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Population Uncertainty and the Timing of
an Urban Transportation Infrastructure Investment[1]

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Abstract

This paper analyzes the impacts of stochastic population changes on the timing of an investment that reduces congestion in an open, monocentric city with fixed boundaries. Congestion pricing cannot be implemented, but a welfare-maximizing planner can buy land and build transportation infrastructure. Under certainty, I derive a rule of thumb to evaluate infrastructure investments that corrects a standard benefit-cost analysis. Under uncertainty, I show that relying on a standard benefit-cost ratio could lead to investing in bad projects, or investing prematurely, or ignoring attractive projects because of population barriers and the impacts of the congestion externality on the land market.

Keywords: transportation infrastructure; congestion; uncertainty; irreversibility; Taylor expansion.
JEL codes: D61, D81, R42.

1.   INTRODUCTION

Apart from political considerations (see the excellent book by Altshuler and Luberoff (2003)), finding the timing of transportation infrastructure investments is difficult for at least three reasons: the net benefits from these investments are inherently stochastic, the decision to invest in infrastructure is largely irreversible, and years go by between the proposal of a project and its completion. Nevertheless, the timing of investing in transportation infrastructure is clearly of interest to elected officials, transportation agencies, private transportation entrepreneurs, and residents of congested areas. Yet, with the exception of Saphores and Boarnet (forthcoming), urban economics papers concerned with congestion relief rely on deterministic models. This literature builds mostly on the seminal papers of Mills and de Ferranti (1971), and Solow and Vickrey (1971); it includes important contributions from Arnott (1979), Braid (1995), Dixit (1973), Hartwick (1980), Hochman (1975), Kanemoto (1975; 1977), Legey, Ripper, and Varaiya (1973), Livesey (1973), Mumy (1980), Oron, Pines, and Sheshinski (1973), Pines and Sadka (1985), Robson (1976), Sheshinski (1973), or Solow (1972; 1973).

This paper extends Saphores and Boarnet (forthcoming) who analyze the timing of a congestion-relief project requiring no urban land when the urban population follows a geometric Brownian motion. By contrast, the congestion-relief project I consider requires urban land, and I propose a general numerical approach for analyzing a large class of population processes in a linear monocentric city where commuters incur both transportation and congestion costs. Following Solow (1972), the city limits are fixed, so residents leave when congestion costs are excessive, which is akin to an upper barrier on city population. The city population is also bounded from below because of political, geographical or historical factors. Unlike Saphores and Boarnet (forthcoming), I account for these barriers throughout this analysis. A social

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planner can buy land and build a time-consuming transportation project to relieve congestion.

When the urban population is constant, I show that following a standard benefit-cost analysis (BCA) would lead to building too much transportation infrastructure. By contrast, when no urban land is required, Saphores and Boarnet (forthcoming) find that a BCA would roughly point to the optimal decision. For the stochastic case, the approach proposed in this paper is illustrated numerically for an urban population that follows a continuous random walk. My results confirm those of Saphores and Boarnet (forthcoming) away from the population barriers. As uncertainty increases, I also find that a project can become undesirable if the optimal population threshold under certainty is close to the upper population barrier; conversely, a project can become more advantageous if the optimal population threshold under certainty is close to the lower population barrier. In addition, I show that relying on a standard benefit-cost ratio could lead to investing in bad projects, investing prematurely, or ignoring attractive projects. These results are driven by interactions between uncertainty and the population barriers, but also by the impact of the congestion externality on the land market.

In the next section, I develop a framework for analyzing the optimal timing under uncertainty of an urban transportation infrastructure investment. In Section 3, a utility-based rule of thumb is derived for assessing the worth of a project under certainty, and the resulting land rents changes are analyzed. Section 4 illustrates the stochastic case numerically, using plausible parameter values when the urban population follows a continuous random walk. Section 5 summarizes conclusions. A summary of key notations can be found in Table 1.

2.   THE MODELING FRAMEWORK

This section introduces key notation, discusses assumptions and summarizes derivations of land rents and indirect utilities before, during, and after a project designed to relieve congestion; expressions related to these different phases are indexed by the subscripts "0," "01," and "1," respectively. My approach relies simply on the equilibrium in the land market and on the equality of the utility of urban residents. After formulating the objective of a social planner, I simplify it without sacrificing generality using Taylor expansions based on empirical evidence (for other economic applications of Taylor expansions, see Dahl and Gonzalez-Rivera (2003); Teulings and Gautier (2004); or Weitzman (1988)). Elements of this framework common with Saphores and Boarnet (forthcoming) are recalled without proof for completeness.

City Geography, Population, Land Rents, and Congestion

Consider an open, monocentric city on a homogeneous plain, where the central business district (CBD) is a strip of land of length *B* that separates the city in two equal rectangular halves. The aerial distance from the CBD to the city edge is normalized to 1 and $h \in [0,1]$ denotes the aerial distance between a point in the city and the CBD (see Figure 1).

City residents are identical; their number, X_t, changes randomly over time according to

$$dX_t = \mu(X_t)dt + \sigma(X_t)dw_t, \tag{1}$$

$\mu(x)$ and $\sigma(x)>0$ are continuous; dt is a small time step; and dw_t is an increment of a standard Wiener process (Dixit and Pindyck (1994)). Herein, *x* is a realization of the random variable X_t.

To avoid complexities linked to land conversion (e.g., see Cappoza and Helsley (1990) or Clarke and Reed (1998)), the city limits are set by natural or political boundaries (as in Solow (1972)). Diseconomies of scale outgrow agglomeration benefits at high densities, so city

residents leave when the urban population is too large and X_t admits an upper reflecting barrier $\bar{x}$ (X_t is reflected downward when it hits $\bar{x}$). Likewise, the urban population admits a lower reflecting barrier $\underline{x}$ (it reflects X_t upward) because agglomeration benefits at low densities overtake negative externalities associated with urban life (congestion, crime, or pollution).

Now consider an urban resident who lives at a distance $h \in [0,1]$ from the CBD. The flow of her utility is given by the logarithm of a Cobb-Douglas function that depends only on her consumption per unit time of land L and of a numeraire good g,

$$U(g,L) = a \ln(g) + (1-a) \ln(L), \tag{2}$$

where $a \in (0,1)$ is the utility elasticity of the numeraire good.

As she commutes to the CBD, she incurs transportation and congestion costs. The former are proportional to h. Since the Vickrey (1969) bottleneck congestion function does not yield explicit land rents here, the average flow of her congestion costs over the city transportation network is assumed proportional to both h and to a function of the city population, x. For simplicity, her income flow is constant and equals 1. Hence, her budget constraint is

$$g + R_0(x,h).L + \lambda h + \omega_0(x)h = 1, \tag{3}$$

where g, $R_0(x,h)$, λh, and $\omega_0(x)h$ are respectively the fraction of her income flow spent on the composite good, rent per unit area, transportation, and congestion; moreover, L is the area of land she rents. From Saphores and Boarnet (forthcoming), the flow of land rents for a unit parcel at distance h from the CBD when the total city population equals x, is:

$$R_0(x,h) = \frac{x[\lambda + \omega_0(x)]}{2B} \frac{\left[1 - \lambda h - \omega_0(x)h\right]^{\nu+1}}{1 - \left[1 - \lambda - \omega_0(x)\right]^{\nu+1}}, \tag{4}$$

where ν is the ratio of the utility elasticity of the numeraire to the utility elasticity of land:

$$v = \frac{a}{1-a}.\qquad(5)$$

Combining (4) with the demand functions for land and the numeraire gives the flow of indirect utility $V_0(x)$ of a city resident, based on indirect utility at the CBD:

$$V_0(x) = a\ln(a) + (1-a)\ln\left(\frac{2(1-a)B}{x}\frac{1-[1-\lambda-\omega_0(x)]^{v+1}}{\lambda+\omega_0(x)}\right).\qquad(6)$$

Transportation Infrastructure and Congestion

Congestion pricing cannot be implemented for technical or political reasons. Instead, consider the second best framework where, to relieve congestion, a social planner needs to decide on the timing of a transportation project that lowers $\omega_0(x)$ to $\omega_l(x)$. Completing this project requires Δ units of time and a rectangular land area of width βB that connects opposite sides of the city through the CBD (see Figure 1). As the urban land available for housing decreases by $2B\beta$, the population displaced by the project relocates elsewhere in the city, and the land market equilibrium is restored at higher prices and densities.

