Title
MEASUREMENT OF T = 2 ELASTIC nn CROSS SECTIONS

Authors
Colton, E.
Malamud, E.
Schlein, P.E.
et al.

Publication Date
1970-11-01

Submitted to Physical Review

UCRL-20053
Preprint

c.2

MEASUREMENT OF T = 2 ELASTIC ππ CROSS SECTIONS

E. Colton, E. Malamud, P. E. Schlein
A. D. Johnson, V. J. Stenger, and P. G. Wohlmut

November 1970

AEC Contract No. W-7405-eng-48

LAWRENCE RADIATION LABORATORY
UNIVERSITY of CALIFORNIA BERKELEY

DISCLAIMER

MEASUREMENT OF T = 2 ELASTIC ππ CROSS SECTIONS

E. Colton
University of California, Lawrence Radiation Laboratory, Berkeley, 94720
E. Malamud
National Accelerator Laboratory, Batavia, 60510
P. E. Schlein[+]
University of California, Los Angeles, 90024
A. D. Johnson, V. J. Stenger, and P. G. Wohlmut
University of Hawaii, Honolulu, 96822

ABSTRACT

Experimental differential cross sections from data on the reaction $\pi^- p \to (\pi^-\pi^-)\Delta^{++}$ at beam momenta of 2.7, 3.0, 3.2, 3.9, and 4.2 GeV/c have been extrapolated to the one-pion-exchange pole to obtain the $\pi^-\pi^-$ elastic scattering cross section. An attempt is made to correct for background due to kinematic overlap with the competing OPE process $\pi^- p \to \rho^0(\pi^- p)$. Analyses done independently on the data in two beam momentum groupings at ~3.0 and ~4.0 GeV/c give consistent results of a roughly constant 7 - 11 mb cross section for $\pi^-\pi^- \to \pi^-\pi^-$ over the dipion mass range 450 - 750 MeV. Our results are compared with available results from other analyses and with several theoretical predictions for the T = 2 s-wave phase shift δ_0^2.

In this note we present a determination of the $\pi^-\pi^-$ elastic scattering cross section by means of a modified Chew-Low[1] extrapolation to the pion-exchange pole in the reaction

$$\pi^- p \to (\pi^-\pi^-)\Delta^{++}(1238) \quad . \qquad (1)$$

It is thereby assumed that the process depicted in Fig. 1(a) (referred to hereafter as process A) plays a significant role in the overall reaction. 21,941 events of the type

$$\pi^- p \rightarrow \pi^- \pi^- \pi^+ p \qquad (2)$$

with beam momenta from 2.7 to 4.2 GeV/c have been used in the analysis. The data were separated into two sets: 10,773 events at 2.7[2], 3.0[3], 3.2[4] GeV/c (referred to hereafter as the 3 GeV/c data) and 11,168 events at 3.9[5] and 4.2[4] GeV/c (referred to hereafter as the 4 GeV/c data). Similar investigations were independently performed on each set.

The peripheral sample of data corresponding to reaction (1) is obtained by applying the simultaneous cuts:

$$\begin{array}{rcll} & m & < 0.75 & \text{GeV} \\ 1.12 < & M & < 1.34 & \text{GeV} \\ & t & < 0.5 & \text{GeV}^2 \end{array} \qquad (3)$$

where t is the square of the momentum transfer from the target proton to the outgoing $\pi^+ p$ system (we take t to be positive in the physical region) and the $m(M)$ denote the $\pi^-\pi^-$ $(\pi^+ p)$ invariant masses. We restrict the discussion henceforth to those events which satisfy the restrictions of Eqs. (3), 1334 at 3 GeV/c and 978 at 4 GeV/c. The differential distributions $d\sigma/dm$, $d\sigma/dM$, and $d\sigma/dt$ are presented separately for the 3 and 4 GeV/c data in Figs. 2(a) - 2(f). The smooth curves drawn through the data in Fig. 2 are the results of a one-pion-exchange-model calculation described below which includes the effect of background due to the process shown in Fig. 1(b) (referred to hereafter as Process B).

The experimental $d\sigma/dt$ for reaction (1) have been extrapolated to the one-pion-exchange pole for four different $\pi^-\pi^-$ mass regions at each beam momentum following the procedure of Ma et al.[6] This procedure differs from the traditional Chew-Low procedure in that $(d\sigma/dt)_{exp.}$ is normalized to the pole equation[7] modified by a form factor F(m, M, t):

$$\frac{d^3\sigma}{dtdmdM} = \frac{1}{4\pi^3 m_p^2 P_L^2} \; \frac{1}{(t+\mu^2)^2} \; \frac{m^2 q(m)\sigma(m)}{(\hbar c)^2} \; M^2 Q(M)\sigma(M) \; F(m,M,t) \tag{4}$$

instead of to the pole equation alone. In Eq. (4) $\mu(m_p)$ are the pion (proton) rest masses, P_L is the laboratory beam momentum, and q(Q) are the magnitudes of the momenta in the $\pi^-\pi^-(\pi^+p)$ rest frames. The σ functions are the on-mass-shell vertex elastic scattering cross sections. All quantities have the units GeV or mb, except F which is dimensionless.

The form factor, F, can be any smooth function which reduces to unity at the pion-exchange pole. We use the phenomenological Dürr-Pilhuhn[8] (referred to hereafter as DP) or Benecke-Dürr[9] factors which have been shown[10,11,12] to approximately summarize experimental Chew-Low distributions for strong-interaction reactions of the classes $Xp \to X\pi^+n$ and $Xp \to X\pi^-\Delta^{++}$ over a large range of beam momenta. The usefulness of this procedure lies in the fact that the complexity of the t-dependence of the function to be extrapolated is minimized, thereby decreasing the order of the polynomial necessary to fit the experimental points.

