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Essays on Quantitative Macroeconomics and Monetary Policy

DISSERTATION

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for the degree of

DOCTOR OF PHILOSOPHY

in Economics

by

Nayib Rene Zamarripa

Dissertation Committee:
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2022

DEDICATION

To Kimi and Lucila, for their wholehearted love and support during this journey.

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ABSTRACT OF THE DISSERTATION

Essays on Quantitative Macroeconomics and Monetary Policy

By

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This dissertation contains three chapters on empirical macroeconomics and monetary policy. In Chapter 1, I test the forecast performance of a small-scale Dynamic Stochastic General Equilibrium (DSGE) model with sentiment shocks. I relax the benchmark assumption of rational expectations and assume instead that economic agents behave in a near-rational fashion: every period they learn and update their beliefs using a constant gain learning algorithm. Sentiment shocks are captured by exploiting observed data on expectations and are defined as the deviations from the model implied expectations due to exogenous waves of pessimism or optimism. The forecast evaluation is accomplished by comparing the root mean squared prediction error of the canonical 3-equation New Keynesian model at different horizons and under different expectation assumptions: rational expectations, learning, and learning with sentiment. The results show that the model with learning and sentiment shocks is not only able to compete with the other two alternatives, but it is generally better to forecast the output gap and the inflation rate.

In Chapter 2, I use a small open economy DSGE model to investigate how Mexico's central bank has conducted its monetary policy in the period 1995-2019. The main objective of the paper is to document the systematic changes in the Bank of Mexico's reaction function by analyzing possible shifts in the parameters of the policy rule. The central bank's policy

is modeled using a Taylor rule that relates the nominal interest rate to output, inflation, and the exchange rate. I employ Bayesian computational techniques and conduct rolling-window estimations to explicitly show the transition of the policy coefficients over the sample period. Furthermore, the paper examines the macroeconomic implications of these changes through rolling-window impulse-response functions. The results suggest that the Bank of Mexico's response to inflation has been steady since 1995, while the response to output and the exchange rate has decreased and stabilized after 2002.

In Chapter 3, I reconsider whether monetary policy in small open economies responds to exchange rates by studying possible parameter instabilities in a DSGE model. The main focus of the paper is to revisit preceding evidence on the response to exchange rate movements by the Bank of England and determine if its reaction function has remained constant throughout the sample. To this end, I estimate a small open economy general equilibrium model using Bayesian econometric techniques over rolling windows. I find overwhelming evidence of shifts in several parameters, including those related to the policy rule. Furthermore, posterior odds tests reveal a time-varying response to exchange-rate fluctuations by the monetary authorities. The results favor the model with the nominal exchange rate embedded in the policy rule for the initial subsamples. However, the evidence steadily evolves across windows and ultimately changes to prefer the model specification with no exchange rate. The paper also documents evident variations in the model dynamics derived by the instability of parameters via rolling-window impulse response functions and variance decomposition analysis.

Chapter 1

Forecast Evaluation of a Small-Scale DSGE Model with Sentiment Shocks

1.1 Introduction

It is not surprising that Dynamic Stochastic General Equilibrium (DSGE) models have become a popular tool used for policy analysis and the study of the business cycle, as not only do they combine a sound theoretical framework and explicit micro-foundations, but also offer an improved fit on macroeconomic time series. The internally consistent interpretation that these models offer and the possibility of implementing policy experiments that are, in principle, not subject to the Lucas critique, have made DSGE models particularly attractive to policymakers. However, when it comes to project economic activity, there is still a debate on whether these models can outperform surveys of professional forecasters or traditional time series models.

Several papers have documented how the forecasts that arise from DSGE models can compete with other models widely used in macroeconomics. Smets and Wouters (2004) and

Smets and Wouters (2007) show that medium-scale New Keynesian models with real and financial frictions compare well with conventional Vector Autoregression (VAR) and Bayesian Autoregression (BVAR) models in out-of-sample forecasting. Del Negro et al. (2007) use a DSGE-VAR approach to study the time series fit of a DSGE model and find strong evidence of miss-specification in New Keynesian models. Nevertheless, they conclude that, although forecast and policy analysis should be interpreted with caution, New Keynesian DSGE models can still generate realistic predictions.

In a real-time data environment, the forecast capabilities of these models have also been examined. Edge et al. (2010) provide a real-time forecast comparison between DSGE forecasts, those from reduced-from time series models, and forecasts from the Federal Reserve staff. The forecast comparison in this paper confirms that sufficiently rich DSGE models are valuable forecasting tools capable to compete with more sophisticated models. In contrast, Rubaszek and Skrzypczyński (2008) compare the quality of forecasts from a small-scale DSGE model, a VAR and BVAR models, and the Survey of Professional Forecasters (SPF). Through a recursive estimation approach, they find that while the DSGE model is not able to significantly outperform the SPF, DSGE forecasts are close to the SPF predictions in terms of accuracy.

Efforts have also been made to investigate the forecast performance of DSGE models in an open economy context. Adolfson et al. (2005) analyze the forecast performance of an open economy DSGE model against a wide range of reduced-form forecasting models and conclude that forecasts generated by the open economy DSGE model perform satisfactorily in comparison with VARs and BVARs models.[1] Similarly, Christoffel et al. (2010) examine the forecast performance of a DSGE model designed for macroeconomic projections at the European Central Bank against nonstructural benchmarks. The forecasting exercise conducted by these authors suggests that the DSGE model performs relatively well when forecasting real variables but is less successful to predict certain nominal variables.

[1] See also Adolfson et al. (2008) and Lees et al. (2011) for more on forecasting comparison between open economy DSGE models and time series models

The literature on the forecast performance of DSGE models has also been extended to study the forecast ability across alternative DSGE specifications. Among the most influential papers in this regard, Del Negro and Schorfheide (2013) illustrate how the forecast accuracy of these models can be improved upon different estimation techniques and alternative model specifications.

This paper adds to this literature by arguing that the forecast quality of DSGE models is sensitive to how expectations are modeled in the micro-foundations. In particular, the paper relaxes the conventional rational expectations hypothesis and evaluates the forecast performance of a DSGE model under three different assumptions in the expectation formation process. In the first one, agents form expectations rationally; they are assumed to know the correct model of the economy, its parameters, and the stochastic structure. In the second one, agents use a constant gain learning algorithm to update their beliefs. In the third one, agents use the same learning algorithm but might deviate from the model-implied expectations due to sentiment shocks.

The deviation from rational expectations is modeled following recent learning literature (see for instance Evans and Honkapohja (2012), Evans and Honkapohja (1999), Preston (2003), Milani (2007), Slobodyan and Wouters (2012), among many others). Agents are assumed to behave as econometricians and form expectations of the forward-looking variables using past information on the model variables. I abstract from making additional assumptions about the knowledge that economic agents have about the state of the economy and employ a prototypical 3-equation New Keynesian model to keep the learning process relatively simple. To my knowledge, this is the first paper to compare the forecast properties of DSGE models between rational expectations and learning mechanisms.

The forecast evaluation is done using out-of-sample data on the output gap, the inflation rate, and the nominal interest rate. Specifically, I evaluate the forecast performance of each model specification in three steps. First, I test if there are statistical differences between the data and the DSGE forecasts that come from each model specification. I then evaluate the relative performance between these alternatives using statistical measures of forecast errors. Lastly, I test for statistical differences in the forecast accuracy between the model specifications. The results suggest that, while no model specification completely outperforms the other two, the model with learning and sentiment shocks performs relatively better when it comes to forecasting the output gap and the inflation rate.

There are two main contributions in this paper. First, the paper aims to argue that psychological factors are important in macroeconomic models and can potentially improve the forecast quality of DSGE models. Second, the paper adds to the learning literature, as it provides new research that compares the forecast performance between rational expectations and learning models.

The rest of the paper is organized as follows. Section 1.2 briefly introduces the model and the assumptions about the learning process. Section 1.3 describes the data and the estimation approach. Section 1.4 explains the empirical results. Section 1.5 presents the forecast evaluation analysis. Lastly, Section 1.6 concludes.

1.2 The model

This paper uses the benchmark 3-equation New Keynesian model to summarize the aggregate dynamics of the economy. The model follows Clarida et al. (1999) and is built under the assumption that economic agents solve an intertemporal optimization problem:

$$x_t = \hat{E}_t x_{t+1} - \sigma(i_t - \hat{E}_t \pi_{t+1}) + g_t \tag{1.1}$$

$$\pi_t = \beta \hat{E}_t \pi_{t+1} + \kappa x_t + u_t \tag{1.2}$$

$$i_t = \rho i_{t-1} + (1-\rho)[\chi_\pi \pi_{t-1} + \chi_x x_{t-1}] + \varepsilon_t \tag{1.3}$$

where x_t is the output gap, π_t is inflation, and i_t is the nominal interest rate.

Equation 1.1 represents the IS curve and is obtained by log-linearizing the intertemporal Euler equation that arises from the households' optimal choice of consumption. Output depends on expected one-period-ahead output gap and the ex-ante real interest rate. The negative response of output to changes in the real interest rate reflects the intertemporal substitution of consumption. In particular, the parameter $\sigma > 0$ represents the intertemporal elasticity of substitution of consumption and measures the response of current consumption to the real interest rate. The mathematical expectations operator E_t is replaced with $\hat{E}_t$ to indicate subjective (possibly non-rational) expectations. g_t is interpreted as a demand or preference shock.

Equation 1.2 is the forward-looking New Keynesian Phillips curve that comes from log-linearizing aggregate pricing decisions from monopolistic firms. Inflation depends on one-period-ahead expectations about inflation and the output gap. The parameter $0 < \beta < 1$ denotes the household's discount factor and κ is a function of other parameters that represents the slope of the New Keynesian Phillips curve. u_t is the cost-push shock and it intends to represent changes in the expected marginal cost.

Equation 1.3 describes the monetary policy of the central bank. The central bank sets the nominal interest rate in response to observed inflation and observed output gap. ρ represents the inertia of the nominal interest rate. χ_π and χ_x denote the policy response to inflation and output, respectively. The policy shock ε_t captures any deviations from the policy rule. Clarida et al. (2000) argued that a forward-looking specification might be a more realistic monetary policy as it allows the central bank to rely on additional information beyond lagged inflation and output. However, as discussed in Milani (2007), such rule needs further assumptions about the knowledge that the central bank has on private expectations and might affect the results under learning.

The shocks g_t and u_t are assumed to follow an AR(1) process:

$$g_t = \rho_g g_{t-1} + \nu_t^g \tag{1.4}$$

$$u_t = \rho_u u_{t-1} + \nu_t^u \tag{1.5}$$

where ρ_g and ρ_u are autorregresive coefficients and $\nu_t^g \sim iid(0, \sigma_g^2)$, $\nu_t^u \sim iid(0, \sigma_u^2)$.

1.2.1 Learning process, expectations formation, and sentiment shocks

The assumption of rational expectations has been criticized in influential papers due to the incredible amount of information that it assigns to economic agents.[2] An alternative approach in the literature instead assumes that agents form their expectations using a learning algorithm. Every period agents use historical data to estimate their Perceived Law of Motion (PLM)

$$Y_t = a_t + b_t Y_{t-1} + \epsilon_t \tag{1.6}$$

where $Y_t = [x_t, \pi_t, i_t]'$ and a_t and b_t are vectors and matrices of coefficients. Agent's PLM has the same endogenous variables as the Minimum State Variable (MSV) solution under rational expectations which intends to capture a small deviation from rational expectations: they have the correct model of the economy but they lack knowledge about the coefficients.[3] As the departure from rational expectations is small, the model can be interpreted as *near-rational* (Milani, 2017).

[2]See for example Sargent (1993), Evans and Honkapohja (1999), Evans and Honkapohja (2012), among others.

[3]It is worth to mention that g_t and u_t also appear in the MSV solution of the system under rational expectations. However, the assumption here is that economic agents do not observe the structural disturbances and therefore those variables do not appear in the PLM under learning.

When additional data becomes available, agents in the model update their beliefs using a constant-gain learning algorithm:

$$\widehat{\phi}_t = \widehat{\phi}_{t-1} + \bar{\mathbf{g}} R_t^{-1} X_t (Y_t - \widehat{\phi}'_{t-1} X_t)' \tag{1.7}$$

$$R_t = R_{t-1} + \bar{\mathbf{g}} (X_t X_t' - R_{t-1}) \tag{1.8}$$

where $X_t = [1, Y'_{t-1}]'$ and $\hat{\phi}_t = [a_t, b_t]'$. Equation 1.7 refers to the agent's updating process regarding the coefficients of Equation 1.6, where $\bar{\mathbf{g}}$ represents the gain parameter and R_t is the precision matrix. Equation 1.8 describes how the precision matrix is updated every period.

The information assumptions in the model are motivated to match the information set typically available to the econometrician and the timing in the Survey of Professional Forecasters.[4] In each period t economic agents observe the variables in Y up to $t-1$ and form expectations of $t+1$, but they do not observe the structural shocks.[5]

After observing the endogenous variables and updating their beliefs, agents use 1.6 to form expectations as follows:

$$\widehat{E}_{t-1} Y_{t+1} = [I + \widehat{b}_{t-1}] \widehat{a}_{t-1} + \widehat{b}^2_{t-1} Y_{t-1} + d\alpha_t \tag{1.9}$$

where d is a selection matrix with elements equal to 1 for expectations for which observations are available, and α_t is a vector collecting the sentiment shocks $\alpha_t = [\alpha_t^x \alpha_t^\pi]'$, assumed to evolve according to a univariate AR(1) processes.[6] α_t captures exogenous deviations from the model implied expectations. These deviations, defined as "sentiment", represent waves of pessimism or optimism about the state of the economy.

[4]In the survey forecasters are asked to provide quarterly projections of a variety of economic variables for five quarters and annual projections for the current and the following year.

[5]Under rational expectations, agents are assumed to observe the structural shocks in period t.

[6]For the model specification under learning but without sentiment shocks, the term $d\alpha_t$ does not enter equation 1.9.

1.2.2 State-space representation

The model can be expressed in its state-space form as:

$$\xi_t = A_t + F_t\xi_{t-1} + G_t\omega_t \tag{1.10}$$

$$Y_t^{OBS} = H\xi_t \tag{1.11}$$

where $\xi_t = [Y_t', g_t, u_t, \widehat{E}_t x_{t+1}, \widehat{E}_t \pi_{t+1}, \alpha_t^x, \alpha_t^\pi]'$, $\omega_t \sim N(0, \sigma_w^2)$, H is a matrix of zeros and ones selecting variables from ξ_t for which observed data is available, and A_t, F_t, G_t are time-varying matrices of coefficients, function of the structural parameters and agents beliefs.[7] Equation 1.10 describes the transition of the state variables and represents the Actual Law of Motion (ALM) of the economy. Notice that under rational expectations $A_t = A = \vec{0}$, $F_t = F$ and $G_t = G$. If the system has a unique (and non-explosive) solution, these matrices can be found using the methodology proposed by Sims (2002). Under learning, these matrices and vectors of coefficients possibly change every period as a result of agent's learning process described by equations 1.7 and 1.8. Similarly, when sentiment is included in the model with learning, A_t, F_t and G_t are obtained using survey expectations in equation 1.9 and the learning process described above.

1.3 Estimation

This paper uses Bayesian methods to estimate the model. The approach resembles that of An and Schorfheide (2007) but is extended to include subjective (possibly non-rational) expectations. Bayesian computational techniques have become a popular tool in recent empirical papers that estimate DSGE models. Fernández-Villaverde (2010) estimated a benchmark DSGE model with real and nominal rigidities using a random walk Metropolis-Hastings al-

[7] Note that under learning but without sentiment shocks, α_t^x and α_t^π are not part of ξ_t in equation 1.10.

gorithm. Smets and Wouters (2007) used likelihood-based Bayesian methods to estimate a medium-scale DSGE model capable to compete with standard VAR and BVAR models. Milani (2008) and Slobodyan and Wouters (2012) provide a good example of Bayesian estimation of DSGE models under learning. In particular, Milani (2017) uses a Bayesian approach to estimate a model under learning that incorporates sentiment shocks.

The model is estimated to fit the following set of observed variables: output gap, inflation rate, nominal interest rate, expected output gap, and expected inflation. The posterior distribution of the parameters is estimated using the random walk Metropolis-Hasting algorithm. 400,000 draws are run, discarding the initial 25% as burn-in. The likelihood of the model is evaluated with the coefficient matrices of equation (2.10) using the Kalman filter. Under learning, the initial beliefs of the agents are estimated using pre-sample data.

The parameters of the model are collected in the vector θ:

$$\theta = \{\sigma, \kappa, \rho, \chi_\pi, \chi_x, \rho_g, \rho_u, \sigma_g, \sigma_u, \sigma_\varepsilon, \bar{\mathbf{g}}, \rho_{\alpha_x}, \rho_{\alpha_\pi}, \sigma_{\alpha_x}, \sigma_{\alpha_\pi}\}$$

where $\bar{\mathbf{g}}$ is the constant-gain parameter, and ρ_{α_x}, ρ_{α_π}, σ_{α_x}, and σ_{α_π} are the autorregresive coefficient and standard deviation of the sentiment shocks of the output gap and the inflation rate, respectively. The choice of jointly estimating the gain coefficient with the other structural parameters is mainly motivated to avoid imposing assumptions about the learning speed.[8]

1.3.1 Data

The model is estimated using quarterly data for the period 1981:Q3 to 2000:Q1. Series for the output gap, the inflation rate, and the nominal interest rate were obtained from the Federal Reserve Bank of St. Louis. Sentiment shocks are captured by exploiting observed

[8] In Milani (2005), Orphanides and Williams (2005), and Orphanides and Williams (2007), the persistence of inflation rises as a result of the learning process.

data on expectations from the Survey of Professional Forecasters (SPF), available at the Federal Reserve Bank of Philadelphia.[9] The forecasts from the SPF are available starting from 1981:Q3, which matches the initial date for the estimation. I use the mean across forecasters for the data on expectations obtained from the SPF.

The output gap is defined as the log difference between real GDP and Potential GDP, the inflation rate as the log differences of the GDP Implicit Price Deflator.[10] The nominal interest rate is measured using the Federal Funds rate. Data on expectations about output correspond to the one-period-ahead real GDP growth from the SPF. Expectations about inflation are obtained using the forecast of the GDP price index, and is defined as the log of the expected two-quarter-ahead inflation minus the log of the expected one-quarter ahead inflation. Because the base year for the GDP price deflator changes over the sample, the series are transformed to maintain the same base year, chosen to be 2009.

Agent's initial beliefs of the coefficients and the covariance matrix are obtained using pre-sample data from 1960:Q1 to 1981:Q2. Lastly, the out-of-sample forecast evaluation is performed using data from 2001:Q2 to 2010:Q4. The sample ending date is chosen to avoid dealing with the zero lower bound period.

