Office of Scientific Research and Development

National Defense Research Committee

Division Six - Section 6.1

THE DISCRIMINATION OF TRANSDUCERS AGAINST REVERBERATION

by

Reverberation Group

University of California Division of War Research
At the U. S. Navy Radio and Sound Laboratory
San Diego, California

UCDWR No. U75

Copy No. 64

R. W. Raitt

File No. 01.40

May 31, 1943

Preface

This report was prepared by the Reverberation Group* of the University of California, Division of War Research, for the National Defense Research Committee under contract OEMsr-30.

Abstract

The directivity index, discussed in Refs. 1 and 2, which is currently used for characterizing the directional properties of transducers, refers primarily to their ability to radiate sound power. When transducers are used in echo ranging, the directivity index is, theoretically at least, of minor interest. Of greater interest is the ability of the transducer to discriminate between the echo from a target at which it is pointed, and the reverberation returned to it from this and other directions. This discrimination is measured by other quantities, called reverberation indices (Ref. 3). One of these concerns volume, the other surface or bottom reverberation. The purpose of the present work was to study the relations between the three indices.

Conclusions drawn from a study of typical projector patterns are as follows:

1. The volume reverberation index and the surface reverberation index of a projector are linearly related to the directivity index, provided that the directivity pattern is reasonably similar to that of a circular piston in an infinite baffle. This condition is found in the echo-ranging projectors studied when they are operated at 24 kc without domes. However, the directivity index does not provide a reliable measure of the reverberation indices when the projector pattern has abnormally strong side lobes.

2. Neither projector housing studied has appreciable effect on reverberation indices.

3. The echo:reverberation ratio depends almost entirely on the shape of the main lobe of the composite directivity pattern (see p. 1) between zero and -6 db. As a result, the reverberation indices

---

* At present, this group consists of R. J. Christensen, R. W. Raitt, C. H. Parker, T. H. Schafer, J. D. Frautschy, and M. J. Sheehy.

of a transceiver can be determined by measuring the width of its directivity pattern at -6 db. Half of this angle will be termed the half-width of the pattern.

4. Since the reverberation indices can be so readily calculated from the half-width, it is recommended that this quantity be specified in describing a transducer. The directivity index usually, but not always, can be calculated from the half-width to within 3 db.

CONTENTS

THE DISCRIMINATION OF TRANSDUCERS AGAINST REVERBERATION

DEFINITIONS

λ = wavelength.

a = effective radius of circular projector, effective side of rectangular projector (Fig. 1).

b = effective side of rectangular projector (Fig. 1).

A = effective area of projector.

S = center of projector face.

SS' = axis of symmetry of directivity pattern.

ϕ, θ, ψ defined by Fig. 1.

q = intensity of projected sound at any distance along ST (Fig. 1), in units of the intensity at the same distance of the sound projected along SS'.

q' = electrical power generated by the projector when sound from a standard source at any distance falls on it along the line ST (Fig. 1), in units of the electrical power generated when the source is in front of the projector on the axis SS' at the same distance.

qq' = composite directivity pattern.

d = directivity factor of projector $= (1/4\pi) \int_0^{2\pi} \int_{-\pi/2}^{\pi/2} q(\theta,\phi) \cos\theta \, d\theta \, d\phi$.

d' = directivity factor of hydrophone $= (1/4\pi) \int_0^{2\pi} \int_{-\pi/2}^{\pi/2} q'(\theta,\phi) \cos\theta \, d\theta \, d\phi$.

$Q = \int_0^{2\pi} q(0,\phi) q'(0,\phi) d\phi$.

