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By

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Abstract

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What happens to pedagogy when a teacher's personal goals of supporting students' productive dispositions toward *learning* collide with her professional identity as a successful teacher whose students *perform* well on standardized tests? This dissertation is a mixed-methods case study that shows how context shapes one teacher's identity and decision-making, such that she seems to be two drastically different teachers in two different instructional contexts – a summer course in which she had complete flexibility over the curriculum, goals, and achievement measures and an academic year course in which she felt bounded by the state standards test. The dissertation examines the very real consequences these pedagogical decisions have for students.

Using qualitative classroom observations and quantitative survey and assessment data, this dissertation examines why, despite the teacher's strong commitment to growth mindset instruction and equity in both contexts, the teacher implemented pedagogical moves that contributed to distinctly different opportunities for students to engage with rich mathematics in each, and what those shifts meant for students' mathematical identities and learning.

The different cultural contexts in the summer and academic years offered the teacher identity resources about what was valued as good teaching, which led to distinct pedagogical decisions that aligned with the salient aspects of her professional identity in each context. Despite her commitment to growth mindset instruction in both contexts, this teacher implemented pedagogical moves that contributed to distinctly different opportunities for students to engage with rich mathematics and develop productive mathematical self-concepts.

This dissertation examines the ways the institutional context shifted and practices changed subtly as a result, and uses these comparisons to unpack which elements of the whole system of teaching for a growth mindset are necessary to contribute to productive changes in student mindsets or dispositions toward mathematics, engagement, and persistence with learning. Using Ms. M as a case study, this dissertation sheds light on the ways in which school contexts - in concert with a teachers' multifaceted identity - contribute to decision-making while setting instructional goals.

In the memory of Randi A. Engle

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Chapter 1: Introduction, Prior Literature, & Methods

> So algebra, once solely in place as the gatekeeper for higher math and the priesthood who gained access to it, now is the gatekeeper for citizenship; and people who don't have it are like the people who couldn't read and write in the industrial age. (Moses & Cobb, 2001, p. 14)

Purpose and Overview

What happens to pedagogy when a teacher's personal goals of supporting students' productive dispositions toward *learning* collide with her professional identity as a successful teacher whose students *perform* well on standardized tests? Many teachers face this dilemma daily. Despite teachers' best intentions, school contexts may shape teachers' classroom decisions in ways that have damaging consequences for students' opportunities to learn (Schoenfeld, 2010). This dilemma has become increasingly strong with the nationwide adoption of the Common Core State Standards in Mathematics (CCSSM) amidst a climate of high stakes testing and merit pay, where teachers are asked to negotiate evolving ideals about supporting students' mathematical learning with the pragmatics of performance-based goals. This dissertation is a mixed-methods case study that shows how context and the options for professional identity made available shaped a teacher's decision-making, such that she seemed to be two drastically different teachers in two different instructional contexts.

In 2011, Ms. M[1] taught a summer middle school course for students identified as previously low achieving. Despite this label, students engaged in challenging mathematics, demonstrated learning gains, and developed productive growth mindsets (Nix, 2012). Analysis traced these changes to opportunities Ms. M provided for students to experience themselves as competent doers of mathematics. Ms. M consistently emphasized that competent mathematical participation meant persisting in the face of challenge, focusing on the development of *learning* goals (Dweck, 1999). She stressed, "I want you to be conscious of the fact that every time you take on a challenge and you tiger up and you learn from it, your brain is actually developing the same way that muscles develop."

Ms. M supported this message by bringing challenging non-routine problems into the classroom and giving students the authority to develop their own ideas about the mathematics. When students struggled, she reinforced, "That is what I was going for […] You're coming to my class to do things that violate you mentally so that you are forced to heal and *learn*." When teachers incorporate challenges and emphasize *learning* goals—highlighting effort as opposed to *performance*—students are likely to adopt a *growth mindset,* believing that their intelligence is malleable and increasing their effort in the face of challenges (Dweck, 1999). In short, Ms. M's explicit focus on what, building on Dweck (2006), I refer to as "teaching for a growth mindset" led to meaningful engagement with and learning of challenging mathematics.

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[1] All teacher, student, and school names are pseudonyms

Contrast this classroom with Ms. M's traditional Algebra class this past academic year. On the first day of school, Ms. M had students guess the percentage of 8th graders from their school who scored "proficient" on the California Standards Test (CST) each year since 2006, highlighting the change that occurred after her first year there. For the year 2013, she wrote a question mark on the board, "That's you. So we have this year to prepare you for the state test at the end of the year, which will be about this [*holds up textbook*]. This is our mission." In this interaction, Ms. M also emphasized and praised growth on the CST over time, but the accompanying goal was *performance* –to perform well on the test. A *performance* orientation can cause students to be concerned with looking smart—a goal aligned with the belief that intelligence is *fixed* (Dweck, 1986). This mindset leads students to seek easy successes in the pursuit of looking smart and to reduce their persistence in the face of obstacles (Dweck, 1999). The teacher's first-day interaction established a single major *performance* goal for students in this course, sending a strong message about the nature of intelligence and what it means to be competent in this class (Dweck, 1999).

These two vignettes reveal seemingly different opportunities for students' mindsets about intelligence in ways that could affect their dispositions toward mathematics and persistence with challenge (Dweck 1999). Despite these distinctly different classroom framings, both of these classrooms had the same teacher, Ms. M, who outlined the same pedagogical goal of teaching for a growth mindset for her Algebra students in the academic year that I observed in her summer course. In her teaching philosophy, Ms. M spoke extensively about Dweck's (2006) work on mindsets and expressed a deep commitment to equitable teaching that provides all students the opportunities to succeed. On all conventional measures, Ms. M has demonstrated tremendous success in increasing student achievement for all her Algebra students, gaining widespread recognition for her teaching. About five years ago, she began weaving elements of growth mindset work into her instruction – most notably by sharing a quote about growth mindset with her students each week and engaging them in a conversation about how it relates to their learning – and she reports noticing changes in students' work since then.

While observing her summer course I also noticed significant changes that motivated a detailed characterization of her pedagogical moves and their effect on students. Student interviews pointed to the power of these moves to affect the mindsets of students with a history of low-achievement and decreased motivation. For example, Tyrone explained, "I like the challenge 'cuz it's like really hard, but […] if you work for it then, you know, you'll get it and I try to work very hard […] I felt like even when I didn't get it, I still would feel like, 'okay, well this is something I need to work on.'" He and other students adopted a growth mindset and learning orientation—if you work hard, you'll eventually understand the problem. Rather than being indicative of any shortcomings, Tyrone came to view his struggles as a sign to increase effort. These mindset shifts were accompanied with increased engagement and persistence on challenging measures of learning. The compelling outcomes from this mindset pedagogy motivated a more comprehensive study of these same moves over the course of an academic year in Ms. M's Algebra class.

The potential of these moves to transform student learning in an Algebra class is particularly significant. As algebra has taken on the role of gatekeeper for citizenship and a modern-day civil right, mastery of algebra has become central to students' future opportunities as citizens and thus a central issue in equity conversations in mathematics education (Moses & Cobb, 2001). Coming into her Algebra class, I expected to observe the same kinds of teaching for a growth mindset moves as in the summer course and to measure the effects of these moves on heterogeneously grouped *Algebra* students. However, despite some surface similarities in the way that Ms. M discussed mindset with her students, I observed a noticeably different set of practices across the two classes.

This study compares the drastically different pedagogies of the same urban middle school mathematics teacher in two distinct teaching contexts with similar populations of students from educationally disadvantaged backgrounds – a summer course in which she had flexibility over the curriculum and goals for students, and an academic year Algebra course in which she felt herself bounded by the end of year state standards test and accompanying curriculum. In the academic year course, this teacher had divided goals; on the one hand she was strongly committed to growth mindset, but her professional identity hinged on her students' success on a procedural standardized state test. As a result, this commitment to her professional identity undermined her work in effectively teaching for a growth mindset. This tension raises the questions addressed in this dissertation.

Using a range of qualitative classroom observations and quantitative survey and assessment data, the goal of my research is to examine why, despite her strong commitment to growth mindset instruction, Ms. M implemented moves that contributed to distinctly different opportunities and what those shifts meant for students. Specifically, this dissertation will address:

1. What happens when a teacher—who has a demonstrated commitment to growth mindset ideologies and the skills to teach for a growth mindset in ways that influence her students' mindset and performance—teaches in two very different contexts: a) a summer course in which there is little accountability for content learning, and in which she chooses challenging content as a means of supporting work toward growth mindsets, and b) a regular academic year Algebra course in which the immense pressure she felt to prepare her students to perform well on a high stakes accountability measure drove her pedagogical choices?
    a) Which pedagogical strategies are implemented, are modified, or disappear in each context?
    b) Why does the teacher make such distinct pedagogical choices in each context?

2. What is the overall impact of the implemented pedagogical strategies on students' a) self concepts and dispositions toward mathematics, b) engagement with challenging mathematics, and c) persistence with learning? In other words, what are the necessary pedagogical elements for effectively teaching for a growth mindset?

The analysis of the two classroom contexts is twofold. The first analysis strand – *pedagogical moves* – aims to qualitatively characterize the growth mindset pedagogy in each context and identify the reasons for divergence between these two pedagogies with the same teacher. The second strand – *student outcomes* – examines how these pedagogical moves in the two contexts enabled and constrained students' developing mathematical identities and behaviors.

The close analysis of the first strand considers how a teacher, who has demonstrated success in teaching for a growth mindset in one context, supports a seemingly contrasting classroom environment in another context. Analysis will characterize the system of moves in each context toward teaching for a growth mindset. This strand examines the intersection of the teacher's multifaceted professional identity and the context in which she negotiates goal setting for her students. The case study of Ms. M combines research on decision-making (e.g., Schoenfeld, 2010) and identity (e.g., Holland et al., 2001) in order to examine the way a change in context can result in different aspects of her multifaceted identity being activated, which results in different consequential pedagogical choices being made. The second analysis strand examines the consequences of these pedagogical moves for student mindsets or dispositions toward mathematics, engagement with challenging mathematics, and persistence in the face of challenge.

The goal of this research is to compare the different figured worlds of these two classes to reveal the ways they afforded different opportunities for students to develop mathematical self-concepts and engage with rich mathematics (Holland et al., 2001). In doing so, I examine the ways the institutional context shifted and practices changed subtly as a result, and use these comparisons to unpack which elements of the whole system of teaching for a growth mindset are necessary to contribute to productive changes in student mindsets or dispositions toward mathematics, engagement, and persistence with learning.

Using Ms. M as a case study, this dissertation will shed light on the ways in which school contexts - in concert with a teachers' multifaceted identity - contribute to decision-making while setting goals (Schoenfeld, 2010). In this case, the contextual emphasis on high stakes tests resulted in the deformation of the pedagogy of a teacher who has demonstrated that she *can* effectively teach for a growth mindset. This dissertation will use detailed qualitative analysis of the classroom and of the teacher's goals and orientations to elaborate on the mechanisms by which this process occurred, and will supplement this analysis with quantitative results to analyze the origins of student self-concepts and their relationship to engagement and learning, toward providing more equitable learning opportunities for all students.

More broadly, this dissertation speaks to a larger argument about the nature of mathematics in the classroom and to a shift in the way teachers are supported in their pedagogy. Research has established that when teachers teach for skills, concepts, *and* problem solving, students will perform as well on skills, but much higher on tests of concept and problem solving than if teachers only teach procedurally to the test; However, as was the case for Ms. M, this shift toward problem solving requires a leap of faith for teachers when their professional identity hinges on their students' success on a standardized test. Without shifting the school context and supporting teachers in taking this leap of faith, teachers' professional identities will continue to undermine their work in providing rich learning opportunities for all students. This dissertation raises and attempts to unpack these issues.

Prior Literature

Teacher Decision-Making and Professional Identity in Context

One way to consider the different pedagogical decisions teachers make is to look at their goals and orientations in context. Research on decision-making shows that resources, goals, and orientations all come into play as people make decisions in context; as people orient their knowledge resources to particular situations, goals are established or prioritized and people make decisions consistent with these goals (Schoenfeld, 2010). This literature offers a broad understanding of why teachers in different routine contexts may establish goals that lead to different pedagogical moves.

Another strand for considering these pedagogical differences is to look at teachers' multidimensional professional identities. In examining practice-linked identities, Nasir and Cooks (2009) argue that as individuals participate in "communities of practice" (cf. Wenger, 1998), they are offered (and negotiate) resources for identity development. This argument reinforces the notion that the cultural context of the school makes available resources that can support the development of a particular professional identity. By providing ideas about a person, their relationships to practice, and what is valued as good, ideational resources made available for teachers in particular school contexts can position teachers into specific roles (that they can then negotiate) (Nasir & Cooks, 2009).

Teachers' developing professional identities can in turn contribute to their pedagogical decisions. If a person is concerned with being seen as a particular kind of teacher, then the pedagogical choices that she makes and the way she uses the cultural forms available for instruction will serve those identity purposes (Langer-Osuna, 2007), and the goals that are salient in two contexts may differ depending on the set of norms, expectations, and ideas of that context (Holland et al., 2001). Building on Schoenfeld (2010), one could say that the identity resources available in a particular context shape the orientations that influence the process of decision-making. By showing the ways in which context and identity in concert shape a teacher's decision-making, this dissertation lies at the intersection of theories of teacher identity and teacher decision-making, and contributes to both.

Pedagogy to Influence Student Dispositions, Engagement, and Persistence

These shifts in pedagogy are important because they can contribute to the kinds of dispositions toward mathematics that students develop by providing varying opportunities for students to experience themselves as competent doers of mathematics (Gresalfi & Cobb, 2006). Research on identity development recognizes that students' mathematical self-concepts derive from their experiences in schools and classrooms that create different opportunities for their development, and shape the ways in which students go about engaging with mathematics (Gresalfi, 2009; Holland et al., 2001).

Students' concepts about their own mathematical intelligence and mindsets about the nature of intelligence can, in turn, have a large effect on their classroom behavior (Dweck, 1986). How students develop a sense of self with respect to mathematics and to their classroom peers inextricably relates to the ways they then engage in mathematical practices and what they learn as a result of that engagement (Gresalfi & Cobb, 2006; Schoenfeld, 1992). These self-concept beliefs can, for example, strongly influence how much effort students put forth or how long they persevere in the face of difficulty (Dweck, 2006), making them consequential for student motivation and learning.

In 2008, the National Mathematics Advisory Panel called for additional research on "interventions that address social, affective, and motivational factors" (p. 32), particularly for African American and Latino students. To that end, researchers have grown increasingly aware of the importance of social contexts and interaction in the classroom for the development of mathematical self-concepts and learning (e.g., Nasir & Cooks, 2009). Consistent with Holland and her colleagues' (2001) work on identity, classrooms can be seen as a type of figured world, or a socially and culturally constructed realm of interpretation of particular characters, activities, and situations. In the same way that school contexts can provide different identity resources for teachers, classroom ecologies constrain and enable the ways students can participate based on the ways students are positioned (and position themselves) in particular roles and the ways activities can be interpreted within a figured world.

Accordingly, students and teachers develop different identities in diverse figured worlds because they are afforded different subject positions in those worlds (with which they can align themselves or reject) (Holland et al., 2001) and are provided differential access to resources for the development of productive discipline-related identities (Nasir & Cooks, 2009). Research shows that practices that invite multiple storylines into classroom and widen opportunities students have to align with the discipline increase the likelihood that students will develop identities as doers of mathematics (Cobb, Gresalfi, Hodge, 2009; Boaler & Greeno, 2000).

This perspective on identity, then, calls for an examination of the moves that teachers can use to influence students' mathematical identities in ways that support their engagement and success with rich mathematics. If the kinds of dispositions toward mathematics that students develop influence students' mathematical behavior, and if classroom practices can create different opportunities to influence these dispositions, then research must also examine these practices. Conceptions of classrooms should not only consider the ideas and skills that students learn, but also the kinds of dispositions toward mathematics that they are developing and the ways specific classroom practices can contribute to this development (Nasir & Cooks, 2009; Gresalfi & Cobb, 2006).

For example, Hand (2009) provides a framework for examining how classroom positioning and framing discourse can influence students' behavior. Participation structures are one aspect of classrooms that can influence student action in more or less productive ways. Specifically, flexible participation structures widen what it means to be a competent participant by allowing for negotiation between teachers and students in defining productive contributions, and support engagement among a variety of learners by leaving different forms of activity undefined and open for negotiation (Hand, 2009).

Like Hand, Gresalfi (2009) has provided a lens for examining how participant frameworks of classrooms can lead to particular student dispositions and actions. Rather than only looking at the engagement and dispositions of individual students, Gresalfi (2009) shifts toward looking at the *individual-with-context* to examine how a participant structure "shapes the ways in which students are expected, obligated, and entitled to participate with content and with others in the classroom" (p. 331). She examines how student behavior is made meaningful in a classroom context, and how dispositions develop and shift in interaction between an individual and classroom practices over the course of a school year. Through this lens, Gresalfi (2009) argues that particular classroom practices can create different opportunities for students to engage with mathematics and to develop dispositions in more productive ways.

Specifically, pedagogical practices that address a) the sociodisciplinary aspects of classrooms that afford opportunities for students to engage competently in mathematics (e.g., norms around what it means to "do" mathematics) (Schoenfeld, 1988; Yackel & Cobb, 1996) and b) the interpersonal opportunities that enhance these affordances (e.g., framing and positioning) (Hand, 2009) can contribute to the development of productive mathematical identities. These practices increase students' opportunities to experience themselves as competent while engaging in worthwhile mathematics and, in doing so, opportunities for students to develop productive dispositions toward mathematics (Gresalfi, 2009).

Student engagement. The pedagogical approach Complex Instruction (CI) consists of these types of pedagogical practices. In particular, CI uses strategies to address the multiple ways students can be seen as competent via both the academic task structure and social participation structures in order to increase engagement and thus, learning (Cohen & Lotan, 1995). CI uses the strategy of assigning competence to students through revoicing in which teachers assign credit to students for their mathematical ideas by reformulating their contributions (O'Connor & Michaels, 1993). This type of discursive positioning move increases the opportunities for students to experience themselves as competent doers of mathematics by positioning them with authority over and accountability to the content (Engle & Conant, 2002). As such, this type of discursive positioning move has the potential to afford the development of productive student self-concepts.

Student mindsets. Furthermore, Dweck's (2006) research on student motivation and beliefs has theorized about how teachers can influence students' beliefs about intelligence and their motivational patterns in ways that affect their persistence with challenging problems (Dweck, 2006). Dweck's (1999) theory of student mindsets asserts that there are two contrasting views of intelligence that students can hold that can influence their behavior in powerful ways. As previously described, this research on student mindsets shows that students who believe their intelligence is malleable (hold a growth mindset) develop learning goals – as opposed to performance goals – which influence them to choose and persist on more challenging tasks that foster learning. When teachers incorporate challenges within a learning-oriented context where students are praised for effort as opposed to achievement, students are likely to develop more adaptive motivational patterns. My goal in coordinating these strands of literature is to use them to analyze the sociodisciplinary and interpersonal aspects in Ms. M's class aimed at increasing students' opportunities to experience themselves as competent learners of mathematics, and the influence of these moves on students' developing mathematical self-concepts.

While there have been many studies that look at the effects of *particular* pedagogical moves (e.g. Cohen & Lotan, 1995; Hand, 2009; Dweck, 1999), there have been few, if any, with a fine-grained analysis of a case study of a teacher implementing such a deliberate treatment of mindset and competence over the course of an academic year. This research is pertinent for practice as, more and more, teachers are committing themselves to the ideas of developing students' growth mindsets, but have expressed not knowing how to connect it to teaching math (Boaler, 2013). I coordinate this literature to analyze the pedagogical moves in Ms. M's classes—summer and academic year—aimed at increasing students' opportunities to experience themselves as competent learners of mathematics.

Methods

Research Questions

1. What happens when a teacher—who has a demonstrated commitment to growth mindset ideologies and is able to make them work in a certain environment, and has demonstrated success in increasing student achievement on standardized measures—is faced with teaching for a growth mindset in the constraints of a traditional Algebra context?

    a) Which pedagogical strategies are implemented, are modified, or disappear in this context?

    b) Why does the teacher make such distinct pedagogical choices in each context?

2. What is the overall impact of the implemented pedagogical strategies on students':

    a) mathematical self concepts,

    b) engagement with challenging mathematics, and

    c) persistence with learning?

Context

Data for this study come from two urban Northern California middle school classrooms with the same 15-year veteran teacher, Ms. M: a 2011 summer school course of rising 8th graders and a 2012-2013 academic year Algebra course. These two sites afford comparisons of the pedagogy of same teacher in two different instructional and school contexts. As mentioned earlier, about five years ago Ms. M began weaving elements of explicit growth mindset instruction into her Algebra classes, and brought them into her summer school course as well. She expressed a commitment to equity-oriented teaching, in which she sees all students as capable of succeeding and provides opportunities for all students to do so.

Summer course. The first data set comes from a five-week summer school course, during which we first observed Ms. M's explicit growth mindset instruction. This data served as a pilot study that motivated the second set of data from Ms. M's Algebra class over the course of the

2012-2013 academic year. The 16 summer school students, rising 8th graders, were recommended to the course by their counselors at the three district middle schools based on students' history of low achievement and decreased motivation. The students were not required to take the course in order to pass to the next grade, but all slots were filled, with additional students joining from a waiting list after a few students did not show up on the first day. There were no costs to the students for enrolling in the course.

The five-week long summer school class met four days a week for an hour and 50 minutes each day; typically, the last 30 minutes was allocated to the computer lab, with students working on school-mandated mathematics learning software**.** As students were not earning required credit for this course, for the other hour and 20 minutes, Ms. M was not confined to a set curriculum and thus had immense flexibility to design course curriculum, goals, and achievement measures. She was not required to keep a particular pace, cover specific topics, or implement state or district assessments.

Each of the first three weeks of the course was developed around a particular problem-solving strategy: guess and check, look for a pattern, and solve an easier related problem. Each day was structured around the students working independently, in groups of three, and as a whole class to solve a challenging non-routine problem of the day that was chosen to help students practice these strategies. For the fourth and half of the fifth week of the course, the goal was to increase students' pre-algebra skills to prepare students for 8th grade Algebra; specifically, the skills that drove the curricular choices were: maintain equality in an equation, and represent linear relationships in tables, graphs, and equations. Similar to the previous three weeks, each day was structured around the students working independently, in groups of three, and as a whole class to explore and define algebraic relationships from various pictorial patterns.

In addition to these larger organizing goals, the curriculum incorporated the sub-goals of adding, subtracting, and multiplying integers, as well as goals for students' math attitudes and the class culture. Specifically, Ms. M stated that she aimed to increase students' awareness of the difference between math exercises and math problems, their openness to word problems, their confidence in their math abilities, their willingness to tolerate confusion without giving up, and their direct relationship with math. Similarly, she wanted to foster a positive, safe class community, collaboration through team talks, and student engagement. These math attitude and class culture goals were brought to fruition in the curriculum through the types of problems chosen and were further fostered through the pedagogical moves I will characterize in this dissertation.

The three middle schools that fed into the summer course collectively had 52% of 8th grade students in 2011 score proficient or advanced on the Algebra 1 California Standards Test (CST). The students in this particular class were identified as low-achievers, and, although their specific CST scores are unavailable, there is reason to believe the percentage of students who scored basic or below basic on the CST was actually much higher for this class.

Across the three feeder middle schools, 50.1% of students received free or reduced-price lunch and demographically, 26.3% of the enrolled were African American, and 22.3% were

Latino/a. In this particular class, a majority of the student population was African American or Latino/a, with at least two English Language Learners.

The summer school course was taught at a fourth school, Bailey Middle School, in a room unfamiliar to both the teacher and the students. Over the course of the summer, Ms. M participated as a teacher-researcher as part of the Algebra Teaching Study, attending research meetings and collaborating with other members of the team regarding her teaching.

Academic year Algebra course. The second data set comes from an 8^{th} grade Algebra course taught by the same teacher over the 2012-2013 academic year at one of the three district middle schools, Frost Middle School. Table 1 shows the demographic characteristics of Frost Middle School. Compared to the state average of 32% in 2012, 75% of 8^{th} graders at Frost Middle School scored proficient or advanced on the Algebra 1 CST. As previously mentioned, Ms. M has demonstrated tremendous success in increasing student achievement on the CST, gaining personal recognition from US Secretary of Education Arne Duncan for her students' Algebra achievement in 2010.

Table 1
Demographic characteristics of the student body at Frost Middle School in 2012

# of students	460
% African American	31%
% Asian	5%
% Latino	34%
% White	20%
% Two or more races	8%
% Socioeconomically disadvantaged	57%
% English learners	22%
% Students with special needs	18%

Compared to the summer, students in the academic year Algebra course were, to a large extent, grouped heterogeneously based on achievement. While there was one period of honors Algebra for students who had demonstrated high-achievement on certain measures, the rest of the student population was divided among the traditional Algebra courses, with some students also enrolled in a second Algebra support class.

While the curricular content may have been peripheral to the goals of the summer course, it was necessarily a central focus of the academic year Algebra class; Ms. M and her students were bounded by the state standards and the accompanying CSTs. In particular, Ms. M explained that Frost Middle School was committed to raising students' test scores. She aimed for her students to be proficient on particular content standards, and taught using a traditional procedures-based Holt Algebra curriculum, where students are taught a sequenced set of procedural methods and then solve a set of similar tasks from the textbook for homework. While

Ms. M assigned fewer problems than typical of the curriculum, taking care to select the most difficult problems from the end of the problem set, she rarely deviated from the textbook.

Procedures

My research is a mixed-methods case study design. A primary data strand is video observations of courses, mixed with student and teacher interviews, and pre-post student measures of learning and self-concept beliefs.

Bailey Middle School data collection. The data set from the summer course at Bailey Middle School includes video observations of every session of the summer school course, teacher and student interviews, and match pre-post content assessments.

Classroom observations. Two researchers videotaped each day of the summer school course. The camera specifically captured the teacher as she talked to the whole group, to small groups of three, or to individual students. One or two additional researchers took fieldnotes to capture what happened both in terms of pedagogical moves and student actions. I analyzed the video and fieldnotes for evidence of student engagement and pedagogical moves the teacher made in the service of providing opportunities for students to experience themselves as competent learners of mathematics.

Teacher interviews. Prior to the first day of the course, two researchers audio-recorded an interview with Ms. M about her goals for the summer, and what pedagogical moves she planned to employ to reach these goals.

Over the course of the summer, a researcher would audio-record both pre- and post-lesson interviews to track Ms. M's goals and progress toward these goals from her perspective. The pre-lesson interviews asked Ms. M about her mathematical and classroom interaction goals for the current lesson, the big mathematical ideas she wanted students to focus on, and how this lesson fit into the unit or class as a whole, including how it built on prior lessons. The post-lesson interviews asked Ms. M how she felt the lesson went according to her goals mentioned in the pre-lesson interview, if anything unexpected or different happened from what she had planned, and how, if at all, the current day's lesson would influence her plans for the future lessons.

At the end of the summer, a researcher audio-recorded an interview assessing how Ms. M felt the summer course went with regard to reaching her goals, as well as what pedagogical moves she employed to reach those goals and her reasons for those moves. Additionally, I audio-recorded an interview with Ms. M after the summer school course ended about the instructional moves she made in service of her math attitudes and classroom culture goals.

Student learning. The Algebra Teaching Study team administered a set of matched pre- and post-assessments to capture changes in student learning over the course of the summer. Both the pre- and post-assessments consisted of 3 open-ended multi-question non-routine tasks adapted from Mathematics Assessment Resource Service (MARS) tasks to assess students' robust understanding of Algebra through problem-solving skills typically learned by the end of Algebra 1. None of the mathematical skills assessed on either part of the pre- and post-assessments were directly taught over the course of the five-week summer program.

Students' assessments were scored along five Robustness Criteria (RC), described further in Chapter 4, using a standardized rubric and were compared from pre- to post-assessment. Students' assessments were also scored for correctness and raw scores were compared from pre to post assessment to measure evidence of increased student learning. The pre- and post-assessments were analyzed for evidence of increased student engagement by measuring the number of attempted problems on both tests. If any work was shown on an item, students were given credit for attempting the problem.

Student post-summer interviews. During the last week of the summer course, I audio-recorded interviews with three students about their prior experiences with mathematics, as well as their experiences in the summer course. The three students were chosen in collaboration with Ms. M for having exhibited low engagement at the beginning of the summer, and because their prior teachers had previously characterized them as disengaged low-achieving students. This interview asked students about their past math experiences, including whether they thought they were good at math, whether they enjoyed math, and their grades in their prior math classes. Additionally, the interview asked students about their work and experiences in the summer school course, including whether it was similar to or different from their prior experiences with math.

Frost Middle School data collection. The data set from the academic year Algebra course at Frost Middle School includes video observations of instruction, teacher and student interviews, matched pre- and post-content assessments, and pre-and post-surveys on student mindsets.

Classroom observations. The primary data source for the study is videotapes and accompanying field notes of the classes. I videotaped and took fieldnotes of the first ten days of instruction and once per week for the remainder of the academic year to capture how the community developed with regards to a growth mindset pedagogy and what student engagement looked like in these beginning weeks. All told, there were 41 video recorded lessons during the academic year. As with the summer course, to the extent possible with lapel microphones, I specifically captured the teacher as she talked to the whole group, to small groups of three, or to individual students. As part of her intentional pedagogy, Ms. M engaged students each week in a discussion of a quote about mindsets. Any of these conversations that occurred on non-observational days were audio recorded by the teacher. Nineteen of these conversations were audio recorded.

Teacher interviews. At the beginning of the academic year, I audio-recorded formal and informal interviews with the teacher. I began by asking her about her philosophy of teaching, how she conceptualized her strategies, and the goals for using these particular pedagogical strategies.