1.3.2 Priors

The priors are selected to facilitate comparability with papers that investigate learning in DSGE models. In particular, most of the priors follow the ones chosen by Milani (2008) due to the similitude of the model and the learning process. However, because the role of sentiment is not exploited in that paper, the priors associated with the sentiment shocks follow Milani (2017).

[9] The data on expectations from the Survey of Professional Forecasters is used to estimate only the model specification under learning and sentiment shocks.

[10] Refers to the CBO's estimate of the output the economy would produce assuming a high use its capital and labor

Table 1.1:
Prior distributions and posterior estimates

Parameter	Prior distributions	Posterior distributions					
		Rational expectations		Learning		Learning with sentiment	
β	0.99	-	-	-	-	-	-
σ	$\Gamma(1, 0.7)$	0.06	[0.01-0.15]	0.08	[0.04-0.13]	0.09	[0.07-0.11]
κ	$\mathrm{N}(0.1, 0.05)$	0.007	[0.001-0.014]	0.012	[0.002-0.027]	0.011	[0.006-0.016]
ρ	$\mathrm{U}(0, 1)$	0.83	[0.73-0.91]	0.79	[0.59-0.90]	0.77	[0.71-0.83]
χ_π	$\mathrm{N}(1.5, 0.5)$	1.80	[1.34-2.27]	1.74	[1.52-1.93]	1.70	[1.35-1.94]
χ_x	$\mathrm{N}(0.5, 0.25)$	0.26	[0.10-0.52]	0.32	[0.25-0.40]	0.40	[0.31-0.47]
ρ_g	$\mathrm{U}(0, 1)$	0.89	[0.84-0.94]	0.79	[0.56-0.90]	0.84	[0.81-0.87]
ρ_u	$\mathrm{U}(0, 1)$	0.82	[0.75-0.88]	0.90	[0.78-0.98]	0.69	[0.64-0.76]
σ_g	$\Gamma^{-1}(0.3, 1)$	0.08	[0.05-0.12]	0.20	[0.08-0.37]	0.08	[0.03-0.18]
σ_u	$\Gamma^{-1}(0.3, 1)$	0.03	[0.02-0.05]	0.08	[0.04-0.13]	0.05	[0.02-0.10]
σ_ε	$\Gamma^{-1}(0.3, 1)$	0.18	[0.15-0.21]	0.17	[0.05-0.35]	0.26	[0.03-0.36]
$\bar{\mathbf{g}}$	$\Gamma(0.031, 0.022)$			0.014	[0.009-0.021]	0.026	[0.014-0.037]
ρ_{α_x}	$\mathrm{U}(0, 1)$					0.74	[0.67-0.86]
ρ_{α_π}	$\mathrm{U}(0, 1)$					0.68	[0.59-0.78]
σ_{α_x}	$\Gamma^{-1}(0.3, 1)$					0.25	[0.21-0.30]
σ_{α_π}	$\Gamma^{-1}(0.3, 1)$					0.11	[0.05-0.23]

Note: Prior distributions reflect the mean and the standard deviation, with the exception of the Uniform distribution, which is expressed instead in terms of its minimum and maximum values. The estimate for the posterior means is computed over 300,000 draws (after the burn-in) of the Metropolis-Hastings.

Table 1.1 summarizes the selection of the priors. As it is standard in the literature, the discount factor β is fixed at 0.99. The intertemporal elasticity of substitution, σ, is assumed to follow a Gamma distribution with mean 1 and standard deviation 0.7. The slope of the New Keynesian Phillips curve, κ, follows a normal distribution with mean 0.1 and standard deviation 0.05. The inertia of the interest rate, ρ, is assumed to follow a uniform distribution over the interval [0,1]. The prior on the policy feedback coefficient on inflation, χ_π, is centered at 1.5 with a standard deviation of 0.5. The policy feedback coefficient on the output gap, χ_x, follows a normal distribution with mean 0.5 and standard deviation 0.25. The prior for the autorregresive coefficients of the demand shock and the cost-push shock are assumed to follow a uniform distribution over the interval [0,1]. An inverse Gamma distribution with mean 0.3 and standard deviation 1 is chosen for the standard deviation of the demand shock, the cost-push shock and the monetary policy shock. The constant-gain coefficient is assumed to follow a Gamma distribution with mean 0.031 and standard deviation of 0.22. Lastly, the sentiment shocks are assumed to follow a beta distribution for their autorregresive coefficients and an inverse Gamma for their standard deviations.

1.3.3 Initial beliefs

A well-known remark about learning models is that the results are very sensitive to the choice of the initial beliefs. The usual approach in the literature is to consider different alternatives about the learning process and select the model that delivers the highest marginal likelihood. In this paper, three different alternatives are considered to determine the initial beliefs.[11] To do so, the model is estimated using pre-sample data from 1960:Q1 to 1981:Q2.

1.3.3.1 Initial beliefs equal to REE from 1960-1981 pre-sample.

In this setup, agents are assumed to form expectations rationally during the pre-sample period. Initial beliefs for the second sample (1981-2010) are then chosen so $\hat{\phi}_0 = \hat{\phi}_{REE}$ and $R_0 = XX'_{REE}$. As stated in Milani (2017), this alternative can be interpreted as a change to a new regime: during the old regime agents had enough time to converge to the rational expectations equilibrium. However, after 1981 the economy moves to a new regime and agents use their initial beliefs to gradually learn about the new structure of the economy.

[11] An alternative approach for the initialization of the learning process is to use regression-based initial beliefs. This approach is considered by Slobodyan and Wouters (2012) and consists of running a regression of the endogenous variables on X_t using the pre-sample data. This alternative is not pursued here.

1.3.3.2 Initial beliefs equal to ending point of learning beliefs from 1960-1981 pre-sample.

In the second setup, agents are assumed to form non-rational expectations during the 1960-1981 period. Their initial beliefs follow an AR(1) process for all the variables in their PLM with an autorregresive coefficients equal to 0.9. This alternative endows agents knowledge about the persistence of macroeconomic variables; however, agents' initial beliefs are left uninformative as the PLM do not considers a more complex synergy between the variables. Consequently, the prevailing beliefs at the ending point of the pre-sample period are chosen as the initial learning process for the second sample.

1.3.3.3 Initial beliefs equal to zero.

This last setup intends to represent fully uninformative beliefs. Here the model is estimated without a pre-sample estimation. Instead, the initial coefficient matrices are set equal to zero and an identity matrix is chosen as the initial precision matrix, $R_0 = I$.

1.4 Estimation Results

Posterior estimates are reported in Table 1.1. The table illustrates different estimation results for each model specification. First, the intertemporal elasticity of substitution increases as rational expectations is relaxed, and again when sentiment shocks are introduced. This result is particularly interesting, as it shows that the model with sentiment shocks estimates a lower value for the coefficient of relative risk aversion, closer to what have been found in empirical research.[12] The estimate for the slope of the New Keynesian Phillips curve in the

[12] See for example Kydland and Prescott (1982), Mehra and Prescott (1985), Altug (1989), and Hildreth and Knowles (1982), among others.

three specifications is low, consistent with many papers in the literature. The interest rate smoothing parameter evidences the inertia of the nominal interest rate, with a value higher than 0.77 in the three specifications. An interesting result is depicted from the posterior estimates for the central banks' response to the output gap and the inflation rate. Under learning with sentiment the results suggest a relatively lower response of the central bank to fight inflation and a relatively higher response for the output gap.

The structural disturbances for the demand and the supply shock are persistent under the three expectation assumptions. In particular, rational expectations and learning exhibits high persistence in the autoregressive coefficient of the demand shock, and learning in the autoregressive coefficient of the cost-push shock, both coefficients being close to 0.9. For the model with learning and sentiment, the structural disturbances are also persistent for both sentiment shocks, with autoregressive coefficients close to 0.7.

A key finding in Milani (2017) is that sentiment shocks are responsible to explain a considerable portion of the US business cycle. To verify this result I proceed by examining the forecast error variance decomposition. The results are presented in Table 1.2.

The general picture that arises from the table is that omitting sentiment shocks increases the role of the demand and the cost-push shocks. The contribution of the demand shock to the variance of the output gap is significantly larger when sentiment shocks are excluded, with values of 0.95 under rational expectations, 0.67 under learning, and 0.46 under learning and sentiment. The variance of inflation is mainly driven by the cost-push shock for rational expectations and for learning, with values above 0.7. When sentiment shocks are included in the model the contribution falls to 0.11. Although the demand shock is still the main driver of the business cycle, sentiment shocks do account for 40% of the fluctuations, with the sentiment of output alone explaining 24% of the shifts. It is also worth mentioning that, in contrast to rational expectation, the monetary policy shock plays a relatively larger role as a contributor of business cycle in both models under learning.

The variance of inflation is mainly driven by the inflation sentiment, accounting for 27% of

Table 1.2:
Forecast error variance decomposition

	$\hat{g}_t$	$\hat{u}_t$	ε_t	α_t^x	α_t^π
Business cycle horizon (4-24)					
REE					
x_t	0.95	0.04	0.01	-	-
π_t	0.24	0.76	0.00	-	-
i_t	0.57	0.38	0.05	-	-
Learning					
x_t	0.67	0.22	0.11	-	-
π_t	0.19	0.73	0.08	-	-
i_t	0.27	0.51	0.22	-	-
Learning with sentiment					
x_t	0.46	0.04	0.10	0.27	0.13
π_t	0.25	0.11	0.11	0.26	0.27
i_t	0.29	0.05	0.17	0.31	0.18

Note: The numbers refer to the mean value of the forecast error across the Metropolis-Hastings draws.

this variable forecast error variance. Together, with the sentiment of the output gap, these shocks explain more than 50% of the variability of inflation.

Sentiment shocks also play an important role explaining the variance of the nominal interest rate. While the demand and the cost-push shock explain a sizable part of the variance under rational expectations and under learning, the contribution of the two shocks is much lower under learning with sentiment, only 34%. Furthermore, sentiment shocks account for nearly half of the variation of the interest rate.

Indeed, the results seem to suggest that sentiment shocks play a crucial role as contributors of aggregate fluctuation.

1.5 Forecast evaluation

This section reports the out-of-sample forecast evaluation results of the small-scale DSGE model under the different expectation assumptions. The forecast evaluation for the three macroeconomic variables is analyzed using recursive forecasts for horizons that span from one up to four periods ahead. This is done using the out-of-sample data and fixing structural parameters at their posterior means. Specifically, the first set of recursive forecasts uses data up to 2000:Q1 (i.e., the end of the estimation sample) and predicts for 2000:Q2, 2000:Q3, 2000:Q4, and 2001:Q1. Similarly, the second set uses data up to 2000:Q2 and predicts for 2000:Q3, 2000:Q4, 2001:Q1, and 2001:Q2. The same process is repeated up until 2009:Q4. Figure 1.1 depicts a graphical representation of this procedure.

The forecast evaluation is performed following Rubaszek and Skrzypczyński (2008). First, I test if the forecasts are biased by regressing actual data (X_t) on the forecasts (X_t^F) for each variable, at different horizons, for the three different expectation assumptions, and testing the null hypothesis that the constant term is ($\alpha = 0$) and the slope is equal to one ($\alpha_1 = 1$):

$$X_t = \alpha + \alpha_1 X_t^F + \varepsilon_t \tag{1.12}$$

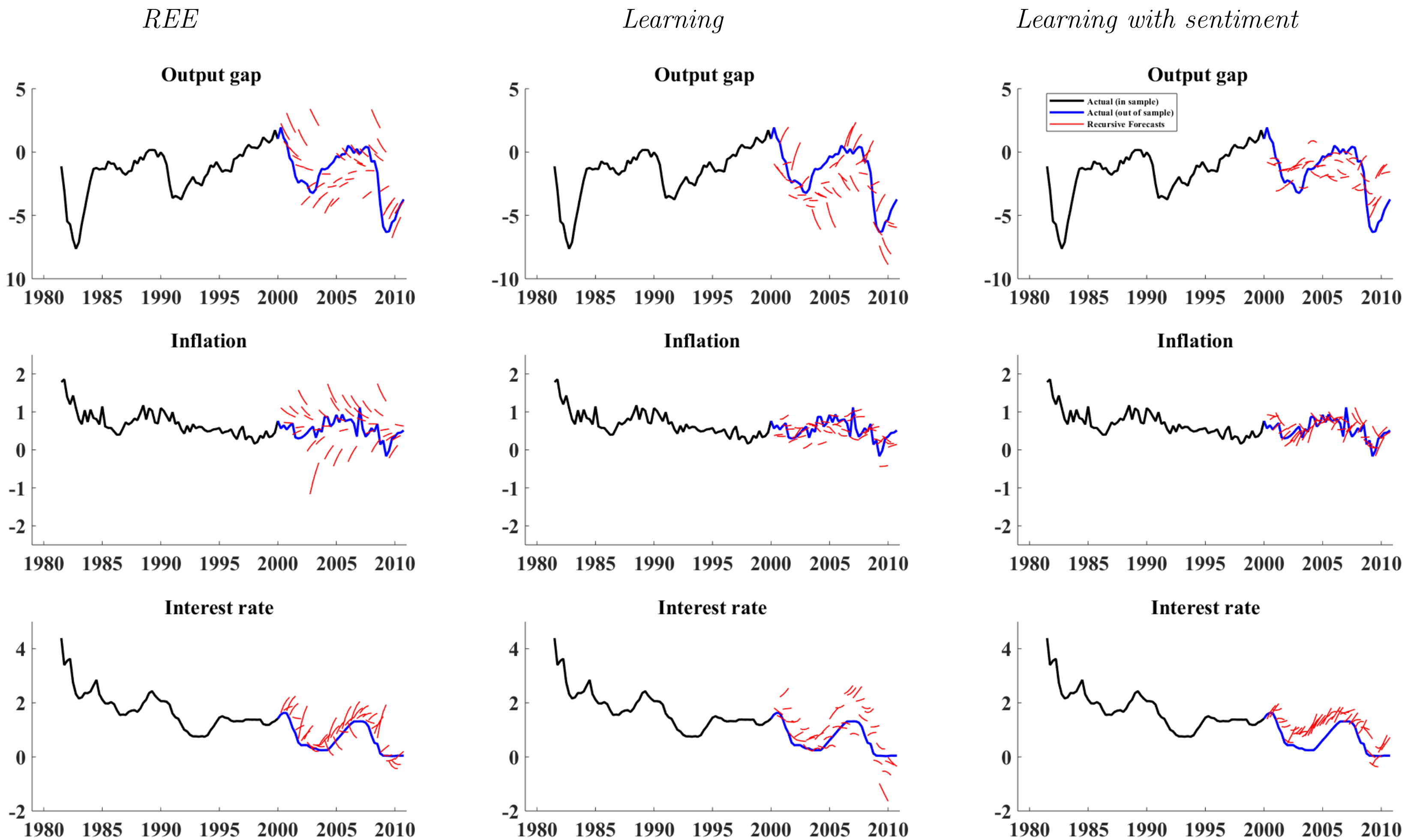
Table 1.3 summarizes the results. In short, at 5% significance level, the null is rejected in all the forecasts for the model under rational expectations and under learning, suggesting that the forecasts are biased. In particular, the constant terms for the output gap are far from zero in most horizons, and the slope for the inflation rate is below 0.1 at all horizons under rational expectations, and below 0.6 under learning. Moreover, all the forecasts for the interest rate under the three specifications are imprecise. An interesting extension of this paper could consider adopting a more sophisticated policy rule, and verify if there is a significant improvement for the forecasts of this variable. As stated before, here I do not

Table 1.3:
Test of forecast unbiasedness

h	Output gap				Inflation				Interest rate			
	$\hat{\alpha}$	$\hat{\alpha}_1$	R^2	p-value	$\hat{\alpha}$	$\hat{\alpha}_1$	R^2	p-value	$\hat{\alpha}$	$\hat{\alpha}_1$	R^2	p-value
REE												
1	-0.859	0.524	0.392	**0.000**	0.464	0.098	0.060	**0.000**	-0.044	0.881	0.945	**0.000**
	(0.307)	(0.102)			(0.056)	(0.061)			(0.032)	(0.033)		
2	-1.045	0.461	0.252	**0.000**	0.539	-0.014	0.001	**0.000**	0.015	0.685	0.772	**0.000**
	(0.350)	(0.125)			(0.065)	(0.078)			(0.065)	(0.059)		
3	-1.299	0.359	0.122	**0.001**	0.484	0.064	0.012	**0.000**	0.109	0.499	0.534	**0.000**
	(0.394)	(0.154)			(0.074)	(0.095)			(0.091)	(0.075)		
4	-1.632	0.198	0.030	**0.000**	0.488	0.059	0.007	**0.000**	0.194	0.361	0.340	**0.000**
	(0.435)	(0.184)			(0.087)	(0.116)			(0.107)	(0.081)		
Learning												
1	0.997	0.978	0.784	**0.000**	0.248	0.600	0.440	**0.000**	0.144	0.551	0.737	**0.000**
	(0.271)	(0.080)			(0.058)	(0.106)			(0.063)	(0.051)		
2	0.103	0.701	0.557	**0.000**	0.376	0.322	0.122	**0.000**	0.167	0.491	0.657	**0.000**
	(0.346)	(0.099)			(0.075)	(0.137)			(0.071)	(0.056)		
3	-0.524	0.510	0.367	**0.000**	0.364	0.330	0.125	**0.000**	0.208	0.408	0.540	**0.000**
	(0.385)	(0.107)			(0.079)	(0.139)			(0.079)	(0.060)		
4	-1.121	0.314	0.168	**0.000**	0.423	0.202	0.048	**0.000**	0.272	0.311	0.395	**0.000**
	(0.422)	(0.114)			(0.085)	(0.146)			(0.085)	(0.062)		
Learning with Sentiment												
1	-0.113	0.956	0.261	0.975	0.329	0.399	0.254	**0.000**	-0.195	0.891	0.693	**0.000**
	(0.509)	(0.251)			(0.064)	(0.107)			(0.099)	(0.093)		
2	-0.291	0.954	0.278	0.724	0.254	0.513	0.269	**0.003**	-0.258	0.847	0.713	**0.000**
	(0.476)	(0.243)			(0.080)	(0.134)			(0.099)	(0.085)		
3	-0.398	0.965	0.278	0.464	0.049	0.834	0.538	0.119	-0.269	0.764	0.681	**0.000**
	(0.472)	(0.249)			(0.076)	(0.124)			(0.106)	(0.084)		
4	-0.611	0.895	0.215	0.282	0.126	0.666	0.313	**0.018**	-0.233	0.658	0.610	**0.000**
	(0.505)	(0.277)			(0.102)	(0.160)			(0.116)	(0.085)		

Note: The table shows the coefficient estimates of regression Equation 1.12. The p-values refer to the test of the null hypothesis that the forecast is unbiased, where bold indicates the rejection of the null at 5% significance level. Standard errors are denoted in parenthesis.

