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THE PROBLEM OF THE RELATION BETWEEN DOUBLE POMERON EXCHANGE IN INCLUSIVE AND EXCLUSIVE EXPERIMENTS

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Abstract : The pionization region of the inclusive single-particle spectrum is accounted for by double pomeron exchange in the absorptive part of a six-point amplitude. In this paper a multiperipheral model for the six-point amplitude with double pomeron exchange is used for continuation by crossing and analyticity to the physical region of the exclusive two particle → four particle production process. The cross section for $\pi^- p \to \pi^-(\pi^+\pi^-)p$ in the double-Regge region is then calculated and compared with the experimental analysis of Lipes, Zweig and Robertson which sets an upper bound to the strength of the double pomeron exchange coupling. This upper bound, coupled with the model for continuation to the inclusive cross section, is shown to give too small a magnitude for the double pomeron exchange in the pionization region. Further avenues for investigation are discussed.

1. Introduction

One of the fundamental questions about the nature of the pomeron singularity is the strength of the coupling of two pomerons to a particle or particles. It is our purpose to investigate how current models relate the strength of this coupling in two of its appearances. One occurence is in the analysis of the central plateau or pionization region [1–3] of the inclusive pion single-particle spectrum $a + b \to c + X$ where the observed pion has c.m. momentum $|q_{\parallel}| < O(s^{\frac{1}{2}})$. The optical-theorem approach of Mueller [4] succintly describes the single-particle spectrum as an absorptive part in missing mass $M^2 = (p_a + p_b - q)^2$ of the forward three-particle → three-particle amplitudes in the $a + b + \bar{c}$ channel. Pionization occurs at high energy by

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the exchange of two pomerons to two pions, fig. 1. This coupling and the existence of the pionization region was indicated by the $\bar{n} \propto \ln s$ growth of the average multiplicity [5] and has been confirmed directly by the recent ISR experiments [6]. In a sense to be detailed in this paper, the coupling of two pomerons here is "strong."

Another occurence of double pomeron exchange is in the exclusive two-particle → four particle production process $\pi^- p \to \pi^-(\pi^+\pi^-)p$, fig. 2, where the $(\pi^+\pi^-)$ pair is at low invariant mass and the subenergies s_{12} and s_{34} are large. This experiment was analyzed exclusively in the double-Regge region by Lipes, Zweig and Robertson [7] (LZR) and no evidence for double pomeron exchange was found.

The three-particle → three-particle amplitude and the two- particle → four-particle amplitude are related by crossing to be analytic continuations of the same six-point function into the different physical regions. The program is to take current models for the six-point amplitude and continue them between the two-particle → four-particle production region and the forward three-particle → three-particle region where we take the $M^2 = (p_a + p_b - p_c)^2$ discontinuity to obtain the pionization spectrum. Since both regions have large subenergies in which double pomeron exchange occurs, we will be able to relate the strengths of the double pomeron coupling in the two regions.

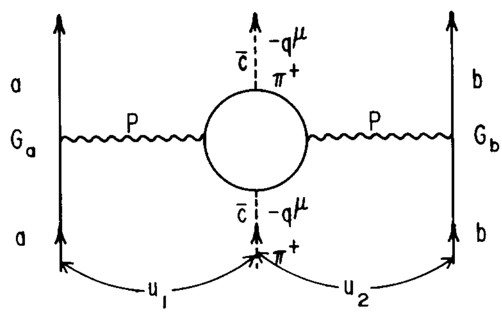

Fig. 1. Double pomeron exchange in the three-particle → three-particle amplitude. The discontinuity in the $ab\bar{c}$ channel gives the pionization spectrum for a + b → c + X.

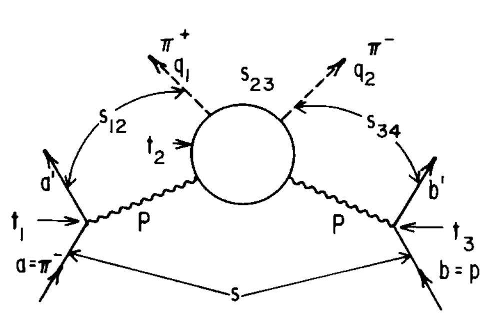

Fig. 2. 2–4 exclusive reaction a + b → a' + c + $\bar{c}$ + b' *via* double pomeron exchange.

In this paper we use a multiperipheral model for the six-point amplitude which has a dynamical structure in terms of intermediate states in the M^2 channel. The model is an analytic continuation [8] of a model for the single-particle spectrum with Regge behavior and exponential damping in momentum transfer previously studied by Silverman and Tan [9] and by Caneschi and Pignotti [10]. The model incorporates double Regge behavior for the exchanged pomerons and possesses an analytic behavior for double Regge coupling which is consistent with the Steinmann relations [11–13]. The amplitude can be explicitly continued between the production and pionization regions. The strength of the coupling in the pionization region is found by taking the absorptive part in M^2 and fitting to the experimental results on pionization.

The continued amplitude is then used to calculate the cross section for the production experiment and the results are compared with the analysis of LZR. We find that the coupling strength from pionization is much too large for the production region. By adjusting the coupling to be consistent with the upper bound on double pomeron exchange set by the LZR analysis, we find that this coupling strength is only about $\frac{1}{70}$ of that observed in pionization.

In a subsequent paper [14] we will again carry out this continuation and comparison with experiment using a dual resonance model for the six-point amplitude with the pomeron exchanges included in a phenomenologically consistent way.

For comparison with the calculations with the continued amplitudes, we also calculate the production cross section using a simple pion-pole exchange in the production amplitude with the strength determined by factorization. The two-particle $\rightarrow$ four-particle production cross section from the pion pole is below that obtained by LZR.