The function which we extrapolate to the pole for each specified Δm interval is

$$"\sigma" = \frac{(d\sigma/dt)_{exp.}}{(d\sigma/dt)_{DP-OPE}} \tag{5}$$

where $(d\sigma/dt)_{DP-OPE}$ is (4) after integration over m and M. The on-shell $\pi^-\pi^-$ cross section σ is set equal to 1 mb in calculating the denominator, so that at the pole $\sigma^{\pi^-\pi^-}_{\text{on-shell}}$ = "σ". A polynomial in t is then fit to the experimental "σ" points. If $(d\sigma/dt)_{\text{DP-OPE}}$ had precisely the same t dependence as $(d\sigma/dt)_{\text{exp.}}$ then "σ" would be independent of t. Thus the presence of linear or higher terms in the polynomial fit allows for departures of $(d\sigma/dt)_{\text{DP-OPE}}$ from $(d\sigma/dt)_{\text{exp.}}$. The DP factors used in calculating $(d\sigma/dt)_{\text{DP-OPE}}$ have the form (see Ref. 10 or 11):

$$F(m,M,t) = \left[\frac{2.3-\mu^2}{2.3+t}\right]^2 \left(\frac{Q_t}{Q}\right)^2 \left[\frac{1+16Q^2}{1+16Q_t^2}\right] \left[\frac{(M+m_p)^2+t}{(M+m_p)^2-\mu^2}\right] \tag{6}$$

where Q_t is the momentum of the incoming target proton in the Δ^{++} rest frame. No correction is made to the $\pi^-\pi^-$ vertex factor. This last assumption (valid according to DP for s-wave vertices) is in disagreement with expected off-shell effects near threshold.[16] By allowing "σ" to depend on t in the extrapolation fits, any objections to the use of this form factor near threshold are satisfied. See further comments on this point below.

The experimental "σ" values given by Eq. (5) are shown as the solid dots in Figs. 3(a) - 3(d) and Figs. 3(e) - 3(h) for four different di-pion mass ranges in the 3 and 4 GeV/c data, respectively. The linear extrapolation function "σ" = a + bt, shown as the solid lines in Fig. 3 has been fit to the data points in each case. The resulting confidence levels, and the best fit values for a and b as well as the extrapolated on-shell cross sections ($\sigma^{\pi^-\pi^-}_{\text{on-shell}}$) are presented in Table I. All of the fits are seen to yield acceptable confidence levels. Also given in Table I are the T = 2 s-wave ππ phase shifts[13] (δ_0^2) obtained from

$$\sigma^{\pi^-\pi^-}_{\text{on-shell}} = 8\,\pi\,\bar{\lambda}^2 \sin^2 \delta_0^2 \tag{7}$$

in which it is assumed that only the s-wave contribution is significant for m < 0.75 GeV. In Eq. (7) the quantity $\bar{\lambda}^2$ is calculated at the central value of the $\pi^-\pi^-$ mass bin. Since the results at both beam momenta are similar, the average cross sections for each di-pion mass range are also presented in Table I.

We turn now to the subject of background[14] from process B and its effects upon the pole extrapolation. For events satisfying the cuts (3), we display in Fig. 4 the differential distributions of $d\sigma/dm_{\pi^-\pi^+}$, $d\sigma/dM_{\pi^-p}$ and $d\sigma/dt_{\pi^-p}$ for each beam momentum. Two combinations are plotted for each event. The curves drawn through the data in Fig. 4 are similar to those presented in Fig. 2 and are discussed below. The strong peripheral $\rho^0(765)$ component which is observed in Figs. 4(a) and 4(d) constitutes evidence for the process B [see Fig. 1(b)]. With the use of the DP-OPE description of this process, which has been demonstrated[10,11,12] to summarize rather well the Chew-Low distributions of available processes of this type, we attempt to subtract this background contribution and redo the pole extrapolation analysis described above.

Assuming only π exchange and neglecting interference terms between the competing processes, the background contribution to a $(d\sigma/dt)$ point is given by $\Delta\sigma_B/\Delta t$ where $\Delta\sigma_B$ is the cross section contribution from process B subject to the cuts on m, M, and t, specified in Eq. (3). The width Δt is the width of the t bin in question. The calculation of $\Delta\sigma_B$ is discussed in a brief appendix to this paper. The function which is extrapolated to the pole for each specified Δm interval is, therefore,

$$"\sigma" = \frac{(d\sigma/dt)_{\mathrm{exp.}} - \Delta\sigma_B/\Delta t}{(d\sigma/dt)_{\mathrm{DP\text{-}OPE}}} \tag{8}$$

As before $\sigma(m)$ is set equal to 1 mb in calculating the denominator so that at the pole $\sigma^{\pi^-\pi^-}_{\text{on-shell}} = "\sigma"$.

The experimental "σ" points, calculated using Eq. (8) are displayed in Fig. 3 as the open circle points. The dashed lines and the open-circle extrapolated cross sections at $t = -\mu^2$ are the results of fits of "σ" = a + bt to these points. The parameters and cross section results are presented in Table II. The cross sections in the mass range 440 - 750 MeV are seen to have a more or less consistent value of 7 - 11 mb.[15]

The results in Table II indicate that large positive b coefficients are still required in the 4 GeV/c data for dipion mass $m < 0.55$ GeV. If we assume that the background subtraction has been properly done and that interference effects are insignificant, the necessity for the non-zero b parameters in the fits to the background subtracted "σ" vs. t points in this mass range indicate that DP-OPE is a poor approximation to the t-distribution for $\pi\pi$ masses just above threshold, as suggested by Lovelace.[16]

As a means of illustrating the degree of overall fit quality of a strict DP-OPE model to the data (<u>which we stress is not assumed in the actual extrapolations</u>, <u>where we permit the linear coefficient b to be non-zero</u>), we show in Figs. 2 and 4 curves calculated assuming an incoherent sum of Processes A and B. The contribution for Process A is assumed given by Eqs. (4) and (6) with $\sigma(m) = 10$ mb for $m < 0.75$ GeV. The contribution from Process B is described in the appendix. The integrated theoretical cross sections [for Process A + Process B subject to the same experimental cuts (3)] of 0.21 mb and 0.15 mb for the 3 and 4 GeV/c data are to be compared with the experimental values of (0.24 ± 0.01) mb and (0.17 ± 0.01) mb, respectively. The integrated theoretical cross sections from Process A account for 55% and 59% of the total, respectively, at the two momenta. The curves describe the data rather well in Figs. 2 and 4, especially in the m and t distributions, however the position of the $\Delta^{++}(1238)$ is shifted to lower mass in the data and the curve does not adequately reproduce the data in the $\Delta^{0}(1238)$ region of the $\pi^{-}p$ mass spectrum of the 3 GeV/c data.

