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DISCLAIMER

A UNIVERSAL LAW FOR HIGH-TEMPERATURE DIFFUSION
CONTROLLED TRANSIENT CREEP

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ABSTRACT

The transient creep at high temperatures is suggested to arise

principally as a result of the dispersal of entanglements by the climb

mechanism. The dispersal of the entanglements is assumed to follow a

unimolecular reaction kinetics with a rate constant that depends on stress

and temperature in the same way as does the secondary creep rate. The

analysis shows that the strain ε, time t relation can be represented by:

$$\varepsilon = \varepsilon_0 + \dot{\varepsilon}_s t + \frac{\beta-1}{K}\left[1 - e^{-K\dot{\varepsilon}_s t}\right]$$

where ε_0 is the instantaneous strain on loading; $\dot{\varepsilon}_s$ the secondary creep

rate, $K\dot{\varepsilon}_s$ is the rate constant and β the ratio of initial to secondary

creep rate. The experimental creep data on several body centered cubic

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and face centered cubic metals and alloys correlate quite well with the

proposed mechanism. The constants β and K were found to be independent

of temperature and stress. The proposed formulation becomes inapplicable

for correlating creep data in polycrystals at low stresses because of the

significant contribution of grain boundary sliding to the total creep at

these stress levels.

I. INTRODUCTION

Most of the early investigations[1] on high-temperature creep showed that

$$\varepsilon = \varepsilon \left\{ t \ e^{\frac{-Q_c}{RT}} \right\} \quad \text{for } \sigma = \text{constant} \tag{1}$$

over both the transient and steady-state stages. Here ε is the total strain, which is the sum of the initial strain ε_0 upon stressing at $t = 0$ and the creep strain up to time t. The apparent activation energy, Q_c, for creep was shown to be insensitive to the stress, strain, grain size and temperature. The activation energy agreed very well with that for diffusion. The subsequent years produced extensive experimental evidence to confirm the validity of Eq. (1), but not much progress was achieved to further advance the understanding of the transient stage of creep. It is only over the last few years that several important observations have been made that have encouraged a more thorough analysis of transient creep.

For many materials the creep strain is found to follow Andrade's empirical equation,[2] namely

$$\varepsilon - \varepsilon_0 = \dot{\varepsilon}_s t + Bt^{1/3} \tag{2}$$

where $\dot{\varepsilon}_s$ is the steady-state creep rate and B is a constant. The formulation often appears to be in fair agreement with high temperature creep data. One objection to the universal application of Andrade's equation concerns its prediction of infinite initial creep rates. Although it is

admittedly difficult to measure the initial creep rates accurately, many

carefully conducted experiments suggest that they are actually finite.

An alternative formulation that has been employed is:

$$\varepsilon - \varepsilon_0 = \dot{\varepsilon}_s t + \varepsilon_T \left(1 - e^{-rt}\right) \tag{3}$$

where ε_T is the limiting transient strain and r is the ratio of the

transient creep rate to the transient creep strain. Equation (3) was

first suggested on a purely empirical basis by McVetty.[3] It has been

applied subsequently by Garafalo,[4] Conway and Mulliken[5] and by Evans and

Wilshire.[6]

It is the purpose of this report to provide a unified analysis for

high-temperature creep that will incorporate the effect of both tempera-

ture and stress on the shape of transient creep curve. Section II of

the report will summarize the pertinent substructural changes that

accompany transient creep and will emphasize the role of dislocation

climb in the dispersal of entanglements. Section III will be used to

develop analytical theory for transient creep that is controlled by climb

of dislocations based on unimolecular reaction-rate kinetics. The

validity of the adoption of unimolecular rate kinetics will be justified

in Section IV from correlation of experimental data.

II.   SUBSTRUCTURAL CHANGES DURING TRANSIENT CREEP

A recent review[7] of high-temperature creep reveals that only those

metals and alloys that undergo initial straining upon stressing exhibit

the usual normal transient stage of creep during which the creep rate

progressively decreases to that for the steady-state. Those alloys which

do not undertake significant initial straining at t = 0 upon application

of stress do not exhibit the usual normal transient stage. Instead, they

display either a brief normal transient (BNT) or a brief inverted tran-

sient (BIT) or no transient behavior at all. Such BNT or BIT behavior,

when present, usually does not extend beyond the first five or ten

minutes of the test.

Therefore, normal transient creep rates appear to result from changes

in the substructure from that which is produced immediately upon stress-

ing to that which pertains to the steady state. The substructures which

are formed upon initial straining at creep temperatures closely resemble

those which are developed during strain hardening over stage III deforma-

tion at lower temperatures.[8-10] Most of the dislocations are arranged

in rough cellular patterns, the walls of which are composed of dis-

location entanglements. Some cells contain a few randomly meandering

dislocations in their interior.

Etch pitting studies[8,11] and electron microscopy[10-12] have shown

that dislocations rearrange themselves during the early part of transient

creep as a result of the extra degree of freedom resulting from dis-

location climb which becomes facile at high temperatures. Consequently,

the entanglements disperse and adjacent cells coalesce so as to produce

more sharply delineated subgrains which have a volume of about eight

times that of the original cells. Over the secondary stage of creep an

invariant substructure persists which depends only on the magnitude of

the applied stress. It consists of subgrains separated from one another

by low angle boundaries within which there exists a few isolated and

randomly meandering dislocations.

The substructural changes that take place during normal transient

creep are extremely complex. This is clearly revealed by the careful

investigations of Clauer, Wilcox and Hirth.[11]   Changes in the different

substructural details over the transient stage are obviously inter-

related. The dispersal of entanglements, the build up of subboundaries,

the changes in density of dislocations and the alterations in misorienta-

tions across subboundaries, etc., are not mutually independent. Among

these various substructural details, only the changes in dislocation

density have as yet been placed on a quantitative basis.