In sect. 2 we formulate the general six-point amplitude with double pomeron exchange and discuss the method of analytic continuation between the production and pionization region. In sect. 3 we present the multiperipheral model with exponential damping in momentum transfer and use it for the analytic continuation. In sect. 4 the absorptive part of the amplitude in M^2 is related to the single-particle spectrum in the pionization region using the Mueller optical theorem. The strength of the absorptive part is found by comparison with the pionization data. In sect. 5 we present the method of performing the phase space integration to compute the $\pi^- p \rightarrow \pi^-(\pi^+\pi^-)p$ cross section from the continued amplitude. In sect. 6 we present the calculated results for the production cross section. We find that a much smaller value for double pomeron coupling than that found in pionization is demanded by the non-observation of double pomeron exchange in the production experiment. Possible sources for this inconsistency are discussed as well as its relation to the proofs of pomeron decouplings.

2. The six-point amplitude with double pomeron exchange

The production process, fig. 2, is a function of eight independent invariants which we choose as

$$s = (p_a + p_b)^2\,, \qquad s_{12} = (p_a' + q_1)^2\,, \qquad s_{34} = (q_2 + p_b')^2\,, \qquad s_{23} = (q_1 + q_2)^2$$

$$t_1 = (p_a' - p_a)^2\,, \qquad t_2 = (p_a' + q_1 - p_a)^2\,, \qquad t_3 = (p_b' - p_b)^2\,, \tag{2.1}$$

$$\eta = -\,s_{12}\,s_{34}/M^2\,,$$

where

$$M^2 = (p_a + p_b - q_2)^2\,.$$

The analytic continuation to the pionization region consists of taking

$$p_a' = p_a\,, \qquad p_b' = p_b\,, \qquad q_1 = -q\,, \qquad q_2 = q\,, \tag{2.2}$$

where q is the momentum of the observed pion in the single-particle spectrum. The variables describing the subenergies in pionization are chosen to be

$$u_1 = (p_a - q)^2\,, \qquad u_2 = (p_b - q)^2\,,$$

which are related to $M^2 = (p_a + p_b - q)^2$ and s in the pionization region by

$$M^2 = s + u_1 + u_2 - m_a^2 - m_b^2 - m^2\,,$$

where m is the pion mass. For the continuation to pionization for large s and large subenergies $u_1 \sim -\,\mathrm{O}(s^{\Delta y})$, $u_2 \sim \mathrm{O}(s^{1-\Delta y})$ with $0 < \Delta y < 1$, the invariants are, to leading order in powers of s,

$$s_{12} \simeq u_1\,, \qquad s_{34} \simeq -u_2\,, \qquad s_{23} = 0\,, \qquad M^2 \simeq s\,,$$

$$t_1 = 0\,, \qquad t_3 = 0\,, \qquad t_2 = m^2\,, \tag{2.3}$$

$$\eta \simeq u_1 u_2/M^2 = q_\perp^2 + m^2\,. \tag{2.4}$$

In constructing our model for the six-point amplitude we are concerned in this study mainly with the part which contains the discontinuity of T in M^2. Additional parts which we have, for example, a pion pole in t_2 are much smaller than the part with the discontinuity in M^2 and their effects will be discussed in sect 6.

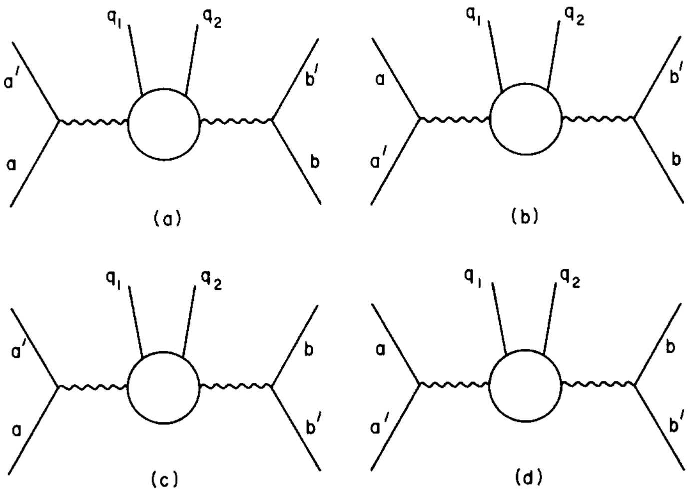

Fig. 3. The four diagrams which produce the signature structure of the six-point amplitude. The complete amplitude also includes the diagrams with q_1 and q_2 interchanged.

The models that we will use are put in the form of a double pomeron exchange amplitude with a function that gives the M^2 discontinuity observed in the pionization region. The double Regge form for T from the diagram, fig. 3a, is

$$T_{ab} = G_a \beta_{a\pi}(t_1)\, G_\pi \frac{\Gamma(-\alpha_1+1)}{\pi} (-s_{12}/s_0)^{\alpha_1}$$

$$\times\, G_b \beta_{b\pi}(t_3)\, G_\pi \frac{\Gamma(-\alpha_3+1)}{\pi} (-s_{34}/s_0)^{\alpha_1} (g_P/s_0) F(\eta)\,, \tag{2.5}$$

where

$$\eta = -s_{12}s_{34}/M^2\,; \tag{2.6}$$

$\alpha_1 = \alpha_P(t_1)$ and $\alpha_3 = \alpha_P(t_3)$ are pomeron trajectories; G_a, G_b and G_π are the zero-momentum transfer couplings of the pomeron to a, b and π; and $\beta_{a\pi}(t_1)$, $\beta_{b\pi}(t_3)$ are the Regge residue damping factors normalised to one at zero momentum transfer. The explicit occurence of G_π is for convenience in comparing the coupling strengths and does not mean to imply that the pomerons (P) in PP $\to \pi\pi$ couple to on-shell pions. g_P is the intrinsic strength of the double pomeron coupling and will be determined in sect. 4. The PP $\to \pi\pi$ coupling function $F(\eta)$ may also depend on s_{23}, t_2, t_1 and t_3 but we will suppress this in the notation.

In an amplitude with double Regge exchange, the M^2 discontinuity occurs through the discontinuity in the variable η [8, 12, 15–17] which involves M^2 (2.6). In the models we will study the function $F(\eta)$ will have a cut for $\eta \geqslant 0$ which occurs in the pionization region where $\eta = q_\perp^2 + m^2$. η is negative in the production region, eq. (2.6).

