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Author

Dale, Reid

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OPERATIONALISM MEETS MODAL LOGIC

REID DALE

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1. INTRODUCTION

In the abstract to *There is No Really Good Definition of Mass*, physicist Eugene Hecht laments that

There seems to be a fairly prevalent belief in the physics community that the basic concepts of our discipline (mass, force, energy, and so forth) are well understood and easily defined. After all, there are dozens of textbooks on every level that supposedly define all the terms they introduce. Apparently, we teachers can pass this wisdom on to our students without any cautionary notes and without any concern for subtleties. Remarkably, this is most certainly not the case, and anyone who has studied the foundational literature in physics over the last several centuries knows that none of the fundamental ideas [are] satisfactorily defined.

Hecht's concerns are not new, but they are alarming; the problem of providing an adequate definition of the dynamical concepts of force and mass in classical mechanics has been the subject of much debate since the subject's inception¹. One of the key historical figures in this debate is Ernst Mach, who wished to divest mechanics from the metaphysical notions of force and mass:

That which in the mechanics of the present day is called *force* is not something that lies latent in the natural processes, but a measurable, actual circumstance of motion... Also when we speak of the attractions or repulsions of bodies, it is not necessary to think of any hidden causes of the motions produced. We signalise by the term attraction merely an actual *resemblance* between events determined by conditions of motion and the results of our volitional impulses. [Mach1919 264]

For Mach, the dynamical terms of *force* and *mass* are to be “determined by conditions of motions,” which is to say by their *kinematics*. Therefore Mach's operationalism challenges:

Using only kinematical data, which we take to be directly observable, define all physical quantities (namely mass) referred to in Newtonian mechanics.

This charge necessitates an answer to the following question:

Is mass definable in Newtonian mechanics?

¹An excellent account of this history can be found in Jammer [Jammer1961]

This question is deceptively simple in its formulation. An answer, whether affirmative or negative, can only be made sensible if we can specify what is meant by

- a. Definability
- b. Newtonian Mechanics (or a scientific theory more generally)
- c. Mass (or a physical quantity or property more generally)

Fixing each of these notions furnishes, as a matter of fact, an answer to the aforementioned question.

It is my view that, in the existing literature, most of the substantive disagreement regarding the definability of mass in Newtonian mechanics arise in the respective authors' conceptions of definability and how it interacts with their views on the methodology of science.

My hope in these pages is to develop a framework that simultaneously allows one to:

- a. give a taxonomy of the differing views underlying differing accounts of definability in science, and
- b. objectively assess the definability of physical quantities within given theories in their own terms.

In the end, the full framework will consist of a modal logic built out of a specification of two parameters:

- I. A physical theory T with a partition of the underlying language \mathcal{L} into *observable* and *theoretical* terms.
- II. The “admissible data” E that reflects the epistemic position of an idealized scientist.

Given this input, we construct an associated modal frame, \mathcal{F}_{TE} , to be interpreted in standard first-order modal logic

Following Bressan [Bressan1972a,b], we may then ask of the frame whether or not the physical quantity in question is “modally definable” in terms of observational terms. A quantity is *modally definable* provided:

- a. For all objects it is possible to perform an experiment that would yield a unique determination of the quantity in question given knowledge of the admissible data.
- b. It is necessary that if the experiment is performed on the same object, the values obtained in each experiment are identical. This criteria for modal definability specializes to the first-order notion of definability in the case of modal collapse, which for instance happens in Pendse’s account [Pendse1939].

In the end, my hope is to provide a first step towards a defense of Mach’s definition by exhibiting a toy model for which it is the case that Mach’s proposed definition is a definition in the modal sense.

2. THE PROBLEM OF MASS IN CLASSICAL MECHANICS

2.1. Setting the Stage. The theory of Newtonian mechanics, T_{Newt} consists of a family of rules which serve as the target of interpretation of many further refinements of the theory. The formal multi-sorted language we will use consists of

- A sort P for particles
- An interval $T \subseteq \mathbb{R}$ of time.
- A two-place function $s(p, t) : P \times T \rightarrow \mathbb{R}^3$ for position.
- A mass function $m(P) : P \rightarrow \mathbb{R}^+$.
- A force function $F_{xy} : P^2 \times T \rightarrow \mathbb{R}^3$

The basic laws of Newton's theory are by-and-large geometrical, interpreting the fundamental concept of *force* as being equal to the product of the vector-valued *acceleration* of an object and the *mass* of the object. In order to make an observational prediction, though, the theory requires one to work in a certain *inertial frame of reference*, which corresponds to picking a distinguished point O in absolute space, called the *origin*, which is acted on by no force. We denote the total force on P as measured in reference frame O as \mathbf{F}_{OP} . For convenience, we identify O with the zero vector in \mathbb{R}^3 .