Let x^* denote the population level at which it is optimal to buy the project land; x^* marks the start of the project in the absence of land banking. Total project costs, $C(x^*)$, consist of two parts: C_C, which includes construction cost lumped with all future maintenance and operation costs, and land costs, $C_L(x^*)$. $C_L(x^*)$ depends on x^* because total city population impacts land rents, and therefore land prices. In a competitive urban land market, the cost of land is the present value of expected rents. For simplicity, assume that the social planner pays the present value of expected rent without the project, i.e.,

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$$C_L(x^*) = E_{x^*} \int_0^{+\infty} S_0(X_t) e^{-\rho t} dt, \tag{7}$$

where E_{x^*} is the expectation with respect to X_t given $X_0 = x^*$; $S_0(X_t)$ is the flow of rents on

project land without the project; and ρ is the social planner's discount rate. Integrating $R_0(x, h)$

over the project land area gives:

$$S_0(x) = \frac{x\beta}{\nu + 2} \frac{1 - [1 - \lambda - \omega_0(x)]^{\nu+2}}{1 - [1 - \lambda - \omega_0(x)]^{\nu+1}}. \tag{8}$$

If $C(x^*) \equiv C_L(x^*) + C_C$ is annualized over infinity and shared equally among city residents,

the ratio of the flow of interests on project costs to the flow of total city income equals

$$\Omega(x; x^*) \equiv \frac{\rho C(x^*)}{x}. \tag{9}$$

After project completion, congestion is reduced, land rents are impacted by the reduced

land supply, and a fraction $\Omega(x; x^*)$ of the income flow of each city resident goes towards

paying for the project. As a result, the budget constraint (3) becomes

$$g + R_1(x, h; x^*).L + \lambda h + \omega_1(x)h = 1 - \Omega(x; x^*). \tag{10}$$

x^* is an argument of $R_1(x, h; x^*)$ because post-project rents depend on the timing of the project

through $\Omega(x;x^*)$ (see (11)). Proceeding as for $R_0(x,h)$, the land rent per unit area becomes

$$R_1(x, h; x^*) = \frac{x}{2B(1-\beta)} \frac{[\lambda + \omega_1(x)][1 - \Omega(x; x^*) - \lambda h - \omega_1(x)h]^{\nu+1}}{[1 - \Omega(x; x^*)]^{\nu+1} - [1 - \Omega(x; x^*) - \lambda - \omega_1(x)]^{\nu+1}}. \tag{11}$$

After simplifications, the post-project flow of indirect utility of a city resident may be written

$$V_1(x; x^*) = a \ln(a) +$$

$$(1-a) \ln \left(\frac{2(1-a)B(1-\beta)}{x} \frac{[1-\Omega(x;x^*)]^{\nu+1} - [1-\Omega(x;x^*) - \lambda - \omega_1(x)]^{\nu+1}}{\lambda + \omega_1(x)} \right). \qquad (12)$$

During construction (indexed by the subscript "01"), unit land rents are $R_{01}(x, h; x^*)$, there is no congestion relief, but city residents must pay for the project, so $R_{01}(x, h; x^*)$ and the flow of indirect utility, $V_{01}(x; x^*)$, are derived from (11)-(12) by changing $\omega_1(x)$ to $\omega_0(x)$.

The Social Planner's Objective Function

A welfare-maximizing social planner interested in the optimal timing of a transportation infrastructure investment that relieves congestion faces a standard stopping problem (Dixit and Pindyck (1994)). Indeed, when the city population is small, congestion is low and per-capita costs of transportation infrastructure improvements are high, so it is best to wait. Conversely, when the city population is large, congestion is high and the per capita costs of infrastructure improvements are low, so investing in congestion relief makes sense. Therefore, the social planner is looking for the population threshold $x^* \in [\underline{x}, \bar{x}]$ that maximizes the net discounted utility gains from investing in congestion relief. The project gets the go-ahead, however, only if the present value of the expected utility of all city residents with the project exceeds the present value of their aggregate expected utility without it, which I assume for now.

If $x_0 \in [\underline{x}, \bar{x})$ is the current city population, suppose the social planner goes ahead with the project at $x \in [x_0, \bar{x}]$. Let E_{x_0} be the expectation operator with respect to X_t given that $X_0 = x_0$, and let τ designate the random time it takes X_t to hit x for the first time. To obtain a convenient form of the social planner's objective function, subtract the present value of the aggregate utility

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of city residents without the project, $E_{x_0} \int_0^{+\infty} X_t V_0(X_t)e^{-\rho t}\,dt$, from their aggregate utility with it,

$$E_{x_0}\left\{\int_0^{\tau} X_t V_0(X_t)e^{-\rho t}\,dt + \int_{\tau}^{\Delta+\tau} X_t V_{01}(X_t;x)e^{-\rho t}\,dt + \int_{\tau+\Delta}^{+\infty} X_t V_1(X_t;x)e^{-\rho t}\,dt\right\}.$$ Then, add and

subtract $\int_{\tau+\Delta}^{+\infty} X_t V_{01}(X_t;x)e^{-\rho t}\,dt$, and rearrange terms to obtain

$$x^* = \arg\max_{\underline{x}\leq x\leq \overline{x}} E_{x_0}\left\{\int_{\tau}^{+\infty} X_t[V_{01}(X_t;x)-V_0(X_t)]e^{-\rho t}\,dt + \int_{\tau+\Delta}^{+\infty} X_t[V_1(X_t;x)-V_{01}(X_t;x)]e^{-\rho t}\,dt\right\}. \quad (13)$$

Now consider the first integral on the right side of (13). Change variables $u=t-\tau$ and take

its expected value given $X_0=x_0$ to get $E_{x_0}(e^{-\rho\tau})E_x \int_0^{+\infty} X_u.[V_{01}(X_u;x)-V_0(X_u)]e^{-\rho u}\,du$; indeed,

only population variations between x_0 and the first time x is reached intervene in the calculation

of $E_{x_0}(e^{-\rho\tau})$, whereas the expected value of the integral term depends only on population

variations after X_t reaches x. Proceed similarly with the second integral in (13) to formulate the

social planner's objective function as

$$x^* = \arg\max_{\underline{x}\leq x\leq \overline{x}}\left\{D_{x|x_0}P(x)\right\}, \quad (14)$$

subject to (1). $D_{x|x_0}$ is the stochastic discount factor (see the Appendix for a derivation)

$$D_{x|x_0} \equiv E_{x_0}(e^{-\rho\tau}), \quad (15)$$

and $P(x)$ is the present value of expected net utility gains when the project starts:

$$P(x) = E_x\left(\int_0^{+\infty} X_t.[V_{01}(X_t;x)-V_0(X_t)]e^{-\rho t}\,dt + \int_{\Delta}^{+\infty} X_t.[V_1(X_t;x)-V_{01}(X_t;x)]e^{-\rho t}\,dt\right). \quad (16)$$

The first integral of Equation (16) captures project costs in terms of utility: $V_{01}(X_t;x) - V_0(X_t)$ is the flow of utility loss of a city resident during project construction (she is paying for the project, less land is available for housing, and there is no congestion relief). The second integral in Equation (16) captures project benefits: $V_1(X_t;x) - V_{01}(X_t;x)$ is the flow of individual utility gains after project completion because project payments are unchanged but congestion is reduced. These utility flows are then aggregated over the city population, summed over time, and expectations are taken in order to find the net expected utility gains from the project.

This formulation of the social planning problem provides an alternative to the real options approach used in Saphores and Boarnet (forthcoming). Let us now derive the first order necessary condition for (14), which summarizes the tradeoff between waiting and investing.

Proposition 1: The first order necessary condition for an interior solution for x^* can be written

$$\varepsilon^D \equiv \left(\frac{-x}{D_{x|x_0}} \frac{\partial D_{x|x_0}}{\partial x} \right)\Big|_{x=x^*} = \varepsilon^P \equiv \left(\frac{x}{P(x)} \frac{\partial P(x)}{\partial x} \right)\Big|_{x=x^*}, \tag{17}$$

where ε^D is the population elasticity of the stochastic discount factor (see (15)) and ε^P is the population elasticity of $P(x)$, the present value of expected net utility gains when the project starts (see (16)). $\square$

Proof: Let $\pi(x;x_0) \equiv D_{x|x_0} P(x)$. Since $\frac{\partial \pi(x;x_0)}{\partial x}\Big|_{x=x^*} = 0$, divide by $D_{x^*|x_0} P(x^*)$, multiply by x^* and arrange terms to get (17). x^* is independent of x_0, since for $x_0 \leq x_1 \leq x^*$, $D_{x^*|x_0} = D_{x^*|x_1} D_{x_1|x_0}$; as x_0 appears only in $D_{x^*|x_0}$, changing x_0 is just akin to multiplying (14) by a constant. $\square$

Taylor Approximations of Land Rents and Changes in Indirect Utility

Empirical evidence (see below) shows that the flows of transportation and congestion costs as well as the flows of infrastructure project costs are typically small compared to the flow of income (i.e., $\omega_i(x) \ll 1$, $\lambda \ll 1$, and $\Omega(x) \ll 1$, respectively). In the rest of the paper, I suppose that the lower population barrier $\underline{x}$ is high enough (so $\Omega(x) \ll 1$ for all $x \in [\underline{x}, \overline{x}]$) and the upper barrier $\overline{x}$ is low enough (so $\omega_i(x) \ll 1$ over $[\underline{x}, \overline{x}]$) that simple Taylor expansions of rents or indirect utility differences over $[\underline{x}, \overline{x}]$ can be used, irrespective of the value of the other model parameter.

