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Cis-lunar Trajectory

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Description

Cis-lunar trajectories encompass all of the orbits revolving around the Earth (circumterrestrial) and Moon (circumlunar), as well as those about the Earth-Moon Lagrange point (libration-point orbits) and the various paths between the Earth and Moon (trans-lunar trajectories and transfers). The scope herein is limited to the later classes of orbits, thereby omitting discussions on near-Earth trajectories from low-Earth orbits to the geosynchronous regime and the bounded selenocentric orbital regions such as low-lunar orbits.

Introduction

New transportation, communication, and logistic infrastructures are being planned and developed for cis-lunar space in the Earth-Moon system. Cis-lunar (alternatively, cislunar) space refers to the orbital regimes about the Earth out to and including the region around the surface of the Moon. A wide variety of dynamical models are employed to approximate the diversity of trajectories in cis-lunar space. Whereas circumterrestrial and circumlunar orbits are largely governed by the perturbed two-body problem [1], in which the effects of the non-spherical gravity field and third-body perturbations on Earth or Moon satellites are often treated in a Hamiltonian formulation (see, e.g., [2]), all other cis-lunar trajectories, including lunar transfers and libration-point orbits, are specific applications of the gravitational n-body problem. The simplest way to model trans-lunar trajectories is by the method of patched

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conics, described in great detail in [3], whereby the Moon's sphere of influence (SOI) separates selenocentric from geocentric motions (Fig. 1).

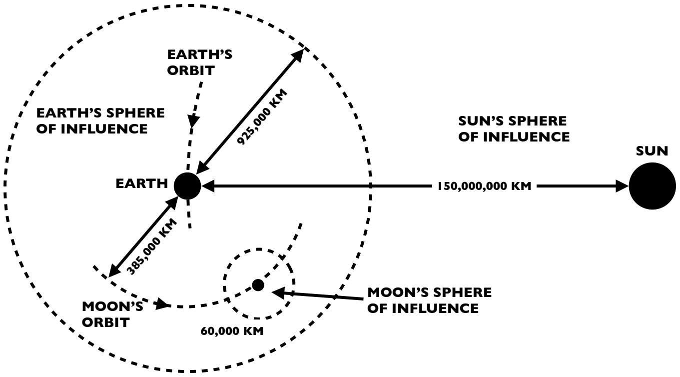

Fig. 1 Approximate sphere of influence of the Earth, Moon, and Sun. (Not to scale.)

More realistically, however, trans-lunar trajectories are governed by the restricted three-body problem, in which the spacecraft of negligible mass is simultaneously affected by the terrestrial and lunar gravitational forces. This framework efficiently captures orbital transfers between the Earth and the Moon [4, 5], models the regions of the Euler-Lagrange equilibrium points shown in Fig. 2 [6], and has generally been the most studied formulation of motion in cis-lunar space.

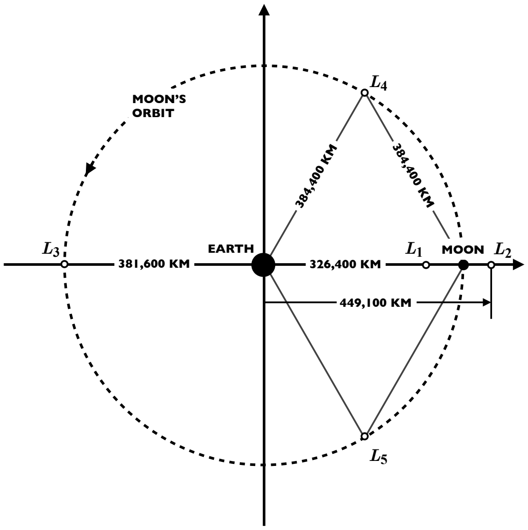

Fig. 2 Approximate location of the five Lagrange points in the Earth-Moon system. (Not to scale.)

