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LIQUIDITY FLOWS AND FRAGILITY OF BUSINESS ENTERPRISES

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Abstract. This paper develops a macroeconomic model in which investable assets flow to entrepreneurs through long-term relationships with lenders. Low asset flows cause relationships to break up due to insufficient liquidity. Multiple Pareto ranked steady states emerge from complementarity between financial intermediation, reflected by the number of relationships, and households' incentives to provide assets. This complementarity also serves as a mechanism for propagating aggregate shocks. Financial collapse may become inescapable if a shock destroys sufficiently many relationships.

1. Introduction

The standard credit market paradigm in macroeconomics presumes that firms borrow on spot markets from anonymous lenders. An important body of evidence has shown, however, that the establishment of long-term relationships between borrowers and lenders is a common feature of credit market trading.[1] In this paper, we demonstrate that frictions associated

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[1] Petersen and Rajan (1994,1995) document the predominant concentration of borrowing by U.S. firms among a few lenders, along with the importance of geographic proximity. Hoshi, Kashyap and Stein (1990,1993), Slovin, Sushka and Polonchek (1993), Berger and Udell (1995) and Gibson (1995) demonstrate links between the financial health of banks and their borrowers. Berger and Udell (1995) and Petersen and Rajan (1995) show that benefits to borrowers increase with the age of their relationships with lenders.

with credit relationships can play a central role in sustaining low-activity steady states and propagating aggregate shocks.

We develop a model in which investable assets flow to entrepreneurs through long-term relationships with lenders. An entrepreneur obtains resources for production and contracting exclusively through his relationship; thus, the lender's asset flow constitutes a *liquidity constraint* on the relationship.[2] Flows are subject to lender-specific randomness, reflecting shocks in producing intermediation services. Further, the entrepreneur must make an effort choice in each period that is necessary for maintenance of the relationship. A low asset flow implies that the relationship becomes insufficiently liquid to sustain a high-effort contract, leading the relationship to break up. Thus, the combination of moral hazard and wealth constraints makes relationships *fragile* in the face of fluctuations in available liquidity.

It follows that the available wealth of the *lender* can have important implications for credit market relationships. In particular, the lender's short-run access to liquidity determines whether the relationship can be sustained in the face of contracting problems.

Lenders and entrepreneurs form relationships through a process of search. The number of relationships is an important state variable in the economy, since it affects the efficiency of financial intermediation and the returns that households can earn on investments. The interaction between the number of relationships and investment gives rise to multiple Pareto-ranked steady state equilibria, where positive-activity steady states coexist with a zero-activity "collapse" steady state. Multiple equilibria emerge from *complementarity* between intermediation and investment. As the number of relationships rises, intermediation becomes more efficient, and the rate of return on investment increases. The higher rate of return induces households to provide more liquid assets in the aggregate, thereby allowing more relationships to be sustained. Aggregate increasing returns therefore emerge from the process of allocating investable assets through long-term relationships. For a range of values of

[2] For simplicity, the model assumes that entrepreneurs do not make use of private wealth for production or contracting; i.e., issues of collateral are ruled out. This paper instead focusses on frictions in external finance.

aggregate liquidity, this effect dominates the usual decreasing returns effect arising from the production function.

Feedbacks between financial intermediation and investment serve as a mechanism for propagating aggregate shocks. When an exogenous shock breaks up a proportion of the relationships, damage is persistent since reforming relationships takes time. More interestingly, destruction of relationships also causes aggregate liquidity to fall due to a decline in the efficiency of intermediation. This in turn leads even more relationships to break up, inducing further declines in aggregate liquidity. In this way, the intermediation-investment complementarity works to propagate the shock.

For a large enough shock, the formation of new relationships can be too slow to offset the ongoing destruction of existing relationships caused by low liquidity. The economy then enters a region in which the collapse state is the *unique* equilibrium. In this instance, collapse is not induced by a sunspot; rather, it becomes unavoidable when the financial structure state variable is too low to support adequate investment incentives.

Our theory sheds light on processes that underlie phenomena such as financial collapses and credit crunches. Outflows of liquidity can damage financial structure by breaking up credit market relationships, thereby generating further outflows. Timely injections of liquid assets by a policy authority can stave off financial damage, and may prevent the economy from collapsing.

The results developed here relate to the large literature on "coordination failure," pioneered by Bryant (1983) and Cooper and John (1988), that stresses the possibility of low-activity steady states sustained by macroeconomic complementarities. We contribute to this literature by proposing a novel source of complementarities stemming from frictions associated with credit relationships. Since the number of relationships can adjust only gradually, our approach to coordination failure offers added insights with respect to macroeconomic dynamics. Importantly, in our model the low-activity steady state may arise as the unique equilibrium following a large shock, rather than as one of several equilibria that are conditioned on a sunspot.[3]

The liquidity flows approach to credit market frictions, which focusses on external finance, represents an alternative to established "internal equity" models of credit frictions that highlight contracting problems created by limited borrower collateral; see Bernanke and Gertler (1989), Kiyotaki and Moore (1997) and Carlstrom and Fuerst (1997). Both the liquidity flows and internal equity approaches provide mechanisms for propagating macroeconomic shocks, and they can be viewed as complementary perspectives. The liquidity flows model offers additional predictions pertaining to aggregate increasing returns and multiple steady states.

Several other papers have considered how lender wealth constraints contribute to credit market frictions. Using static models with adverse selection, Farmer (1988a,1988b) shows that limited access to liquidity can affect the efficiency of contracting and the extent of factor utilization. Diamond (1984) and Holmstrom and Tirole (1997a) present microeconomic models that link financial intermediation to the lender's wealth position. There have been a number of previous theoretical models of long-term relationships in credit markets; see Freixas and Rochet (1997, chapter 4). These models have focussed on properties of the contract between borrower and lender.[4]

Our analysis draws on formal methods used in the labor literature, particularly Mortensen and Pissarides (1994) and Ramey and Watson (1997). The modelling of liquidity allocation to lenders is closely connected to the case of "costly capital adjustment" considered in Den Haan, Ramey and Watson (2000), where the level of a capital input must be chosen before a relevant shock is realized. For both liquidity allocation and costly capital adjustment, the key idea is that inputs cannot adjust costlessly.

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[3] Cooper and Corbae (1997) propose a model of financial collapse based on coordination failure in financial intermediation. In their paper, households must simultaneously commit to payments in order to finance the fixed costs of intermediation, and collapse occurs when households believe that other households will not contribute. Periodic collapse outcomes are tied to a sunspot process.

[4] Dell'Ariccia and Garibaldi (1998) have recently developed a matching model of bank lending, considering how matching frictions and breakup costs affect dynamic responses to short-term interest rate shocks.