For $0 \leq y \ll 1$ and $\alpha \neq 0$, I repeatedly use the truncated Taylor expansions (Sprecher (1970))

$$(1+y)^\alpha \approx 1 + \alpha y + \alpha(\alpha-1)\frac{y^2}{2} + \alpha(\alpha-1)(\alpha-2)\frac{y^3}{6}, \tag{18}$$

$$\ln(1+y) \approx y - \frac{y^2}{2}. \tag{19}$$

Approximating first the flows of land rents, $R_{01}(x,h;x^*)$, leads to

$$R_{01}(x,h;x^*) \approx \frac{x}{2B}\frac{1}{\nu+1}\left\{(1+\beta)\left(1 - \Omega(x;x^*) + (\lambda + \omega_0(x))\left[\frac{\nu}{2} - (1+\nu)h\right]\right) + \right.$$
$$\left. +\beta^2 + \frac{\nu}{2}(\lambda + \omega_0(x))^2\left[\frac{\nu+2}{6} + (\nu+1)h(h-1)\right]\right\}. \tag{20}$$

For approximating $R_1(x,h;x^*)$, replace $\omega_0(x)$ with $\omega_1(x)$ in (20); for $R_0(x,h)$, set β and $\Omega(x;x^*)$ to 0. Integrating $R_0(x,h)$ gives $S_0(x)$, the flow of rents on project land without the project (see (8))

$$S_0(x) \approx \frac{x\beta}{\nu+1}\left\{1 - \frac{1}{2}(\lambda + \omega_0(x)) + \frac{\nu}{12}(\lambda + \omega_0(x))^2\right\}. \tag{21}$$

It is also necessary to approximate differences in indirect utilities that appear in (14). I find

$$V_{01}(x;x^*) - V_0(x) \approx \frac{1}{1+\nu}\left\{-\beta - \frac{\beta^2}{2} - \nu\Omega(x;x^*)\left[1 + \frac{\lambda}{2} + \frac{\omega_0(x)}{2} + \frac{\Omega(x;x^*)}{2}\right]\right\}, \tag{22}$$

$$V_1(x;x^*) - V_{01}(x;x^*) \approx \frac{v}{1+v} \frac{\omega_0(x) - \omega_1(x)}{2} \left\{ 1 + \Omega(x;x^*) + \frac{4-v}{12} [2\lambda + \omega_0(x) + \omega_1(x)] \right\}. \quad (23)$$

As λ appears only as a second-order term in (22)-(23), small values of λ have little impact on x^*.

Let us now analyze the deterministic case to benchmark the impact of uncertainty.

3.   THE DETERMINISTIC CASE

In this section, let us suppose that the city population is constant and equals x, so x and x^* can be omitted as arguments of land rents and utility functions.   In this context, it is optimal to undertake the project if the indirect utility of a city resident with the project exceeds her indirect utility without it, i.e. if $[V_{01} - V_0] + e^{-\rho\Delta}[V_1 - V_{01}] \geq 0$.   Keeping only the first order Taylor expansion of $V_{01} - V_0$ and $V_1 - V_{01}$ from (22) and (23) leads to the rule of thumb (recall that $\Omega = \rho C/x$):

Proposition 2. When the city population is constant, a transportation infrastructure project designed to relieve congestion must verify

$$ae^{-\rho\Delta}x\frac{\omega_0 - \omega_1}{2} \geq a\rho C + (1-a)\beta x \quad (24)$$

in order to increase welfare. □

To interpret (24), calculate Φ_0, Φ_{01}, and Φ_1, which are respectively the total flows of congestion costs before, during, and after the project. Starting with the latter, let $N_1(h)$ denote the number of city residents who live within distance h from the CBD. In the 2 parallel strips of length $B(1-\beta)$ and thickness dh located at h from the CBD, there are $N_1'(h)dh$ city residents.

Multiply this quantity by the individual demand function for land to get the total demand for land there. The corresponding land supply is $2B(1-\beta)dh$, so equilibrium in the land market requires

$$N_1'(h) = (1+v)x \frac{[\lambda + \omega_1][1 - \Omega - (\lambda + \omega_1)h]^v}{[1-\Omega]^{v+1} - [1-\Omega - \lambda - \omega_1]^{v+1}}. \tag{25}$$

The total flow of congestion costs after the project is then $\Phi_1 \equiv \int_0^1 \omega_1 h N_1'(h)dh$. After integrating Φ_1, a first order Taylor expansion based on (18) gives

$$\Phi_1 \approx x \frac{\omega_1}{2}. \tag{26}$$

Proceeding similarly for $\Phi_i \equiv \int_0^1 \omega_0 h N_i'(h)dh$, with $i=0$ or $i=01$ leads to

$$\Phi_0 \approx x \frac{\omega_0}{2}, \quad \Phi_{01} \approx x \frac{\omega_0}{2}. \tag{27}$$

The left side of (24) is thus the present value of project benefits multiplied by the utility elasticity of the numeraire: from (27), the total flow of congestion costs is roughly unchanged during the project, but it drops by $x \frac{\omega_0 - \omega_1}{2}$ once the project is complete. These project benefits need to be discounted since congestion relief occurs only after Δ time units, the duration required to complete the project. The factor 2 reflects that the average individual flow of congestion costs is approximately half of its maximum value at the city edge.

The right side of (24) is a weighted sum of costs: the first term is the flow of interests on project costs weighted by the utility elasticity of the numeraire, and the second term is total city income (remember that individual income is normalized to 1) multiplied by the fraction of urban land required for the project, weighted by the utility elasticity of land.

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By contrast, a standard benefit-cost analysis (BCA) would recommend building the project if $e^{-\rho\Delta}x\dfrac{\omega_0-\omega_1}{2}\geq\rho C$, which ignores elasticity terms that reflect preferences and the impact of the project on the land market. A standard benefit-cost ratio would then underestimate costs by $v^{-1}\beta x$ (recall that $v=a/(1-a)$), so it would lead to building too much transportation infrastructure unless $a=1$ (city residents don't get utility from land) or $\beta\approx 0$ (as for an elevated highway or an underground metro.) Kanemoto (1977) finds a similar bias with a BCA near the CBD, but the reverse at the city edge if the price elasticity of compensated housing demand is less than one. However, Kanemoto (1975) shows that following a BCA would lead to under-investing in roadways in a von Thünen framework.

Let us now examine how land rents vary with the project. From (4) and (11), we already know that $R_0(h)$, $R_{01}(h)$ and $R_1(h)$ decrease as h increases. In addition, we have:

Proposition 3. Suppose that (24) holds, so building the project increases welfare. Then:

1) During construction, land rents increase if $\beta>\Omega$ (i.e., if the project is relatively land intensive); this increase is maximum at the CBD and minimum at the city edge. Conversely, if $\beta<\Omega$, land rents decrease throughout the city. Finally, if $\beta=\Omega$, land rents increase at the CBD but decrease at the city edge.

2) After construction, land rents change almost linearly from the CBD to the city edge; they decrease at the CBD, and increase at the city edge. They are unchanged at $h=0.5a$.

3) Overall (comparing rents with and without the project), land rents decrease at the CBD and increase at the city edge. □

Proof: 1) To compare rents before and during construction, use Equation (20) to get:

$$R_{01}(h) - R_0(h) \approx \frac{x}{2B}\frac{1}{v+1}\left\{\beta - \Omega + \beta\left(\beta - \Omega + [\lambda + \omega_0]\left[\frac{v}{2} - (v+1)h\right]\right)\right\}. \qquad (28)$$

Hence, the first-order impact of construction, $\frac{x}{2B}\frac{1}{v+1}\{\beta - \Omega\}$, is independent of h and has the

sign of β-Ω. The other terms are linear in h, with a maximum at $h=0$ and a minimum at $h=1$.

2) To compare rents during and after project completion, use again Equation (20) to get:

$$R_1(h) - R_{01}(h) \approx \frac{x}{2B}\frac{\omega_0 - \omega_1}{1 - \beta}\left\{h - \frac{a}{2}\right\}. \qquad (29)$$

Equation (29) is linear in h with $R_1(0) - R_{01}(0) < 0$, $R_1(1) - R_{01}(1) > 0$, and $R_1(\frac{a}{2}) - R_{01}(\frac{a}{2}) \approx 0$.