Changes in density of dislocations during transient creep have been

studied by several investigators.[8,11,13]   It is difficult to accomplish

this with satisfactory accuracy because of the major changes that take

place in the geometrical distribution of dislocations. Also, it is not

immediately obvious how the density of dislocations should be partitioned

among those entangled in disappearing cell-walls, those joining the grow-

ing low-angle boundaries and those that are more or less randomly dis-

tributed. As the transient creep rate approaches that for the steady

state, it is customary to count only those dislocations that are within

the subgrains. But as shown by Clauer, Wilcox and Hirth,[11] moving tilt

boundaries form a significant part of the substructure during transient

creep of single crystals of Mo and consequently the dislocations in the

subboundaries also deserve special attention. Although the state of

knowledge concerning substructure is admittedly unsatisfactory, the

available experimental data suggest that dislocation densities decrease

only slightly during the dispersal of the entanglements in transient

creep. The decrease in creep rate during normal transient stage might

be assumed to indicate that strain hardening is taking place. Strain

hardening, however, usually has the implication of increased density of

dislocations. It therefore appears that the term recovery strengthening

due to dispersal of entanglements by dislocation climb is preferable.

III. ANALYSIS

The substructural changes taking place during transient creep are

so complex as to preclude any detailed mechanistic analysis of the

process at this time. Although the dislocation density decreases over

the transient stage of creep, it alone cannot account for the much

greater percentage decrease in the creep rate. The major substructural

change, as pointed out by Gupta and Strutt,[8] centers about the dispersal

of entanglements but the relationship between such dispersal and the

transient creep rate is not yet understood. Undoubtedly all sub-

structural changes during high-temperature transient creep are inter-

related and dependent upon the dislocation climb mechanism. We there-

fore suggest that transient as well as steady-state creep is controlled

by the rate of climb of dislocations. This is consistent with the

validity of Eq. (1). Furthermore, we also assume that reactions taking

place obey the laws of unimolecular kinetics. This assumption will be

verified a posteriori.

Assuming that dislocation climb kinetics control transient creep

the rate constant can be given in terms of the temperature and stress as

$K\dot{\varepsilon}_s$ where $\dot{\varepsilon}_s$ is the steady-state creep rate as controlled by climb and K

is a constant. As shown by Webster, Cox and Dorn[14] the unimolecular can

be formulated as:

$$\frac{d(\dot{\varepsilon} - \dot{\varepsilon}_s)}{dt} = -K\dot{\varepsilon}_s(\dot{\varepsilon} - \dot{\varepsilon}_s) \tag{4}$$

Integrating Eq. (4) twice one obtains:

$$\varepsilon = \varepsilon_0 + \dot{\varepsilon}_s t + \frac{\dot{\varepsilon}_i - \dot{\varepsilon}_s}{K\dot{\varepsilon}_s}\left(1 - e^{-K\dot{\varepsilon}_s t}\right) \tag{5}$$

where $\dot{\varepsilon}_i$ is the initial creep rate at $t = 0$, ε_0 is the initial strain upon stressing and

$$\frac{\dot{\varepsilon}_i - \dot{\varepsilon}_s}{K\dot{\varepsilon}_s} = \varepsilon_T \tag{6}$$

where ε_T is the transient strain. Since the transient and steady-state stages have the same energy of activation and also the same kinetics of reaction as implied by Eq. (1) it follows necessarily that $\dot{\varepsilon}_i = \beta \dot{\varepsilon}_s$ where β is a constant greater than unity. Therefore the total transient strain

$$\varepsilon_T = \frac{\dot{\varepsilon}_i - \dot{\varepsilon}_s}{K\dot{\varepsilon}_s} = \frac{\beta-1}{K} \tag{7}$$

where β, K are constants independent of stress and temperature. A significant implication of Eq. (7) is that although $\dot{\varepsilon}_i$ and $\dot{\varepsilon}_s$ are both functions of stress, ε_T is a constant and like β and K is independent

of stress and temperature.

According to the above analysis, the following must apply:

1. The initial strain upon stressing, ε_0 is athermal and depends on the original state of the metal or alloy and the value of σ/G where σ is the stress and G is the shear modulus.

2. The ratio of the initial to secondary creep rates β, is a constant independent of stress, temperature and initial strain.

3. The rate of dispersal of the entanglements depends on the same function of stress and temperature as does the secondary creep rate.

4. For a given metal or alloy the transient strain ε_T is constant independent of stress or temperature.

5. For a given metal or alloy there exists a universal high temperature transient and steady-state creep curve of the form

$$\varepsilon - \varepsilon_0 = f(\dot{\varepsilon}_s t) = \phi \left[(\sigma/G)^n \frac{DGb}{kT} t \right] \qquad (8)$$

where the function f is derived from the general equation

$$\frac{\dot{\varepsilon}_s kT}{DGb} = \text{Constant } (\sigma/G)^n$$

as shown to be valid by a general survey[15] on secondary creep. Here D is the diffusivity, b is the Burgers vector, n is the stress dependence of secondary creep rate and kT has the usual meaning of Boltzmann constant times absolute temperature. The relationship depicted in Eq. (8)

is an extension of Eq. (1), which now incorporates the effects of stress

as well as temperature on the shape of transient creep curve.

IV. CORRELATION WITH EXPERIMENTAL DATA

A careful survey and reanalysis was made of all the experimental

results on high-temperature transient creep where the pertinent data on

diffusivity and shear modulus were also available. A summary of all

the examples that have been analyzed is given in Table I.