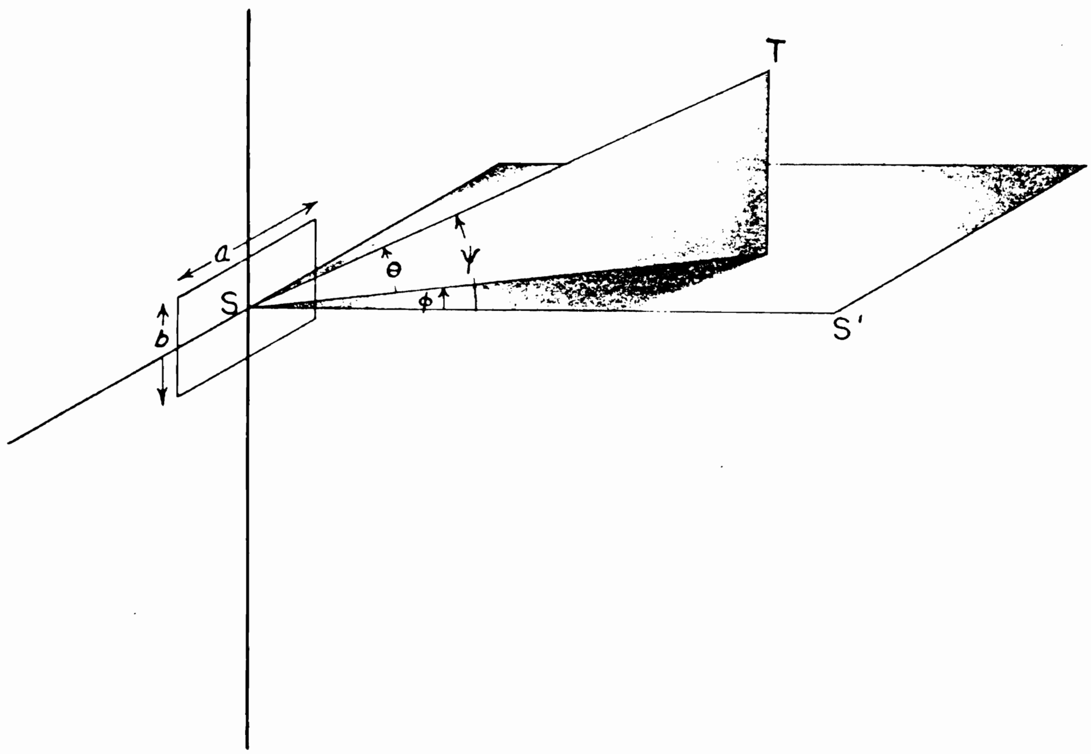
$P = \int_0^{2\pi} \int_{-\pi/2}^{\pi/2} q(\theta,\phi) q'(\theta,\phi) \cos\theta \, d\theta \, d\phi$.

D = directivity index of projector $= 10 \log_{10} d$.

D' = directivity index of hydrophone $= 10 \log d'$.

J_s = surface reverberation index $= 10 \log Q/2\pi$.

FIG. 1



University of California
Division of War Research
San Diego, California No. U75-1

J_v = volume reverberation index = 10 log $P/4\pi$.

y = half the angle in degrees in the plane $\theta = 0$ between the two rays of the composite directivity pattern on which qq' is 6 db below unity. The angle y is called the half-width of the pattern.

$Q_1 = Q$ of rectangular projector with side a in the plane $\theta = 0$.

$Q_2 = Q$ of rectangular projector with side b in the plane $\theta = 0$.

$u = (2\pi a/\lambda) \sin \psi$, circular projector.

$u = (\pi a/\lambda) \sin \phi$, rectangular projector.

$v = (\pi b/\lambda) \sin \theta$, rectangular projector.

NOTE: D depends only on q and D' depends only on q', whereas P and Q depend on both q and q'. The patterns q and q' should be identical and can be considered so for ideal transducers. However, when a transducer is driven at large amplitudes it may vibrate in a different way than at low amplitudes, perhaps due to nonlinearity in the characteristic of the crystals. The geometric mean $q'' = (qq')^{\frac{1}{2}}$ could be used for both projector and receiver; but since the pattern q''^2 from which P and Q would then be computed is just qq', the simplest way to treat empirical patterns is to define P and Q in terms of the measured patterns, as though projector and receiver were separate units.