3) Finally, the overall impact of the project on land rents is given by

$$R_1(h) - R_0(h) \approx \frac{x}{2B}\frac{1}{1+v}\left\{\beta - \Omega + (\omega_0 - \omega_1)\left[(1+v)h - \frac{v}{2}\right]\right\}. \qquad (30)$$

From (24), $\beta + v\Omega - \frac{v}{2}(\omega_0 - \omega_1) \le 0$ (recall that $\frac{a}{1-a} = v$ and drop $e^{-\rho\Delta}$), so for $h=0$ (at the

CBD), the terms in brackets in (30) are $\beta - \Omega - (\omega_0 - \omega_1)\frac{v}{2} < \beta + v\Omega - (\omega_0 - \omega_1)\frac{v}{2} \le 0$. At $h=1$

(at the city boundary), the terms in brackets in (30) equal $\beta - \Omega + (\omega_0 - \omega_1)\left[\frac{v}{2} + 1\right] > 0$, since (24)

implies that $\frac{1}{2}(\omega_0 - \omega_1) > \Omega$. The result follows. $\square$

These results are intuitive based on the objective function (14). (24) guarantees a welfare

improvement for renters, so let us examine now if landowners benefit from congestion reduction.

Proposition 4. If (24) holds, on aggregate landowners also benefit from congestion reduction.

Proof: Let H_0, H_{01}, and H_1 be respectively the total flows of rent for the urban land dedicated to housing before, during, and after the project. H_{01} can be obtained by integrating the first-order terms of (20) over the urban land area available for housing; we get:

$$H_{01} \approx \frac{x}{v+1}\left\{1-\Omega-\frac{1}{2}[\lambda+\omega_0]\right\}. \tag{31}$$

For H_0, set Ω to 0 in (31); for H_1, just replace ω_0 with ω_1. Now let us compare the total flow of urban rents with the project, including the construction phase, to the flow of urban rents without the project. This leads, after simplification, to

$$H_{01}\int_0^\Delta e^{-\rho t}dt + H_1\int_\Delta^{+\infty} e^{-\rho t}dt - H_0\int_0^{+\infty} e^{-\rho t}dt \approx \frac{1}{\rho}\frac{x}{v+1}\left[e^{-\rho\Delta}\frac{\omega_0-\omega_1}{2}-\frac{\rho C}{x}\right], \tag{32}$$

which is clearly positive because of (24). $\square$

4.   NUMERICAL ILLUSTRATION OF THE STOCHASTIC CASE

Functional Forms

To explore the properties of the solution of Equation (14), let us make two additional assumptions. First, assume that the city population follows a continuous random walk

$$dX_t = \sigma dw_t, \tag{33}$$

where dw_t is an increment of a standard Wiener process (Dixit and Pindyck (1994)). This is the simplest continuous-time process and its deterministic counterpart is a constant population, so the deterministic results above can serve as benchmarks.

Second, let us suppose that the fraction of the individual income flow spent on congestion costs by a resident living at the city edge is

$$\omega_i(x) = \gamma_i x^2. \tag{34}$$

Data

Although the model discussed above is highly stylized, it is important to obtain plausible estimates of key parameters in order to establish the relevance of our numerical results.

Social discount rate. I use an annual discount rate of 7%, as required by federal guidelines for transportation benefit-cost analyses (Circular No. A-94 Revised from the OMB), for a 10-year effective rate of $\rho=1.07^{10}-1=0.967$; calculations are repeated with a 4% annual discount rate, which was used in the past for transportation projects.

Time unit and population variability. I adopt a unit time scale of 10 years to reflect the frequency of U.S. censuses and consider a city whose population varies between $\underline{x}=1$ and $\bar{x}=2$ (in millions). To cover a wide range of cases, σ (in millions divided by the square root of a time unit) varies over [0; 0.3]. To understand the implications of this choice, (A.8) is used to generate 10 years ahead 95% confidence intervals for X_t given a current population of 1.5 million; for $\sigma=0.15$ and 0.30, I respectively find [1.21, 1.79] and [1.07, 1.93] (in millions).

Congestion costs. According to the Texas Transportation Institute (2002), congestion costs per driver in 2000 averaged $1,160 in all 75 metropolitan statistical areas studied, and $1,590 in the largest ones. The 2000 median household and per capita incomes were $42,228 and $21,587, respectively, so plausible values of congestion as a fraction of income range from 0.027 ($1,160 / $42,228) to 0.074 ($1,590 / $21,587). For our base case, I set $\omega_0(\bar{x})$ to 0.05.

Magnitude of congestion reduction. Following Weisbrod, Vary, and Treyz (2001), typical congestion reductions from infrastructure investments are 2.5% to 6.3% of total travel costs. Targeted infrastructure investments might lead to larger reductions, so I consider total

travel cost reduction as large as 10%. For the base case, I select a 7% congestion reduction (so $\omega_1(\bar{x}) = 0.93 \; \omega_0(\bar{x}) = 0.0465$) and explore reductions between 3.5% and 10%.

Project costs. A review of construction costs for six recent major highway or rail transit corridor investments in U.S. metropolitan areas (Boarnet, DiMento, and Macey (2002); Howell (1996); Martin (2001); Pae (1995); SDMTDB (2002); and Zamichow (1993)) shows that, excluding maintenance, $\Omega_C(x) \equiv \rho C_C /(mx)$ ranges from $5.6 \; 10^{-5}$ to $8.9 \; 10^{-4}$. Assuming maintenance adds 50% to annualized construction costs, I pick $\Omega_C(\bar{x}) = 2 \; 10^{-4}$ for the base case.

Other parameters. I set the utility elasticity of the numeraire to $a=2/3$ (people living by the CBD spend 2/3 of their income on the numeraire), so $\nu=2$ (see (5)); I also consider $\nu=1$ ($a=1/2$) and $\nu=3$ ($a=3/4$). The project duration is $\Delta=5$ years (Δ varies between 3 and 15 years) and requires 0.025% of the urban land (so $\beta=0.00025$). Finally, Equations (22) and (23) show that λ has only a second-order impact on changes in indirect utility, so I set λ to 0.

Results

Results are illustrated on Figures 2 to 5B. They were generated on a PC with MathCAD, a symbolic computation software. The numerical solution is sketched out in the Appendix.

Optimum timing. Figure 2 illustrates how x^* varies with the population volatility σ for different levels of congestion reduction. It is useful to consider Figure 2 simultaneously with Figures 3A and 3B, which show the present values at x^* of expected utility losses and gains from the project (the first and the second integral on the right side of (13) respectively).

First, we see as expected that the project should start later, if at all, when benefits from the project (congestion reduction) are smaller.

Second, at low levels of population uncertainty (for σ small), x^* increases with σ because expected utility gains increase faster than expected utility losses (see Figures 3A and 3B); a higher population threshold x^* is also reached faster with a higher σ. Except if X_t is close to $\underline{x}$ or $\overline{x}$, the probability density function of X_t is concentrated around the deterministic population threshold, so the population barriers $\underline{x}$ and $\overline{x}$ play almost no role.

This also holds when x^* is close to the middle of the interval $[\underline{x},\overline{x}]$, even at moderate values of σ. As in Saphores and Boarnet (forthcoming), but for a different population process, x^* first increases and then decreases with σ ("Congestion -7%" on Figure 2). As they explain, the impact of uncertainty is ambiguous because it affects both the benefits and the costs of waiting.

At higher values of σ, however, x^* is influenced by the impact of $\underline{x}$ and $\overline{x}$ on the tails of the distribution of X_t. When x^* is close to the upper barrier $\overline{x}$ (for a 3.5% congestion cut), an increase in σ lowers the expected city population and, therefore, also expected utility gains from the project, because $\overline{x}$ truncates the upper values of X_t; expected project costs (through the cost of land) also decrease, although much less (Figure 3A). It is then optimal to wait for higher values of x^*, but when X_t reaches $\overline{x}$, delaying is no longer possible and the project becomes worthless. Conversely, when x^* is close to the lower barrier $\underline{x}$ (for a 10% reduction in congestion), more uncertainty augments the likelihood of a higher population because lower values of X_t are truncated by $\underline{x}$. Expected utility gains increase faster than expected utility losses (Figure 3B) and it becomes optimal to act earlier (immediately when $x^*=\underline{x}$); a kink appears in the present value of expected utility gains as σ keeps on increasing while x^* is stuck at $\underline{x}$.

A sensitivity analysis (results not shown) shows that the discount rate has a fairly steady impact on x^*; as expected, decreasing the discount rate increases the present value of the project

benefits and causes the investment in congestion reduction to take place earlier. Likewise, x^* increases with project costs. Increasing Δ, the time to buy land and build the project, augments the present value of congestion costs during construction and decreases the present value of project benefits. In addition, a larger utility elasticity of the numeraire a leads to reducing congestion sooner as it reflects a preference for a reduction in congestion costs over land.

Impacts of congestion relief on land rents. It is also of interest to examine how land values (the present value of expected land rents) change with the project. As for the deterministic case, land values decrease during construction everywhere in the city; this decrease is greatest at low population levels, and it is more pronounced away from the CBD. After the project (compared with the construction phase), land values decrease from $h=0$ to $h=0.5a$ (see Proposition 3) and increase thereafter; changes augment with the urban population. This follows from the Taylor expansion of $R_1(h)-R_{01}(h)$ (see (29)).