In the middle of the academic year, I audio-recorded an interview with Ms. M, asking in more detail how she conceptualized the work she was able to do over the summer, and to compare and contrast that work with what she had done during the academic year up until that point. I asked her which aspects of the teaching for a growth mindset pedagogy she had outlined at the beginning of the year she felt were carrying over successfully, and which were not, asking her to reflect on specific moves such as assigning competence. I also asked her how she conceptualized teaching for a growth mindset within the context of her success in teaching the California standards and how she sees this work shifting, if at all, as she works toward teaching the new Common Core Standards in Mathematics.

Finally, I audio-recorded an interview at the end of the year, asking similar questions to the mid-year interview toward reflecting on the teaching for a growth mindset work as it played out in the academic year.

Student self-concepts. At the beginning of the school year, students completed a survey asking about their self-concepts as doers of mathematics, their beliefs about the nature of mathematics, and their overall self-theory beliefs with regards to mathematics and learning in general. This survey was adapted from existing instruments, including Dweck's (2006) Mindset survey, and had been piloted the year before. These surveys were supplemented with student interviews with a set of 2 to 4 focal students at the start and end of the year. The focal students were selected to represent a range of learning histories. Students completed a similar survey administered at the end of the school year that, in addition to the exact same survey items from the pre-survey, also asked about students' perceptions of Ms. M's pedagogical strategies.

Student learning. Finally, students took matched pre- and post-content assessments, jointly developed based on Ms. M's content goals. In the development, Ms. M aimed to align the assessment with her conception of the new Common Core Standards in Mathematics by measuring students' problem solving strategies and persistence. Ultimately, however, the tasks still reflect specific problem types such as "working together" or "proportional reasoning," that students were directly taught how to do during the academic year. These word problems, however, were still seen as the most challenging problems students worked on during the academic year.

Students' assessments were scored for correctness and raw scores were compared from pre- to post-assessment to measure evidence of increased student learning. The pre and post assessments were also analyzed for evidence of increased student engagement by measuring the number of attempted problems on both tests. Because the test was not designed to align with the Robustness Criteria used as a measure in the summer assessment, the academic year assessments were not scored along these criteria.

Dissertation Guide

This dissertation is broken into four sections. Through fine-grained qualitative analysis of Ms. M's pedagogy in the summer course, Chapter 2 will provide examples of what effectively Teaching for a Growth mindset can look like, and will examine the context and teacher decision-making that supported the pedagogy. In a similar fine-grained analysis, Chapter 3 will compare the pedagogy of the academic year to the summer, and will examine the teacher's pedagogical decision making, considering the context and professional identity resources made available as explanatory frames for the differences in growth mindset pedagogy. Chapter 4 will use both qualitative interviews and quantitative surveys and assessments to consider the impact of these two distinct pedagogies on students' developing mathematical self-concepts, engagement with rich mathematics, and persistence with learning. Finally, Chapter 5 will provide a discussion of the findings, considering the implications of this research on the effectiveness for teaching for a growth mindset, as well as directions for future research.

Chapter 2: "Tiger-up!" Teaching for a Growth Mindset in a Middle School Summer Algebra Course

With the nationwide adoption of the Common Core State Standards in Mathematics (CCSSM) while high stakes testing still permeates the landscape, teachers now have to negotiate evolving ideals about supporting students' mathematical *learning* with the pragmatics of *performance*-based outcome goals. This dissertation examines how context and the professional identity made available in two different contexts shaped a teacher's decision-making, such that she seemed to be two drastically different teachers in two different instructional contexts.

While the data gathered in the academic year revealed the challenges Ms. M faced as she aimed to implement growth mindset instruction in that traditional context, her summer school course the year prior exemplified Ms. M's success with Teaching for a Growth Mindset in ways that influenced students mathematical self concepts, engagement with rich mathematics, and persistence with learning. Analyzing the summer school data reveals not only Ms. M's capacity to successfully teach for a growth mindset and the potential for what that pedagogy *can* look like, but it also contributes to the understanding of the optimal conditions for implementing these instructional moves.

The goal of this chapter is to characterize the nature of Ms. M's pedagogy in her summer middle school mathematics course, which was aimed at providing opportunities for students to experience themselves as competent *doers* and *learners* of mathematics in ways that successfully influenced the students' self concepts and motivated them to engage with challenging mathematics. This analysis will show the four major strands of pedagogy that she employed to successfully implement a growth mindset pedagogy. In doing so, this chapter will begin to unpack the major differences in pedagogy and accompanying instructional affordances between the two classroom contexts. Chapter 3 will build on this comparison by characterizing the pedagogy as it shifted in her academic year course.

The students in Ms. M's summer course at Bailey Middle School had all been identified by their prior teachers as previously low-achieving. Yet, despite this label and the slower-paced and restricted curricula often accompanying the label (Burris et al., 2006; Oakes & Lipman, 2003; Boaler & Staples, 2008), Ms. M tailored her instruction in such a way that students engaged in rich problem solving, demonstrated persistence on learning assessments, and developed productive growth mindsets. More specifically, she implemented an approach aimed at influencing her students' mindsets about their own intelligence that successfully motivated them to engage with challenging mathematics. Chapter 4 will outline the student outcomes measured across the two classrooms; this current chapter will build on prior analyses of the summer classroom to illustrate the pedagogy that contributed to these outcomes in productive ways and will describe the context and professional identity resources that reinforced the effective implementation of the teacher's goals.

In service of achieving shifts in student behaviors and outcomes, Ms. M began both the summer school course at Bailey Middle School and the academic year course at Frost Middle School with the pedagogical goal of providing opportunities for students to experience themselves as *smart* in mathematics. Ms. M was explicitly familiar with Carol Dweck's (2006) book *Mindset*, and she explicitly aimed to teach a growth mindset to her students. Specifically, Ms. M believed, "a growth mindset is the ignition for learning" (Ms. M, 2011). She argued that by instilling in students the ability to "consciously change their own mindset in the face of new and difficult problems" to recognize that "if they want to be good at something it's a matter of effort and not just a matter of figuring out what it is that you're good at," students will have more sustained engagement and perseverance in challenging problem-solving tasks and will learn more as a result (Ms. M., 2011 & 2012).

Building from this framework, Ms. M designed her lessons and pedagogical strategies to teach a growth mindset that would increase students' endurance in taking on challenge without giving up, and provide opportunities for the students to experience themselves as *smart* in mathematics. Her use of the term *smart* in this sense did not imply a static conception of having or not having a particular amount of intelligence, but instead centered on students feeling capable of engaging in challenging mathematics because they can increase their intelligence by working hard or "tigering up."

Her pedagogy in the summer course at Bailey Middle School reflected these goals and accordingly, students showed evidence of shifts in mindset that corresponded with more sustained engagement with challenging mathematics and an increase on learning measures from the beginning to the end of the short summer program. As we will see in Chapter 4, the same shifts did not occur in the academic year course at Frost Middle School. After characterizing the classroom pedagogy of the summer course, this chapter will describe how the specific context of the summer course – in which Ms. M had flexibility to design course curriculum, goals, and achievement measures without a set curricula – provided fertile grounds that supported Ms. M's goal of teaching for a growth mindset.

Bailey Middle School Classroom Context

The five-week long summer school class for rising 8th graders met four days a week for 110 minutes each day. Typically, the last 30 minutes were allocated to the computer lab, with students working on school-mandated mathematics learning software**.** Given that students were not required to take this course for specific course credit, the teacher had flexibility in determining the curriculum, goals, and achievement measures.

Recall from Chapter 1 that the first three weeks of the course were each developed around a particular problem-solving strategy: guess and check, look for a pattern, and solve an easier related problem. For a week and a half at the end of the course, the instructional goal was to improve students' pre-algebra skills to prepare them for 8th grade Algebra. Students were organized to work independently, in groups of three, and as a whole class to solve a challenging non-routine "problem of the day" that supported the development of the weekly strategy or to explore and define algebraic relationships from various pictorial patterns.

In addition to these larger organizing goals, Ms. M set sub-goals of adding, subtracting, and multiplying integers, as well as goals for students' math attitudes and the class culture. Before the course started, Ms. M expressed an explicit desire to increase students' awareness of the difference between math exercises and math problems, their openness to word problems, their confidence in their math abilities, their willingness to tolerate confusion without giving up, and their direct relationship with math. Similarly, she wanted to foster a positive, safe class community, collaboration through team talks, and student engagement. These math attitude and class culture goals interacted with the curricular content goals to produce a particular set of enfranchising pedagogical moves characterized in the following sections.

Pedagogical Strategies Toward Teaching for a Growth Mindset – Summer

Open coding with the corpus of video data and transcripts generated an extensive list of codes of pedagogical moves that provided opportunities for students to experience themselves as competent *learners* of mathematics. With the help of undergraduate researchers, I consolidated these codes into key themes through an iterative process that we then took to the broader data for analysis and refinement. By coordinating these themes with prior literature on pedagogical moves to develop productive student mindsets and support engagement with challenging mathematics, I developed comparison dimensions to characterize the opportunities Ms. M provided for her students to experience themselves as competent mathematical participants in ways that would support or constrain the development of a growth mindset.

To that end, four major dimensions emerged that, building on Dweck (2006), characterize Ms. M's differences in pedagogy with respect to "Teaching for a Growth Mindset." These dimensions are: 1) Framing Success (What are the long-term goals for students? What gets defined as competent mathematical participation and how? What are the messages about what it means to work competently?); 2) Treatment of malleable intelligence; 3) The nature of the mathematics students are asked to work on; and 4) Agency and Authority. None of these dimensions are mutually exclusive, and many examples of the teacher's pedagogy cut across multiple dimensions.

As a productive example of a "Teaching for a Growth Mindset" over an extended period of time, we shall first consider the summer course, analyzing the characteristics of the pedagogy along those four major dimensions and their sub-strands (Table 2). This chapter will argue that the schooling context of the summer course afforded particular resources that contributed to this effective pedagogy. Chapter 4 will provide evidence of productive shifts in student mindsets, engagement, and learning to document the efficacy of the "Teaching for a Growth Mindset" pedagogy employed in the summer course.

Table 2
Summer Pedagogy for Productively Teaching for a Growth Mindset

1) Framing Success: Teacher and students co-constructed competent mathematical participation as supporting the learning-based goal of persisting in the face of challenges	A. Student goals: Teacher set long-term personal learning growth goals for students B. Teacher's explicit statements around what it means to work competently on mathematics in this class were learning-based C. Competent participation: *Students co-constructed competent participation as persisting in the face of challenge, and student creativity and contributions were valued as competent mathematical participation.*
2) Treatment of malleable intelligence: Teacher emphasized malleable intelligence	A. Teacher shared a quotation each week about learning and intelligence, and engaged students in relating the quotes to their own experiences as learners. B. Teacher communicated growth mindset messages throughout the term.
3) The Mathematics was primarily non-routine and challenging	A. Teacher incorporated challenge into the classroom by assigning difficult non-routine mathematics problems
4) Agency and Authority: Students had intellectual agency over the mathematics and were positioned with authority over their mathematical habits and ideas	A. *Intellectual agency* i. Students and teacher shared authority over the mathematics; students were "authorized" to come up with their own mathematical ideas. B. *Authority and positioning* i. Students had a sense of authority over their own behavior ii. Teacher positioned students with authority for productive mathematical habits or behaviors iii. Teacher positioned students with authority over the mathematics by revoicing/reformulating mathematical ideas

1) Framing Success: Teacher and Students Co-Constructed Competent Mathematical Participation as Supporting the Goal of Persisting in the Face of Challenges

A. Student goals: Teacher set long-term personal learning growth goals for students. In the summer school course, Ms. M framed the long-term goals for the students as *learning* oriented from the outset. Ms. M's work of framing the course started from a potentially volatile context: students had been selected for the course based on their previous low achievement, but not because they had failed. With this challenging starting point, she framed the class of students not as failures, but instead as students with potential to grow and learn.

Rather than defining the end goal for the course as performance on an exam, she framed students' course goal as readiness to learn in their Algebra course the following year and discussed the importance of this goal through the lens of equity. On the very first day of school, Ms. M began, "My goal is to prepare you for Algebra […] My inspiration is this guy Robert

Moses, and Robert Moses says that Algebra is a civil right. And he believes that we won't have equality until everybody has access to Algebra." On the second day, she continued to frame the selection of students by saying,

> I want you to all understand that you were recommended for this class, but because it was believed that you could actually make some great improvements to get that extra advantage to do really well next year. […] They gave me students who they thought would actually learn a lot this summer and then go back and be stars back in September.

While the long-term goals she framed for students were still content-based around Algebra readiness, they were framed as goals for *learning* rather than *performing.* In coding for evidence of goals, goals strictly around *performance* did not show up in the summer course in the ways that they did in the academic year, as will be shown in Chapter 3.

She continued to emphasize this framing throughout the first couple weeks of the course. For example, on the sixth day of the course, Ms. M reiterated,

> It is a class not designed…For those of you who didn't get this message earlier, it is not a class that is designed for people who failed a course and they need to make it up. This is a class designed to give you an advantage, to give you a boost, so that you can be stars next year. So this is not the kind of summer school class that sometimes people think of. This is a positive course. And our tiger, Bruce, reminds us to be fierce when it comes to math. When you get confused, tiger up. No kitty cats.

In an interview before the course began, Ms. M explicitly stated that her goal for her summer students was for them to "have increased endurance to handle taking on a challenge without giving up. And I want them to have increased confidence." The ways that she framed the long-term goals of the course for the students directly reflected these pedagogical goals.

In this framing, Ms. M positioned the students as competent learners, thus setting the stage for students to experience themselves as smart in mathematics, rather than beginning from a place of failure (Cohen & Lotan, 1995). In doing so, Ms. M influenced both the expectations for competence students have for themselves, as well as for each other as they embarked on this course together (Cohen & Lotan, 1995). Furthermore, by emphasizing a learning orientation, she contributed to a growth mindset in a way that may have helped students develop more adaptive motivational patterns by increasing their effort and persistence when faced with challenges (Dweck, 1999).

B. Ways of working: Teacher's explicit statements around what it means to work competently on mathematics in this class were learning-based. Throughout the summer school course, Ms. M consistently made explicit statements that framed learning and hard work as what it meant to work competently in this class. In these framing statements, Ms. M proposed what it means to be competent in this math class as learning-based, where understanding is the goal and confusion is part of the process.

In her explicit statements regarding what it means to work competently, Ms. M consistently stated that hard work was a necessary component for participating in this class. For example, she would make statements such as, "So that's what we're going for now – how fierce you are in the face of challenges […] so what we all want to do is work our hardest so we can be our smartest" (110727, Day 7). Her use of the term "smart" referred not to a static conception of performance, but as something fluid that can increase through hard work, where learning and increasing "smartness" represents what it means to work competently.

In addition to hard work, the day-to-day goals she set for students were not about performance, but instead about understanding and *learning.* For example, on the 12th day of summer school, Ms. M reiterated that learning was the goal and that it was one that required hard work,

> So we're about half - we're more than halfway through the days, but we're about halfway through the learning. So as well as you have done, I just want you to know that we have a lot of work ahead of us and you're going to leave here way smarter than you came. That's our goal every day - to get smarter than we were the *day before*.

In statements such as this one, Ms. M reiterated that to work competently in this class meant learning more than what you knew the day before.

Building on these goals, Ms. M further expressed to students that confusion was not only acceptable, but a necessary component of the process of hard work and learning. When students were frustrated on a particular problem, instead of scaffolding away the challenge with specific steps, Ms. M embraced the struggle as important for learning. She consistently made statements such as, "You're not supposed to know how. That's the whole point. That I'm supposed to give you things you've never done before. You're supposed to feel pain and frustration. And then you're supposed to tiger up" and "I hope you get confused, I hope you get frustrated. And then I hope you feel some joy as you figure them out. No pain, no gain." In these messages to students, she emphasized that to participate competently in this class meant that the work would require frustration, perseverance, and hard work.

Within her framing of competent mathematical participation, Ms. M emphasized that students' goal is to learn. Statements like those above positioned working hard in the face of challenge as competent participation. As in the classroom analyzed by Gresalfi and colleagues (2009), "in this classroom the system of competence that was established […], what counted as competent here involved […] a process that was both challenging and time-consuming," as opposed to getting the right answer quickly (p. 58). Through these learning-based statements, Ms. M framed the mathematical activities as something that required hard work, but also something doable (Gresalfi et al., 2009).

C. Competent participation: Students co-constructed competent participation as persisting in the face of challenge, and student creativity and contributions were valued as competent mathematical participation. In addition to the explicit statements concerning ways of working competently in this class, Ms. M and the students in her class co-constructed what it meant to be smart in that summer school course. In doing so, they co-constructed the framing norms that organized the participation structures for competent mathematical behavior (Hand, 2009). According to Gresalfi and colleagues (2009), behaviors or contributions get positioned as competent most often by the teacher – the person who generally holds the power in classrooms to determine competence. However, the competent positioning by the teacher then gets negotiated as students interact with the activity systems in the classroom. Thus, the teacher and the students both participate in shaping the construction of competence in the classroom (Gresalfi et al., 2009).

This same process occurred in Ms. M's summer school course, as competent participation in the mathematical discipline was defined as learning-oriented through negotiation between the teacher's framing and the students' participation and contributions.

This happened through two processes. First, while Ms. M made explicit statements highlighting learning as what it means to work competently, the students were also provided opportunities to contribute to the class-community in ways that defined competent participation in the mathematics discipline. Second, in addition to having the opportunity to define competent participation, a range of activities that included student creativity and ideas were valued as competent mathematical participation.

Two examples illustrate how students were provided opportunities to co-construct definitions of competence. In the first example, the teacher and students co-constructed the notion of *tigering up.* This notion stemmed from a stuffed tiger that was positioning in the corner of the summer school classroom. During the first week of the course, a student, Marquan, named the tiger *Bruce*. Rather than positioning this contribution as non-mathematical, Ms. M accepted this contribution as competent and used Bruce as a symbol for working hard for the rest of the summer. For example, on the eighth day, Ms. M said,

> Marquan named our tiger Bruce for us. And what's important is that Bruce reminds us to be fierce. I gave you a problem that you should not have been able to do easily. It should have felt really challenging to you. That was my whole point, I wanted to give you something really challenging—what I wanted you to do was to be fierce like Bruce and attack it, rather than cowering like a small kitten (110711, Day 8).

From this interaction stemmed the notion of *tigering up*, or persisting in the face of challenges, which Ms. M used consistently throughout the summer to remind students to work hard on challenging mathematics with learning as the end-goal. In one uptake of the concept, Ms. M said,

> That is part of how we tiger up, because if you decide that since you can't do something you're just dumb, of course you're going to give up every time. But if you just remember, I just haven't learned this yet, this is something that I need to figure out, then you tiger up (110726, Day 17).

In this statement, Ms. M created a flexible participation structure by providing the opportunities for Marquan to co-construct what it means to contribute productively to classroom learning (Hand, 2009).

In a second example, Ms. M silently counted down from five using her fingers on alternate hands in order to re-gain the class' attention after working in their small groups. On the second day, several students followed along, adding a fist bump at the end of the count down to signify zero. The teacher picked up on this contribution and in all subsequent countdowns over the course of the summer utilized a fist bump at the end. It wasn't until the eighth day that the teacher verbally acknowledged this contribution as different from her routine participation structures when she said "you're my first class to insist on the fist at the end. I like it. I like it. Because it's kind of fierce…" (110711, Day 8).

During an interview at the end of the summer, one student, Tyrone, recognized this student contribution as unique and one that contributed to a classroom community,

> And, I guess, in a way, like in a way, like you know, a family, because we all because like when she be counting down, we go like that (makes fist bump), and stuff, and I guess that we're the only class that does that, but this is a really cool class.

By framing the students' fist bump as related to the notion of being fierce and *tigering up*, Ms. M positioned the students' contributions as competent mathematical participation that further supported the learning-oriented objective of persisting in the face of challenges. As these two examples demonstrate, although Ms. M was the one who initially framed *persistence in the face of challenges* as competent participation, the teacher and the students both participated in shaping the appearance of competent mathematical behavior in the classroom.

In addition to providing opportunities for students to negotiate the larger frames of what it means to be successful, a range of activities that included *student creativity and ideas* were framed as competent mathematical participation. While in the third dimension we will consider the mathematics content and in the fourth dimension we'll consider the ways the teacher assigned competence for particular contributions and how authority is distribute in the classroom, this dimension category considers the range of opportunities the teacher provided for students to participate competently in the mathematics.

In many math classrooms, competent student participation is defined through a series of initiation-response-evaluation (IRE) sequences, whereby a student's short contribution after responding to a simple teacher-driven factual question is quickly evaluated as competent by the teacher (Mehan, 1979). Unlike these traditional classrooms, in the summer school course, Ms. M provided an array of opportunities for students to participate competently that included student creativity and ideas. Competent participation in this summer school class took many forms ranging from a) working independently and in teams on challenging problems before they were aired in whole class discussions, b) explaining their thinking, c) having multiple acceptable ways to approach a problem, and d) inventing their own problems.

After a problem of the day was shared with the class, the students were given the chance the work independently and then in teams before the mathematics was aired in whole class discussions. While working in teams, the teacher encouraged students to share *their* ideas with one another, a move that valued student ideas and creativity.

Once the students came back as a whole class to consider the mathematics, the teacher provided opportunities for students' ideas to have the floor. For example, on the sixth day, Ms. M brought everyone's attention to the front of the room,

> I want Josiah to explain his thinking. Everyone needs to look up at the board now. Josiah's about to explain how he got this row, so make sure that you're listening to this because some of you had questions for me and I think Josiah is about to answer your questions. Go ahead.

In airing the key mathematical ideas in the whole class discussion, Ms. M provided a space where participating competently meant coming up with one's own ideas about the mathematics. Students were not simply asked to repeat lessons directly taught by the teacher.

Similarly, the students were not simply confined to using one prescribed method, but were encouraged to use creativity to consider multiple ways to do a problem. After Josiah shared his method, Ms. M observed that Josiah's method was one of many potential methods:

> So a lot of you were seeing there's so many patterns in here, we could spend a week on it. But Josiah found a different way […] So, you can still work with your diagonals, but you also have an additional way of looking at it now from Josiah.

An additional way that students could participate competently in the summer course was by authoring their own algebra problems. During the last week of the course, when groups had finished solving assigned algebraic equations, Ms. M encouraged them to create their own problems, challenging them to create even more difficult problems than they had already encountered. The next day, Ms. M provided a forum for one of the groups to present their challenging problems for the rest of the class to solve.

These examples show that Ms. M provided a wide range of ways for students in the summer to experience themselves as competent in mathematics. Ms. M allowed student-centered activities to be considered as competent participation, thus widening the repertoire of what it means to be smart to include students' own experiences and ideas (Hand, 2009; Gresalfi et al., 2009). By widening these definitions of competence, Ms. M created a flexible participation structure in her classroom that allowed for broad engagement in the learning-based goals she set for her student (Hand, 2009). Competence was defined as learning-based and negotiated to include students' experiences of doing mathematics in the definition of what it means to participate in this class (Gresalfi et al., 2009).

Furthermore, similar to other positioning moves we will see in dimension four, by providing space for students to create and share their own problems Ms. M recognized students as authors of relevant mathematical ideas. This recognition is not only an important element for students' mathematical reasoning (Engle & Conant, 2002; Hand, 2009), but also could productively contribute to student's self-theories about their own abilities in mathematics as they are being positioned with the same amount of competence as teachers and textbook authors who typically create mathematics problems.

2) Treatment of Malleable Intelligence: Teacher Emphasized Malleable Intelligence

A. Teacher shared a quotation each week about learning and intelligence, and engaged students in relating the quotes to their own experiences as learners. In the summer course, Ms. M emphasized malleable intelligence in two distinct ways. First and most notably, Ms. M shared a growth mindset quotation each week about learning and intelligence. All but one of these quotes came from Dweck's (2006) *Mindset*. When sharing a quote, she engaged students in relating the quotes to their own learning experiences. These discussions provided a lens for students to regard learning mathematics, thus expanding the ways that students could experience themselves as smart in this context to include simply working hard toward learning. An example of such instructional interaction from the 12th day follows.

Quote: "When you learn new things, tiny connections in the brain multiply and get stronger. The more you challenge yourself to learn, the more your brain cells grow. The result is a stronger, smarter brain" - Carol Dweck

Ms. M: This week, I want you to be conscious of the fact that every time you take on a challenge and you tiger up and you learn from it, your brain is actually developing the same way that muscles develop. […] The whole deal with weight training is that you are actually damaging your muscles on purpose, and this is reminding me of Charlie's comment last week, when you weight train, when you lift weights, what you are doing is you are destroying, you are tearing up your muscles, you are ripping them apart, and then you rest [...] then the muscles grow back to where they were before, but they're actually stronger than they were before [...] do you see a connection to learning?

Student: When you learn, you get smart. Your muscles get stronger, it's like learning, you get smarter by learning.

In this example, Ms. M provided systematic instruction on malleable intelligence in order to provide a lens for students to regard their everyday experiences with learning mathematics. Three other quotes discussed over the course of the summer were:

- "It's not always the people who start out the smartest who end up the smartest" -Carol Dweck
- "Nobody laughs at babies and says how dumb they are because they can't talk, they just haven't learned yet" –Carol Dweck
- "I don't divide the world into the weak and strong or the successes and the failures, I divide the world into the learners and the non-learners" –Benjamin Barber

All of these quotes explicitly emphasize Ms. M's guiding framework for the course: a growth mindset is the ignition for learning, and only by working hard and taking on challenges can one get smarter. These quotes support Ms. M's dynamic framing of what it means to be *smart* in mathematics, and thus, through explicit instruction on malleable intelligence, Ms. M provided a lens through which students could experience themselves as *smart* simply by working hard and learning.

B. Teacher communicated growth mindset messages throughout the term. The second way that Ms. M emphasized malleable intelligence was by communicating growth mindset messages throughout the summer. Ms. M consistently communicated that learning occurs only through hard work, and through hard work, students *will* improve over time.

Many of Ms. M's statements that defined what it meant to work competently on mathematics also communicated the growth mindset message that learning requires hard work. Through this reinforcement, Ms. M was able to influence students' uptake of the challenging problems discussed in the next section, and subsequently, their beliefs about intelligence as malleable.

For example, on the eleventh day, a student said that he was struggling with the problem and the following interaction ensued.

Student: This problem is mentally violating me. Really, like this problem is violating me mentally. I can't breathe.

Ms. M: I love that. That is what I was going for. As I've told you before, you're not coming to my class to do things you've done before. You're coming to my class to do things that violate you mentally so that you are forced to heal and *learn*.

Through this explanation, in addition to defining what it means to work competently in this class, as shown in the previous dimension, Ms. M explained that *learning* occurs through hard work. By emphasizing a learning orientation, where the goal is to increase learning, Ms. M implicitly emphasized the notion that intelligence is malleable through hard work.

In addition to communicating the growth mindset message that learning requires hard work, she also consistently emphasized that through hard work, students *will* learn and improve over time. For example, on the second day, after asking students to engage in their first team talk about mathematics, Ms. M shared, "that was quite good - *you're going to get better at it*, but I am really impressed with how you just started that." In this statement she supported the notion that students' intelligence is malleable and will grow over time.

Again, on the eighth day, after introducing a new strategy game, Ms. M explained, "We are all beginners are this game. I am not good at this game yet. I am a beginner at this game too. This game is new to me. So, over the course of the week, we should all get better at this game together." Again, Ms. M explicitly emphasized that students will improve over time, thus supporting the notion that intelligence and ability are not fixed entities, but are indeed malleable.

Finally, in the last week, Ms. M explicitly emphasized that students had, in fact, made progress over time, "admit it, did you know how to do that yesterday? So, are you smarter than you were yesterday [Students: yes]. Okay, this is your goal. Everyday, you need to get smarter. And remember, you are so young, if you get smarter everyday, you will have the opportunity to be brilliant adults." Through communicating with students that they *will* (and have) improve(d) over time, Ms. M explicitly emphasized and provided evidence that students' intelligence can grow, further supporting the growth mindset by which students can experience themselves as *smart* by continuing to learn.

By emphasizing malleable intelligence in these growth-oriented statements, Ms. M was contributing to the development of what Dweck (1999) calls a growth mindset. This growth mindset instruction teaches students that intelligence can be increased through learning, and thus influences them to develop learning goals (Dweck, 1999). As described previously, these learning goals cause students to come to value getting smarter by learning new things, which has been shown to unleash students' motivation to take risks on challenging problems (Dweck & London, 2004; Dweck, 1986; Dweck, 1999; Dweck, 2007a; Dweck, 2009). Dweck (1999) argues that students who have a growth mindset will "readily sacrifice opportunities *to look smart* in favor of opportunities to learn something new" (emphasis added).

This explicit instruction in a growth mindset thus cultivates the belief that failure or challenge is simply a sign that the right strategies have yet to be found, and more effort is required (Dweck, 1999). Therefore, students with a growth mindset develop a hardy response to failure because their goal is *getting* smarter, rather than *appearing* smarter (Dweck, 1999). Thus, by explicitly emphasizing malleable intelligence, Ms. M was contributing to students' mindsets in a way that could productively influence their beliefs about their own abilities and the efficacy of their efforts, their achievement goals, and their effort (Dweck, 1999; Dweck & London, 2004).

3) The Mathematics was Primarily Non-Routine and Challenging

A. Teacher incorporated challenge into the classroom by assigning difficult non-routine mathematics problems. In addition to framing success in the classroom as learning based and explicitly teaching students about a growth mindset, Ms. M also provided opportunities for her summer school students to *experience* a growth mindset by incorporating challenge into the classroom. Ms. M incorporated challenge by assigning difficult non-routine mathematics problems and creating room for students to productively struggle with the mathematics.