In conclusion we compare our results for $\sigma^{\pi^-\pi^-}_{\text{on-shell}}$ with previous determinations.[12,17-20] Figure 5 shows most available values. The dashed error bars in Fig. 5 represent the uncertainties in the smooth curve at the $\pi\pi$ mass value in question. In those cases in which only δ_0^2 was given (e.g., Baton et al.[17]), $\sigma^{\pi^-\pi^-}_{\text{on-shell}}$ was calculated using Eq. (7). Similarly we present in Fig. 6 the set of related δ_0^2 values as well as the prediction (dashed curve) of Arnowitt[21] from current algebra and the prediction (solid curve) of Wagner[22] who utilized a unitarized Veneziano formula. While our results are in good agreement with theoretical expectations, the rather large divergence of available experimental results suggests that unknown systematic uncertainties exist in many determinations. A high statistics electronics experiment on the more background-free $\pi^{+}p \rightarrow \pi^{+}\pi^{+}n$ reaction at higher beam momentum should permit a more reliable determination of δ_0^2 and of the as yet unknown contributions of d-wave and higher angular momentum states.

APPENDIX

The calculation of the curves shown in Figs. 2 and 4 was performed with a Monte-Carlo program.[23] The integrations were taken over the full kinematic range of variables subject to the cuts of Eq. (3). In order to calculate the reflection of Process B on the histograms relevant to Process A, and vice versa, it is necessary to include information about the angular distribution in each vertex center-of-mass. In all cases this was approximated by the on-shell angular distribution.[24] The π-proton angular distributions were calculated from the CERN phase shift analysis[25] and the $\pi\pi$ angular distributions reconstructed from the phase shift analysis of Malamud and Schlein[26] (the results are insensitive to the choice of solution) for $m < 1$ GeV and for $m > 1$ GeV from Wolf.[12] The Dürr-Pilkuhn correction at the π^-p vertices are identical to those used by Colton et al.[27] in an analysis of $pp \to (p\pi^-)(p\pi^+)$, in which it was demonstrated that these corrections are unnecessary for $M \geq 1.6$ GeV.

REFERENCES AND FOOTNOTES

* Supported in part by the United States Atomic Energy Commission.

\+ J. S. Guggenheim Fellow, on leave 1969-1970, CERN, Geneva.

1. G. F. Chew and F. E. Low, Phys. Rev. 113, 1640 (1959).
2. P. R. Klein et al., Phys. Rev. 150, 1123 (1966).
3. A. W. Key et al., Phys. Rev. 166, 1430 (1968).
4. S. U. Chung, Lawrence Radiation Laboratory Report No. UCRL-16881-Rev. (1967), see also Phys. Rev. 165, 1491 (1968).
5. K. Abe et al., Phys. Rev. Letters 22, 251 (1969).
6. Z. Ming Ma et al., Phys. Rev. Letters 23, 342 (1969).
7. See e.g., E. Ferrari and F. Selleri, Supp. Nuovo Cimento 24, 453 (1962).
8. H. P. Dürr and H. Pilkuhn, Nuovo Cimento 40, 899 (1965).
9. J. Benecke and H. P. Dürr, Nuovo Cimento 56, 269 (1968). At the low values of t used in this analysis, these form factors are numerically almost identical to the Dürr-Pilkuhn factors.
10. G. Wolf, Phys. Rev. Letters 19, 925 (1967).
11. P. Schlein, Meson Spectroscopy, eds. C. Baltay and A. H. Rosenfeld (W. A. Benjamin, Inc., 1968), p. 161, and *Proceedings of the Conference on $\pi\pi$ and $K\pi$ Interactions*, Argonne National Laboratory, May 14-16, 1969, pps. 1 and 446.
12. G. Wolf, Phys. Rev. 182, 1538 (1969).
13. For the sign of the phase shift see e.g. J. P. Baton and G. Laurens, *Proceedings of the Conference on $\pi\pi$ and $K\pi$ Interactions*, Argonne National Laboratory, May 14-16, 1969, p. 131.

14. P. G. Wohlmut, et al., Nucl. Phys. **B18**, 505 (1970). See also Ref. 5.

15. We note that these numbers are in agreement with the preliminary quoted value of ~ 10 mb quoted by E. Malamud and P. E. Schlein, *Proceedings of the Conference on $\pi\pi$ and $K\pi$ Interactions*, Argonne National Laboratory, May 14-16, 1969, p. 93. In the extraction of the phase shift δ_0^2 at the K-mass from the cross sections, a factor of 2 was inadvertantly left out in *The Proceedings*, causing our value of $|\delta_0^2|$ at the K-mass to come out $(17 \pm 3)^\circ$ instead of the value $(12 \pm 3)^\circ$ which more properly goes with a $\pi^-\pi^-$ cross section of 10 mb.

16. C. Lovelace, Ibid, p. 562. See also the Comments on p. 18.

17. J. P. Baton, G. Laurens, and J. Reignier, Nucl. Phys. **B3**, 349 (1967).

18. W. M. Katz, T. Ferbel, P. F. Slattery, and H. Yuta, *Proceedings of the Conference on $\pi\pi$ and $K\pi$ Interactions*, Argonne National Laboratory, May 14-16, 1969, p. 300.

19. G. V. Beketov et al., Institute for Theoretical and Experimental Physics Report No. ITEP-767, (1970) unpublished.

20. W. D. Walker et al., Phys. Rev. Letters **18**, 630 (1967).

21. R. Arnowitt, *Proceedings of the Conference on $\pi\pi$ and $K\pi$ Interactions*, Argonne National Laboratory, May 14-16, 1969, p. 619.