Figure 4 illustrates the percentage change in unit land values before and after the project as a function of distance to the CBD for $\sigma=0.10$ (solid lines) and $\sigma=0.30$ (dashed lines). As expected, land values decrease at the CBD and increase at the city edge; they are unchanged around $h=0.5a$. A higher city population amplifies these changes, which remain small ($\leq 1\%$) for the parameters considered. A higher σ amplifies land value changes when total city population is close to the lower population barrier $\underline{x}$, but the reverse is true when X_t is close to $\bar{x}$.

In addition, property owners benefit on average from congestion relief at x^* (the converse of Proposition 4; results not shown). Expected overall gains increase with total city population, and with σ at low population levels (a higher population is more likely because of $\underline{x}$) but they decrease with σ for X_t close to $\bar{x}$ because a lower population is more likely in that case.

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Comparison with a naïve benefit-cost analysis. Infrastructure projects are frequently justified by benefit-cost analyses, so let us now contrast a standard benefit-cost ratio at x^*, denoted by $\text{SBC}(x^*)$, with $\text{UBC}(x^*)$, the ratio at x^* of the present value of expected utility gains to the present value of expected utility losses. Figures 5A and 5B show these ratios for different values of σ; $\text{SBC}(x^*)$ is represented by dashed lines and $\text{UBC}(x^*)$ by solid lines. Since a standard benefit-cost analysis is static, assume for evaluating $\text{SBC}(x^*)$ that the city population is constant and equals x^*; note that, when X_t verifies (33) and if population barriers are ignored, then $E[X_t \mid X_0 = x^*] = x^*$. Ignoring the impact of the congestion externality on the land market, the numerator of $\text{SBC}(x^*)$ is the present value of the decrease in the flow of congestion costs and its denominator is $C(x^*)$, total project costs. Proceeding as for Φ_0, Φ_{01}, and Φ_1 (see (25) to (27)), a first-order approximation of the numerator of $\text{SBC}(x^*)$ is given by $e^{-\rho\Delta}(x^*)^{\delta+1}(\gamma_0 - \gamma_1)/2\rho$.

At low levels of σ, $\text{SBC}(x^*)$ is consistently higher than $\text{UBC}(x^*)$; they differ under certainty because $\beta x^* /(\rho v)$, the impact of the congestion externality on the land market (the last term in (24) after dividing throughout by a), is approximately one-third of direct project costs. As σ increases, however, $\text{SBC}(x^*)$ may become lower than $\text{UBC}(x^*)$. For a 3.5% congestion reduction (Figure 5A), $\text{SBC}(x^*)$ suggests that the project is increasingly attractive, whereas the reverse is true; in fact, the project should never be built for $\sigma > 0.23$ (recall Figure 2). By contrast, for a 10% congestion reduction (Figure 5B), $\text{SBC}(x^*)$ wrongly indicates that the project should not be pursued at higher levels of σ, when $x^* \approx \underline{x}$ because $\text{SBC}(x^*)$ falls below 1 whereas $\text{UBC}(x^*)$ is clearly greater than 1. Overall, basing the decision to invest on $\text{SBC}(x^*)$ would lead to building too early at lower levels of uncertainty; conversely, at higher levels of σ, this may result in considering investments without merit or in ignoring valuable projects.

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5.   CONCLUSIONS

This paper contributes to the urban economics literature by analyzing the impacts of uncertainty, irreversibility, and project duration on the timing of a congestion-relief urban infrastructure project that requires urban land. To date, this literature typically has considered a deterministic framework with a static population.

To benchmark the impact of uncertainty, I first revisit the deterministic case. I derive a utility-based rule of thumb to assess the merits of a project and show that a standard benefit-cost ratio would lead to building too much infrastructure. This result derives from the impact of the congestion externality on the land market. Population uncertainty further undermines the validity of a "naïve" cost-benefit analysis (CBA), i.e. a static and deterministic comparison of direct costs and benefits. Indeed, a numerical illustration based on plausible parameter values show that following a CBA could result in investing too early, investing in projects without merit, or in ignoring attractive projects. These results are driven by the time lag needed to complete the project (see Bar-Ilan and Strange (1996) or Saphores and Boarnet (forthcoming)) combined with the impact of the population barriers, which are present for economic and political reasons. This to the importance of correctly accounting for population dynamics in a framework that captures the irreversibility of infrastructure investments; they also highlight the need to obtain accurate estimates of project costs and duration, whereas urban mega-project are often late and over budget (see Altshuler and Luberoff (2003) for notorious examples).

Future work could explore the value of buying land for future infrastructure projects; analyze the timing of infrastructure investments in expanding urban areas; consider the impact of state and federal subsidies; or jointly analyze timing and capacity choice.

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APPENDIX

This appendix summarizes the procedure followed to solve the stochastic objective function (14) in order to find the population threshold x^*.

For $y \in [\underline{x}, \overline{x}]$, let us first derive an alternative expression for the functional

$$F(y) \equiv E_y \left\{ \int_0^{+\infty} f(X_t) e^{-\rho t} dt \right\}, \text{ which occurs in the objective function (see Equation (16)); here}$$

$f(x)$ is a continuous real function. This alternative expression enables us to replace the expectation and the indefinite integral with a single integral on a finite interval when X_t follows Equation (33). From Karlin and Taylor (1981), we know that $F(y)$ verifies

$$\frac{\sigma^2(y)}{2} \frac{d^2 F(y)}{dy^2} + \mu(y) \frac{dF(y)}{dy} - \rho F(y) + f(y) = 0 \qquad (A.1)$$

with boundary conditions (see Saphores 2002)

$$\frac{dF(\underline{x})}{dy} = \frac{dF(\overline{x})}{dy} = 0. \qquad (A.2)$$

The key result is:

Lemma 1. Let $F_1(y)$ and $F_2(y)$ be two independent solutions of the homogeneous equation associated with (A.1) (.i.e, (A.1) without $f(y)$). Then a particular solution of (A.1) is

$$F_P(y) = F_1(y) \int_{\underline{x}}^{y} \frac{F_1(\xi) f(\xi)}{\sigma^2(\xi) s(\xi)} \int_{\xi}^{y} \frac{-2s(\eta)}{F_1^2(\eta)} d\eta d\xi, \qquad (A.3)$$

where $s(\eta) = \exp \left\{ \int_{\underline{x}}^{\eta} \frac{-2\mu(\xi)}{\sigma^2(\xi)} d\xi \right\}$. In addition, the solution of (A.1)- (A.2) is given by

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$$F(y) = F_P(y) - \frac{dF_P(\bar{x})}{dy} \frac{\dfrac{dF_1(\underline{x})}{dy} F_2(y) - \dfrac{dF_2(\underline{x})}{dy} F_1(y)}{\dfrac{dF_1(\underline{x})}{dy} \dfrac{dF_2(\bar{x})}{dy} - \dfrac{dF_2(\underline{x})}{dy} \dfrac{dF_1(\bar{x})}{dy}}. \qquad (A.4)$$

Proof: Calculate the first and second derivatives of $F_P(y)$ and substitute in (A.1). As a linear combination of functions that verify (A.1), (A.4) does also (see Farlow (1993)). Direct calculations show that (A.2) is also verified. $\square$

When X_t follows (33), it is easy to check that

$$F_1(y) = \exp\left(\frac{\sqrt{2\rho}}{\sigma} y\right), \quad F_2(y) = \exp\left(-\frac{\sqrt{2\rho}}{\sigma} y\right), \qquad (A.5)$$

$$F_P(y) = \frac{1}{\sigma\sqrt{2\rho}} \int\limits_{\underline{x}}^{y} f(\xi) \left[\exp\left(\frac{\sqrt{2\rho}(\xi - y)}{\sigma}\right) - \exp\left(\frac{\sqrt{2\rho}(y - \xi)}{\sigma}\right) \right] d\xi. \qquad (A.6)$$

To transform the other integral in Equation (16), a change of variables leads to

$$E_x\left(\int\limits_{\Delta}^{+\infty} X_t.[V_1(X_t;x) - V_{01}(X_t;x)]e^{-\rho t} dt \right) =$$

$$e^{-\rho\Delta} \int\limits_{\underline{x}}^{\bar{x}} p(\xi,\Delta;x) E_\xi\left(\int\limits_{0}^{+\infty} X_t[V_1(X_t) - V_{01}(X_t)]e^{-\rho t} dt \right) d\xi, \qquad (A.7)$$

where $p(\xi,\Delta;x)$ is the value at ξ of the probability density function of X_Δ given that $X_0 = x$ (X_t is Markovian so its future distribution depends only on the time elapsed since the last known value of X_t). To calculate (A.7) when X follows (33), we use the Lemma above with (A.5)-(A.6) and the following result derived in Saphores (2005)

$$p(y,t;x) = \frac{1}{\bar{x} - \underline{x}} \left\{ 1 + 2\sum\limits_{n=1}^{+\infty} \exp\left(-\left(\frac{n\pi\sigma}{\bar{x} - \underline{x}}\right)^2 \frac{t}{2} \right) \cos\left(n\pi \frac{x - \underline{x}}{\bar{x} - \underline{x}} \right) \cos\left(n\pi \frac{y - \underline{x}}{\bar{x} - \underline{x}} \right) \right\}. \qquad (A.8)$$