The validity of Eq. (5) is shown by the creep curves depicted in

Fig. 1 (a) through (f) for experimental data on Al,[16] Ag,[17] purified

Fe,[18] Fe containing carbon,[19] Cu,[20] plain carbon steel,[21] Mo single

crystals,[22] low carbon Ni,[24] Pt[26] and stainless steel.[12] In general,

the data spanned a range of stress levels and in the case of data on

Pt, low carbon Ni, Al, Ag, and Fe containing carbon, a range of tempera-

tures as well. The creep curves clearly reveal that regardless of

temperature and stress, the data for each material falls well on a

single curve which agrees well with the solid theoretical curve of

Eq. (5). Figure 2 shows the initial strain, ε_0, as a function of (σ/G)

for the cases that were analyzed. These data refer to the modulus-

adjusted athermal stress-strain curves at high temperatures. These data

show considerable scatter, primarily because of difficulties in deter-

mining the initial strains immediately following stressing. The

secondary creep rates $\dot{\varepsilon}_s$, for the metals and alloys in question are

shown in Fig. 3, where log $\dot{\varepsilon}_s kT/DGb$ is plotted as a function of log σ/G.

The curves agree fairly well with similar curves that were presented in

a recent review.[7] There is some scatter in the data, particularly that

for Pt and to a lesser extent in Al, both of which showed somewhat

higher stress dependence of the secondary creep rates than were re-
ported[7] earlier. The plot for Ag on the other hand revealed a lesser
stress dependence of $\dot{\epsilon}_s$. In general, however, the plots clearly fall
into the category of creep as controlled by the dislocation-climb
mechanism.

A typical example of the universal creep curve for the case of
polycrystalline Ni is shown by the datum points of Fig. 4(a). Regard-
less of stress and temperature all data fall well on a single curve.
The solid line refers to the theoretical expression given by Eq. (8).
An even more convincing proof of the validity of the universal creep
curve is shown in Fig. 4(b) for Nb, where the experimental data cover
ranges of temperature as well as stress and the secondary creep rates
vary by two orders of magnitude. The remaining cases that were examined
were about equally consistent and illustrated the good agreement of the
theory with the experimental results.

The stress dependence of $\dot{\epsilon}_i$ and $\dot{\epsilon}_s$ is shown in Fig. 5 where
$\dot{\epsilon}kT/DGb$ is plotted against σ/G in dimensionless units. The datum points
refer to creep of Ni[23] at one temperature and five stress levels and to
creep of Nb[25] for a series of temperature or stress levels. The figure
illustrates that the initial creep rates, $\dot{\epsilon}_i$, for a series of stresses
and temperatures are greater than the secondary creep rates, $\dot{\epsilon}_s$, by a
constant factor β. Both $\dot{\epsilon}_i$ and $\dot{\epsilon}_s$ are dependent on the substructural
details that develop and hence are functions of stress. As shown in
Fig. 5, their difference is a constant regardless of stress and tempera-
ture and is not a function of state.

Values for the transient strain ε_T were deduced from curves in Figs. 1(a-f) and Fig. 4(a-b). The rate constant K for different cases was determined at one half of the transient strain ε_T where Eq. (5) becomes $K = \ln 2/\dot{\varepsilon}_s t$. The calculated values for the constants K, β and ε_T are shown in Table I. Whereas the transient strain ε_T exhibits rather pronounced variations from case to case, the rate constant K seems to be somewhat less variable. Undoubtedly both depend on significant substructural details . Up to the present, however, no consistent trends in variations of K or β have been uncovered relative to effect of grain size, stacking-fault energies, or any other pertinent structural details.

Evans and Wilshire[6] noted that Eq. (5) becomes inaccurate for creep of polycrystalline iron at low stresses. Similar discrepancies, which are observed in the creep of austenitic stainless steel, have been shown by Webster, Cox and Dorn [14] to arise from the increased contribution of grain boundary sliding to the total creep rate at the low applied stresses. (The data for stainless steel by Garofalo were analyzed here only at higher stress levels where the contribution of grain boundary sliding is less than 0.04 of the total creep strain.) When grain boundary sliding prevails, $K\dot{\varepsilon}_s$ is no longer the pertinent rate constant since it refers to a creep mechanism resulting from climb alone. Once grain boundary sliding is better understood quantitatively, this factor might also be taken into consideration so as to completely account for high temperature transient creep at low stresses in polycrystals as well.

V. CONCLUSIONS

1. It has been shown that the creep equation

$$\varepsilon = \varepsilon_0 + \dot{\varepsilon}_s t + \varepsilon_T \left[1 - e^{-K\dot{\varepsilon}_s t} \right]$$

can be derived from the assumption that transient creep follows a first order kinetic reaction rate theory, with a rate constant that depends on stress and temperature in the same way as does the secondary creep rate.

2. High-temperature transient creep is suggested to result from substructural modifications due to stress and diffusion controlled climb of dislocations.

3. For a given metal or alloy there exists a universal high temperature transient and steady-state creep curve that incorporates the effect of temperature as well as stress in the general form of

$$\varepsilon = \varepsilon_0 + f(\dot{\varepsilon}_s t) = \phi \left[(\sigma/G)^n \cdot \frac{DGb}{kT} t \right] + \varepsilon_0$$

4. For any given metal or alloy the transient strain ε_T is constant independent of stress or temperature.

5. The initial creep rate $\dot{\varepsilon}_i$ for a given metal or alloy is a constant multiple, β, of the steady-state creep rate $\dot{\varepsilon}_s$, regardless of stress and temperature.

6. The initial strain ε_0 depends on the original state of the metal or alloy and the value of (σ/G).

7. In polycrystals at low stress levels, $K\dot{\epsilon}_s$ is no longer the pertinent rate constant and the effect of grain boundary sliding need yet be taken into consideration.

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FIGURE CAPTIONS

Fig. 1. Experimental data for Eq. (5) as $\epsilon - \epsilon_0$ versus $\dot{\epsilon}_s t$.

Fig. 2. Initial strain ϵ_0, versus (σ/G).

Fig. 3. The stress dependence of the secondary creep rate $\dot{\epsilon}_s$ plotted as

$$\text{Log } \frac{\dot{\epsilon}_s kT}{DGb} \quad \text{versus Log } (\sigma/G).$$

Fig. 4. The Universal Creep Curve.