Figure 1.1:
Recursive forecasts and actuals.



Note: Recursive forecasts are calculated at the posterior mean across the Metropolis-Hastings draws.

pursue this alternative mainly to keep the model simple and comparable with other papers. Finally, for the model under learning and sentiment, the forecasts for the output gap at all horizons and for the inflation rate at the three-quarter horizon are unbiased.

I then continue to compare statistical measures of forecast errors between the three model specifications. Table 1.4 presents the mean errors (ME), the mean absolute errors (MAE), and the root mean squared errors (RMSE) for the forecasts of three variables at the different horizons. The results for the MAE and RMSE reveal that, with the exception of the one-quarter horizon, the model under learning and sentiment reports the lowest value for the output gap and the inflation rate, and the lowest RMSE for the nominal interest rate for the three- and four-quarter horizons. Furthermore, the difference in the RMSE gets significantly larger after the second horizon when comparing between the model with sentiment and the other two alternatives. It is also worth to mention that the traditional model with learning also exhibits lower RMSE at all horizons for the output gap and the inflation rate. However, the model under rational expectations is superior to forecast the nominal interest rate regarding the MAE and the RMSE for the one- and two-quarter ahead forecasts.

Lastly, I proceed by examining whether there is significant statistical difference in the accuracy of the forecasts previously analyzed. To accomplish this goal I use Harvey et al. (1997) modified version of Diebold and Mariano (1995) and test the null hypothesis of equal expected forecast performance of two competing models. The results are presented in Table 1.5. The RMSE forecast comparison shows that, with the exception of the four-quarter horizon between learning and learning with sentiment, no model specification is significantly better than the other two to forecast the output gap. For the inflation forecasts, learning with sentiment outperforms rational expectations at all horizons, and learning at the three-quarter ahead forecast. Furthermore, the inflation forecasts of learning also outperform those of rational expectations at all horizons. For the nominal interest rate forecasts, the results suggest that rational expectations is superior than learning with sentiment at the

Table 1.4:
Forecast evaluation

h	Output gap			Inflation			Interest rate		
	REE	Learning	LS	REE	Learning	LS	REE	Learning	LS
ME									
1	-0.072	1.057	**-0.040**	-0.148	0.061	**0.025**	**-0.139**	-0.276	-0.299
2	**-0.147**	0.919	-0.217	-0.126	0.053	**-0.008**	**-0.272**	-0.322	-0.419
3	**-0.257**	0.783	-0.343	-0.131	**0.032**	-0.046	-0.395	**-0.377**	-0.541
4	**-0.353**	0.682	-0.454	-0.131	**0.017**	-0.075	-0.504	**-0.428**	-0.659
MAE									
1	1.513	**1.202**	1.448	0.501	**0.173**	0.219	**0.171**	0.422	0.350
2	1.663	1.474	**1.433**	0.474	0.243	**0.199**	**0.333**	0.486	0.442
3	1.836	1.818	**1.478**	0.396	0.249	**0.152**	**0.504**	0.570	0.562
4	1.964	2.252	**1.544**	0.325	0.278	**0.190**	**0.643**	0.677	0.682
RMSE									
1	4.149	**2.081**	3.288	0.396	**0.051**	0.083	**0.038**	0.271	0.171
2	4.638	3.145	**3.099**	0.351	0.093	**0.062**	**0.170**	0.361	0.251
3	5.279	4.618	**3.091**	0.242	0.092	**0.034**	0.383	0.502	**0.380**
4	6.004	7.049	**3.391**	0.197	0.114	**0.057**	0.609	0.708	**0.548**

Note: Bold indicates the minimum absolute values for the MEs, MAEs and RMSEs. LS stands for the model under learning with sentiment.

one-quarter horizon, and than learning for the one- and two-quarter horizon. However, at longer horizons, no model seems to dominate the other two to forecast the nominal interest rate.

1.6 Conclusions

This paper analyzed the forecast performance of small-scale DSGE model with sentiment shocks by comparing the quality of forecasts to those under the benchmark assumption of rational expectations, and the traditional model under learning. With the usage of recursive forecasts and out-of-sample data for the output gap, the inflation rate, and the nominal

Table 1.5:
Test for equal forecast accuracy

h	Output gap		Inflation		Interest rate	
	HLN	p-value	HLN	p-value	HLN	p-value
REE vs Learning						
1	1.648	0.107	4.163	**0.000**	-4.529	**0.000**
2	1.111	0.273	3.560	**0.001**	-2.676	**0.011**
3	0.343	0.733	2.676	**0.011**	-1.042	0.304
4	-0.472	0.640	2.514	**0.016**	-0.598	0.553
REE vs Learning with Sentiment						
1	0.603	0.550	3.434	**0.001**	-5.575	**0.000**
2	1.035	0.307	3.760	**0.001**	-1.197	0.238
3	1.194	0.239	3.956	**0.000**	0.021	0.984
4	1.372	0.178	2.844	**0.007**	0.276	0.784
Learning vs Learning with Sentiment						
1	-1.502	0.141	-1.222	0.229	1.716	0.093
2	0.037	0.971	1.927	0.061	1.042	0.303
3	1.055	0.298	2.596	**0.013**	0.772	0.444
4	2.748	**0.009**	1.836	0.074	0.630	0.533

Note: The table presents the Diebold-Mariano test modified by Harvey, Leybourne, and Newbold (HLN). A positive value of the HLN statistic indicates that the RMSE of A is higher than that of B, where A and B stand for Rational Expectations, Learning or Learning with Sentiment. Bold indicates the rejection of the null hypothesis of equal forecast accuracy at the 5% significance level.

interest rate, the paper was able to illustrate how expectation assumptions matter for forecasting.

The forecast evaluation was conducted in three steps: a test of forecast unbiasedness, a RMSE comparison, and a test for equal forecast accuracy. The results show that the output gap forecasts were unbiased at all horizons for the model with learning and sentiment shocks, but were all biased for rational expectations and learning. However, for the inflation rate and the nominal interest rate, the forecasts turn out to be imprecise in the three methods, with the only exception at the three-quarter- ahead forecast of the inflation rate for the model under learning and sentiment. The RMSE comparison evidenced that learning with sentiment is relatively better to forecast the output gap and the inflation rate. For the nominal interest rate the results are inconclusive, learning with sentiment is superior only at the three- and four-quarter horizons while rational expectations performs better at the one- and two-quarter horizons. The test for equal forecast accuracy revealed that learning with sentiment performs better at all horizons to forecast the inflation rate. For the other variables, the test shows that learning with sentiment is outperformed only for the forecast of the nominal interest rate at the one-quarter ahead horizon.

Overall, the results illustrate how the model with learning and sentiment shocks is able to forecast as well or even better than the two alternatives previously mentioned, which speaks about the importance of embedding behavioral factors in economic models, particularly in the forecasting dimension.

Chapter 2

Estimating the Bank of Mexico's Reaction Function in the Last Three Decades: A Bayesian DSGE approach with Rolling-Windows

2.1 Introduction

How do central banks in emerging market economies conduct monetary policy during (and after) disinflation episodes? This paper studies the case of Mexico and addresses this question by conducting rolling-window estimations of a small open economy DSGE model. I employ a Bayesian methodology and estimate the model a number of times over rolling-samples to show possible drifts in the coefficients of the monetary policy rule. I model the Bank of Mexico's reaction function using a Taylor rule that responds to inflation, output gap, and changes in the exchange rate. Moreover, the paper also analyzes to what extent, a different reaction to these variables affect the model dynamics by computing rolling-window impulse responses.

The main finding of the paper is that the Bank of Mexico has decreased its response to exchange rate changes and the output gap while maintaining stable that to the inflation rate. Through the rolling-window approach, I provide evidence of apparent shifts in the posterior distributions of the former parameters and remarkably stable distributions for the latter. These findings are consistent with the idea that central banks in developing countries move interest rates in response to exchange rate changes when the pass-through in prices is high. As the relative importance of outside forces decrease, monetary authorities respond less to the depreciation of the domestic currency. In particular, the lower (but still positive) response to the exchange rate is in line with that found in Best (2013). Nevertheless, I interpret these results as policy required to achieve the inflation targets, instead of 'fear of floating'.[1] In terms of economic implications, I compute rolling-window impulse response functions to study the effects of the changes in the policy parameters. In short, the results reveal a higher degree of stability on the main domestic variables.

The paper is closely related to Lubik and Schorfheide (2007), Liu and Mumtaz (2011), and Best (2013), who have estimated small open economy DSGE models using Bayesian computational techniques. In particular, these papers have employed and estimated these models to study the conduct of monetary policy in different countries. Lubik and Schorfheide (2004) estimate the monetary policy rule for Australia, Canada, New Zealand, and the UK to analyze if these banks are responding to exchange rate movements when setting monetary policy. Liu and Mumtaz (2011) use a Markov-switching framework to explore potential changes in the model parameters using data for the UK. Best (2013) applies a regime-switching approach using Mexican data for the period 1981-2005 to document instabilities in the policy response coefficients because of the 1994-crisis.

[1] Using policy to stabilize the exchange rate. For a detailed discussion on 'fear of floating', please refer to Calvo and Reinhart (2002)

There are two main contributions of this paper. First, I employ a rolling-window approach to exhibit possible systematic shifts in the Bank of Mexico's policy feedback parameters during and after the disinflation episode of 1995-2003. This choice was carefully selected in light of the transition that the Mexican economy experienced during that period, instead of dramatic changes in policy. At the beginning of this period, the Mexican economy endured a financial crisis that consequently caused a notorious depreciation of the peso and a rise in the inflation rate. As a stabilization measure to keep inflation low and controlled, the Central Bank of Mexico adopted a new intervention mechanism focused on reducing the cost of fighting it. The new framework consisted of announcing annual inflation targets and setting a long-run inflation target of 3 percent to be reached by the end of 2003. This new approach intended for monetary policy to gain credibility across the economic agents and, ultimately, to make inflation easier to control and for stability to propagate across the economy. From 1995 to 2004, the monetary authorities relied on the Accumulated Balances Regime, a mechanism that allowed them to control the general level of prices and maintaining stability by supplying part of the money demanded at interest rates above the market.[2] After 2004, the Bank of Mexico migrated from the Accumulated Balances Regime and adopted the overnight interbank funding rate as the policy instrument.

Secondly, to my knowledge, this is the first paper to document the evolution of monetary policy parameters during the disinflation episode of an emerging market economy. Although the DSGE framework has been extensively used to understand how central banks set monetary policy, this paper adds to the literature by considering the case of a small open economy. In particular, the empirical results provide new evidence of parameter instabilities on the parameters that govern the Taylor-rule and bring light to our understanding of the conduct of monetary policy in emerging market economies. Moreover, the findings also resemble the importance of considering that structural parameters may not be completely invariant to policy changes.

[2] Also known as 'Corto'.

The rest of the paper is organized as follows. Section 2.2 describes the structural small-open economy model employed in the empirical analysis. Section 2.3 discusses the rolling-window approach and the Bayesian econometric procedure used to estimate the model. Section 2.4 presents the estimation results and their implications, as well as a robustness exercise. Section 2.5 concludes.

2.2 The model

I assume that the aggregate dynamics of the small open economy can be outlined using a small-scale DSGE model. The model is taken from Lubik and Schorfheide (2007), which is a simplified version of Gali and Monacelli (2005), and is governed by the following equations:[3]

$$\begin{aligned} y_t = {} & E_t y_{t+1} - \big[\tau + \alpha(2-\alpha)(1-\tau)\big](R_t - E_t\pi_{t+1}) \\ & - \alpha\big[\tau + \alpha(2-\alpha)(1-\tau)\big]E_t\Delta q_{t+1} + \alpha(2-\alpha)\frac{1-\tau}{\tau}E_t\Delta y^*_{t+1} + g_t \end{aligned} \tag{2.1}$$

$$\pi_t = \beta E_t\pi_{t+1} + \alpha\beta E_t\Delta q_{t+1} - \alpha\Delta q_t + \frac{\kappa}{\tau + \alpha(2-\alpha)(1-\tau)}(y_t - \bar{y}_t) + u_t \tag{2.2}$$

$$R_t = \rho_R R_{t-1} + (1-\rho_R)\big[\psi_\pi \pi_t + \psi_y y_t + \psi_e \Delta e_t\big] + \varepsilon^R_t \tag{2.3}$$

$$\pi_t = \Delta e_t + (1-\alpha)\Delta q_t + \pi^*_t \tag{2.4}$$

$$\Delta q_t = \rho_q \Delta q_{t-1} + \epsilon^q_t \tag{2.5}$$

Equation 2.1 is the log-linearized consumption Euler equation, obtained from the households optimization problem. This forward-looking open economy IS-equation resembles its

[3] The model presented in this paper follows the reduced-form version of Lubik and Schorfheide (2007) proposed by Best (2013).

closed-economy counterpart as it relates the (domestic) output gap, y_t, with one-period-ahead expectations of future output, the ex-ante real interest rate, and a demand shock, g_t. However, notice that domestic output also responds to the one-period-ahead expected change in the terms of trade, Δq_t, defined as the relative price of exports in terms of imports, and to expectations about changes in the world's output, y_t^*. The parameter $0 < \alpha < 1$ represents the import share and can be viewed as a degree of trade openness, and τ is the intertemporal elasticity of substitution. It is important to notice that setting $\alpha = 0$, yields to its closed economy counterpart.

Equation 2.2 is the open economy New Keynesian Phillips Curve that is derived from log-linearizing firm's pricing decisions around a zero-inflation steady-state and assuming firms set prices in a staggered fashion (see Calvo (1983)). The inflation rate is a function of one-period-ahead expectations of the inflation rate and the terms of trade, current changes in the terms of trade, the domestic output gap, and the world's output. β denotes the discount factor, $\bar{y}_t = -(\alpha(2-\alpha)(1-\tau)/\tau)y_t^*$ is the equilibrium level of output in the absence of nominal rigidities and conditional on the world output, and κ is a function of other structural parameters that capture labor demand and supply elasticities and price stickiness. The cost-push shock, u_t, represents exogenous changes in the domestic marginal cost, and can be interpreted as a supply shock.

Equation 2.3 embodies the central bank's monetary policy. The monetary authority is assumed to follow a Taylor-type rule with partial adjustment, setting the nominal interest rate, R_t, in response to the changes of the inflation rate, the output gap, and the exchange rate. The structural parameter ρ_R features the interest rate smoothness. Coefficients ψ_π, ψ_y, and ψ_e depict the policy response to inflation, output, and to the exchange rate, respectively. ε_t^R is the policy shock, which can be considered as an exogenous (non-systematic) component of monetary policy.

Equation 2.4 results from the log-linearization of the CPI formula using the purchasing power parity and assuming that the law of one price holds. The resulting equation links the inflation rate to the first difference of both the nominal exchange rate and the terms of trade. π_t^* is a world inflation shock.

The law of motion of the terms of trade is modeled through Equation 2.5, where ρ_q is an auto regressive coefficient. It is worth mentioning that this alternative ignores the possible market power of firms, and therefore takes the prices of international products as exogenous. This choice of modeling the terms of trade growth rate by adding a structural shock is mainly motivated from Lubik and Schorfheide (2007), as estimating the fully structural model is too restrictive and might yield implausible estimates.[4] Furthermore, this enables a fair comparison with similar papers.

Lastly, g_t, u_t, y_t^*, and π_t^* are assumed to evolve according to AR(1) processes, with autorregresive coefficients ρ_g, ρ_u, ρ_{y^*}, and ρ_{π^*}, and innovations ε_t^g, ε_t^u, $\varepsilon_t^{y^*}$, and $\varepsilon_t^{\pi^*}$, respectively. These innovations, together with ε_t^R and ε_t^q, are modeled as *i.i.d.*'s with mean equal to zero and a corresponding standard deviation, σ_i.

2.2.1 State-space representation

To summarize, the linear rational expectations model is conformed by an open-economy IS-curve (2.1), an open economy Phillips curve (2.2), a monetary policy rule (2.3), an expression for the nominal exchange rate (2.4), and a law of motion for the terms of trade (2.5). The structural model, including the shocks processes and the expectation formation expressions, can be rewritten in its state-space form as:

$$\Gamma_0 X_t = \Gamma_1 X_{t-1} + \Psi \epsilon_t + \Pi \eta_t \tag{2.6}$$

[4]The terms of trade endogenous relationship ($[\tau + \alpha(2-\alpha)(1-\tau)]\Delta q_t = \Delta y_t^* - \Delta y_t$) asserts that an increase in the world output raises the terms of trade, as it increases the demand for domestic goods. The effect of domestic output is the opposite. This specification is not used in the estimation.

where X_t is a 13×1 state vector that includes the endogenous variables, the AR(1) disturbances, and the expectation terms, ϵ_t is a 6×1 vector of exogenous innovations, and η_t is a 4×1 vector of expectational errors, defined as $\eta_t = X_t - E_{t-1}X_t$ and such that $E_t\eta_{t+1} = 0$. The model is solved using the methodology proposed by Sims (2002).[5] If the equilibrium exists and it's unique, the model solution under rational expectations can be expressed in state-space as:

$$X_t = FX_{t-1} + G\epsilon_t \tag{2.7}$$

where the matrices F and G are functions of the parameters of the model.

Equation 2.7 represents the transition equation of the state-space model and it expresses the state variables only as functions of their past values and the exogenous shocks.

2.3 Estimation strategy

In this section, I shall describe the econometric approach used for the estimation of the model, as well as the choice of prior distributions for the model parameters, and the data set used for the empirical analysis.

2.3.1 Econometric methodology

I follow the growing literature that estimates DSGE models using Bayesian computational techniques (see for example An and Schorfheide (2007), Del Negro et al. (2007), Del Negro and Schorfheide (2011), Fernández-Villaverde (2010), Lubik and Schorfheide (2005), Milani (2017), Rabanal and Rubio-Ramírez (2005), Smets and Wouters (2007), among many others).

[5] The model solution can also be found using alternative methods, for example Blanchard and Kahn (1980) and Uhlig (1999).