Second, we need an expression for the discount factor $D_{x|y}$ with a reflecting barrier at $\underline{x}$;

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it is easily derived. Indeed, Karlin and Taylor (1981) show that $W(y) \equiv D_{x|y}$ verifies the linear,

2nd order, ordinary differential equation

$$\frac{\sigma^2(y)}{2}\frac{d^2W(y)}{dy^2} + \mu(y)\frac{dW(y)}{dy} - \rho W(y) = 0. \tag{A.9}$$

The two conditions needed to fully define $D_{x|y}$ are given in Saphores (2002):

$$\begin{cases} W(x) = 1, \\ \dfrac{dW(y)}{dy}\Big|_{y=\underline{x}} = 0, \end{cases} \tag{A.10}$$

If X_t follows (33), direct calculations show that, for $0 < y < x$,

$$D_{x|y} = \frac{e^{\frac{\sqrt{2\rho}}{\sigma}(y-\underline{x})} + e^{-\frac{\sqrt{2\rho}}{\sigma}(y-\underline{x})}}{e^{\frac{\sqrt{2\rho}}{\sigma}(x-\underline{x})} + e^{-\frac{\sqrt{2\rho}}{\sigma}(x-\underline{x})}}. \tag{A.11}$$

We can now find x^*. To proceed, first assume a value for $C_L(x^*)$; then maximize the objective function (14) using the Lemma above, with numerical integration and the conjugate gradient method in MathCad; once an approximate value for x^* is obtained, calculate a new value for $C_L(x^*)$ using (7) and (21); iterate until x^* and $C_L(x^*)$ change by less than 1% between consecutive iterations. MathCad allows controlling the precision of all of calculations; results with 2nd order Taylor expansions are typically within 1% of results with 1st order Taylor expansions. Finding x^* on a PC only takes a few seconds.

Table 1: Key Parameters and Notation.

| Name	Description
a, $1-a$	Utility elasticities of the numeraire good and of land.
B	City width (see Figure 1).
β	% of urban land used for the project.
$C_L(x^*)$	Cost of buying project land when the city population is x^*.
C_C	Cost of building the project, including future maintenance and operation.
$C(x^*) \equiv C_L(x^*) + C_C$	Total direct project cost.
g	Flow of individual income spent on the composite numeraire good.
h	Aerial distance from the CBD to a point in the city; $h \in [0,1]$ (see Figure 1).
λ	Ratio of the flow of transportation costs to the flow of individual income.
ν	$\nu \equiv a / (1-a)$ is a ratio of elasticities.
ρ	Social discount rate.
σ	Population volatility (in millions per $\sqrt{\text{unit of time}}$).
$\omega_1(x)$	Ratio of the flow of congestion costs at the city edge to the flow of individual income.
$\Omega_L(x;x^*) \equiv \dfrac{\rho C_L(x^*)}{x}$	Ratio of project land payments to the flow of total city income.
$\Omega_C(x) \equiv \dfrac{\rho C_C}{x}$	Ratio of construction and M&O payments to the flow of total city income.
$\Omega(x;x^*)$	$\equiv \Omega_L(x;x^*) + \Omega_C(x)$
$\underline{x}$, $\overline{x}$	Lower and upper bounds on the city population (in million).
x^*	Population threshold for building the project.
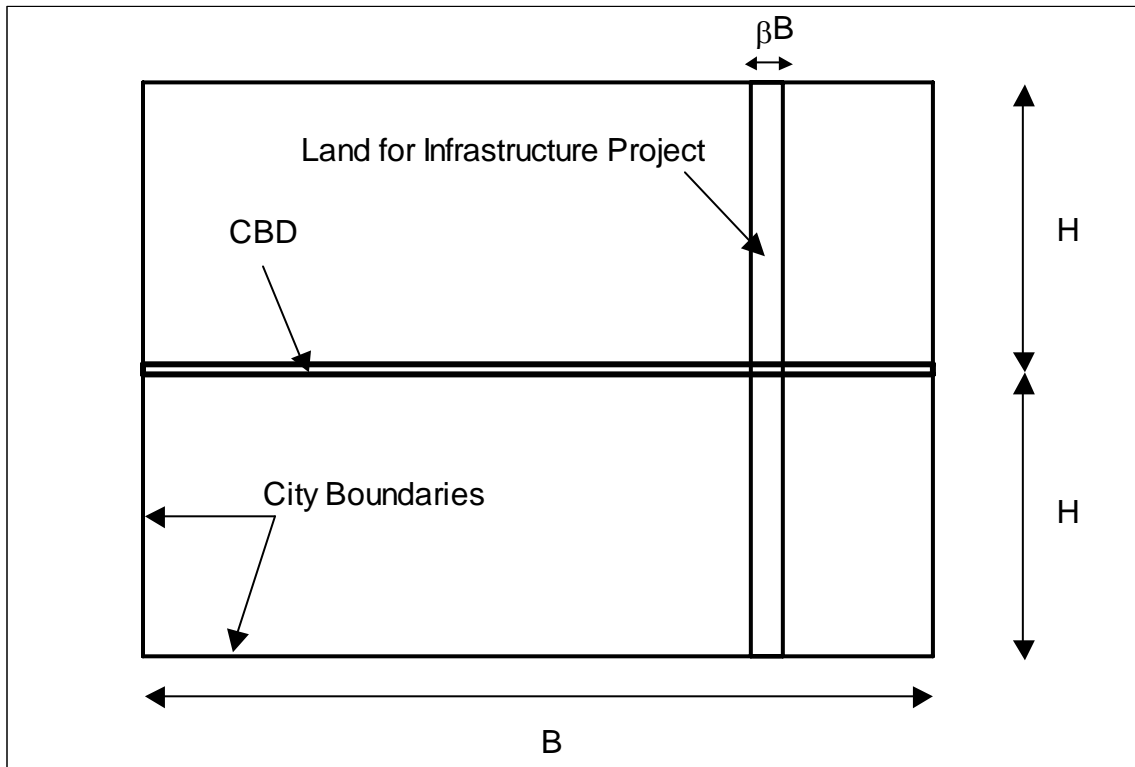
Figure 1: City map.
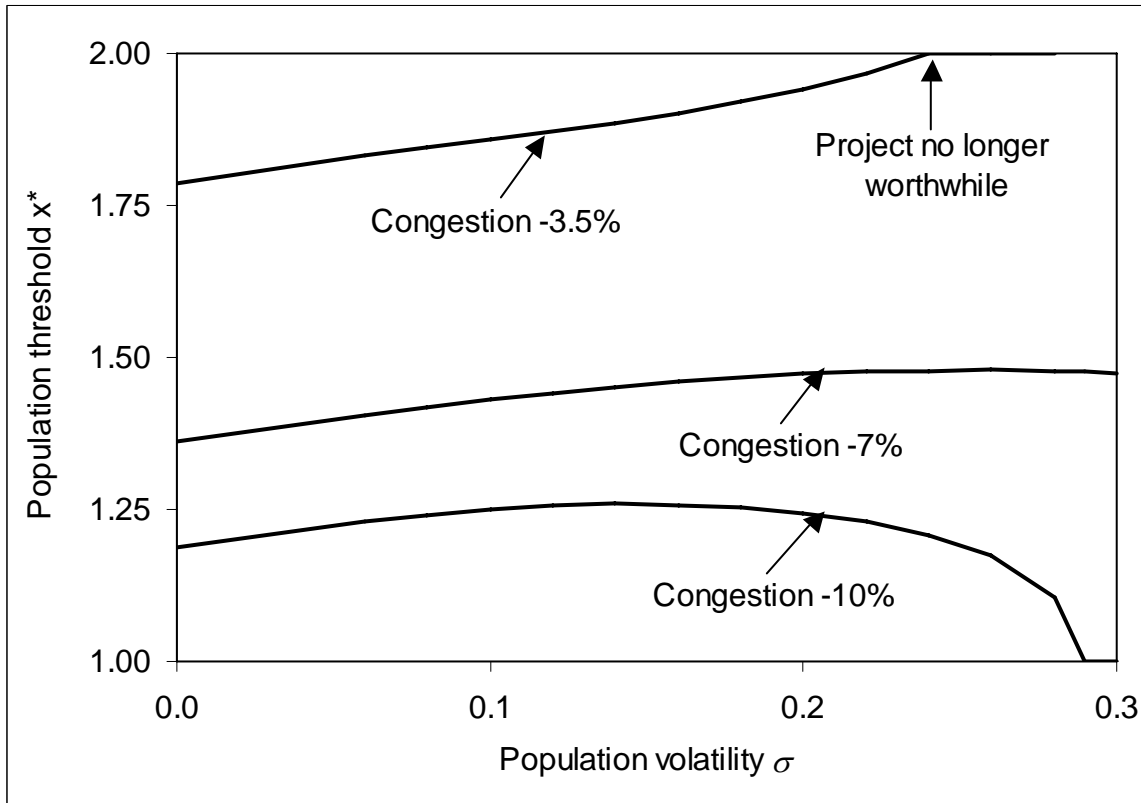
Figure 2: Population threshold x^* versus population volatility σ.