The signature factors for the Regge exchanges occur by adding the three diagrams with interchanges of $(p_a \Leftrightarrow - p_a')$, $(p_b \Leftrightarrow - p_b')$ and $(p_a \Leftrightarrow - p_a', p_b \Leftrightarrow - p_b')$. These are each computed from the same function T_{ab}, eq. (2.5), but with the appropriate subenergy variables resulting from interchanging the momenta in the definitions of s_{12}, s_{34} and M^2, eq. (2.1): for $p_a \Leftrightarrow - p_a'$,

$$s_{12} = (p_a' + q_1)^2 \to (-p_a + q_1)^2 \simeq -s_{12} , \tag{2.7}$$

and for $p_b \Leftrightarrow - p_b'$,

$$s_{34} = (p_b' + q_2)^2 \to (-p_b + q_2)^2 \simeq s_{34} .$$

The interchanges in M^2, however, produce invariants for different channels :

$$\text{fig. 3a } (T_{ab}): \quad M_{ab}^2 = (p_a + p_b - q_2)^2 = M^2 \simeq s , \qquad \eta_{ab} = -\frac{s_{12}s_{34}}{M_{ab}^2} = \eta ,$$

$$\text{fig. 3b } (T_{a'b}): \quad M_{a'b}^2 = (-p_a' + p_b - q_2)^2 \simeq -s , \qquad \eta_{a'b} \simeq \frac{s_{12}s_{24}}{M_{a'b}^2} ,$$

$$\text{fig. 3c } (T_{ab'}): \quad M_{ab'}^2 = (p_a - p_b' - q_2)^2 \simeq -s , \qquad \eta_{ab'} \simeq \frac{s_{12}s_{34}}{M_{ab'}^2} ,$$

$$\text{fig. 3d } (T_{a'b'}): \quad M_{a'b'}^2 = (-p_a' - p_b' - q_2)^2 \simeq s , \qquad \eta_{a'b'} \simeq -\frac{s_{12}s_{34}}{M_{a'b'}^2} . \tag{2.8}$$

Adding these four diagrams gives the signature factor in the production region, since all of the four η's are approximately numerically equal. Since our choice of η is not symmetric in q_1 and q_2, we must add the same terms with q_1 and q_2 interchanged. The complete six-point amplitude in the production region is then

$$\begin{aligned} T = {} & G_a \beta_{a\pi} G_\pi \frac{(e^{-i\pi\alpha_1} + 1)}{\pi} \Gamma(-\alpha_1 + 1) (s_{12}/s_0)^{\alpha_1} \\ & \times G_b \beta_{b\pi} G_\pi \frac{(e^{-i\pi\alpha_3} + 1)}{\pi} \Gamma(-\alpha_3 + 1) (s_{34}/s_0)^{\alpha_3} \\ & \times \frac{g_P}{s_0} F(\eta) + (q_1 \Leftrightarrow q_2) . \end{aligned} \tag{2.9}$$

Since η is negative in the production region, it is not necessary to distinguish if it is above or below the η cut in the various terms.

In taking the discontinuity in the channel $M^2 = M^2_{\mathrm{ab}} = (p_{\mathrm{a}} + p_{\mathrm{b}} - q_2)^2$ for pionization, only the first of the four terms, T_{ab}, contributes since the other η's do not contain M^2, but variables which refer to other channels. In the four additional terms obtained from $q_1 \Leftrightarrow q_2$, however, the term arising from $T_{\mathrm{a'b'}}$, call it $\widetilde{T}_{\mathrm{a'b'}}$, contains $\widetilde{M}^2_{\mathrm{a'b'}} = (-p'_{\mathrm{a}} - p'_{\mathrm{b}} - q_1)^2$ from exchanging $q_2 \Leftrightarrow q_1$ in $M^2_{\mathrm{a'b'}}$. Now from four-momentum conservation

$$\widetilde{M}^2_{\mathrm{a'b'}} = (p_{\mathrm{a}} + p_{\mathrm{b}} - q_2)^2 = M^2 \;, \tag{2.10}$$

so this term gives an M^2 discontinuity identical to the first term T_{ab} in the pionization region.

3. The multiperipheral model with exponential damping in momentum transfer

The model we take for the functional form of $F(\eta)$ arises from an analytic continuation [8] of the Caneschi-Pignotti [10] model for the single-particle spectrum which was studied analytically by Silverman and Tan [9]. It views production of a particle in the central region by the peripheral form, fig. 4, where any number of particles may be emitted to the left or right of the observed particle. It is assumed that the sums of these inclusive particles behave like total cross sections which are Regge behaved and pomeron dominated at large s_{l}, s_{r} (fig. 4). In order to obtain exponential damping in $q_\perp^2$, exponential damping is assumed in the momentum transfers t_{l} and t_{r}. Using the described form for the amplitude in fig. 4, the single particle spectrum is given by

$$\frac{\mathrm{d}\sigma}{\left(\frac{\mathrm{d}^3 q}{E}\right)} = \frac{c}{s} \int \mathrm{d}s_{\mathrm{l}}\, \mathrm{d}s_{\mathrm{r}}\, \mathrm{d}t_{\mathrm{l}}\, \mathrm{d}t_{\mathrm{r}}\, J \,|\, s_{\mathrm{l}}^{\alpha_1} s_{\mathrm{r}}^{\alpha_3}\, \mathrm{e}^{\Omega_{\mathrm{l}} t_{\mathrm{l}} + \Omega_{\mathrm{r}} t_{\mathrm{r}}} \,|^2 \;, \tag{3.1}$$

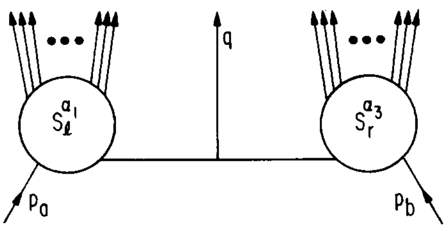

Fig. 4. Diagram for peripheral production of a particle in the central region.