The econometric procedure consists of jointly estimating the model parameters, which I represent in an 18×1 parameter vector θ:[6]

$$\theta = [\tau, \alpha, \kappa, \rho_R, \psi_\pi, \psi_y, \psi_e, \rho_{y^*}, \rho_g, \rho_u, \rho_{\pi^*}, \rho_q, \sigma_R, \sigma_{y^*}, \sigma_g, \sigma_u, \sigma_{\pi^*}, \sigma_q]'$$

Draws are generated from the posterior distribution using Markov Chain Monte Carlo (MCMC) sampling methods. As an initial step, I calculate the posterior mode with standard numerical optimization routines. Then, for each estimation, I run 300,000 draws using a random-walk Metropolis-Hastings algorithm and discard the initial 30% as burn-in. At each iteration, I evaluate the likelihood of the model with the solution matrices of Equation 1.7 using the Kalman-Filter. Consistent with the rest of the literature, multiple equilibria solutions[7] are ruled out from the estimation by discarding draws that fell in the indeterminacy region.[8]

I explore the systematic changes in the Bank of Mexico's reaction function by estimating the model over a number of samples. This rolling-window approach has become a popular tool to evaluate possible changes in the model's structure as it allows parameters to evolve over time. Several papers have adopted this strategy to document instabilities and drifts in structural parameters. Canova (2009) performs rolling-window estimations on a small-scale New Keynesian model to investigate why output and inflation volatility have fallen in the US. His analysis provides evidence of changes in most of the model parameters, especially those that describe the private sector's behavior. Moreover, he finds that changes in the parameters that govern the policy rule and the covariance of the shocks explain best the Great Moderation episode. Similarly, Canova and Ferroni (2012) use rolling estimation techniques to study the time-varying nature of the structural parameters of a medium-scale

[6] As it is standard in the literature, I fix the discount rate β at 0.99.

[7] I refer the reader to Llosa and Tuesta (2008) for a complete analysis of the determinacy conditions that govern this New Keynesian small open economy model.

[8] Only an insignificant proportion of the draws (less than 0.0005% after the burn-in) fell in the indeterminacy region.

New-Keynesian model. They find structural changes to explain the evolution of US inflation in the last four decades. Castelnuovo (2012) implements this methodology to estimate both a small and a medium-scale DSGE model and show the time-dependence role of trend inflation shocks and the instability of the model parameters across the sub-samples. Hurtado (2014) documents the parameter drifts of the Smets and Wouters (2007)'s model by employing a rolling-window estimation. The paper also explores the policymaking implications of these changes by analyzing how the impulse-response functions vary over the sample period.

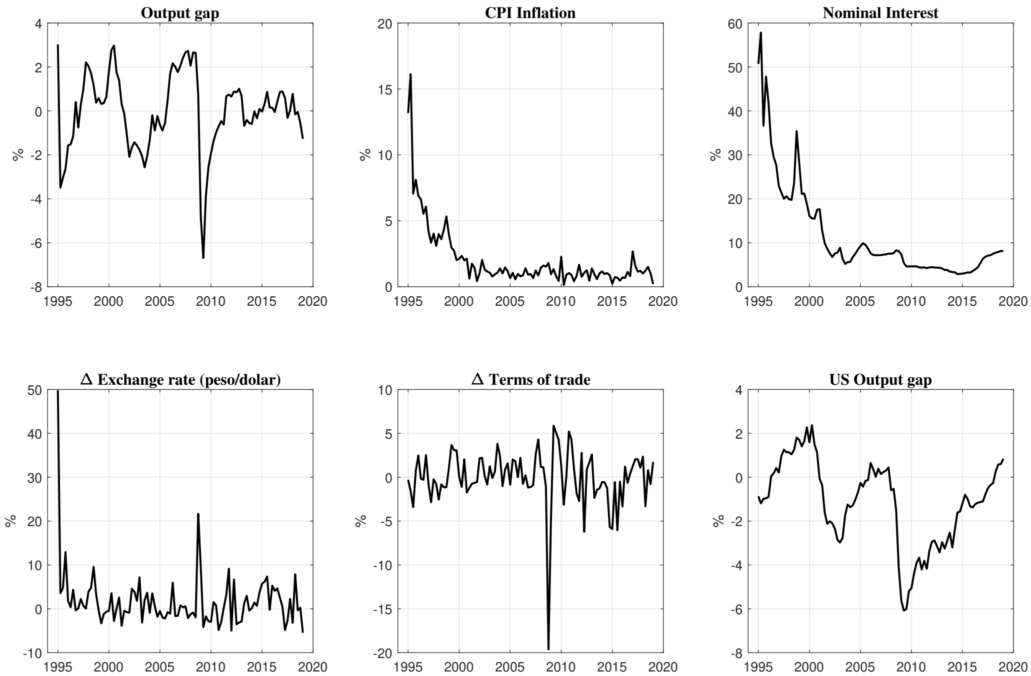
An alternative well-accepted approach to examine the evolving nature of parameters is to model the economy accounting for the possibility of different regimes over the period analyzed. This technique has been widely used in recent literature to detect parameter instabilities associated with abrupt changes in monetary policy. One of the most recognized papers in this context is the one from Clarida et al. (2000). They estimate a single forward-looking monetary policy reaction function to document the systematic changes in the U.S. monetary policy after 1979 (when Paul Volcker was appointed Chairman of the Federal Reserve). Lubik and Schorfheide (2004) use Bayesian techniques to study whether U.S. monetary policy changed in this same period. The paper estimates a standard 3-equation New Keynesian model and provides evidence of a much less active monetary policy before 1979 versus an aggressive one during the Volcker-Greenspan period. Sims and Zha (2006) analyze this episode working with a Markov-Switching model that allows for time-varying coefficients. In contrast to similar research, they find that the best-fitting model exhibits no change in the coefficients of the policy rule, but rather on the variance of structural disturbances. Similarly, estimating a New Keynesian model with Bayesian methods, Milani (2008) shows that there is weak evidence of a regime switch in the U.S. monetary policy if the rational expectation's hypothesis is relaxed. Bianchi (2012) addresses the subject using a Markov-Switching medium-scale DSGE model where agents are allowed to recognize the possibility of regime changes. His findings support the idea of a dovish behavior by the Federal Reserve in the 1970s and a hawkish one post-1979. However, the results suggest that the rise in inflation cannot be solely explained by the conduct of monetary policy but by a lack of confidence from the economic agents to move back to the hawkish regime. In a small open-economy context, Liu and Mumtaz (2011) use a Markov-Switching DSGE model fitted to UK data and test if the structural parameters of the model had stayed constant over the 1970-2009 period. In short, their results suggest that a change in the UK policy rule and the volatility of the structural shocks is needed to match the economy's performance during that time. Best (2013) employed a Bayesian approach to estimate the open-economy DSGE model worked in this paper and finds that, while fear of floating is still exhibited in the data, the Bank of Mexico's response to the exchange rate has decreased among the regimes.

While this approach is attractive to capture different phases in the data, it requires the researcher to assume fundamental changes in the economic environment, not to mention the inevitable pressure of the model to fit the data within a limited number of states. I abstract from these complications and instead let the data speak by employing a rolling-window approach.[9]

The model is therefore estimated throughout sub-sample windows using quarterly data that spans the period 1995:Q1 to 2019:Q1. In particular, the rolling-window approach is performed considering windows of ten years with increments of one year between the estimations. For instance, the first estimation uses data from 1995:Q1-2005:Q1, the next estimation employs data from 1996:Q1-2006:Q1, and consequently, the last estimation covers the period 2009:Q1-2019:Q1. This procedure is repeated every year of the sample period, which means that the model is estimated a total of fifteen times. The window width chosen is consistent with similar research focused on time-varying structural changes in DSGE models. Later in the paper, in the robustness analysis, I investigate the sensitivity of the results to different window widths. Lastly, I maintain the window size constant to minimize differences among

[9]It should be noted, however, that this strategy assumes that economic agents take parameter variations as exogenous when forming expectations. Thus, agents are neglected from employing information on parameter's drifts on the future state of the economy.

Figure 2.1: Data series



Note: Nominal variables are shown prior to subtracting the mean for illustration purposes.

the estimations.

2.3.2 Data

The model is fitted to match the following set of observables: output gap, inflation rate, nominal interest rate, exchange rate changes, terms of trade changes, and US output gap. All series are seasonally adjusted and demeaned prior to the estimation. The sample comprehends the period 1995Q1 to 2019Q1.[10]

The output gap is obtained using series for seasonally adjusted real GDP, from the Na-

[10] Among other reasons, I chose 1995Q1 to be the starting date for the estimation exercise as that period is commonly associated with a regime switch in the Mexican economy. It follows immediately after the 1994-crisis and after the adoption of a free-floating exchange rate. Additionally, data series before 1995 are characterized by high instability, which difficult the estimation.

tional Institute of Statistics and Geography (INEGI), and potential GDP, estimated with the Hodrick-Prescott filter;[11] and it is defined as the log difference between real GDP and potential GDP. I use the monthly National Consumer Price Index (INPC) from INEGI to calculate the inflation rate, defined as quarter-to-quarter percentage changes of the INPC.[12] The original series was converted to quarterly-basis by sampling the end of each quarter. Nominal interest rates are computed using series for the federal Certificates of the Treasury (CETES) at 91 days, which is the oldest instrument of debt issued by the Mexican government. The series is obtained from the Bank of Mexico's website and was converted to quarterly frequency by taking the average of the period. To compute the change in the exchange rate, I use the nominal exchange rate series available at monthly frequencies from the Bank of Mexico's website. I convert the series by taking the average of the period and use it to calculate exchange rate changes by taking the log difference (scaled by 100). Similarly, I use the terms of trade monthly series from the Bank of Mexico's database (transformed to a quarterly frequency by taking the period's average) and converted in log differences (scaled by 100) to obtain the percentage change of the terms of trade. Lastly, I use data on U.S. output as a measure of the world output gap. The U.S. output gap is defined as the log difference between U.S. real GDP and potential GDP, both available at the Federal Reserve Bank of Saint Louis.

Figure 2.1 provides a graphical representation of the data series. A particular remark that is worth mentioning is the relatively high inflation rate that governed the beginning of the sample. During this disinflation episode, the Bank of Mexico started reporting annual inflation targets to help the economy transition to its long-run level by 2003, chosen to be 3 percent and maintained since that year. I believe these annual reports provide more valuable information when de-meaning the inflation and the nominal interest rate, rather

[11] Potential GDP is derived in these lines to match the output gap estimates in the Bank of Mexico's official inflation reports. The specific methodology can be found in the quarterly inflation report of 2009 (April-June), available at the Bank of Mexico's website.

[12] I decided to use percentage changes in the consumer price index instead of the traditional log differences measure to best mimic the official inflation rate numbers reported by the Bank of Mexico.

Table 2.1:
Bank of Mexico annual inflation targets

Monetary policy program	Inflation targets (%)	Inflation rate (%) (end of year)
1995	19	51.97
1996	20.5	27.7
1997	15	15.72
1998	12	18.61
1999	13	12.32
2000	10	8.96
2001	6.5	4.4
2002	4.5	5.7
2003-2019	3	4.2 (on average)

Note: The inflation targets were obtained from the monetary policy annual programs published on the Bank of Mexico's website. The inflation targets refer to the CPI inflation rate between December of the previous year and December of the current year. The last column reads the actual inflation rate reported at the end of the year. Monetary policy annual programs are typically available to the public at the end of January of the corresponding year.

than subtracting the inflation's sample mean. Hence, I exploit official data on the Bank of Mexico's annual inflation targets (divided by 4 to yield quarterly values) and use it to de-mean both variables. These target rates are briefly summarized in Table 2.1. As a robustness check, I will later use an alternative measure to de-mean the two variables and verify the validity of the results.[13]

2.3.3 Prior distributions

Prior distribution for the estimation of the model are reported in Table 2.2. Most of the priors follow Best (2013), mainly because of the similarities estimating Mexico's small open economy. I center the policy response to the inflation rate and output gap at values regularly linked to the Taylor-rule. ψ_π is assumed to follow a Normal distribution with a mean of

[13]The main results are also robust to demeaning the data with the sample mean.

Table 2.2:
Prior distributions

Parameter	Domain	Distribution	Mean	SD
β	0.99	-	0.99	-
τ	[0, 1)	Beta	0.50	0.20
α	[0, 1)	Beta	0.50	0.20
κ	$\mathbb{R}^+$	Gamma	0.50	0.20
ρ_R	[0, 1)	Beta	0.50	0.20
ψ_π	$\mathbb{R}$	Normal	1.50	0.25
ψ_y	$\mathbb{R}$	Normal	0.50	0.25
ψ_e	$\mathbb{R}$	Normal	0.50	0.25
ρ_{y^*}	[0, 1)	Beta	0.80	0.10
ρ_g	[0, 1)	Beta	0.80	0.10
ρ_u	[0, 1)	Beta	0.80	0.10
ρ_{π^*}	[0, 1)	Beta	0.80	0.10
ρ_q	[0, 1)	Beta	0.80	0.10
σ_R	$\mathbb{R}^+$	InvGamma	8.00	6.00
σ_{y^*}	$\mathbb{R}^+$	InvGamma	1.00	0.70
σ_g	$\mathbb{R}^+$	InvGamma	7.00	4.89
σ_u	$\mathbb{R}^+$	InvGamma	7.00	4.89
σ_{π^*}	$\mathbb{R}^+$	InvGamma	10.00	6.00
σ_q	$\mathbb{R}^+$	InvGamma	7.00	4.89

1.5 and a standard deviation of 0.25 and ψ_y a Normal distribution with mean 0.5 and standard deviation of 0.25.[14] As in Best (2013), I originally modeled the monetary response to the exchange rate, ψ_e, with a loose prior and use a Uniform distribution with support [-2, 2]. Nonetheless, after estimating all the sub-samples, the estimations for 1996-2006 and 1997-2007 turned out to be problematic and yielded implausible parameter estimates. The apparent reason is that using a non-informative prior for the exchange rate's response creates a complication by not guiding the posterior to plausible regions of the parameter space. Therefore, I repeated the estimation exercise for all the sub-samples using instead a Normal distribution as a prior for the exchange rate's response and centered it around the

[14] As a robustness check, I've also estimated the model using Gamma priors for the policy parameters and obtained near-identical results.

posterior estimate found by Best (2013). As a consequence, posterior estimates for the 1996-2006 and 1997-2007 sub-samples were much in line with the other windows, with virtually the same posteriors found before for the other sub-samples. Thus, the estimation results reported correspond to the latter specification. The interest rate smoothing parameter, ρ_R, and the trade openness coefficient, α, are modeled using a Beta distribution with mean 0.5 and standard deviation of 0.2. I specify the priors for the slope coefficient in the Phillips curve, κ, and the intertemporal elasticity of substitution, τ, following Lubik and Schorfheide (2007), with a Gamma distribution centered at 0.5 and a Beta distribution with mean 0.5, respectively. The autoregressive coefficients are all assumed to follow Beta distributions with mean 0.8. Finally, the standard deviations of the shocks are modeled using loose priors to allow for some variation and follow Inverse Gamma distributions.

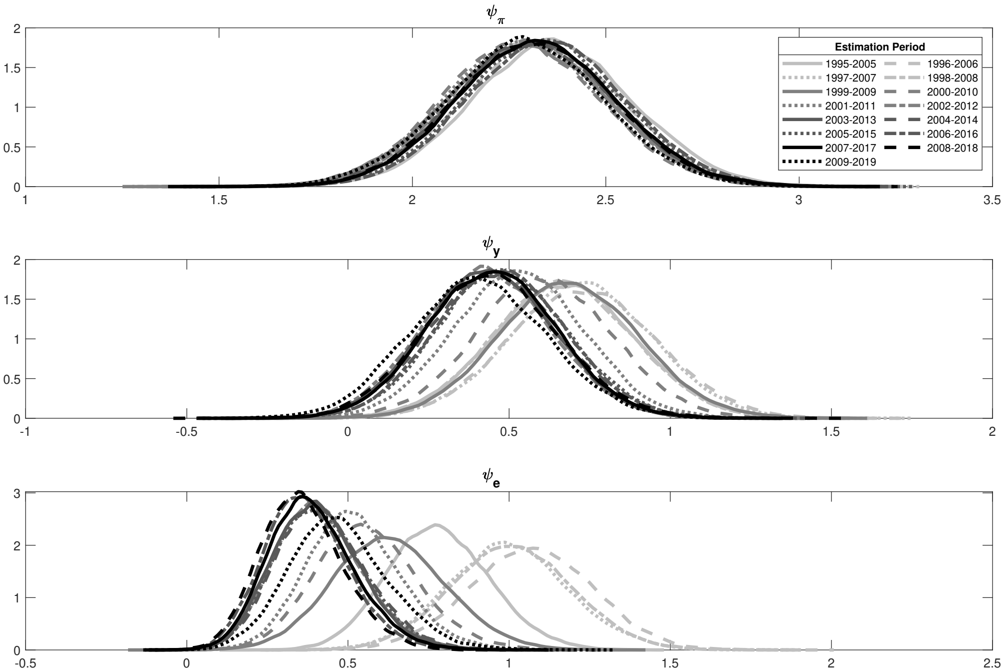
2.4 Empirical results

2.4.1 Posterior distributions

I present the evolution of the Bank of Mexico's reaction function policy coefficients in Figure 2.2. The figure overlaps the posterior densities across the sub-samples. A striking result from the graph is that it exposes the stable (and relatively high) response held by the central bank towards the inflation rate across all the estimations. One interpretation of this finding is that the Bank of Mexico has maintained a high degree of inflation aversion that followed the inflation episode of the mid-90s. Alternatively, the result is also consistent with the idea of commitment by the central bank to help anchor inflation expectations, and thus bring the economy close to the long-run inflation level. The policy response to the output gap, however, displays a different picture. Posterior densities for this parameter show an evident transition to a lower output response that stabilizes after the 2002-2012 estimation. Two main explanations give support to this result, one that speaks about the drift and one about the stabilization. Regarding the former, the lower output response resembles the convention that large interest rate cuts are needed to stimulate the economy when nominal interest rates are high. As for the latter, the image seems to capture the notion that the monetary authorities started responding less aggressive to economic growth when the inflation rate began to reach its long-run target. Concerning the exchange rate response, the figure portrays a similar transition as of the output response. However, there are a few differences that are worth mentioning. The evolution of this parameter depicts a more evident shift to a lower response in the first sub-samples, that settles too after the 2002-2012 period. Moreover, the posterior densities that followed this period's estimation reveal a higher degree of certainty regarding the central bank's response to the exchange rate. The general picture that arises from these results seems to be in line with the findings in Best (2013), that after 1994 the monetary authorities in Mexico adopted the inflation rate as the new nominal anchor of the economy and, therefore, have decreased the response to the exchange rate.

