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ABSTRACT

Apparent ambiguities of definitions of masses and lifetimes of unstable particles that depend either on the introduction of unperturbed Hamiltonians and their eigenstates, or on an assumed correspondence between resonances and elementary fields, are noted. The S-matrix definition is unambiguous; the positions of poles in the first unphysical sheets are given by the zeros of the Fredholm denominator function, which is a function only of an appropriate center-of-mass energy. The mass and lifetime of a particle are consequently independent of the variables of the scattering process or of the particular process to which the particle contributes. The invariance of the Fredholm denominator under charge conjugation, which is a consequence of CPT invariance, ensures the equality of masses and lifetimes of relatively conjugate antiparticles.

Unstable particles are closely akin to stable ones; by the factorization of the residues of unstable-particle poles, unstable-particle scattering functions quite analogous to ordinary scattering functions can be unambiguously defined. Like ordinary scattering functions they are defined only on the mass shell, the fixed masses of the unstable particles being well-defined complex numbers. The needed factorizability of the residue is an immediate consequence of Fredholm's second theorem. The rigorous existence of the Fredholm functions required for continuation, by means of unitarity, through the multiparticle physical cuts onto the first unphysical sheets is not yet established in the generality needed for the widest applicability of the results.

UNSTABLE PARTICLES IN S-MATRIX THEORY*†

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Various definitions of the mass and lifetime of an unstable particle have been proposed. Lüders and Zumino[1] in their proof of the equality of the masses and lifetimes of conjugate antiparticles use the position of a pole in a diagonal matrix element of the formal resolvent of the exact Hamiltonian. The matrix element used corresponds to the single-particle eigenstate of an unperturbed Hamiltonian. The existence of such quantities, in a rigorous sense, is a matter of doubt. In field theory the free-particle eigenstates do not appear generally to lie in the space over which the exact Hamiltonian is defined. Also there is a practical question of whether the definition is unique. Zumino has proposed elsewhere[2] that the unperturbed vacuum state be used in place of the unperturbed one-particle eigenstate. Finally, the apparent dependence on fictitious unperturbed states and unperturbed Hamiltonians detracts from any claim of universality.

A definition that avoids these problems is the one by Peierls.[3] He proposes the use of a pole in the one-particle propagator, the two-particle Green's function. This definition has the advantage of not depending on an unperturbed Hamiltonian or eigenstate. However, it raises the obscure question of the connection between particles and fields. The Green's function depends on which field is used. But the work of Nishijima[4] and Zimmerman,[5] and more recently of Low,[6] has cast serious doubt on the practicability of establishing a unique correspondence between particles and fields, a doubt that is even more acute for the case of unstable particles and resonances.

An important virtue of the S-matrix approach is the simple unambiguous treatment it provides in questions regarding unstable particles. The mass and lifetime of an unstable particle are defined by the position of a pole in an unphysical sheet of the S matrix. In both the Peierls and the Lüders-Zumino definitions the matrix elements involve states that carry no internal parameters, and the function considered is consequently a function of energy alone. The S matrix, on the other hand, is a function of many variables, and the question of the dependence of the position of the pole on these variables is crucial.

To the extent that the inverse of S exists, the S matrix on the second sheet can be determined from its value on the first by the equation

$$(S_N^N(E^*))^\dagger = (S^N(E))^{-1} , \qquad (1)$$

where variables other than the center-of-mass energy E are suppressed matrix indices. The subscript N on $S_N(E)$ designates the value of $S(E)$ on the sheet obtained by clockwise continuation through the N̲th interval of the energy axis, and the superscript N means the restriction to the subspace of configurations that are open in the N̲th interval. Equation (1) together with the assumption of maximal analyticity, which states that S is an analytic function in the physical sheet, except for stable-particle poles, implies that S is also analytic in the unphysical sheet obtained by reflection of the physical sheet through the physical cut, except at images of the points where the inverse of $S^N(E)$ fails to exist.

For the two-particle cuts it is easy to show that the requirement of maximal analyticity is sufficient to guarantee the existence of the numerator and denominator functions of the Fredholm solution for the inverse,

$$(S^N(E))^{-1} = \frac{\text{adj } S^N(E)}{\det S^N(E)} \quad . \tag{2}$$

Here the numerator and denominator represent the usual absolutely convergent Fredholm series.

Inspection of the Fredholm solution shows that the poles of $S^N(E)$ in first unphysical sheets can occur only at the zeros of $\det S^N(E)$. Since $\det S^N(E)$ is a function of E alone, not of the suppressed matrix index variables, the position of the pole is also a function of E alone, independent of the scattering variables and of the particular process. The position of the pole therefore provides an unambiguous definition of the mass and lifetime of an unstable particle. As it is a pole in the S matrix itself, rather than in some less observable quantity, the physical effects associated with this pole are more directly connected to observable phenomena.

The Fredholm denominator function depends only on traces of powers of the kernel of S. Using this fact, one can show that invariance under CPT implies also invariance under charge conjugation. Thus the equality of the masses and lifetimes of conjugate antiparticles follows much as for Lüders and Zumino.

Unstable particles are closely akin to stable particles. Reversing Eq. (1) used above, one finds that the stable particles correspond to zeros of the determinant on the first unphysical sheet.

For stable-particle poles the unitarity conditions imply that the residue of the pole of the scattering function is a product of two other scattering functions. This fact is, indeed, the basis of the contact between S-matrix theory and experiment; the pole contributions correspond to particles that propagate on their mass shell between independent interactions.

The factorizability of this residue ensures that the zero of the Fredholm denominator is a simple zero. If there are several particles of the same mass, such as particles in different spin states, then there is, of course, neither factorizability nor a simple zero except in the appropriate submatrix; one must diagonalize J_z , for instance, and consider a diagonal submatrix in order to find the simple zero corresponding to a single particle.

Generalizing, let us define a "single-particle pole" to be a pole corresponding to a simple zero of a Fredholm denominator. The factorizability of the residue then follows from Fredholm's second theorem. This theorem says that if $\det S = 0$ then for any V' the function

$$V = (\mathrm{adj}\ S)V' \tag{3}$$

is a solution of the equation

$$SV = 0 . \tag{4}$$

Moreover, if $\mathrm{adj}\ S \neq 0$, then, aside from a multiplicative factor, V is independent of V' . This implies that the matrix elements of $\mathrm{adj}\ S$ have the form

$$\langle \alpha \mid \mathrm{adj}\ S \mid \beta \rangle = -i V_\alpha W_\beta^\dagger , \tag{5}$$

which is just the statement of factorizability. If $\mathrm{adj}\ S(E_r)$ is zero, then analyticity in E requires that each matrix element contain at least one power of $(E - E_r)$. But then $S^{-1}(E_r)$ is regular if $\det S(E)$ has only a simple zero. From the anti-Hermitian analyticity property of the M functions one also finds in (5) that, to within an arbitrary scale factor which can be taken to be unity, $V = W$.

The factors of the residue are quite analogous to the S-matrix elements for physical particles. They are defined only over the manifold constrained by the conservation laws and mass conditions. However, the fixed mass of the unstable particle is complex rather than real. The factorizability property is of signal importance; it says that the dependence of the residue on the initial and final states separates into two factors, one depending only on the initial particles, the other on the final particles of the channel in whose center-of-mass energy the pole occurs.

Difficulties with disconnected parts of scattering functions have so far prevented extension of proof of the applicability of Fredholm theory to the continuation through multiparticle cuts. It is hoped that the difficulties will be overcome by the time of the Conference.

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