Finally, Holmstrom and Tirole (1997b) have considered a model in which low entrepreneurial collateral can lead to termination of projects. In their model, financial intermediaries transfer wealth between entrepreneurs in order to avoid terminations, and aggregate liquidity can be insufficient when entrepreneurs' wealth is highly correlated. Our model, in contrast, considers external finance rather than collateral, and shows that insufficiency of aggregate liquidity can be brought on by damage to financial structure.

Section two presents the model, and section three lays out the equilibrium conditions. Multiple steady state equilibria are derived in section four, propagation of shocks is considered in section five, and section six concludes.

2. Model

2.1. Liquidity Flows. Consider an economy in which there is a single asset that may be used for investment and contracting. Let H_t denote the aggregate quantity of this asset at the start of period t. Households determine H_t in making their consumption/savings decisions at the end of period $t-1$. For simplicity, we assume that households maximize the expected present value of consumption, where r is the discount rate. Let $\beta \equiv 1/(1+r)$ denote the implied discount factor.

Households are not able to invest in a spot asset market. Rather, investment must flow through middlemen, called *lenders*, who operate on behalf of the households. Assume that there is a unit mass of lenders. Each lender obtains a portion of the aggregate liquid assets, subject to random influences. Let h_t denote the quantity of the asset received by a particular lender at the start of period t. For this analysis, we abstract from details of the asset allocation process, and assume simply that h_t is determined by a reduced-form *liquidity allocation rule*. In particular, h_t is drawn according to the continuous distribution function $\nu(h_t|H_t)$, which is increasing in H_t according to first-order stochastic dominance. The support of $\nu(h_t \mid H_t)$ is bounded above by $h^u(H_t)$, where we assume $\lim_{H_t \to 0} h^u(H_t) = 0$. Assume further that $h_t = 0$ is contained in the support of $\nu(h_t|H_t)$.[5] At the end of period

[5]This specification makes the simplifying assumption that, conditional on aggregate liquidity, idiosyncratic liquidity fluctuations are i.i.d. across lenders and over time. The key property is that liquidity is not

t, the lender remits to the household the liquidity allocation together with any net returns from current period operations.

2.2. New Projects. Lenders invest their liquidity allocations in projects operated by entrepreneurs. In each period, a lender is either matched in an ongoing relationship with an entrepreneur, or else the lender is searching for a new entrepreneur in whom to invest. There is an infinite mass of potential entrepreneurs, each of whom may choose to promote new projects. To promote a new project, an entrepreneur must incur an effort cost of $c > 0$ per period. Let V_t denote the mass of promoting entrepreneurs in period t.

The probability that an unmatched lender identifies a promoting entrepreneur in a given period depends on how scarce new projects are relative to the total number of unmatched lenders. Let U_t denote the mass of unmatched lenders at the start of period t, and let $\theta_t \equiv V_t/U_t$. A given unmatched lender identifies a new project in period t with probability $\lambda(\theta_t)$, which is continuous, strictly increasing and satisfies $\lambda(0) = 0$. When a new project is identified, the lender and entrepreneur begin an ongoing relationship in the following period. The aggregate flow of new projects in a period is given by $U_t\lambda(\theta_t)$. The probability that a promoting entrepreneur is matched with a lender, $\lambda(\theta_t)/\theta_t$, is assumed to be strictly decreasing in θ_t, and satisfies $\lim_{\theta_t\to 0} \lambda(\theta_t)/\theta_t = 1$ and $\lim_{\theta_t\to\infty} \lambda(\theta_t)/\theta_t = 0$.

2.3. Ongoing Relationships. Lenders and entrepreneurs in ongoing relationships negotiate contracts and engage in production in each period. Production requires both investable assets and effort by the entrepreneur. Entrepreneurial effort is also necessary for maintenance of the project. Entrepreneurial effort may be either high or low. If the liquidity allocation is h_t and the entrepreneur chooses high effort, then output produced in the period is given by $f(h_t)$. Assume that $f(h_t)$ is strictly increasing, strictly concave, and satisfies $f(0) = 0$.

allocated perfectly across lenders. More realistic specifications would allow liquidity allocations to be based on information about the lender (e.g., whether or not the lender is currently in a relationship). Positive autocorrelation in allocations could also be considered. Our conclusions carry over, however, as long as some chance of liquidity misallocation remains. See the conclusion for further discussion.

The choice of high effort further implies that the project is maintained, and the relationship continues into the following period.

If low effort is chosen, then zero output is produced, and instead the entrepreneur obtains a private effort benefit of $x > 0$. Moreover, low effort causes the project to fail, and the relationship is severed. In this case, it is assumed that the lender cannot be rematched with a new entrepreneur until the following period.[6]

At the start of the period, the lender and entrepreneur observe the current-period realization of h_t, and following this they negotiate a contract that determines the division of joint surplus, along with the entrepreneur's effort choice. Total contractible assets for the period consist of the liquidity allocation plus any output produced. In particular, for simplicity we assume that the entrepreneur does not have private assets that can be transfered as part of the contract. The lender is assumed to appropriate the liquidity allocation and output, and the contract specifies payments to the entrepreneur conditional on his effort choice. Contract negotiation consists of a first and final offer by the lender, which the entrepreneur may either accept or reject. If the entrepreneur rejects the offer, then the relationship is severed, and the lender becomes unmatched. The lender may also opt to sever the relationship in lieu of making an offer. In either of these cases, the lender may be rematched in the current period.

To rule out the possibility that the liquidity allocation rule generates increasing returns in the aggregate, we impose a joint restriction on $\upsilon(h_t|H_t)$ and $f(h_t)$. For given $z \geq 0$, let $\mu(z|H_t)$ be defined by

$$\mu(z|H_t) \equiv \int_z^{h^u(H_t)} f(h_t) d\upsilon(h_t|H_t).$$

Then $\mu(0|H_t)/H_t$ is taken to be strictly decreasing in H_t. Assume also that $\lim_{H_t \to 0} \mu(0|H_t)/H_t = \infty$ and $\lim_{H_t \to \infty} \mu(0|H_t) = 0$. We show below that aggregate increasing returns arise once the breakup of relationships is considered.

[6] As an alternative to severance of the relationship, it can be assumed that low effort leads to a "breakdown" that causes the project to produce zero output for some number of periods. Under this assumption, λ can be specified as the (constant) probability of project recovery. It is straightforward to rework the model and extend the results to this case.