Bayesian posterior estimates for all the model parameters are presented in Figure 2.3. The figure displays the median across the Metropolis-Hastings draws along with 95% posterior probability intervals. In general, most of the parameters exhibit a steady behavior across the sub-samples. There are, however, some parameter instabilities. The intertemporal elasticity of substitution appears to grow over time, increasing its value in the initial periods and stabilizing after the 2002-2012 estimation. The trade openness parameter and the slope of the Phillips curve shift up and down in the initial estimations, but rather show an erratic behavior around a constant value opposed to an evolving nature. As noted before, the central bank's response to the output gap and the exchange rate both decrease over time. Most of the autoregressive coefficients present high and steady persistence across the estimations. The exception being the disturbance associated with the domestic demand shock, which initiates at a highly persistent value and consistently decreases as the window moves towards recent

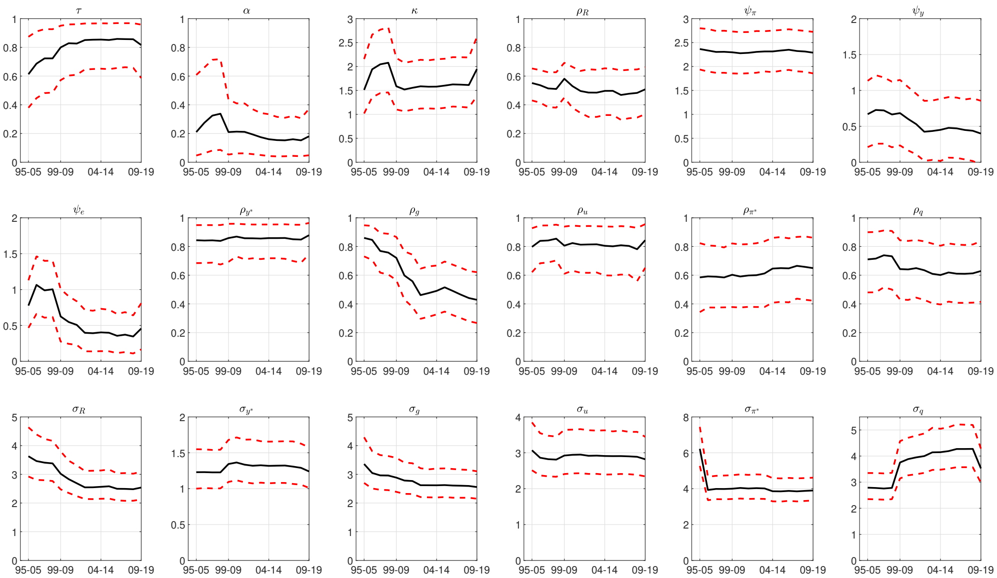
Figure 2.2: Posterior distributions of policy coefficients



data. The autoregressive coefficient on the terms of trade shows a downward shift after the 1999-2009 estimation, but a steady behavior after that.

A few remarks can be made concerning the volatilities of the shocks. First, the volatility of the monetary policy shock and that of the domestic demand shock display an evident downward trend. Second, the volatility of the world inflation shock exhibits a steady behavior if the 1995-2005 estimation is excluded. This result seems to be associated with how the world inflation shock enters the model in Equation 2.4. As a result of the high inflation rate and exchange rate depreciation reported in 1995, the world inflation shock seems to absorb this volatility by yielding a high standard deviation. Third, there is an irregular movement that resembles a hump-shaped pattern in the volatility of the terms of trade shock. And fourth, the standard deviation of the world output innovation reveals a steady nature across the sample. I will discuss further the implications of the time-varying effect of

Figure 2.3: Evolution of model parameters



Note: The figure shows the posterior median (solid line) of each sub-sample across the Metropolis-Hastings draws, along with 95% Bayesian credible interval bands (dashed lines).

the innovation's volatilities with the impulse response functions.

To summarize the distribution characteristics of the econometric exercise across the sub-samples, I also present rolling-window estimates for the model parameters. The results are expressed in terms of the posterior median and are reported in Table 2.3. Although the table cannot be used to make inference regarding the time-varying nature of the model parameters, it serves as a quick reference to examine the stability across the sub-samples.

2.4.2 Recursive impulse response functions

I proceed to evaluate the model dynamics and macroeconomic implications of the policy parameters' change by computing impulse response functions for each of the sub-samples and analyzing how they change over time. The discussion is first briefly focused on the

Table 2.3:
Rolling-window estimates for model parameters

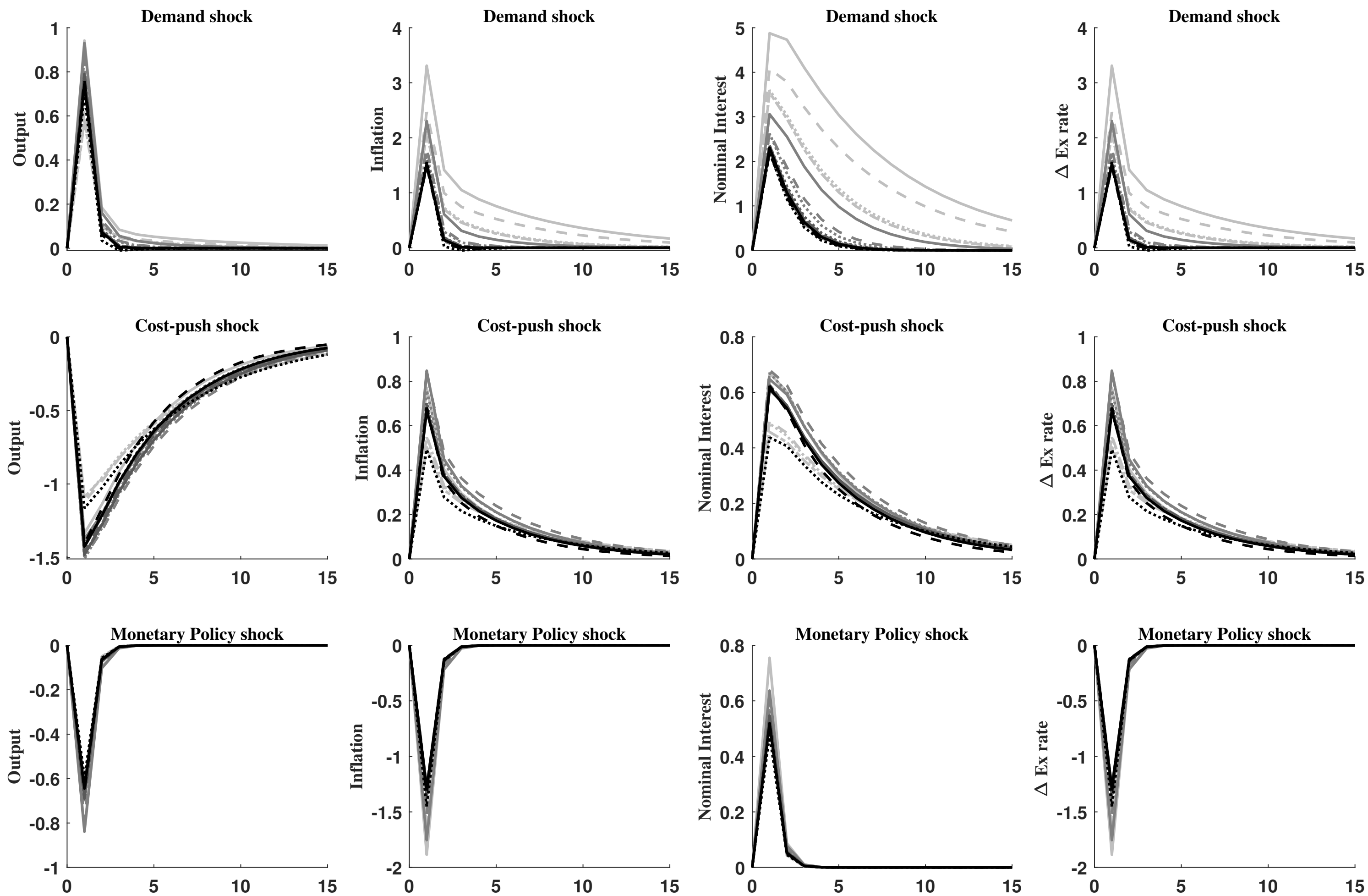
Parameter	Rolling-window estimates (posterior median)			
	Min	Median	Mean	Max
τ	0.64	0.83	0.81	0.87
α	0.17	0.22	0.25	0.44
κ	1.22	1.30	1.34	1.57
ρ_R	0.48	0.51	0.52	0.59
ψ_π	2.28	2.31	2.31	2.35
ψ_y	0.30	0.39	0.47	0.67
ψ_e	0.36	0.48	0.59	1.07
ρ_{y^*}	0.84	0.86	0.85	0.88
ρ_g	0.44	0.53	0.60	0.87
ρ_u	0.80	0.83	0.83	0.85
ρ_{π^*}	0.54	0.60	0.62	0.67
ρ_q	0.58	0.61	0.63	0.74
σ_R	2.48	2.56	2.84	3.65
σ_{y^*}	1.22	1.31	1.29	1.37
σ_g	2.60	2.67	2.82	3.38
σ_u	2.94	3.03	3.03	3.19
σ_{π^*}	3.86	3.97	4.10	6.20
σ_q	2.74	3.94	3.69	4.26

Note: Rolling-window posterior medians of the model's parameters are estimated over the ten-year window for each year of the evaluation sample.

sign-effect from the structural shocks, and then on the observed shifts of these functions. Figure 2.4 depicts the impulse responses to one standard deviation shocks.

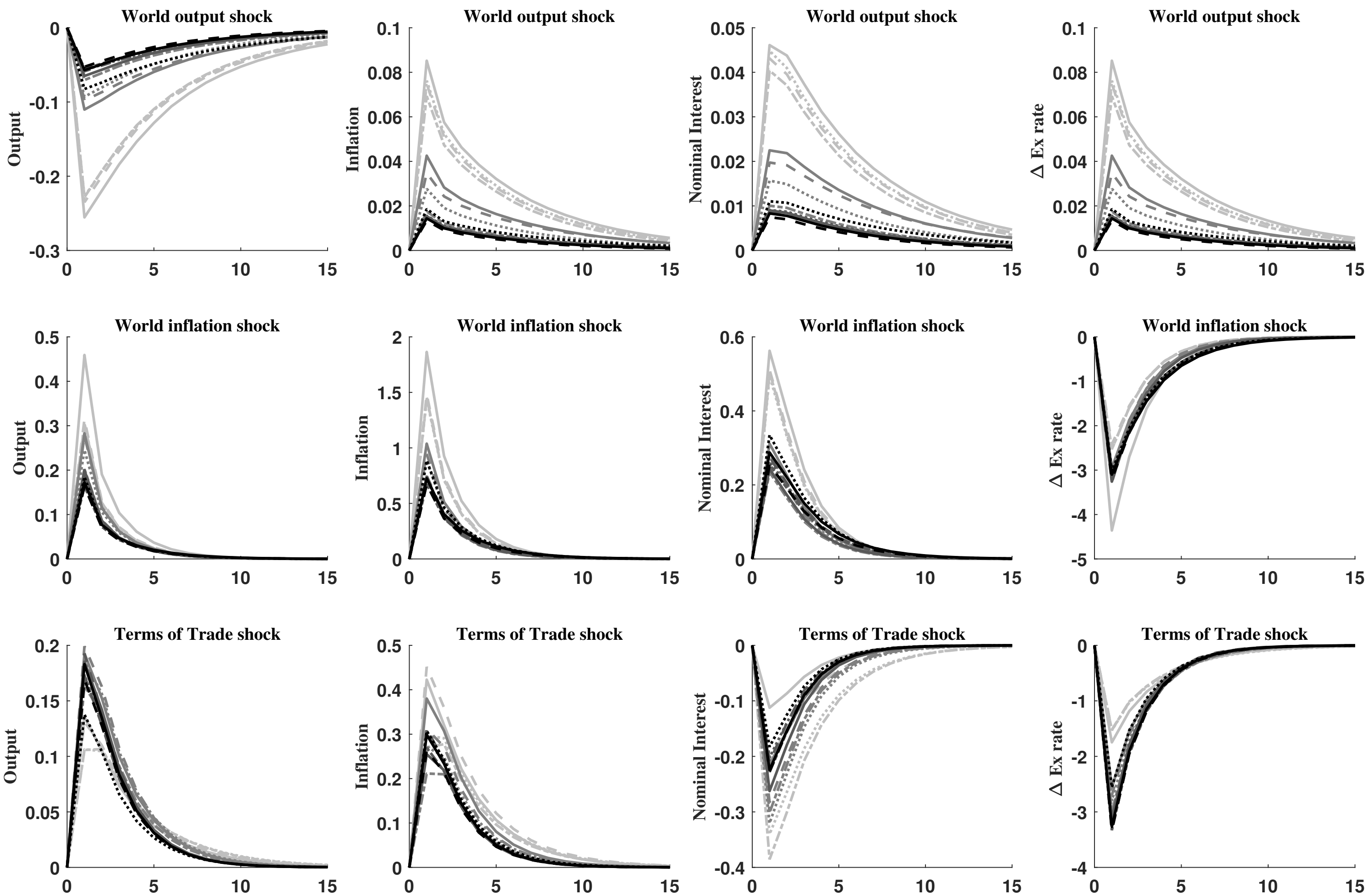
According to the results, all the impulse response functions exhibit the expected sign effect from the structural shocks and are consistent with similar research on open economies (for example Lubik and Schorfheide (2007), Adolfson et al. (2007), Liu and Mumtaz (2011), Best (2013), among others). The domestic demand shock increases domestic output, inflation, nominal interest rates, and the exchange rate. The supply shock has an opposite effect on output and inflation, decreasing the former and increasing inflation, nominal interest

Figure 2.4: Rolling-window Impulse Response Functions



Note: Rolling-window impulse response functions are calculated using the last 50,000 draws from the MCMC. The impulse responses represent the median of each period across draws. Bayesian probability intervals omitted for clarity purposes. Legend is the same as in Figure 2.2.

Figure 2.4: (cont.)



Note: Rolling-window impulse response functions are calculated using the last 50,000 draws from the MCMC. The impulse responses represent the median of each period across draws. Bayesian probability intervals omitted for clarity purposes. Legend is the same as in Figure 2.2.

rates, and the exchange rate. A contractionary monetary policy shock lowers output and inflation and raises nominal interest rates. The economy's response to the rest of the world's demand shock is more intriguing. Domestic output declines as a reaction to the positive world output shock, while both inflation and the exchange rate rise. As discussed in Lubik and Schorfheide (2007), this result occurs due to the negative effect that the shock generates on domestic potential output.[15] As a consequence, the excess demand created stimulates inflation and pushes the central bank to increase interest rates. Positive world inflation shocks appreciate the domestic currency and increase domestic inflation rate and output. Finally, an improvement in the terms of trade appreciates the domestic currency, which induces the central bank to lower interest rates. The expansionary monetary policy enhances output and increases the inflation rate.

I now move to analyze the economic implications of the change in the policy parameters. I will abstract to mention overlapping probability intervals and focus instead on the dynamic behavior of the impulse responses. There is an evident impact effect from the demand shock on the inflation rate, nominal interest, and exchange rate, that not only decreases over time but reverses to zero faster. This result is in line with the decline previously found on the output's response policy coefficient. The central bank responds to less severe demand shock effects by reducing its reaction to output. By looking at the supply shock responses, I find no evidence of drifting effects. The lines rather oscillate around one another, which again is consistent with the constant response towards inflation by the central bank. Contractionary monetary policy shocks exhibit a decreasing initial impact effect on all the variables (in absolute terms). In particular, the nominal interest rate effect can be interpreted as a decrease in the non-systematic component of monetary policy.

Rolling-window impulse responses for world output and world inflation shocks display un-

[15] The effect is conditional on $\tau < 1$. When this is the case, domestic and foreign goods act as substitutes, and the variables behave countercyclically. The relationship can be easily noticed by looking at Equation 2.2.

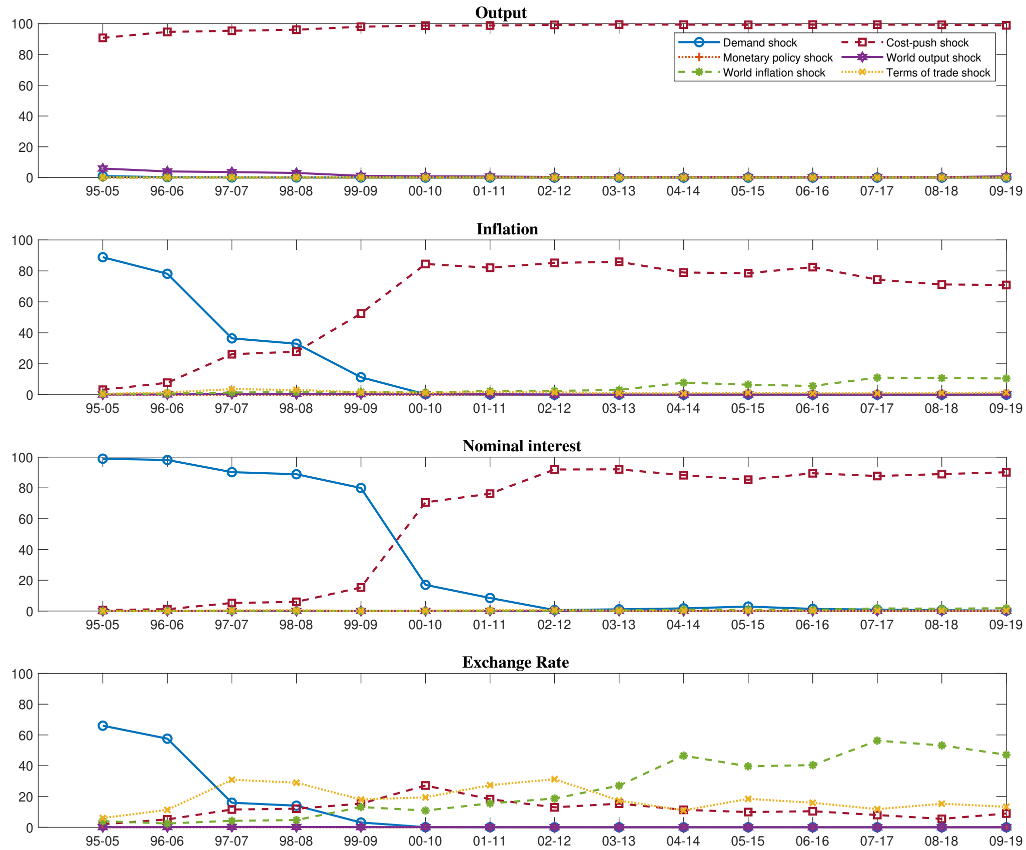
ambiguous shifts that expose a periodic transition towards lower impact effects (in absolute value). Moreover, world output shocks seem to dissipate faster across all the variables. World inflation shock shows similar shifts in the domestic output, inflation, and interest rate impulse responses. The initial impact decreases as the rolling window moves and stabilizes after the 2002-2012 sub-sample. The only exception of this behavior is captured by the world inflation shock effect on the exchange rate. In contrast to the other results, the impact effect appears to increase over time. This is strongly related to the central bank's reaction to the changes in the exchange rate. As world output and world inflation shocks start to have lower impacts on domestic variables, the depreciation of the currency represents a smaller risk. In response, the central bank reacts by decreasing its attention to exchange rate changes.