DIRECTIVITY AND REVERBERATION INDICES

The directive property of echo-ranging transducers is currently expressed by the directivity index D. This index is defined as ten times the logarithm of the directivity factor d. In turn, this is defined as the ratio of the total sound power emitted to the total sound power which would be emitted if the intensity in all directions were equal to that on the axis. A mathematically identical quantity D' is used when the transducer is operated as a receiver, but D' has no simple physical meaning. While the directivity index is sufficient to characterize a transducer used as a projector or as a receiver, two indices can be defined which, in some cases, more accurately characterize the discrimination of a unit against reverberation when it is used as a transceiver of echoes. The purpose of this study was to investigate the relation among these various indices under practical circumstances.

It can be shown from the definition of d that

$$d = (1/4\pi)\int_0^{2\pi}\int_{-\pi/2}^{\pi/2} q(\theta,\phi)\cos\theta \, d\theta \, d\phi, \tag{1}$$

where θ and ϕ are angles defined by Fig. 1, and $q(\theta,\phi)$ is the projector directivity pattern. The directivity factor of a receiver, d', is the same integral except that the receiver directivity pattern $q'(\theta,\phi)$ is substituted for $q(\theta,\phi)$.

As shown in Ref. 3, p. 8, under ideal conditions volume reverberation intensity is proportional to an integral P which depends on the directivity patterns of the transceiver. Similarly, the intensity of surface and bottom reverberation is proportional to a factor Q which depends on the directivity patterns. These are given by the integrals

$$P = \int_0^{2\pi}\int_{-\pi/2}^{\pi/2} q(\theta,\phi)q'(\theta,\phi)\cos\theta \, d\theta \, d\phi \tag{2}$$

$$Q = \int_0^{2\pi} q(0,\phi)q'(0,\phi)d\phi. \tag{3}$$

The quantity qq' is called the composite pattern of the transceiver. (See Note on p. 2.) Q is the Q(0) of Ref. 3.

The condition for complete nondirectionality is $q = q' = 1$, whence $d = 1$, $P = 4\pi$, and $Q = 2\pi$ for a nondirectional transceiver. Thus, the ratio of the volume reverberation experienced to that which would be experienced were the transceiver nondirectional is $P/4\pi$, and the corresponding quantity for surface and bottom reverberation is $Q/2\pi$. These quantities could be termed volume reverberation factor and surface reverberation factor by analogy with directivity factor. The corresponding quantities on the db scale are called volume reverberation index, J_v, and surface reverberation index, J_s. Summarizing,

$$D = 10 \log d \tag{4}$$

$$D' = 10 \log d'$$

$$J_v = 10 \log P/4\pi \tag{5}$$

$$J_s = 10 \log Q/2\pi \tag{6}$$

It is difficult to evaluate d, P, and Q analytically, even in the case of ideal patterns; but there are several numerical methods available. A graphical method is described in Ref. 2, p. 11. All data for this report were obtained by numerical integration, using the trapezoidal rule. This method consists in converting the pattern to be integrated from db vs 1 to intensity, at equal intervals; multiplying by the appropriate trigonometric factor; adding up the products; and multiplying the sum of the products by the interval in radians between consecutive ordinates. A method of numerical integration for a three-dimensional pattern to find P when the only information at hand consists of directivity patterns in the planes $\theta = 0$ and $\phi = 0$, is given in Ref. 3, pp. 14-17. A three-dimensional pattern is needed to find d, d', and P, but these quantities can be calculated approximately from one plane pattern by assuming axial symmetry.