Fig. 5. The stress dependence of $\dot{\epsilon}_i$ and $\dot{\epsilon}_s$ for Nb and Ni plotted in

dimensionless units.

ACKNOWLEDGMENT

Table I. Summary of Analyzed Transient Creep Data

Metal (Polycrystalline)	Crystal Structure	Grain Size (mm)	ε_T	K	β	Chemical Composition	Ref
Al	F.c.c.	0.21	0.105	82	9.56		16
Ag	F.c.c.	0.017	0.013	99	2.28		17
Alpha-Fe	B.c.c.		0.053	92.5	5.9	C, N <50 p.p.m., total impurity <100 p.p.m.	18
α - Fe	B.c.c.	0.025	0.017	82.5	2.4	C: 0.054, N: 0.0005, O: 0.0005, Si: 0.013, S: 0.0026, P: 0.001, Mn: 0.001, Al: 0.006	19
Cu	F.c.c.	0.03	0.0085	277	3.35	Fe: 40 p.p.m, S: 8, Ag: 6, Ni: 5, Pb: 4	20
Plain C - Steel	B.c.c.	austenitic grain size 0.21-0.104	0.024	116	3.77	C: 0.23, Si: 0.13, Mn: 0.60, S: 0.036, P: 0.036, cr: 0.092 Ni: 0.17 and Cu: 0.125%	21
Mo [S] Single crystal	B.c.c.		0.052	173	10.00		22
Ni	F.c.c.		0.042	346	12.9		23
low c - Ni	F.c.c.		0.036	330	12.88	C: 0.01, Mn: 0.03, Si: 0.1	24
Nb	B.c.c.	0.17 - 0.36	0.032	173.2	10.2	C: <0.015, O: 0.076, Ta: <0.25	25
Pt	F.c.c.		0.01	138.6	2.39	Traces of Fc, Pd <0.01 wt %	26
Stainless Steel	F.c.c.	0.09	0.065	34.60	3.26		12
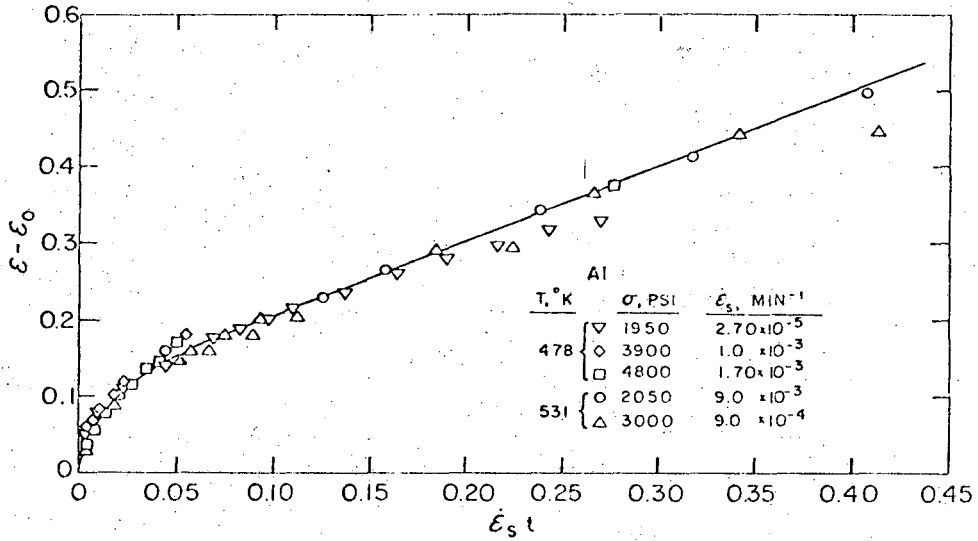
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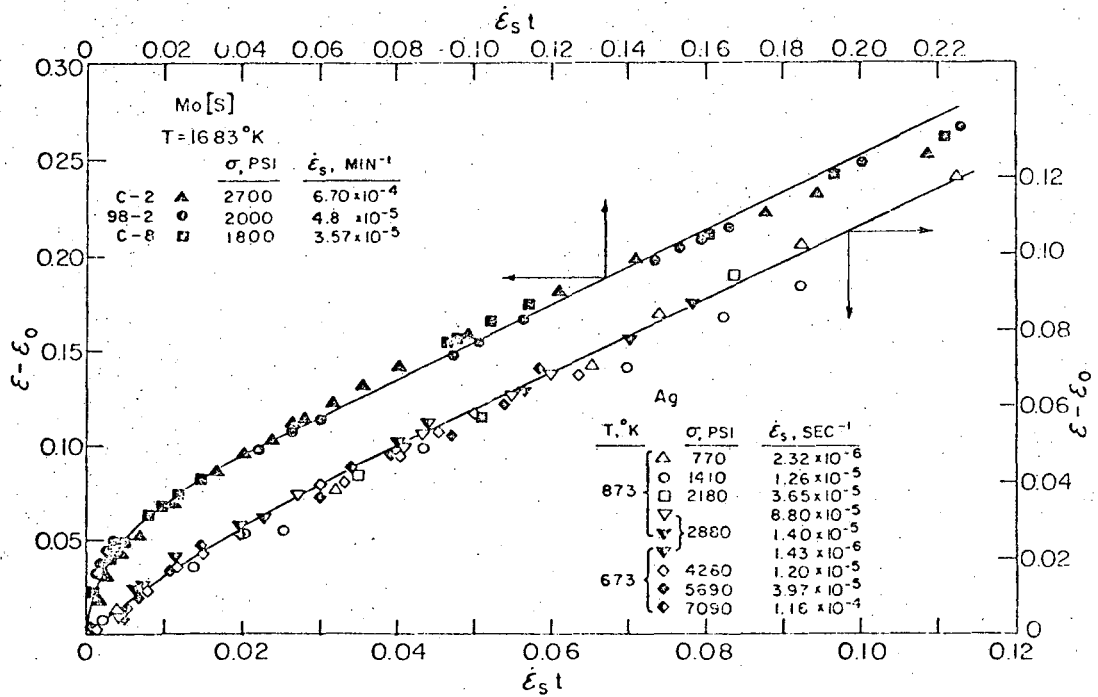
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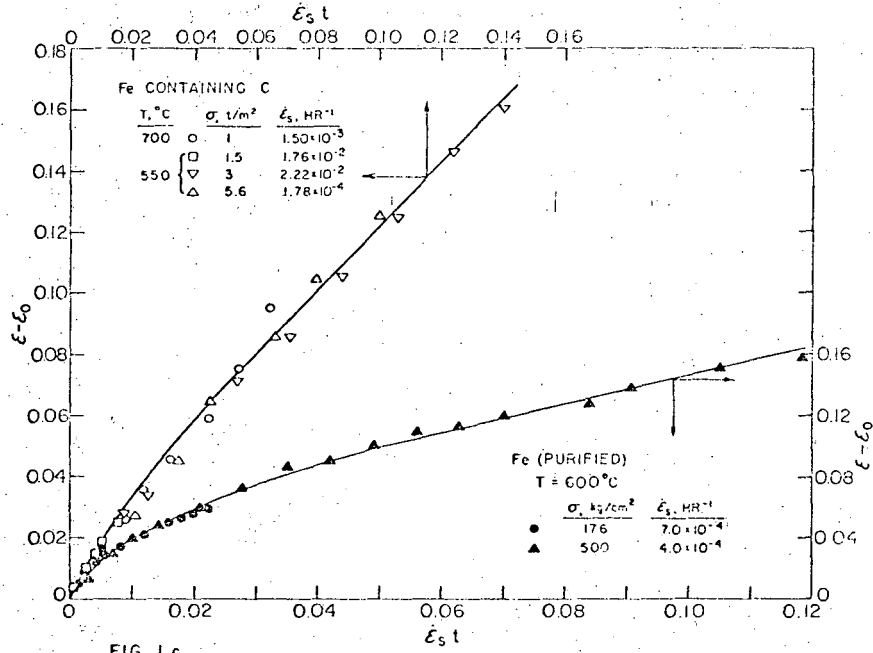
XBL 701-182