Bringing in challenging non-routine problems and giving students space to struggle with the mathematics conveys to students that intelligence is malleable through hard work. Rather than assigning short exercises, Ms. M organized each lesson around a different problem of the day in order to help students practice one of three mathematics strategies. After setting students up to work on the mathematics productively (i.e., answering questions and having the class agree as a whole what the problem was asking), students would spend the rest of the day working independently, in groups, and then as a whole class on these deep non-routine problems, with student ideas guiding each participation structure. The following 10-card pick up problem is a representative example of the types of problems students worked on:

> *Ten cards are dropped on the floor. Bruce may have eaten none, one, ten, or any number of cards in between. How many different combinations are there for what Bruce ate?*

These problems were designed to challenge students, so as to incorporate struggle into their repertoire of what it means to be smart in mathematics. In addition to assigning challenging tasks, Ms. M maintained the high cognitive demand of the task by emphasizing that learning occurs only through hard work, rather than scaffolding the problems in ways that reduced the cognitive demand when students experienced struggle.

Even when a problem took longer than expected, the teacher found ways to guide students toward organizing their thoughts. She then made explicit that this struggle is what makes a mathematics problem a "good" one. For example, when students struggled with the 10-card pick-up problem for multiple days, she solicited student ideas to suggest how they could simplify the problem and then build it up to solving the 10-card pick-up. She then let students again work on solving it on their own before soliciting their ideas as a whole class. When referring back to the problem at the beginning of the period, she said, "today's day 6 on this problem, which tells me it's a good problem," thus making clear that *challenge* is an essential part of what it means to do math in this classroom. In doing so, Ms. M increased the propensity for students to engage in these high level tasks by not reducing the complexity of the tasks (Henningsen & Stein, 1997; Stein et al., 1996).

In their TRU Math Conversation Guide, Baldinger and Louie (2014) draw attention to the importance of engaging students in centrally important mathematics through productive struggle, whereby students understand the challenges they confront while still having room to make their own sense of those challenges. Both the nature of the mathematics students are asked to work on and the cognitive demand of that math are important elements for robust student learning. In this summer course, Ms. M did not have a set curriculum she had to follow and had immense flexibility to design the course in such a way that brought in challenging problems *and* provided time for students to struggle and *learn* as a result.

By incorporating struggle into students' repertoire for what it means to do mathematics, Ms. M provided opportunities for students to *experience* that hard work leads to learning – to *experience* the growth mindset they were learning about. Through this productive struggle, students had more opportunities to experience themselves as smart in mathematics, since being a competent participant wasn't restricted to having the right answer. Boaler (2013) argues, "When tasks are more open, offering opportunities for learning, students can see the possibility of higher achievement and respond to these opportunities to improve" (p. 146). These moves could thus influence the resulting ideas that students form about the nature of mathematics, and about themselves as doers of mathematics (Gresalfi, 2009).

Through engaging students in challenging mathematics problems and repeatedly emphasizing the importance of hard work for learning, Ms. M positioned the tasks as ones that require hard work, but are indeed still doable (Gresalfi et al., 2009). What counted as competent mathematical behavior was not about getting the right answer quickly, but about evaluating the sensibility of mathematical choices and working with other students, a process involving hard work over time (Gresalfi et al., 2009). Through this positioning, Ms. M expanded what counts as competent mathematical behavior by including hard work and struggles in this repertoire, thus providing increased opportunities for students to experience themselves as smart in mathematics.

In this sense, students in this summer school course did not simply learn about the importance of having a growth mindset, but they got to *experience* struggle and see the learning that occurs as a result of that struggle.

In doing so, these moves – combined with Ms. M's emphasis that students *are* making progress through their hard work – could directly contribute to students' development of a growth mindset and more adaptive motivational patterns (Dweck, 1999). Incorporating and emphasizing challenge and hard work, as opposed to success or innate ability, within a learning-oriented context can lead students to view challenge as a boost to self-esteem rather than a threat, and to subsequently seek more challenging tasks (Dweck, 1986; Dweck, 1999; Dweck, 2009; Dweck 2010). These moves can help students develop productive self-theories in which they view intelligence as malleable, develop learning goals, and thrive on challenge (Dweck, 1999).

Additionally, incorporating challenge into the classroom can lead to the teacher expectancy effect, whereby students' performance tends to fall in line with the teachers' expectancies, and as a result, students perform higher (or lower) than they would have otherwise (Dweck, 1986). This effect occurs because people's beliefs about their abilities, stemming from their teacher's input, influence the amount of effort they put forth and how long they persevere in the face of challenges (Bandura, 1994). Specifically, Bandura's self-efficacy theory argues that when given only easy routine mathematics tasks, students tend to develop the belief that they are not capable of more challenging tasks. This message then contributes to students' low self-efficacy and a sense of learned helplessness such that, by only experiencing easy successes, students come to expect quick results and give up in the face of challenges, avoiding tasks they see as personal threats (Bandura, 1994). In the opposite case, by incorporating the notion of challenge into what it means to be smart in mathematics, Ms. M contributes to high self-efficacy, which can positively influence students' persistence and performance on challenging problems. Thus, by giving challenging mathematics problems, Ms. M may have also been contributing to increased student performance.

4) Agency and Authority: Students had Intellectual Agency over the Mathematics and were Positioned with Authority over their Mathematical Habits and Ideas

The nature of the mathematics described above provided a productive space for the teacher and the students to share authority over the mathematics and for the teacher to position students as competent doers of mathematics in meaningful ways. If the mathematics in the summer course had been rote or solely textbook-based without opportunities for the students to draw connections on their own, there would not have been the same productive space for the teacher to share authority over the mathematics or to acknowledge and position students for their mathematical contributions. In this way then, the nature of the open mathematics Ms. M brought into the summer course fostered an environment where the students and teacher could share authority over the mathematics, student had agency to make valuable mathematical contributions, and the teacher could position students with authority for those contributions.

A. Intellectual agency.

i. Students and teacher shared authority over the mathematics; students were "authorized to come up with their own ideas. Unlike what often happens in traditional math classrooms guided by direct instruction, in which the teacher or the textbook distributes knowledge and is the arbiter of correctness (Mehan, 1979), Ms. M did not give direct instruction on how to solve particular types of problems in the summer course. By structuring the course around challenging non-routine problems, Ms. M instead provided opportunities for students to come up with their own ideas and, as described in dimension 1, these student ideas were central to the mathematical conversations in the summer course. Instead of telling students the "most correct" way to do a problem, even when students got frustrated, she gave them opportunities to come up with their own ideas, which in turn allowed students to have agency over the mathematics.

As previously described, the summer course was structured such that students would first have an opportunity to come up with mathematical ideas on their own and in teams before coming to work as a whole class. For example, on the tenth day, after introducing the problem, Ms. M began by asking, "do you have any idea about how you're going to get started with this? How are you going to figure out how many squares are here?" She then solicited student strategies for solving the problem. After students had a chance to think about it themselves, Ms. M would always begin whole class discussions by solicit student ideas or, as will be described later, airing student ideas she heard in small groups with credit back to the students.

Through these moves, Ms. M shared the authority over the mathematics with her students, rather than maintaining the position that the arbiter of truth or correctness is solely the teacher's or textbook's taught methods. Gresalfi and Cobb (2006) argue that the ways classroom organizations position students with respect to agency and authority can influence their interest, motivation, and feelings of competence. Specifically, they argue that classrooms that provide opportunities for students to exercise conceptual agency, in which students can choose and develop conceptual meaning – as opposed to disciplinary agency, which involves only applying established methods – widen opportunities for students to participate in mathematical practices and to develop feelings of competence with respect to the discipline (Gresalfi & Cobb, 2006; Cobb et al., 2009; Schoenfeld, 1988; Engle, 2011).

In the summer course, students had the opportunity to exercise conceptual agency in this way; they were able to make decisions about the interpretation of tasks, the reasonableness of solution methods, and the legitimacy of solutions, and they had opportunities to conjecture, explain, and make mathematical arguments (Cobb et al., 2009; Schoenfeld, 2013). Engle (2011) refers to this conceptual agency as the first level of authority called *intellectual agency* in which learners "are 'authorized' to share what they actually think about the problem in focus rather than feeling the need to come up with a response that […] matches what some other authority like a teacher or textbook would say is correct" (p. 8). In giving students intellectual agency to come up with their own ideas, Ms. M increased the opportunities for her students to experience themselves as competent doers of mathematics (Engle, 2011; Cohen & Lotan, 1995), which potentially influenced students to develop productive mathematical self-concepts or beliefs about themselves as doers of mathematics (Hand, 2009; Boaler, 2010)

Incorporating the challenging mathematics problems described in strand 3 *and* authorizing students to come up with their own mathematics can also contribute to increased student engagement (Engle & Conant, 2002). By organizing each challenging non-routine and open-ended problem around a big problem solving idea of the content approached from different contexts and asking students to share *their* ideas first, Ms. M provided multidimensionality. Multidimensionality affords multiple opportunities for students to develop understandings or contribute ideas (Cohen et al., 1999; Boaler, 2010). These challenging mathematics problems also increased students' access to mathematical sense-making rather than relying on didactic approaches to simplifying the mathematics (Hand, 2009). Accordingly, the problems established a flexible participation structure and a system of competence that supported engagement among a *variety* of learners, thus further expanding the opportunities for students to experience themselves as smart in mathematics (Hand, 2009; Gresalfi et al., 2009).

The ways Ms. M provided a space for students to have conceptual agency to develop their own ideas while working on non-routine tasks also increased the need for students to interact with one another to draw upon each other's strengths (Cohen et al., 1999; Cohen, 1996). Similar to the classroom observed by Boaler (2010), when students were stuck on a problem, Ms. M would reiterate the importance of being stuck, "and would leave groups to work through their understanding rather than providing them with small structured questions that led them to the correct answer" (p. 44). According to theories of Complex Instruction (Cohen, 1996) these pedagogical moves lead to increased discourse, and thus increased learning. By ceding agency and authority to students on multidimensional challenging mathematics tasks that required group interaction, Ms. M contributed to increased equitable engagement by broadening the *ways* for students to experience themselves as smart in mathematics.

B. Authority and positioning. In addition to authorizing students to have the conceptual agency to come up with their own mathematical ideas, Ms. M also gave students a sense authority over their own behavior and publicly credited students for their mathematical practices and contributions, creating the opportunities for students to become socially recognized as an authority on a particular mathematical topic/idea. These moves positioned students as mathematically competent and as authors of important ideas, thus allowing them to contribute to and negotiate what counts as competent mathematical behavior (Gresalfi et al., 2009; Hand, 2009).

i. Students had a sense of authority over their own behavior. Ms. M shared a sense of authority in the classroom by giving the students the opportunity to feel responsible for their own behavior in the classroom. Rather than policing students for off-task behavior, Ms. M created a card chart with students' names that she and the students could use to keep track of students' behavior.

Every day, students' cards began at green, but if a student was not following the classroom expectations for learning, a card could get turned to yellow warning, an orange warning, and finally a red referral to the office. While this negative consequence was shared once on the first day, she regularly emphasized the positive consequences of the entire class having a "green day." While Ms. M sometimes turned cards, silently without verbal explanation, when students were off task she would also ask students to tell her if and when they needed a card to be turned. In this way, the card chart served as a visual reminder to students that whatever they were doing was not contributing to learning. By giving students responsibility over the card chart, students could feel a sense of authority to contribute to decisions concerning what appropriate classroom behavior looked like.

ii. Teacher positioned students with authority for productive mathematical processes, effort, or improvement. Ms. M positioned both the class and individual students as competent by recognizing their process or effort. Ms. M positioned the entire class as competent by continuously recognizing and praising the effort or behavior of the class as a whole. For example, on the second day of the course, after students had been talking about their mathematical ideas in teams of three without much instruction about the content, Ms. M said to the class,

> I want to say, that I am impressed, for your first team talk that was quite good. So it seems to that you have had math talks before because you seem to have some experience with talking to other people about math. That was quite good - you're going to get better at it, but I am really impressed with how you just started that.

In this representative example, Ms. M praised the class for their math talk process and behavior.

Beyond just praising the class for their mathematical processes, Ms. M also recognized *improvement* in these mathematical behaviors. For example, on the sixth day of the course, Ms. M exclaimed, “There has been improvement in peoples' ferocity? Ferociousness? Fierceness? Which of those are words, I'm not sure. But you are definitely *tigering up* more this week than last week.” In this exclamation Ms. M not only praised the class for the students’ mathematical effort, but also recognized their improvement over time.

In addition to positioning the class as a whole as competent, Ms. M positioned individual students as competent by recognizing their mathematical behaviors or effort. For example, on the sixth day, after the teacher called on a student who did not know the answer, but persisted to eventually solve it. Ms. M verbally recognized that student for her mathematical process,

> Good, Daniela. And that was a good example of Daniela tigering up and getting her participation points. When I first called on her, she didn't know, but she didn't give up, she worked with me, and she gets full credit. Okay, so if that ever happens to you, do what Daniela did, that was good.

Rather than praising this student for her success, or ability, Ms. M recognized her for her mathematical process and effort.

By recognizing process or effort, as opposed to ability, Ms. M supported students in developing a growth mindset (Dweck, 2007a). This type of praise sends motivational messages to students that these processes, effort, and perseverance are what is valued in mathematics, rather than the end product or simply appearing smart in the traditional sense by getting correct answers quickly (Dweck, 2007b; 2009). Furthermore, emphasizing effort over ability sends strong messages about the nature of high achievement in mathematics as one that is a product of working hard rather than innate ability (Boaler, 2010). This positive message can, in turn, have the effect of enhancing students’ performance and motivation on challenging mathematics problems (Dweck, 2007b; 2009). For example, in one study, Kamins and Dweck (1999) found that students who received effort or strategy praise were significantly less likely than children who received praise in the form of evaluation, traits, or abilities to display helpless reactions when met with setbacks.

Through recognizing improvement in the mathematical processes, Ms. M further contributed to the notion of malleable intelligence; students *can* and *are* improving over time. When students focus on improving, Gresalfi (2009), they are more likely to, in turn—just as those with a growth mindset would—"feel more comfortable making mistakes and revising their thinking" (p. 329)

Even further, by identifying the appropriate mathematical behaviors with particular students, Ms. M also *assigned competence* to that student by publically recognizing a specific, relevant, and intellectual contribution of the student (Cohen et al., 1999; Boaler 2010). Assigning competence by praising specific students for these behaviors positions students with authority over their mathematical practices. Assigning competence in this way can further influence students' self-theories about their ability as well as their expectations for other students' competence, thus providing opportunities for students to experience themselves as smart in mathematics (Cohen & Lotan, 1995).

By praising students' process or behavior, Ms. M contributed to a flexible participation structure. Specifically, Ms. M emphasized that through effort and engaging in mathematical behaviors, regardless of ability or accuracy, students can productively participate. In this way, students were given agency with which they were positioned to do the mathematics, and their mathematical experiences were incorporated into the official discourse of what it means to participate in mathematics (Gresalfi et al., 2009). This praise then widens what it means to contribute productively to the classroom learning and in turn the opportunities students had for experiencing themselves as smart in mathematics, as well as *who* can be defined as competent (Hand, 2009; Boaler, 2010).

iii. Teacher positioned students with authority over the mathematics by revoicing/reformulating their mathematical ideas. In addition to recognizing and praising students for their process, Ms. M also positioned students with authority over the mathematics by revoicing individual students' mathematical ideas publically.[2] In a post-summer interview, Ms. M explained this pedagogical strategy as one in which she listens to students' ideas and attempts to find and revoice a really *smart* contribution to the class in a more formal way, all the while giving students ownership for their mathematical ideas by crediting the student for the idea.

[2] I recognize that there is a large literature concerning the ways that people position themselves in discourse (e.g. Van Lagenhove & Harré, 1999; Taylor et al., 2003). Although potentially useful for examining one's role in one's own self-concept formation, this perspective does not afford an explanation of the influence of pedagogical strategies on students' self-concept development. For that reasoning, I am strictly examining the ways that teachers can position students, while still acknowledging that the ways that students position one another and themselves may also play a role in students' developing mathematical self-concepts.

Much as in Cohen and colleagues' (1999) theory of Complex Instruction (CI), Ms. M would publically position students as competent participants by giving them ownership for their mathematical contributions, a move that CI refers to as part of "assigning competence." Revoicing students' ideas in a more mathematically acceptable way, while still giving students credit for the idea, can position students as competent doers of mathematics in ways that influence their self-concepts, engagement, and persistence (O'Connor & Michaels, 1996).

This pedagogical move was the most salient and common in observing Ms. M's teaching, but only one example will be shared to illustrate this move. This example is characteristic of the pedagogical move in action. On the third day of class, while students were working independently on a non-routine problem, Ms. M assigned credit to a student for her mathematical idea of using the strategy guess-and-check.

First, as she often does when assigning credit to students for their ideas, Ms. M started by recognizing that other students or groups probably had the same idea,

> Okay, what I see on some papers here, here, here, I think here, here […] maybe 7 papers is. [Ms. M shows everyone Lily's paper] What does it look like Lily's strategy is? What is she doing? How is she approaching the problem? Can you tell by looking at this work what she was trying to do? [.....]What I see Lily doing is guessing and checking […] This is what I would have done.

After crediting Lily for this idea, Ms. M continued to publically refer to organized guess-and-check by crossing out incorrect answers as "Lily's strategy" for the rest of the class period, even suggesting that other students try "Lily's strategy of crossing it out if it does not work." In this interaction, Ms. M publically assigned competence to Lily by giving her credit for the mathematical idea of organized guess-and-check.

In this interaction, Ms. M reformulated Lily's mathematical strategy in terms more widely recognizable as a "guess-and-check" strategy (O'Connor & Michaels, 1993). With this reformulating, Ms. M aligned Lily with the mathematics and created an opportunity for students to hear and appreciate Lily's strategy, while simultaneously introducing students to a mathematical discourse community with particular thinking practices (O'Connor & Michaels, 1993). In this move, Ms. M created an opportunity for Lily to be positioned as a competent doer of mathematics.

Research has begun attending to the nature of these types of interpersonal positioning moves in social classroom discourse and their potential for influencing student dispositions, behavior, and learning in both productive and unproductive ways (Hand, 2009; Wagner & Herbel-Eisenmann, 2009). This specific positioning move, revoicing by reformulating, allows the teacher to reframe a student's contribution in terms most useful for the group's consumption, while still crediting the student for his or her contribution and providing an opportunity for the student to clarify their intention. One way, they describe, that teachers can employ this strategy is by subtly reformulating a student's contribution in terms more recognizable by a wider audience, while still maintaining the student's ownership over the reformulation (O'Connor & Michaels, 1993). By implementing this strategy, a teacher potentially positions students as competent through their ability to contribute mathematical ideas, and also links student ideas and experiences with the conventional knowledge of the mathematical community at large (O'Connor & Michaels, 1993). O'Connor and Michaels (1993 & 1996) suggest that over time reformulation brings students to see themselves as legitimate mathematical participants.[3]

These reformulation moves that Ms. M regularly implemented can position students relative both to the content (academic) and to their peers (social) (Greeno, 2011; Gresalfi & Cobb, 2006). Specifically, in mathematics classrooms, Greeno (2011) argues that students can be positioned semantically in relation to the concepts and mathematics, and systemically in relation to other students and the teacher in the class. These ways of positioning provide affordances for how students are entitled, expected, and obligated to participate and interact in relation to others in the classroom (Greeno, 2011; Gresalfi & Cobb, 2006). Revoicing students' ideas can potentially position students both semantically and systemically. Additionally, these reformulation moves have the potential to afford positions for not just one, but multiple students to occupy simultaneously.

For example, in the post-summer interview, Ms. M explains that she listens to students' ideas and attempts to find and reformulate a contribution to the class in a more formal way, all the while crediting the student for the idea (O'Connor & Michaels, 1996). This practice of reformulating can "bring them [students] to see themselves and each other as legitimate participants in the activity of making, analyzing, and evaluating claims, hypothesis, and predictions" (O'Connor & Michaels, 1996, p. 78). As a result, reformulating students' ideas can contribute to the semantic positioning of students with respect to the mathematics (e.g. Lily as an originator of mathematical strategies).

However, Ms. M not only gave credit to Lily, but also recognized other students who potentially had the same idea. In the post-summer interview Ms. M described this action to be intentional, "I try as much as possible to acknowledge the good thinking [of others] so that I don't pretend like [Student X] is the only person in the room who had this idea, she just happens to be the one who is called on." In this way, Ms. M assigns competence to individual students, but not at the expense of providing opportunities for other students to experience themselves as competent in mathematics.

More specifically, by saying "Okay, what I see on some papers here, here, here, I think here, here […] maybe [on] 7 papers is […] What I see Lily doing is guessing and checking […] This is what I would have done […this is] Lily's strategy" Ms. M reformulated Lily's guess-and-check strategy, giving Lily credit for the mathematical idea, while still recognizing the potential competence of others who may have had the same idea. In doing so, this move contributed to not only the semantic positioning of students with respect to the mathematics, but also systemic positioning of students with respect to one another (e.g. Lily and others with the same idea as

[3] Just because a teacher's discourse move affords a particular position for a student to occupy, does not mean the student will occupy this position. Instead, it depends upon the individual student's history as a doer of mathematics, the history of trust between the teacher and the student, as well as the dynamic intentions established between the teacher and students in this particular classroom context (Gresalfi et al., 2009).

competent doers of mathematics). Through reformulating, then, the student is positioned as having elevated status – both with respect to the content and with respect to his/her classmates – but not at the expense of these peers (O'Connor & Michaels, 1996). She positioned one student as competent, without positioning other students as not competent. Research suggests that this type of positioning leads to a flexible participation structure in which multiple ideas are valued as competent and productive, where many students have the opportunity to experience themselves as competent in mathematics (Hand, 2009; Boaler, 2010).

These revoicing and reformulating moves also serve to distribute authority to students. Engle (2011) outlines multiple levels of intellectual authority that build on each other, starting with the first stage of intellectual agency described in the section above. She argues that for students to be truly engaged with mathematics, they need to have some degree of intellectual authority in which they are "allowed, encouraged, and responsible for intellectually engaging with the problems as themselves" (p. 8). Beyond having intellectual agency, authority is strengthened as students are publicly recognized as authors of their own ideas (*authorship*), their authoring influences others' ideas or the learning environment (*contributorship*), and they become socially recognized as an *authority* about some topic(s), which happens as their ideas become increasingly influential with others. Engle (2011) argues that as students are positioned with increasing authority, "they are increasingly expected by themselves and others to be able to engage knowledgeably with it" and will thus show more disciplinary engagement.

Through these revoicing moves, Ms. M publically and socially recognized students as authoring and justifying important mathematical ideas, which could contribute to students' increasing engagement in rich mathematics (Engle & Conant, 2002; Hand, 2009; Engle, 2011). By providing opportunities for students to author important mathematical ideas, students are also in turn provided opportunities to contribute to and negotiate what gets constructed as a mathematically competent through this discourse (Gresalfi et al., 2009). Thus, as in the prior examples, this co-construction of competence provides students even more opportunities to experience themselves as competent in mathematics (Gresalfi et al., 2009).

Engle & Conant (2002) also claim that giving students more authority in this way may productively influence their identities. In short, these positioning moves provided what Nasir and Cooks (2009) call "ideational resources," or "the ideas about oneself and one's relationship to and place in the practice and the world, as well as ideas about what is valued or good" (p. 47). Through explicitly positioning students with the high level of socially recognized mathematical authority by using student ideas in increasingly influential ways in the classroom, Ms. M made these ideational resources available in ways that could influence students' mathematical self-concepts. With these ideational resources placing value on effort and ideas, as opposed to ability, Ms. M also supported students in developing a growth mindset in a way that could enhance their persistence and motivation on challenging mathematics problems (Dweck, 2007a; Dweck, 2007b; Dweck, 2009[4]).

[4] For more detailed explanation of the mechanisms by which positioning can influence students' mathematical self concepts see Nix (2012).

Prior research on Complex Instruction further explains how assigning competence through revoicing can positively influence student learning and engagement (Cohen & Lotan, 1995). Ms. M's pedagogical move of assigning competence, like that described in CI, was public, intellectual, specific, and relevant student feedback (Boaler, 2010). Similar to CI, when assigning competence, Ms. M either brought a student's intellectual contribution to the class' attention, asked a student to present an idea, or publically praised a students' work in a specific way (Boaler, 2010). Through this move, a teacher, as a high-status person, can alter the status processes in the classroom by influencing the expectations for competence students hold for themselves, as well as those they hold for their peers (Cohen & Lotan, 1995). In doing so, students will begin to believe that their ability is consistent with the teacher's evaluation, which can then influence their engagement and perseverance. For example, Boaler (2010) describes an example in which students shared a response to a teacher's question, and the teacher replied by saying, "Oh that is like Ivan's idea, you're building on that," thus raising the status of Ivan's contribution (p. 43). This teacher's statement influenced Ivan to then visibly sit up straight and lean forward (Boaler, 2010).

Cohen and colleagues (1999) recognize that assigning competence in this way is "especially important and effective to focus attention on low-status students" (p. 85). Although Ms. M gave all students credit for coming up with mathematical ideas, she also intentionally used strategies to raise the status of low-status students. First, she intentionally chose students who had low-status or low-persistence on challenging problems to give credit for smart mathematical ideas. For example, during a pre-lesson interview on the eighth day, Ms. M explained, "I'm wanting to assign status to Shakia because she's still fragile around persevering in difficult problems, but I think if she gets recognized […], I think that will inspire her to boost her effort." Then, during class that day, Ms. M publically shared Shakia's great idea about the problem, thus intentionally assigning competence to a student with low-status and persistence.

Assigning competence by revoicing and crediting students for mathematical ideas leads to higher participation, especially for low-status students (Cohen & Lotan, 1995). Cohen and Lotan's (1995) research shows that "the frequency of the teacher's use of status treatments had a statistically significant positive effect on the participation rate of the low-status students" (Cohen & Lotan, 1995, p. 111). This pedagogical move has also been shown to transfer to new situations, in which students who had previously been assigned competence continue to hold high expectations for themselves (Cohen & Lotan, 1995). Thus, by addressing status through assigning competence, Ms. M could significantly lessen the influence of status, in ways that increase the participation and engagement for all students, which in turn leads to more learning.

The data in this summer school course the year prior exemplified Ms. M's success with Teaching for a Growth Mindset in, as will be shown in Chapter 4, ways that influenced student's mathematical self-concepts, engagement with rich mathematics, and persistence toward learning. The analyses in this section reveal Ms. M's capacity to successfully Teach for a Growth Mindset and the four pedagogical strands that are necessary for the successful implementation of a growth mindset pedagogy.

Teacher Professional Identity and Decision-Making in the Summer Context

As will be shown by Ms. M's significantly different pedagogy in her traditional academic year context (see the next chapter), simply looking at the teacher's instructional intentions doesn't tell the whole story about *why* the classrooms looked different. We must also examine the optimal conditions for implementing these instructional moves and the ways the context might foster particular pedagogical decisions.

Transcript analyses of teacher interviews suggest that the summer context may have fostered the growth mindset moves by providing different identity resources for Ms. M than during the academic year (Nasir & Cooks, 2009). Specifically, a change in school context may have resulted in different aspects of Ms. M's multifaceted identity being activated, contributing to her distinct pedagogical decisions.

In the summer course, Ms. M had complete flexibility over curriculum, goals, and achievement measures. She was not required to keep a particular pace, cover specific topics, or implement standardized tests. In a mid-year interview during the academic year, Ms. M reflected on the summer course, describing the content as "fun and not grade level specific and not, not structured, and it wasn't a high-pressured situation." This statement reveals that during the summer, she did not feel any pressure from the school or its external context about teaching a particular set curriculum and measuring students on that content in a particular way. In fact, in a pre-summer interview, she explicitly described that the curricular goals of the summer were not about "here's the math you should already know, do you know it, let's make sure you do. It's more of a let's think mathematically and this will benefit you no matter where your skills are." These pre-summer remarks showed that she thought about content of the summer course as removed from external pressures of covering a set of content standards, which gave her the flexibility to incorporate non-routine challenging problems into the content of the course.

While in the retrospective interview she described the content as not "grade-level specific," in the pre-summer interview, she described the content as specifically *setting* students up for Algebra:

> The problems that I picked are actually problems that students will learn how to solve in Algebra, but I want them to solve them now without Algebra. So it's the same type of problem […] I'm picking problems that Algebra will help them solve later. Also, this is my motivation for thinking about this week of actually solving equations and what does it mean to be a solution, so that it directly prepares them for Algebra. (Pre Summer Interview)

While she didn't feel constrained by external course level goals or tests, she still set academic goals for her students that *were* aligned with grade-level mathematics. Ms. M considered these non-routine problems as the same type of math that students would be able to solve the following year in Algebra, yet, as we will see in the next chapter, these problems were significantly different than the routine mathematics students were actually asked to work on in the academic year.

As a result of not having to follow a set of standards or having to meet particular standardized test benchmark goals, Ms. M had the flexibility to prioritize goals that aligned with her own orientation toward the importance of growth mindset (Schoenfeld, 2010). One way to consider the different pedagogical decisions teachers make is to look at their goals and orientations in context; as people orient to particular situations, goals are established and prioritized and people make decisions that are consistent with these goals (Schoenfeld, 2010). Schoenfeld (2010) describes orientations as a term that encompasses beliefs, dispositions, values, tastes, and preferences; I build on this to add that orientations can include an individual's identity in a particular context. For example, if a teacher views her professional identity as a successful teacher as being tied to her students' performance on standardized tests, the goals she establishes and the decisions she makes will be consistent with those priorities – what matters to her for her own professional identity and in what ways (Schoenfeld, 2010)?

A person's identity is multidimensional and not durable across contexts – it is made up of composites of many, often contradictory, self-understandings that develop as people participate in particular *communities of practice,* and use the cultural resources and subject positions afforded in particular *figured worlds* or situations (Wenger, 1998; Holland et al., 2001). Thus, a person's professional identity can differ from context to context, as different contexts offer diverse resources for identity development (Nasir and Cooks, 2009). In other words, different aspects of a person's identity can be activated depending on the present situation.