In the inertial frame of reference O , Newton's Laws may be written as

- (1) (Inertia) $(\forall x \in P)(\forall t_0 \in T) \frac{d\mathbf{v}_x}{dt}|_{t=t_0} = 0$ if and only if $\mathbf{F}_{Ox}(t_0) = 0$.
- (2) (Definition of Force) $(\forall x \in P)(\forall t_0 \in T) \mathbf{F}_{Ox}(t_0) = \frac{d(m_P \mathbf{v}_P)}{dt}|_{t=t_0} = m_P \frac{d\mathbf{v}_x}{dt}|_{t=t_0}$.
- (3) (Reaction) $(\forall x \in P)(\forall t_0 \in T) \mathbf{F}_{xy}(t_0) = \mathbf{F}_{Ox}(t_0) - \mathbf{F}_{Oy}(t_0)$.

I call the above theory **Vanilla Newtonian Mechanics**. You may notice that in this account there is no mention of gravity or any other forces. This is due to the fact that the usual (classical) theories *interpret* the above theory in a straightforward way. If the world were, at a fundamental level, Newtonian and we have a list of its fundamental forces, then the interpretation of force given by

$$F_{total} = \sum F_i$$

for all particles x at all times t yields a model of Newtonian mechanics.

These interpretations of complex phenomena into the basic framework of Newtonian mechanics yield inter-theoretical property inheritance relations and facilitate inferences about definability and operationality. For instance, if in T_{Newt} mass was operationally definable and the interpretation of T into T_{Newt} were also operational, one would have an operational definition of mass in T . Conversely, while Vanilla Newtonian mechanics may not define mass, stronger theories that interpret Newtonian mechanics *may* define mass.

2.2. Mach's Definition. One of Mach's main goals in axiomatizing classical mechanics was to provide an *operational definition* of mass. What constitutes an operational definition, and what rules govern the analysis of scientific concepts in terms of their operational definability? Moreover, do notions of definability in science bear any resemblance to notions of definition encountered in logic?

Mach defines mass ratios between particles *kinematically*, assuming at the outset that Newton's laws hold for the derived concept of force, realized as mass times acceleration.

Following Mach (1919, 243), the idea is as follows: take two particles A and B forming an isolated system and measure their accelerations, provided that they are not zero. Newton's third law says that

$$m_{AA}(A, t) = -m_{BA}(B, t);$$

but as Mach does not avail himself to masses but only mass ratios, the correct formulation would be that

$$a(A, t) = -m_{BA}a(B, t)$$

where m_{BA} is the mass ratio of A in B . Since a unique scalar exists, this mass ratio is uniquely determined by the experiment. In order to handle the case of an n -particle system, he prescribed repeating the experiment for other pairs of particles as well.

2.3. Pendse's Objection. In 1938, C. G. Pendse problematized Mach's proposed definition of mass on epistemic grounds by showing that, while

Mach's definition of mass may be a dynamical one, it does not enable an observer to determine the ratios of the masses of a system—an 'isolated system'—of bodies (particles) from his observations if the system be composed of more than a certain number of bodies (particles) (Pendse1939 1013).

That is, Pendse maintains that Mach's view on the definition of mass is flawed.

To show this, Pendse reconstructs Mach's definition of the mass-ratio of a pair of particles as follows:

Mach defined the ratio of two particles as the negative inverse ratio of their mutually produced accelerations, and he postulated that that ratio was a constant (Pendse1939 1016).

Pendse latches on to the highly idealized circumstances to which Mach's proposed definition applies: it defines

the ratio of the masses of *two* bodies (particles)" under the assumption that "the bodies (particles) form an 'isolated' system relative to the frame of reference. Hence it is necessary to examine Mach's definition

from a theoretical standpoint when the system is composed of more than two bodies (particles) (Pendse1939, 1016).