22. F. Wagner, Nuovo Cimento **64A**, 189 (1969).

23. J. H. Friedman, Lawrence Radiation Laboratory Report No. UCRL-19206-Rev., June 1969.

24. Colton et al.,[27] have shown in a series of papers on pp studies that this is a rather good approximation for peripherally produced π-proton systems. See also E. Colton and P. Schlein, *Proceedings of the Conference on $\pi\pi$ and $K\pi$ Interactions*, Argonne National

Laboratory, May 14-16, 1969, p. 1.

25. A. Donnachie, R. G. Kirsopp, and C. Lovelace, CERN-TH 838, Addendum; 1967.

26. See Reference 15.

27. E. Colton, P. E. Schlein, E. Gellert, and G. A. Smith, University of California, Los Angeles, Report No. UCLA-1036 (1970) to be published in the Physical Review (Vol. D1-1).

Table I. Results of fits of the experimental solid "σ" points shown in Fig. 2 to the assumed forms "σ" = a + bt.

		$\pi^-\pi^-$ Mass Range (GeV)			
Data set	Fit quantities	0.28-0.44	0.44-0.55	0.55-0.65	0.65-0.75
	Con. lev.(%)	84	45	93	78
3 GeV/c	a (mb)	3.4± 2.0	6.6± 1.8	9.7± 1.9	11.9± 2.1
	b (mb/GeV^2)	231.1±33.2	128.2±19.7	63.1±13.7	29.9±12.1
	σ_{extrap}(mb)	-1.1± 2.5	4.1± 2.2	8.4± 2.1	11.3± 2.3
	δ_0^2 (Deg.)	0[a]	-7.7± 2.0	-14.3± 1.8	-20.2± 2.1
	Con. lev.(%)	17	71	85	29
4 GeV/c	a (mb)	4.6± 2.6	6.5± 2.0	10.7± 1.8	7.0± 1.8
	b (mb/GeV^2)	279.4±45.1	169.5±24.0	55.2±14.4	58.9±14.1
	σ_{extrap}(mb)	-0.9± 3.4	3.2± 2.3	9.6± 2.0	5.9± 2.1
	δ_0^2 (Deg.)	0[a]	-6.7± 2.4	-15.3± 1.7	-14.5± 2.6
Average value of the 2 sets	Con. lev.(%)	50	58	89	53
	a (mb)	3.8± 1.6	6.55± 1.3	10.2± 1.3	9.1± 1.4
	b (mb/GeV^2)	248.5±26.8	145.0±15.2	59.5± 9.9	42.3± 9.2
	σ_{extrap}(mb)	-1.0± 2.0	3.7± 1.6	9.0± 1.4	8.4± 1.5
	δ_0^2 (Deg.)	0[a]	-7.3± 1.5	-14.8± 1.2	-18.0± 1.6

[a] Since $\sigma_{extrap} \propto \mathrm{Sin}^2\, \delta_0^2$ and σ_{extrap} was less than zero the phase shift has been set to zero.

Table III. Results of fits of the experimental open circle "σ" points shown in Fig. 2 to the assumed form "σ" = a + bt.

		$\pi^-\pi^-$ Mass Range (GeV)			
Data set	Fit quantities	0.28-0.44	0.44-0.55	0.55-0.65	0.65-0.75
	Con. lev.(%)	80	40	72	91
3 GeV/c	a (mb)	1.2± 2.4	8.9± 1.9	10.4± 1.9	10.9± 2.1
	b (mb/GeV^2)	97.5±36.7	27.0±20.5	13.5±14.0	0.4±12.3
	σ_{extrap}(mb)	-0.7± 3.0	8.4± 2.2	10.1± 2.1	10.9± 2.3
	δ_0^2 (Deg.)	0^a	-11.0± 1.5	-15.7± 1.7	-19.9± 2.2
	Con. lev.(%)	2	65	29	18
4 GeV/c	a (mb)	5.1± 2.7	7.2± 2.0	10.9± 1.9	6.1± 2.0
	b (mb/GeV^2)	155.3±47.1	91.0±24.6	-5.9±15.0	29.3±15.0
	σ_{extrap}(mb)	2.1± 3.5	5.4± 2.4	11.1± 2.1	5.5± 2.2
	δ_0^2 (Deg.)	-3.3± 2.8	-8.7± 1.9	-16.5± 1.6	-14.0± 2.9
Average value of the 2 sets	Con. lev.(%)	41	52.5	50.5	54.5
	a (mb)	2.9± 1.8	8.1± 1.4	10.7± 1.4	8.4± 1.4
	b (mb/GeV^2)	119.5±29.0	53.2±15.7	4.5±10.2	12.1± 9.5
	σ_{extrap}(mb)	0.5± 2.3	7.0± 1.6	10.6± 1.5	8.1± 1.6
	δ_0^2 (Deg.)	-1.3± 2.3	-10.1± 1.2	-16.1± 1.2	-17.8± 1.8

a. Since $\sigma_{extrap} \propto \sin^2 \delta_0^2$ and $\sigma_{extrap} < 0$ the phase shift has been set to zero.

FIGURE CAPTIONS

Fig. 1. The reactions considered in this paper, A is the one we attempt to isolate in order to study $\pi^-\pi^-$ elastic scattering. Process B, which is the dominant mode of $\rho^0\pi^-p$ production at the beam momenta considered here, is a background source for the study of Process A.

Fig. 2. Experimental differential cross sections: (a) $d\sigma/dm$ where m is $\pi^-\pi^-$ mass; (b) $d\sigma/dM$ where M is π^+p mass; (c) $d\sigma/dt$ where t is the momentum transfer to the π^+p system for the data at 3 GeV/c. (d) - (f) are the same for the data at 4 GeV/c. The smooth curves drawn are the predictions of the strict Dürr-Pilkuhn OPE model described in the text.

Fig. 3. Experimental "σ" values plotted as a function of t. The solid data points are calculated using Eq. (5) whereas the open circle data points are corrected for background with the use of Eq. (8) as described in the text. The solid and dashed lines represent the best fits of the linear expression "σ" = a + bt to the data. The extrapolated cross sections at the left side of each plot are the values of this best fit function at $t = -\mu^2$.