In examining practice-linked identities, Nasir and Cooks (2009) argue that as individuals participate in "communities of practice" (cf. Wenger, 1998), they are offered (and negotiate) resources for identity development. This argument reinforces the notion that the cultural context of the school makes available resources that can support the development of a particular professional identity. By providing ideas about a person, their relationships to practice, and what is valued as good, ideational resources made available for teachers in particular school contexts can position teachers into specific roles (that they can then negotiate) (Nasir & Cooks, 2009).

Teachers' developing professional identities can in turn contribute to their pedagogical decisions and the high-priority goals they set for their students. If a person is concerned with being seen as a particular kind of teacher (e.g. successful), then the pedagogical choices that she makes will serve those identity purposes (Langer-Osuna, 2007) and the resulting high-priority goals will different from context to context based on the set of norms, expectations, and ideas of that context that shape identity formation (Holland et al., 2001).

With complete flexibility over the classroom goals and without external pressures on curriculum and testing, Ms. M's professional identity as a successful teacher in the summer course was strictly tied to the goals *she* set for her students – what was valued as good teaching in this context was simply whether and to what extent she achieved the goals she set for her students and this course.

In a pre-summer interview, she defined part of these goals as helping students develop productive dispositions by "giving them difficult things and showing them that they can actually have some success with that." In other words, the measure of her own success as a teacher was whether or not her students came out "with new attitudes toward themselves and toward math"

(Pre-summer Interview) – whether they could take on challenges without giving up and whether they developed a growth mindset that supported them in doing so. Accordingly, the pedagogical decisions described above – in which she 1) Framed the mathematical goals of the course as learning-based and persisting the face of challenges, 2) Provided an explicit treatment of malleable intelligence, 3) Organized the course around non-routine challenging mathematics, and 4) Provided opportunities for students to develop agency and authority over the mathematics – directly served those high-priority goals.

Conclusion

As will be shown in Chapter 3, Ms. M's teaching for a growth mindset pedagogy looked significantly different in the academic year. As Chapter 4 will show, this contributed to significantly different experiences for her students.

Drawing from the pioneering work of Dweck (2006), when preparing to teach for a growth mindset, most teachers consider sharing information about the importance of a growth mindset with students, praising students for their growth and effort versus their performance, and even having a learning-orientation and framing instruction around long-term learning goals. These elements cover the first two strands of the four strands of pedagogy Ms. M implemented in her summer course: 1) Framing Success (What are the long-term goals for students? What gets defined as competent mathematical participation and how? and What are the messages about what it means to work competently); 2) Treatment of malleable intelligence.

Many teachers have read Dweck's 2006 bestseller *Mindset* and aim to implement the book's ideas in their classrooms, but don't know what it means for their discipline-specific teaching, in particular for mathematics – a subject where fixed ability messages proliferate (Boaler, 2013). As we will see in Chapter 3, the communication of growth mindset through orientations and growth mindset messages are not enough to shift student's mindsets in ways that affect their engagement and behavior.

Instead, as shown in the examples above and as Boaler (2013) has also found, teaching for a growth mindset requires and examination of all aspects of teaching, including 3) the tasks and 4) the agency and authority with which students are set up to work on the mathematics. Students need to be given challenging non-routine open mathematics problems and they need to be "authorized" to come up with their own mathematical ideas so they can experience themselves as competent participants that have productive mathematical habits, behaviors and ideas. As shown above, Ms. M's practice in the summer course incorporated all four strands, and as Chapter 4 will show, this pedagogy was met with productive shifts in student mindsets, engagement, and persistence toward learning.

Chapter 3: Pedagogy that Enables and Constrains the Development of a Growth Mindset in a Traditional Academic Year Algebra Course

What happens to pedagogy when a teacher who has demonstrated a commitment to and success with teaching for a growth mindset in one context is faced with the constraints of high stakes testing and set traditional curriculum in another? In the current mathematics education climate, teachers are asked to balance their evolving ideals around what it takes to push students toward rich mathematical *learning* and the strong emphasis on *performance* goals that proliferates the K-12 school context.

This conflict has never been more salient than it is currently with the nationwide shift toward the Common Core State Standards in Mathematics. Teachers are expected to negotiate these research-based instructional practices that put student thinking and *learning* as central goals with the required high student performance on standardized tests and a saturated curriculum that requires teachers to fit 180 lessons into less than 180 instructional days. What teachers do with these potentially conflicting goals can have very real outcomes for student mindsets, engagement, and learning.

This dissertation examines the ways these contextual constraints and the professional identity resources made available in two different contexts shaped one teacher's decision-making – such that she implemented drastically different pedagogy in two different instructional contexts. After having success with teaching for a growth mindset in her summer course, Ms. M took the same goals and strategies into her academic year Algebra course the following year. Yet, she didn't anticipate the difficulties she'd face in implementing these strategies in a climate of high stakes testing. The data in this study tell the story of a teacher navigating tensions between her ideals about supporting students' mathematical *learning* and identities within a climate that reveres *performance*-based goals - it tells the complex story of a growth mindset philosophy with a performance orientation mitered onto it.

While the summer school data reveal Ms. M's capacity to successfully teach for a growth mindset and the potential for what that pedagogy *can* look like, comparing that pedagogy with that of the academic year reveals not only the necessary components for effectively teaching for a growth mindset, but also the optimal conditions that support these instructional moves. The goal of this chapter is the characterize the nature of Ms. M's pedagogy in her academic year Algebra course in service of providing or not providing opportunities for students to experience themselves as competent doers and *learners* of mathematics in ways that that could influence students' self concepts, engagement, and ultimately their learning. To illustrate the key differences between the two courses, this chapter will analyze Ms. M's academic year pedagogy employing the four dimensions of pedagogy used to characterize her pedagogy of the summer course. After characterizing the pedagogy in the academic year, this chapter will consider the constraints of the school context to examine why, despite her strong commitment to growth mindset instruction, Ms. M implemented moves that contributed to distinctly different opportunities for students.

Frost Middle School Classroom Context

The yearlong 8th grade academic year class met four days a week for 48 minutes each day and one day a week for 39 minutes. Students took this course for credit in Algebra 1, and classes were held accountable by the school and district to perform well on the California Standards Test (CST). Accordingly, Ms. M had a traditional textbook and curriculum she was expected to follow.

Just as she did in the summer course, Ms. M began the academic year with growth mindset goals for her students and a commitment to teach in a way that would contribute to the development of a growth mindset. As described in Chapter 2, Ms. M was familiar with Carol Dweck's (2006) *Mindset*, and aimed to teach about a growth mindset to her students in ways that would encourage them to take on challenges and persevere in the face of those challenges (Ms. M, 2013) In service of these goals, just as she did in the summer course, Ms. M shared a quote about growth mindset with her students once a week and continually emphasized that hard work and perseverance were necessary components to *learning* and success. However, beyond this explicit mindset instruction, these goals did not play out the same way in the academic year.

While she outwardly set growth mindset goals for her students, in practice she also set *performance* goals for her students, with an emphasis of outperforming previous years on the standardized tests throughout the year. In service of these performance goals, she tailored her instruction to align with the traditional Algebra curriculum the students would be tested on, leaving little room for the rich problem solving and student authority that was manifest in the summer course. Accordingly, while Ms. M's explicit statements regarding what it meant to work competently in this class were both *learning* and *performance* based, competent participation was limited to short closed problems with correct answers as the end goal and student voices in whole class mathematical discussions were largely limited to short turns in response to direct teacher questions.

While Ms. M emphatically believes in *learning* and set growth-oriented *learning* goals for her students, these learning goals often lost out to highly prioritized *performance* goals relating to the California Standards Test (CST). While learning goals support the development of a growth mindset and increasing effort in the face of challenges, these *performance* orientations can support the notion that intelligence is fixed and can lead students to seek easy successes in the pursuit of looking smart and to give up when faced with challenges (Dweck, 1986 & 1999). As Chapter 4 will show, these shifts in pedagogy had consequences for students. The mindset interventions in the academic year did not correspond with productive shifts in student mindsets, engagement, or persistence toward learning. So why did the academic year look different despite outwardly expressing the same goals?

Pedagogical Strategies Toward Teaching for a Growth Mindset – Academic Year

Using the extensive list of codes of pedagogical moves that provided opportunities for students to experience themselves as competent *learners* of mathematics that was developed using summer data as a starting point, I expanded and refined this list to examine these opportunities in the academic year, as well as to capture the ways in which these opportunities may have been missing along the same four dimensions. With the help of undergraduate researchers, these codes were consolidated into the four dimensions introduced in Chapter 2. As described in Chapter 2, by coordinating these themes with prior literature, I developed comparison dimensions to characterize the opportunities Ms. M provided for her students to experience themselves as competent mathematical participants in ways that would support or constrain the development of a growth mindset. Ultimately, we analyzed both the summer and the academic year data using the same set of rubrics to compare the ways in which Ms. M's pedagogy supported or constrained the development of a growth mindset across the two contexts.

The four major dimensions that were used to characterize Ms. M's pedagogy with respect to "Teaching for a Growth Mindset" in Chapter 2 are also used to capture the ways this practice. These dimensions are: 1) Framing Success (What are the long-term goals for students? What gets defined as competent mathematical participation and how? and What are the messages about what it means to work competently?); 2) Treatment of malleable intelligence; 3) The nature of the mathematics students are asked to work on; and 4) Agency and Authority (Table 3). As you saw in Chapter 2, these dimensions are not mutually exclusive, and many examples of Ms. M's pedagogy cut across multiple dimensions.

After having considered the summer course as a productive example of "Teaching for a Growth Mindset" over an extended period of time, this chapter will use the same four dimensions to compare that pedagogy to that of the academic year. This analysis will show that the sub-strands for each dimension look quite different in the academic year. Accordingly, I will argue that the schooling context of the academic year provided a different set of resources that constrained Ms. M's goal of teaching for a growth mindset in ways that were not productive for students. Chapter 4 will build on this to show how these differences in pedagogy were not met with the same productive shifts in student mindsets, engagement, and learning. By providing a contrast, this analysis will show the *necessary* elements for effectively "Teaching for a Growth Mindset" in a middle school mathematics classroom.

Table 3
Academic Year Pedagogy to Enable and Constrain Teaching for a Growth Mindset

1) Framing Success: Teacher constructed competent mathematical participation as both learning *and* performance oriented	D. Student goals: Teacher set long-term learning growth and comparative performance goals for students E. Teacher's explicit statements around what it means to work competently on mathematics in this class were both learning and performance based F. Competent participation: Competent student participation was limited to short closed problems with correct answers and brief initiation-response-evaluation sequences
2) Treatment of malleable intelligence: Teacher emphasized malleable intelligence, but it was unintentionally coupled with performance-based fixed mindset messages	C. Teacher shared a quotation each week about learning and intelligence, and engaged students in relating the quotes to their own experiences as learners. D. Teacher communicated growth mindset messages throughout the year, coupled with performance goals
3) The Mathematics was primarily routine and procedural	B. The mathematics was primarily routine and procedural, lacking opportunities for students to struggle productively
4) Agency and Authority: Authority was distributed primarily to the teacher or the textbook, and there were few opportunities for students to be positioned as competent in meaningful ways	C. *Disciplinary agency* i. The teacher and the textbook were in charge of the mathematics and were the arbiters of mathematical facts. Students only had disciplinary agency to apply established methods. D. *Authority and positioning* i. Students had a sense of authority over their own behavior ii. Teacher praised or complimented habits/effort, but it was not specific

1) Framing Success: Teacher Constructed Competent Mathematical Participation as Both Learning *and* Performance Oriented

A. Student goals: Teacher set long-term learning growth goals and comparative performance goals for students. From the outset of the course, Ms. M framed the long-term goals for her students as both *learning* oriented and *performance* oriented. Consider the vignette at the beginning of Chapter 1. On the first day of school, Ms. M introduced the course by saying, "Starting with Algebra 1, I hope that your experience is that math makes sense and that things are connected and you start to understand why you're learning the things that you're learning." Just as she did in the summer course, she went on to discuss the importance of learning Algebra through the lens of equity by sharing Bob Moses's book *Radical Equations*. She described that Bob Moses called Algebra 1 the civil rights issue of today, and after learning how schools were not doing a good job of teaching Algebra 1, she became inspired to teach Algebra.

Following this emphasis on equity and *learning* Algebra, however, she went on to discuss the school's history of performance on standardized exams. She asked students to guess the percentage of proficient 8th graders from their school on the California Standards Test (CST) each year since 2006, highlighting the change that occurred after her first year teaching at Frost Middle School. To emphasize how good the scores were for the year 2011, she told students that congresswoman Barbara Lee and Secretary of Education Arnie Duncan came to her classroom to recognize the school's success of having 67% of students score proficient on the CST. Then, for the year 2013, she wrote a question mark on the board, "That's you. So we have this year to prepare you for the state test at the end of the year, which will be about this [*holds up textbook*]. This is our mission."

While Ms. M shared her goals for equity and emphasized and praised growth on the CST over time, the accompanying goal in this interaction was of *performance* – to perform well on the test. Dweck's (1986 & 1999) work describes that a *performance* orientation can cause students to be concerned with looking smart—a goal aligned with the belief that intelligence is *fixed* (Dweck, 1986). This mindset leads students to seek easy successes in the pursuit of looking smart and to reduce their persistence in the face of obstacles (Dweck, 1999). This interaction established a single major *performance* goal for students in this course, sending a strong message about the nature of intelligence and what it means to be competent in this context (Dweck, 1999).

These goals reappeared throughout the year as conversations related to the standardized testing pervaded the classroom dialogue. For example, toward the end of the year, during the standardized tests, Ms. M pressed students to double and triple check their work so they wouldn't make mistakes. In response a student raised his hand to ask, "how many can you miss in order to still be proficient?" When Ms. M responded that students could miss 9, the student pressed on, "What if you get like 10 wrong? That is not proficient?" What is evident in this interaction is that through the framing of the goal of the course as performing on the CST, students came to see the CST as a symbol of whether or not they succeeded.

However, these interactions were not all performance-based. In the midst of this particular end-of-the-year conversation around proficiency, Ms. M also highlighted a growth-oriented message, "Everyone is moving up. That's the important thing. That everyone is doing better than they have before." In the first day of school interaction and this interaction, and as was typical in the majority of interactions around the long-term goals for the course, students received a mixed message that the goal is *learning*, but their *performance* on the end-of-the-year CST will concretely evaluate whether or not they learned.

This dual framing is not surprising given the goals Ms. M set for her students. In an interview prior to the start of the year, Ms. M described her goals for her students. The first goal she set was around proficiency and performance on the CST:

> I want my students to be proficient in the content of Algebra 1 … One of my goals is to get my kids proficient in the content that they're supposed to learn. I take that really seriously! They're supposed to know this, they're going to be tested on that, it's my job to deliver.

First and foremost, as with most great teachers, Ms. M set the goal of proficiency. For her, proficiency was defined by what students will be tested on. Accordingly, the CSTs played a significant role in the dialogue around goals for students throughout the entire year. The second goal she set for her students concerned the development of a growth mindset:

> I want students to … have an attitude towards their own skills and learning that they can do whatever they work hard at doing. If they want to be good at something its a matter of effort … I want them have a growth mindset and to see themselves as always able to get smarter and as a result I want them to be more willing to take on challenges.

While proficiency on the set curriculum and performance on the standardized test was a central goal for Ms. M, so was developing a growth mindset and a *learning* orientation that pushes students to take on challenges and work hard in order to succeed. While learning goals were central to the discussions in her course, these goals were often coupled with conversations around performance and testing.

The school-wide grading system also highlighted this dual focus on the standardized tests and hard work. Students earn two grades for every class – one for "Standards Based Performance" and another for "Habits of Work." In an ideal world, these two grades would be correlated in such a way to support the notion of a growth mindset that hard work leads to learning. However, it still ultimately sets *performance* as a dual goal along with *hard work*, rather than measuring habits of work and students' individual *learning* gains.

The salient messages around long-term course goals were often fused in such a way that did not solely emphasize either the goal of *performance* or the importance of *learning*. As a result, the framing of the course did not fully support the development of a growth mindset in the way that the non-performance-based learning-oriented summer course did.

B. Teacher's explicit statements around what it means to work competently on mathematics in this class were both learning and performance based. Throughout the academic year, Ms. M made explicit statements that framed learning and hard work as what it meant to work competently in this class, as well as ones that framed accuracy and performance as what it meant to work competently. In these framing statements, Ms. M proposed conflicting messages regarding what it means to be competent in this math class as both learning-based *and* performance-based. At times she framed accuracy as the goal and at other times, understanding as the goal and confusion as part of the process.

The most explicit and pervasive way these conflicting messages surfaced was through the daily conversations concerning the ways in which students were expected to engage with their homework. On the one hand, she framed homework as a *learning* activity, in which students' credit was earned based on how much they learned. Ms. M would consistently reinforce this goal by saying things like,

> Your homework is not just to do something but it is to *learn* something. So what you are telling me if you say full credit is that you fully understood all of the problems. If there is

something you don't understand, you should ask for half credit. Come to tutoring, learn the problem, and then upgrade. You can get those points later. Don't ask for full credit, until you fully understood.

Through these statements, she supported the notion that to work competently on their homework meant to strive for understanding and learning.

However, at the same time, she would just as frequently refer to the goal of students' homework as striving for perfection and correct answers (as determined by the back of students' textbook). A primary part of their homework each night was to score themselves with a red pen against the answers in the back of the book. So while credit was earned for *learning*, learning also meant accuracy and correct answers – sending a strong *performance* message at the same time. These performance messages were reinforced with statements such as,

> I'll stamp it full credit if your homework was perfect in every way. Half credit if it wasn't quite perfect. And no stamp at all if it isn't even close.

> You should have your red pens out, lots of people do, go ahead and score #1. If you got that right on the first try, give yourself 2 stars. That is, if you have the work and the correct answer, then you deserve 2 stars. If you did something wrong and you cross it out and you do it over in red pen, you start over, and then you understand it - you get 1 star. If you … still don't understand this problem, you need to put an X. … You cannot give yourself a star of any kind if you have not mastered the skill.

> This needs to be scored with red pen. The answers are in the back of the book … you have to check your answers … You actually have to go in the back of the book to make sure that you're understanding. You don't know that you're understanding the math unless you can check the answers. So your homework is actually to learn something and you don't know if you're leaning it if you don't know if you're right. … if you do not understand something, you just put an x and that's a problem that you need to come to tutoring to get help with.

Statements like those above highlight the role of the textbook as an external feedback mechanism that relies on a merit-based system of competence. Potentially because of the nature of the short, closed mathematics questions students are asked to complete, students are expected to consider their work on a right/wrong system of competence that favors the former as the end goal. The issue with asking students to fit an ideal mold of striving for accurate and precise answers and as those answers determining a students' understanding, is that it highlights a fixed-mindset with no room for students' individuality, creativity, or authority (as will be discussed further in a later section) (Dweck, 2006).

While on the one hand these statements reinforce that the goal of homework is to have correct answers, these statements also support the idea that accuracy and correct answers are a sign of understanding. While these statements may support the message that mistakes are a necessary and welcomed part of the process, they also define *understanding* as one's ability to produce correct answers – in the end, there is no place for errors. Accordingly, the messages related to homework most saliently highlight the conflicting and concurrent nature of both learning and performance-oriented messages regarding what it means to work completely in this mathematics classroom.

Beyond the messages concerning homework, Ms. M consistently made statements around what it means to work competently in this class that were both *learning*-based and *performance*-based. In particular, she often indicates that confusion, struggle, and perseverance through that confusion are normal and necessary elements of working on math in this course. For example, she makes many statements similar to the one below:

> It's normal in this class to be confused. That it's important to be confused. I hope you get confused every day. Because if you don't get confused, you can't grow. So what I want you to do is to come to math class, get confused, struggle, struggle don't give up, work at it, figure it out, and at the end of the class you will be smarter than you were when you started. So I don't want you, the first time you get confused to go "wha?" and just give up. I want you to think, "oh, I'm supposed to get confused. That's my job. To get confused and then figure it out" Confusion is normal and important.

In these statements, Ms. M makes it clear that confusion is part of the process toward learning. These statements de-emphasize performance, and instead focus on practice, study, persistence and effort – processes that all support the development of a growth-orientation (Dweck, 2006). Similar to the summer course, Ms. M emphasized that students' goal is to learn, a process that requires hard work. However, in the academic year, these statements are also made in situations in which students aren't given an opportunity to experience confusion on non-routine problems. This statement above is made in the first week of school as Ms. M outlines the norms for her class. The concept appears again, for example, when Ms. M asks students if they experienced confusion after a quiz that consisted of three exact word problems students had previously seen. In a context in which students are asked to complete straightforward non-routines tasks, statements referring to confusion can be considered a form of encouragement rather than a clear growth mindset oriented statement. Furthermore, on multiple occasions when a student expressed confusion during a whole class exposition, Ms. M quickly addressed the confusion by re-explaining the procedures, but not highlighting the importance of confusion as she did in the summer course.

While bringing in messages of learning and hard work, Ms. M still emphasized external measures of performance. In particular, she told students that if they score 90% or higher on a quiz, she'd post their quiz on the bulletin board for all to see. This sort of emphasis on test scores is a form of praising students' performance and intelligence, a move that can harm students' motivation and harm their future performance (Dweck, 2006). Like in the summer course, in the academic year what counted as competent involved a process that was challenging and time-consuming (Gresalfi, 2009), but unlike in the summer course, the system of competence placed a high premium on getting the right answers and *performing.* These two messages of what it means to work competently were often at odds with one another in ways that did not fully allow the development of a learning orientation.

C. Competent participation: Competent student participation was limited to short closed problems with correct answers and brief initiation-response-evaluation sequences.

While the messages concerning what it meant to work competently in this Algebra class were often learning-based, the opportunities that students were actually given for working competently were much more limiting. In the summer course, students co-constructed competent participation as persisting in the face of challenge, and student creativity and contributions were valued. Contrastingly, in the academic year, students still had ample opportunities to participate competently, but the *ways* in which they were expected or obliged to competently participate were limited to working short closed problems with correct answers and brief initiation-response-evaluation sequences (Mehan, 1979). As will be discussed in more detail in strand 4, Ms. M retained the most authority in this classroom, which also gave her the most power to determine competence (Gresalfi et al., 2009), and students' opportunities to contribute to the class-community in ways that defined competent participation were limited to short responses.

This process of defining competent participation in such a limiting way happened along two major processes in the academic year. First, the course was primarily structured with Ms. M at the board modeling mathematics problems with student participation limited to short initiation-response-evaluation sequences (IRE: the teacher Initiates the sequence by asking a typically short, factual question; the student Responds; and the teacher briefly Evaluates the response as competent and the cycle continues) (Mehan, 1979). Second, only a small scope of student activities – that primarily included following the teacher's or textbook's given methods – were valued as competent mathematical participation.

The following sequence provides an example of the first process in which the students' participation in their learning was limited to short IRE sequences led by Ms. M. To read a larger transcript of this interaction, see Appendix A.

Teacher: I'm going to make a t-table, but notice in my t-table the right side is wider than the left. That's on purpose … So in our t-table, usually you see people write x and y, but I'm not going to write y, I'm going to write what y equals. Instead of y, I'm going to say what y is equal to. And what is y equal to?

Students: Negative x plus 4

Teacher: Negative x plus 4. So really this is x and this is y, I just wrote y in a different way because I happen to know from the equation that y is the same as this. So this is x and this is y. Everybody okay so far?

Teacher: Next. We're going to create our own domain. Our own list of x values. So I'm going to create a list of x values, and I'm going to show you my favorite domain, and I recommend it very highly and I'll tell you why. My favorite domain is -3, -2, -1, 0, 1, 2, and 3. I like this domain because we have a little bit of negatives, a little bit of positives, and 0 and they're all easy numbers to deal with. So you're going to use this domain a lot.

Teacher: Now the next thing that I want to show you is that what we're going to do is, when x is negative 3, we're going to find out what y equals by substituting negative 3 in for x. But I offer you this fantastic way of doing this – whenever you substitute, I've told you to substitute with parenthesis, so instead of negative x + 4, I'm going to write negative parentheses plus 4. Negative parenthesis plus 4. …[*chanting 7 times for each number in the domain*] So I want you to always do this, whenever you're going to substitute in for a number, start by replacing the letter with parentheses (*continues* *to explain without student interaction how to fill out a t-table*.)

Teacher: Alright, so here we go. The first x value we're going to substitute is…
Students: Negative 3
Teacher: Negative 3. The next one?
Students: Negative 2
Teacher: That's right. Go ahead and put the x values in.

This direct instruction looked like most of the whole class discussions in which students were given limited ways to participate competently. On most days, the class would begin with a warm-up while Ms. M stamped homework. Ms. M would then go over the warm-up on the board using IRE sequences to solicit student steps and answers. Finally, Ms. M would present the lesson for the day with several examples in which, generally after she modeled the first example, she would call on students or the class as a whole in an IRE format like the one above to talk her through solving the problems. Note that each of the teacher's questions guide students through the appropriate steps so students are only asked to apply routinized procedures to deliver quick answers.

This interaction structure makes sense given the context of Frost Middle School, in which Ms. M feels bounded by set curriculum and accountable to the state standardized test, with only a 48-minute class session. In fact, Ms. M was acutely aware that she was engaging in these proceduralized IRE sequenced instructional practices and that they might not be supporting her students in the most effective way. In a mid-year interview, Ms. M explained:

> I feel forced to just deliver strategies for them, to tell them what to do. So it's – I have so many standards to cover, and it's so many skills that I need to teach, and I end up a lot of the time doing a very good job, I would say, of saying "here's the kind of problem you need to be able to do, here's a way or two that you can do it, and here's why it works and how it connects to some things you've learned before." But, I don't at all believe that that is *the* best way to teach and to learn math.

In the interview, as will be discussed later in this chapter, Ms. M reveals that she felt she did not have sufficient time to provide opportunities for students to participate in ways that would allow them to authentically experience themselves as doers and learners of mathematics. As a result of this limitation, Ms. M recognized that this instructional practice is not the most effective way to teach.

In addition to limiting the *ways* in which students could participate competently in learning mathematics through the interaction structures, it was also the case that only a small scope of student activities – which primarily included following the teacher's or textbook's prescribed methods – were valued as competent mathematical participation. In the third dimension below, we will consider the mathematical *content* in more detail; in the fourth dimension, we will discuss the ways in which Ms. M was the primary holder of authority over the mathematics in the academic year. This current dimension category considers only the range of opportunities Ms. M provided for students to participate competently in mathematics.

First, as was alluded to in the example above, the classroom activities frequently involved the teacher at the board, with students' expected role being to follow the methods presented by the teacher. In one lesson, Ms. M makes this expectation particularly clear by saying, "I am going to be modeling. I am going to be modeling how I do math and you are going to be taking notes watching me and then homework is your turn." This process of Ms. M modeling with only IRE sequences as the option for students to competently participate was a regular occurrence in the academic year.

When students were asked to work on mathematics independently and in groups, competent participation was likewise defined by following the teacher or the book's taught methods with an emphasis on working quickly. With this emphasis, following prescribed methods and procedures in service of speed and accuracy was valued as competent mathematical participation to the detriment of student creativity and ideas. Common statements to students such as, "You're right! You're doing it the book way. The way I taught you was to use y=mx+b. So either way - you're allowed to use whatever one you want" and "So you use a rectangle to get four terms and then simplify, the answer is $6n^2 + 17n + 12$. And if you don't have that right it's probably because you didn't do the rectangle [method]" both reveal the emphasis on students following the methods they learned from the teacher or the textbook in order to participate as a competent doer of mathematics.

Even when students had these opportunities to work on problems independently or in groups, Ms. M often encouraged them to "catch up," noting when they were "so far behind." These kinds of reinforcements revealed that in this class, competent participation meant working quickly using given methods and responding to teacher-driven questions with short routinized answers. These messages highlighted speed and correctness as aspects of competent participation, rather than thoughtful exploration.

These examples show that in the academic year, students had a restricted range of ways to experience themselves as competent participants in their mathematical learning. The type of classroom created by these constraints provides and example of what Hand (2009) calls a polarized participation structure. In a polarized participation structure – as opposed to the flexible participation structure described in the summer course – what it means to be competent is defined by teacher-sanctioned behaviors and diminished access to mathematical sense-making with an emphasis on didactic approaches to instruction can lead to an oppositional culture in the classroom with fewer and fewer students actively engaged (Hand, 2009).

Similarly, Gresalfi, Martin, Hand, & Greeno (2009) examined the ways that what counts as competent is constructed through participation and discourse in particular classrooms. They analyzed the discourse of two middle school mathematics classrooms to examine the ways that contrasting systems of competence develop. In one classroom, the system of competence was constructed in what Hand (2009) calls a polarized participation structure that included a system of competence that did not create space for students' own ways of participation, but instead reinforced a process following directions and using the procedures or methods modeled by the teacher to complete problems (Gresalfi et al, 2009). Like the academic year, the classroom Gresalfi et al (2009) studied limited the ways student had for participating competent by reducing the agency and authority with which they were positioned to do that work.

Furthermore, as will be discussed in dimension four, student authority is diminished as teachers and textbook authors are considered the arbiters of truth. This practice of limiting the ways in which students can be seen as competent doers and learners of mathematics can contribute to students' beliefs about mathematics and, in turn their identities as doers of mathematics. Lampert (1990) notes that students abstract their mathematical worldview from their experiences in their mathematics classroom – in which "*doing* mathematics means following the rules laid down by the teacher; *knowing* mathematics means remembering and applying the correct rule when the teacher asks a question; and mathematical truth is *determined* when the answer is ratified by the teacher" (p. 32).