FIG. 1 c
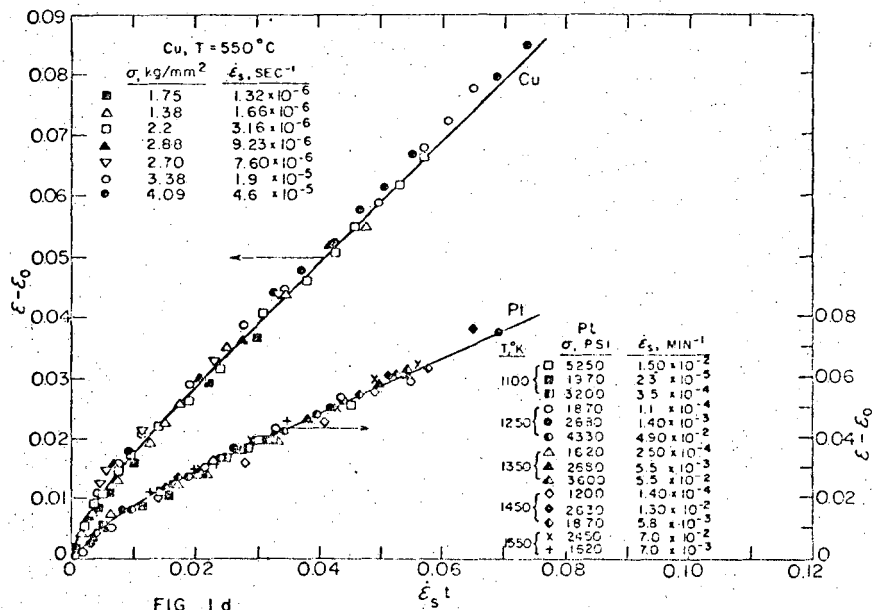
FIG. 1 d

XBL 701-181

FIG. I e

FIG. I f
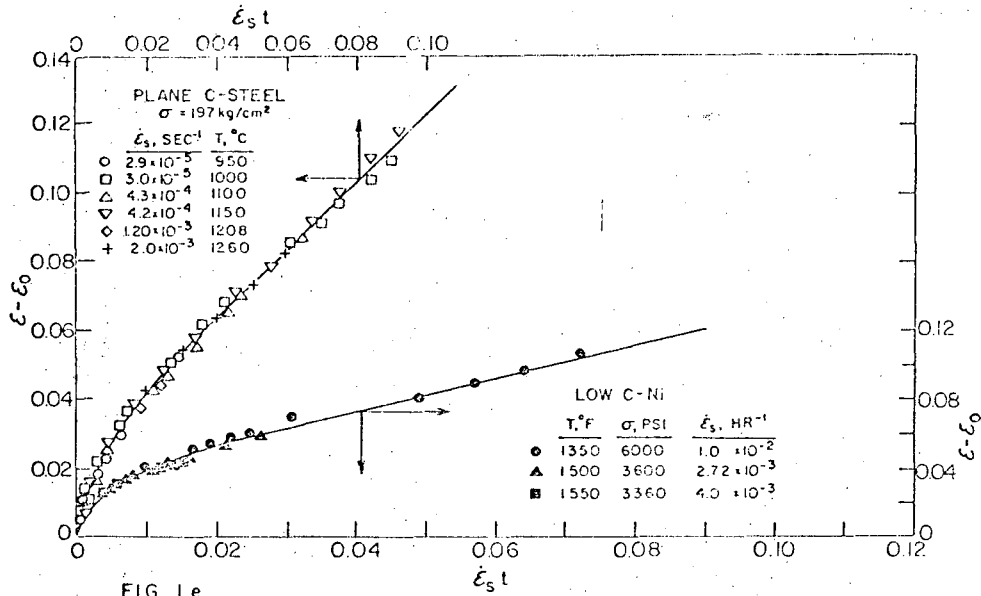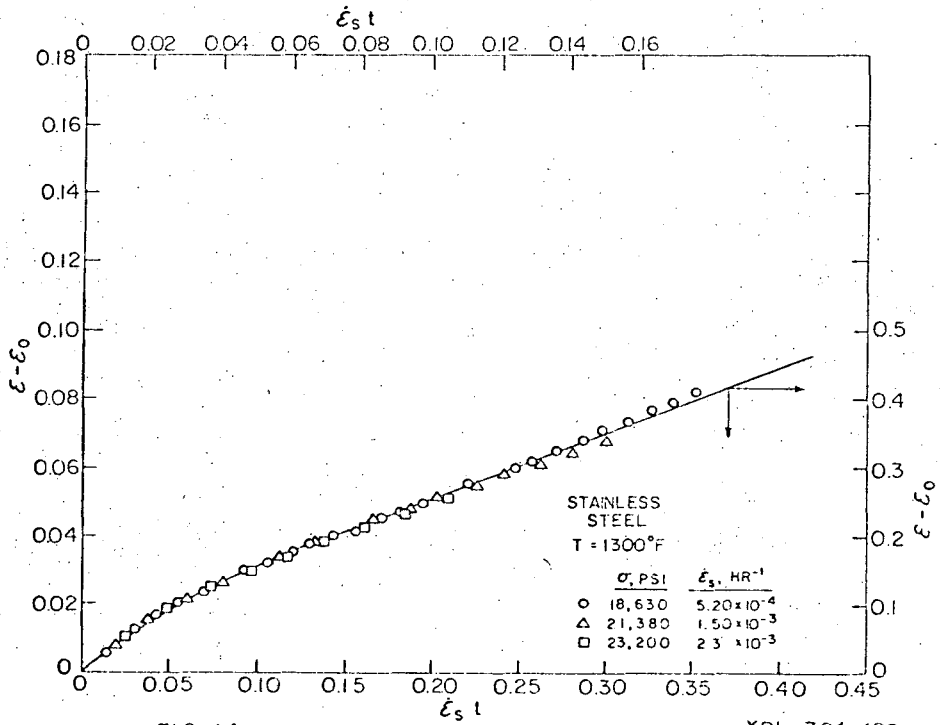
XBL 701-183

FIG. I   EXPERIMENTAL DATA FOR EQUATION (5) AS $\epsilon - \epsilon_0$ vs. $\dot{\epsilon}_s t$

FIG 2 INITIAL STRAIN, ε, vs. σ/G.