Main Text

An immense and hardly surveyable variety of cis-lunar trajectories is possible. The earliest attempts to classify these came during the Sputnik and Apollo era and included systematic investigations on passive trajectories leading to lunar impact, the possible trajectories for circumnavigating the Moon and returning to Earth, and other important characteristics of cis-lunar-flight dynamics [4, 5]. The simplest lunar transfer is one that collides with the Moon on either a hyperbolic or highly eccentric elliptical collision trajectory. The first of the Soviet Union's Luna program, Luna 1, was launched on the former unbounded path and, having missed its target by roughly 6,000 km, became the first spacecraft to be placed in heliocentric orbit [7]. The United States' counterpart, Pioneer 4, passed within the ∼60,000 km lunar SOI on its Earth-escape trajectory. Also of note was the Russian flyby mission, Luna 4, that entered a distant, highly eccentric, 89,250 by 694,000 km equatorial orbit, which is believed to have been later perturbed into a heliocentric orbit. The first successful impact trajectories on approximate Hohmann transfers were Luna 2 in 1959 and Ranger 4 in 1962. A more interesting trans-lunar trajectory, exemplified by Luna 3, is when the probe's elongated orbit takes it just within the lunar SOI to circumnavigate the Moon. While the Moon cannot capture the spacecraft into selenocentric orbit without subsequent propulsion, on leaving the Moon's SOI, the space vehicle will enter a new and somewhat unpredictable elliptical orbit about the Earth. Luna 3 twice suffered close approaches with the Moon, and despite having an initial perigee outside of geosynchronous orbit (GEO), after 11 revolutions it plummeted back into the Earth's atmosphere [8, 9].

Fig. 3 A Soviet postage stamp commemorating the Luna 3 space mission. The main image is a chronological description of the probe's translunar-lunar flight path.

Luna 3 demonstrated how the passage of a space vehicle through the Moon's SOI can be exploited to produce orbits of a very different kind (Fig. 3). The Interstellar Boundary Explorer (IBEX) [10] and the Transiting Exoplanet Survey Satellite (TESS) [11], two modern Luna 3-like orbits both from NASA's Explorers program, are distinguished by their high apogee distances and lunar mean-motion resonance (MMR) phasing. IBEX, with its nominal mission lasting only 2 years, had to change its operational orbit for its extended mission to avoid violating altitude and eclipse mission constraints. Its nominal orbit turned out to be chaotic and unpredictable beyond 2.5 years, as a result of significant lunar perturbations, and IBEX was subsequently maneuvered using a gravity assist from the Moon into a novel 3:1 MMR with orbital period ∼9.1 days. Following suit, TESS orbits in a ∼13.7 day, 2:1 lunar MMR, which was also established using a lunar swing-by maneuver (Fig. 4). Also of note in this regard is the lunar flyby rescue of AsiaSat-3/HGS-1 [12], a communications satellite initially thought stranded in an unstable, highly eccentric and inclined, geostationary-transfer orbit.

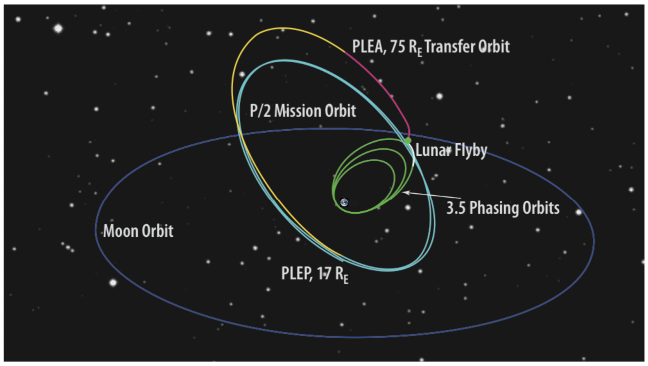

Fig. 4 Schematic of the maneuvers and lunar flyby leading to the final TESS P/2 mission orbit [14] PLEA and PLEP are the post-lunar-encounter apogee and perigee, respectively.

The remaining Soviet and American cis-lunar mission of the 1960's included impacts, flybys, orbiters, and landers from the Luna, Zond, Ranger, Surveyor, and Explorer programs. Crewed missions to the Moon were conducted between 1968 and 1972 as part of the United States' Apollo program [13]. Missions before Apollo 13 used a free-return trajectory (Fig. 5), where the trans-lunar injection would lead to a lunar flyby that would swing the spacecraft back to Earth without need for further propulsive maneuvers (for mathematical details, see [3]). For later missions, a mid-course correction maneuver was implemented and a hybrid trajectory that did not create such a gravitational slingshot was used that enabled greater mission flexibility.