Pendse's concerns seem to hinge upon an argument of the following type:

(Unique Determination Principle) An operational definition of a physical quantity must uniquely determine the value from given observational data.

(Underdetermination of Mass) Mach's proposed definition of mass is not guaranteed to uniquely determine mass ratios for systems with sufficiently many particles.

(Conclusion) Therefore, Mach's proposed definition of mass ratios is not an operational definition.

This form of argument depends on what is meant by observational data. For Pendse, observational data seems to be bounded above by *merely recording* the kinematical facts of particles in the system. Pendse wonders

will it be possible for an observer fixed in a Newtonian frame for the system of particles... to determine the ratios of the masses of the particles by observing their motions? It is assumed that the observer can record the path of each particle... [and therefore] know the motion of every particle in the system. He cannot observe the ratios of the masses of the particles or the reactions between them: he can only aspire to infer them [Pendse1939 1017].

A caveat needs to be made: Pendse considers only *finite* numbers of observations, as evidenced by the statement of his theorems:

Theorem 2.1 (Pendse 1939). *In models of Newtonian Mechanics with at least 8 particles (P) and an arbitrary finite number r of observations of kinematical data, mass is in general not uniquely recoverable. That is, there exist at least two assignments of masses to particles $p \in P$ consistent with Newtonian Mechanics.*

A corollary of this theorem is the first-order undefinability of mass in Vanilla Newtonian Mechanics from kinematical information:

Corollary 2.2. *In Vanilla Newtonian Mechanics together with any finite quantifier free sentence in the language consisting of kinematic terms, the mass function m is not first-order definable.*

Noticeably missing from Pendse's setup is the possibility of *intervention* by a Newtonian observer. That Pendse does not allow for this maneuver is evident by his rejection of the following line of reasoning:

(Intervention Hypothesis) Any two bodies may be placed in an isolated system and be made to mutually produce accelerations with one another.

(Mach's Definition of Mass) In an isolated system comprised of two bodies, the mass ratio of the two bodies is the negative inverse ratio of their mutually produced accelerations.

(Conclusion) Mach's definition of mass uniquely determines the mass ratio between any two particles and is therefore an operational definition.

Therefore Pendse holds a restricted view on what data is *admissible* in the inferences meant to determine the values of mass ratios between bodies.

3. MODAL DEFINABILITY

The underdetermination results of the previous section seem to bar the *first-order* definability of mass from observational data. However, why demand that the definition be first-order in the first place? In this chapter we investigate the possibility of defining mass (and other theoretical terms) with a *modal* definition.

To this end, we study the desiderata for a modal definition of quantities as articulated by Bressan (1972a) and exhibit Kripke frames that furnish such a modal definition. In general, we construct out of a given *scientific theory* T and collection of *epistemically admissible sentences* E a Kripke frame \mathcal{F}_{TE} and corresponding modality \Box_{TE} to yield the *intersection modality* of a certain epistemic modality and a physical possibility modality. This approach has many advantages:

- a. This approach is *flexible* enough to describe many potential views on modal definitions in science.
- b. By focusing on the *semantic* side of the logic and interpret it in standard first-order modal logic, we avoid the complicated process of building up a logic from scratch.

3.1. Modally Definable Functions. The notion of definability that we will use here is a notion of definability from **modal logic** similar to Aldo Bressan's notion [Bressan1972a]. This notion is meant to capture the intensional content of the performance of an experiment that yields, as a function of its input, the data the experiment was meant to provide.

Definition 3.1. Let \mathcal{F} be a Kripke frame on the class of models of a theory T . A function f^2 is modally definable in an \mathcal{L} -theory T in a reduct \mathcal{L}_{red} of \mathcal{L} if there is an \mathcal{L}_{red} -formula $\psi(\bar{x}, \bar{y})$ such that

²Here, I really mean for all families of functions $(f_W)_{W \in \text{Mod}(T)}$

- (Coincidence) For all $W \models T$ and for all $\bar{x} \in W$

$$(W \models \psi(\bar{x}, \rho)) \implies f(\bar{x}) = \rho$$

- (Possibility) For all $W \models T$ it is always *possible* to evaluate the function:

$$W \models \forall x \exists \rho (\diamond \psi(\bar{x}, \rho))$$

- (Uniqueness in Worlds) For all $W \models T$ the function is *necessarily* well-defined when witnessed in a world:

$$W \models \Box \forall x \forall \rho \forall \rho' ((\psi(\bar{x}, \rho) \wedge \psi(\bar{x}, \rho')) \rightarrow (\rho = \rho'))$$