3. Equilibrium

3.1. Equilibrium Contract and Breakup Margin. The equilibrium contract in an ongoing relationship is derived as follows. Let p_t denote the payment to the entrepreneur if high effort is chosen. Clearly, the equilibrium contract will specify that the entrepreneur receives nothing if low effort is chosen, since negative payments are not possible as a consequence of limited entrepreneurial liability. The entrepreneur selects high effort if and only if

$$p_t + g_t^e \geq x, \tag{1}$$

where g_t^e indicates the present value of the entrepreneur's expected future payments if the relationship continues. Note that the entrepreneur obtains a future return of zero if the relationship breaks up, since potential entrepreneurs dissipate all the rents from promoting new projects. Thus, the right-hand side of (1) reflects only current-period private benefit of low effort.

In negotiating a contract that induces high effort, the lender will offer the smallest value of p_t that satisfies (1). Moreover, p_t must be nonnegative, since the entrepreneur has no assets. Thus, p_t is given by

$$p_t = \max\{x - g_t^e, 0\}. \tag{2}$$

The contract is further constrained by the available contractible assets; i.e., lender cannot draw on the future value of the relationship to make current payments to the entrepreneur. This means that the following liquidity constraint must be satisfied:

$$f(h_t) + h_t \geq p_t. \tag{3}$$

We next consider whether the lender and entrepreneur benefit from continuing the relationship. The lender prefers to offer a contract that induces high effort, as opposed to severing the relationship at the start of the period, if and only if the following condition holds:

$$f(h_t) + g_t - (p_t + g_t^e) \geq w_t, \tag{4}$$

where g_t indicates the present value of expected future joint returns from continuing the relationship, and w_t denotes the present value of the lender's expected future returns from entering the pool of unmatched lenders in period t; both terms are net of future liquidity allocations. The left hand side of (4) constitutes the share received by the lender, consisting of the the joint returns, $f(h_t) + g_t$ less current and future payments to the entrepreneur.

Further, the lender never offers a contract that induces low effort. Such a contract would give the lender a current-period net return of zero, while the present value of expected future returns would be βw_{t+1}, based on commencing search for a new entrepreneur at the start of the next period. In any equilibrium, this is less than the return of w_t that the lender obtains from breaking up the relationship at the start of the period. Thus, the lender prefers to continue the relationship if and only if (4) holds. Condition (1) implies that the entrepreneur always prefers to continue the relationship as long as the contract induces high effort.

In summary, the relationship continues into the next period if and only if (3) and (4) hold, where p_t is given by (2). If either (3) or (4) are violated, then the relationship breaks up at the start of the period.

Observe that both constraints (3) and (4) become weaker as h_t is increased: $f(h_t)$ is an increasing function of h_t; and, conditional on the path of aggregate liquidity, the future values g_t, g_t^e and w_t are independent of h_t. Thus, there exists a *breakup margin* $\underline{h}_t$ having the property that the relationship continues if $h_t \geq \underline{h}_t$, and breaks up if $h_t < \underline{h}_t$. Combining (2), (3) and (4), it follows that the the following expression determines the breakup margin:

$$f(\underline{h}_t) + \min\{\underline{h}_t - \max\{x - g_t^e, 0\}, g_t - \max\{x, g_t^e\} - w_t\} = 0. \tag{5}$$

3.2. Future Returns. The present value of expected future joint returns, g_t, is determined by

$$g_t = \beta E_t[\mu(\underline{h}_{t+1}|H_t) + (1 - \nu(\underline{h}_{t+1}|H_{t+1}))g_{t+1} + \nu(\underline{h}_{t+1}|H_{t+1})w_{t+1}]. \tag{6}$$

The present value of the entrepreneur's expected future returns, g_t^e, satisfies

$$g_t^e = \beta E_t[\int_{\underline{h}_{t+1}}^{\infty} (p_{t+1} + g_{t+1}^e) d\nu(h_{t+1}|H_{t+1})]. \tag{7}$$

As for the present value of the lender's expected future returns from entering the pool of unmatched lenders, w_t, we have

$$w_t = \lambda(\theta_t)(g_t - g_t^e) + (1 - \lambda(\theta_t))\beta E_t[w_{t+1}]. \tag{8}$$

3.3. Matching. Let N_t denote the mass of lenders who enter period t in ongoing relationships. The mass of lenders who seek new projects in period t, U_t, may be expressed as

$$U_t = (1 - N_t) + \nu(\underline{h}_t|H_t)N_t. \tag{9}$$

The first term on the right-hand side of (9) indicates lenders who enter the current period unmatched, while the second term captures lenders whose relationships have broken up at the start of the current period.

The ratio of new projects to unmatched lenders, θ_t, is determined by the following rent dissipation condition:

$$\frac{\lambda(\theta_t)}{\theta_t} g_t^e = c. \tag{10}$$

It is possible to satisfy (10) if $c/g_t^e \leq 1$; otherwise, $\theta_t = 0$ holds.

The law of motion for the mass of lenders in ongoing relationships is given by

$$N_{t+1} = (1 - \nu(\underline{h}_t|H_t))N_t + U_t\lambda(\theta_t), \tag{11}$$

where the first term on the right-hand side captures ongoing relationships from the preceding period, and the second term reflects newly-formed relationships.

3.4. Aggregate Liquidity. Let R_t denote the one-period aggregate net rate of return on investment:

$$R_t \equiv \frac{N_t \int_{\underline{h}_t}^{\infty} (f(h_t) - p_t) d\nu(h_t|H_t)}{H_t}, \tag{12}$$

where p_t is determined by (2). Households demand a rate of return of r on their investments. Thus, H_t gives equilibrium aggregate liquidity if

$$R_t = r. \tag{13}$$

Additionally, if $R_t < r$ at $H_t = 0$, then $H_t = 0$ gives an equilibrium value.[7]

To summarize, given an initial number of relationships N_1, equations (5)-(13) jointly determine $\underline{h}_t$, g_t, g_t^e, w_t, U_t , θ_t, N_{t+1}, R_t and H_t for $t = 1, 2, \ldots$.

3.5. Entrepreneur's Future Value.

The following lemma derives a constraint on the entrepreneur's future value that greatly simplifies the analysis of equilibria.