Fortunately, it is not necessary to integrate to obtain P and Q to sufficient accuracy, in any case studied so far. Only in the presence of unusually high side lobes will it be necessary to resort to numerical integration to obtain d and d'. It is shown in the appendix, Eqs. (23) and (30) that for large ($a > 2\lambda$)* circular and rectangular pistons in an infinite baffle

$$Q = 0.0264y, \tag{7}$$

where y is the half-width of the composite directivity pattern in the plane $\theta = 0$ (see definitions). It is also shown in the appendix, Eqs. (22) and (37), that under the same conditions

$$P = Q^2 \text{ for a circular piston} \tag{8}$$

and

$$P = Q_1 Q_2 \text{ for a rectangular piston,}$$

where Q_1 and Q_2, respectively, are values of Q for the side a and the side b in the plane $\theta = 0$. Thus P and Q can both be readily calculated from the half-width for two very common projector shapes.

Also, d is related to Q. For a large circular piston in an infinite baffle, Eqs. (21) and (26) give

$$d = 0.169Q^2 \tag{9}$$

---

* This criterion for a large diaphragm is quite conservative, and Eqs. (10), (11), and (12) are probably valid for any transducer which does not have abnormally high side lobes.

Equations (7), (8), and (9) can be used with Eqs. (4), (5), and (6) to express the directivity and reverberation indices in terms of the half-width:

$$J_s = 10 \log y - 23.8 \qquad (10)$$

$$J_v = 20 \log y - 42.6 \qquad (11)$$

$$D = 20 \log y - 39.9 \qquad (12)$$

Sixteen standard echo-ranging projector patterns were studied to see how well the above relations, derived for an ideal case, hold in practice.

The values of J_s, J_v, and D which were obtained by Eqs. (10), (11), and (12) are compared in Table I with the values obtained by numerical integration. The directivity indices in Col. 9 are taken from Refs. 4 and 5, except Cases 12-16. Important facts shown for the cases studied by Table I are listed below:

1. Reverberation index values computed from the half-width (Cols. 6 and 8) are not significantly different from values found by integrating over the directivity pattern (Cols. 5 and 7).

2. In most cases, the directivity index computed by numerical integration (Col. 9) is within 3 db of the value predicted by Eq. (12). However, reverberation indices are much less affected by the side lobes than is the directivity index, as can be seen from the fact that in computing the former the directivity pattern is squared before integration. Cases 4, 7, and 16 illustrate this very clearly.

3. Neither type of projector housing studied has appreciable effect on reverberation indices.

TABLE I

1	2	3	4	5	6	7	8	9	10
Case	Reference*	Projector	Freq.	J_s		J_v		D	
			kc	Num. Int.	Eq. (10)	Num. Int.	Eq. (11)	Num. Int.	Eq. (12)
1	ES701730	QC with hood	24	-14.7	-14.9			-19.5	-21.7
2	ES701731	QC, no hood	24	-13.5	-13.5	-22.4	-22.1	-19.0	-19.3
3	ES701733	QC, combination	24	-15.6	-15.6	-26.3	-26.4	-22.4	-23.5
**4	ES701870	JK, combination	12.5	-14.0	-15.1	-24.7	-25.4	-13.8	-20.3
5	ES701871	JK, combination	25	-15.7	-15.8	-26.7	-26.7	-23.1	-23.7
6	ES701872	JK, combination	30	-16.3	-16.3			-24.1	-24.9
**7	ES701873	JK, combination	35	-18.7	-19.0			-21.3	-29.7
8	ES701874	JK, combination	50	-18.8	-18.8			-26.0	-29.9
9	ES701875	JK, combination	65	-20.1	-20.3			-28.9	-32.5
10	USRL449	QC, no dome	24	-17.1	-17.0			-24.2	-26.5
11	USRL453	JK, no dome	24	-16.4	-16.4			-23.8	-25.1
12	USRL477	JK, 0° with dome	24	-16.2	-16.2			-22.4	-24.7
13	USRL478	JK, 45° with dome	24	-15.5	-15.5			-21.2	-23.3
14	USRL479	JK, 90°	24	-16.9	-16.9			-23.7	-26.1
15	USRL480	JK, 135°	24	-16.1	-16.1			-22.7	-24.5
16	USRL481	JK, 180°	24	-18.8	-18.8			-18.2	-29.9
17		Jasper JK 1/25/44	(24)		-16.0				

* Numbers in this column identify projector patterns given in Refs. 4 and 5.

