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Author

Zholents, A.

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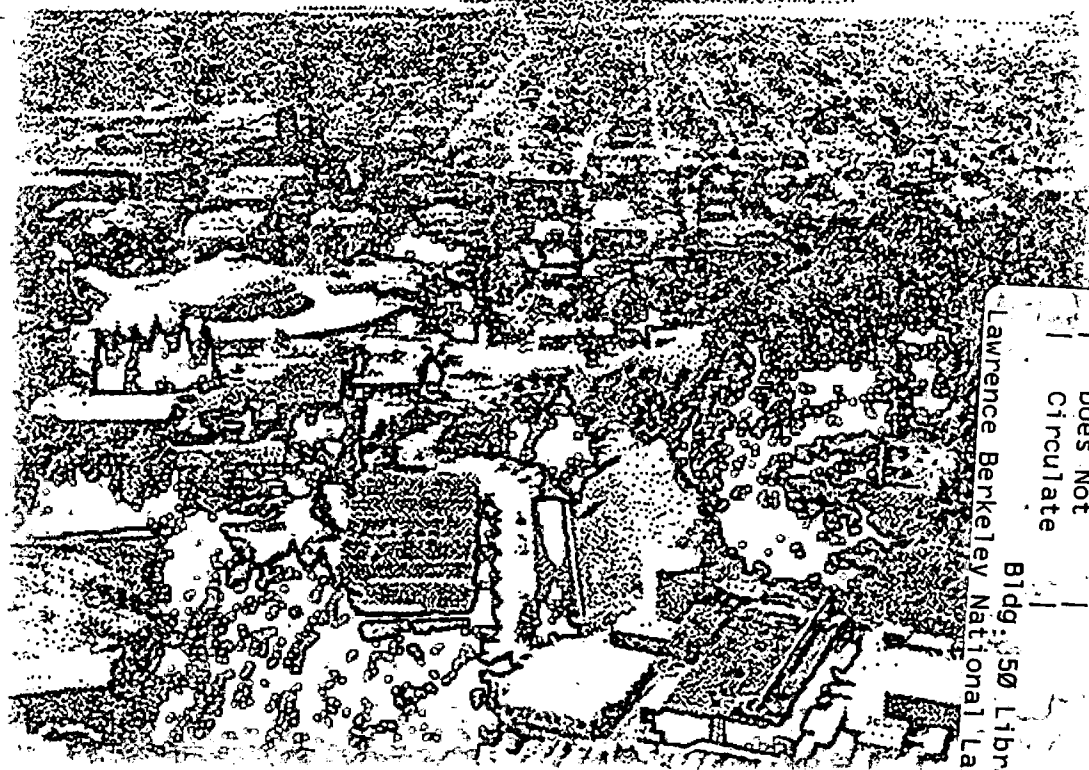


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A. Zholents and M. Zolotarev
Accelerator and Fusion
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and Energy Spread***

A. Zholents and M. Zolotarev
Lawrence Berkeley National Laboratory
University of California
Berkeley, California 94720

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Correlation Technique for Measurements of Beam Emittance and Energy Spread

A. Zholents and M. Zolotarev

August 6, 1996

Abstract

When a beam of charged particles passes through a lattice with bending magnets, then mixing of the relative longitudinal positions of particles can take place. This mixing is caused by the energy spread of beam particles and by the spread of the particle betatron coordinates and angles and has the effect of changing the time structure of the fields radiated by the beam at different points of its trajectory. It is shown in the paper that by measuring the correlation function of the radiation fields at two points of the beam trajectory one can determine the beam emittance and the beam energy spread. A detailed analysis of this diagnostic technique with a few illustrative examples, is provided.