XBL 701-175

-22-
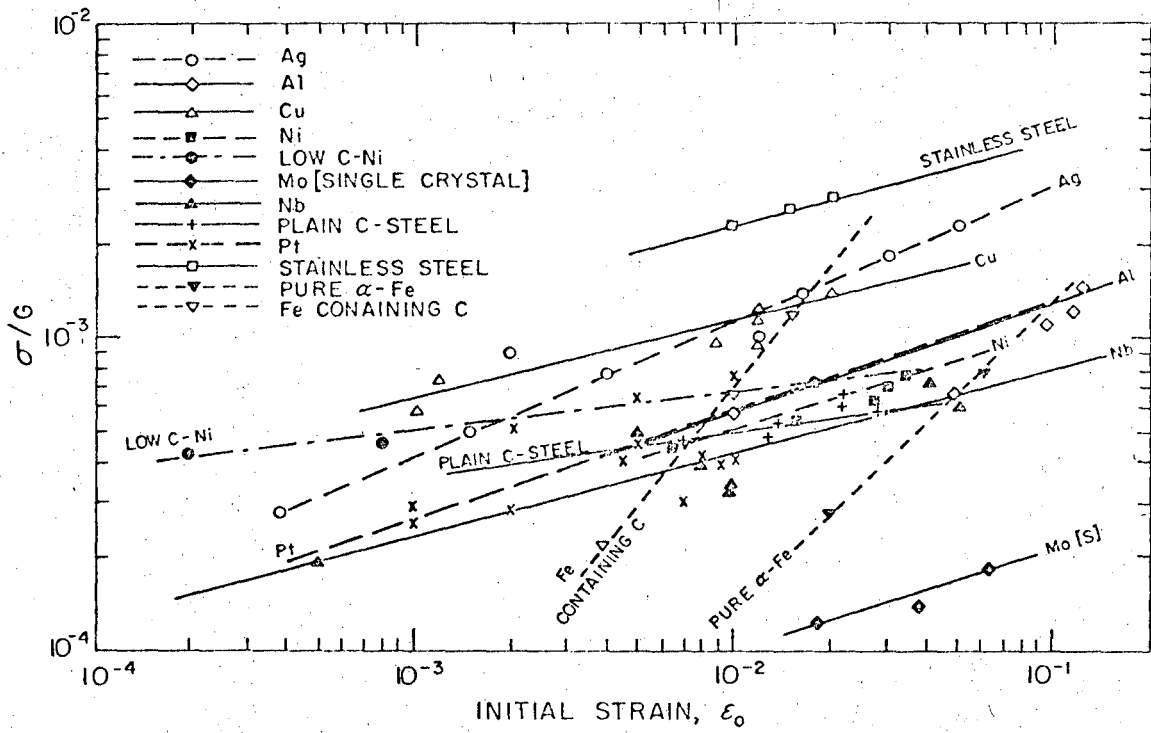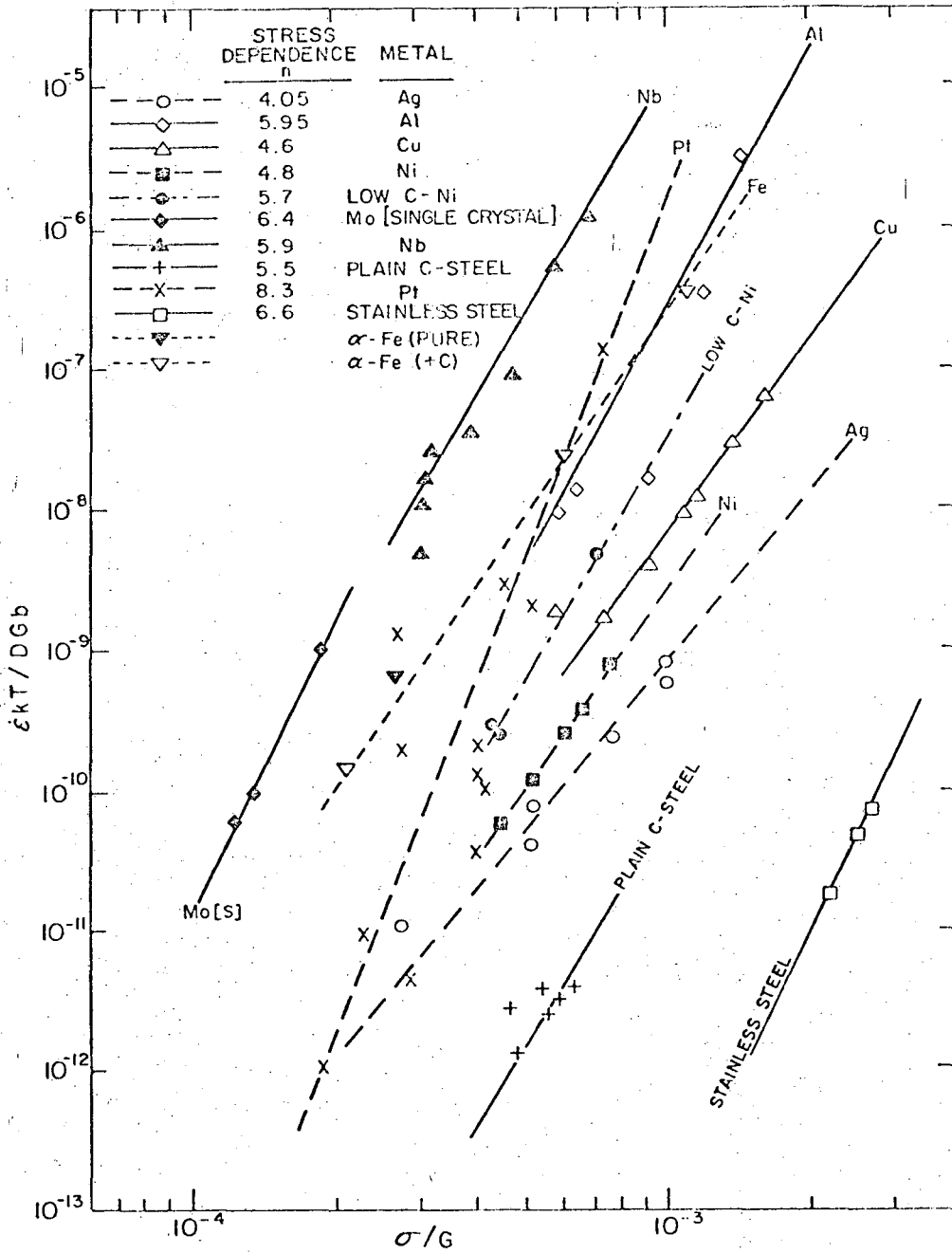