where J is the Jacobian of the phase space integrals and c is a constant. This has been computed for $\alpha_1 = \alpha_3 = 1$ in ref. [9], and for arbitrary α_1, α_3 in the three-particle → three-particle scattering region in ref. [8]. In the pionization region the result is a function only of $K = k\eta$ where $k = 2\Omega_1\Omega_r/(\Omega_1 + \Omega_r)$. The result is also the absorptive part in K of the six-point amplitude in the pionization region. It may be extended to an anlytic function in K in a way consistent with the Steinmann relations (discussed below). We use the result of the continuation in K (ref. [8]):

$$F(K) = (-K)^{-\alpha_1}\, U(-\alpha_1, -\alpha_1 + \alpha_3 + 1, -K)\,, \tag{3.2}$$

where U is the confluent hypergeometric function [18] (also called Ψ [19]), as the basic model for the K continuation in this paper. It should be noted that the entire amplitude is not analytically continued to the production region, but only the K dependence is abstracted from the forward three-particle → three-particle amplitude. If the entire amplitude were continued to the production region some of the internal momentum transfers can become positive, and the increasing exponential dependence would be a severe overestimate.

It is seen that $F(K)$ has a cut for $K \geqslant 0$ which gives the absorptive part in the pionization region. The same function arises in the six-point function in the dual resonance model [14, 15] where K is replaced by $x = \alpha'\eta/z(1-z)$ in the pionization region and an integration over z is performed. This significantly alters the continuation and will be discussed in the paper on the dual resonance model [14]. The analytic structure is directly expressed [18] in terms of the entire function M (also called Φ [19]):

$$F(K) = \frac{\pi}{\sin \pi(-\alpha_1 + \alpha_3 + 1)} \left\{ \frac{(-K)^{-\alpha_1} M(-\alpha_1, -\alpha_1 + \alpha_3 + 1, -K)}{\Gamma(-\alpha_3)\,\Gamma(-\alpha_1 + \alpha_3 + 1)} \right.$$

$$\left. - (-K)^{-\alpha_3} \frac{M(-\alpha_3, -\alpha_3 + \alpha_1 + 1, -K)}{\Gamma(-\alpha_1)\,\Gamma(-\alpha_3 + \alpha_1 + 1)} \right\}. \tag{3.3}$$

The Steinmann relations state that the double discontinuity of two overlapping variables must vanish in the physical region. Considering the three-particle → three-particle scattering in the physical region of forward $a + b + \bar{c} \to a + b + \bar{c}$ we have $u_1 > 0, u_2 > 0$ and the double Regge form

$$(-u_1)^{\alpha_1}\, (-u_2)^{\alpha_3}\, F(K) \tag{3.4}$$

from one of the terms in (2.9). Since u_1 and u_2 are overlapping variables, the Steinmann relations state that we cannot have simultaneous singularities in these. So the form of $F(K)$ contains $(-K)^{-\alpha_1}$ or $(-K)^{-\alpha_3}$ which gives respectively

$$(-u_1)^{\alpha_1}(-u_2)^{\alpha_3}(-K)^{-\alpha_1} \propto (-u_2)^{\alpha_3-\alpha_1}(-M^2)^{\alpha_1}\,,$$

$$(-u_1)^{\alpha_1}(-u_2)^{\alpha_3}(-K)^{-\alpha_3} \propto (-u_1)^{\alpha_1-\alpha_3}(-M^2)^{\alpha_3}\,. \tag{3.5}$$

The $1/\Gamma(-\alpha_3)$ in the coefficient of $(-K)^{-\alpha_1}$ prevents simultaneous poles in the amplitude in α_3 and u_2 since they are overlapping variables, and similarly for the $1/\Gamma(-\alpha_1)$ term in the coefficient of $(-K)^{-\alpha_3}$. The function $F(K)$ does not in fact contain poles at $\alpha_1 = \alpha_3$.

We now perform the continuation to the pionization region eqs. (2.2) and (2.3). We recall that the M^2 or $\eta = \eta_{ab}$ discontinuity is only present in the term T_{ab} in (2.8) and the term $\widetilde{T}_{a'b'}$ with $q_1 \Leftrightarrow q_2$. Then in the pionization region we have

$$\begin{aligned}\mathrm{Im}_{M^2}\, T &= \mathrm{Im}_{M^2}\,(T_{ab} + \widetilde{T}_{a'b'})\\ &= \frac{2}{\pi^2}\, G_a G_b G_\pi^2 \alpha_1 \alpha_3 (-u_1/s_0)^{\alpha_1} (u_2/s_0)^{\alpha_3}\\ &\times \frac{g_P}{s_0}\, \Gamma(-\alpha_1)\,\Gamma(-\alpha_3)\, \mathrm{Im}_K F(K)\,.\end{aligned} \tag{3.6}$$

In taking the discontinuity of T_{ab} it is important to keep α_1 and α_3 not equal to 1. T_{ab} has a pole at these points and since M^2 is an overlapping variable, T_{ab} could not have an M^2 discontinuity in a residue of these poles. It is perhaps simplest for analytic purposes to consider the pomeron intercept as slightly less than one, and in the later numerical evaluations take $\alpha_1 = 1$ and $\alpha_3 = 1$ as a good numerical approximation.

Evaluating $\mathrm{Im}_\eta\, F(K)$ is simplest from the form (3.3) where the discontinuity arises from the terms $(-K)^{-\alpha_1}$ and $(-K)^{-\alpha_3}$. After using the Kummer transformation [18]

$$M(a, b, z) = \mathrm{e}^z M(b-a, b, -z)\,, \tag{3.7}$$

and rearranging gamma functions we can recombine terms to obtain for $K \geqslant 0$

$$\mathrm{Im}_K\, \Gamma(-\alpha_1)\,\Gamma(-\alpha_3)\, F(K) = -\pi\, \mathrm{e}^{-K} K^{-\alpha_1}\, U(\alpha_3+1, -\alpha_1+\alpha_3+1, K)\,. \tag{3.8}$$

The continuation can also be performed and the absorptive part taken when $\alpha_1 = \alpha_3$ as occurs at $t_1 = t_3 = 0$, by using the appropriate form of the hypergeometric function for this case [18, 19]. The results are the same as obtained from (3.8) by setting $\alpha_1 = \alpha_3$.