After Apollo and the Luna probes of the early 1970's, there had been a comparative lull in further exploration of the moon, with the Luna 24 sample return representing the last lunar mission for a decade and a half. Nevertheless, cis-lunar space activity beyond GEO carried on unhindered, being an ideal site for many astronom-

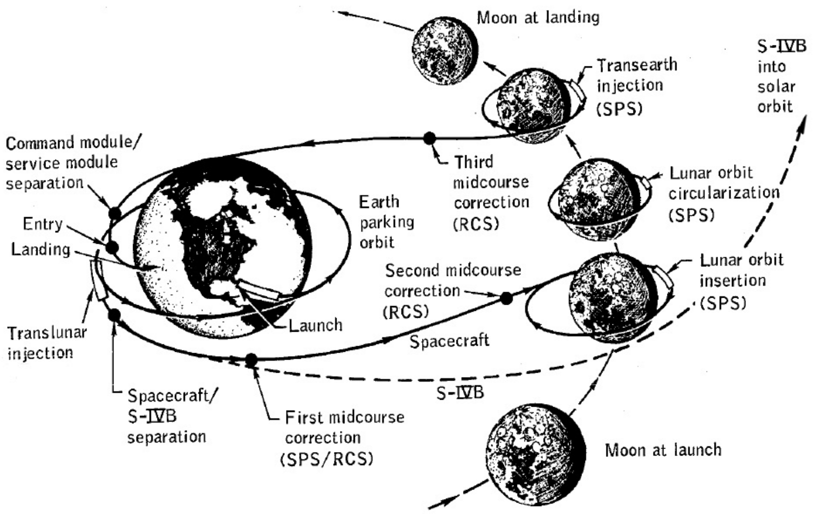

Fig. 5 Schematic of the Apollo free-return trajectory (Image Credit: NASA).

ical space observatories. For the investigation of the Earth's magnetosphere and the interplanetary space outside of it, satellites with orbits of high eccentricity, large semi-major axis, and multi-day period are often used [10, 15, 16, 17, 18, 19, 20]. Under the influence of the Moon and the Sun, a highly eccentric orbit of a cis-lunar space probe can become nearly circular or a nearly circular orbit might become eccentric, while orbital inclination may also exhibit large shifts [21, 22, 23]. Thus, the Sun and the Moon may provide a substantial boost in perigee height for the satellite under properly chosen circumstances. For other conditions the perturbation may be minimized to obtain a relatively stable orbit. Explorer 12 (1961 Upsilon) was the first cis-lunar satellite to be launched at a time that was preselected for a minimum orbital lifetime of one year, utilizing lunisolar perturbations to ensure stability [24].

Japan's Hiten spacecraft, which initially used the conventional Hohmann-like transfer to reach the Moon, was the first robotic lunar probe since Luna 24 in 1976. While the main Hiten spacecraft was designed to only circumnavigate the Moon, a new ballistic capture trajectory was implemented based on weak stability boundary theory [25] that used solar perturbations to enable the probe to be temporary captured into circumlunar orbit. This novel low-energy transfer ushered in a new era of space-mission design that was no longer simply predicated on Keplerian motion [26, 28]. The GRAIL Earth-Moon transit (Fig. 6) was based on these notions, as was the low-energy trajectories for the two ARTEMIS spacecraft, relying on space-manifold dynamics [27].

The past two decades have witnessed a dramatic growth of interest with new American, European, Japanese, Chinese, Indian, and Israeli lunar missions. Of special note was ESA's SMART-1 low-thrust spiraling trajectory to the Moon (Fig 7), which used a combination of multiple coast arcs [29] and multiple weak lunar

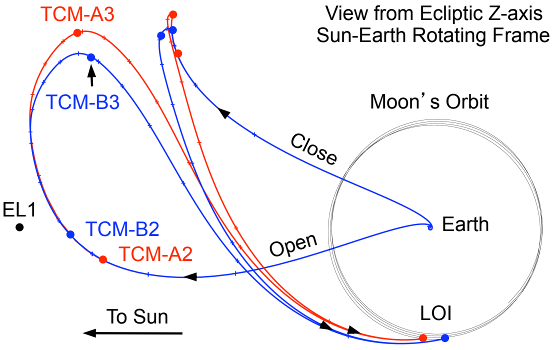

Fig. 6 Trans-lunar cruise phase trajectories for GRAIL's mission design (Image Credit: NASA). The low-energy trajectories leave Earth following a path towards the Sun, passing near the interior Sun-Earth Lagrange point L_1 before heading back towards the Earth-Moon system. The mission design is based on weak stability boundary theory.

gravity assists. Other recent trans-lunar trajectories, including the Chandrayaan and Chang'e missions, Clementine and Beresheet, instead employed conventional chemical propulsion with staging to outwardly spiral towards the Moon.

