq's are nonassociative. As such not only may they describe natural languages, under certain conditions they have properties that define a new class of mathematical languages. A common technique of social and cultural anthropology is to describe a culture by first describing the language of the bearers of that culture. Use of peq's to describe natural languages leads in a direct way to recognizing kinship terminologies as subsets of natural languages, and in turn to characterizing relationships among such terminologies and marriage rules. The Dravidian classes of isotopic quasigroups which, are also compete terminologies and grammatical structures of natural languages, provide good illustrations.

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