Finally, after taking the discontinuity we set $\alpha_1 = 1, \alpha_3 = 1$ at $t_1 = 0, t_3 = 0$. From (3.8)

$$\mathrm{Im}_K\, \Gamma(-\alpha_1)\,\Gamma(-\alpha_3)\,F(K) \underset{\substack{\alpha_1\to 1\\ \alpha_3\to 1}}{\to} -\pi\,\frac{\mathrm{e}^{-K}}{K}\,U(2,1,K)\,. \tag{3.9}$$

We can express this in terms of simpler functions by using eq. (13.4.23) of ref. [18]:

$$U(2,1,K) = U(1,1,K) + K\,\frac{\mathrm{d}}{\mathrm{d}K}\,U(1,1,K)\,, \tag{3.10}$$

and also eq. (13.6.30)

$$U(1,1,K) = \mathrm{e}^K\,E_1(K)\,, \tag{3.11}$$

where E_1 is the exponential integral. We find then

$$\mathrm{Im}_K\, \Gamma(-\alpha_1)\,\Gamma(-\alpha_3)\,F(K) \underset{\substack{\alpha_1\to 1\\ \alpha_1\to 1}}{\to} -\frac{\pi}{K}\,[(1+K)E_1(K) - \mathrm{e}^{-K}]\,. \tag{3.12}$$

Finally, combining with (3.6) and using $K = ku_1u_2/M^2$ we find for the M^2 absorptive part in the pionization region

$$\mathrm{Im}_{M^2} = \frac{2M^2 G_\mathrm{a} G_\mathrm{b} G_\pi^2}{\pi s_0^2\, k}\left(\frac{g_\mathrm{P}}{s_0}\right)\,[(1+K)\,E_1(K) - \mathrm{e}^{-K}]\,. \tag{3.13}$$

This form for the pionization spectrum appears in the previous calculations of the exponentially damped multiperipheral model with $\alpha_1 = 1, \alpha_3 = 1$ [9].

To complete the specification of our amplitude in the production region, we recall that the general pomeron-pomeron to $\pi\pi$ amplitude F in (2.5) can also have additional dependence on s_{23}, t_2, t_1 and t_3. Lacking further dynamical knowledge, we assume the smoothest behavior possible. First, we assume that the t_1 and t_3 dependence has been largely removed in the elastic scattering residue factors $\beta_{\mathrm{a}\pi}(t_1)$, $\beta_{\mathrm{b}\pi}(t_3)$ which will be taken to be

$$\beta_{\mathrm{a}\pi}(t) = \beta_{\mathrm{b}\pi}(t) = \mathrm{e}^{\Omega t}\,, \tag{3.14}$$

with $\Omega = 5\ \mathrm{GeV}^{-2}$ as given by elastic scattering data. This magnitude of damping is consistent with that observed in the production experiment for a single pomeron exchange [7]. Second, we assume no explicit dependence in s_{23} in continuing from pionization, $s_{23} = 0$, to production, $s_{23} \geqslant 4m^2$. Resonance structure would only enhance the production amplitude, which is not needed. In the dual resonance model [14] we include the effects of resonances in s_{23}. Finally, since all hadron ampli-

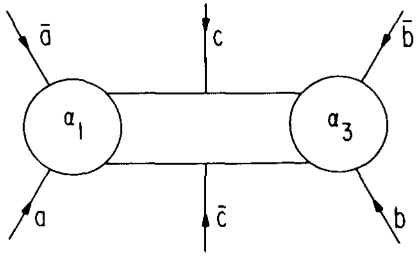

Fig. 5. The exponentially damped multiperipheral model as a six-point amplitude.

tudes appear to be damped in momentum transfer, we will estimate the effects of possible damping in the variable t_2 by including in the amplitude the form

$$e^{a(t_2-m)^2} \,. \tag{3.15}$$

This does not affect pionization where $t_2 = m^2$.

In this paper we have abstracted only the K dependence in the pionization region from the multiperipheral model with exponential damping. We have also considered the subenergy dependence of the model in the three-particle → three-particle non-forward region as indicated in fig. 5. The diagram is explicitly symmetric under ($a \Leftrightarrow \bar{a}$), ($b \Leftrightarrow \bar{b}$) and ($c \Leftrightarrow \bar{c}$), and the result of the calculation is the same function $F(K)$ used in (3.2) but with K replaced by the symmetric combination

$$K' = \frac{k(s_{a\bar{c}} + s_{\bar{a}c})(s_{b\bar{c}} + s_{\bar{b}c})}{4M^2_{ab\bar{c}}} \,. \tag{3.16}$$

Also the Regge powers are now

$$\left(\frac{s_{a\bar{c}} + s_{\bar{a}c}}{2}\right)^{\alpha_1} \left(\frac{s_{b\bar{c}} + s_{\bar{b}c}}{2}\right)^{\alpha_3} \,. \tag{3.17}$$

With this definition for K' and the Regge powers we need to keep only the four signature generating interchanges of $a \Leftrightarrow \bar{a}$ and $b \Leftrightarrow \bar{b}$ since the sum is then already symmetric under $c \Leftrightarrow \bar{c}$. The amplitude in the pionization region is identical to (3.13). We also computed the cross section in the production region with these variables. The results did not differ significantly from those obtained from (2.9) with $K = k\eta$.