1 Introduction

In this paper we propose a new technique for measuring the emittance and the energy spread of a relativistic charged particle beam. This technique can be used in both beam transport lines and circular accelerators. Based on the same principle, measurements can be done in many different ways, depending on the practical constraints and beam characteristics. In this paper, we describe only the idea of the measurement technique, but not its realization, although we mention few examples of possible measurement set ups. The most suitable application of our diagnostic technique would be measurements of the emittance of low emittance beams when it is difficult to do these otherwise. Measurements of the beam energy spread in storage rings and in the beam transport lines are routinely done through the measurement of the transverse beam size in the dispersive section. But it is hard to do accurate measurements when the beam energy spread contribution to the transverse beam size is small. In this case, measurements of the beam energy spread with a new technique would be useful.

2 Description of the working principle

The diagnostic technique that we are going to discuss here is based on the analysis of the correlation function of the fields that are radiated by charged beam particles at different

points of the beam trajectory. For diagnostic purposes, it is possible to use any kind of radiation providing that this radiation is incoherent, i.e., the bunch length of the beam is larger than the typical radiation wavelengths.

In its simplest form, the basic idea of the measurement technique can be explained using the example of the measurement of the beam energy spread. Imagine that the radiation of the beam particles is observed from two identical points of the beam trajectory separated by an achromat section of the lattice.¹ The far field beam radiation can be considered as a superposition of the radiation fields of the individual particles. Therefore, a temporal structure of the radiation field is defined by the particle positions in the beam in the longitudinal direction at the moment of the radiation. Imagine now that the particle positions do not change when the beam comes to the second radiation point. Then, the far field of the beam radiation at the second radiation point will be exactly the same as at the first radiation point. This means that the two radiation fields are coherent. If, on the contrary, particles change their longitudinal positions relative to each other between the first and the second radiation points (we call this process longitudinal mixing), then the two radiation fields are only partially coherent or perhaps not coherent at all. The degree of coherence can be determined by measuring the correlation function of the two radiation fields. For an achromat lattice, the loss of coherence is determined by the beam energy spread, because the longitudinal mixing in the achromat lattice occurs due to the difference in the path lengths of the particle trajectories with the different energies.² Therefore, the beam energy spread can be deduced from measurements of the correlation function of the two radiation fields.

Along with the longitudinal mixing, there is also mixing of particle transverse coordinates and angles when the beam passes through the lattice between the first and the second radiation points. In general, this transverse mixing could also affect the correlation function. However, in most practical cases the effect of the transverse mixing is negligible and can be ignored. To make further analysis as simple as possible, we do not include transverse mixing in the calculations. We consider the validity of this approach in section 4.

3 Analysis

In this section we perform a rigorous analysis of the correlation measurement technique described above, and show how this technique can be applied not only to measurements of the beam energy spread but also to the measurements of the beam emittance. In what follows, we find the correlation function (the degree of the coherence) of two radiation

¹Such points often exist in modern storage rings, since most of them are built from achromatic blocks such as the triple-bend achromat, double-bend achromat, etc. Even a traditional FODO lattice is now generally built in a such way that a few standard cells create an achromat with a 360 degree betatron phase advance.

²More precisely, the longitudinal mixing in the achromat lattice depends on the product of the beam energy spread and a constant defined by the lattice parameters.

fields customary defined in normalized form to be:

$$\gamma_{12}(\tau) = \frac{\langle E_1(t) E_2^*(t + \tau) \rangle}{[\langle |E_1(t)|^2 \rangle \langle |E_2(t)|^2 \rangle]^{1/2}}, \quad (1)$$

where γ_{12} is the correlation function, $E_1(t)$ is the far field beam radiation at the first point, and $E_2(t)$ is the far field beam radiation at the second point. Averaging, denoted by the brackets $\langle \dots \rangle$, involves integration over the large time interval.