Lemma 1. *In any equilibrium, $g_t^e \leq x$ for every t.*

Proof. From (2), we see that the most an entrepreneur can obtain in any single period is x. Thus, $g_t^e \leq \sum_{k=1}^{\infty} x\beta^{k-1} = x/(1-\beta)$. Suppose there is a contingency (including realizations of h_t and other variables describing the economy) under which $g_t^e > \alpha$, for some $\alpha > x$. From (7) it follows that there must be a contingency in this relationship, occurring with positive probability, for which $p_{t+1} + g_{t+1}^e > \alpha/\beta$. Note that (2) implies $p_{t+1} + g_{t+1}^e = \max\{x, g_{t+1}^e\}$. If $p_{t+1} + g_{t+1}^e = x$, then we have $x > \alpha/\beta$, contradicting $x < \alpha$; thus, $p_{t+1} + g_{t+1}^e = g_{t+1}^e > \alpha/\beta$. Iterating this argument, observe that for each k there is a contingency occurring in period $t+k$ under which $g_{t+k}^e > \alpha/\beta^k$. This yields a contradiction for large k. *Q.E.D.*

The lemma is a consequence of the fact that the entrepreneur's current payment would be zero if his future returns exceeded x in any period.

4. Multiple Steady States

In this section we demonstrate that the model possesses multiple steady state equilibria (SSE), associated with differing levels of aggregate activity. Making use of Lemma 1, steady

[7] The appendix presents a more detailed model of household investment decisions that yields these equilibrium conditions.

state versions of equilibrium conditions (5)-(11) may be written as follows:

$$f(\underline{h}) + \min\{\underline{h} - x + g^e, g - x - w\} = 0, \tag{14}$$

$$g = \beta[\mu(\underline{h}|H) + (1 - \nu(\underline{h}|H))g + \nu(\underline{h}|H)w], \tag{15}$$

$$g^e = \beta(1 - \nu(\underline{h}|H))x, \tag{16}$$

$$w = \lambda(\theta)(g - g^e) + (1 - \lambda(\theta))\beta w, \tag{17}$$

$$U = (1 - N) + \nu(\underline{h}|H)N, \tag{18}$$

$$\frac{1}{\theta}\lambda(\theta)g^e = c \text{ if } \frac{c}{g^e} \leq 1,\ \theta = 0 \text{ if } \frac{c}{g^e} > 1, \tag{19}$$

$$N = (1 - \nu(\underline{h}|H))N + U\lambda(\theta). \tag{20}$$

As for condition (13), let

$$R = \frac{N}{H}[\mu(\underline{h}|H) - (x - g^e)(1 - \nu(\underline{h}|H)]. \tag{21}$$

Equilibrium aggregate liquidity satisfies the following:

$$\text{Either } R = r, \text{ or } H = 0 \text{ and } R < r. \tag{22}$$

For purposes of establishing existence of SSE, the eight equilibrium conditions (14)-(22) may be conveniently compressed into three equations. To do this, three functions are defined. First, let $\psi^A(\underline{h}, H, \theta, x)$ be given by

$$\psi^A(\underline{h}, H, \theta, x) \equiv f(\underline{h}) + \min\{\underline{h} - (1 - \beta(1 - \nu(\underline{h}|H))x,$$
$$\frac{(1 - \lambda(\theta))\beta\mu(\underline{h}|H) + \lambda(\theta)\beta(1 - \nu(\underline{h}|H))x}{1 - \beta(1 - \nu(\underline{h}|H))(1 - \lambda(\theta))} - x\}.$$

The function $\psi^A(\underline{h}, H, \theta, x)$ is obtained by substituting (15)-(17) into (14). Second, $\psi^B(\underline{h}, H, x)$ is defined implicitly by

$$\frac{\lambda(\psi^B(\underline{h}, H, x))}{\psi^B(\underline{h}, H, x)}\beta(1 - \nu(\underline{h}|H))x = c. \tag{23}$$

If (23) cannot be satisfied by any $\psi^B(\underline{h}, H, x) > 0$, then set $\psi^B(\underline{h}, H, x) = 0$. Note that this function derives from substituting (16) into (19). Third, define $\psi^C(\underline{h}, H, \theta, x)$ by

$$\psi^C(\underline{h}, H, \theta, x) \equiv \frac{\lambda(\theta)}{\lambda(\theta) + (1 - \lambda(\theta))\nu(\underline{h}|H)}[\frac{\mu(\underline{h}|H)}{H}$$

$$-\frac{(1-\beta(1-\nu(\underline{h}|H)))(1-\nu(\underline{h}|H))x}{H}].$$

In this case, (16), (18) and (20) are substituted into the first part of (22).

Lemma 2. *Suppose $\underline{h}$, H and θ satisfy the following conditions:*

$$\psi^A(\underline{h}, H, \theta, x) = 0, \tag{24}$$

$$\theta = \psi^B(\underline{h}, H, x), \tag{25}$$

$$\min\{r - \psi^C(\underline{h}, H, \theta, x), H\} = 0. \tag{26}$$

Then there exists a SSE having equilibrium values $\underline{h}$, H and θ.

Proof. Let $g - w$ and N be given by

$$g - w = \frac{(1-\lambda(\theta))\beta\mu(\underline{h}|H) + \lambda(\theta)\beta(1-\nu(\underline{h}|H))x}{1-\beta(1-\nu(\underline{h}|H))(1-\lambda(\theta))}, \tag{27}$$

$$N = \frac{\lambda(\theta)}{\lambda(\theta) + (1-\lambda(\theta))\nu(\underline{h}|H)}. \tag{28}$$

Let g^e and U be determined by (16) and (18), respectively. Then (24) implies (14), (25) is equivalent to (19), and (26) assures that (22) is satisfied. Finally, using (27) it is possible to define g and w that satisfy (15) and (17), while (18) and (28) together imply (20). *Q.E.D.*

We now state conditions under which there exists a SSE with $H > 0$, meaning that there is productive activity in the economy. Households are willing to supply positive aggregate liquidity only if they earn a sufficiently high return. From the definition of R, (21), it may be seen that the return depends on the size of payments to the entrepreneur: returns may be too low if $x - g^e$ is too high. Since payments to the entrepreneur are based on x, it follows that small x is needed to sustain a SSE with $H > 0$. According to (16), however, potential entrepreneurs have little incentive to promote new projects when x is low. As a consequence, for given values of x, we must specify values of c that are sufficiently low to support $\theta > 0$, and thus $N > 0$, in equilibrium.

It follows that low values of x must be accompanied by sufficiently low values of c in order to ensure that $\theta > 0$. The following proposition uses a technically tractable condition on c that is sufficient for this.

Proposition 1. *Choose any $\omega \in (0,1)$, and let c be given by $c = \omega\beta x$. There exists a SSE with $H > 0$ if x is sufficiently small.*

Proof. Let $\underline{h}^L(H,x)$ denote the smallest value of $\underline{h}$ that satisfies

$$f(\underline{h}^L) + \underline{h}^L - (1 - \beta(1 - \nu(\underline{h}^L|H)))x = 0. \tag{29}$$

Thus, (14) and (16) together ensure that $\underline{h} \geq \underline{h}^L(H,x)$ in any SSE. Note that for all $H \geq 0$, $x > 0$ implies $\underline{h}^L(H,x) > 0$, while $\lim_{x\to 0} \underline{h}^L(H,x) = 0$. Further, for a given interval $[H^l, H^u]$, $\underline{h}^L(H,x)$ is a continuous function of H on $[H^l, H^u]$ if x is sufficiently small.