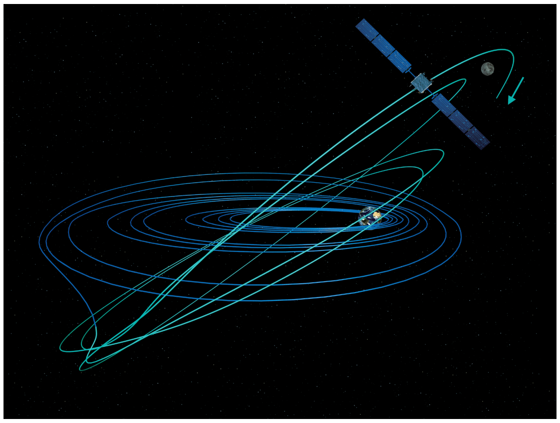

Fig. 7 Schematic of the SMART-1 orbital path to the Moon (Image Credit: ESA)

Of the myriad of cis-lunar trajectories, the distant retrograde orbits (DROs) [30] and near-rectilinear halo orbits (NRHOs) [31, 32], shown in Fig. 9, have assumed a special recent significance. DROs, which are stable geocentric orbits that resemble large quasi-elliptical retrograde orbits around the Moon in the rotating frame, have been proposed as parking orbits for interplanetary missions and for the Asteroid Redirect Mission [33]. Halo orbits about L_1 and L_2 have been proposed for payloads supporting lunar exploration and communication [6], including the Lunar Gateway, and are currently being used by the CNSA's lunar relay satellite, Queqiao (Fig. 8).

This mathematical formulation for treating all of the enumerated trajectories and missions describes motion in a rotating reference frame and centered at the Earth-Moon barycenter, with possible perturbations from the Sun. For representation of motion in cis-lunar space, we can state a more general framework than is typically used, one that can be tailored to specific problems of relevance and interest [34, 35]:

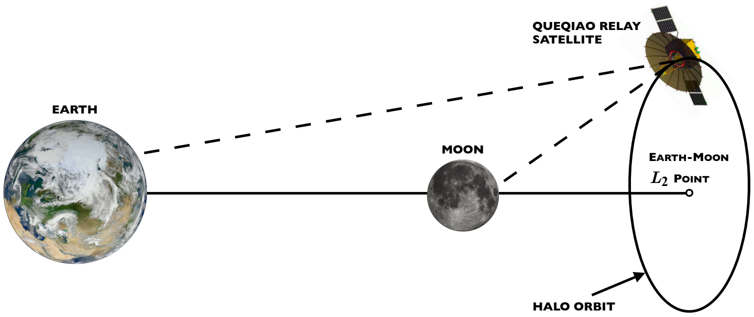

Fig. 8 Schematic of the Queqiao lunar relay satellite in an L_2 halo orbit.

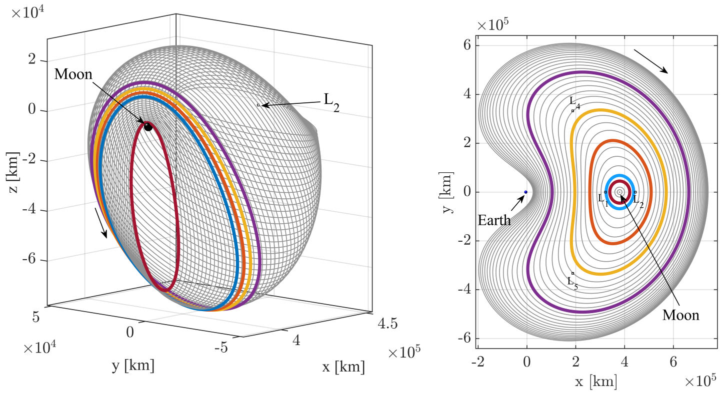

Fig. 9 Periodic orbits of recent interest for cis-lunar dynamics in the Earth-Moon rotating frame [32]. *Left*: Southern L_2 halo family with NRHOs being the subset between the *maroon* and *purple* highlighted orbits. *Right*: The distant retrograde orbit family with select orbits denoted.