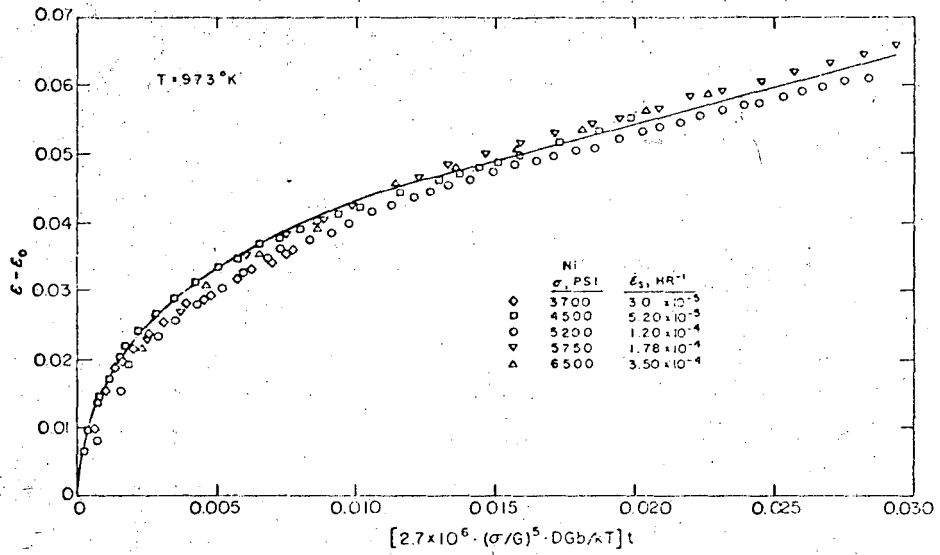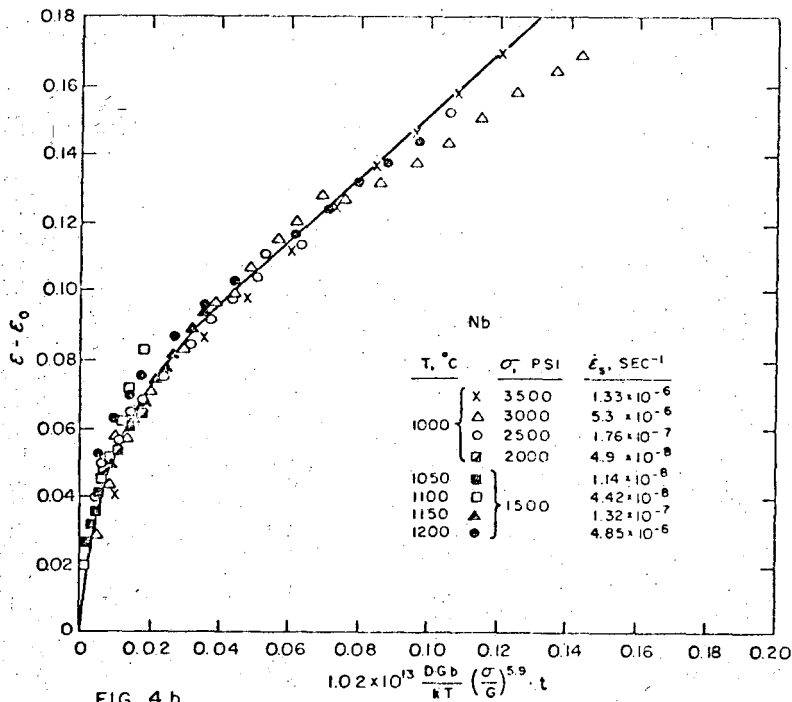
FIG. 3   STRESS DEPENDENCE OF SECONDARY CREEP RATE.