Define the following:

$$\nu^L(H,x) = \nu(\underline{h}^L(H,x)|H),$$

$$\theta^L(H,x) = \psi^B(\underline{h}^L(H,x), H, x).$$

These are continuous functions of H on $[H^l, H^u]$ when x is sufficiently small, based on continuity of $\underline{h}^L(H,x)$. Note also that $\lim_{x\to 0} \nu^L(H,x) = 0$. Further, using (23) we have

$$\frac{\lambda(\theta^L(H,x))}{\theta^L(H,x)}\beta(1 - \nu^L(H,x))x = c = \omega\beta x.$$

From this it follows that $\lim_{x\to 0} \theta^L(H,x) = \theta' > 0$. Observe that (25) is satisfied by setting $\theta = \theta^L(H,x)$.

Next, let H^* be the unique solution to

$$\frac{\mu(0|H^*)}{H^*} = r. \tag{30}$$

Note that $H^* > 0$. Choose values $H^l \in (0, H^*)$ and $H^u > H^*$. Suppressing arguments, we have, for all $H \in [H^l, H^u]$:

$$\lim_{x\to 0} \frac{(1 - \lambda(\theta^L))\beta\mu(\underline{h}^L|H) + \lambda(\theta^L)\beta(1 - \nu^L)x}{1 - \beta(1 - \nu^L)(1 - \lambda(\theta^L))} = \frac{(1 - \lambda(\theta'))\beta\mu(0|H)}{1 - \beta(1 - \lambda(\theta'))} > 0.$$

This implies, for sufficiently small x:

$$\underline{h}^L - (1-\beta(1-\nu^L))x < \frac{(1-\lambda(\theta^L))\beta\mu(\underline{h}^L|H) + \lambda(\theta^L)\beta(1-\nu^L)x}{1-\beta(1-\nu^L)(1-\lambda(\theta^L))}. \tag{31}$$

Moreover, continuity of the functions $\underline{h}^L$, ν^L and θ^L means that (31) will hold for all $H \in [H^l, H^u]$ when x is small. Applying (29) and (31), it follows that for sufficiently small x, $\psi^A(\underline{h}^L, H, \theta^L, x) = 0$ for all $H \in [H^l, H^u]$. Thus, (24) is satisfied.

Finally, observe that

$$\lim_{x\to 0} \frac{\lambda(\theta^L)}{\lambda(\theta^L) + (1-\lambda(\theta^L))\nu^L} = 1,$$

which implies

$$\lim_{x\to 0} \psi^C(\underline{h}^L, H, \theta^L, x) = \frac{\mu(0|H)}{H}.$$

Under our assumptions, we have

$$\frac{\mu(0|H^l)}{H^l} > r > \frac{\mu(0|H^u)}{H^u}.$$

Thus, the following is obtained when x is sufficiently small:

$$\psi^C(\underline{h}^L, H^l, \theta^L, x) > r > \psi^C(\underline{h}^L, H^u, \theta^L, x).$$

Letting x be small enough to assure that $\psi^C(\underline{h}^L, H, \theta^L, x)$ is continuous in H on $[H^l, H^u]$, it follows that $\psi^C(\underline{h}^L, H', \theta^L, x) = r$ for some $H' \in [H^l, H^u]$. Thus, (26) holds. Setting $\underline{h} = \underline{h}^L(H', x)$, $H = H'$ and $\theta = \theta^L(H', x)$, and invoking Lemma 2, we obtain a SSE with $H > 0$. *Q.E.D.*

The proof makes use of the fact that when $x = 0$, there exists a unique SSE, having aggregate liquidity $H^* > 0$ determined by (30). The proof constructs an equilibrium that is a perturbation of this SSE. The assumption $c = \omega\beta x$ is used to rule out the possibility that N collapses to zero when x is small.

In the positive-activity equilibrium of Proposition 1, $\underline{h} > 0$ will hold as a consequence of $x > 0$, and thus there is a positive probability that a low liquidity allocation will cause a given ongoing relationship to break up in any period. Contracting problems, in the form of wealth constraints combined with entrepreneurial moral hazard, cause credit market relationships to be fragile in the face of fluctuations in available liquidity. This financial fragility is manifested in the ongoing failure of established relationships, offset in equilibrium by the creation of new projects.

When x is strictly positive, there will also exist a SSE in which $H = 0$, meaning that all economic activity ceases. The following proposition establishes the existence of this "collapse" equilibrium.

Proposition 2. *There exists a SSE with $H = 0$ if $x > 0$.*

Proof. Fix x and suppose that $\inf_{H \geq 0} \underline{h}^L(H, x) = 0$. Take a sequence $\{H^n\}_{n=1}^{\infty}$ with $\lim_{n\to\infty} \underline{h}^L(H^n, x) = 0$.

$$\lim_{n\to\infty} \{f(\underline{h}^L(H^n, x)) + \underline{h}^L(H^n, x) - (1 - \beta(1 - \nu(\underline{h}^L(H^n, x)|H^n)))x\}$$

$$= -x(1 - \beta) < 0,$$

which contradicts the condition (29) that defines $\underline{h}^L(H, x)$. Thus, we may choose $H' > 0$ that satisfies

$$h^u(H') < \inf_{H \geq 0} \underline{h}^L(H, x).$$

It follows that for all $H < H'$, $\mu(\underline{h}|H) = 0$ and $\nu(\underline{h}|H) = 1$ for any $\underline{h}$ that is consistent with (14) and (16). Thus, $R = 0$ for all such $\underline{h}$, for all $H < H'$, and (26) holds for $H = 0$. Given this value of H, (24) holds when $\underline{h}$ is determined by $f(\underline{h}) + \underline{h} - x = 0$, and setting $\theta = 0 = \psi^B(\underline{h}, 0, x)$ completes the proof, in view of Lemma 2. *Q.E.D.*

In the collapse equilibrium of Proposition 2, entrepreneurs have no incentive to promote new projects, since no liquid assets are available for ongoing relationships. As relationships are not created, households obtain a zero return on any liquid assets given to the lenders, and so nothing is invested. The key credit market imperfection in this case is that households cannot channel liquidity directly to new entrepreneurs; rather, liquidity must flow through lenders who are subject to frictions in allocating funds to entrepreneurs.