** Measurements made at San Diego on another unit of the same type did not show abnormal side lobes. But whether or not these patterns are characteristic of the JK type, they illustrate the small effect which high side lobes have on J_s and J_v, as compared with their effect on D.

APPENDIX I

CIRCULAR PISTON IN AN INFINITE BAFFLE

A. Volume Reverberation

For the special case of circular symmetry, Eq. (2) reduces, by Ref. 3, p. 8, to

$$P = 2\pi \int_0^{\pi} q^2(\psi) \sin \psi \, d\psi, \tag{13}$$

where ψ is an angle defined by Fig. 1. Because of the postulated baffle, the upper limit of integration must be $\pi/2$. From Ref. 6, p. 255, cited in Ref. 9,

$$q^2(\psi) = (2J_1(u)/u)^4, \tag{14}$$

where

$$u = (2\pi a/\lambda) \sin \psi.$$

Also,

$$d\psi = \lambda du/(2\pi a \cos \psi).$$

At $\psi = \pi/2$, $u = 2\pi a/\lambda$; however, the integrand converges so rapidly to zero (Fig. 2A) that the integral from 0 to $u = 3$ is within 2% of the integral from 0 to ∞. Therefore, if $2\pi a/\lambda \geq 3$, the upper limit of integration may be replaced by 3. Since,

$$1/\cos \psi = [1 - (\lambda u/2\pi a)^2]^{-\frac{1}{2}}$$

$$= 1 + 0.5(\lambda u/2\pi a)^2 + \ldots . \tag{15}$$

it follows that if the piston is large, P is very nearly equal to

$$(\lambda^2/2\pi a^2) \int_0^3 [2J_1(u)/u]^4 u \, du. \tag{16}$$

Integrating $[J_1(u)/u]^4 u$, Table II, by the trapezoidal rule

$$P = 0.149(\lambda/a)^2 \tag{17}$$

B. Surface Reverberation

From Ref. 3, p. 13, for circular symmetry

$$Q = \int_0^{2\pi} q^2(\psi)d\,\psi. \tag{18}$$

Because of the baffle, integration is carried over $-\pi/2 \leq \psi \leq \pi/2$. By pattern symmetry, the integral from $-\pi/2$ to 0 equals the integral from 0 to $\pi/2$. Hence,

$$Q = 2\int_0^{\pi/2} q^2(\psi)d\,\psi. \tag{19}$$

By the argument used above, (see Fig. 2A for convergence of the integral) if the piston is large this is approximately