Notes. Model parameters for Figure 2: $\underline{x} = 1.0 \text{ million}$, $\bar{x} = 2 \text{ million}$, $\nu = 2$, $\Delta = 5$ years, $\beta = 0.00025$, $\Omega_C(\bar{x}) = 0.0002$, $\omega_0(\bar{x}) = 0.05$, $\lambda = 0$, and $\rho = 7\%$ per year; σ is in millions per $\sqrt{\text{time}}$, and a time unit is 10 years. See Table 1 for parameter definitions. x^*, the population threshold above which the project should be built, is in millions of inhabitants.
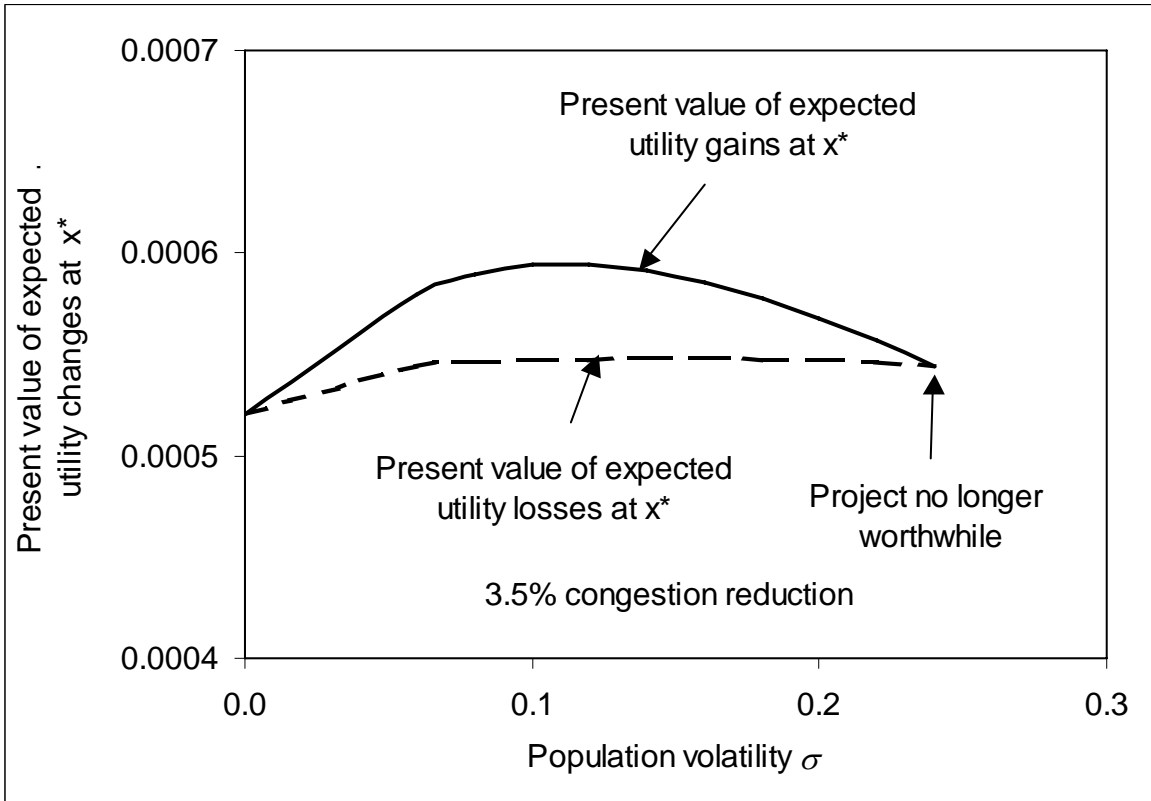
Figure 3A: Present value of expected utility changes versus σ for a 3.5% congestion reduction.
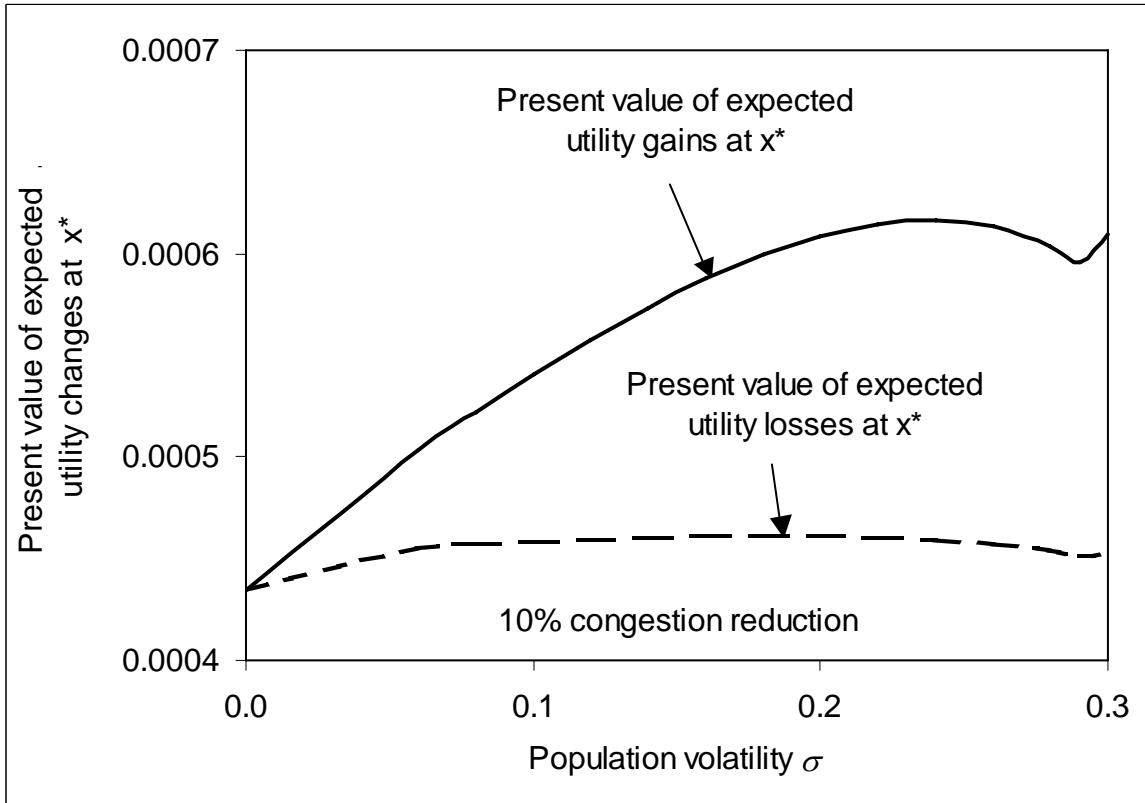
Figure 3B: Present value of expected utility changes versus σ for a 10% congestion reduction.

Notes. Model parameters for Figures 3A and 3B: $\underline{x} = 1.0$ million, $\bar{x} = 2$ million, $\nu=2$, $\Delta=5$ years, $\beta=0.00025$, $\Omega_C(\bar{x})=0.0002$, $\omega_0(\bar{x})=0.05$, $\lambda=0$, and $\rho=7\%$ per year; σ is in millions per $\sqrt{\text{time}}$, and a time unit is 10 years. See Table 1 for parameter definitions.
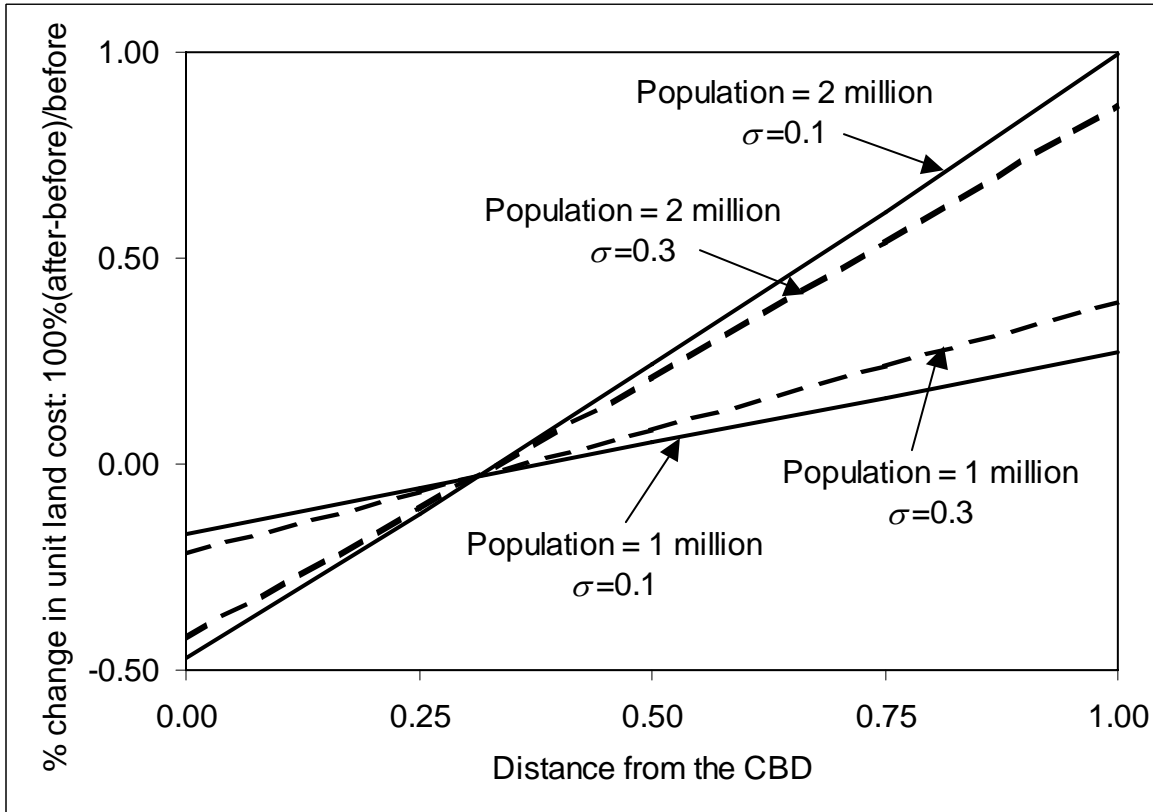
Figure 4: Percentage change in unit land costs versus distance to the CBD