Propositions 1 and 2 together imply that the model possesses multiple SSEs for small positive values of x: a positive-activity equilibrium coexists with the collapse equilibrium. The existence of multiple equilibria reflects *complementarity* between the structure of financial intermediation, represented by the number of ongoing relationships, and investment. High values of N bring about more efficient intermediation, leading to high values of R and a positive choice of H by the households. Low values of N, in contrast, cause intermediation to be less efficient. Since R is low as a consequence, households do not wish to invest, and the collapse outcome obtains.

The relationship between aggregate liquidity and financial intermediation is illustrated in Figure 1. For convenience, the example uses a liquidity allocation rule having a two-point support, with one of the points being $h_t = 0$.[8] The figure reports the values of the number of relationships, N, the lender matching probability, $\lambda(\theta)$, and the breakup probability, $\nu(\underline{h}|H)$, that emerge from solutions to (20), (24) and (25) for given values of aggregate liquidity, H. Note first that for a region of very low H, no relationships form; thus, financial intermediation cannot occur at all unless aggregate liquidity exceeds a minimum threshold. For a range of high values of H, the equations have a single solution, with positive N and $\lambda(\theta)$, and values of $\nu(\underline{h}|H)$ lying below unity. In this case, an increase in H leads to a larger number of relationships, a higher matching probability, and a lower breakup probability. Finally, for a middle range of H, positive-valued solutions coexist with zero-valued solutions.

Figure 2 plots values of the average return, R, associated with the positive-valued solutions in Figure 1; average returns are derived using (21). The figure illustrates the existence of multiple SSE's. The region of low H for which $R = 0$ may be noted, and point A, at the origin, indicates the collapse equilibrium derived in Proposition 2. For a middle region of H, average returns rise with H, as the favorable effect of higher liquidity on the number

[8] For the examples, the support of $\nu(h_t \mid H)$ is taken to consist of 0 and $h^u(H)$, where $\nu(0|H) = 1 - \gamma H^{0.2}$ and $h^u(H) = H(1 - \nu(0|H))^{-1}$. Although this specification does not satisfy the assumption that $\nu(h_t|H)$ should be continuous, the discrepency is inessential, since we can regard the specification as approximating a continuous distribution function that increases sharply at $h = 0$ and $h = h^u(H)$. The examples use the parameters $v(h_t) = h_t^{0.33}$, $x = 3$, $\beta = 0.96$, $\rho^x = 0$, $c = 0.342$, $m(U_t, V_t) = 0.25U_t^{0.5}V^{0.5}$, and $\gamma = 0.401$.

of ongoing relationships outweighs the effect of diminishing returns within individual relationships. Complementarity between H and N thus generates aggregate increasing returns on this region. For high H, diminishing returns come to dominate. These conflicting effects give rise to a pair of equilibria with positive H, at points B and C. Returns for the $x = 0$ economy, in which all lenders are matched in every period, are given in the upper curve. In contrast to the fragile economy, the $x = 0$ economy exhibits diminishing returns for all H, and the unique steady-state equilibrium lies at point D.

5. Propagation of Shocks

This section shows how complementarity between the structure of financial intermediation and investment helps to propagate aggregate shocks. To illustrate how shocks are propagated, we consider a shock that takes the form of a surprise reduction in the number of ongoing relationships. This may be interpreted as a negative productivity shock. The economy is assumed to enter period 1 in a positive-activity SSE, with N giving the number of ongoing relationships. At the start of the period, however, N drops to a lower level N_1, reflecting exogenous breakup of relationships.

Figure 3 presents numerically calculated equilibrium values associated with a negative shock to the number of relationships. Observe that the number returns only gradually to its SSE value; thus, the shock has a persistent effect on the structure of intermediation. The need for lenders to gradually rematch is one source of this persistence. In addition, there is a large and persistent decline in aggregate liquidity following the shock, reflecting the investment response to lower levels of N_t and higher levels of $\nu(\underline{h}_t|H_t)$. Correspondingly, the breakup probability $\nu(\underline{h}_t|H_t)$ remains persistently above its steady-state level, further slowing the return of N_t to the steady state.

The resulting effects on output are shown in Figure 4, which compares output in the equilibrium to the path that would emerge if H_t were held fixed at its SSE level, so that propagation would be driven solely by lender rematching. Observe that intermediation-investment feedbacks serve to magnify the shock on impact, and overall they roughly double the output loss in this example.

The credit market response to the shock involves two competing effects. On one hand, structure is repaired via matching, as relationships are reformed. On the other hand, adverse feedbacks raise the rate at which relationships break up along the adjustment path. The latter effect can dominate, so that the market becomes unable to escape the collapse outcome. The next proposition gives conditions under which this situation can arise.

Proposition 3. *If c lies sufficiently close to βx, then for N_1 sufficiently small, equilibrium is unique and $H_t = N_{t+1} = 0$ for all t.*

Proof. Applying Lemma 1 to (2) and (7), we have $g_t^e = \beta(1 - \nu(\underline{h}_t|H_t))x$. Thus, the following holds in any equilibrium:

$$\min\{\underline{h}_t - x + g_t^e, g_t - x - w_t\} \leq \underline{h}_t - (1-\beta)x.$$

Let $\underline{h}'$ be defined by

$$f(\underline{h}') + \underline{h}' - (1-\beta)x = 0.$$

It follows that $\underline{h}_t \geq \underline{h}'$ must hold in equilibrium. Now let $H' > 0$ satisfy $h^u(H') < \underline{h}'$, and let $N' > 0$ satisfy

$$\frac{N'\mu(0|H')}{H'} < r.$$

If $N_1 < N'$, then either $H_1 < H'$, implying $\mu(\underline{h}_1|H_1) = 0$ and $\nu(\underline{h}_1|H_1) = 1$, or $H \geq H'$, meaning that the return is less than r even when $\underline{h}_t = x = 0$. Thus, when $N_1 < N'$, it follows that $R_1 < r$ for all H_1, and the only value consistent with equilibrium is $H_1 = 0$.

Next, condition (10) may be written

$$\frac{\lambda(\theta_t)}{\theta_t} = \frac{c}{\beta(1 - \nu(\underline{h}_t|H_t))x}.$$

As long as $c/\beta x$ is sufficiently close to unity, we can be sure that θ_t lies as close to zero as desired, so that $\lambda(\theta_t) < N'$ holds.