2.4.3 Variance Decomposition

The paper also examines the contribution of the structural shocks to the main variables of interest and how they evolved over the different windows with a forecast error variance decomposition analysis. The results are presented in Figure 2.5 and refer to the median forecast error variance shares at the 8-quarters ahead horizon.

Output fluctuations are mainly driven by the cost-push shock, explaining over 90% of the output forecast error variance across all the subsamples. For the other endogenous variables, the contribution of the structural shocks is not stable over the sample, with unambiguous differences and drifts as the windows move towards the end of the sample. Inflation is largely explained by the domestic demand shock in the initial windows, being the key driver of the fluctuations for the first four windows but accounting for a lower contribution as the windows move towards the end of the sample. After the 1999-2009 window, the cost-push shock becomes the main source of inflation variability, with shares that fluctuate between 55% and 85%. The share of the world inflation shock increases steadily over the rolling-

Figure 2.5: Evolution of variance decomposition



Note: The figure shows the window-specific median forecast-error variance shares across MH draws at the 8-quarters ahead horizon.

windows, accounting initially from 0.5% to 10.5% in the last sub-sample. Nominal interest rate fluctuations share a similar story to the variance decomposition of the inflation rate. The first five windows identify the domestic demand shock as the main driver of the variability of the interest rate, explaining at least 80% of the fluctuations. As the windows transition to the 2000-2010 window and afterward, the contribution of the cost-push shocks rises to ranges between 76% and 92%. Lastly, exchange rate fluctuations are explained by a combination of shocks. The domestic demand shock contributes the most in the first two windows with over 57% but then its share decreases to near 0% as the windows move. Both the cost-push shock and the terms of trade shock play an important role in explaining the variability of the exchange rate over the entire sample, accounting for ranges between 2-27% and 6-31%, respectively. The world inflation shock becomes the key driver of the fluctuations after the 2003-2013 window, with contributions that range between 27% and 56% across the subsamples.

2.4.4 Robustness

2.4.4.1 Time-varying inflation target

In this section, I check the robustness of the results when allowing for a time-varying inflation target in the policy rule. The analysis portrays the notion that the central bank's inflation target has been drifting and explores the effect on the policy feedback coefficients over the rolling-windows. As such, I follow the expanding literature that models policy-makers' to be concerned with a time-varying inflation target. For instance, Ireland (2007) develops a DSGE model that features a Taylor-rule that allows the monetary authorities' inflation target to adjust in response to supply shocks. The paper finds evidence that the Federal Reserve has responded to short-run pressures by moving its inflation target, which helps explain the inflation episodes in the U.S. during the 1960s and 1970s. Cogley and Sbordone (2008) formulate a version of a New Keynesian Phillips Curve that includes time-varying trend inflation to study the inflation dynamics in the U.S. They find that indexation or a backward-looking component are not needed to explain U.S. inflation episodes when trend-inflation is included in the model. Kozicki and Tinsley (2009) use Greenbook briefing forecasts to estimate an effective inflation target and find substantial variations in the Federal Reserves' inflation target. Cogley et al. (2010) employ a VAR with drifting parameters and a simple New Keynesian DSGE model to study the differences in U.S. monetary policy during the Great Inflation and the Volcker's disinflation episodes. They identify long-run inflation targets as the key element to explain inflation volatility and persistence. In an effort to investigate the role of trend inflation shocks to the U.S. economy, Castelnuovo (2010) uses a New Keynesian model that incorporates inflation targeting. His results hint at the importance of trend-inflation shocks to explain the volatility of inflation and policy rates.[16]

For comparability with the previous estimation results, I will abstract from complicating the model's structure and follow the conventional assumption of a zero trend inflation steady-state. The central banks' reaction function is now modeled to set the nominal interest rate responding to deviations from a time-varying inflation target:

$$R_t = \rho_R R_{t-1} + (1-\rho_R)\big[\psi_\pi(\pi_t - \tilde{\pi}_t) + \psi_y y_t + \psi_e \Delta e_t\big] + \varepsilon_t^R \tag{2.8}$$

where $\tilde{\pi}_t$ represents a time-varying inflation target.

Following Ireland (2007), I model the inflation target as an exogenous random process, and allow it to respond to the domestic supply shock:[17]

$$\tilde{\pi}_t = \rho_{\tilde{\pi}}\tilde{\pi}_{t-1} - \delta_u \varepsilon_t^u + \varepsilon_t^{\tilde{\pi}} \tag{2.9}$$

where $\rho_{\tilde{\pi}}$ is an autoregressive coefficient, δ_u is a parameter chosen by the central bank, and $\varepsilon_t^{\tilde{\pi}}$ is an *i.i.d.* process with mean equal to zero and a standard deviation $\sigma_{\tilde{\pi}}$.

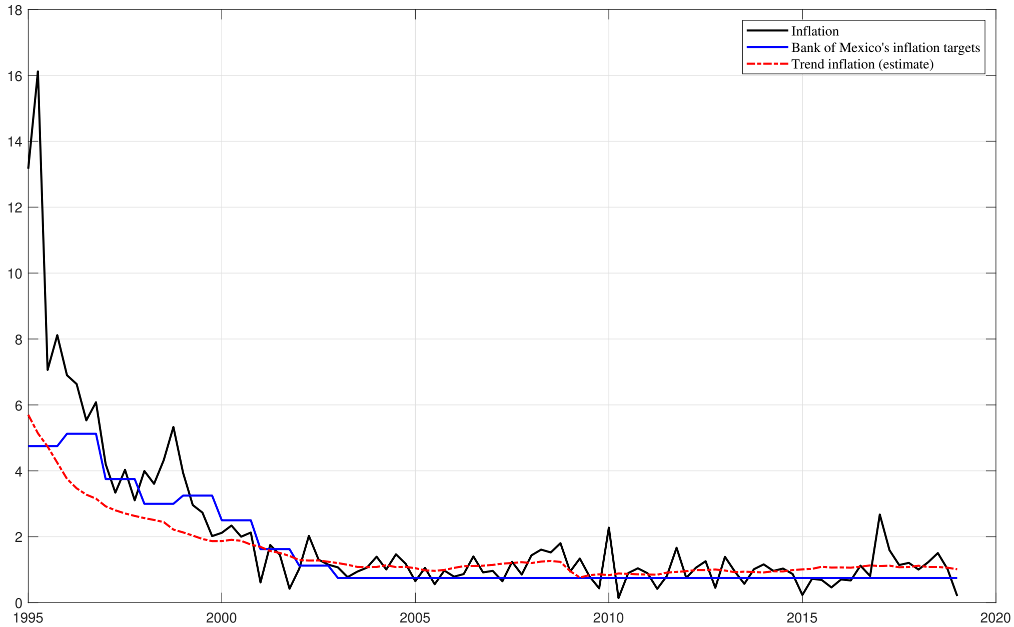
The new system of equations differs from the original specification by replacing Equation 2.3 with Equation 2.8 and introducing Equation 2.9. As a result, the model now includes an additional endogenous variable and an exogenous innovation.[18] With these alterations, the objective is first, derive an estimate of trend-inflation and use it as a measure to de-mean the

[16] Among other equally relevant papers in this literature, please refer to Svensson (2000), Ascari (2004), Coibion and Gorodnichenko (2011), Del Negro and Eusepi (2011), Ascari and Sbordone (2014), Castelnuovo et al. (2014), Milani (2019).

[17] This decision was motivated for calibration purposes only, as the trend-inflation estimate replicated best the inflation episodes when this response (although small) was included.

[18] Not to mention the three additional auxiliary parameters $\rho_{\tilde{\pi}}$,δ_u, and $\sigma_{\tilde{\pi}}$.

Figure 2.6: Model-implied trend inflation (full sample)



Note: The trend-inflation Kalman estimate is computed using the rolling-window posterior medians from Table 2.3.

inflation rate, and second, repeat the estimation exercise and compare the results obtained.

However, a difficulty that arises from the first objective is that a single trend-inflation estimate is needed to avoid differences in the data across the sub-samples and evidently, estimating the model over the entire sample is inconsistent with the rolling-window exercise.[19] To work around this problem, I instead use the rolling-window posterior medians (see Table 2.3), as I believe they provide a better representation of the model parameters' behavior over the whole period. Lastly, I fix the parameters associated with trend inflation[20] to values commonly found in the literature and use the Kalman Filter to compute the trend inflation estimate.[21] Figure 2.6 presents the resulting model-implied trend inflation.

[19]I confirmed this by estimating the model using the whole sample and found huge differences in the posterior estimates with implausible estimates for some parameters.

[20]An attempt was made to jointly estimate these parameters, but the results were very sensitive to the choice of priors and often produced implausible estimates.

[21]Specifically, I set $\rho_{\tilde{\pi}} = 0.995$, $\delta_u = 0.04$, and $\sigma_{\tilde{\pi}} = 0.01$.

Some observations are in order regarding the figure. First, the estimate appears to move close to the Bank of Mexico's inflation targets for most years, with slight exceptions during the deflation episode in the second half of the 1990s. Second, the line responds to the financial crisis of 2008 by reducing the estimate and maintaining it near the official inflation target for the following years. And third, the trend inflation estimate slightly departs from the long-run inflation target in the last years of the sample as a result of the inflation shocks that the economy experienced throughout 2017.

I then continue to de-mean the inflation rate using the trend inflation generated and proceed to estimate the model using the rolling-window approach. The Bayesian estimation strategy is repeated using the prior distributions from Table 1.1 and maintaining the same calibration previously discussed for the trend inflation parameters. Figure **??** depicts the evolution of the reaction function's policy parameters under this specification.

As the figure illustrates, the posterior densities for the policy response to the output gap and the exchange rate are much in line with those previously reported. In particular, the central bank's response to output appears to decrease over time and stabilizes after the 2002-2012 estimation. The monetary policy response to the exchange rate features the same smooth transition towards smaller values. Nonetheless, the posterior densities for the inflation policy coefficient now suggest a decreasing response that settles after 2002. This result is not surprising in light of the model assumptions under the last specification. The central bank is now assumed to set its policy responding to the gap between the inflation rate and the inflation target. As the inflation rate starts to reach the inflation target, monetary authorities maintain the response fixed.

Posterior estimates for the rest of the parameters are displayed in Figure 2.8. In short, the results resemble the ones from Figure 2.3, with few subtle differences in the point estimates, but ultimately evidencing the same evolving nature found before on the model parameters.

Figure 2.7: Posterior distributions of policy coefficients (Model with trend inflation)

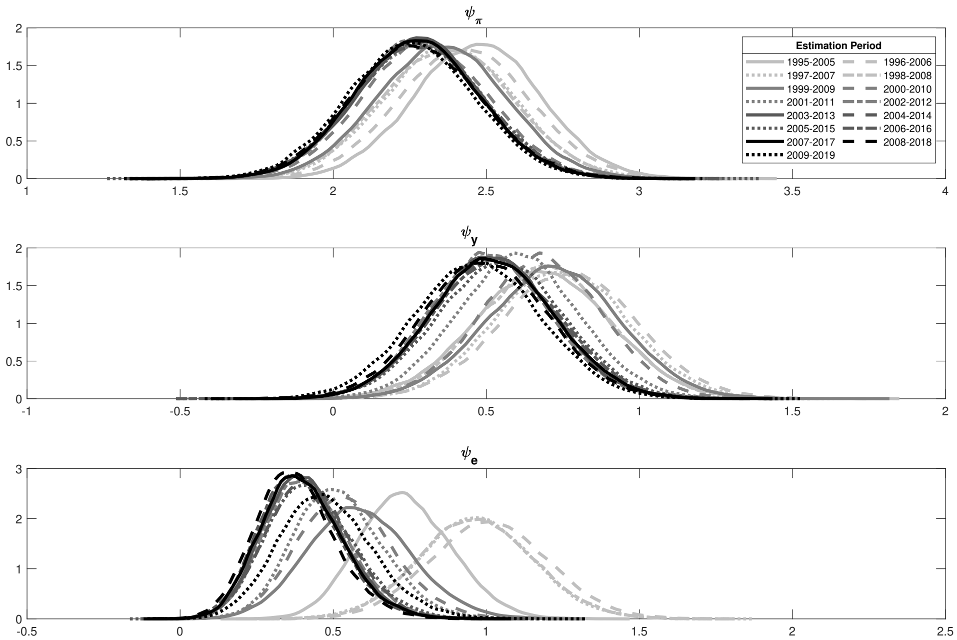
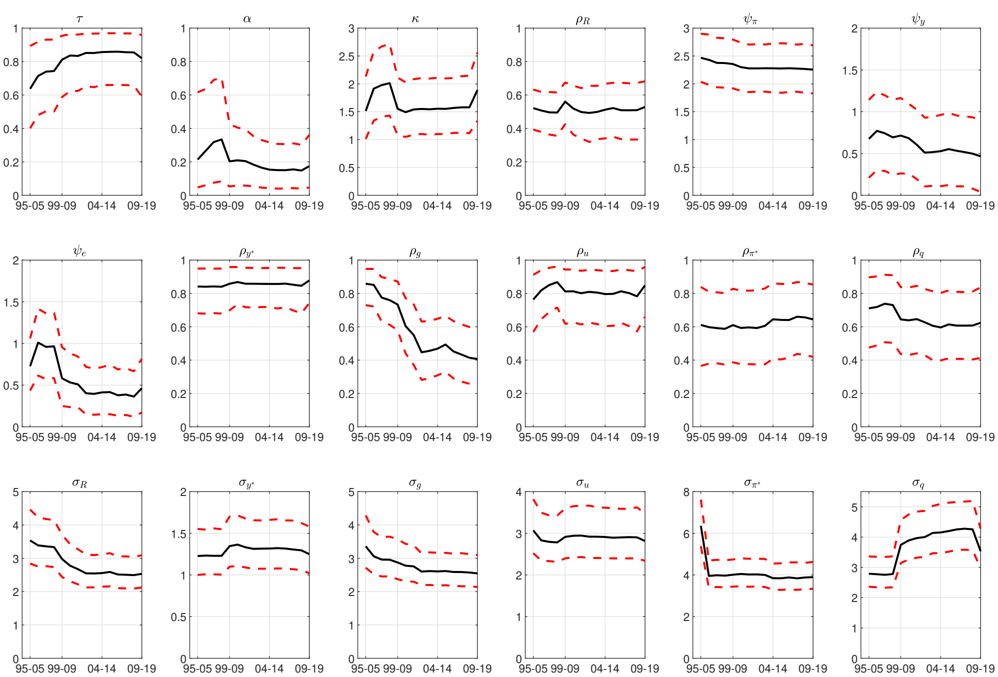
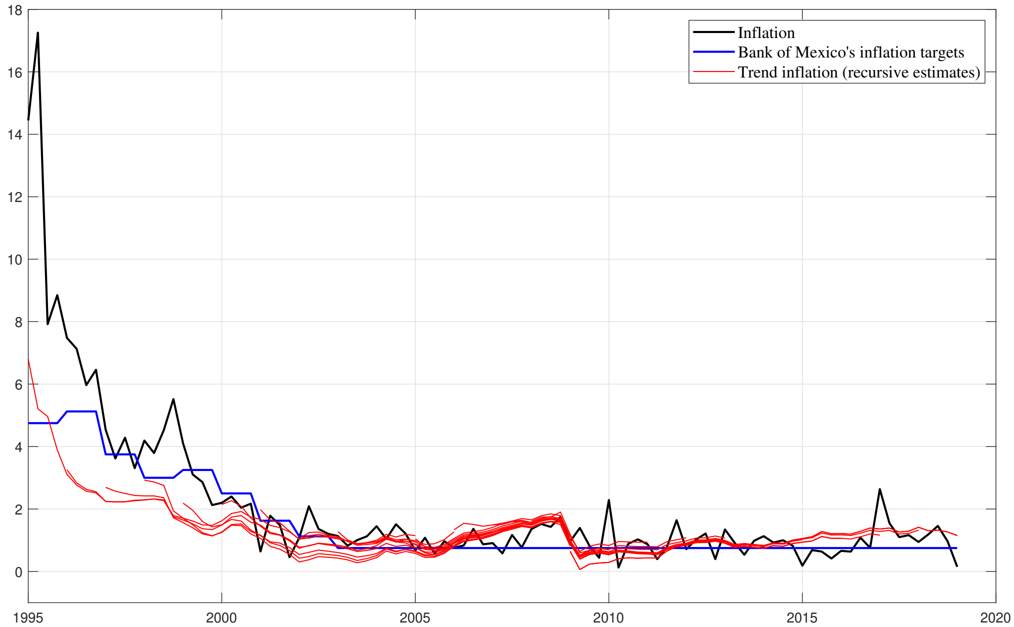


Figure 2.8: Rolling-window posterior estimates (Model with trend inflation)



Note: The figure shows the posterior median (solid line) of each sub-sample across the Metropolis-Hastings draws, along with 95% Bayesian credible interval bands (dashed lines).

Figure 2.9: Model-implied trend inflation (sub-samples)



Note: The figure displays trend-inflation Kalman estimates for each rolling-window of the sample using the last 50,000 draws from the Metropolis-Hastings. Red lines represent the median estimate of each period and across draws. Bayesian credible bands omitted for clarity purposes.

Lastly, I use the Metropolis-Hastings draws from each sub-sample and generate window-specific trend inflation estimates. The objective of this step is to verify that these estimates are in line with the one depicted in Figure 2.6 and used to de-mean the inflation rate. In other words, this is done to assert the validity of using the rolling-window posterior medians to portray the parameters' behavior over the whole sample.

Figure 2.9 displays the trend inflation rolling-window estimates. As the graph illustrates, sub-sample estimates reveal a similar behavior to the one from Figure 2.6. All the lines show a decreasing trend during the disinflation episode of the late 90s, move along the Bank of Mexico's inflation target, and respond to the financial crisis of 2008. On the other hand, the model seems to overestimate the inflation trends of the second half of the sample. This result was expected, as δ_u and $\sigma_{\tilde{\pi}}$ were maintained fixed on all the windows. Ideally, both the central bank's response to the domestic inflation shock and the standard deviation of the trend inflation shock should be jointly estimated along with the other parameters. However, the evolution of these parameters and the importance of trend inflation shocks go beyond the scope of this paper and, therefore, are left for future research.

2.4.4.2 Alternative window widths

I also check the validity of the results to the widths of the rolling windows and re-estimate the canonical model employing alternative window widths. The aim is to verify that the main findings are not unique to the window size previously chosen. I address this issue by repeating the estimation exercise using 12-year and 15-year windows. Posterior densities for the policy parameters under these specifications are illustrated in Figure 2.10 and Figure 2.11, respectively.

The results are qualitatively similar to the ones presented in the previous sections. The central bank's response to the output gap and the nominal exchange rate exhibit a decreasing pattern over the rolling-windows that again stabilize after the economy reaches the long-run inflation target. Although the posterior distributions for the policy response to the inflation rate are not as steady across the estimations (relative to the previous findings), there is no apparent pattern that emerges from the picture. Instead, the densities rather smoothly oscillate with no evidence that signals a drifting parameter. Lastly, while the figures are in line with previous results, the posterior densities for the policy parameters now show a higher degree of certainty that is reflected by narrower distributions. This result is not surprising given the increase in the window's width.