4. Comparison of the absorptive part with pionization

We now relate $\mathrm{Im}_{M^2} T$ to the single particle inclusive cross section and evaluate g_P and k by comparison with the pionization experiment. We use covariant normalization in which

$$\langle \boldsymbol{P}' | \boldsymbol{P} \rangle = (2\pi)^3\, 2E\, \delta^3(\boldsymbol{P} - \boldsymbol{P}') \,, \tag{4.1}$$

and the density of states is $\mathrm{d}^3P/2E(2\pi)^3$. The cross section is given by

$$\mathrm{d}\sigma = \frac{(2\pi)^4}{v_{12} \prod_{\mathrm{in}} (2E)} \prod_{\mathrm{out}} \frac{\mathrm{d}^4 p}{(2\pi)^3}\, \delta^+(p^2 - m^2)\, \delta^4 \Big(\sum_{\mathrm{in}} p - \sum_{\mathrm{out}} p\Big) |T_{\mathrm{if}}|^2 \,, \tag{4.2}$$

where at high energy

$$v_{12} \prod_{\mathrm{in}} (2E) = 2s \,.$$

The S-matrix is

$$S_{\mathrm{fi}} = \delta_{\mathrm{fi}} + i(2\pi)^4\, \delta^4(p_{\mathrm{i}} - p_{\mathrm{f}}) T_{\mathrm{if}} \,, \tag{4.3}$$

and unitarity for a forward amplitude is

$$\mathrm{Im}\, T_{\mathrm{ii}} = \frac{(2\pi)^4}{2} \sum_n \int \prod_{k=1}^{n} \frac{\mathrm{d}^4 p_k}{(2\pi)^3}\, \delta^+(p_k^2 - m^2)\, \delta^4(\mathrm{E} p_k - p_i) \,|T_{n\mathrm{i}}|^2 \,. \tag{4.4}$$

The single particle inclusive spectrum for a + b → c + X is obtained from (4.2) by removing the $\mathrm{d}^3q/2E$ integration for the observed particle c of momentum q :

$$\frac{\mathrm{d}\sigma}{\left(\frac{\mathrm{d}^3 q}{E}\right)} = \frac{1}{2(2\pi)^3} \frac{1}{2s} \sum_X \prod_k \frac{\mathrm{d}^4 p_k}{(2\pi)^3}\, \delta^+(p_k^2 - m^2)(2\pi)^4$$

$$\times\, \delta^4\Big(\sum_X p_k - p_a - p_b + q\Big) \,|T_{\mathrm{a+b\to c+}X}|^2 \,. \tag{4.5}$$

The sum over X includes all states in the a + b + $\bar{\mathrm{c}}$ channel with $M^2 = (P_\mathrm{a} + P_\mathrm{b} - q)^2$ Crossing relates $T_{\mathrm{ab}\to\mathrm{c}X}$ to $T_{\mathrm{ab}\bar{\mathrm{c}}\to X}$ and we then use the unitarity relation (4.4) for the sum over the states in M^2 obtaining the optical theorem used by Mueller [4] :

$$\frac{\mathrm{d}\sigma}{\left(\frac{\mathrm{d}^3 q}{E}\right)} = \frac{1}{2s(2\pi)^3}\, \mathrm{Im}_{M^2}\, T_{\mathrm{ab}\bar{\mathrm{c}}\to\mathrm{ab}\bar{\mathrm{c}}} \,. \tag{4.6}$$

Using the M^2 absorptive part of the model (3.13) we find for the behavior of the pionization spectrum

$$\frac{d\sigma}{\left(\frac{d^3q}{E}\right)} = \frac{2G_a G_b G_\pi^2}{(2\pi)^4 k s_0^3} g_P f(K) , \tag{4.7}$$

where

$$f(K) = [(1 + K) E_1(K) - e^{-K}] , \tag{4.8}$$

and $K = k(q_\perp^2 + m^2)$.

The single particle spectrum for $pp \to \pi^\pm + X$ has been obtained in the pionization region ($x = 0$) at values of s up to 2820 GeV^2 at ISR by the Saclay/Strasbourg collaboration for the range $0.2\ \mathrm{GeV} \leqslant q_\perp \leqslant 0.8\ \mathrm{GeV}$ [6]. The form (4.7) gives an excellent fit to the data with the parameters [8]

$$\frac{d\sigma}{\left(\frac{d^3q}{E}\right)} = 50 \frac{\mathrm{mb}}{\mathrm{GeV}^2} f(K) , \qquad k = 2.7\ \mathrm{GeV}^{-2} . \tag{4.9}$$

To evaluate g_P we now need only G_P and G_π, the pomeron residues at zero momentum transfer. With the pomeron exchange contributing

$$T_{ab}^{el}(s, t) = G_a G_b \beta_{ab}(t) \{1 + e^{-i\pi\alpha_P(t)}\} \frac{\Gamma(-\alpha_P(t) + 1)}{\pi} \left(\frac{s}{s_0}\right)^{\alpha_P(t)} , \tag{4.10}$$

the optical theorem gives for the total cross section

$$\sigma_{ab}^{tot} = G_a G_b / s_0 . \tag{4.11}$$

Taking $s_0 = 1\ \mathrm{GeV}^{-2}$, we find from pp and πp total cross sections

$$G_\pi = 6.2 , \qquad G_P = 10.0 . \tag{4.12}$$

The value of the double pomeron coupling now follows from (4.7), (4.9) and (4.12) for $pp \to \pi^\pm + X$

$$g_P = \frac{(2\pi)^4 k s_0^3}{2G_P^2 G_\pi^2} \times 50\ \mathrm{mb/GeV}^2 , \tag{4.13}$$

$$g_P = 70 . \tag{4.14}$$

Considering the definition of g_P as in (2.9), we see that it is the enhancement of the pomeron-pomeron to $\pi\pi$ interaction over the on-shell coupling of pomerons to pions G_π^2. In this quantitative sense the double pomeron exchange in pionization is very strong.

5. Cross section for two-pion production via double pomeron exchange

In this section we will evaluate the cross section for two pion production *via* double pomeron exchange using the model for T in sect. 3 with the values of the coupling obtained in sect. 4. In order to carry out the phase space integration for the two-particle → four-particle cross section and to compare with the experimental analysis of LZR [7], we convert to the subenergies

$$s_1 = (p'_a + q_1 + q_2)^2 , \qquad s_2 = (p'_b + q_1 + q_2)^2 . \tag{5.1}$$

Then by considering the two produced pions as a body of mass-squared $s_{23} = (q_1 + q_2)^2$ we can consider the final state as a pseudo-three-body final state with subenergies s_1, s_2 and momentum transfer t_1, t_3. In the limit $t_1 \simeq 0$, $t_3 \simeq 0$ enforced by damping of pomeron exchange the amplitude and phase space will factorize into a function of (s_1, s_2, t_1, t_3) times a function of (s_{23}, t_2). This approximation will be used to simplify the calculation.