Now observe that $N_1 < N'$ implies $\nu(\underline{h}_1|H_1) = 1$, and so $U_1 = 1$ and $N_2 = \lambda(\theta_1) < N'$. In turn, $N_2 < N'$ implies $H_2 = 0$, by the argument used above. Thus, $\nu(\underline{h}_2|H_2) = 1$, and in fact $\theta_1 = 0$; then $N_2 = H_2 = 0$ is implied. By induction, this result can be extended to all

t. It may be verified, as in the proof of Proposition 2, that these values give an equilibrium. *Q.E.D.*

The key point is that collapse of the credit market emerges as the *unique equilibrium* for a sufficiently large shock to the structure of financial intermediation. In contrast to the existing literature, coordination failure in this case does not entail equilibrium selection or sunspot arguments that serve to align agents' expectations. What happens instead is that a big shock does so much damage to financial structure that recovery becomes impossible. Proposition 3 identifies a condition on the entrepreneur's project promotion cost, c, that is sufficient to ensure that the rate at which relationships are rematched is too slow to offset the ongoing increase in their destruction due to investment-intermediation feedbacks.[9] The collapse outcome becomes an absorbing state, and only a sustained exogenous injection of liquidity can restore credit market activity.

6. Conclusion

Long-term relationships between borrowers and lenders are a common feature of credit market trading. This paper considers a model wherein relationships constitute channels through which investable assets flow from households to entrepreneurs. Relationships become liquidity constrained when lenders receive low asset flows, causing them to break up. As the number of relationships falls, financial intermediation becomes less efficient, and the returns earned by households decline. Thus, financial intermediation and household investment are complementary in the aggregate. Because of this complementarity, multiple steady state equilibria may exist, including a low activity "collapse" outcome. For a sufficiently large shock to financial structure, collapse becomes the unique equilibrium.

[9] It should be noted that steady-state equilibria with positive investment can exist under conditions supporting Proposition 3. In particular, assume that for given $\underline{h}_t$, $\nu(\underline{h}_t|H_t)$ may be made arbitrarily small by taking H_t sufficiently large. Then positive-investment equilibria will exist under the conditions of the theorem as long as $f(h_t)$ is sufficiently large for h_t outside of a neighborhood of zero, i.e., for a sufficiently high level of productivity.

Our results have implications for policy responses to financial crises. Importantly, outflows of liquid assets associated with crises may cause lasting damage to financial structure, inhibiting any subsequent inflows of assets by making investment less attractive. These feedbacks to financial structure make for slower recovery from crisis episodes, and may drive economies into persistent low-activity states. Policy authorities can potentially prevent financial damage through interventions designed to support aggregate liquidity. Such interventions are motivated by the need to preserve valuable channels through which external investment may flow back into the economy. Policy delays can be costly in this context, since damage continues as long as aggregate liquidity remains low.

The model relies on a number of simplifying assumptions that make the analysis tractable, but that are not essential for our main conclusions. In particular, we assume that liquid assets are allocated in a manner that treats all lenders symmetrically from the ex ante standpoint. This assumption may be weakened in a number of ways. The model could be extended to include institutions that seek to obtain a better ex ante allocation. Households may invest effort in directing asset flows toward more favorable lenders, for example. Lenders may attempt to swap liquidity after observing their allocations and their success at locating new projects.[10] The key assumption, however, is that such processes do not eliminate all allocation errors. As the structure of intermediation weakens, errors rise, leading to lower returns and reduced investment, thereby weakening structure further. Thus, the fundamental complementarity driving the results is robust to allowing for richer liquidity allocation structures.

The assumption that entrepreneurs do not use personal wealth for investment or contracting may also be weakened without undermining the main results. In such an extended model, internal and external finance would combine to determine the breakup margin. High reliance on internal finance can be viewed as a mechanism for insuring against the fragility problems that we highlight. Constructing a model that combines limited collateral with

[10]Holmstrom and Tirole's (1997b) model of financial intermediation incorporates a form of such ex post liquidity exchange.

relationship-based liquidity flows represents an important topic for future research.

APPENDIX

In this appendix we provide a more detailed model of household investment that supports the equilibrium conditions for aggregate liquidity. Let $S > 1$ be an integer that indicates the total mass of households. At the start of period t, each household selects an investment level, H_t/S, and then transfers these assets to a randomly chosen lender. If the lender is not matched with an entrepreneur in period t, then the household obtains a net return of zero for the period. If the lender is currently matched, then the household obtains a gross return that is proportional to its investment.

If a given household selects investment level H_t'/S, while all other households choose H_t/S, then the net proceeds to the household from the matched lender are

$$\xi(\frac{H_t'}{S_t} + (S-1)\frac{H_t}{S})\frac{H_t'}{H_t' + (S-1)H_t},$$

where

$$\xi(H_t) \equiv \int_{\underline{h}_t}^{\infty} (f(h_t) - p_t) d\nu(h_t|H_t).$$

Thus, the household's overall discounted expected net return is given by

$$-\frac{H_t'}{S_t} + \beta[N_t\xi(\frac{H_t'}{S_t} + (S-1)\frac{H_t}{S})\frac{H_t'}{H_t' + (S-1)H_t} + \frac{H_t'}{S}].$$

Maximizing this return with respect to H_t', and imposing the equilibrium condition $H_t' = H_t$, yields the following first-order condition:

$$-1 + \beta[N_t\frac{\partial\xi(H_t)}{\partial H_t}\frac{1}{S} + N_t\xi(H_t)\frac{S-1}{S^2 H_t^2} + 1] = 0.$$

This condition becomes equivalent to (13) as S approaches infinity. It can be checked that the household's second-order conditions are satisfied at $H_t' = H_t$.

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Figure 1 : Effect of liquidity on steady-state values.