Figure 2.10: Posterior distributions of policy coefficients (12-year window)

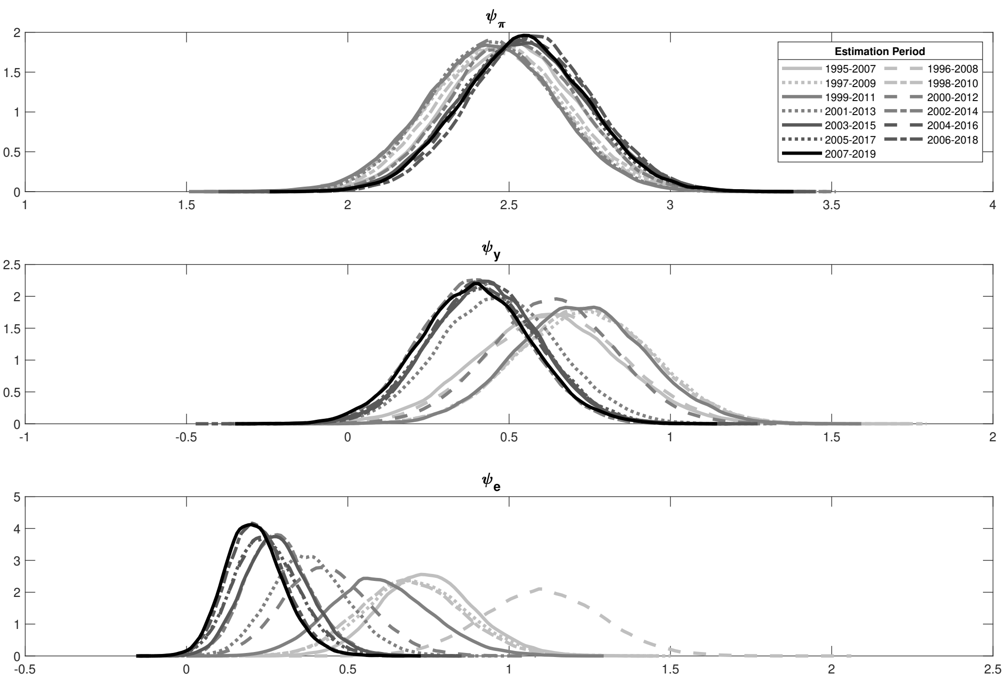
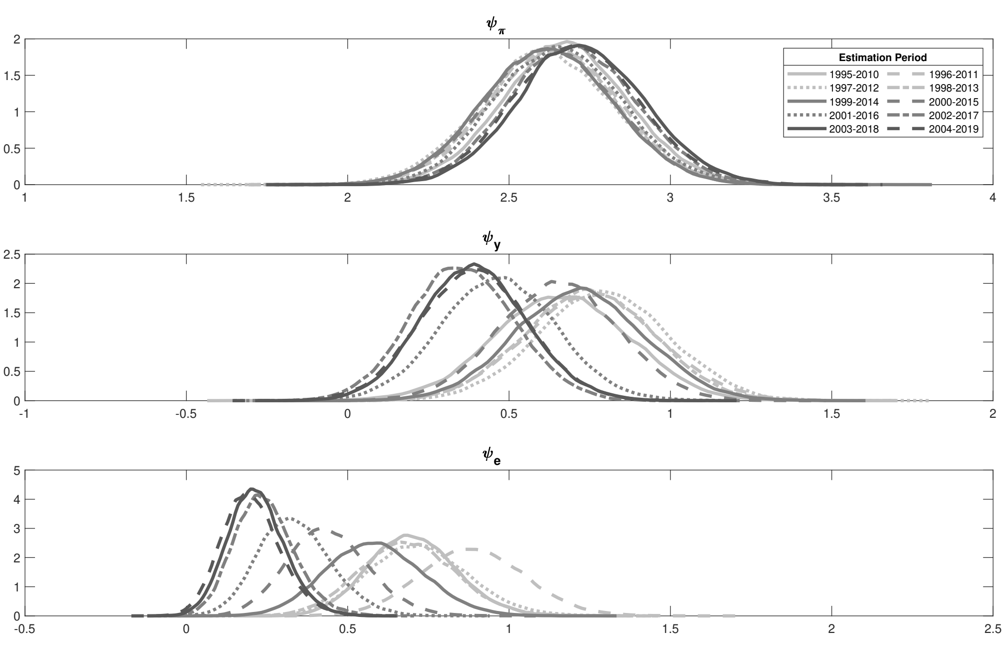


Figure 2.11: Posterior distributions of policy coefficients (15-year window)



2.5 Conclusions

This paper used a rolling-window approach to estimate an open-economy DSGE model and provide evidence of parameter drifts in the Bank of Mexico's reaction function. Employing full-information Bayesian techniques over rolling-windows, I have shown that the policy feedback coefficients on output and the exchange rate transitioned to a lower value, while the response to the inflation rate has remained stable. I find the results consistent with the disinflation episode that the Mexican economy experienced during 1995-2003.

The paper also investigated the macroeconomic implications of the policy parameters' shifts by computing rolling-window impulse response functions. I found compelling evidence of lower impulse-response effects on the main domestic variables due to world demand and world inflation shocks. I attribute this result to the disconnection of the exchange rate as the main nominal anchor of the economy. Indeed, by reducing its response to exchange rate changes, the Bank of Mexico reduced the effect of outside forces on the domestic economy. Nevertheless, this effect is balanced with the increasing impulse effect of world inflation shocks to the exchange rate. Impulse responses for the domestic demand shock exposed a lower impact on the inflation rate and the interest rate, consistent with the lower policy response to the output gap by the monetary authorities.

I have also estimated a version of the paper by changing the reaction function allowing the central bank to respond to deviations from an inflation target. This specification was motivated in light of the information provided by the Bank of Mexico's annual reports. By repeating the estimation strategy, I obtained an invariant narrative pointing towards a reduction of the policy coefficients on output and the exchange rate, that settled when the inflation rate started to reach the long-run inflation target.

Chapter 3

Parameter instabilities and monetary policy in a Small Open Economy: Evidence from an estimated model for the UK

3.1 Introduction

This paper aims to study the following problems: How stable are the structural parameters of a small open economy Dynamic Stochastic General Equilibrium model (DSGE)? If there are parameter instabilities, what are the macroeconomic implications that emerge from this time-varying behavior? Has the Bank of England (BoE) updated its response to exchange rate fluctuations?[1] To answer these questions, I study the evolving nature of the structural parameters of a small open economy DSGE model using data for the United Kingdom (UK).

[1] As I will expand below, there is mixed evidence in the literature on whether monetary authorities in the UK respond to the nominal exchange rate when determining the policy rate.

The model is estimated using Bayesian techniques over rolling samples. By comparing the estimates of each rolling sample, I show that there is strong evidence of parameter variations. Further, I find that these parameter instabilities modify the role of the nominal exchange rate in the BoE's reaction function across estimations. I accomplish this by performing rolling-window posterior odds tests against two specifications: one where exchange rate movements are part of the BoE's policy rule and an alternative specification where the response to this variable is restricted to be zero. I also document how possible parameter shifts may alter the model dynamics by conducting window-specific impulse response functions and study if the relative importance of shocks changes over time via forecast error variance decomposition.

In the paper, I adopt a rolling window approach to depict the possible variations in the model parameters over the sample period and show how the macroeconomy responds to these changes. In contrast to other methodologies that recognize the possibility of time variation in parameters, this procedure allows me to expose possible instabilities using conventional estimation techniques without forcing the data to fit between a finite set of states nor limiting the number of time-varying parameters.[2] Indeed, this strategy helps illustrate changes in all the parameters and facilitates a narrative on the evolving behavior of monetary policy, not to mention that it goes beyond the scope of this paper to test for different regimes in the UK economy. Nonetheless, a possible caveat to this approach is that it assumes economic agents are unaware of possible parameter drifts and believe the structure of the model to be invariant over time when they form expectations. While this could be partly correct, an alternative interpretation is that agents do recognize the instability of parameters, but the possible changes are of unknown form to them.

The paper contributes to several strands of literature, some of which inherently overlap with each other. First, I use a structural model to document the conduct of monetary policy by monetary authorities. DSGE models have become the benchmark framework of recent

[2]Namely, structural models with time-varying parameters or DSGEs with Markov-Switching regime changes in structural parameters or stochastic volatilities.

macroeconomic research for monetary policy analysis. For instance, in the US context, these models have been widely used to investigate possible structural changes that the economy has experienced, such as differences in monetary policy during the high-inflation episode in the 1970s and subsequent periods,[3] or to estimate the Fed's inflation target.[4]

Second, I study the case of the UK and extend previous research on the response of central banks in small open economies to exchange rate fluctuations.[5] Having DSGE models become the standard tool in modern macroeconomics, it is not a surprise the development and uprising adoption of these models to study monetary policy in the open economy context[6]. A large body of literature, for example, has investigated the role of exchange rate movements in central banks' decisions when setting monetary policy. Arguably, Lubik and Schorfheide (2007) has been one the most influential papers in this arena. They use a small open economy structural model to investigate the premise that central banks consider information on exchange rates to determine the policy rate. One of the main findings of their paper is that the central banks of Australia and New Zealand do not react to this variable, though the Bank of Canada and the Bank of England do.

Consequently, numerous papers have built upon this result to explain monetary episodes in different economies or to test the robustness of this result when changing the properties of the model or the estimation technique. This paper falls in the latter alternative. In this regard, the present inquiry resembles recent work that re-evaluates whether the BoE reacts to exchange rate fluctuations using alternative approaches. Dong (2013) extends Lubik and Schorfheide (2007)'s framework and finds that monetary policy by the Bank of Canada, the

[3]See Lubik and Schorfheide (2004), Eo (2009), Milani (2008), Mavroeidis (2010), Coibion (2012), Traum and Yang (2011), Bhattarai et al. (2012), Elias (2020), among other relevant contributions

[4]For example, Ireland (2007), Cogley and Sbordone (2008) Cogley et al. (2010), Del Negro and Eusepi (2011), Milani (2020).

[5]In this setup, the UK has been an attractive case study to researchers due to the structural and economic changes that the economy has experienced over the last decades, not to mention its major participation in the world economy. See De Lipsis (2021) for a thorough list of these historical events.

[6]Influential theoretical contributions in this area include Smets and Wouters (2003), Gali and Monacelli (2005), Adolfson et al. (2007), Justiniano and Preston (2010b)

Reserve Bank of New Zealand, and the BoE is not responsive to exchange rate movements when a limited exchange rate pass-through is introduced into the model. Caraiani and Gupta (2020) use a frequency-components approach and find evidence that not only does the BoE respond to exchange rate changes, but they focus on long-term depreciation movements.[7]

Third, although the paper focuses on the coefficients linked to the central banks' reaction function, I also document the time-varying nature of all the model parameters and shed light on the mechanisms that drive the model dynamics of open economy models. Indeed, numerous papers have considered parameters deviations for the study of monetary policy, although most are applications to the closed economy DSGE counterpart. Fernández-Villaverde et al. (2007) find large variations in several parameters by estimating a medium-scale DSGE model where agents understand and are allowed to respond to policy changes. Galvao et al. (2016) build on Smets and Wouters (2007) and develop a time-varying DSGE with an added financial sector to evaluate how macroeconomic responses to financial friction shocks change over time.

In contrast, Bianchi (2012), Davig and Doh (2014), and Debortoli and Nunes (2014) use a regime-switching approach to analyze the Federal Reserve's behavior during the postwar period. In their findings, these papers support the common belief of a change in US monetary policy that started with the tenure of Paul Volcker as Chairmen of the Fed. However, Bianchi (2012) warns that the Federal Reserves' behavior is better described by a back and forth between passive and active regimes instead of a one-time-only regime change. On the open economy front, Liu and Mumtaz (2011) estimate a Markov switching open economy structural model to examine possible changes in the UK's macroeconomic dynamics and find substantial evidence of parameter variations.

This paper is more closely related to empirical work that uses a rolling window strategy. For

[7]An important remark to mention is that the sample is extended to include the zero lower bound period, which may likely affect the underlying comparison. I revisit this result in Section 3.4.3.

example, Canova (2009), Canova and Ferroni (2012) Castelnuovo (2012), Hurtado (2014), and Ilabaca and Milani (2020) consider parameter instabilities in closed economy DSGE models by performing rolling window estimations. Furthermore, the closest paper that resembles the present analysis is Zamarripa (2021), who performs rolling window estimations on a small open economy DSGE model to document the conduct of monetary policy by the Bank of Mexico during the disinflation episode of 1995-2003. The paper shows that the policy feedback coefficients on output and the exchange rate systematically transitioned to lower values, while the response to inflation remained invariant. In this paper, however, I fully address the possibility that monetary authorities removed the nominal exchange rate from their reaction function. To my knowledge, this is the first application of rolling window estimations using UK data.

In general, the results derived from the present inquiry are of particular relevance for policy analysis as they provide new evidence on the importance of considering that 'structural' parameters may exhibit a time-varying component. For instance, monetary authorities could assign incorrect weights to the parameters that govern the policy rule when pursuing economic objectives and inadvertently create unintended macroeconomic effects. Similarly, failing to consider parameter instabilities would likely yield an inaccurate study of the propagation and relative importance of structural shocks or generate relatively poor forecasts.

The main empirical results of this paper are as follows. First, I find conclusive evidence of drifts in several model parameters. Most of these changes display clear transitions over the rolling windows. In particular, monetary policy is more assertive toward inflation in the initial samples and becomes more passive in the latter ones. The opposite is true for the authorities' behavior to the output gap: monetary policy steadily becomes more reactive. As for the response to exchange rate movements, posterior odds tests reveal a time-varying response to exchange-rate fluctuations by the monetary authorities. The results favor the model with the nominal exchange rate embedded in the policy rule for the initial samples.

However, the evidence weakens and even suggests otherwise in the latter ones. This result is remarkably interesting, as it sides with both fronts of the debate on whether central banks in small open economies respond to the exchange rate by showing that monetary policy is not an invariant process. Mainly, the findings are of interest to the literature on optimal policy design in small open economies. Justiniano and Preston (2010b) show that it is not optimal for policymakers to respond to exchange rate variations within a class of generalized Taylor rules, suggesting an adapting behavior by the BoE. In terms of macroeconomic implications, the results show evident changes in the model dynamics. For instance, rolling-window impulse response functions expose periodic transitions and different degrees of persistence, while the relative contributions of forecast-error variance shares show transitory changes across the samples.

The rest of the paper is structured as follows. Section 3.2 outlines the structural Small Open Economy model. Section 3.3 describes the data, the rolling window procedure, and the estimation strategy. Section 3.4 presents the main estimation results by providing evidence of parameter instabilities, assessing the corresponding macroeconomic implications, and unveiling whether the BoE's has updated its response to exchange rate fluctuations. Section 3.5 concludes. Supplemental materials concerning the estimations are presented in the Appendix.

3.2 The model

The model specification is taken from Justiniano and Preston (2010b), which is a generalization of Monacelli (2005) and Gali and Monacelli (2005), but allowing for complete asset markets, habit formation, and indexation of prices to past inflation. Here, I summarize the reduced form equations, referring the reader to Justiniano and Preston (2010b) for a detailed derivation of the log-linearized model.[8]

$$(1+h)c_t = hc_{t-1} + E_t c_{t+1} - \sigma^{-1}(1-h)(i_t - \mathbb{E}_t\pi_{t+1}) + \sigma^{-1}(1-h)(\varepsilon_t^g - \mathbb{E}_t\varepsilon_{t+1}^g) \quad (3.1)$$

$$y_t = (1-\alpha)c_t + \alpha\eta(2-\alpha)s_t + \alpha\eta\psi_{F,t} + \alpha y_t^* \quad (3.2)$$

$$\Delta s_t = \pi_{F,t} - \pi_{H,t} \quad (3.3)$$

$$q_t = \psi_{F,t} + (1-\alpha)s_t \quad (3.4)$$

$$\Delta e_t = \Delta q_t + \pi_t - \pi_t^* \quad (3.5)$$

$$(1+\beta\delta_H)\pi_{H,t} = \delta_H\pi_{H,t-1} + \theta_H^{-1}(1-\theta_H)(1-\theta_H\beta)mc_t + \beta\mathbb{E}_t\pi_{H,t+1} \quad (3.6)$$

$$(1+\beta\delta_F)\pi_{F,t} = \delta_F\pi_{F,t-1} + \theta_F^{-1}(1-\theta_F)(1-\theta_F\beta)\psi_{F,t} + \beta\mathbb{E}_t\pi_{F,t+1} + \varepsilon_t^{cp} \quad (3.7)$$

$$\pi_t = \pi_{H,t} + \alpha\Delta s_t \quad (3.8)$$

$$\mathbb{E}_t\Delta q_{t+1} = (i_t - \mathbb{E}_t\pi_{t+1}) - (i_t^* - \mathbb{E}_t\pi_{t+1}^*) + \chi a_t + \varepsilon_t^\phi \quad (3.9)$$

$$c_t + a_t = \beta^{-1}a_{t-1} - \alpha(s_t + \psi_{F,t}) + y_t \quad (3.10)$$

$$i_t = \rho_i i_{t-1} + (1-\rho_i)[\psi_\pi\pi_t + \psi_y y_t + \psi_{\Delta e}\Delta e_t] + \varepsilon_t^M \quad (3.11)$$

Eq. (3.1) denotes the log-linear approximation to the domestic household's Euler equation.

[8] An overview of the microfoundations of the small open economy model is also available in the Appendix.

Log of current consumption, c_t, is a function of expected future consumption, past consumption (via the assumption of habit formation in the household's preferences), the ex-ante real interest rate, $(i_t - \mathbb{E}_t\pi_{t+1})$, and a preference shock, ε_t^g. The parameters h and σ indicate the degree of habit persistence in consumption and the inverse of the intertemporal elasticity of substitution, respectively. Notice that disabling habit formation in the model, setting $h = 0$, returns the usual Euler equation.

Eq. (3.2) is derived by log-linearizing the goods market-clearing condition. Domestic output, y_t, is the sum of equilibrium domestic consumption and three elements of foreign variations (which, in turn, describe the foreign demand for the domestically produced good): the terms of trade, s_t, deviations from the law of one price, $\psi_{F,t} \equiv (e_t + p_t^*) - p_{F,t}$, and foreign output, y_t^*. In contrast to Monacelli (2005), import retailers are assumed to retain a small degree of pricing power when determining the domestic currency price of the imported good, thus leading to a violation of the law of one price. α is the import share (the share of foreign goods in the domestic consumption bundle), and η denotes the elasticity of substitution between domestic and foreign goods.