Some remarks are in order. First, for the purposes of operationalism we demand that the graphs of the functions $(f_W)_{W \in \text{Mod}(T)}$ are “represented” by a single first-order formula ψ to capture the idea that ψ provides an *intensional* way of tracking the function $(f_W)_{W \in \text{Mod}(T)}$. That is, we can think of ψ as the (partially-realized) process *generating* the values making up $(f_W)_{W \in \text{Mod}(T)}$. Second, we do not demand that in *any* single world W_0 the predicate $\psi(\bar{x}, \bar{y})$ is a function. In natural language, that would amount to requiring that the world W_0 materially witnesses the process described by ψ being carried out on *all* objects in the domain of f_{W_0} . This is undesirable as, in general, there are processes which render impossible the realization of other processes. Finally, the notion of modally-definable functions generalizes the notion of (first-order) definable functions assuming that the Kripke frame we work with is *reflexive*. First-order definable functions arise as the modally-definable functions in the trivial reflexive frame:

- Proposition 3.2.** a. Let \mathcal{F} be any reflexive Kripke frame and f a (\emptyset) -definable function in T . Then f is a modally-defined function.
b. There is a Kripke frame K on the set of worlds $\text{Mod}(T)$ such that for all functions f , f is modally definable in \mathcal{L}_{red} if and only if f is first-order definable in \mathcal{L}_{red} .

Proof. a. Suppose that \mathcal{F} is reflexive. If f is a (\emptyset) -definable function, there is a formula $\psi_f(\bar{x}, \bar{y})$ such that

$$W \models \psi_f(\bar{x}_0, \bar{y}_0) \leftrightarrow f(\bar{x}_0) = \bar{y}_0$$

so that the coincidence axiom is satisfied. The possibility axiom is satisfied by the definition of definable function, as

$$T \vdash \forall \bar{x} \exists \bar{y} \psi_f(\bar{x}, \bar{y})$$

and therefore for all $W \models T$

$$W \models \forall \bar{x} \exists \bar{y} \psi_f(\bar{x}, \bar{y})$$

Finally, the uniqueness in worlds axiom is satisfied since, by definition, definable functions satisfy

$$T \vdash \forall \bar{x} \forall \bar{y}, \bar{y}' (\psi(\bar{x}, \bar{y}) \wedge \psi(\bar{x}, \bar{y}') \rightarrow \bar{y} = \bar{y}')$$

which implies that for all $W \models T$

$$W \models \forall \bar{x} \forall \bar{y}, \bar{y}' (\psi(\bar{x}, \bar{y}) \wedge \psi(\bar{x}, \bar{y}') \rightarrow \bar{y} = \bar{y}')$$

as desired.

- b. Consider the Kripke frame where each model $W \models T$ accesses *only* W . Then f_W is modally defined by a formula $\psi(x, y)$ in \mathcal{L}_{red} if and only if f_W is first-order defined by \mathcal{L}_{red} . □

3.2. The Frames \mathcal{F}_{TE} . With the notion of modal definability in mind, we introduce a family of Kripke frames that reflect a naïve scientific mood: the possible worlds are those that are *physically reasonable* (with respect to a background theory T) and *epistemically possible*. This mood is formalized as follows, first by defining the notion of an ideal scientist's *epistemic access function* and then by defining the frames in question:

Definition 3.3. Let T be a first-order theory and \mathbb{M} a *monster model*³ of T . We define the notion of an *epistemic access function* on T to be a map

$$E : \text{Mod}(T) \rightarrow \mathcal{P}(\text{Sent}_{\mathcal{L}(\mathbb{M})})$$

that maps a model M to a set $S_M \subset \text{Sent}_{\mathcal{L}(M)}$ of sentences true in M :

$$S_M \subset \text{Th}_{\mathcal{L}}(M).$$

This definition is very flexible; in fact, at this level of generality there's very little one can say about it. However, its intended interpretation is as follows. Suppose that the world of the ideal scientist is in fact a model $W \models T$. We then think of $E(W)$ as the set of true sentences that the ideal scientist *knows* about the world W . While I leave the definition very broad, some natural candidates for E tailored to a Newtonian worldview are apparent:

Definition 3.4. The *FST* (formula, spacetime) epistemic functions are parametrized by the following conditions:

- (1) (Formula) A family $F \subset \mathcal{L}$ of formulas of the full language.⁴
- (2) (Spacetime) A region $R \subset \mathbb{R}^3 \times \mathbb{R}$.