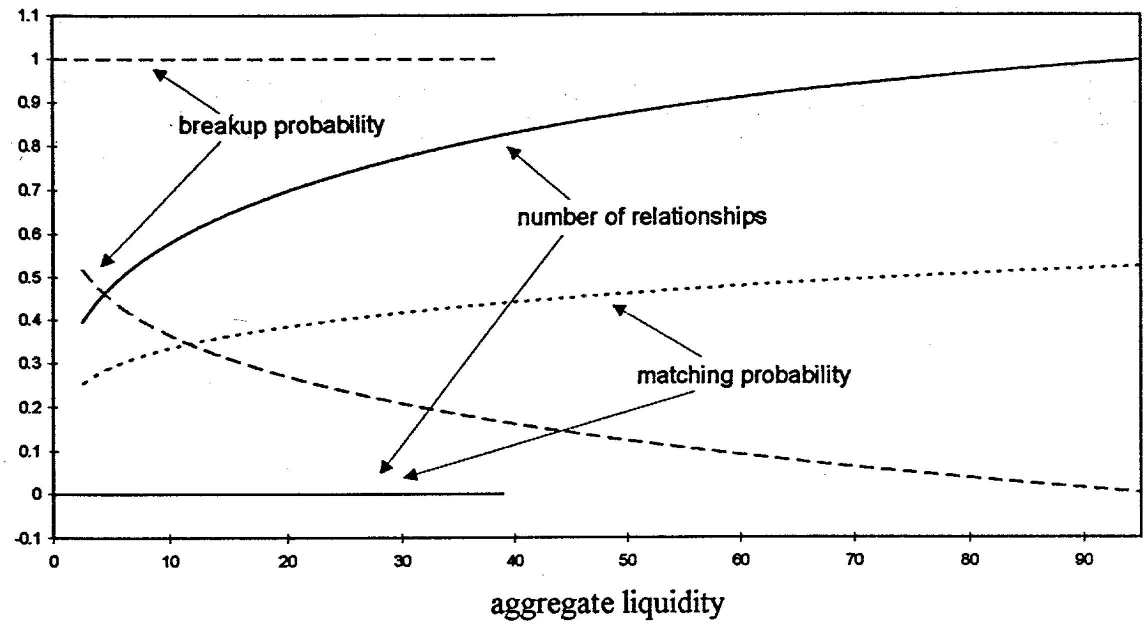

Figure 2: Effect of liquidity on average returns.

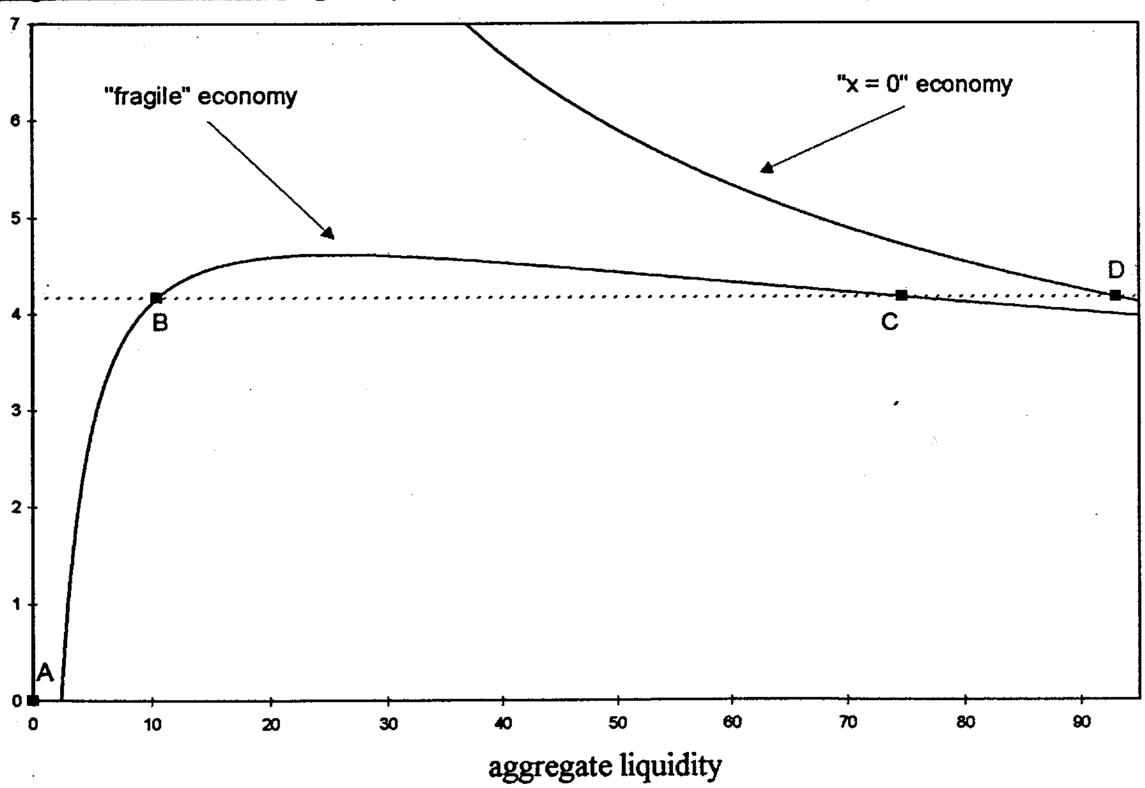

Figure 3: Effect of financial structure shock.

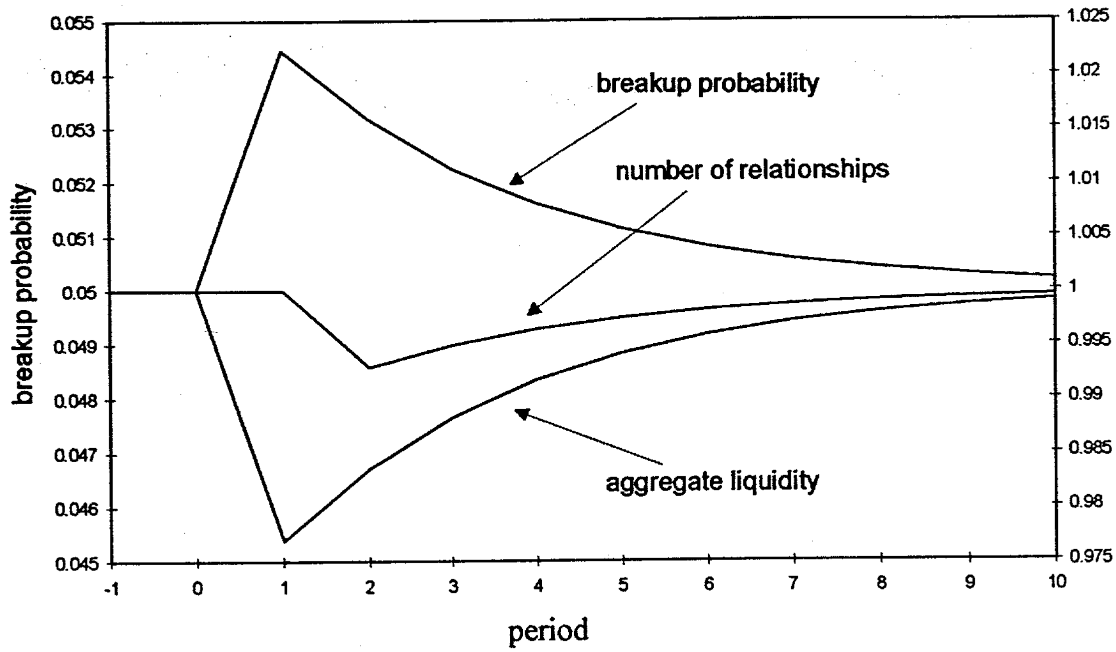

Figure 4: Implications for output.