Fig. 4. Experimental differential cross sections for events satisfying the selection criteria defined in Eq. (3) for the data at 3 GeV/c: (a) $d\sigma/dm_{\pi^-\pi^+}$, (b) $d\sigma/dM_{\pi^-p}$, (c) $d\sigma/dt_{\pi^-p}$. (d) - (f) are the same for the 4 GeV/c data. Two combinations are plotted for each event. The smooth curves are the predictions of the strict Dürr-Pilkuhn OPE model described in the text.

Fig. 5. Comparison between $\sigma^{\pi^-\pi^-}_{\text{on-shell}}$ determined in this analysis with (open-circle points) and without (solid dots) the background subtraction procedure described in the text, and other published values. The solid curve is from Reference 12 and the symbols ■, □, ▼, and ▲ refer to data from References 17-20, respectively. In those cases where δ^2_0 values only are published (References 17, 20) the corresponding cross sections are derived using Eq. (7) in the text.

Fig. 6. Comparison between δ^2_0 determined in this analysis with (open-circle points) and without (solid dots) the background subtraction procedure described in the text, and other published values. The dashed and solid curves are theoretical predictions from References 21 and 22, respectively. The symbols ■, □, ▼, and ▲ refer to data from References 17-20, respectively. In the case where only $\sigma^{\pi^-\pi^-}_{\text{on-shell}}$ values are published (Reference 19) the corresponding phase shifts are derived using Eq. (7) in the text.