Because of the sharp damping in t_1 and t_3 (3.14), we calculate the pomeron trajectories effectively at $\alpha_1 = 1$, $\alpha_3 = 1$. We then obtain for $F(K)$ in (3.2) the simple form [18]

$$F(K) = -\frac{1}{K}\, U(-1, 1, -K) = \frac{1}{K} + 1 . \tag{5.2}$$

The Regge powers in (2.9) then combine with this as

$$s_{12} s_{34} \left(\frac{1}{K} + 1\right) = s \left(\frac{1}{k} + \frac{s_{12} s_{34}}{s}\right) . \tag{5.3}$$

For converting to s_1, s_2, we take the approximation of $t_1 = 0$, $t_3 = 0$ in treating the term $s_{12}s_{34}/s$ in (5.3). At large s, s_{12}, s_{34} with fixed s_{23} we obtain

$$s_{12} = \frac{s_1 (m^2 - t_2)}{s_{23}} , \qquad s_{34} = \frac{s_2 (m^2 - t_2)}{s_{23}} , \qquad s_1 s_2 = s_{23}\, s . \tag{5.4}$$

Combining these gives for the second term in (5.3)

$$\frac{s_{12} s_{34}}{s} = \frac{(m^2 - t_2)^2}{s_{23}} . \tag{5.5}$$

For the terms with q_1 and q_2 interchanged we substitute for t_2 the crossed variable t_2' which at $t_1 = 0$, $t_3 = 0$ is related by

$$s_{23} + t_2 + t_2' = 2m^2. \tag{5.6}$$

The final factorized form is obtained for the amplitude for $\pi^- p \to \pi^-(\pi^+\pi^-)p$ by using (2.9) and (5.2–6):

$$T_{24} = G_\pi G_p G_\pi^2 \frac{2g_P}{s_0}\left(\frac{s}{s_0}\right) e^{\Omega(t_1+t_3)}\, \Psi(s_{23}, t_2)\,, \tag{5.7}$$

where

$$\Psi(s_{23}, t_2) = \frac{1}{s_0}\left(-\frac{1}{k} + \frac{(t_2 - m^2)^2}{2s_{23}} + \frac{(t_2' - m^2)^2}{2s_{23}}\right). \tag{5.8}$$

We now examine the phase space integrals and convert the produced $(\pi^+\pi^-)$ to a body of momentum $Q = q_1 + q_2$ in order to approximately factorize the phase space. The production cross section is

$$\sigma_{2\to4} = \frac{(2\pi)^4}{2s}\prod_{i=1}^{4}\left[\int \frac{d^4 p_i}{(2\pi)^3}\,\delta^4(p_i^2 - m_i^2)\right]\delta^4(p_a' + p_b' + q_1 + q_2 - p_a - p_b) \tag{5.9}$$

$$\times\, |T_{2\to4}|^2\,.$$

Converting to the d^4Q integration and using T_{24} in (4.7) and (4.8) we obtain

$$\frac{d\sigma}{ds_{23}} = \frac{G_\pi^6 G_p^2 (2g_P)^2}{2(2\pi)^8 s_0^2}\, I_3 I_2\,, \tag{5.10}$$

where I_3 is the pseudo-three-body phase-space integration for $p_a + p_b \to p_a' + p_b' + Q$:

$$I_3 = \frac{s}{s_0^2}\int ds_1 \int ds_2 \int d^4p_a' d^4p_b' d^4Q\ \delta(Q^2 - s_{23})\,\delta(p_a'^2 - m_a^2) \tag{5.11}$$

$$\times\,\delta(p_b'^2 - m_b^2)\,\delta((p_a' + Q)^2 - s_1)\,\delta((p_b' + Q)^2 - s_2)\,\delta^4(p_a' + p_b' + Q - p_a - p_b)$$

$$\times\,\exp\{2\Omega(t_1 + t_3)\}\,.$$

I_2 is the two-body phase-space integral for pomeron + pomeron $\to \pi\pi$:

$$I_2 = \int d^4q_1\, d^4q_2\, \delta(q_1^2 - m^2)\, \delta(q_2^2 - m^2)\, \delta^4(Q - q_1 - q_2)\, |\Psi|^2 . \tag{5.12}$$

In order to complete the factorization of phase space we have deleted the dependence on t_1 and t_3 in the limits of integration of I_2 by making the effective approximation of $t_1 = 0$, $t_3 = 0$. I_2 is then a function only of s_{23} :

$$I_2 = \frac{\pi}{2s_{23}} \int_{t^-}^{t^+} dt_2\, |\Psi(s_{23}, t_2)|^2 , \tag{5.13}$$

where

$$t^{\pm} = -\tfrac{1}{2}s_{23} + m^2 \pm \tfrac{1}{2}[s_{23}(s_{23} - 4m^2)]^{\frac{1}{2}} . \tag{5.14}$$

In I_3 the inner integrals for three-body phase space with exponential momentum transfer damping have been performed exactly by Chan, Kajantie and Ranft [20] * with the result

$$I_3 = \frac{\pi^2}{4s_0^2} \int ds_1\, ds_2\, \frac{e^A \sinh X}{X} , \tag{5.15}$$

where

$$A = \Omega[s_1 + s_2 - 2s + 3m_a^2 + 3m_b^2$$
$$-\frac{1}{s}(m_b^2 - m_a^2)(s_2 - s_1 - m_a^2 + m_b^2)] , \tag{5.16}$$

$$X^2 = \frac{\lambda(s, m_a^2, m_b^2)}{s^2}\, \Omega^2\, \{\lambda(s, s_1, m_b^2) + \lambda(s, s_2, m_a^2)$$

$$+ 2s(s - s_1 - s_2 - m_a^2 - m_b^2 + 2s_{23}) - 2(s_1 - m_b^2)(s_2 - m_a^2)\} , \tag{5.17}$$

$$\lambda(x, y, z) = x^2 + y^2 + z^2 - 2xy - 2yz - 2zx . \tag{5.18}$$

The boundary of integration is given by the Kibble function

$$G(s_1, s_2, s, s_{23}, m_a^2, m_b^2) = 0 , \tag{5.19}$$

* The integral is, in fact, $\frac{1}{4}$ of the value given in this reference.