$$Q = (\lambda/\pi a) \int_0^3 (2J_1(u)/u)^4 du. \tag{20}$$

Integrating by the trapezoidal rule,

$$Q = 0.387\lambda/a. \tag{21}$$

Comparing Eqs. (17) and (21),

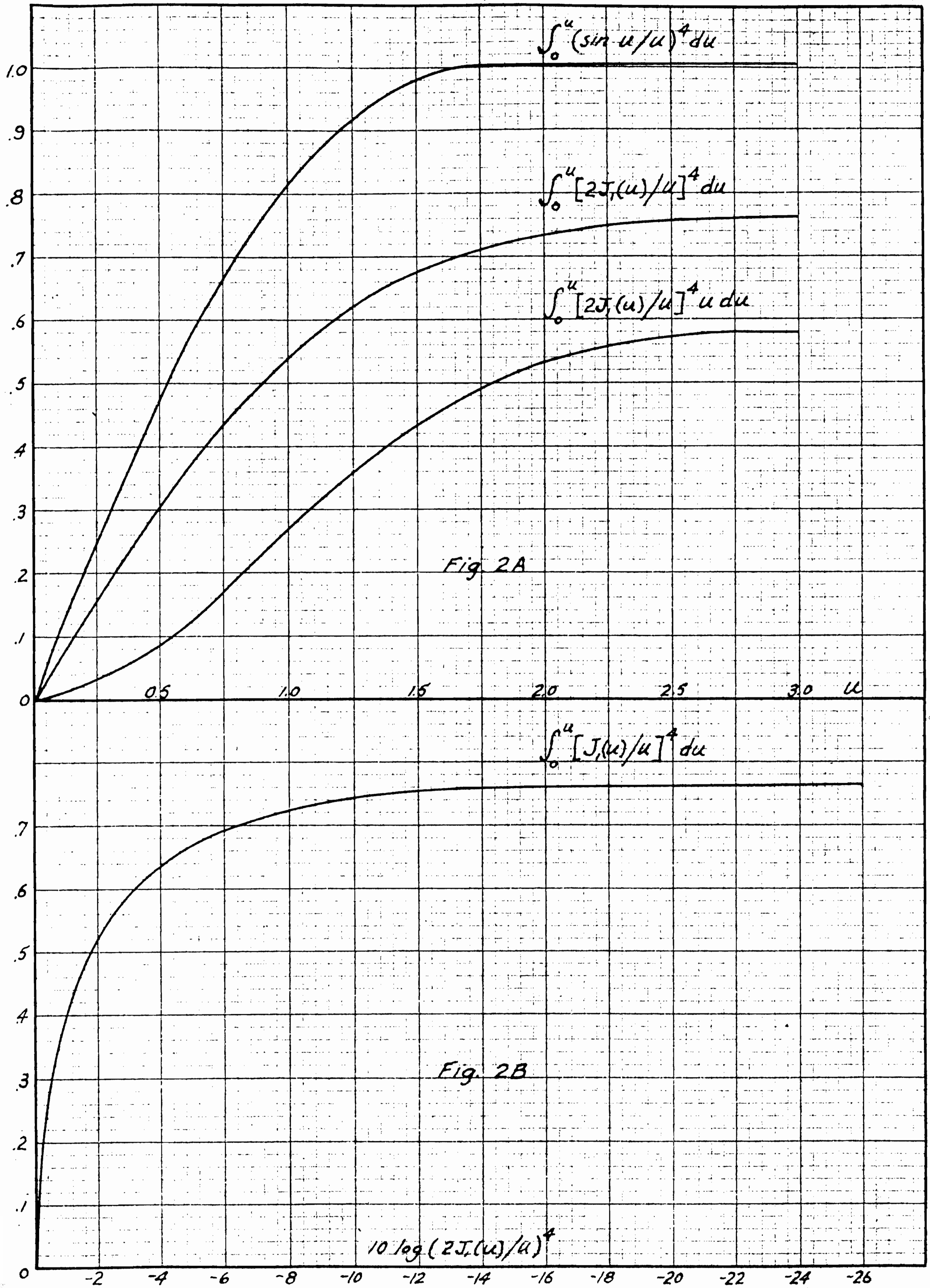
$$P = Q^2. \tag{22}$$

Q can be determined from any point (u_1, q_1) on the main lobe of the composite pattern. The point must be so chosen that $\int_0^u (J_1(u)/u)^4 du = \int_0^3 (J_1(u)/u)^4 du$, nearly; but it must not be chosen so far down from the peak that the irregularity of empirical patterns will be likely to affect the measurement of u_1. Figure 2B is a plot of $\int_0^{u_1} (J_1(u)/u)^4 du$ against the pattern level $10 \log (2J_1(u)/u)^4$. At 6 db the integral is within one db of its final value, and the rate of rise is beginning to diminish rapidly. Therefore, the point $(u_1, 0.25)$ was chosen. From a table of $q = (2J_1(u)/u)$, u_1 is found to be 1.61. By definition,

$$u_1 = (2\pi a/\lambda) \sin \psi_1$$

$$= 2\pi\psi_1 a/\lambda,$$

Fig. 2.