$$\ddot{\boldsymbol{r}} + \dot{\boldsymbol{\Omega}} \times \boldsymbol{r} - 2\boldsymbol{\Omega} \times \dot{\boldsymbol{r}} + \boldsymbol{\Omega} \times \boldsymbol{\Omega} \times \boldsymbol{r} = \frac{\partial U}{\partial \boldsymbol{r}} + \boldsymbol{u} \tag{1}$$

where $\boldsymbol{r}$ is the position vector of a satellite relative to the Earth-Moon barycenter and

$$\begin{aligned} U(\boldsymbol{r},t) = {} & \mathscr{G}(M_E + M_M)\left[\frac{1-\mu}{\|\boldsymbol{r}-\boldsymbol{r}_E\|} + \frac{\mu}{\|\boldsymbol{r}-\boldsymbol{r}_M\|}\right] \\ & + \mathscr{G}M_S\left[\frac{1}{\|\boldsymbol{r}-\boldsymbol{r}_S\|} - \frac{\boldsymbol{r}\cdot\boldsymbol{r}_S}{r_S^3}\right] \end{aligned} \tag{2}$$

is the gravitational potential with M_E, M_M, M_S and $\boldsymbol{r}_E$, $\boldsymbol{r}_M$ and $\boldsymbol{r}_S$ representing the masses and barycentric positions of the Earth, Moon and Sun, respectively. The parameter $\mu = M_M/(M_E + M_M)$ is the mass fraction of the Earth-Moon system (~ 0.012), and $\boldsymbol{\Omega}$ is the instantaneous angular velocity of the Earth and Moon about each other ($\sim 2.66 \times 10^{-6}$ rad/s). This formulation of the problem allows for the perturbation of the Sun on the Earth-Moon system and on the satellite (e.g., that which is captured in the simplified bi-circular problem [36]), but does not necessarily assume that the Earth and Moon orbit in a circular, or even Keplerian, orbit. It should be noted that the gravitational attraction of either the Moon or the Earth can be replaced with a more accurate gravity field expansion, should the application warrant it. The final term $\boldsymbol{u}$ represents either solar radiation pressure (SRP) or the commanded thrusting of a spacecraft.

While the above general form can be quite useful, especially when understanding the connections between cis-lunar space and the Earth-Sun system, in many cases the more standard circular, restricted, three-body problem (CR3BP) formulation of the problem is employed. In the cis-lunar context, the CR3BP ignores the Sun and forces the Earth and Moon to revolve around each other in a circular orbit (making $\dot{\boldsymbol{\Omega}} = 0$). Applying the usual non-dimensionalization (unit of length equal to the distance between the Earth and Moon and the unit of time being the mean motion of the Earth and Moon about each other) reduces the equations to the simpler form:

$$\boldsymbol{r}'' - 2\hat{\boldsymbol{z}} \times \boldsymbol{r}' + \hat{\boldsymbol{z}} \times \hat{\boldsymbol{z}} \times \boldsymbol{r} = \frac{\partial U}{\partial \boldsymbol{r}} + \boldsymbol{u} \tag{3}$$

where the $()'$ denotes differentiation with respect to the mean anomaly of the Earth-Moon orbit and the non-dimensional acceleration potential is simplified to

$$U(\boldsymbol{r}) = \left[\frac{1-\mu}{\|\boldsymbol{r} + \mu \hat{\boldsymbol{x}}\|} + \frac{\mu}{\|\boldsymbol{r} - (1-\mu)\hat{\boldsymbol{x}}\|} \right] \tag{4}$$

Here we note that if the control acceleration is zero, this system has an integral of motion called the Jacobi integral, which is analogous to energy conservation in the rotating frame. In this regard, we also obtain the Lagrange equilibrium points, shown in Fig. 2, on which so much of modern cis-lunar mission design is based.

Conclusions

There are significant future lunar missions scheduled or proposed by over a dozen nations or organizations to be launched in this decade. Cis-lunar space, outside the confines of the traditional near-Earth satellite orbits, is poised to serve as a new high ground for space operations, and, like its circumterrestrial counterpart, must be sustained against risk from debris. It is precisely the distinctive and multi-faceted dynamical features of this regime, highlighted herein, that complicates space situational awareness efforts and represents significant challenges for space sustainability. While the theory for plotting a trajectory in cis-lunar space is well understood, despite over 60 years of activities, our knowledge of the multi-timescale dynamics of orbits in this regime is still incomplete. It is both an exciting endeavor from a research and exploration standpoint to enable safe cis-lunar space operations for many years to come.

Cross-References

- Apollo Program
- ARTEMIS Mission
- Chandrayaan-1 Mission
- Chang'e Missions
- GRAIL Mission
- Luna Missions
- Ranger Missions
- Surveyor: The Spacecraft
- Zond Missions

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