where

$$G(x, y, z, u, v, w) = xy(x + y - z - u - v - w) + zu(z + u - x - y - v - w)$$

$$+ vw(v + w - x - y - z - u) + xzw + yzv + xuv + yuw\,. \qquad (5.20)$$

The boundary is also subject to the additional constraints imposed on the data by LZR [7] to isolate the multi-Regge region

$$s_1 \geqslant 2\mathrm{GeV}^2\,, \qquad s_2 \geqslant 4\mathrm{GeV}^2\,, \qquad s_3 = (p'_a + p'_b)^2 \geqslant 4\mathrm{GeV}^2\,. \qquad (5.21)$$

LZR also impose cutoffs on the momentum transfers t_1 and t_3 which we have neglected since they are effectively included by the damping in t_1 and t_3.

6. Results and conclusions

Our most striking result is the enormous value of the cross section $\sigma_{2\to4}$ in the LZR region (5.21) computed using the continuation function (3.2) with the large double pomeron exchange strength $g_P = 70$. Computing I_2 and I_3 numerically and using (5.10) we find for the cross section in the LZR region of phase space

$$\sigma_{2\to4} = \int_{4m^2}^{2.25\,\mathrm{GeV}^2} \frac{\mathrm{d}\sigma}{\mathrm{d}s_{23}}\,\mathrm{d}s_{23} \simeq 30\ \mathrm{mb}\,. \qquad (6.1)$$

This is even larger than the total $\sigma_{2\to4} \simeq 5\mathrm{mb}$ and very much larger than the cross section in the LZR region of phase space $\sigma_{2\to4}^{\mathrm{LZR}} \simeq 90\,\mu\mathrm{b}$. This is a reflection of the occurence of the large double pomeron exchange strength g_P linearly in the pionization region but quadratically in the two-particle → two-particle production cross section. Even exponential damping in t_2 like $\mathrm{e}^{a(t_2-m^2)}$ in the continuation only reduces $\sigma_{2\to4}$ by order $1/a$.

An alternative and perhaps more reasonable way of presenting this result is to find what is the largest value of g_P consistent with the LZR analysis and comparing this with the coupling strength found in pionization. The LZR analysis find for $\sigma_{2\to4}$ in the double Regge region (5.21) integrating over s_{23} from $4m^2$ to 2.25 GeV^2 the value $\sigma_{2\to4} = 90\mu\mathrm{b}$. From studying the distributions in s_1 and s_2 in LZR, which show that the cross section comes mainly from (P, P′) and (P,ρ) exchange, we can estimate that the double-pomeron contribution is at most a tenth of this or 10μb. To reduce (5.1) to this value requires g_P to decrease to

$$g_P \leqslant 1 \qquad (6.2)$$

for LZR. This g_P would only give about $\frac{1}{70}$ of the observed pionization.

The continuation and calculation of the cross section has also been carried out with the symmetrized form (3.16) and (3.17). This form only alters Ψ from (5.8) to

$$\Psi = \frac{1}{s_0}\left(-\frac{1}{k}+\frac{(t_2-t_2')^2}{4s_{23}}\right) . \tag{6.3}$$

The result for $\sigma_{2\to 4}^{\mathrm{calc}}$ are about 50% higher for this form than for (5.8), which is not a significant difference.

To check that the problem of reconciling the pionization and production experiments lies in the continuation model and not in the double Regge structure, we have calculated the production cross section in the LZR region from pion pole exchange in the coupling pomeron + pomeron $\to \pi^+ + \pi^-$. Since the on-shell pomeron $-\pi-\pi$ coupling has already been exhibited in (2.9), we simply replace g_{P} by 1 and replace $1/s_0F$ by the pion propagator $1/(t_2-m^2)$ to obtain the pion-pole dominated two-particle $\to$ two-particle amplitude. With the term $q_1 \leftrightarrow q_2$ added, the calculation is carried out as before but with Ψ in (5.8) replaced by

$$\Psi_{\mathrm{pole}} = -\tfrac{1}{4} . \tag{6.4}$$

The resulting cross section for the LZR region (5.21) is $\sigma_{2\to 4}^{\mathrm{pole}} = 50\mu\mathrm{b}$, which is below the $90\mu\mathrm{b}$ observed by LZR.

In addition to the model for $F(K)$ studied in this paper, the formalism presented above for calculating the magnitude of the six-point amplitude from pionization and for computing the two-particle $\to$ four-particle cross section can be applied to other models of the continuation from pionization to production. In another paper we carry out the continuation with the dual resonance model using the formalism of this paper. The results of the dual resonance model also give too large a value for $\sigma_{2\to 4}$ using the magnitude of g_{P} from pionization.

The importance of the inconsistency presented here is that it uses current dynamical models of inclusive hadron production which successfully predict the qualitative behavior of those experiments, but it demonstrates that they are still quantitatively unsuccessful. Some calculations on the multiperipheral model used in this paper by inserting physical cross sections are quantitatively below the observed magnitude of pionization by about an order of magnitude. This is reflective of our result of accounting for only $\frac{1}{70}$ of the observed pionization using $g_{\mathrm{P}} \leqslant 1$ consistent with LZR. A related problem is that using only the physical coupling strength in the multiperipheral model will not give an output trajectory near 1.

Another source of the inconsistency may be the analyticity structure in K used to satisfy the Steinmann relations. Dynamically it was the $(-K)^{-\alpha} \to -1/K$ term in $F(K)$ that led to the numerically large result for $\sigma_{2\to 4}$. This analyticity structure has been verified for double Regge coupling only in planar graphs and in dual resonance models, which are just the type that lead to the inconsistency observed above. Recently, Brower and Weis [21] have shown that this type of analytic form for double Regge coupling would cause a pomeron of intercept 1 to decouple from total cross sections. Further investigations of the analyticity structure in K are suggested both by the Brower and Weis result and by the numerical inconsistency demonstrated in this paper.

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