$\int_0^u (\sin u/u)^4 du$
$\int_0^u [2J_1(u)/u]^4 du$
$\int_0^u [2J_1(u)/u]^4 u\, du$
Fig. 2A
1.0
.9
.8
.7
.6
.5
.4
.3
.2
.1
0
0.5
1.0
1.5
2.0
2.5
3.0
u
$\int_0^u [J_1(u)/u]^4 du$
Fig. 2B
$10 \log (2J_1(u)/u)^4$
-2
-4
-6
-8
-10
-12
-14
-16
-18
-20
-22
-24
-26

approximately, if ψ_1 is in radians. Putting for a/λ its value 0.387Q from Eq. (21) and replacing ψ_1 by y, its equivalent in degrees,

$$Q = 0.0264y. \tag{23}$$

It is easily shown (Ref. 1, Card 17) that for a directivity pattern which is symmetrical about the axis

$$d = 0.5 \int_0^{\pi} q(\psi) \sin\psi \, d\psi. \tag{24}$$

For a circular piston in an infinite baffle,

$$d = 0.5 \int_0^{\pi/2} (2J_1(u)/u)^2 \sin\psi \, d\psi. \tag{25}$$

By Ref. 7, p. 98, which is cited in Ref. 2, if $a > \lambda$ it can be shown that

$$d = (\lambda/2\pi a)^2. \tag{26}$$

APPENDIX II

RECTANGULAR PISTON IN AN INFINITE BAFFLE

In the study of reverberation made by the Reverberation Group, much of the data was taken with rectangular projectors. It was necessary to work out pattern relations analogous to the foregoing to determine the projector constants $Q(0)$ and P, and the effective dimensions of the projector.

A. Surface Reverberation

If $a \neq b$, Q_1, the value of Q with the side a horizontal (as in Fig. 1), is different from Q_2, the value of Q with the side b horizontal. By Ref. 3, p. 13,

$$Q = \int_0^{2\pi} q(0,\phi)q'(0,\phi)d\phi. \qquad (27)$$

It can be shown (Ref. 8, p. 100) that for a rectangular piston in an infinite baffle $q(0,\phi) = (\sin u/u)^2$, where $u = (\pi a/\lambda)\sin\phi$. For an ideal transducer $q' = q$. Because of the baffle and because $q(0,\phi)$ is symmetrical, the integral from 0 to 2π equals twice the integral from 0 to $\pi/2$. Hence,

$$Q_1 = (2\lambda/\pi a)\int_0^{\pi a/\lambda}(\sin u/u)^4 du/\cos\phi. \qquad (28)$$

Since $1/\cos\phi = 1 + \frac{1}{2}(\lambda u/\pi a)^2 + \ldots$, and because the integrand converges rapidly (Fig. 2A),

$$Q_1 = (2\lambda/\pi a)\int_0^{3}(\sin u/u)^4 du,$$

approximately, if the piston is large. Integrating $(\sin u/u)^4$, Table II, by the trapezoidal rule

$$Q_1 = 0.665\ \lambda/a \qquad (29)$$

$$Q_2 = 0.665\ \lambda/b = Q_1 a/b$$

By definition, $(\sin u_1/u_1)^2 = 0.5$ if u_1 is the half-width. Assuming that the pattern is so narrow that $\sin\phi = \phi$, $u = \pi\phi a/\lambda$. From a table of $(\sin u/u)^2$, $u_1 = 1.39$. Putting its value $0.665/Q$ for a/λ and converting the angle ϕ_1 corresponding to u_1 to y in degrees,

$$Q = 0.0262y \tag{30}$$

B. Volume Reverberation

From Ref. 3, p. 8,

$$P = \int_0^{2\pi}\int_{-\pi/2}^{\pi/2} q^2(\theta,\phi)\cos\theta\, d\theta\, d\phi. \tag{31}$$

Because of the baffle, and by symmetry,

$$P = 4\int_0^{\pi/2}\int_0^{\pi/2} q^2(\theta,\phi)\cos\theta\, d\theta\, d\phi. \tag{32}$$