More specifically, the types of practices in which the students are given opportunities to participate competently – such as using a taught procedure at the correct time – can contribute to the belief that mathematical methods will be bestowed upon them and neither discovery nor understanding has a role in this process (Schoenfeld, 1988 & 1992). The traditional procedural approach to mathematics education can limit students' opportunities for learning to simply reproducing and adhering to mathematical rules and procedures, suppressing original creative thought. These practices then become part of students' mathematical learning identity, as passive receivers rather than users of mathematics (Boaler 2002; Boaler & Greeno, 2000).

2) Treatment of Malleable Intelligence: Teacher Emphasized Malleable Intelligence, but it was Unintentionally Coupled With Performance-Based Fixed Mindset Messages

A. Teacher shared a quotation each week about learning and intelligence, and engaged students in relating the quotes to their own experiences as learners. Much as in the summer course, Ms. M emphasized malleable intelligence by sharing a growth mindset quotation each week about learning and intelligence and engaging students in a conversation around how the quote related to their own learning experiences. While in the first half of the year, the quotes came from Dweck's (2006) *Mindset* like in the summer course, in the second half of the year she also used quotes from a variety of other sources, including athletes, politicians, musicians, and once a student in her class. These discussions served to give students a lens for considering what is required to learn mathematics to include working hard toward learning. Bringing in the student quote is the one distinct way that Ms. M provided space for students to contribute to the rhetoric of defining competent mathematical participation, albeit in a way still curated by the teacher. An example of a non-Dweck quote from April of the second semester is below.

Ms. M: This quote does not come from the mindset book, but it *definitely* makes a mindset point. So here we go. Today's quote comes from a violinist from Spain. His name was Pablo de Sarasate. And he said this. I'm going to read it twice and leave it to you to figure out what he meant by that, what his point was …

Quote: "A genius? For 37 years, I've practiced 14 hours a day and now they call me a genius?"

Ms. M: What's his point? Team talk, what's his point?
(*students talk in teams of three for 2 minutes while Ms. M walks around talking to teams)*

Ms. M: Alright so, I've heard *many* teams say this theory. So I'm going to say this and then I'm going to ask you to raise your hand if you have a *different* idea. So many people said they think that his point is, "I've been practicing for 14 hours a day for 37 years and you finally get that I'm a genius." That's *one* theory. Raise your hand if you have a *different* theory. Something different from that most popular theory? Yes, Khalil

Khalil: I said, I didn't know which way he was saying. Whether he was saying, "yeah, I've been a genius" or whether he was saying, "yeah, I've been practicing for 14 hours a day but I'm still not a genius."

Ms. M: So Maybe. Khalil's introducing a new idea. Maybe he's saying I'm still not a genius. Is his point that he is a genius or that he isn't a genius? Alright, we're getting warm. And Lizzie you had a different theory?

Lizzie: If you consider yourself a genius, you're still not a genius because you haven't learned everything. The definition of genius is a person that knows everything (inaudible).

Ms. M: So maybe a genius is supposed to know everything and he doesn't know everything yet. Keyonna?

Keyonna: (inaudible) growth mindset. Doing practicing 14 hours a day isn't making you a genius, it's just what he does to be better.

Ms. M: Okay, we have a winner here. Keyonna, I think you've hit the nail on the head. At least, this is what I think his point is and this is what I want you to consider maybe his point was. So Keyonna, would you repeat that so that people can hear because it's subtle.

Keyonna: That because he's been practicing 14 hours a day doesn't make him a genius, but because he has a growth mindset, he's just doing what he has to do to be better.

Unlike in the summer course, the mindset quotes in the academic year were regularly coupled with a few minutes of team talk in which students discussed what the quote meant before the class came together as a whole to discuss it. In the example above, Ms. M allowed students' ideas to guide how students interpreted the quote, still guiding how the conversation went by airing an idea that might be incomplete and asking for *other* ideas. By providing this space, students had the opportunity to develop a lens to regard their experiences with learning mathematics toward a growth mindset – a way to frame what it means to learn (and struggle with) mathematics. In this way, it makes sense that in the student interviews discussed in Chapter 4, students are very articulate about what a growth mindset is and the importance of it for learning. In short, students had many rich opportunities to learn and talk about a growth mindset, even if, as will be discussed in dimension three, they didn't have the same rich opportunities to *experience* a growth mindset.

A list of all of the recorded quotes that Ms. M shared and discussed over the course of academic year can be found in the appendix. All of these quotes illustrate one of Ms. M's goals for her students: to encourage the development of a growth mindset that would motivate them take on challenges and persevere in the face of these challenges. These quotes are the primary explicit way the growth mindset goals materialized in students' experiences in the academic year, and provided a lens for students to view hard work and learning as the goals to strive for in this class.

B. Teacher communicated growth mindset messages throughout the year, coupled with performance goals. Much as in the summer course, Ms. M also communicated growth mindset messages throughout the academic year. However, these messages about learning requiring hard work were also coupled with performance goals that emphasized right answers and scores on tests. Often the clashing messages came together in the same interaction.

Ms. M's statements defining what it meant to work competently in this class – described in dimension one – also communicated the growth mindset message that learning requires hard work. These statements were meant to influence students' engagement with challenging mathematics and their perseverance toward learning. However, a majority of the statements that communicated the necessity for a growth mindset were coupled with an emphasis on performing on tests. All of the following quotes come from different days of the academic year in which Ms. M shared messages that would support the development of a growth mindset.

> The three problems on this quiz are word problems that you have seen before. We've been studying systems, we've been studying mixture and you have been tigering up to take on problems that used to be really hard for you but now shouldn't be so hard anymore.
>
> Now, you have a test in two days. People who do well on it are not just the people who right at this moment are ready. It is the people who work the hardest between right now and test time. The successful people are the ones who are going to work hard.
>
> Okay so if you want to be good at math, it just takes practice. Come to tutoring. All of you are capable of being great in Algebra and it is just a matter of time and effort. So if you are struggling, step up your game and come to every single tutoring session, find a friend who can help you etc. How well you do [on your test], depends on how well your work.
>
> I want to remind you that we have made great gains so far this year in our proficiency. We have 47 people proficient on the quarter 1 test and that went up to 64 people on the quarter 2 test … So now we're looking toward quarter 3 test …We have 2 months to prepare … Aiming for 75.
>
> How well you do on that exam, and therefore how well you do in this class and your grade and also how you get placed in high school into your math class depends on how hard you work (inaudible) between now and then.
>
> You have a final exam this week and the way to do well on it is to decide that you want to do well on it, get inspired, and then do the hard work to study and prepare.
>
> So you have one test left this year. It is the final exam and it is the same material that you studied for in the State test. So we are going to continue practicing the same concepts. So you have one more test and that one does count in your SPP grade and it counts for your placement [in high school]. … Basically, here is what we do. We race to cover all the

> material before the state test because everything is on the state test, as you now know. But now we have some time to review and slow down before the final exam that actually counts for your grade and your placement. So you have basically seen everything. You want to get at least 70 in order to go onto the next level. And if you want to score advanced, and get placed into an honors class, then 90.

These messages work to communicate that learning requires hard work, and through hard work, students *will* and *have* improved. Yet, it's important to note that all of these *growth*-oriented mindset quotes are still inextricably tied to *performance* on tests.

While Ms. M's growth-oriented statements potentially contributed to the development of a growth mindset that considers intelligence as malleable (Dweck, 2006), these statements were also coupled with very concrete performance goals that may have impeded the effects of the growth mindset messages seen in the summer course. Rather than motivating students to take risks on challenging problems, performance goals cause students to be concerned with their intelligence, with *looking* smart, and thus inhibit their willingness to take on challenge (Dweck, 1999).

Instead, Dweck suggests that rather than focusing on *performing* well, teachers and parents should de-emphasize scores on tests and say something like this instead, "It must be a terrible thing to feel that everyone is evaluating you and you can't show what you know. We want you to know that we are not evaluating you. We care about your learning, and we know that you've learned this stuff. We're proud that you've stuck to it and kept learning" (Dweck, 2006, p. 180). By emphasizing malleable intelligence in the context of performance goals, the intent of these moves may have been stymied in ways that impeded students from fully internalizing a growth mindset that would influence their engagement and persistence with learning.

3) The Mathematics was Primarily Routine and Procedural

A. The mathematics was primarily routine and procedural, lacing opportunities for students to struggle productively. While some differences in the ways that Ms. M framed competence and emphasized malleable intelligence in the academic year have already been described, the major difference in pedagogy between the academic year and the summer course lies in the nature of the mathematics students were asked to work on. While, as shown in dimension two above, Ms. M's *explicit* use of weekly growth mindset quotes carried over from the summer course, it was no longer accompanied by the challenging non-routine problems that students in the summer worked on. Instead, in the academic year, the mathematics was primarily routine and procedural, lacking opportunities for students to struggle productively. The academic year was characterized by traditional instruction in procedural mathematical content aligned with the California Standards Test (CST). In other words, students did not have the opportunity to *experience* a growth mindset.

As briefly described in dimension one, new content in the academic year was typically presented by the teacher in a "demonstrate and practice" format, by employing IRE sequences (Mehan, 1979). After completing a series of worked examples using an IRE participation structure, students were occasionally given time to solve similar problems individually or in pairs. A snippet of such interaction is below.

Ms. M	[*Writes '4x ≥ 7x + 6' on the board*] Now we are going to be doing problems just like the warm up, except the warm up was equations and now we're going to be doing…
Students	Inequalities
Ms. M	Inequalities. But it turns out solving inequalities is very similar to solving equations. So I'm going to rewrite this so I don't mangle the original. And Tyree I haven't called on you today so it's your turn. How are you going to deal with the fact that we have x's on both sides. What would you like to do.
Tyree	Subtract.
Ms. M	Sure. Subtract what?
Tyree	4x from 7
Ms. M	Subtract 4x from both sides or subtract 7x from both sides? It's your choice.
Tyree	4x
Ms. M	Okay, let's subtract 4x's from both sides. That is a legal move. And when I say 'on both sides,' you know I'm saying 'both sides of the inequality.' So what Tyree has done is he's created a zero on the left. So what do we have now? Paul, you haven't had a chance today.
Paul	Uhhh – so you have one x
Ms. M	So I have four xs and I take away my 4xs
Paul	So zero
Ms. M	So I have nothing. It's the same thing – by the way, side note – zero is the same thing as zero x. … Go on, Paul
Paul	Is greater than or equal to 3x plus 6.
Ms. M	Thank you. And today, I don't think I've gotten to Omar. Did I get you? Okay, then, I got almost everyone. Oh, Dalia, did I get you? Okay, then I got almost everyone. Okay, Mattrel. Mattrel, how would you like to. [*Student disrupts and asks for her card to be turned]*
Mattrel	Subtract 3x from both sides.
Ms. M	That's a legal move. And I can see why he wanted to do that. He created a zero. He also set it up so we would have our letters on one side and our constants on the other, which is very handy. So what will we have now? Go ahead José.
José	Negative 3x is greater than or equal to positive 6
Ms. M	Thank you and Farris, now what would you like to do?
José	Divide -3 on both sides
Ms. M	Ohhh – did you just say divide both sides by a negative?
Students	Switch the signs
Ms. M	Yess [*sings to the tune of "If you're happy and you know it"*] If you divide both sides by a negative switch the sign
Students	Switch the sign!
Ms. M	If you divide both side by a negative switch the sign
Students	Switch the sign!
Ms. M	I am signing a silly song, so you won't get the problems wrong. If you divide both sides by a negative switch the sign!
Students	Switch the sign!

By introducing the mathematics primarily through skills-oriented IRE sequences and giving students opportunities to engage only in skills-based ways, Ms. M did not incorporate challenge in the academic year as she did in the summer. Accordingly, in their experiences with mathematics in the academic year, students were not given opportunities to engage in centrally important math through productive struggle, which The Algebra Teaching Study's TRU Math Scoring Rubric and its supporting documents argue is a centrally important strand for teaching for robust understanding (Schoenfeld et al., 2014a & 2014b). Instead, the reverence toward speed and perfection in mathematics is the enemy of difficult learning (Dweck, 2006). As such, students were not provided opportunities to *experience* that hard work leads to learning – to *experience* the growth mindset they were learning about.

In addition to students not experiencing challenging mathematics in their classroom activities, their homework and tests similarly did not provide these opportunities. Students' homework consisted of textbook problems that directly matched or built on the day's lesson. While Ms. M aimed to assign homework problems primarily from the challenge section of textbook sub-chapters, these problems were still typically very formulaic in requiring students to apply the same procedures they learned in the lesson, even though they may have involved words or more steps than the easier textbook problems. Finally, when giving tests, Ms. M often gave a review sheet that matched the test and was very explicit with students that the problems were identical to the ones they had been taught in class, saying things like, "As always I'm giving you the exact 8 problems that are just like the real test so if you master these you will do really well on the chapter test." In this framing, the expectation was set that students were to follow the specific procedures laid down by the teacher.

With traditional mathematics classes often consisting of a series of closed questions with right or wrong answers in this way, mathematics becomes the subject area that communicates the strongest fixed ability messages to students (Boaler, 2010). Treating mathematics as a series of right or wrong short procedural questions can send strong fixed mindset messages to students – "if students are working on short, closed questions that have right or wrong answers, and they are frequently getting wrong answers, it is hard to maintain a view that high achievement is possible with effort" (Boaler, 2013). Likewise, if students only experience easy successes they can develop the fixed-mindset belief that you are only smart if you can succeed without effort, which can deter students from seeking challenge or persisting in the face of struggle (Dweck, 2010).

This effect of students' self-concepts on their mathematical behavior is consistent with Bandura's (1994) theory on self-efficacy. Much like Dweck's findings on the influence of challenging work on their motivational patterns, Bandura's (1994) theory claims that people's beliefs about their abilities strongly influence how much effort they put forth and how long they persevere in the face of difficulty. For example, Bandura claims that when students are given only routine unchallenging tasks, they come to expect easy successes and are easily discouraged by failure or difficulty. This message contributes to a sense of learned helplessness such that, by only experiencing easy successes, students come to expect quick results and give up in the face of challenges, avoiding tasks they see as personal threats (Bandura, 1994). Through the kinds of mathematical knowledge that students had opportunities to build and to use in the academic year, the mathematical content and the cognitive demand of the tasks did not support the growth mindset goals Ms. M so diligently and intentionally set out to achieve with her students (Gresalfi et al., 2009; Stein et al., 2000).

Holding students accountable only to routine procedural problems, such as those found in a traditional textbook, conveys strong messages to students that influence their self-concepts as doers of mathematics and their beliefs about the nature of mathematics in ways that have direct impact on their engagement and persistence. In their TRU Math Conversation Guide, Baldinger & Louie (2014) argue that students often experience math as a set of isolated procedures to be memorized and applied. However, research indicates that when math is taught in this way, students' knowledge is fragile and often accompanied by of a series of counterproductive beliefs and practices.

More specifically, when students are given non-problematic tasks through studying traditional skills-oriented curricula, they come to learn mathematics through memorizing rather than understanding, which unintentionally reinforces students' beliefs that they should have a ready-made solution to a given problem that should produce an answer in less than five minutes. (Schoenfeld, 1992 & 2008; Kilpatrick et al., 2001). Schoenfeld (1992) argues that in traditional mathematics classrooms where students are given many procedural tasks each night for homework, students come to expect that problems should take no more than a few minutes to complete. This belief can have negative consequences on engagement, such that students are more likely to give up on a problem if it takes more than a few minutes to complete, even though they may have solved it if they had persisted (Schoenfeld, 1992).

Furthermore, with traditional instruction, where well-organized, step-by-step, correct answers are emphasized over sense making, students come to expect that they can only solve problems they have been taught how to solve – that mathematical methods will be bestowed upon them and neither discovery nor understanding has a role in this process – a belief that can reduce their persistence in the face of obstacles (Schoenfeld, 1992). Boaler and Greeno argue that this procedural approach to mathematics education limits students opportunities for learning to simply reproducing and adhering to mathematical rules and procedures—suppressing original creative thought—which then becomes part of students' mathematical learning identity, as passive receivers rather than users of mathematics (Boaler 2002; Boaler & Greeno, 2000).

Unlike in the summer course, Ms. M did have a set curriculum in the academic year that she felt compelled to follow. Even so, as a veteran and highly regarded teacher at her school, she had immense flexibility to design the course in such a way that brought in challenging problems and provided time for students to struggle and *learn* as a result. These types of problems would emphasize learning – they would be open-ended, allow for multiple ways of seeing and multiple entry points or pathways to solving (Boaler, 2013). However, as will be shown in the second half of this chapter, Ms. M's professional identity as a successful teacher was directly tied to the curriculum she was tasked to implement. This constraint stifled her willingness to challenge that curriculum in ways that might have supported her growth mindset goals more effectively.

In classroom conversations, Ms. M worked to expand what counts as competent mathematical behavior by redefining mathematics as a subject requiring struggle, persistence, and hard work. However, these messages and conversations were not accompanied with opportunities for students to experience those characteristics, which, as will be described in Chapter 4, did not support students in developing a growth mindset in ways that influenced their engagement and persistence. While prior research argues that "just learning about the growth mindset can cause big shifts in the way people think about themselves and their lives" (Dweck, 2006, p. 216), the results in Chapter 4 will show that *learning* about the growth mindset must also be accompanied by opportunities for students to *experience* a growth mindset.

4) Agency and Authority: Authority was Distributed Primarily to the Teacher or the Textbook and there were Few Opportunities for Students to be Positioned as Competent in Meaningful Ways

By incorporating only routine mathematics and limiting opportunities for students to struggle with mathematics, as described in the section above, Ms. M did not create a productive space for her and her students to share authority over the mathematics, for students to make valuable mathematical contributions, or for Ms. M to position students with authority for those contributions in meaningful ways. Because Ms. M *told* students *the* correct steps and methods to solving particular problem types, rather than "authorizing" students to explore or come up with their own ideas, there were not the same rich opportunities for Ms. M to position students with authority by revoicing their ideas.

A. Disciplinary agency.

i. The teacher and the textbook were in charge of the mathematics and were the arbiters of mathematical facts. Students only had disciplinary agency to apply established methods. Unlike in the summer course in which the students and teacher shared authority over the mathematics, the teacher and the textbook held authority over the mathematics and were the arbiters of mathematical facts during the academic year (Mehan, 1979). By giving direct instruction on how to solve particular types of problems, Ms. M did not position students with authority over the content or process of mathematics. Instead students were only held accountable for checking their work on routine problems either with the teacher or in the back of the book, in the service of getting accurate answers (Engle & Conant, 2002). As described in strand one above, this positioning narrowed the meaning of being a competent doer of mathematics to getting the right answer.

As described above, the academic year was structured such that Ms. M would introduce new mathematical concepts through worked examples with direct instruction and only short student contributions. This direct instruction was framed as though, without being taught the exact way to do it, students would not be able to work through the problems. For example, when preparing for the state test in the spring, Ms. M framed a particular lesson by saying,

> It's on the state test. I want you to have the opportunity to get it right on the state test. If I don't show it to you, then when the state test comes, you'll just have to guess on those and I want you to have a chance to get these right.

This framing explicitly contributes to the notions that a) the teacher has the authority over how to do the mathematics and b) if you have not been taught how to do a particular type of problem, you will not be successful. This belief about students' role in the mathematics can stifle students from persevering when faced with challenging problems (Schoenfeld, 1992).

As described in strand 1 above, when students had opportunities to work on mathematics alone or in teams, the expectation was that students were to follow the procedures presented during the lecture or else to compare answers until the team agreed. For example, early in the year when students were asked to work on some problems independently, Ms. M framed their work by saying, "Let me be clear that you must do these four problems the way I taught yesterday. It's okay to use your notes, it's fine to use your notes, but I want to see that you learned yesterday's methods of adding and subtracting. So it's okay to use your notes to do these four problems." In this framing, she set the expectation that students were only accountable to following the teacher's methods taught and their creativity or ideas were not useful here. On another day, when students were asked to get into teams, they were asked to "go over your answers as a team." In this situation, the teamwork was not a space for students' ideas to be heard, but instead where they were accountable to correct answers. Later in the year after going over one of the problem's answer step by step on the board, Ms. M added, "the answer is $6n^2 + 17n + 12$, and if you don't have that right, it's probably because you didn't do the rectangle [method]," a statement that again reiterated for students that they were accountable to following the prescribed methods and that those methods were the only appropriate way to solve the problem.

Finally when students were given homework, the teacher and the textbook were again positioned as the authority over the mathematics. As described in strand 1 above, Ms. M positioned students' homework as performance-based by requiring that they check their answers in the back of the book. In doing so, she set the textbook as the arbiter of truth, in which students don't know how well they're doing or how much credit they deserve if they don't have the answers from the textbook or the teacher. Additionally, she also set the expectation that the goal of students' homework was the follow the directions and methods taught during the class – "I am going to be modeling how I do math and you are going to be taking notes, watching me, and then homework is your turn." Even when the homework was framed as challenging, the challenge was framed around following the directions, "you're going to have to tiger up not so much for the math of the homework, but just to follow the directions carefully. The math on this homework is not going to be challenging for you, but you're going to have to use your tigering up just to make sure you're following the directions carefully." By holding students accountable to following the methods taught by the teacher and checking their final answers against those in the back of the book, the teacher and the textbook were positioned with authority over the mathematics in students' homework.

Likewise, when given tests and quizzes, students were not given authority to come up with mathematical ideas but instead were expected to repeat the steps taught by the teacher. For example, statements like "this quiz has no surprises on it. It is exactly what I told you it would be. It looks like the practices we've been doing" were often used to introduce tests. In this and the other examples just described, the teacher and the textbook were positioned as the authority in the classroom – the ones who determined *how* to do the math and also the legitimacy of solutions.

In the ways students were set up to work on mathematics in the classroom and in homework, authority was "distributed only to the teacher, who was solely responsible for determining the legitimacy of responses" (Gresalfi & Cobb, 2006, p. 51). I would extend this argument to suggest that authority was distributed to the teacher *and* the textbook. In either case, students only had room to exercise disciplinary agency in which they were expected to apply an established method (Gresalfi & Cobb, 2006). Without conceptual agency to choose and develop conceptual meaning, students had fewer opportunities to participate in mathematical practices and to develop feelings of competence with respect to the discipline (Gresalfi & Cobb, 2006; Cobb et al., 2009; Schoenfeld, 1988; Engle, 2011). Accordingly, students in the academic year did not have intellectual agency, which Engle (2011) argues is necessary to foster productive engagement with the discipline.

B. Authority and positioning. Since students were only provided opportunities to replicate the mathematics demonstrated by the teacher, they did not have opportunities to develop higher levels of authority (Engle & Conant, 2002). The only authority students were given in the academic year was to feel responsible for their own behavior. However, because students were not given significant opportunities to contribute meaningfully, Ms. M did not publicly credit students for their mathematical practices and contributions. Instead, she praised or complimented students' habits or effort, but she was not specific when doing so.

This lack of opportunity to develop authority over the mathematics may have inadvertently mitigated Ms. M's efforts to support students in developing a growth mindset in which they see themselves as competent learners of mathematics. Without authority, students are not given autonomy as doers and learners of mathematics. Accordingly, while students might feel competent at applying the teacher's or textbook's math, students may not have also developed identities as competent learners who come up with and apply their own ideas.

i. Students had a sense of authority over their own behavior. While Ms. M did not share authority over the mathematics with students, she did give students a sense of authority to monitor their own behavior in the classroom. Similar to in the summer course, Ms. M created a card chart with students' names that she and her students used to keep student on task.

Just as it worked in the summer course, students' cards began at green every day, but if a student was not following the classroom expectations for learning, a card would get turned to a yellow warning, an orange warning, and finally a red referral. Students could work off their card turns by coming in after school and cleaning the teacher's white boards. While these negative consequences were shared on the first day, the card chart was primarily referred to as a positive incentive, in which if all students in a class "go green" for five days, every student gets a late homework pass, which can be used or can be saved up for an end of the year raffle.

When introducing the card chart during the first week of school, Ms. M explained that students shared authority over the card chart,

> I don't want this to be a situation where the students are trying to get away with things and it's the teacher's job to be the boss of everyone. I would like you to be your own boss, so if you make a mistake and you talk at a time you're not supposed to be talking, like

> when you enter the room, I would like for you to just point to the card chart - you don't even have to tell me why - and ask for your own warning. That way we don't have this teacher/student working against each other thing. You handle your own business. If you make a mistake, please ask for your own card.

Then throughout the year, she regularly emphasized student's responsibility for their own behavior, with statements such as "can you all just check yourselves and if you make a mistake please ask for your own card" and "some of you should ask for your own card … if you're gonna talk, take responsibility." Just as it functioned in the summer course, the card chart served as a silent reminder that whatever a student was doing was not contributing to learning, and by having the responsibility to make decisions about when they deserved a card turn, students could have a sense of authority to make decisions about what appropriate classroom behavior looked like. This sense of authority, however, was not coupled with authority over the mathematics, as it was in the summer course.

ii. Teacher praised or complimented habits/effort, but it was not specific. The traditional didactic instruction of the academic year did not provide fertile ground for the kinds of mindset moves Ms. M accomplished in the summer. More specifically, the coding of the observed lessons in the academic year reveal few, if any, instances of Ms. M giving credit to students for coming up with their own mathematical ideas, which can easily be attributed to the fact that students had few opportunities to come up with their own mathematical ideas. In fact, in the mid-year interview quoted above, Ms. M noticed that, despite her plans, she had not been giving students credit for their mathematical ideas in the same way she had been in the summer (cf. Cohen & Lotan, 1995). She made a comparison to how she imagined her practice might change with the CCSM and attributed this exclusion to her current practice of *telling* students appropriate strategies rather than having them struggle with challenging problems.

Instead, Ms. M often praised or complimented students' performance. For example, after a recent test, Ms. M called on a student and announced, "continue for us Dalia, who did really well on your test I'm remembering, your test was fantastic." In this example, Ms. M praised a student for her mathematical success rather than her process or effort, a move that can contribute to students' development of a fixed mindset, where they come to develop a performance orientation (Dweck, 2006). Dweck warns that parents and teachers should refrain from a particular *kind* of praise – one that implies they're proud of them for their intelligence or their performance rather than how hard they worked, their strategies, or their persistence (2006). Instead, Dweck (2006) would suggest that instead of focusing on *performing* well, teachers should de-emphasize the score and say something that emphasizes hard work like, "you really studied for that test and your improvement shows it!" (p. 177).

While Ms. M did not position students with authority over the mathematics by revoicing or reformulating their mathematical ideas, she often praised the class or students for their processes or behavior, but in ways that did not develop students' mathematical authority. For example, Ms. M verbally recognized the class for its productive learning habits in the beginning of the year by saying,

> I just want to say - some of you are writing notes that I did not write. That is brilliant. I will often say things that are important that I don't write down and it's really smart of you to be writing them down.

Again toward the second half of the year, Ms. M again recognized the class for their habits of work by saying,

> I am very proud of the way that students read the directions to this new worksheet and made sense of the wording that was different from the wording that you usually see. Secondly, many of you encountered a problem with a number that isn't an integer and you handled that beautifully. … So I'm really proud of the way that you tigered up.

In one final example, Ms. M praised a specific student by name for his classroom behavior. After students were talking in teams, Ms. M said to the whole class,

> I also just want to compliment Nikhil for your wonderful manners you have such nice manners. As I passed by he was careful to not trip me with his stuff, I appreciate your thoughtfulness it does not go unnoticed.

In all of these examples Ms. M recognized the class or an individual student for their classroom behavior. However, as illustrated in all of these examples, the praise did not contribute to students' mathematical authority. Instead, the praise was around processes that subverted student mathematical authority by emphasizing the teacher's authority or students' accountability toward following directions. The final example recognized a student in a public way, but not one that was intellectually or mathematically relevant, both key tenets of the Complex Instruction practice of assigning competence that pervaded her instruction in the summer course (Boaler, 2010).

Unlike those examples from the summer course, the praise for process or behavior in the academic year did not broaden what it means to contribute productively to classroom learning in ways that increase the opportunities for students to experience themselves as smart in mathematics. Instead, being smart in mathematics still meant employing demonstrated methods and being well behaved. Accordingly, Ms. M's pedagogy in the academic year did not serve to strengthen students' authority in the classroom.

While Ms. M implemented an explicit and focused treatment of malleable intelligence for her students, she also framed success along both learning growth and comparative performance strands and accordingly communicated both growth mindset and fixed mindset messages throughout the year. The treatment of growth mindset was also not accompanied by mathematical opportunities for students to experience a growth mindset. Accordingly, authority was retained primarily by the teacher and the textbook with few opportunities for students to be positioned as competent in meaningful ways. The next section will examine some of the causes of these major differences in pedagogy between the summer and the academic year – changes that occurred even though the teacher began the academic year with the same commitment to teach in a way that would contribute to the development of a growth mindset.

Teacher Professional Identity and Decision-Making in Academic Year Context

Comparing the above analysis of significantly different pedagogical decisions in the academic year to that of the summer course does not filly explain *why* Ms. M's pedagogy changed between the two contexts. The analysis must then consider the conditions of the two contexts that may have interacted with her growth mindset pedagogy efforts in more or less productive ways.

While the transcript analysis of the teacher interviews in Chapter 2 suggests that the summer context may have fostered her growth mindset moves, the analysis of teacher interviews and pedagogy in the academic year suggest that this context may have thwarted these goals by offering different identity resources for Ms. M (Nasir & Cooks, 2009). Specifically, this shift in school context may have contributed to different aspects of Ms. M's multifaceted identity being activated in the academic year, which resulted in different prioritization of goals and accompanying pedagogical decisions.