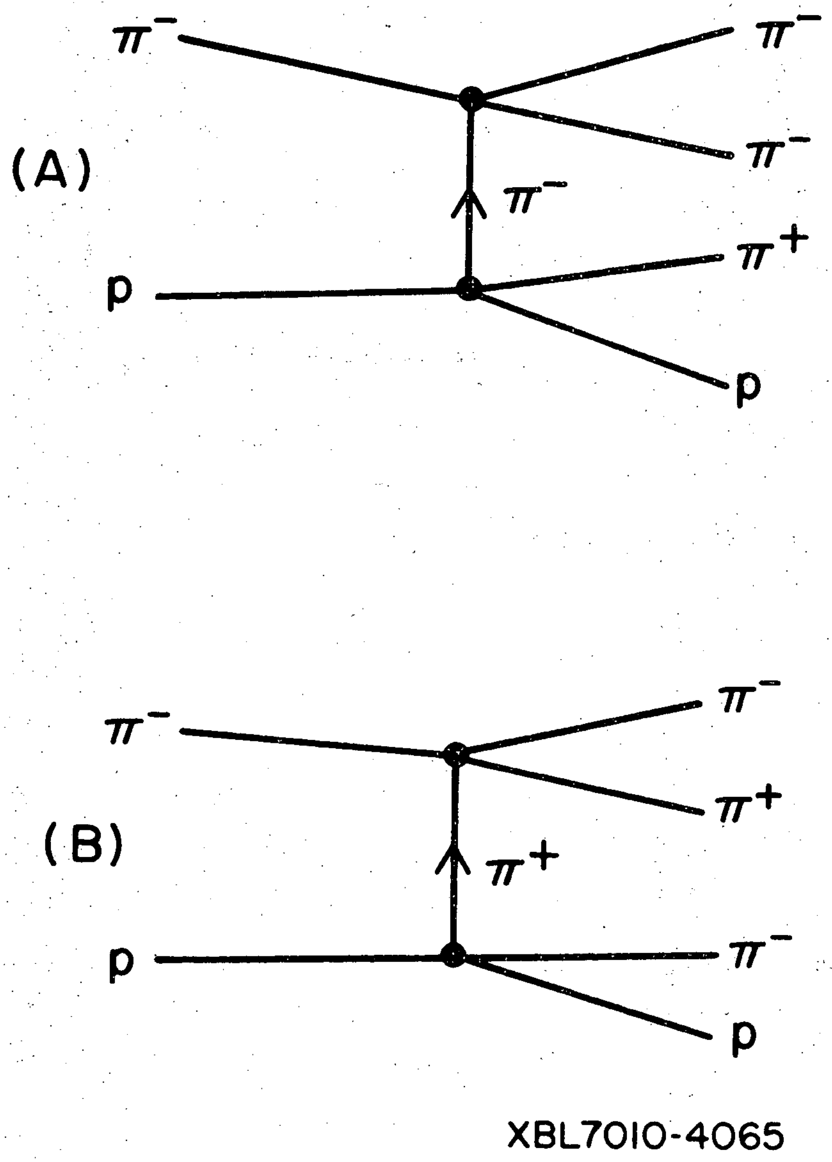


FIGURE 1

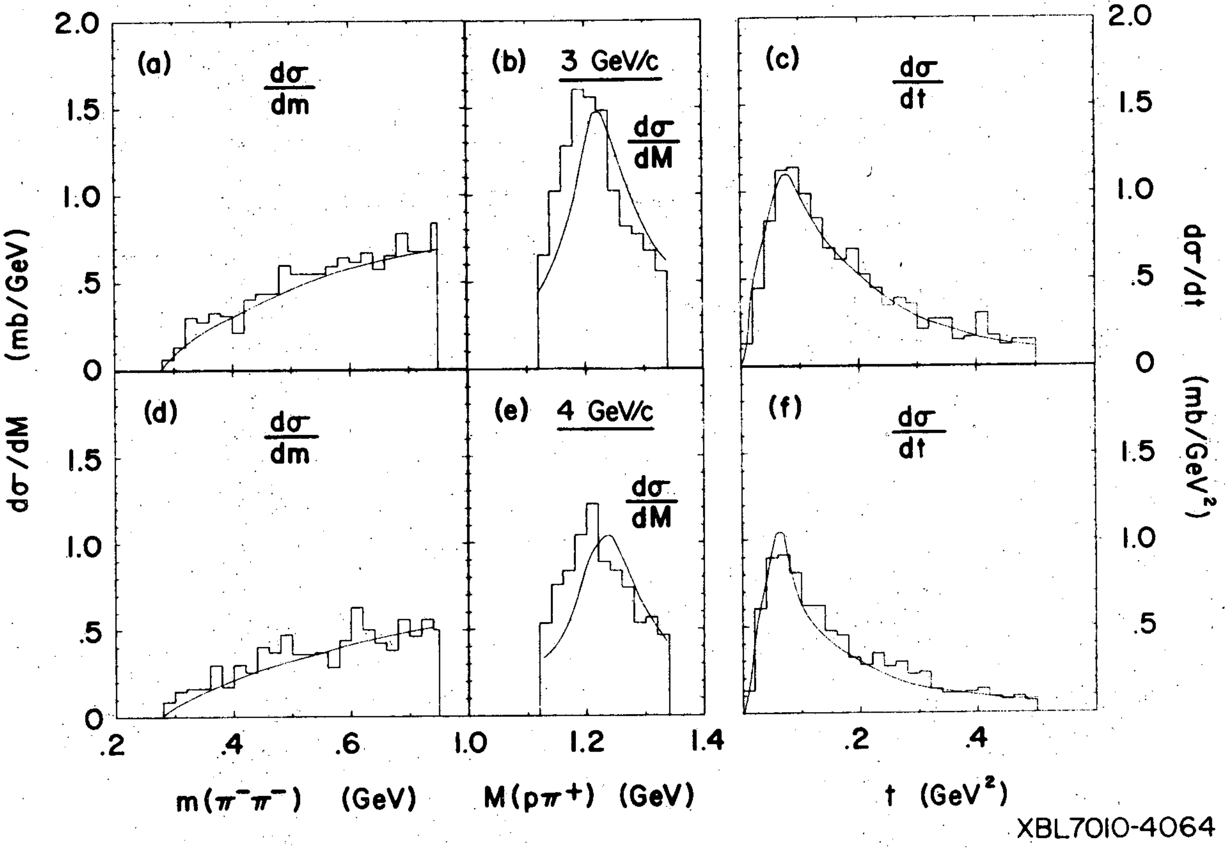