Notes. Model parameters for Figure 5: $\underline{x}=1$ million, $\bar{x}=2$ million, $a=2/3$ so $v=2$, $\Delta=5$ years, $\beta=0.00025$, $\Omega_C(\bar{x})=0.0002$, $\omega_0(\bar{x})=0.05$, $\omega_1(\bar{x})=0.0465$, $\lambda=0$, and $\rho=7\%$ per year; σ is in millions per $\sqrt{\text{time}}$, and a time unit is 10 years. See Table 1 for parameter definitions.
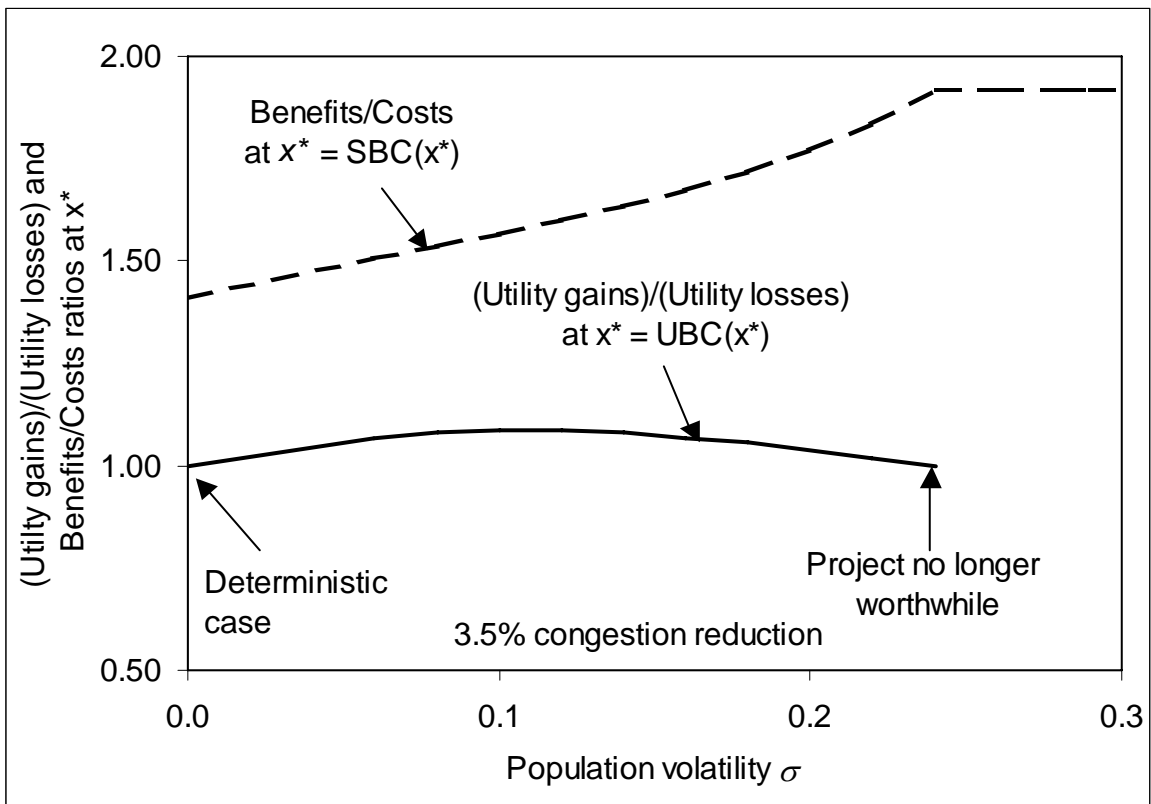
Figure 5A: Utility gains/losses and benefits/costs at x^* versus σ for a 3.5% congestion reduction.
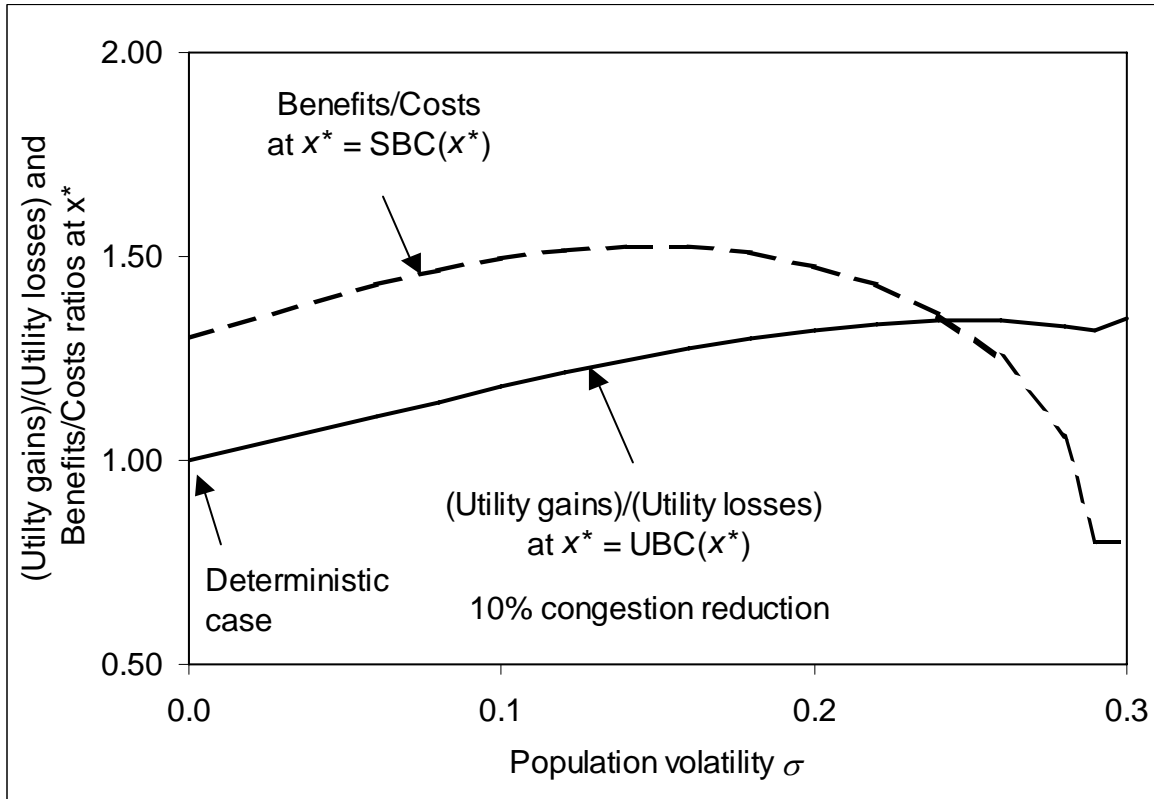
Figure 5B: Utility gains/losses and benefits/costs at x^* versus σ for a 10% congestion reduction.

Notes. Model parameters for Figures 5A and 5B: $\underline{x}$=1 million, $\bar{x}$=2 million, ν=2, Δ=5 years, β=0.00025, $\Omega_C(\bar{x})$=0.0002, $\omega_0(\bar{x})$=0.05, λ=0, and ρ=7% per year; σ is in millions per $\sqrt{\text{time}}$, and a time unit is 10 years. x^* (in millions) is the population threshold for building the project. See Table 1 for definitions.