While the curriculum was peripheral to the goals of the summer course, it was necessarily a central focus in the academic year; Ms. M felt bounded by the California Standards Test and the assigned curriculum. In an interview prior to the start of the academic year, Ms. M explained that she felt pressured to abide by the standards on the California Standards Test, "One of my goals is to get my kids proficient in the content that they're suppose to learn. I take that really seriously! They're suppose to know this, they're going to be tested on that, it's my job to deliver." Even though she explained that one of the aspects of her practice wanted to work on was "focusing much more on math practices and open-ended problems and problem solving," she explained that would be "a little bit tricky for me this year because of the fact that I take standards very seriously." Again in the mid-year interview, Ms. M explained that she felt tremendous pressure to get through the content, "And now it's, now it's try to fit it into this race, we have like two standards per day, I mean, sorry, two days per standard, and a day is 45 minutes." Finally in an interview after the end of the academic year, Ms. M explained her frustrations with this pressure,

> I'm given a responsibility to teach these standards in this amount of time …the most efficient way to do that is this traditional dumb textbook style without interesting problems. So and I'm not even able to do um, formative assessment, I don't have the time like if you didn't get it I, I can only teach you about negative exponents in one day. If you didn't get it, that's too bad. It's not coming back, I'm not teaching it again during class because I have to go on to the next topic

A common thread in all of Ms. M's interviews was the immense pressure she felt to deliver the all of the content that she was asked to teach – the content that would be tested on the CST. Accordingly, she did not feel she had the flexibility to design the curriculum in a way that best supported her growth mindset pedagogy.

This same pressure from the external context in the academic year played out in the ways she set goals for her students, as described in strand 1 above. From the first day of class goals around performance on the CST to constant statements similar to "we have a lot to get through" to specific comments about the state test, Ms. M's instruction similarly revealed the immense pressure she felt to teaching the content students would be tested on the CST first and foremost. For example, toward the end of the year, Ms. M made many statements to students similar to the sentiment of the one below:

> Okay, so I have so many more things I want to do with you today so I want to remind you that you have a test on Friday but also the state test is coming in 3 weeks. And this is the one that has – the State test is the one that got the secretary of education Arne Duncan - The United States Secretary of Education … He came here along with congresswoman Barbara Lee. They came here because of our state test scores. They came here for this. We are good at preparing for this state test and today starts the test prep. …We need to review everything from the beginning of the year. We have a lot to review.

Ms. M stated in interviews and to her students that she felt pressure for her students to perform well on the CST – pressure that came from the school, the district, and even the Secretary of Education.

Accordingly, Ms. M set pedagogical goals that aligned with these external pressures. In addition to the growth mindset goals she articulated, Ms. M described a second set of goals. In fact, when asked what her primary goals for her students were just prior to the start of the academic year, she first said, "I want my students to be proficient in the content of Algebra 1 … I take that really seriously. They're supposed to know this, they're going to be tested on that, it's my job to deliver." Only after describing this goal did she begin articulating her other goal of "having students develop a relationship with math directly, so they will be motivated to continue to study math beyond my class …[and] I want them to have a growth mindset and to see themselves as always able to get smarter and as a result I want them to be more willing to take on challenges." In articulating these two sets of goals, Ms. M did not originally see them as conflicting, but as two simultaneous threads to her instruction.

However, as a result of feeling pressured to follow a set of standards and perpare her students for the CST, Ms. M did not have the same flexibility to prioritize goals that aligned with her own orientation toward the importance of growth mindset (Schoenfeld, 2010). Schoenfeld's (2010) research on human decision-making provides a useful lens for at looking at the different pedagogical decisions teachers make: as people orient to particular situations, goals are established and prioritized and people make decisions consistent with these goals. However, just looking at this model does not explain why Ms. M prioritized one set of goals over the other, as demonstrated by her commitment to performance on the standardized test often to the detriment of her commitment to teaching for a growth mindset.

As described in Chapter 2, to consider the prioritization of goals requires examining a teacher's identity in a particular context as part of her orientations. What matters for a teacher in each context (and in what ways) will influence the ways she prioritizes goals and the accompanying pedagogical decisions she makes. Made up of composites of many, often conflicting, self-understandings that develop as people participate in particular *communities of practice* or as they use cultural resources and subject positions in different *figured worlds,* identities are multidimensional and can shift across contexts (Wenger, 1998; Holland et al., 2001). Accordingly, different context can offer different resources for identity development, and different aspects of one's identity can be activated in different contexts (Nasir & Cooks, 2009).

More specifically, the cultural context of the school made salient and available resources that can support the development or activation of a particular professional identity. Through the values about what is considered good teaching and how a teacher relates to that practice, ideational resources made available for teachers in each school context can position teachers into specific roles – roles that they can then negotiated (Nasir & Cooks, 2009).

While the curriculum was peripheral to the goals of the summer course, it was a central focus in the academic year; Ms. M was bounded by the CST. Even though Ms. M approached each classroom context with the same knowledge resources for teaching for a growth mindset, and seemingly the same set of goals to influence students' mindsets in productive ways, what was valued as successful teaching in each context influenced her professional identity in ways that affected her prioritization of goals. The cultural context in which Ms. M taught during the academic year carried a different set of norms, expectations, and ideas than the summer context – and these constrained and enabled different teacher moves (Holland et al., 2001).

This school context offered Ms. M ideational resources—ideas about herself, her relationship to the community of practice in which she was participating, and what was valued as good teaching (Nasir & Cooks, 2009). For a teacher who had US Secretary of Education Arne Duncan and congresswoman Barbara Lee recognize her for her students' success on the standardized test, scores were necessarily and rightfully a salient aspect of her identity. The scores were what the school, district, and state measured teacher success by, so the scores were how Ms. M made sense of her own professional identity as a successful teacher. When introducing these performance goals for her students, Ms. M said, "So, this is a big, big year for you *and for me*, and I hope that you'll understand that this is important, and I will have very high expectations for you, but it's because we're on a mission together. That we're going to work together to get you proficient in Algebra 1." This statement articulates just how central Ms. M saw these goals as being to her own identity as an effective teacher.

In a mid-year interview, Ms. M recognized how salient these resources for her identity were for her practice, but that the resulting traditional curriculum and instruction was not complementing her intended mindset work:

> It would be different if I had a different personality … when I'm given a job and I'm told here are these standards … and here is the tests and when the tests are going to be, I just-I just take that seriously … I could just say, 'who's going to fire me, who's going to stop me,' but I would have to deal with my test scores and I'm not—I'm attached to that success and I'm attached to setting goals and meeting them and being transparent with it and telling the kids, 'this is what we're doing.' … When my job matches what I believe to be best, I will be at more peace in my career. But the way it's been, it's not been like that at all. … the state test is still here. So as long as I have that – you know, we've charted our scores over time and, I just feel … [Interviewer: if it dropped] - what would that *mean*? That would hurt.

Interaction with the identity resources offered in this academic year context formed a particular identity trajectory for Ms. M; as a result of test pressures and without flexibility over the curricular content, Ms. M's professional identity as a successful teacher in this context was directly tied to her students' success on the CST. As Ms. M oriented to this context, this aspect of her identity became salient, leading to pedagogical decisions that aligned with the prioritized goals of that context.

In alignment with these identity resources, Ms. M's goal of getting students to score proficient on the CST was prioritized, and the pedagogical decisions leading to her teaching traditional content and instruction straight from the textbook at a fast pace directly served those high-priority goals.

Conclusion

As illustrated in comparison to the analysis in Chapter 2, Ms. M's teaching for a growth mindset took very different form in the academic year than it did in the summer. This chapter examined the differing sociocultural processes in two distinct classrooms that aimed to contribute to the development of a growth mindset. What it offered was a detailed analysis of four strands of pedagogy that can contribute to effectively teaching for a growth mindset: 1) Framing success 2) Treatment of malleable intelligence, 3) The mathematics, and 4) The agency and authority with which students are set up to work on the mathematics.

The second half of this chapter examined the conditions under which this pedagogy can fail to flourish. By providing restrictive professional identity resources for successful teaching, the school context in the academic year influenced Ms. M to prioritize goals that aligned with her professional identity. Accordingly, she made pedagogical decisions that aligned with those goals – namely, traditional content and instruction – that were incongruent with her growth mindset goals.

While many teachers are beginning the process of applying the research of Dweck (2006) to their classrooms, there is a lack of research on what it takes to effectively teach for a growth mindset. The analysis in this chapter reveals that framing success and directly teaching about growth mindset are only two pieces of the puzzle. As Chapter 4 will show, these are not enough to shift students' mindsets in productive ways. This chapter considered the multiple factors that came into play as Ms. M set out to teach for a growth mindset in the academic year and the ways she negotiated them so they were not conflicting for herself. What happened in this process resulted in direct instruction on growth mindset and an emphasis on working hard, coupled with skills-based procedural curriculum with test performance as the goal and few opportunities for students to develop authority in the classroom. In short, students did not have the opportunity to *experience* a growth mindset – and as Chapter 4 will show, this pedagogy was not met with the same productive outcomes for students as the summer pedagogy. Just as we do not expect students to be able to learn mathematics if they do not have a chance to practice, we cannot expect students to develop a growth mindset if we do not give them an opportunity to practice having one.

What this and the previous chapter reveal is that teaching for a growth mindset requires challenging mathematics *and* students' agency and authority to come up with their own ideas about the mathematics, so they can experience themselves as competent mathematical learners. The next chapter will examine the resulting shifts in student mindset, engagement, and evidence of learning that occurred to differing degrees across the two contexts in order to support the argument for the necessary elements to effectively teach for a growth mindset pedagogy in the classroom.

Chapter 4: The Effect of Growth Mindset Pedagogy on Middle School Algebra Students' Mindsets, Engagement, and Persistence Toward Learning

The differences in the pedagogical moves that played out in these two contexts are only noteworthy if they are influential for students. The potential of growth mindset pedagogy to transform student learning – particularly in Algebra classes positioned as gatekeepers that often lock certain students out – is significant for current equity conversations. This chapter analyzes whether this type of pedagogy can shift students' mindsets in ways that have meaningful consequences for student engagement and persistence with learning.

Learning is fundamentally related to both sociocultural process and cognitive processes. Thus, conceptions of classrooms should not only consider the ideas and skills that students learn, but also the kinds of dispositions toward mathematics that they are developing (Nasir & Cooks, 2009; Gresalfi & Cobb, 2006). Current research on growth mindset in classrooms argues that "just learning about the growth mindset can cause big shifts in the ways people think about themselves and their lives" (Dweck, 2006, p. 216). However, as shown in the four strands of analysis in Chapters 2 and 3, despite students *learning about growth mindset* in both contexts, the two distinct classrooms reveal drastically different opportunities for students' mindsets about intelligence, in ways that could affect their dispositions toward mathematics and persistence with challenge (Dweck, 1999).

This chapter will show that while the students in the summer course showed evidence of shifts in mindset that influenced both their engagement with rich mathematics and their persistence with learning, the students in the academic year had different outcomes. More specifically, in the academic year, while students could talk about growth mindset and its importance during interviews, student surveys revealed that their self-concepts as doers of mathematics did not significantly shift toward having a growth mindset over time. Accordingly student engagement with rich mathematics and student persistence toward learning, as measured through analysis of classroom videotape and student assessments, also did not increase over time.

As the findings in this chapter indicate, in order for growth mindset pedagogy to influence students' mindsets in meaningful ways, students need to not only *learn about growth mindset.* Students must also *learn to have a growth mindset,* which necessarily includes there being opportunities for students to have authority over their own learning and to experience a growth mindset while persisting through challenges. Only through these experiences will students learn to develop identities as lifelong learners capable of surmounting the learning challenges they face. The extent to which a student develops this identity has been shown to directly influence whether and to what extent s/he persists with challenging mathematics (Dweck, 1986).

Bailey Middle School– Summer Course

Student Mindsets

Analyses of post-summer student interviews provide evidence the notion that at the end of the five-week summer course, students had developed productive growth mindsets. In these interviews, no evidence was found of negative or unproductive student self-theory mindsets. Although only two students' descriptions will be used as examples here, these students did not stand out as anomalies in the class, but instead were indicative of the norm for students in the course.

The student interviews revealed that these students had developed a growth mindset in which they viewed intelligence not as a fixed trait, but as something that can be cultivated through learning (Dweck, 1999). As theorized by Dweck (1986, 1999), these students in turn developed a learning-orientation, with which they became comfortable with challenge. Rather than viewing failure as indicative of low ability, these students began to interpret struggles as a sign that more effort was needed (Dweck, 1999; Dweck & London, 2004). In sum, students had begun developing the full range of effects of a productive growth mindset. For example, consider the excerpts from one student, Tyrone's interview:

> Well it's just like, it's like sometimes I like the challenge 'cuz it's like really hard, but at the same time, you know, if you work really, if you work for it then, you know, you'll get it and I try to work very hard … I felt like even when I didn't get it, I still would feel like, "okay, well this is something I need to work on" and once I got it I would feel a lot better, like at the end of the day or whenever I got it. … I'm not a really big person on homework, but at the same time, you know, I like it when teachers sort of make it fun because you, like in a way, like I don't like it when they just give you homework and it's not like thinkable homework and you just do it and you already know. I like the homework where you give it and then you go home and you tryina figure it out because like you know the answer, but like something, like you missing a piece and you don't know what that piece is and until you get that piece right, you know … It didn't make me mad that I did all the work because when I did all the work it like, you know, it was like cool, because like it shows that I could like dedicate through it you know?

In this excerpt, Tyrone stated that he believed if he worked hard he would ultimately understand a problem, thus revealing his growth mindset and belief in malleable intelligence. Further, he explained that when he struggled, it was not indicative of his own shortcomings, but instead a sign to increase his effort, providing evidence for his learning-orientation and hardy response to failure. Finally, he said that he preferred homework that was challenging because then he could persist through it and learn in the end. With this growth mindset, he explained that he began to thrive on challenge.

Another student revealed a similar self-theory mindset. In her interview, she stated that she appreciated the strategies Ms. M shared with the class, as well as the challenging problems she assigned, "she showed us … new strategies, new ways to figure out math. Then at the same time, she was making it hard for us to do the math so we can learn from our mistakes and make better choices." In this excerpt, Keyonna said not only the fact that she appreciated these challenges, but also that she operated with a growth mindset and learning orientation (Dweck, 1999).

Later in her interview, Keyonna claimed, "I see myself, like I see how sometimes you guys are like 'wow she's really great at math and she's really uhh interested in learning this new stuff' and so from hearing you guys say that, that makes me more proud of myself to do better at math and get better and better." When questioned what she meant by "you guys," Keyonna listed the teacher and two researchers who had spent the most time in the classroom. The noteworthy aspect of this exclamation is that there is no evidence in the data of the teacher or of any researcher ever explicitly expressing to Keyonna that she was great at math. It is possible that Keyonna interpreted the researchers taking fieldnotes as a sign that she is great at math, but the more likely interpretation relies on Ms. M's instructional strategies. From this perspective, Keyonna's statement provides evidence that Ms. M's pedagogical moves sent a strong message to Keyonna about her mathematical abilities by providing several opportunities for her to experience herself as smart in mathematics. Accordingly, Keyonna developed a growth mindset with the goal to increase her knowledge by working hard.

Changes in Student Engagement with Rich Mathematics

Quantified coding comparisons of classroom activity structures as students worked on difficult non-routine problems at the start and end of the summer course revealed an increase in engagement with challenging mathematics. Analysis of student engagement at the beginning of the summer revealed that there was work to be done in building the classroom community that would function the way Ms. M wanted. In subsequent weeks, students were more fully engaged with rich mathematics in ways that supported their learning and required fewer teacher interventions.

To provide evidence of early disengagement, I first quantify a representative sample lesson from the beginning of the summer, and then describe some vignettes from this lesson to indicate the character of the challenge Ms. M faced at the beginning of the summer. I then quantify and characterize a representative sample lesson from the end of the summer to provide an indication of how students' participatory behavior changed as the teacher implemented her pedagogical strategies.

Disengagement on day 3, first week. On the third day of the course, students were asked to solve the following problem as a warm-up problem:

> *Maria subtracted two numbers and got 10. Allison multiplied the same two numbers and got 651. What were the two numbers?*

The problem was written on the board at the beginning of class, and, after the task was introduced, students were asked to work independently on the problem. The teacher then led students through a series of participation structures that included independent work time, team talk time, and whole group talk time. The problem and the participation structures of this class had affordances for students to engage meaningfully with rich mathematics.

Table 4 provides a summary that chunks the time the students and teacher worked on the problem based on the nature of the participation structures, with quantifications that characterize the nature of discourse for each chunk. Counts of discourse interactions that occurred in each of these participation structures provide evidence of student engagement with rich mathematics to the extent that increased discourse is indicative of increased student engagement (Cohen, 1996;

Erickson, 1982). However, to control for non-productive discourse, these counts have been separated by interaction type. For example, re-engagement prompts, while discourse, are indicative of students being off-task and thus, not engaging with rich mathematics, whereas mathematical interactions are indicative of increased student engagement with the rich mathematics that was offered. I will use Table 4 as a guide to describe the student engagement on this task.

Table 4
Quantifying Interactions on Third Day – Summer

Participation Structure	**Duration**	**Evidence of Engagement**	**Evidence of Disengagement**
Independent Work	6:00	--	Teacher 2 re-engagement prompts
Whole Group Talk	4:54	Students 2 students answer questions when called on	
Independent Work	4:25	Students 1 confirmation independent question Teacher 1 math interaction	Teacher 3 re-engagement prompts
Team Talk	3:06	Teacher 1 math interaction	Teacher 3 re-engagement prompts
Whole Group Talk	2:22	Students 1 student answers question when called on	Students Several students chatting about non-math topics 2 students make unrelated jokes 1 off-topic comment Teacher Questions remain unanswered or only answered with non-serious answers
Total	20:47		

First, when the problem was introduced, students did not ask any clarifying questions. Instead, many students did not immediately engage. For example, after four minutes, six out of ten students captured on camera were not actively working on the task. These students were either not writing on or looking at their paper, were staring into space, had their arms tucked inside their shirt, or were standing up. During the first six minutes, Ms. M made two attempts to re-engage students in the task. Specifically, Ms. M responded to two students who were not on-task by saying, “The good morning always tells you your warm-up” and “I don’t even see the problem written down.”

After six minutes, Ms. M attempted to gather the class’ attention to work on the problem as a whole group. During this time, only two students spoke, and only to answer a procedural question when they were randomly called on by Ms. M. During this time, no students asked clarifying questions or volunteered their ideas.

Students were then given time to work independently again. Two minutes into the independent work time, four out of the six students captured by the camera were not actively engaged with the mathematics. These students were either lying down, laughing aloud, or were talking about non-math topics with students in other groups. During this independent work time, only one student asked a question, but only to confirm whether or not his answer was correct. Most notably, Ms. M made three interactions to attempt to re-engage students in the mathematical task. For example, Ms. M approached a student who had been sitting staring into space, but had not yet written anything on his paper.

> Teacher: Did you get it?
>
> Student: No
>
> Teacher: Oh, then why'd you stop?
>
> Student: ‘cuz I was thinking.
>
> Teacher: Alright, so show your thinking on paper so I can actually see because I can't read your mind. I want to see your thinking.

Note that in her re-engagement prompt, Ms. M did not reprimand the student for being off-task, but instead provided an opportunity for him to re-engage with the mathematics, as if he had never been disengaged. This conversation had potential for becoming a mathematical conversation if the student had shared what he was thinking, but instead he simply shrugged at Ms. M’s request. Even though Ms. M made several re-engagement prompts, she was only able to have one math interaction with a student, where she spoke with the student about her strategy for solving the problem.

Following independent time, students were provided the opportunity to talk in their teams of three about their work on the problem. During this time, Ms. M performed three additional re-engagement prompts with students who were off-task. For example, when one group was talking about non-math topics, Ms. M said, “Hey, Marquan, please lead your team.” Later, when these same students were again off-task, she said to the group, “consistently, not just when I’m looking, but even when I’m not looking.” Throughout this team talk time, Ms. M had to remind students to be on task, and, as a result, was only able to have one mathematical interaction with students. In this mathematical interaction, a student shared what he was thinking with Ms. M and she reminded him of one of the constraints of the problem that he had disregarded.

The chunk of time where students’ disengagement is perhaps most salient occurred as Ms. M again attempted to bring the class back together to work on the problem as a whole group. Ms.

M first called on one student to provide the answer to the problem, then she guided the class through verifying the solution by subtracting the two numbers to get 10 and multiplying them to get 651. Throughout this entire chunk of whole group time, many students were having side conversations about non-math topics. After the answer has been provided, the following interaction ensued:

Teacher: Anybody who's not sure can check now. To make sure that this actually is the solution to the problem. Have we found the two numbers that Maria and Allison were thinking of?
Student A: Yes
Teacher: Now, Marquan, do you agree? *[no response – students are chatting amongst themselves]*
Teacher: Is it possible that there is another answer?
Students: Yes, no, a bunch, yes *[Students just shouting out random answers, but not engaging in conversation]*
Teacher: Do you think? Do you think that there are two other numbers that when subtracted will make 10 and when multiplied will make 651?
Student B: 650 + 1 *[student is bouncing in his seat]*
Teacher: Hmm
Student C: But if you do 650 minus 1, it's not gonna equal 10
Student D: Yes it is. *[class laughs]*
Teacher: Are we sure we know Maria and Allison's numbers?
Student E: *[Student says inaudible joke about "Maria" and the rest of the class laughs]*
Teacher: And by the way Maria and Allison are friends. I used friends' names for this problem. Two teacher friends. Well actually, Maria is in the room. Coincidentally, Ms. Sanchez happens to be here right now. I didn't expect that. It's a total coincidence, she just caught me using her name in a problem.
Student F: She's gonna sue you.
Teacher She's probably okay with it. Alright, so um, I actually am convinced that these are the two numbers, but I think it's an interesting thing for us to think about - is it possible for a different two numbers to add up to the same thing and also multiply to the same thing?
Student C: Could they be different? Or they have to be the same numbers?
Teacher: Exactly, they have to be the same numbers
Student C: We could flip it around. Oh, wait that wouldn't be the same
Teacher Oh, yeah, so, good. If we did that we would have to talk absolute value or something like that. Yeah, we could tweak it a little bit, some of you are thinking that. But I'm satisfied that their numbers are 31 and 21. What I'd like you to do now is please close your spiral, put your pencils down, shown me good posture and prepare for my announcements.

In this interaction, Ms. M asks several questions to try to get the class to determine if the answer they have found is the only possible solution, but her attempts remain unanswered or are answered with non-serious non-focused responses. For example, students simply shout out non-focused random answers to a yes or no question, and another student, while bouncing in his seat, shouts out, "650+1" as a non-serious solution. Not disregarding this student's contribution, Ms. M says "Hmm" which appears as an attempt to get more students to engage with this suggestion. One student takes up this suggestion and gives a mathematical reason why it will not work. However, another student makes a joke about this reason, claiming that 650 – 1 equals 10, which causes the rest of the class to laugh.

Ms. M again attempts to direct the conversation back to the math problem, but another student makes an inaudible joke about Maria, one of the names used in the problem, which again makes the class laugh. Ms. M re-appropriates the students' joke by telling the class that Maria is actually one of her friends, who happened to be in the class, thus not positioning the student's remark as opposition. In response to the teacher's re-appropriation, another student makes a comment that attempts to steer Ms. M off-topic, by saying, "she's gonna sue you." Instead, Ms. M again attempts to direct the conversation back to the mathematics, and only one student, the same student who engaged earlier, responds. Finally, after being unsuccessful at engaging the students in this mathematical conversation, Ms. M closes the task, and moves on.

These vignettes and quantifications of types of interaction during a task on the third day are representative of the first three days of the course and reveal the challenges that Ms. M faced in engaging the students in mathematics at the beginning of the summer. These illustrations reveal the particular challenge of getting students engaged in *any* schoolwork let alone rich mathematics that Ms. M faced – a struggle that is common to many low-achieving classrooms. Prior to being given this task, the students had not had any formal Algebra instruction, and consequently, some might argue that any evidence of student disengagement exists because this problem is beyond the students' ability. In what follows, I will contrast student engagement on this task with a task given during the third week, which was inarguably much further beyond their ability level.

Engagement on day 11, third week. On the eleventh day of the course, students were asked to solve the following problem as a warm-up:

> *Ten cards are dropped on the floor. Bruce* [the class stuffed tiger] *may have eaten none, one, ten, or any number of cards in between. How many different combinations are there for what Bruce ate?*

The warm-up problem was intended as a shortened version of a larger problem in order to emphasize the problem solving strategy of the week, solving an easier problem. The larger problem that this 10-card pick up was adapted from was the same basic problem but with an entire 52-card deck dropped on the floor and cards being picked up rather than eaten by the class tiger. The warm-up problem, Ms. M predicted, would take only about 20 minutes, but ended up lasting an hour and five minutes of that class period, and on into 46 minutes of the following class session. The 52-card problem was written on the board at the beginning of class, and after introducing the problem solving strategy of the week, solving and easier problem, Ms. M wrote the 10-card pick up problem on the board and asked students to write down the problem before working independently. The students' behavior on this day was typical and representative of their behavior during the latter half of the summer school session.

Since this problem solving session lasted significantly longer than the 22 minutes and 20 seconds of the day three session, I have selected representative chunks for each type of participation structure of similar length for quantifying the interaction types. Table 5 provides a summary of these chunks and the quantifications that characterize the nature of engagement for each chunk, analogous to Table 4. The chunks shown in Table 5, although they occurred chronologically relative to one another, also had other chunks between them that are not shown in order to do a cross-day comparison of similar lengths of each participation structure. I will again use this table as a guide to describe the student engagement on this task.

Table 5
Quantifying Interactions on 11th Day – Summer

Participation Structure	**Duration**	**Evidence of Engagement**	**Evidence of Disengagement**
Independent Work	10:46	Students 2 clarifying questions Teacher 2 math interactions	--
Team Talk	4:08	Students 1 clarifying question Teacher 3 math interactions	--
Whole Group Talk	2:06	Students 1 volunteered idea	--
Whole Group Talk	6:42	Students 2 students answer questions when called on 3 volunteered ideas Several students shout out and debate strategies	--
Total:	23:02		

As on the third day, Ms. M began by introducing the problem to the class, then asking students to work independently on the task. However, unlike the third day, as soon as students were set free to work independently, students asked clarifying questions. In particular, two students asked Ms. M what cards were included in the set of ten cards used in this task. Although inconsequential to the answer to the problem, students were engaged with determining which ten cards were being used – for example, whether there were four kings, four queens, and two jacks or whether the cards were all hearts. These two clarifying questions reveal that even when students felt confused, they were engaged enough to ask for help, rather than to turn toward off-task behavior, which may have been the case on the third day.

In answering these independent questions, Ms. M spoke loud enough so other students who may have a similar question could hear the answer and perhaps ask even further clarifying questions. Her response to these questions in this manner could serve to engage students who may also be stuck on which ten cards she meant, or could also be ignored by students who were working independently already. Thus, although Ms. M responded by speaking to the whole class, a majority of the students continued working independently. In fact, four minutes into the independent work time, all ten of the ten students on camera were actively working on the task, by writing in their notebooks.

In addition to these two clarifying questions, Ms. M had two in-depth mathematical interactions with two different students. The first interaction began with Ms. M pointing out to a student, Khalil, that he had not written anything on his paper. However, rather than this comment turning into a failed re-engagement prompt, Khalil responded by describing exactly what he was thinking about.

Teacher:	Khalil, you haven't written anything. What are you thinking?
Khalil:	Uh, I don't know.
Teacher:	Yeah. Just tell me, what are you thinking at this point?
Khalil:	I'm thinking of the cards order, I did one, two, three, four, five, six, seven, eight, nine, and then, jack. Then I crossed each other out from start to start and see (inaudible) 5 and 6, but I dunno.
Teacher:	So one possibility is 5 and 6. That's a possibility. What's another possibility?
Khalil:	7 and 1
Teacher:	Maybe. And you're trying to figure out how many combinations there are. You just named two of them.
Khalil:	Well how many [inaudible] Oooo

Rather than the conversation becoming about the students' behavior as it did in a similar interaction on day 3, the conversation became about the mathematics because Khalil shared what he was thinking, rather than shrugging his shoulders at Ms. M's initial comment. This interaction revealed that although Khalil may not have written anything, he was actively engaged in thinking about the problem.

The second mathematical interaction that occurred during this independent work time began in a similar way when Ms. M asked a student who had stopped writing on her paper what she was thinking. Again, rather than the conversation becoming about Kathy's behavior, Kathy responded by explaining to Ms. M exactly what she was thinking, and revealing that she was similarly confused by which cards were included in the task. Accordingly, the conversation became about the mathematics, and thus similarly revealed that although Kathy had stopped writing, she too was actively engaged in thinking about the problem. During this independent work time, Ms. M did not have to re-engage any students in the task.

After working independently, students were given the opportunity to share ideas and work together in their teams. During a segment of this team talk time similar in length to the third day, Ms. M again did not have any re-engagement prompts. As a result, she was able to engage in three deep mathematical interactions, in which students discussed their strategies and Ms. M asked probing questions to help the students think further about their strategies. Two of these interactions occurred in the same team as two students shared different ideas about how to approach the problem. The third interaction occurred in a separate team with a student sharing his organized list strategy and asking for help on what he could do to make it simpler. In addition to these three mathematical interactions, one student asked a clarifying question about which cards could be included in the set of ten, revealing that even when students were stuck, they were engaged enough to ask questions to propel themselves forward.

In the midst of students working in teams, Ms. M brought the class back together as a whole group to check in with everyone's progress, and to share teams' ideas with the larger group. In a part of this whole group interaction, one student voluntarily shared his strategy with the whole group, describing how it was different from the ideas Ms. M shared that she had observed students employing. The students were quiet and listening while he shared his idea and Ms. M reiterated it to the whole group. After this whole group episode, students were given time to work in teams before coming back to a whole group again.