³The notion of monster model comes from model theory. A monster model \mathbb{M} can be thought of as a large, saturated, homogeneous model of T .

⁴For instance, a reduct \mathcal{L}_{red} of \mathcal{L} .

Then $E_{FST}(M)$ is the set of all true quantifier-free F -sentences of M with parameters from R .

Remark 3.5. Some clarifications of the above definition are called for.

- (1) Why include the “Formula” parameter? There are two immediate benefits. First, it allows one to restrict to exclusively observable predicates. Second, it allows one to restrict the scope of generalizations along the axis of *complexity*. For example, one may wish to avoid Π_1 generalizations such as “every particle is one of x_1, \dots, x_n ,” formalized as

$$\forall x \bigvee_{i \leq n} (x = x_i).$$

- (2) One interpretation of the “Spacetime” parameter is one in which the ideal scientist restricts her epistemic access only to her *lab environment*. This kind of idealization is the kind that Mach often employs.
- (3) In the present paper we restrict to *quantifier-free* sentences because they represent the measurements taken in an “ideal lab notebook,” absent any form of induction.

Given a specification of the theory T and epistemic function E , we can finally define the modal frame \mathcal{F}_{TE} and the associated modal operators \Box_{TE} and \Diamond_{TE}

Definition 3.6. The (T, E) -frame \mathcal{F}_{TE} consists of the following:

- (1) The class of possible worlds of \mathcal{F} is the class of models $M \models T$.
- (2) There is an edge from M to N , written $M \leq_{TE} N$, provided that $N \models T \cup E(M)$. In other words, N is consistent with the scientist’s knowledge of M .

Its corresponding necessity and possibility operators are denoted \Box_{TE} and \Diamond_{TE} respectively.

3.3. Towards a Vindication of Mach. We define a frame amenable to Machian physics of the sort \mathcal{F}_{TE} by specifying a theory T and epistemic accessibility function E .

For the theory, we let T_{Mach} be Vanilla Newtonian Mechanics together with the following physical hypotheses:

- (1) (Almost-Always Separation) This condition intuitively says that every particle is almost always separated from all other particles by some small distance. It is important that this condition allow for *contact* between particles, hence why it holds only “almost everywhere.” Formally, it can be expressed as follows:

$$\forall p \forall t \forall \delta \exists t' (\exists \epsilon_t > 0) ((|t - t'| < \delta) \wedge ((\forall s \in [t' - \epsilon_p, t' + \epsilon_p]) (\neg \exists q) q \in B_{\epsilon_p}(p(t))))$$

- (2) (Object Permanence) In this setting we assume that no particles can be annihilated. This assumption is crucial if one wishes to use a *weaker* frame.

$$\forall x \forall t (\exists z \in \mathbb{R}^3) p(x, t) = z$$

- (3) (Only Contact Forces) This postulates that the *only* forces are *contact* forces that act at the level of points.⁵

For our epistemology, we consider *any* *FCST* epistemic accessibility function E_{Mach} of the sort

- \mathcal{L}_{ob} the *observable* formulas in the Newtonian language, namely, the kinematic predicates.
- A **bounded** region $R \subset \mathbb{R}^3 \times \mathbb{R}$ of spacetime.

For notational convenience, set

$$\mathcal{F}_{Mach} = \mathcal{F}_{T_{Mach}, E_{Mach}}$$

Given this, we may state the following theorem:

Theorem 3.7. *Let \mathcal{F}_{Mach} be a frame of the form described above. Then mass is modally definable.*

Proof. To begin, note that Vanilla Newtonian Mechanics proves the *Uniqueness in Worlds* criterion of a modal definition.

It therefore suffices to verify the *Possibility* criterion. Let M be a model of T_{Mach} and select two particles x and y in M . Let M' be a world such that after a finite number of collisions by particles outside of the region R , $p(x, t) = p(y, t)$ for some $t > t_0$. This results in a collision, and the mass-ratio of x and y is given in M' . By construction, $M \leq_{Mach} M'$, and so the possibility criterion for modal definition is satisfied. \square

Dropping the assumption of Almost-Always Separation allows counterexamples such as *swampy worlds*.