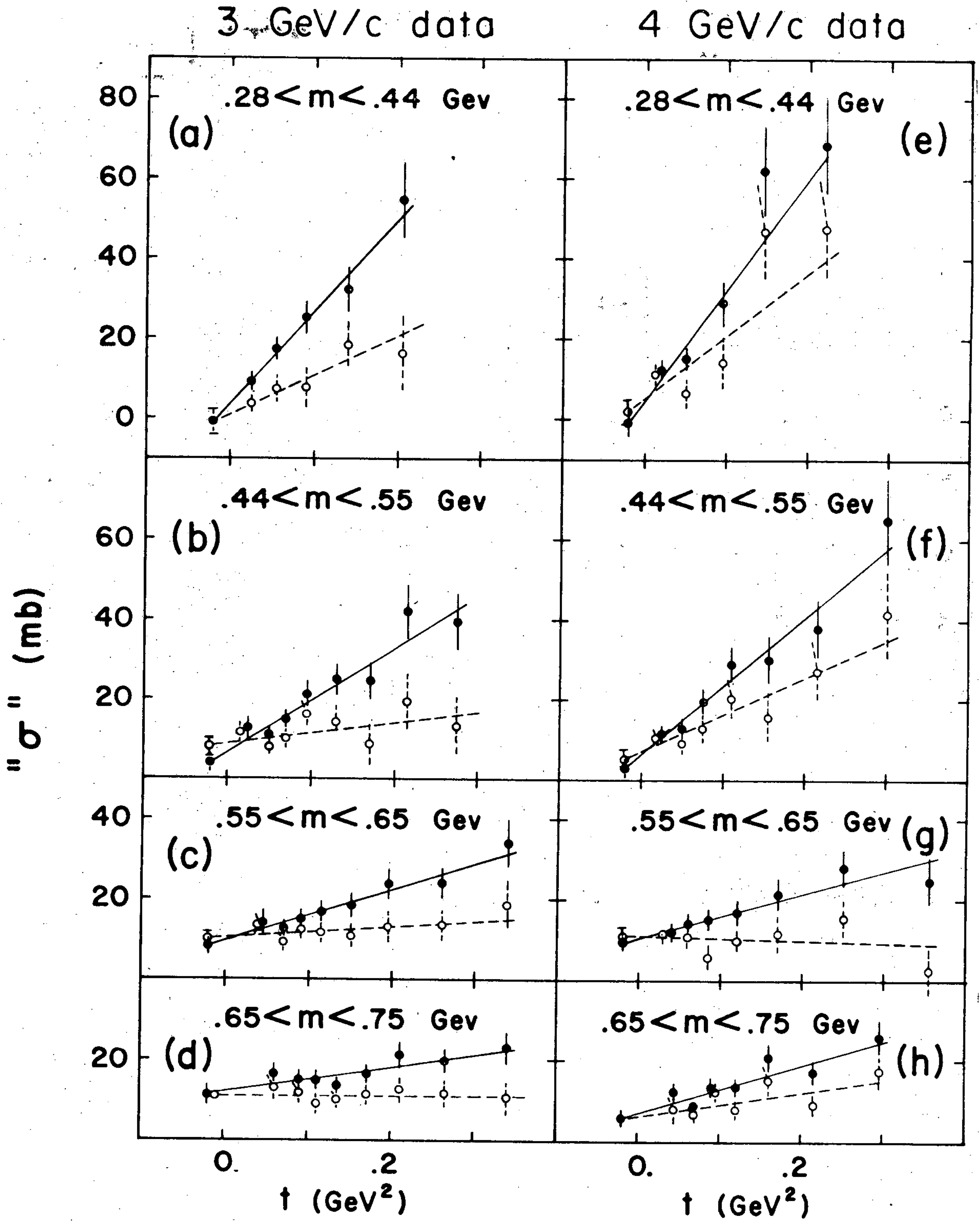
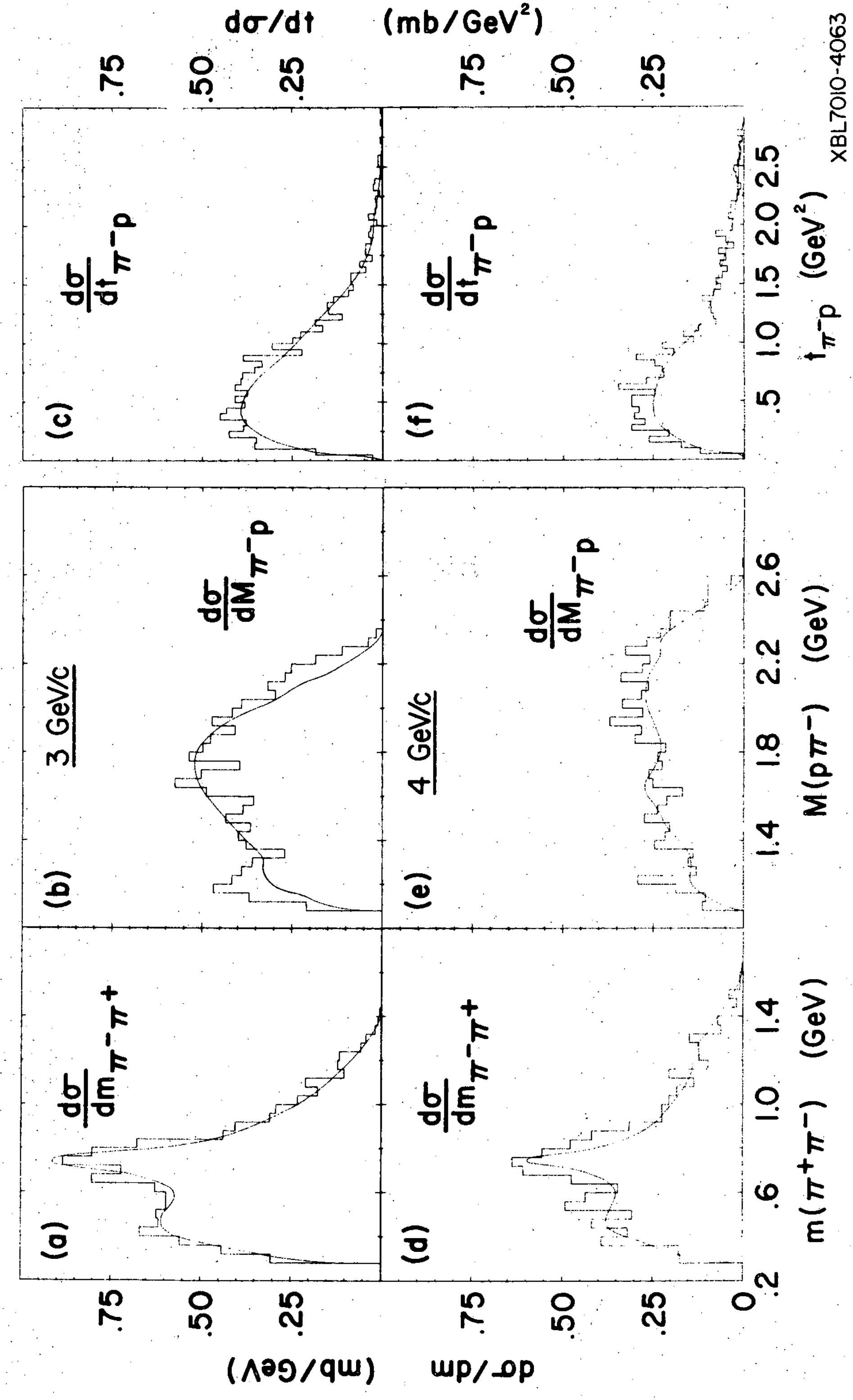