XBL 701-180

FIG. 4a
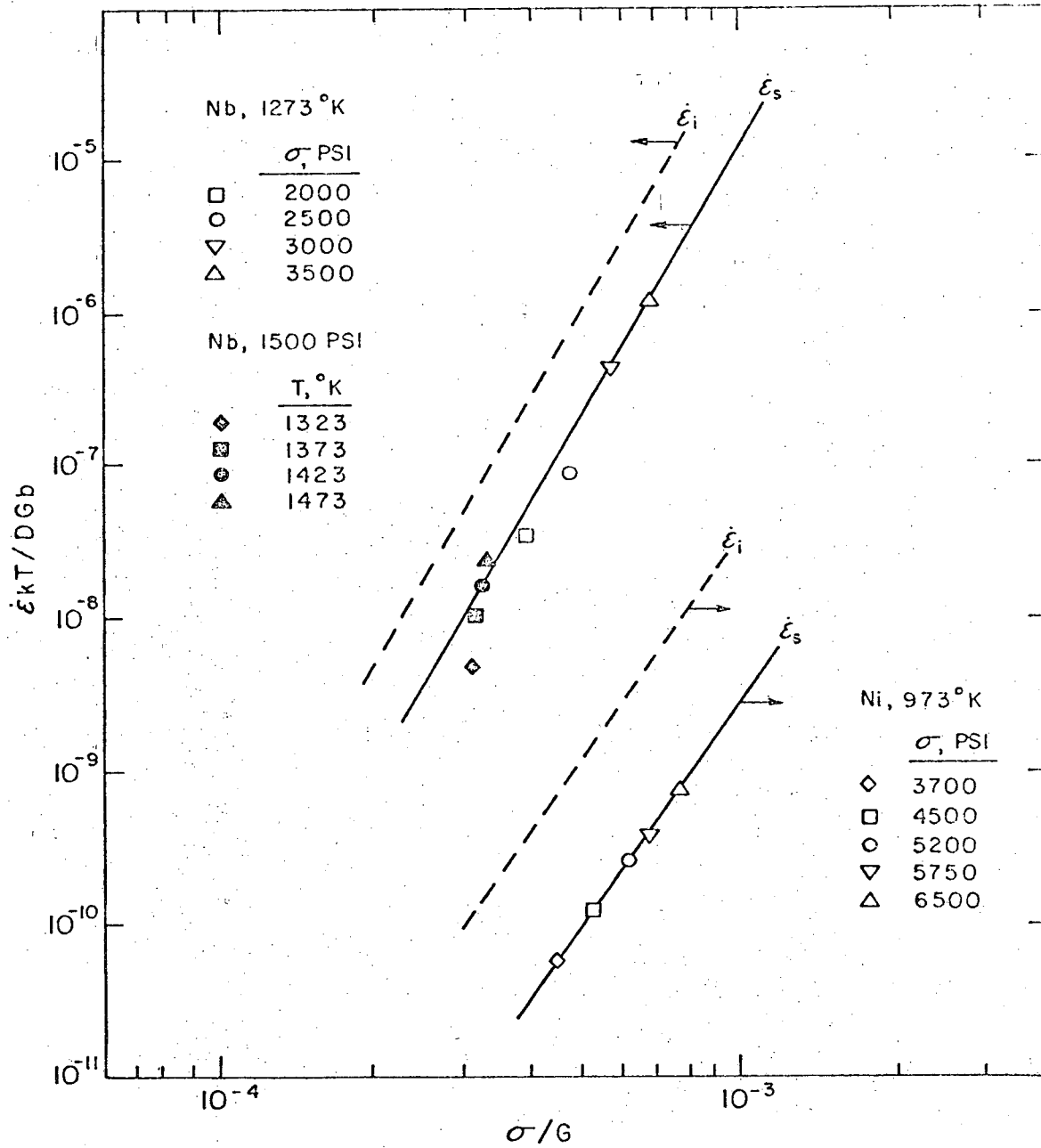
FIG. 4b

XBL 701-184

FIG. 4 THE UNIVERSAL CREEP CURVE.

FIG. 5 THE STRESS DEPENDENCE OF INITIAL AND SECONDARY
CREEP RATE FOR Nb AND Ni.

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