In the following analysis we consider the radiation field of the beam as a superposition of the radiation fields of the individual particles $e(t)$:

$$E_{1,2}(t) = \sum_{i=1}^N e(t - \tau_i^{(1,2)}), \quad (2)$$

where N is the number of particles in the bunch, $\tau_i^{(1)}$ is the time of the radiation for the i -th particle at the first point and $\tau_i^{(2)}$ is the time of the radiation for the i -th particle at the second point. Note, that

$$\tau_i^{(2)} - \tau_i^{(1)} = \ell_i/v \quad (3)$$

where ℓ_i is the pathlength of the particle trajectory between two points and v is the particle velocity. The distribution function of the pathlengths of all beam particles can be described by a Gaussian probability function:

$$P(\ell_i) = \frac{1}{\sqrt{2\pi}\Delta\ell} \exp\left\{-\frac{(\ell_i - \ell_0)^2}{2\Delta\ell^2}\right\}, \quad (4)$$

where ℓ_0 is the average pathlength and $\Delta\ell$ is the rms deviation of the distribution.

We also assume a Gaussian distribution for the spectral function of the radiation intensity:

$$I(\omega) = \frac{c}{4\pi} \langle |E_1(\omega)|^2 \rangle = \frac{c}{4\pi} \langle |E_2(\omega)|^2 \rangle = \frac{I_0}{\sqrt{2\pi}\Delta\omega} \exp\left\{-\frac{(\omega - \omega_0)^2}{2\Delta\omega^2}\right\}, \quad (5)$$

where c is the speed of light, ω_0 is the central frequency and $\Delta\omega$ is the effective spectral bandwidth of the radiation field behind the filter of the measuring system.

By writing the Fourier transform of $e(t)$ in the form:

$$e(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \hat{e}(\omega) e^{-i\omega t} dt, \quad (6)$$

and by plugging Eq's.(5) and (2) into Eq.(1), we get:

$$\gamma_{12}(\tau) = \sum_{i,j=1}^N \exp\left\{-\frac{\Delta\omega^2 (\tau_j^{(2)} - \tau_i^{(1)} + \tau)^2}{2}\right\} \exp\{i\omega_0 (\tau_j^{(2)} - \tau_i^{(1)} + \tau)\}. \quad (7)$$

Since we choose to use incoherent beam radiation, then a reciprocal of $\Delta\omega$ is much shorter than the duration of the radiation pulse. In this case, we can neglect most of the terms with $j \neq i$ due to the first exponent in Eq.(7). Now, by averaging $\gamma_{12}(\tau)$ over the ensemble of particles with the probability function of Eq.(4), we get:

$$\overline{\gamma_{12}(\tau)} \simeq \exp\left\{-\frac{k^2\Delta\ell^2}{2}\right\} \exp\left\{-\frac{\Delta\omega^2\tau^2}{2}\right\} \exp\{i\omega_0\tau\}, \quad (8)$$

where $k = \omega_0/c$ is the wave number.

The second exponent in Eq.(8) shows that the coherence drops with the characteristic time scale $1/\Delta\omega$. The first exponent in Eq.(8) contain all the information about the beam characteristics. It shows that the coherence drops with increasing particle longitudinal mixing, $\Delta\ell$, during the beam passage between the two radiation points. In order to measure $\Delta\ell$ by measuring γ_{12} one needs to select the spectral range of the radiation centered around the wavelength $\lambda \sim 2\pi\Delta\ell$.

For the achromat lattice, discussed above, the value of $\Delta\ell$ is equal to:

$$\Delta\ell = \sigma_e |I_D| = \sigma_e \left| \int_0^{\ell_0} \frac{D(s)}{\rho} ds \right|, \quad (9)$$

where the integral I_D is taken along the beam trajectory in the achromat lattice, D is the dispersion function excited in the achromat lattice, ρ is the bending radius in the magnets and σ_e is the rms beam energy spread. Thus,

$$\sigma_e = \frac{\Delta\ell}{|I_D|}, \quad (10)$$

where $\Delta\ell$ is the measured parameter and I_D is either calculated from the lattice or measured parameter. (I_D can be measured, if necessary, by measuring the variation of the path length of the beam trajectory between the two radiation points as a function of the beam energy).