The second whole group time episode in Table 5 occurred at the end of the period as Ms. M attempted to synthesize students' work on the problem, similar to the last whole group chunk analyzed from day three. In whole group talk, two students responded to a series of questions or prompts posed by Ms. M as they worked through the task as a whole group. Additionally, when students realized they would need to add the numbers from one to ten in order to find the solution, several students (if not every student) were shouting out their strategies and arguing with one another about the best way to count up the numbers quickly. After this initial debate, three students voluntarily shared their ideas for how to add the numbers from one to ten efficiently. In addition to these three students, several other students raised their hands to share their strategies, and Ms. M had to remind them, "If I don't call on you, don't hate. Congratulate." When a student revealed the final answer, again the class erupted with the roar: "I told you so!" Although there was shouting, as in the whole group interaction on the third day, this shouting was indicative of students being on task and engaged, as they were sharing their strategies and answers with one another. There were no off-task student comments in this whole group talk, nor did Ms. M have to redirect the conversation in order to engage the class in the math.

These vignettes and quantifications of types of interaction during a task on the 11th day are representative of the increased engagement with rich mathematics that was observed as the course progressed. Table 6 shows the counts of interaction types by each participation structure comparing the third day and the 11th day. By the 11th day, students had still not had any formal Algebra instruction. Additionally, this task was clearly more difficult than the one given on the third day, so much so that the class was unable to solve the task in the entire hour-long period even with the teacher's assistance. After this session, Ms. M even sat down with the ATS researchers to figure out what went wrong and how she could redirect students toward successful problem solving during the next session. Despite having had no formal training and being faced with a more difficult problem than on the third day, students remained engaged in the task, as is evidenced by a decrease in re-engagement prompts and off-topic student comments, and an increase in math interactions, student clarifying questions, and volunteered ideas (Table 6 and 7).

Table 6
Counts of Interaction Types by Each Participation Structure - Summer

		Disengagement		Engagement		
		Re-engagement prompt	Off-topic Student Comment	Math Interaction	Student Clarifying Question or Volunteered Idea	Response to Teacher Prompt
Independent Work	*Day 3*	5	-	1	0	-
	Day 11	0	-	2	2	-
Team Talk	*Day 3*	3	-	1	0	-
	Day 11	0	-	3	1	-
Whole Group Talk	*Day 3*	-	3	-	0	3
	Day 11		0		3	2

Table 7
Counts of Disengagement and Engagement Indicators by Day - Summer

	Disengagement Indicators	Engagement Indicators
Day 3	11	5
Day 11	0	13

Student Persistence Toward Learning

Any study of student engagement must also examine evidence of student persistence toward learning, to document the effectiveness of this engagement. The extent to which students engage in mathematical practices influences what they learn as a result of that engagement (Gresalfi & Cobb, 2006; Schoenfeld, 1992). I use the phrase "persistence toward learning" to acknowledge that any assessment results are mediated by the effort and persistence with which students engage with the assessment; therefore, what students show on an assessment is as much evidence of their willingness to persist toward learning as it is their actual learning over time.

Student's self-concept beliefs and mindset can directly influence the effort students put forth or how long they persevere in the face of difficulty (Dweck, 2006), and thus, the extent to which they persist toward learning. Simply by an increase in the number of attempted problems, the student assessment data reveals an increase in student persistence toward learning.

Results from the pre- and post-summer assessments reveal an increase in student scores. Recall that the assessment included 3 open-ended multi-question non-routine algebraically rich tasks assessing a variety of algebraic skills, none of which were directly taught over the course of the five-week summer program. Before comparing students' pre- and post-test scores, a two-tailed t-test for unequal variances was run to show the equivalence between the two pre-test versions. The overall mean score for version a was 2.57 with a standard deviation of 2.64, and the overall mean score for version b was 2 with a standard deviation of 1.55; there was not a statistically significant difference between the two versions ($t(11)= 0.46$, $p=0.65$). As a result of these statistical tests and because the two versions were counter-balanced from pre-assessment to post-assessment, with the same amount of students taking each version, the two versions were treated together as one assessment for purposes of pre-post comparisons.

A paired t-test was run to analyze any class changes from pre- to post-summer assessment based solely on raw student score. For all t-tests all students who did not complete both the pre-test *and* the post-test were removed. A paired one-tailed t-test showed a statistically significant ($t(12)=1.90$, $p<0.05$) increase in student scores from pre-test to post-test. These findings are notable particularly because students showed a statistically significant increase in content learning despite none of the content on the assessment being directly taught. Cohen (1996) argues that increased student engagement results in increased classroom and individual discourse, which subsequently leads to more learning, especially for low-achieving students. The data presented in the section above reveal that over the course of the summer session, students became more engaged as more productive classroom discourse occurred. These findings provide one explanation for the increase in student learning.

An alternative explanation is that the increase in student engagement with rich mathematics contributed to greater persistence on the post-test itself. To test this hypothesis, a paired t-test was run to analyze changes in the number of problems students attempted from pre- to post- summer assessment, as measured by *any* work shown on a problem. A paired one-tailed t-test also showed a statistically significant increase in the number of problems students attempted on the assessment ($t(12)= 2.058$, $p<0.05$).

In addition to measuring correctness and attempted problems, students' assessments were scored along five Robustness Criteria (RC) shown below using a standardized rubric and were compared from pre- to post-assessment. These robustness criteria were developed as part the Algebra Teaching Study TRU Math project and established to represent student abilities that are believed to allow students to successfully solve algebraically rich tasks, as supported by the literature (Schoenfeld et al., 2014a). Analysis along these strands was chosen to account for the fact that the assessment on the exam was not specifically taught in the summer course.

Robustness Criteria (RCs)
A. **Navigating Language**: Students are able to navigate the language in a problem statement to make sense of the problem situation. B. **Identifying Relevant Quantities and Relationships**: Students are able to identify which quantities are relevant to the problem situations and are able to articulate the mathematical relationships between quantities. C. **Representing Quantitative Relationships**: Students are able to generate appropriate mathematical representations and are able to interpret and make connections between representations. D. **Executing Procedures and Checking Solutions**: Students are able to execute algebraic procedures and arithmetic calculations and check the plausibility of their results by attending to the problem context and considering their solution methods. E. **Explaining and Justifying Reasoning**: Students are able to clearly and thoroughly explain and justify their reasoning.

A paired t-test was run to analyze any class changes in robustness criteria from pre- and post-summer assessment. The overall mean across all five RC measurements was 4.77 with a standard deviation of 3.63 on the pre-test and was 7.85 with a standard deviation of 6.49 on the post-test. A paired t-test showed that this increase was not statistically significant ($t(12)=1.61$, $p=0.07$).

In addition to a paired comparison of students' overall scores across all five of these RC measurements, a paired t-test was run for each measurement, as shown in Table 8. These tests revealed that the statistically significant gains occurred within RC 3, students are able to generate appropriate mathematical representations and are able to interpret and make connections between representations, and RC 5, students are able to clearly and thoroughly explain and justify their reasoning. Although this change in RC 3 suggests a potential improvement in students' content knowledge of representations, the increase in RC 5 might also suggest that these gains are a result of changes in the way students approached the assessment through changes in mindset and engagement. Regardless, these findings from the pre- and post-summer assessment reveal that despite only five weeks of instruction and no direct instruction on the content assessed, the class as a whole showed evidence of increased persistence toward mathematical learning.

Table 8
Significant Test Results Across Robustness Criteria

	Pre-test Mean (SD)	Post-test Mean (SD)	Paired T-test
Overall:	**4.77 (3.63)**	**7.85 (6.49)**	***t*(12)=1.61, p=0.066**
RC 1: Students are able to navigate the language in a problem statement to make sense of the problem situation.	1.77(1.01)	1.85(1.46)	*t*(12)= 1.18, p=0.870
RC 2a: Students are able to identify which quantities are relevant to the problem situations and are able to articulate the mathematical relationships between quantities.	0.77(1.01)	1.11(1.47)	*t*(12)=0.26, p=0.401
RC 3: Students are able to generate appropriate mathematical representations, and interpret and make connections between representations.	1.15 (1.21)	2.08 (1.85)	*t*(12)=1.91, p<0.05
RC 4: Students are able to execute algebraic procedures and arithmetic calculations, and check the plausibility of their results by attending to the problem context and considering their solution methods.	.54 (0.66)	1.5 (1.92)	*t*(12)=1.53, p=0.076
RC 5: Students are able to clearly and thoroughly explain and justify their reasoning.	0.54 (0.97)	1.31 (1.32)	*t*(12)=1.81, p<0.05

Frost Middle School– Academic Year Algebra Course

Student Mindsets

Analyses of post-academic year student interviews revealed that many of these students in the academic year were able to talk about the importance of working hard and struggling. However, these statements were also accompanied by an emphasis on right answers or a concession that the student was not actually working his hardest. When these interviewes are accompanied by the pre- post- student surveys that used real world prompts to measure students' mathematical self-concept, it becomes clear that while students may have been able to talk about the importance of a growth mindset, their mindsets and self-concepts did not actually shift to become more productive over the course of the year.

The student interviews revealed that these students could, for the most part, recognize and talk about the importance of particular features of having and working toward a growth mindset. Below are illustrative quotes from each of the four academic year students that support the notion that all four students knew at least some of the important aspects relating to developing a growth mindset and could articulate these:

Tevin: *Well, you have to be hardworking, um, ask questions and um – to be good at math—well, I just think it's just to be hard working and to be—to maybe figure out the question sometimes and maybe not able to, but—but you'll learn the next time you try.*

Keith: *The most important thing in her class? Uh, struggling. She likes it when kids struggle … 'Cause she feels that we're learning* [Interviewer: and what does struggling have to do with that?] *'Cause, like it—struggling—I dunno, she just likes it when we struggle. That's—that's just her I guess.*

Brendon: *I've learned a lot in Ms. M's Algebra class and it has impacted me a lot in a good way and has me thinking and stuff, 'cause like, through her quote, it says, "If you aren't confused, you aren't learning." So you know, if I aren't confused, I'm not learning.*

Amanisha: *To be good at math, I feel like its learning and understanding the work … If they put in the time and effort to like really try and learn and understand what they're doing then—yeah.*

One student, Tevin, explicitly stated that he knew what a growth mindset was and the importance of having one, but he admitted that he had not been working his hardest in this class, providing evidence that he had not actually developed a growth mindset in ways that would influence his behavior. Even though the questions were specifically tailored to address growth mindset, Tevin was the only student who talked about the idea of having a particular "mindset," but even so, he did not specifically mention "growth mindset." After confessing that he was not working his hardest in this class, he was asked what he might change. He responded, "I would change my my my my work ethic. Um, I know I can do better. Like all these teachers say I can do better, and yeah, I get that but, I, I know I have to change my mindset and my work ethic to become better." In this statement, Tevin articulates that while he understands the importance of hard work (as evidenced by his statement above), he has not actually fully developed a growth mindset. Instead, he explains, "I don't have a mindset that I can't do the math, I uh—I have a it in between where I can do some math and some math I can't do and sometimes I just don't do." Tevin's interview most clearly articulates the tension that students in Ms. M's class may have been dealing with—knowing the importance of working hard on learning, but not knowing how to shift their mindsets in ways that would support a productive shift in behavior.

Furthermore, despite understanding the importance of confusion and struggle, Keith and Brendon, still hung onto performance-based goals as indicators of success. Specifically, along with effort, both students also cited knowing how to do a problem and getting the right answer as evidence that a person is good at math. When asked how a person knows if they are good at math, Keith said, "that they're getting answers right and they're getting it, and they're trying." This statement also points to the tension between *performance* and *effort* that students in Ms. M's class felt. When asked whether he thought he was good at math and why he thinks that, he said, "I'm okay. 'Cause sometimes I get it and sometimes I don't." This expression points out that while he may understand the importance of trying, ultimately the measure of success is based on quick and accurate performance. Finally, when asked how he became okay at math, he said, "Ms. M … she taught me, I guess." Rather than focusing on the influence of working hard on learning, Keith saw any learning as a direct result of the teacher.

Like Keith, Brendan also articulated the tension between performance and *effort* when asked how a person knows whether they're good at math. Brendan responded, "By just looking at the equation and thinking in their mind 'I know how to do that!' and then just giving some of their time to solve it." When asked whether he thought he was good at math and why, he also articulated, "Partly … Yeah, um, I just—by just me looking at the equation and knowing I know how to do it, but some equations I'm not all good at." Like Keith, this expression shows that while Brendan may understand the importance of putting in effort over time, the measure of success is likewise based on having ready-made solution processes when coming across a problem. In both Keith and Brendan's interviews, they spent nearly equal time talking about knowing how to do the problems correctly and more learning-oriented processes like working hard and struggle.

Only one of the four students, Amanisha – the student chosen for interview by Ms. M based on her perception of the student's productive mindset – seemed to articulate clearly the importance of hard work and effort toward learning. When asked how a person knows whether they're good at math, Amanisha explained, "If they put in the time and effort to like really try and learn and understand what they're doing." As described above, this statement provides evidence that Amanisha saw *learning* and hard work as the measure of one's success. When asked what she would do if she faced a problem that she didn't know how to solve, she explained her process for increasing her effort, "I would probably come to tutoring, or I'd ask somebody else in my math class or someone in advanced math and then come ask Ms. M." Later, when asked what Ms. M would say if she was working on a problem and got really frustrated and gave up, Amanisha said, "I feel like—a lot of times like I can't—I get—sometimes I get really frustrated when I can't figure something out, but I know that if I really keep trying to think about it, I will eventually figure it out." Like the students in the summer, Amanisha did not view struggle as indicative of low ability, but simply as a sign that she needed to increase her effort (Dweck, 1999; Dweck & London, 2004). Of the four students interviewed at the end of the year, Amanisha was the only one who seemed to fully develop a growth mindset in ways that may influence her behavior. However, as will be described later in this chapter, her engagement and persistence with challenge – as evidenced by the number of problems she attempted on the pre assessment to the post assessment – stayed the same over time. On both the pre-assessment and the post-assessment, Amanisha attempted 10/12 problems. This may suggest that Amanisha's mindset and self-concept was productive already before the start of her Algebra class with Ms. M.

While these interviews suggest that most students were able to articulate the importance of particular learning-orientated features, the pre- and post- student surveys that used real world prompts to measure students' mathematical self-concepts suggest that their mindsets and self-concepts did not actually shift to become more productive over the course of the year in ways that would influence their behavior.

An examination of the development of students' conceptions of their mathematical ability must begin by defining the construct being analyzed. The construct being measured, *mathematical self-concept*, attempts to define people's views of themselves as doers of mathematics as a dialectical relationship between an individual's global assessment rooted in sociohistorical experiences and the specific context in which the assessment is made. Therefore, *Mathematical Self-Concept* is defined as people's comprehensive view of themselves as doers of mathematics. More specifically, their beliefs of a) their ability to participate in a mathematical learning community, b) their ability to understand math and math problems, c) their ability to solve challenging math problems, d) their ease or difficulty of doing and learning math, and e) the malleability of their intelligence through hard work. Accordingly, for the purposes of the framework developed here, this paper takes students' mathematical self-concepts (MSC) as depending on their *mathematical* self-assessment along two interrelated cultural strands: 1) Global MSC: student's internalized history-in-person, or general self-assessment that stems from sociohistoric processes as they experience themselves as doers of mathematics ontogenetically over time (Holland et al., 2001), and 2) Contextual MSC: student's situated self-assessment within a *given* class or task context.

The student surveys then set to measure student's mathematical self-concepts with respect to mathematics by using real-world examples to ask students about their behaviors in particular general and contextualized examples. Likert-scale responses to items fell along a continuum from unconstructive to constructive self-concepts. As the distance between the categories was the same, I mapped the ordinal Likert responses to interval scores so I could run a t-test to check for changes in student's mathematical self-concept for each item on the survey. I ran the analyses for the entire class for each survey item, and to account for bias from different students, I also ran paired analyses only for students who completed both the pre-survey and the post-survey.

For the first ten items on the survey that related to global MSC, none of the items showed a statistically significant productive change in students' global mathematical self-concept or mindset for both the class aggregate and the matched subset. For the rest of the items related to contextual MSC, only two out of 18 items showed a statistically significant productive change in students' contextual mathematical self-concept for the paired subset. Both items asked "How much do you agree with the following statements about yourself and math?" The two statements that showed productive shifts were "Math problems can have more than one right answer" ($t(6)= 2.291, p<0.05$) and "Some people are good at math and others will never be no matter how hard they try." ($t(7)= 2.049, p<0.05$). Both questions had likert scale response options from strongly agree to strongly disagree. In the larger aggregate comparison between the independent class samples, only the effect for the first of these two items was statistically significant ($t(31)= 3.175, p<0.01$). The second item is the only item from the survey that seemed to suggest that students' understanding of the tenets of a growth mindset became stronger over time. However none of the items about students' behavior or their self-concepts supported a shift toward the development of a growth mindset in ways that influenced their perceptions of self and engagement.

One additional item showed a statistically significant shift in the other direction for the paired subset. The pre-test question asked, "If your scores on a math test in your class last year were compared to the scores of your classmates, where do you think you would be?" with categorical answers ranging from "in the lowest scores" to "in the top scores." The post-test question asked, "If your scores on a math test in this class were compared to the scores of your classmates, where do you think you would be" with the same answer options. There was a significant shift in the post survey ($t(7)= 2.049, p<0.05$) with students on the post survey reporting lower responses than in the pre survey. This effect could simply be explained by the fact that Algebra 1 is a more difficult course than 7th grade math and not because of a shift in students' perceptions of their own mathematical ability. This effect was not perceptible in the aggregate class comparison.

In addition to item-by-item analysis, I put each student's responses to the 28 survey questions about their mathematical self-concepts together to create a self-concept score, and used a one-tailed paired t-test to compare mean scores. The analysis showed that there was not a statistically significant difference in student's mathematical self-concepts from pre survey to post survey ($t(7)= 0.2588$, $p=.4016$). There was also not a statistically significant different in students mathematical self-concepts when comparing students' self-concept scores along the aggregate independent samples $t(31)= 0.5035$, $p=.3091$). While these findings should not be over-interpreted given this is a new survey with a small sample size, the fact that statistically reliable effects between pre- and post- survey were not found is consistent with the argument that students' mathematical self-concepts and mindsets did not become more productive from the beginning to the end of the academic year.

Changes in Student Engagement with Rich Mathematics

Analysis of student engagement from the beginning to the end of the academic year did not show the same increase in engagement with rich mathematics that occurred in the summer course. To compare shifts in student engagement from the beginning to the end of the year, I conducted coding comparisons of student and teacher interactions in typical lessons from the beginning and end of the year.

In the analysis of these lessons, I first quantify a representative lesson from the beginning of the year, describing some vignettes from this lesson to characterize the sense of student engagement from the onset of the academic year. I will then quantify and characterize a representative sample lesson from the end of the summer, comparing the nature of student engagement from that lesson to the beginning of the year.

To give the greatest chance of seeing a shift in student engagement, a lesson with typical course structure but content that could potentially provide opportunities for students to engage with rich mathematics from the end of the year was chosen to compare with a representative lesson from the first week of school. Even so, this word-problem based lesson still took place with the teacher at the board modeling mathematics problems with student participation limited to short initiation-response-evaluation sequences.

While the academic year did not see significant shifts in student engagement, these results could simply be due to the fact that, as shown in Chapter 3, students did not have opportunities to work on challenging non-routine problems. As a result of the limitations in lesson structure, the analysis of the two lessons occurred on one interaction type: whole group talk.

Student engagement on August 5th, day 5, first week. During the academic year, the lessons were not structured around an overarching problem of the day. On the fifth day of school, Ms. M was still spending some of class time setting up the structures for the class, and on this particular day, the students began class by taking a "supply quiz" in which they got credit for having each of the supplies required on their syllabus (e.g. ruler, red pens, and a pencil). Immediately prior to beginning the lesson, Ms. M passed out notebooks and gave explicit instructions on how to fill out their notes in their binders. In these instructions, she told students,

> I want you to have the self-discipline to make yourself write but also be thinking at the same time about what you're writing, otherwise you wont be getting smarter, you'll just have some notes written down. I want you to actually understand it and write it, both, and that's a challenge.

Despite these instructions in which Ms. M aimed for students to think meaningfully about the mathematics, students were ultimately only held accountable to writing out what she wrote on the board. To match the chunk of lesson analyzed in the summer data, the 12-minute chunk of this course was selected based on the lesson's alignment toward content goals (in other words, processes around taking notes or a supply quiz are not aligned with content goals and were thus not included).

The lesson for this day involved defining a variable as "a letter that represents a fixed or changing number," translating basic English sentences into abstract algebraic expressions for each of the four operations (addition, subtraction, multiplication, and division), and then working through the following example problem:

Evaluate the expression for the replacement set {2, 4, 5.7}

1) $x + 8$

The problem was written on the board and Ms. M worked through the example, asking only for student input in response to quick, answer-based questions that followed an IRE sequence (Mehan, 1979). The process of direct instruction followed by one to three worked examples with IRE sequences was a typical structure of the lessons in the beginning of the academic year.

Since the participation structure of this representative lesson only took the form of Whole Group Talk, Table 9 provides a summary that instead chunks the time the students and the teacher worked on this lesson by the three lesson goals, with quantifications that characterized the nature of discourse for each chunk. Unlike in the summer analysis, however, counts of discourse interactions do not directly provide evidence of student engagement with rich mathematics because the mathematics and pedagogy in this academic year lesson did not inherently provide opportunities for meaningful engagement. Therefore, the analysis of the academic year lesson examines evidence of on-task discourse, meaningful engagement with rich mathematics, and disengagement. This analysis will look at the extent to which students were given opportunities to engage meaningfully with rich mathematics, the extent to which they then took up any such opportunities, and the extent to which there was evidence of disengagement.

Table 9

Quantifying Interactions on Fifth Day – Academic Year

Whole Group Talk Goal	Duration	On-Task Discourse	Engagement with Rich Mathematics	Evidence of Disengagement
Defining a variable	3:29	Students 1 idea in response to teacher question 1 whole-class choral response	Students 1 clarifying question	--
Translating English into algebraic expressions	6:08	Students 10 responses to short answers to direct questions 1 whole class choral response	Students 1 clarifying question	--
Worked example of evaluating a replacement set	2:39	Students 1 idea in response to teacher question 7 choral response with one number answers 1 clarifying question about what is written	--	--
Total	12:16			

When beginning the direct instruction, Ms. M wrote the definition of a variable on the board and asked students to copy it down. She then provided one opportunity for students to respond with their own ideas by asking, “So a fixed number, what do you think it is?” However, because the question was constrained to the text of the definition versus asking for a novel mathematical idea, this opportunity is not counted as one in which students could engage meaningfully with rich mathematics, and accordingly, the student’s response. “it’s a number that’s already been placed in the answer” is coded simply as on-task engagement discourse. When Ms. M provided an example of a constant, 12, one student asks a clarifying question, “I don’t understand why … [inaudible]” and the teacher responds quickly by saying, “12 can only be 12. 12 can only equal 12. It's constantly 12, it's constant." This student's question reveals that she was attempting to engage with the mathematics in a way that foregrounded understanding, and as such, it is coded as an example of engaging with rich mathematics, albeit a weak example.

When the lesson switched to the next goal of translating English sentences such as "the sum of a and b" into algebraic expressions, the interactions looked similar. Ten students responded with short one to two word answers in response to direct questions such as "how do I write times without the word times?" These interactions modeled IRE sequences in which Ms. M initiated a question, a student responded quickly, and Ms. M evaluated the response. Accordingly, there were 11 teacher affirmation statements such as "thank you," "uh huh," and "product is correct" in this lesson chunk. Again, a student asked a clarifying question, "Um can you put …[inaudible]" but this time is cut off by the teacher's response, "I think that's actually what Sean was thinking too – Z divided by 2. This is the same thing except we're using a diagonal fraction bar instead of a horizontal fraction bar." For a similar reason, this question was coded as engagement with rich mathematics because the student attempted to offer up her own idea via the question, even though the opportunity was not pursued.

In the final lesson goal in which the teacher worked through the quick example given above, the interactions again looked similar to the previous two lesson chunks. The following interaction occurs:

Teacher: I think you have seen this before, but I want to remind you. We want to evaluate this expression [*points at x+8*] for this [*points at {2, 4, 5.7}*] replacement set. Raise your hand if you think you know what they are talking about with all the fancy language. Two people think they know what is going on here? Three, four, five?

Teacher: Alright. Mikali, what do you think they want here?

Mikali: You have to replace it each time by one of those numbers in the replacement set numbers.

Teacher: What am I going to replace these with...

Mikali x

Teacher: Yes! I am going to replace x with these values. Evaluate means find the value. I am going to find the value of this expression when I replace x with these 3 numbers. So this has three parts. [*Draws a tree diagram from the x+8 equation on the board with three branches*].

Teacher: When I substitute, I always use parentheses. So instead of x+8, I am going to have ()+8. [*repeats this 2x more while writing it under each branch of the tree diagram*]. Joe?

Joe: The last one is 5.7?

Teacher: Correct. 5.7, good. So not, we are going to replace the first x with...

Students: 2

Teacher: 2 and evaluate that. 2+8?

Students: 10

Teacher: Okay. Replace the second x with...

Students: 4

Teacher: And evaluate that.

Students: 12
Teacher: And replace the last x with...
Students: 5.7
Teacher: ... and evaluate that.
Students: 13.7
Teacher: 13.7. You will see some problems like that in your homework and now you know what it is to evaluate an expression for a replacement set. It sounds so fancy, I know. Alright so would you please now close the notes, lock them in your binder, and produce your homework calendar.

This interaction primarily consisted of a series of IRE sequences, each of which lasted a few seconds. Only one student had the opportunity to offer an idea that was not a brief one-word answer, but this was still only about the directions and procedure and not about rich mathematics. Note that there were no instances of disengagement indicators in the entire lesson. These data might suggest that Ms. M did not have the same work to be done to get the class to function in the way she wanted as she did in the summer course. However, there were also not opportunities for students to engage meaningfully with rich mathematics. The lesson was so constrained that there were not opportunities for students to disengage. Throughout the entire lesson on the fourth day, students were not given opportunities to come up with their own mathematical ideas or provide mathematical explanations or justifications, and there were only two opportunities where students had the opportunity to share ideas beyond short right/wrong answers. However, these ideas were still constrained to short sentences, were in response to the directions or text of the lesson, and were not taken up in a meaningful way by the teacher. On the whole, students were not engaged as sense-makers, problem solvers or creators of mathematical ideas, and instead were constrained by what the teacher or worksheet said and did (Baldinger & Louie, 2014).

Student engagement on March 19th, second semester. A lesson from the eighth month of school, March 18th, was chosen for comparison as a representative lesson from the second half of the year. This lesson was chosen because it represented a structure that was typical to Ms. M's everyday teaching, but involved content regarding word problems that could potentially provide more opportunities for students to engage with rich mathematics. As was typical in her academic year class, this class period began with students working on some review-based warm-up problems independently and silently while Ms. M walked around stamping students' homework. After eight and a half minutes, students were allowed to team talk about their warm-ups, while Ms. M finished stamping students' homework for another five minutes. She then went over the warm-up problems with student participation limited to short IRE sequences for seven minutes. Now 20 minutes into the class, she spent the next 12 minutes giving announcements (including reiterating the importance of the quarter three district test as a competition against the other middle schools).

The lesson segment analyzed for comparison here began 30 minutes into the class session and was similarly selected to compare to the beginning of the year based on the segment's alignment toward content-based goals. Since, as was the case in the beginning of the year, the participation structure of this representative lesson only took the form of Whole Group Talk (with the exception of 15-seconds of team talk), Table 10 provides a summary that chunks the time the students and the teacher worked on this lesson into four episodes based on their goals. First, Ms. M began the lesson with direct instruction on defining "working together problems," moved into 15 seconds of team talk, discussed student ideas for solving "working together problems", and finally, worked through an example of this problem type.

Table 10
Quantifying Interactions on March 19th – Academic Year

Whole Group Talk Goal	Duration	On-Task Discourse	Engagement with Rich Mathematics	Evidence of Disengagement
Defining "Working together problems"	1:29	--	--	Students 2 off-topic comments Teacher 1 card move
Team talk	0:15	--	--	Students 1 off-topic comment Teacher 1 re-engagement prompt
Student ideas for solving	4:29	Students 2 whole class choral responses	Students 3 ideas surfaced	Students 1 re-engagement prompt 1 off-topic comment Teacher 4 re-engagement prompts
Worked example of "working together problem"	5:58	Students 5 responses to short answers to direct questions 6 whole class choral responses	--	Students 1 loud yawn interruption
Total	12:11			

As she did in the beginning of the year lesson and most lessons throughout the year, Ms. M began the direct instruction by providing a definition – in this case, a working together problem as a problem where two people or two things are working together at the same time. She shares the big idea that when people or machines work together, the job gets done faster. Ms. M goes on to provide an example, interrupting the student chatter to remind students to listen carefully. She provides the following example to support the definition:

It takes me two hours to do the dishes. We're talking like a restaurant situation. It takes me two hours to do the dishes and it takes Amira an hour and a half to do the same amount. Silently think – how long will it take us to work together?

While she shares the example, two students interrupt her in the middle with non-math distractive comments saying "Daaaaang!" and "Not bad!" Meanwhile, several students mumble side comments while Ms. M is talking. As a result of the off-topic chatter, Ms. M moves a card on the card chart as a warning for off-task behavior.

She then asks students to team talk about how long the job will take if they both work together, but only provides 15 seconds of team talk time. In those 15 seconds, one student tries to get Ms. M to have a conversation about something else and Ms. M tries to re-engage him by saying, "not now, I have a new topic." This interaction ends the team talk.