Remark 3.8. Call a world *swampy* if for all times t and all positions $s \in \mathbb{R}^3$ there exists a particle x such that $\text{pos}(x, t) = s$. In such a world, between every two particles x and y there is a particle z lying on the line between x and y by the axioms of a real closed field together with the swampy hypothesis. Thus, no two distinct particles can be collided in swampy worlds: it is not possible for x to reach y . \square

A natural objection to these results goes as follows:

⁵This assumption is, of course, physically unrealistic. However, it allows one to compute masses *exactly* without the problem of dealing with forces that act at a distance.

Envision Mach attempting to collide two particles, A and B , in world W in order to compute their mass ratio, by performing the above experiment. A Machian demon, sitting outside the spatial region accessible within $E(W)$, decides to play spoiler. Every time Mach sends a particle P to collide with A , Mach's demon sends a particle \hat{P} to collide with P , preventing a collision between A and B . In this scenario, the experiment E *cannot* happen. Does this not rule out Mach's definition of mass as a modal definition?

To respond to this line of objection, special attention must be paid to the notion of *cannot* invoked in the argument. First, we show *how* this argument fails to show that mass is (modally) undefinable under the assumptions of the previous section. Let W_{demon} be *any* world satisfying the constraints of the above. Then

$$W \leq_{TE} W_{demon}.$$

This does in fact establish that

$$W \models \Diamond_{TE} \neg(\exists \rho, t) \text{Exp}(A, B, \rho, t)$$

However, even in W_{demon} , it is *possible* in the sense of \Diamond_{TE} for Mach's experiment to be carried out. This follows by *boundedness assumptions*: because $E(W_{demon})$ allows epistemic access only to a bounded $R \subseteq \mathbb{R}^3$, upwards-bounded

$$T \subseteq (-\infty, t_0] \subsetneq \mathbb{R},$$

and quantifier-free information, it is indeed possible for there to be a time $t_1 > t_0$ and particle $P(x, t)$ outside of R for all $t < t_0$ to collide with A , which then collides with B . Call such a world W' . Thus

$$W_{demon} \models \Diamond_{TE}(\exists \rho, t) \text{Exp}(A, B, \rho, t).$$

and, by upwards transitivity,

$$W \models \Diamond_{TE}(\exists \rho, t) \text{Exp}(A, B, \rho, t).$$

What are we to make of this? The crucial technical point is that $E(W_{demon})$ *does not see* that there is a Machian demon.

A natural, imprecise conjecture is that:

Conjecture 3.9. Under assumptions more physically reasonable than the conjunction of Almost-Always Separation, Object Permanence, and Only Contact Forces, mass is modally definable.

4. REMARKS ON THE SUPPES-BRESSAN DEBATE

In a pair of papers presented at the Biennial Meeting of the Philosophy of Science Association, Suppes and Bressan discussed the role of modal concepts in science. Suppes position is that

modal concepts... are essential to standard scientific talk. Yet, in a majority of cases the modal concepts remain implicit in that talk, and their logic is scarcely used in either theoretical or experimental analyses of empirical phenomena. [Suppes1972 305]

In the case of spacetime, Suppes justifies this position by pointing to the *eliminability* of modal language in the definition of a physical spacetime manifold:

I shall not attempt to examine the technical details of constructing the space-time manifold as the set of possible paths of particles or bodies. The simpler constructive geometry, based upon the two operations of intersection and laying-off of segments, provides a conceptual model of the kind of I have in mind. [Suppes1972 313]

Suppes seems to locate the role of modal concepts in science as providing a mental model for some scientific concept, which scientists then eliminate in favor of direct extensional constructions.

In the case of spacetime, Suppes seems to be largely successful. However, the above results on the various forms of *definability* of more sophisticated concepts puts pressure on Suppes' position: there is a precise sense in which certain scientific concepts (such as mass) cannot have their modal definitions be eliminated in favor of direct (first-order) definitions.

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REID DALE, UNIVERSITY OF CALIFORNIA AT BERKELEY, GROUP IN LOGIC AND THE METHODOLOGY OF SCIENCE, EVANS HALL, BERKELEY, CA 94720, USA

E-mail address: reiddale@math.berkeley.edu

URL: <https://math.berkeley.edu/~reiddale>