(a)
$\frac{d\sigma}{dm}$
(b) 3 GeV/c
$\frac{d\sigma}{dM}$
(c)
$\frac{d\sigma}{dt}$
(d)
$\frac{d\sigma}{dm}$
(e) 4 GeV/c
$\frac{d\sigma}{dM}$
(f)
$\frac{d\sigma}{dt}$
2.0
1.5
1.0
.5
0
(mb/GeV)
$d\sigma/dM$
$d\sigma/dt$
(mb/GeV^2)
.2
.4
.6
1.0
1.2
1.4
m($\pi^-\pi^-$) (GeV)
M($p\pi^+$) (GeV)
t (GeV^2)
XBL7010-4064

FIGURE 2



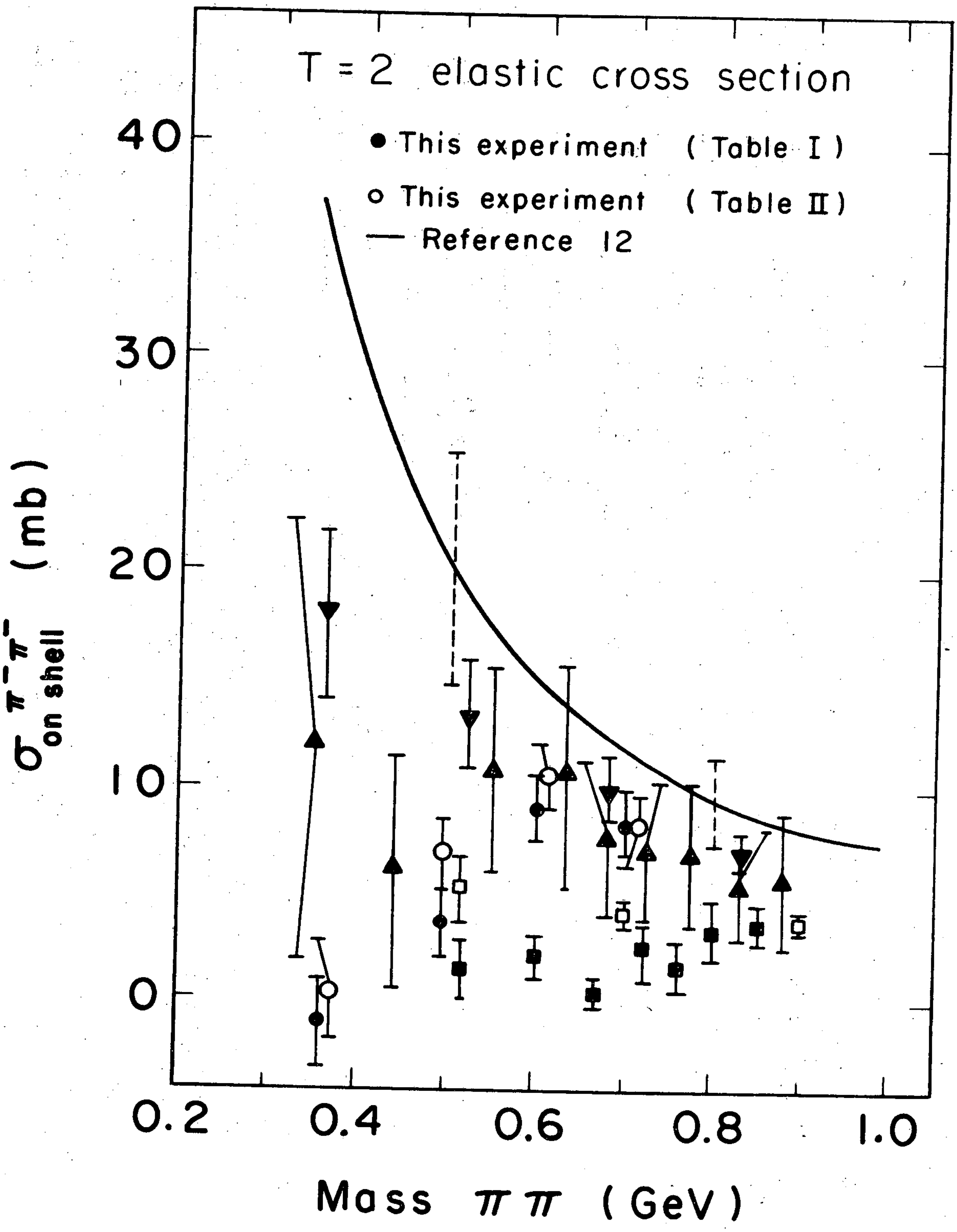
XBL 7010-4066

FIGURE 3



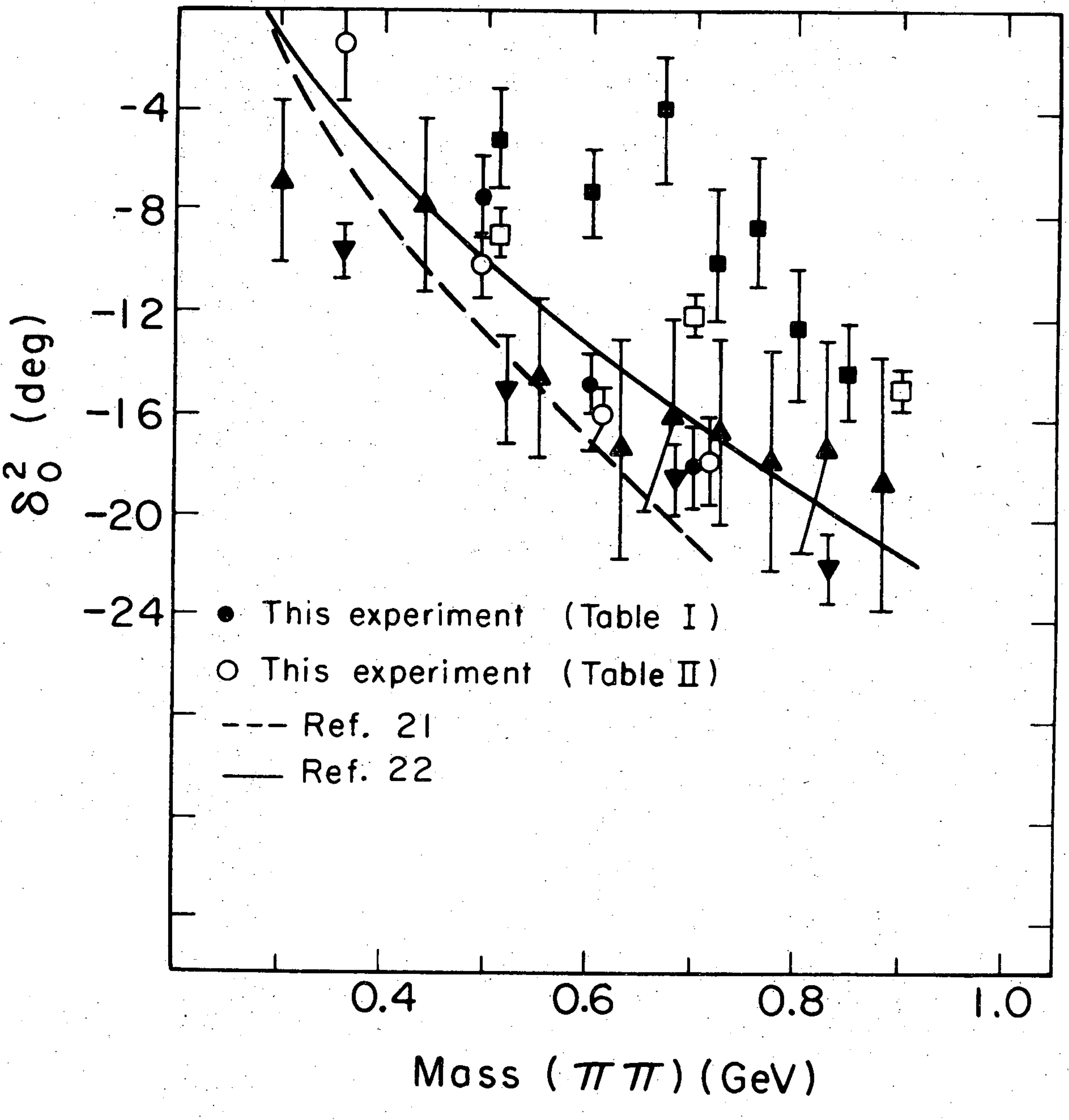
XBL7010-4063

FIGURE 4



XBL 7010-4062

FIGURE 5



XBL7010-4061

FIGURE 6

TECHNICAL INFORMATION DIVISION
LAWRENCE RADIATION LABORATORY
UNIVERSITY OF CALIFORNIA
BERKELEY, CALIFORNIA 94720