In the third chunk of the lesson, student ideas are surfaced. Ms. M allows three students to provide their theories on how to figure out how long the job will take if they both work together. Before the first student can talk, another student tries to get the class to quiet down by saying "Hey, everybody pay attention – shhhh!" One student suggests finding the difference between their two times, another suggests finding the average, and a third suggests estimating. The only one of these ideas Ms. M engages beyond just accepting the idea is the second one, to which Ms. M responds, "So your theory would be it would take us an hour and 45 minutes to do the job, but she could do it by herself in an hour and a half. So, if I'm helping, it shouldn't slow her down." During this interaction in which students provide their rich mathematical ideas, many students have their heads down, are chattering amongst themselves, or even make a joke by saying "maybe, you like, slow her down!" and laughing. At the end of this chunk, Ms. M tries to explain that speed is what you do in how much time, but many students are talking over her. She ends this chunk by turning to a student and saying, "Dahlia, check yourself. Stop trying to clown around."

Finally, in the last chunk of the lesson, Ms. M walks students through an example of a worked example problem on a worksheet she passed out, limiting students' participation to IRE sequences. In this interaction, five students answer direct questions with short one-word responses and there are six instances in which several students chorally shout out the short responses to direct questions. Also in this interaction, two out of the ten students on camera have their heads down and a third student yawns very loudly.

This vignette from the second half of the class was representative of the engagement observed as the course progressed, except that, if anything, it provided more opportunities for students to engage meaningfully with rich mathematics simply because it involved content that was more open-ended. Despite these opportunities for student ideas to surface, only a few students actually engaged in rich mathematics for a short chunk of time. However, there are also significantly more students disengaging in the lesson than in the beginning of the year. On the whole, the entire class was not supported in engaging meaningfully with rich mathematics, and when students were off-task, Ms. M constrained the lesson again so there were not as many opportunities for students to disengage. Table 11 shows the accounts by day of the extent to which students were given opportunities to engage meaningfully with rich mathematics, the extent to which they then took up any such opportunities, and the extent to which there was evidence of disengagement. As Table 11 indicates, even in a lesson where there may have been more potential for students to engage with *rich* mathematics, this did not occur, and, in fact, there was more disengagement with the lesson as a whole toward the end of the year. While these are just two example lessons, what they indicate is that there was not a dramatic shift in students' engagement with rich mathematics as there was in the summer course.

Table 11
Counts of Discourse Type by Day – Academic Year

	On-Task Discourse	Engagement with Rich Mathematics	Evidence of Disengagement
August 5th	22	2	0
March 19th	13	3	12

Student Persistence Toward Learning

Implementing pedagogy that shifts student engagement in more or less productive ways is consegential for students' persistence toward learning. In the summer, the growth mindset pedagogy shifted students' mindsets in ways that increased student engagement and persistence toward learning. The academic year pedagogy was documented to be different, providing fewer opportunities for students to *experience* a growth mindset and to feel authority over their own learning. Despite the instruction about growth mindset, the academic year was not met with the same shifts in student mindset and engagement. Accordingly, the student assessment data from the academic year reveals neither an increase in student learning nor persistence toward learning.

Recall that the assessment in the academic year was developed in collaboration with the Ms. M based on her content goals, and each problem type was directly taught during the academic year. Unlike in the summer course, results from the pre- and post- academic year assessments did not show an increase in student scores over time. A paired t-test was run to analyze any class changes from pre- to post-summer assessment based solely on raw student score. As in the summer, for all t-tests all students who did not complete both the pre-test *and* the post-test were removed. A paired one-tailed t-test did not show a statistically significant ($t(13)=1.69$, $p>0.05$) increase in student scores from pre-test to post-test. These findings are notable despite all of the content on the test being directly taught.

An alternative explanation is that the lower student engagement contributed to lower persistence toward learning during the course, reflected by lower persistence on the post-test. To test this hypothesis, a paired t-test was run to analyze changes in the number of problems students attempted from pre- to post- academic year assessment, as measured by *any* work shown on a problem. A paired one-tailed t-test also failed to showed a statistically significant increase in the number of problems students attempted on the assessment ($t(13)= 0.43$, $p>0.05$). In fact, the mean number of problems students attempted actually decreased from the beginning to the end of the year, though this decrease was not statistically significant. These findings support the claim that Ms. M's pedagogy in the academic year did not contribute to shifts in student mindset in ways that would influence their engagement and persistence toward learning.

While these findings should not be over-interpreted given that the assessment in the academic year differed from the one in the summer, the fact that statistically reliable effects were found on the assessment on which students were not taught any of the content and no effects were found on the assessment where students *were* directly taught all of the content on the assessment is consistent with the changes in Ms. M's growth mindset pedagogy between the two contexts.

Conclusion

As documented in the comparison of pedagogy of the summer to that of the academic year in Chapters 2 and 3, Ms. M's growth mindset pedagogy looked very different in the academic year than in the summer. In particular, the two classrooms provided drastically different opportunities for students to develop growth mindsets. These differences, however, are only meaningful if they had real consequences for students. The results in this chapter showed that while students in the summer course showed evidence of shifts in mindset that influenced their engagement with rich mathematics and their persistence toward learning, the students in the academic year did not show evidence of changes in mindset, increases in engagement with rich mathematics, or persistence toward learning.

In the summer course, the student interviews showed evidence that students had developed productive mindsets. The students in the summer course became more fully engaged with rich mathematics in ways that supported their learning over the course of the short summer session. These shifts in mindset and changes in engagement with rich mathematics were also met with increases in student scores on the pre- post- assessments, as well as increases in their persistence on the challenging assessment, as evidenced by the number of problems they attempted. These changes in student persistence toward learning occurred even despite none of the content assessed being directly taught during the summer. Contrastingly, in the academic year, student interviews and mindset surveys did not show the same productive shift in student mindsets. Additionally, the students did not become more fully engaged with rich mathematics over the course of the academic year – a fact that can be attributed in part to students' fewer opportunities to engage with rich mathematics because of the nature of the mathematics. Accordingly, student assessments did not show an increase in either score or persistence, despite all of the content assessed being directly taught in the academic year.

The analysis in this chapter reveals that framing success and directly teaching about growth mindset are two pieces of the puzzle, but they alone are not enough to shift students' mindsets in productive ways. When this pedagogy is decoupled from a rich mathematics curriculum that gives students opportunities to productively struggle and provides opportunities for students to develop authority in the classroom, the growth mindset instruction is productive. In order for the growth mindset pedagogy to be productive for students, classrooms need to not only *teach about growth mindset*, but also provide opportunities for students *learn to have a growth mindset* by having authority over their own learning and experiencing a growth mindset while persisting through challenges.

Chapter 5: Summary & Discussion

The effect of growth mindset on student learning is a topic of significant and growing importance in educational research. However, there are few studies that look at the ways in which teachers can implement growth mindset pedagogy in real classrooms. This dissertation compared one teacher's pedagogy toward *teaching for a growth mindset* in two contexts in an attempt to better understand the ways in which context influences pedagogy and to unpack the nuances of what it takes to effectively teach for a growth mindset in these contexts. In brief summary, context plays an important role for pedagogy and learning: *in order for students to develop a growth mindset, students must not only learn about growth mindset, but must also have opportunities to engage in practices that support its development.* In this discussion chapter I start with a summary of the findings, connecting the analyses in the three data chapters into a cohesive story. Further, I draw implications from these findings by connecting them to the broader research literature. Last, I discuss the limitations of the present study and outline potential directions for future research.

Summary of findings

Recall that the research questions addressed in this dissertation are as follows:

3. What happens when a teacher—who has a demonstrated commitment to growth mindset ideologies and the skills to teach for a growth mindset in ways that influence her students' mindset and performance—teaches in two very different contexts: a) a summer course in which there is little accountability for content learning, and in which she chooses challenging content as a means of supporting work toward growth mindset, and b) a regular academic year Algebra course in which the immense pressure she felt to prepare her students to perform well on a high stakes accountability measure drove her pedagogical choices?
    a) Which pedagogical strategies are implemented, are modified, or disappear in ach context?
    b) Why does the teacher make such distinct pedagogical choices in each context?

4. What is the overall impact of the implemented pedagogical strategies on students' a) self concepts and dispositions toward mathematics, b) engagement with challenging mathematics, and c) persistence with learning? In other words, what are the necessary pedagogical elements for effectively teaching for a growth mindset?

In addressing these research questions, this dissertation examined a teacher's pedagogy that aimed to contribute to the development of a growth mindset in two contexts - a summer course in which she had complete flexibility with the curriculum, and an academic year course in which she felt bound to the California Standards Test (CST) and the accompanying curriculum. Chapters 2 and 3 provided detailed analyses of each context along four strands of pedagogy that can contribute to effectively teaching for a growth mindset: 1) Framing success (What are the long-term goals for students? What gets defined as competent mathematical participation and how? What are the messages regarding what it means to work competently?) 2) Treatment of malleable intelligence (How is malleable intelligence discussed? What are the mindset messages communicated to students?), 3) The mathematics (What is the nature of the mathematics students are asked to engage with?), and 4) The agency and authority with which students are set up to work on the mathematics.

In the summer course Ms. M:

1) Framed the mathematical goals of the course as learning-based and persisting the face of challenges,
2) Provided an explicit treatment of malleable intelligence,
3) Organized the course around non-routine challenging mathematics, and
4) Provided opportunities for students to develop agency and authority over the mathematics.

In the academic year, the first two strands of pedagogy looked somewhat similar, but Ms. M added an emphasis on performance. In the academic year, Ms. M

1) Framed the mathematical goals of the course and competent participation as both *learning* and *performance* oriented, and
2) Provided an explicit treatment of malleable intelligence that was also coupled with performance-based fixed mindset messages.

The nature of the last two strands of pedagogy, however, looked drastically different in the academic year than in the summer. In particular, in the academic year, Ms. M 3) Organized the course around routine procedural mathematics and 4) Retained most of the authority in the classroom.

Chapter 4 revealed that these differences in pedagogy resulted in with drastically different outcomes for student mindsets, engagement with rich mathematics, and persistence toward learning. While in the summer course students' mindsets shifted in ways that increased their engagement with rich mathematics *and* their scores on pre- and post-assessments, the students in the academic year showed no such productive shifts in mindset, engagement, or learning. In sum, the pedagogy in the summer, with growth mindset instruction that was coupled with challenging mathematics and opportunities for students to share authority over their learning, was more effective at productively influencing students' mindsets and performance. When students were deprived of the opportunity to *experience* a growth mindset by only working on proceduralized routine mathematics in which they had no authority to come up with their own ideas, their mindsets did not shift in ways that were productive for engagement or learning.

I will discuss the factors that contributed to Ms. M's shift in pedagogy between the contexts in the next section, while also talking about the larger implications of these findings.

Implications

Teacher Professional Development and Decision Making

This case study of Ms. M teaching for a growth mindset shows how context shapes one teacher's identity and decision-making, such that she seems to be two drastically different teachers in two different instructional contexts. The analysis tells both the story of the professional identity resources that influenced Ms. M's different pedagogical decisions, and also what these different classroom opportunities meant for students' developing mindsets. This research provides an understanding of the ways teachers' negotiation of school contexts with their multifaceted professional identities can have meaningful impacts on student learning, and it considers the factors necessary to successfully teach for a growth mindset in a way that productively influences student engagement and learning.

The case study of Ms. M combines research on human decision-making (e.g., Schoenfeld, 2010) and identity (e.g., Holland et al., 2001) to examine the way a change in context can result in different aspects of her multifaceted identity being activated, which results in different consequential pedagogical choices being made. Specifically, this dissertation sheds light on how school contexts interact with teachers' multifaceted identity in ways that can contribute to teachers' decision-making when setting classroom goals (Schoenfeld, 2010).

The different contexts offered Ms. M identity resources about what was valued as good teaching, which led to distinct pedagogical decisions that aligned with the salient aspects of her professional identity in each context. Despite her commitment to growth mindset instruction in both contexts, Ms. M implemented pedagogical moves in those two contexts that contributed to distinctly different opportunities for students to engage with rich mathematics and to develop productive mathematical self-concepts.

As a result of the immense pressure Ms. M felt to support her students to perform well on the CST, Ms. M's professional identity in the academic year setting was tied to her students' success on these standardized tests, which ultimately deformed her practice of teaching for a growth mindset in problematic ways. Rather than considering only the *communities of practice* (Wenger, 1998) in which Ms. M participated, this dissertation shows that beliefs and contexts can shape one's practice in fundamental ways. This dissertation considered the multiple factors that came into play as Ms. M set out to teach for a growth mindset in the academic year and the ways she negotiated them so they were not conflicting for her. What happened in this process resulted in direct instruction on growth mindset and an emphasis on working hard, coupled with skills-based procedural curriculum with test performance as the goal and few opportunities for students to develop authority in the classroom. While she still explicitly shared information about growth mindset through quotes each week and spoke often about learning-goals, the traditional mathematical content and the ways it was implemented within a performance-oriented context in Ms. M's academic year classroom wound up depriving the students of opportunities to engage in productive struggle and to experience how a growth mindset, combined with hard work, can produce significant learning.

Ms. M's pedagogy in the academic year was undergirded by the belief that to get the students to perform well on the CST, she had to teach for *performance*. This instruction took a form that was largely procedural and did not provide room for students' critical thinking or productive struggle. As a result, students' opportunities to develop productive mathematical identities were scarce and the growth mindset work did not have space to take hold.

Yet, Ms. M's mid-year reflection on her growth mindset practices in the academic year did not initially reveal that she perceived a conflict from the shift in context that constrained her ideological practice. By the end of the reflection, and with guidance from the interviewer, Ms. M acknowledged that her practice looked different than in the summer in ways that made teaching for a growth mindset more difficult, but did not see this as conflicting and did not use this reflection to shift her practice in the second half of the year. It seemed that Ms. M negotiated the tension between these performance-based goals and her learning-based ideological commitment toward growth mindset to implement pedagogy that resolved the conflict between these goals for her. That is, she told her students about the importance of growth mindset, while adopting a more performance-oriented pedagogical stance. However, the resulting student outcomes reveal that the ways. Ms. M negotiated these tensions for herself actually depressed the effectiveness of her teaching on student mindsets, engagement, and learning.

Various theories on pedagogical strategies that influence students' developing mathematical identities have been extensively applied and their effects studied in actual classrooms (e.g. Cohen & Lotan, 1995; Boaler & Staples, 2008). However, there has been little, if any, work that has examined how particular contexts and the goals that emerge from those contexts afford and constrain opportunities for teachers to implement these strategies in real mathematics classrooms.

This study highlights the importance of teachers' professional identities in ways that have implications for the implementation of the new Common Core State Standards in classrooms and professional development. In the academic year classroom, this teacher had divided loyalties. While the teacher was deeply committed to teaching growth mindset ideologies, her professional identity hinged on her student's performance on a procedurally oriented state test. The result was that her commitment to this part of her identity undermined her work toward teaching for a growth mindset. Understanding these pedagogical shifts raises questions that need to be exported and addressed. In particular, how do contexts define teacher success in ways that provide spaces for teachers to simultaneously implement reform pedagogies and support their developing professional identities? This work contributes to a nuanced understanding of how contexts and teachers' professional identities can contribute to pedagogical decisions toward more equitable learning opportunities for all students.

Growth Mindset Research

Additionally, this dissertation contributes to the literature on growth mindset instruction by expanding the evidence of what it looks like to effectively teach for a growth mindset in the context of a classroom. There has been a recent interest among teachers in teaching students about growth mindset. Many teachers have read Dweck's 2006 bestseller *Mindset* and aim to implement the book's ideas in their classrooms, but don't know what it means for their discipline-specific teaching, in particular for mathematics – a subject where fixed ability messages proliferate (Boaler, 2013). Drawing from the pioneering work of Dweck (2006) when preparing to teach for a growth mindset, many teachers consider it important to share information about having a growth mindset with their students, praising students for their growth and effort versus their performance, and framing instruction around long-term learning goals. There has been little work, however, that examines what it takes to effectively teach for a growth mindset in the context of a full-year mathematics classroom, and that teases out which elements of teaching for a growth mindset are necessary for contributing to students' mindsets in ways that influence their engagement and, consequently, their learning of rich mathematics.

The analysis in this dissertation reveals that directly teaching about growth mindset and framing success and competence are only two pieces of the puzzle. As revealed by the different student outcomes in both contexts, *direct growth mindset instruction alone is not enough to shift students' mindsets in productive ways*. This pedagogy must be coupled with content that provides opportunities for students to struggle – *opportunities for students to experience a growth mindset* – and gives them the authority to come up with their own ideas and develop identities as doers and learners of mathematics. The two classrooms analyzed in this dissertation – same teacher, different contexts – reveal that it is not enough to bring a "growth mindset package" into a classroom and expect it to work without also considering the content and the ways students are positioned to work on that content. Just as we do not tell someone how to tie their shoelaces and expect them to be able to do so, we cannot *tell* students about growth mindset and expect them to develop a growth mindset.

Limitations and Future Research Directions

One of the major pedagogical differences between the academic year and the summer was Ms. M's primary emphasis on the CST and performance goals in the academic year, and the ways she emphasized these goals often conflicted with her growth mindset learning goals. However, it is also important to note the role these performance goals have for students. In Gutierrez's (2002) argument toward shifting the equity conversations to focus on the distinctions between *dominant* mathematics – that which is valued in high-stakes testing and curriculum and that "privileges a static formalism in mathematics" (p. 150) – and *critical* mathematics – that which is built around students' cultural identities in order to take on social/political issues – she suggests the two are not entirely in conflict with one another. Gutierrez (2002) suggests that schools should strive to achieve both, such that "the learning of dominant mathematics may serve as an entrance for students to critically analyze the world (using mathematics)" (p. 152). Using this same argument, performance goals and growth mindset learning goals do not have to be incompatible with one another. Setting standards and supporting students in reaching those standards plays a critical role in classrooms as an indicator of equity. If teachers do not teach students to perform well on the ways in which they will be evaluated, they are doing a disservice to students. The question then is in what ways teachers negotiate these tensions so both goals can work to support one another.

More broadly than teaching for a growth mindset, this dissertation speaks to a larger issue of supporting teachers in negotiating the tensions between teaching skills to perform on a procedural test and – what research has shown is more effective for student learning – teaching procedures, concepts, *and* problem solving. Ms. M's case shows how this tension can undermine effective equity-oriented pedagogy. When teachers' professional identity hinges on their students' success on a standardized test, school contents must support teachers in taking a leap of faith in their classrooms if the goal is to provide rich learning opportunities for students.

Future research and work with teaching for a growth mindset should consider the ways to support teachers in negotiating these tensions such that students reach performance goals while still having rich mathematical learning experiences. This study compares the practices of only one teacher who set out to weave elements of explicit growth mindset instruction into her Algebra classes. To further unpack the nuances of growth mindset, similar studies would need to be replicated in other contexts with more teachers. Finally, while the effects of growth mindset in the summer course are promising, one of the primary goals of growth mindset instruction is to create lifelong learners. A longitudinal study that follows students into their future mathematics classrooms over time would begin to clarify the lasting effect of these moves on students' global mathematical self-concepts.

Final Thoughts

Based on the work in this dissertation, I posit that as research begins to see social and motivational factors as influential to student learning, using real classrooms to unpack the nuances of what it takes to effectively teach for a growth mindset – both in terms of contextual resources for teachers and the pedagogical practices in classrooms – should be a primary focus of research and instruction on student learning of mathematics. The more we elaborate and clarify these processes for schools and for teachers in terms of classroom realities, the more prepared we are to support teachers in equitably supporting students amidst a nationwide adoption of the Common Core State Standards in Mathematics (CCSSM).

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Appendix A: Transcript of IRE Sequence

Teacher: Let's begin with a big idea. The big idea for today is that [*writes on the board*] we can graph a 2-variable equation. We can graph a 2-variable equation by creating a t-table. [inaudible student question] On graph paper, yes, you are taking all notes on graph paper today and then the graph paper will get attached. So only write on your graph paper today. We can graph a 2-variable equation by creating a t-table.

Teacher: Today we're going to practice taking an equation, creating a t-table, and graphing the solutions from the t-table onto…umm…onto a grid. So I understand this to be 7th grade --

Teacher: Oh, there's one more. Thank you for reminding me. One more announcement is about the rules. Please don't do anything with a ruler that will make me look at you. The most famous things to do are helicopters - that's not okay. Or also, for some reason people like to do this [*hits her chin with a ruler*] and then obviously, you know [*dances with ruler*] There are lots of fun things you can do with a ruler, but please don't do anything with a ruler that will distract me from teaching because I need to move. So José, please, your tricks are cute, but I want you to save them for after school. Show me all your tricks later. Okay? Not now. So I know you have a trick eating the ruler, but not during class. Alright.

Teacher: So I understand this to be a 7th grade skill – taking what are all going to be called linear equations. Taking a linear equation, creating a t-table, and graphing the solutions from the t-table, but it is such an important topic that I am going to dedicate this entire day to it because I need to make sure that you know how to do this. Alright. So here we go. We're going to do number 1, which is…

Teacher: To graph y=-x+4. *[writes it on the board*] 1. y=-x plus what did I say?

Student: Four

Teacher: Four. Graph y=-x+4. Now, here's a big…another big idea in math. If you're going to solve an equation that has just one letter, you might get an answer like x=2 or y=7 or b=1 or z=-1/2. The answer if there's only one letter can have just one part. The solution just has one part. But if you have an equation that has two variables in it, then a solution cannot be something as simple as x is 5 because if x is 5 that doesn't make this true. If x is 5, this would be -5 and -5 +4 is negative 1, so a solution to this would have to be a pair. We would have to have an x value and a y value that together make this true. If there are two letters, then the solution has 2 parts. If there are two letters, then a solution will have two parts. We need an x and a y pair that makes this true. So for example, when x is 5, y is -1. So -5, -1 is a solution. So the solution will have two parts.

Teacher: Does this make sense to you? A solution will have two parts if there are 2 letters. So creating a t-table is finding x y combinations that are solutions to this. So here we go. Watch how I do this.

Teacher: I'm going to make a t-table, but notice in my t-table the right side is wider than the left. That's on purpose. That's on purpose. All on graph paper, everything on graph paper today. Alright. So in our t-table, usually you see people write x and y, but I'm not going to write y, I'm going to write what y equals. Instead of y, I'm going to say what y is equal to. And what is y equal to?

Students: Negative x plus 4

Teacher: Negative x plus 4. So really this is x and this is y, I just wrote y in a different way because I happen to know from the equation that y is the same as this. So this is x and this is y. Everybody okay so far?

Teacher: Next. We're going to create our own domain. Our own list of x values. So I'm going to create a list of x values, and I'm going to show you my favorite domain, and I recommend it very highly and I'll tell you why. My favorite domain is -3, -2, -1, 0, 1, 2, and 3. I like this domain because we have a little bit of negatives, a little bit of positives, and 0 and they're all easy numbers to deal with. So you're going to use this domain a lot.

Teacher: Now the next thing that I want to show you is that what we're going to do is, when x is negative 3, we're going to find out what y equals by substituting negative 3 in for x. But I offer you this fantastic way of doing this – whenever you substitute, I've told you to substitute with parenthesis, so instead of negative x + 4, I'm going to write negative parentheses plus 4. Negative parenthesis plus 4. … [*chanting 7 times as she writes each number in the domain*] So instead of –x we're going to have negative parenthesis. So I want you to always do this, whenever you're going to substitute in for a number, start by replacing the letter with parentheses and then we're going to take our value for x and put it into the parentheses. That will save you so many errors. I see UC Berkeley student Tom in the back nodding his head yes. It's very helpful to substitute with parenthesis. So we do this for a reason.

Teacher: Alright, so here we go. The first x value we're going to substitute is…

Students: Negative 3

Teacher: Negative 3. The next one?

Students: Negative 2

Teacher: That's right. Go ahead and put the x values in.

Sam: (inaudible) distribute the negative?

Teacher: Yes, you can see it that way. So Sam is look at this as distributing the negative into the parentheses. I'm going to say it a different way, which is -x means the opposite of x [*writes that on the board*]. Negative x does not necessarily mean you're going to get a negative. What it means is the opposite of x. It's the opposite of x. So Sam to answer your question, um actually it wasn't a question, you were suggesting that we think of it as distributing the negative in. It's the opposite of negative 3. So get ready because I'm going to start calling on people. So this one goes to everyone.

Teacher: What's the opposite of negative 3?

Students: 3

Teacher: and now add 4

Students: 7

Teacher: So Sam, What's the opposite of negative 2?

Sam: 2

Teacher: Plus 4.

Sam: 6

Teacher: Thank you. Hassiem, what's the opposite of negative 1?

Students: 1

Teacher: and add 4. Okay, so what you're noticing Sam – negative x, this is just a sidenote for you, negative x is just the same as negative 1x – that negative 1 that you're noticing is in the equation, it's not a coincidence.

Teacher: Alright, so Raheem, the opposite of negative 0?

Raheem: 0

Teacher: And add four.

Raheem: 4.

Teacher: Paul, the opposite of 1

Paul: 1

Teacher: And add four.

Paul: 5.

Teacher: Paul, the opposite of 1

Paul: -1

Teacher: And add four.

Paul: 3.

Teacher: Isaiah, the opposite of 2. And I already got the (inaudible)

Isaiah: -2

Teacher: And add four.

Isaiah: 2.

Teacher: And Jameelah, the opposite of 3

Jameelah: -3

Teacher: And add four.

Jameelah: 1.

Teacher: Thank you. Alright, so we've completed our T-table. Now our t-table, as I said earlier, is going to give us a list of solutions. And the solution have two parts. So when x equals -3, what's y?

Students: 7

Teacher: negative 3 comma 7 is a solution to this equation [*continues with IRE writing out all of the solutions and then talking students through how to graph the ordered pairs in the same way*]

Appendix B: Mindset Quotes from Academic Year

- "Champions and not born champions they are relatively ordinary people who work hard to stretch beyond their ordinary abilities." – Dweck, 2006
- "A person's true potential is unknownable and it's impossible to foresee what can be accomplished with years of passion, toil and training." – Dweck, 2006
- "It's not always the people who start out the smartest who end up the smartest." – Dweck, 2006
- "Many people think of the brain as a mystery, they don't know much about intelligence and how it works. When they do think about what intelligence is, many people believe a person is born either smart, average, or dumb, and stays that way for life. But new research shows that the brain is more like a muscle. It changes and gets stronger when you use it." – Dweck, 2006
- "Nobody laughs at babies and says how dumb they are because they can't talk. They just haven't learned yet." – Dweck, 2006
- "In one study, 7th grades described how they would respond to getting a poor test grade. Students with a growth mindset said they would study harder for the next test. Students with a fixed mindset said they would study less for the next test." – Dweck, 2006
- "In a fixed mindset effort means you're not smart or talented. In a growth mindset effort is what makes you smart or talented." – Dweck, 2006
- "In the fixed mindset, both positive and negative label can mess with your mind. When you're given a positive label you're afraid of losing it. And when you're hit with a negative label you're afraid of losing it." – Dweck, 2006
- "You can always be better even when you're the best!" – Student Quote
- "I don't divide the world into the weak and the strong, or the successes and the failures. I divide the world into the learners and the non-learners." – Benjamin Barber, quoted in Dweck, 2006
- "Most kids if they're not sure of an answer will not raise their hand to answer the question, but what I usually do is raise my hand because if I'm wrong my mistake will be corrected. Just by doing that I am increasing my intelligence" – Anonymous 7th grader, quoted Dweck, 2006
- "Some people enjoy doing what's easy for them. They like to do things they've already mastered. Other people enjoy doing hard things, things they've never learned before. Which of these preferences describes you?" – Dweck, 2006
- "I don't think I'm the greatest soccer player and because of that someday I just might be." – Mia Hamm
- "If I wasn't dyslexic, I probably wouldn't have won the games. If I had been a better reader, then that would have come easily, sports would have come easily, and I never would have realized that the way you get ahead in life is hard work." - Bruce Jenner
- "Just keep pumping your arms she instructed herself, it's not that bad so keep going. You can make it, you have enough air. You've got this, just run as hard as you can." – Jackie Joyner-Kersee, quoted in Dweck, 2006
- "Jim Marshal, former defensive player for the Minnesota Vikings, once ran the wrong way and scored for the opposing team on national TV. He actually ran the wrong way. And scored, what is called a safety for the other team. Accidentally. So, at halftime, he

decided to do something about it. In the second half, he played spectacularly and helped his team win the game." – Unknown Source

- "Attitude is like a pair of eyeglasses. Positive minded people see life around them through rose-tinted or clear-lenses. While those who are negative, squint through glasses that are dark and gloomy. Both types of people can look at the same event or situation and see it in two different lights." – Mind Gym, 2001
- "I firmly believe that the only disability in life is a bad attitude." – Scott Hamilton
- "Talent is never enough, with few exceptions the best players are the hardest workers." – Magic Johnson
- "We all want to win. Every athlete wants to succeed, but the ones who do succeed are those who separate 'wanting' from being willing to make the sacrifice that winning demands." – Mind Gym, 2001
- "A persons motivational level may be a better predictor of academic achievement than their IQ score. Believing our intelligence is substantially under our control is a good start to making us smarter." – Richard E Nisbett's from *Intelligence and How to Get it,* quote I Dweck, 2006
- "In psychology, the term self-efficacy is the belief in one's own ability to be successful. Simply believing in yourself doesn't mean you're always going to win, but believing in yourself can help enable you to put yourself into a position to win." – Mind Gym, 2001
- "It's common for students to turn off to school and adopt an era of indifference. But we make a mistake if we think any student stops caring." – Dweck, 2006
- "A genius? For 37 years, I've practiced 14 hours a day and now they call me a genius?" – Pablo de Sarasate
- "If people knew how hard I work to gain my mastery, it would not seem so wonderful at all." – Michelangelo
- "When I was a young man, I observed that 9 out of 10 things I did were failures. I didn't want to be a failure, so I did 10 times more work." – George Bernard Shaw
- "Hard work beats talent when talent fails to work hard." – Kevin Durant
- "It is only a problem if you make it a problem. I don't view it as a problem, so it isn't." – Riley Quinn
- "There are no secrets to success. It is the result of preparation, hard work, and learning from failure." – Colin Powell
- "Genius is 1% inspiration and 99% perspiration." – Thomas Edison