In the general case, the longitudinal mixing is caused by effects related not only to the particle energy deviation from the nominal value, but to the particle coordinate and angular deviations from the reference trajectory due to betatron oscillations [1]:

$$\Delta\ell = \left[\sigma_{xb}^2 I_C^2 + \sigma_{x'b}^2 I_S^2 + \sigma_e^2 \left(\eta_1^2 I_C^2 + (\eta_1')^2 I_S^2 + I_D^2 \right) \right]^{1/2}, \quad (11)$$

where σ_{xb} and $\sigma_{x'b}$ are the horizontal betatron beam size and the horizontal betatron angular beam size in the first radiation point, η_1 and η_1' are dispersion function and its derivative at the first radiation point, and I_C , I_S are the following integrals:

$$I_C = \int_0^{\ell_0} \frac{C(s)}{\rho} ds, \quad I_S = \int_0^{\ell_0} \frac{S(s)}{\rho} ds, \quad (12)$$

where $C(s)$ and $S(s)$ are two independent cosine-like and sine-like solutions of the homogeneous equation of the motion of the charged particle.

As follows from Eq.(11), one can do measurements of the horizontal beam emittance by measuring the correlation function providing that (i) $\eta_1 = \eta'_1 = 0$ and (ii) $I_D = 0$ between the first and the second radiation points. Then,

$$\Delta\ell = \epsilon_x^{1/2} \left[\beta_x I_C^2 + \left(\frac{1 + \alpha_x^2}{\beta_x} \right) I_S^2 \right]^{1/2}, \quad (13)$$

where β_x is the horizontal beta function at the first radiation point, $\alpha_x = -\frac{1}{2}\beta'_x$ and ϵ_x is the horizontal beam emittance.

It is also possible to do measurements of the horizontal beam emittance in a scheme that involves only one bending magnet. In this case the first radiation point is taken at the beginning of the dipole magnet where η_1 and η'_1 are small and the second radiation point is taken inside the magnet at a distance s_0 from the first radiation point. Then, Eq.(11) can be written as:

$$\Delta\ell \simeq \left\{ \epsilon_x \beta_x \phi^2 \left[1 + \left(1 + \alpha_x^2 \right) \frac{s_0^2}{4\beta_x^2} \right] + \sigma_e^2 \phi^4 \frac{s_0^2}{36} \right\}^{1/2}, \quad (14)$$

where $\phi = s_0/\rho$ is the bending angle. In the most practical cases Eq.(14) can be further reduced to:

$$\Delta\ell \simeq \phi \sqrt{\epsilon_x \beta_x}, \quad (15)$$

giving the result:

$$\epsilon_x = \frac{\Delta\ell^2}{\phi^2 \beta_x}, \quad (16)$$

where $\Delta\ell$ is the measured parameter and β_x is again either a calculated or a measured parameter.

The measurement of the vertical beam emittance can be done in a similar way providing that a bending magnet with the vertical bend of the beam trajectory is used.

4 Examples and Limitations

The technique described above offers the maximum sensitivity when $\Delta\ell$ and λ satisfy a condition $\lambda/\Delta\ell \simeq 2\pi$. This condition can be realized either by choosing a convenient spectral range for the measurements and setting up the lattice between radiation points that gives a corresponding $\Delta\ell$, or by choosing a convenient location of the radiation points and using a beam radiation mechanism that gives an adequate yield of the radiation at the appropriate wavelength. For example, four standard cells of the Low Energy Ring of the PEP-II B-Factory [2] create a perfect achromat with $I_C = I_S = 0$ and $I_D = 130$ mm. If radiation points are taken at the beginning and the end of the achromat, then the rms longitudinal mixing between them for $\sigma_e \simeq 8 \times 10^{-4}$ is approximately 0.1 mm. Therefore, measurements should be taken in the spectral range centered around $\lambda \simeq 0.6$ mm. This is still not too long a wavelength; the coherent component of the radiation at this wavelength

is negligible since the rms bunch length is approximately 1 cm. The radiation mechanism that can be used for a beam radiation in this spectral range is Smith-Purcell radiation [3] of electrons moving over a diffraction grating. The relationship between the grating period, d , the radiation wavelength, and the angle of emission, θ , is written [4]:

$$d = \lambda \frac{\beta}{1 - \beta \cos \theta}, \quad (17)$$

where $\beta = v/c$ is the relative velocity of the electron. The average power radiated by an electron beam per unit length of grating, ds , and per unit solid angle, $d\Omega$, is [4]:

$$\frac{d^2 P}{ds d\Omega} = \frac{4 e^2 m I_b}{\pi \lambda^2 e} \frac{\sin^2 \theta}{\beta^2 (1 - \beta \cos \theta)} \exp \left[-\frac{4\pi b}{\beta \gamma \lambda} \right], \quad (18)$$

where e is the electron charge, γ is the Lorentz factor, b is the half aperture of the vacuum chamber, m is the number of bunches and I_b is the average bunch current. Using $\lambda = 0.6$ mm, $\theta = 60^\circ$ and the following B-Factory parameters: $\gamma = 6 \times 10^3$, $b = 25$ mm, $m = 100$ and $I_b = 1$ mA, we calculate:

$$\frac{d^2 P}{ds d\Omega} \simeq 3 \times 10^{-6} \frac{\text{W}}{\text{cm} \cdot \text{sr}} \quad (19)$$

Although, the power level is modest, it exceeds by a large margin the black-body emission taken in the same spectrum within a 10% bandwidth.

Apart from the longitudinal mixing, there is also mixing of the particle's transverse positions in the beam that we did not consider so far. Whether this mixing affects the coherence of the two radiation fields or not depends on the beam transverse sizes and angular divergences at the radiation points. Actually, all kinds of charged particle radiation possess some angular divergence, which we define as θ_R . If (i) θ_R is larger than the angular spread of the beam particles and (ii) a diffraction-limited size of the radiation source, $\lambda/4\pi\theta_R$, is larger than the beam transverse sizes, then the radiation fields of the particles with the same longitudinal coordinates are in phase in the far field region. Therefore, transverse mixing does not show up in the measurement of the coherence.³ Performing the measurements in these conditions is, in fact, the most appropriate use of the technique we describe here, because in the opposite case a conventional measurement technique can be applied.

For a numerical example, consider measurements of the beam emittance $\epsilon_x \simeq 10^{-11}$ m-rad in the visible light spectrum ($\lambda \simeq 0.6\mu\text{m}$). Such an emittance is, currently, considered as a design goal for the next generation, diffraction limited, hard X-ray synchrotron light sources [5]. Using Eq.(14) and assuming $\beta_x = 10$ m and $\Delta\ell \simeq \lambda/2\pi$, we get $\phi \simeq 10$ mrad. Thus, one bending magnet with a bending angle of the beam trajectory greater than 10 mrad would be sufficient to observe the longitudinal mixing associated with the beam emittance $\epsilon_x \simeq 10^{-11}$ m-rad.

³In general, transverse mixing can be ignored when the photon beam emittance, $\lambda/4\pi$, is larger than the horizontal and vertical emittances of a charged particle beam.

All measurements can be done either in a single-shot regime or by the integration over a long period of time and over many pulses. For a single-shot measurement it is necessary to have a condition where the number of photons in the coherence volume is much greater than one. Then we deal with a classical radiation field and quantum fluctuation noise is negligible. In both examples above, these conditions are satisfied by large margins.

5 Conclusion

It has been shown that measurements of the correlation function of the radiation fields radiated by a beam of charged particles at two points of the beam trajectory can be used for measurements of the beam emittance and the beam energy spread. As an illustrative example, the application of the proposed diagnostics technique to the measurement of the energy spread in the Low Energy Ring of the PEP-II B-Factory has been considered. In this ring, the measurement of the beam energy spread is a particularly difficult problem for a traditional measurement technique due to the dominant contribution of the horizontal beam emittance to the horizontal beam size even in places with a large dispersion function. Application of our measurement technique to the measurement of the beam emittance has been considered in the example of the emittance measurement on a next-generation synchrotron light source. It has been shown that emittance 1×10^{-11} m-rad can be measured in the visible light spectrum with our approach.

6 Acknowledgment

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