The space pattern of a rectangular piston in an infinite baffle, from Ref. 8, p. 100, is

$$q(\theta,\phi) = \frac{\sin^2[(\pi a/\lambda)\cos\theta\sin\phi]}{[(\pi a/\lambda)\cos\theta\sin\phi]^2}\cdot\frac{\sin^2[(\pi b/\lambda)\sin\theta]}{[(\pi b/\lambda)\sin\theta]^2} \tag{33}$$

If it is assumed that $\cos\theta = 1$ for all appreciable values of the integrand, then

$$q(\theta,\phi) = (\sin u/u)^2(\sin v/v)^2 \tag{34}$$

where

$$u = (\pi a/\lambda)\sin\phi \text{ and } v = (\pi b/\lambda)\sin\theta.$$

$$P = (4\lambda^2/\pi^2 ab)\int_0^{\pi b/\lambda}\int_0^{\pi a/\lambda}(\sin u/u)^4(\sin v/v)^4 du\, dv/\cos\phi \tag{35}$$

$1/\cos\phi$ can be expanded as before. If $a > 2\lambda$, it can be assumed that $\cos\phi = 1$ for all appreciable values of the integrand and that the upper limits of integration may be replaced by 3 (Fig. 2A). Then because a definite integral is a function only of the limits and not of the variable of integration,

$$P = \frac{4\lambda^2}{\pi^2 ab}\left\{\int_0^a \left[\frac{\sin u}{u}\right]^4 du\right\}^2 \tag{36}$$

$$= Q_1 Q_2 \tag{37}$$

$$= .442\lambda^2/ab. \tag{38}$$

P is related by about the same constant to the wavelength λ and the effective area A of the projector in the two cases studied. For the circular projector,

$$P = 0.461\lambda^2/\pi a^2 = 0.461\lambda^2/A. \tag{39}$$

For the rectangular projector,

$$P = 0.442\lambda^2/ab = 0.442\lambda^2/A. \tag{40}$$

TABLE II

u	$(J_1(u)/u)^4$	$(\sin u/u)^4$
0.0	0.06250	1.00000
.1	.06219	.99321
.2	.06126	.97366
.3	.05975	.94160
.4	.05768	.89832
.5	.05512	.84531
.6	.05214	.78430
.7	.04880	.71736
.8	.04519	.64653
.9	.04139	.57387
1.0	.03750	.50136
1.1	.03358	.43087
1.2	.02973	.36392
1.3	.02600	.30182
1.4	.02246	.24548
1.5	.01914	.19555
1.6	.01609	.15233
1.7	.01334	.11578
1.8	.01089	.08568
1.9	.00875	.06153
2.0	.00691	.04273
2.1	.00536	.02855
2.2	.00439	.01824
2.3	.00358	.01105
2.4	.00221	.00627
2.5	.00156	.00328
2.6	.00108	.00155
2.7	.00072	.00063
2.8	.00046	.00020
2.9	.00028	.00005
3.0	.00016	.00000

REFERENCES

1. A Practical Dictionary of Underwater Acoustical Devices (CUDWR, July 25, 1942).

2. Directivity Considerations for Echo-Ranging Projectors, E. Dietze and L. L. Foldy (CUDWR, April 30, 1943).

3. Reverberation at 24 Kc, Reverberation Group (UCDWR No. U7; November 23, 1942).

4. Analysis of QC and JK Type Projectors (Bell Telephone Laboratories, April 10, 1942).

5. Calibration of Submarine Signal Company Combination WEA-2 Projector and Dome, E. Dietze and N. Bailey (CUDWR, USRL; November 16, 1942).

6. Vibration and Sound, P. M. Morse (McGraw-Hill, 1936).

7. Bessel Functions for Engineers, N. W. McLachlan (Oxford University Press, 1934).

8. Loud Speakers; Theory, Performance Testing and Design, N. W. McLachlan (Oxford, The Clarendon Press, 1934).

9. Report on Directivity Patterns of Sound Sources (Underwater Sound Laboratory, Harvard University, April 29, 1942).

RG:eas
6/29/43
65 MMC