Eq. (3.3) is obtained by time differencing the bilateral terms of trade (i.e., the price of the foreign country's goods in terms of home goods). Differences in the terms of trade are a function of domestic price inflation, $\pi_{H,t}$, and domestic currency import price inflation, $\pi_{F,t}$. As usual, the mathematical operator Δ is used to denote first differences.

Eq. (3.4) portrays the relationship between the terms of trade and the real exchange rate. In particular, the real exchange, q_t, is explained by the deviations of the foreign price from the domestic currency price of imports and the heterogeneity of consumption bundles between the domestic and foreign economies. Time differencing this expression yields Eq. (3.5),[9] where Δe_t measures changes in the nominal exchange rate, π_t stands for CPI inflation, and

[9] This is easier to notice from the original expression $q_t = e_t + p_t^* - p_t$. Eq. (3.4) is then derived using the fact that $p_t^* = p_{F,t}^*$ implied from the treatment of the rest of the world as a closed economy.

π^* is foreign inflation.

Eq. (3.6) is obtained by log-linearizing the optimality conditions that arise from solving the domestic firms' price-setting problem. The resulting Phillips Curve implies that domestic price inflation is defined by the most recent observed inflation rate, the current marginal cost, $mc_t = \varphi y_t - (1+\varphi)\varepsilon_t^a + \alpha s_t + \sigma(1-h)^{-1}(c_t - hc_{t-1})$, where ε_t^a is an exogenous technology shock, and next-period inflation expectations, $\mathbb{E}_t \pi_{H,t+1}$. Compared to the closed-economy setup, domestic goods prices also respond to sources of foreign variation, namely the terms of trade, foreign output, and the deviations from the law of one price.[10] The structural parameter δ_h depicts the degree of indexation to past inflation, θ_H is the fraction of firms that cannot optimally adjust their price each period, φ is the inverse elasticity of labor supply, and β is the traditional discount factor.

Similarly, a log-linear approximation of the retailers' optimality conditions renders Eq. (3.7). In this Phillips curve for import prices, domestic currency import price inflation is a function of its lag, deviations from the law of one price, expectations about the next period's inflation, $\mathbb{E}_t \pi_{F,t+1}$, and a cost-push shock, ε_t^{cp}, that captures inefficient variations in mark-ups. The parameter δ_F represents the indexation to previous import prices, and θ_F is the number of retail firms that cannot adjust prices.

Eq. (3.8) summarizes how domestic CPI and home goods prices are related. More precisely, and by substituting the terms of trade from Eq. (3.3), CPI inflation is defined as the weighted difference between domestic and imported goods price inflation (captured by the trade openness parameter).

Eq. (3.9) represents the log-linear version of an uncovered interest rate parity condition, which introduces the assumption of incomplete asset markets. The difference between the one-period-ahead expected and the current real exchange rate, $\mathbb{E}_t \Delta q_t$, depends on the gap

[10] This occurs directly through the marginal cost, and indirectly via the marke-clearing condition (see Eq. (3.2)).

between the domestic and foreign ex-ante real interest rates, the level of foreign assets position, a_t, and a risk premium shock, ε_t^{ϕ}. The debt elasticity with respect to the interest rate premium is governed by the structural parameter χ. Eq. (3.10) summarizes the foreign assets budget constraint.

Lastly, Eq. (3.11) embodies the BoE monetary policy reaction function. Monetary authorities set the nominal interest, i_t, rate following a Taylor-type rule that features persistence in nominal interest rates but also responds to current CPI inflation, domestic output, changes in the nominal exchange rate, and a monetary policy shock, ε_t^M. Parameters ψ_π, ψ_y, and $\psi_{\Delta e}$ represent the central bank's response to inflation, output, and the changes in the nominal interest rate, respectively. ρ_i is the interest-rate smoothing term.

Thus, Equations (3.1)-(3.11) characterize the domestic block of the model and describe the aggregate dynamics of the small open economy. In contrast, the foreign economy is assumed to be exogenous to the domestic economy[11] and is specified to follow an autoregressive process of order one:

$$\pi_t^* = \rho_{\pi^*}\pi_{t-1}^* + \varepsilon_t^{\pi^*} \tag{3.12}$$

$$y_t^* = \rho_{y^*}y_{t-1}^* + \varepsilon_t^{y^*} \tag{3.13}$$

$$i_t^* = \rho_{i^*}i_{t-1}^* + \varepsilon_t^{i^*} \tag{3.14}$$

Briefly, Equations (3.12)-(3.14) describe the paths for foreign inflation, foreign output, and foreign interest rates, with their associated shocks, $\varepsilon_t^{\pi^*}$, $\varepsilon_t^{y^*}$, $\varepsilon_t^{i^*}$, and corresponding autoregressive parameters, ρ_{π^*}, ρ_{y^*}, ρ_{i^*}, respectively. Together, the domestic and foreign blocks, along with the expectation terms and the exogenous disturbances (also assumed to evolve

[11] Strictly speaking, in Monacelli (2005) the underlying model assumes a world of two asymmetric economies, with one of them being small relative to the other (and which equilibrium is taken as exogenous).

according to univariate autoregressive processes[12]), comprise a linear rational expectations model that can be rewritten in its state-space form as:

$$\Gamma_0 X_t = \Gamma_1 X_{t-1} + \Psi \epsilon_t + \Pi \eta_t \tag{3.15}$$

where X_t is a state vector that collects the domestic and foreign endogenous variables, the expectation terms, and the AR(1) disturbances, $\epsilon_t = [\varepsilon_t^M, \widehat{\varepsilon}_t^a, \widehat{\varepsilon}_t^g, \widehat{\varepsilon}_t^\phi, \widehat{\varepsilon}_t^{cp}, \varepsilon_t^{\pi^*}, \varepsilon_t^{y^*}, \varepsilon_t^{i^*}]'$ is a vector of i.i.d. exogenous innovations with mean zero and a corresponding standard deviation, σ, and η_t is a vector of expectation errors.

Given the state-space representation from Eq. (3.15), I use Sims (2002)'s procedure to solve the model under rational expectations. The algorithm then renders the solution in the form of:

$$X_t = F(\Theta) X_{t-1} + G(\Theta) \epsilon_t \tag{3.16}$$

where the matrices $F(\Theta)$ and $G(\Theta)$ are functions of the parameters of the model.

Eq. (3.16) represents the transition equation of the DSGE model. It expresses the state variables solely as functions of their lags and exogenous innovations. The transition equation, combined with a measurement equation, can then be used to evaluate the likelihood function and estimate the model parameters with the Kalman filter.

[12]Namely $\varepsilon_t^a, \varepsilon_t^g, \varepsilon_t^\phi$, and ε_t^{cp}, such that $\varepsilon_t^j = \rho^j \varepsilon_{t-1}^j + \widehat{\varepsilon}_t^j$ for $j = \{a, g, \phi, cp\}$.

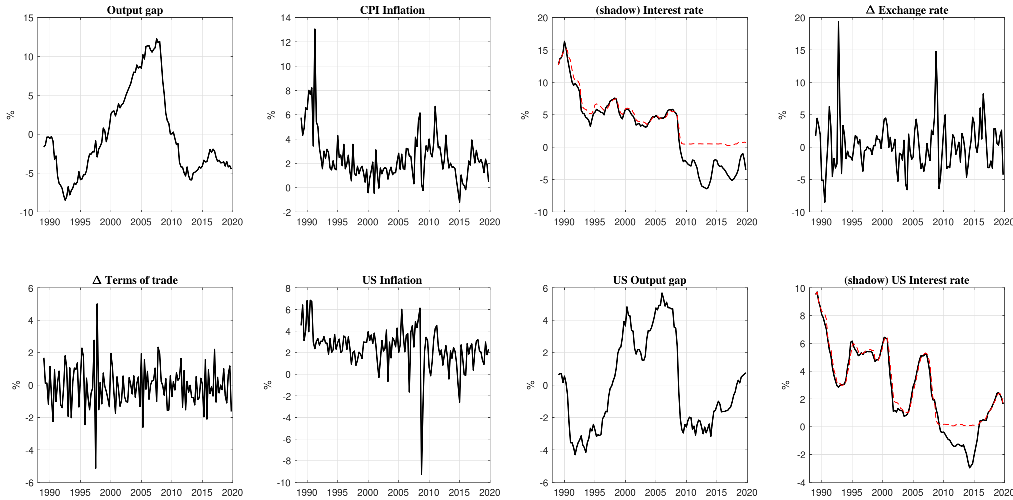
3.3 Estimation approach

3.3.1 Data description

The empirical analysis uses quarterly observations on output, inflation, interest rates, real exchange rate changes, and terms of trade changes. I employ data that spans the years from 1989Q1 to 2019Q4 for the estimation exercise. Data series for the UK are obtained from the Office for National Statistics and the Bank of England databases. Output corresponds to real GDP per capita in log deviations from a linear trend. I calculate the inflation rate taking log difference of the Consumer Price Index (all goods) and scaled it by 400 to obtain annualized percentage rates. Real exchange rate changes are constructed using the UK/US bilateral nominal exchange rate and each country's CPI and then taking log differences to yield percentage changes. Terms of trade changes are computed using the ratio of import and export price indexes, also converted in log differences to obtain percentage changes. For the foreign block, I follow the standard approach of using US observables to approximate foreign variables. These data series are retrieved from the Federal Reserve of St. Louis (FRED) database. As with the domestic counterpart, foreign output is US real GDP per capita in log deviations from a linear trend, while foreign inflation corresponds to log differences of the US Consumer Price Index (scaled by 400). To deal with the zero lower bound situation imposed by the monetary policy instrument in both the UK and the US, I use Wu and Xia (2016)'s shadow interest rate equivalent (i.e., the nominal interest rate when the zero lower bound is not binding).[13] Lastly, all data series are seasonally adjusted and rescaled to have a zero mean. Figure 3.1 depicts a visual representation of the data.

[13] The UK and the US's shadow interest rates are available starting in 1990 throughout the end of the sample. I used the regular interest rate before that. The two series were converted from monthly to quarterly frequencies by taking the period's average.

Figure 3.1: Data series



Note: Red dashed line represents the effective interest rates for the UK and the US, respectively. Series are shown before being mean-zero rescaled.

3.3.2 Bayesian methodology and rolling-windows

As described before, the main objective of this paper is to document possible parameter instabilities, especially those embedded in the monetary policy rule. To this end, I illustrate the evolving behavior of the model parameters by conducting rolling-window Bayesian estimations. The rolling-window approach consists of repeated estimations of the model over different subsamples. Specifically, I use fifteen-year windows for the benchmark results, which I later compare to a twenty-year window exercise as a robustness check. The window size is maintained constant and considers increments of one year between each estimation. This means that the first rolling-window uses data from 1989Q1 to 2003Q4; the second rolling-window uses data from 1990Q1 to 2004Q4; and so forth. Repeating this process over the full sample implies that the model is re-estimated seventeen times for the 15-year-window analysis, and twelve times when using twenty-year windows.

For the estimation of the DSGE model, I follow the Bayesian framework from Schorfheide

(2000) and An and Schorfheide (2007). This approach is suitable to characterize the posterior distribution of the structural parameters that govern the small open economy model over each subsample. These parameters are jointly estimated using Bayesian methods and collected in the parameter vector Θ:

$$
\begin{aligned}
\Theta =&[\alpha, \sigma, \varphi, \theta_H, \theta_F, \eta, h, \delta_H, \delta_F, \rho_i, \psi_\pi, \psi_y, \psi_{\Delta e}, \rho_a, \rho_g, \rho_\phi, \rho_{cp}, \rho_{\pi^*}, \rho_{y^*}, \rho_{i^*},\\
&\sigma_M, \sigma_a, \sigma_g, \sigma_\phi, \sigma_{cp}, \sigma_{\pi^*}, \sigma_{y^*}, \sigma_{i^*}]'
\end{aligned}
$$

For each rolling-window, draws from the posterior distribution are generated using the random-walk Metropolis Hasting algorithm. I compute the posterior mode and the corresponding Hessian matrix employing standard optimization routines. Subsequently, I run 200,000 iterations and discard the initial 25% as burn-in. The Hessian is scaled accordingly to maintain a target acceptance rate between 25 and 30% on each subsample. For each draw, I obtain the likelihood of the model using the Kalman Filter and the state-space matrices derived from the rational expectations solution (see Eq. (3.16)).

3.3.3 Prior distributions

Table 3.1 summarizes the prior distributions for the model parameters. The choice of priors is based on Justiniano and Preston (2010b) and Lubik and Schorfheide (2007), although extended to consider Liu and Mumtaz (2011) as this paper offers a better and appropriate point of comparison in terms of the estimation approach and the country analyzed.

I follow Justiniano and Preston (2010b) and fix the discount factor and the debt elasticity to the interest rate premium coefficients at values of 0.99 and 0.01, respectively. However, I estimate the trade openness parameter jointly with the other model parameters using a Beta distribution centered at the average share for exports and imports to GDP in the UK

Table 3.1:
Prior distributions

Parameter	Domain	Density	P(1)	P(2)
β	0.99	-	-	-
χ	0.01	-	-	-
α	[0,1)	Beta	0.25	0.10
σ	$\mathbb{R}^+$	Gamma	1.20	0.40
φ	$\mathbb{R}^+$	Gamma	1.50	0.75
θ_H	[0,1)	Beta	0.50	0.10
θ_F	[0,1)	Beta	0.50	0.10
η	$\mathbb{R}^+$	Gamma	1.50	0.75
h	[0,1)	Beta	0.50	0.25
δ_H	[0,1)	Beta	0.50	0.25
δ_F	[0,1)	Beta	0.50	0.25
ρ_i	[0,1)	Beta	0.50	0.25
ψ_π	$\mathbb{R}^+$	Gamma	1.50	0.30
ψ_y	$\mathbb{R}^+$	Gamma	0.25	0.13
$\psi_{\Delta e}$	$\mathbb{R}^+$	Gamma	0.25	0.13
ρ_a	[0,1)	Beta	0.80	0.10
ρ_g	[0,1)	Beta	0.80	0.10
ρ_ϕ	[0,1)	Beta	0.80	0.10
ρ_{cp}	[0,1)	Beta	0.50	0.25
ρ_{π^*}	[0,1)	Beta	0.80	0.10
ρ_{y^*}	[0,1)	Beta	0.80	0.10
ρ_{i^*}	[0,1)	Beta	0.80	0.10
σ_M	$\mathbb{R}^+$	Inverse Gamma	0.50	4.00
σ_a	$\mathbb{R}^+$	Inverse Gamma	0.50	4.00
σ_g	$\mathbb{R}^+$	Inverse Gamma	1.50	4.00
σ_ϕ	$\mathbb{R}^+$	Inverse Gamma	0.50	4.00
σ_{cp}	$\mathbb{R}^+$	Inverse Gamma	0.50	4.00
σ_{π^*}	$\mathbb{R}^+$	Inverse Gamma	0.50	4.00
σ_{y^*}	$\mathbb{R}^+$	Inverse Gamma	1.50	4.00
σ_{i^*}	$\mathbb{R}^+$	Inverse Gamma	0.50	4.00

Note: P(1) and P(2) refers the mean and standard deviation for the Beta and Gamma distributions, and scale and shape for the inverse gamma distribution.

over the sample period.[14] I assume the intertemporal elasticity of substitutions σ to follow

[14]Nonetheless, I had to impose a small standard deviation to avoid obtaining unreasonable low estimates.

a Gamma distribution centered at 1.2 with a standard deviation of 0.4. The priors for the inverse Frisch elasticity of labor supply φ and the elasticity of substitution between domestic and foreign goods parameters η are described using Gamma distributions with a mean of 1.5 and a standard deviation of 0.75. Calvo pricing parameters, θ_H and θ_F, are set to follow Beta distributions centered at 0.5 with standard deviations of 0.1. I specify the priors for the habit persistence parameter h and the indexation parameters, δ_H and δ_F, using Beta distributions with a mean of 0.5 and a standard deviation of 0.25. Regarding the parameters that govern the Taylor rule, I adopt the traditional assumption of using Gamma distributions to describe the policy response parameters. The authorities' response to inflation ψ_π is centered at 1.5, with a standard deviation of 0.30. Both the response to output ψ_y and exchange rate changes $\psi_{\Delta e}$ are set to have a mean of 0.25 and 0.13. For the interest rate smoothing parameter ρ_i, I use a Beta distribution mean centered around 0.5 with a standard deviation of 0.25. Given the nature and focus of this paper on studying the central bank's evolving behavior, I verify the robustness of the results by repeating the estimation exercise using an alternative set of priors on the policy rule. Lastly, the autoregressive coefficients of the structural shocks are all assumed to be persistent and follow Beta distributions, while the standard deviations of these disturbances are modeled to follow Inverse Gamma distributions.[15]

In preliminary estimations, I calibrated this parameter to the period's average to check for robustness and found similar overall results.

[15] As additional robustness checks, I also estimated all windows following the prior specifications from Liu and Mumtaz (2011) for both the autoregressive coefficients and the standard deviations of the exogenous disturbances and found near-identical results.

3.4 Evidence of parameter drifts: empirical results

3.4.1 Posterior estimates and policy responses

I present the rolling window Bayesian estimates of all the model parameters in Figure 3.2. The figure shows the posterior median and 95% credible bands for each model parameter across subsamples. Roughly speaking, the parameters are displayed following this order: structural (with policy coefficients at the end) first, and then those that describe the exogenous shocks (autoregressive elements first, followed by the standard deviation of the innovations).

The trade openness parameter shows a steady decrease across the different windows, with posterior medians ranging between 0.18 and 0.22. In contrast, the UK's observed average share of exports and imports to GDP increased over the subsamples, moving from 0.23 in the first window to around 0.28 in the last one. Nevertheless, as previous research suggests (Lubik and Schorfheide (2005), Lubik (2006), and Justiniano and Preston (2010a)) this result is not surprising: an attempt to estimate the openness coefficient leads the estimation algorithm to choose parameter values that match the volatility of the data while complying with the cross-equation restrictions; thus, resulting in relatively lower estimates.

The inverse of the intertemporal elasticity of substitution shows an interesting behavior across the windows. In the first half of the estimations, the posterior medians range between 0.18-0.36 and then shift to higher values between 0.41-0.57 in the second half. One possible explanation of this transition to lower degrees of consumption growth responsiveness could be the relatively low interest rates that characterize the Zero Lower Bound period. These results seem to be consistent with similar research on small open economies.[16] Likewise, the

[16] In the UK context, for instance, Lubik and Schorfheide (2007) report a posterior mean of 0.36 (available in their working paper version), Caraiani and Gupta (2020) a posterior mean of 0.23. Liu and Mumtaz (2011) document values that range between 1.76-2.23 across alternative regimes. However, note that these estimates are derived using different sample periods and model specifications.

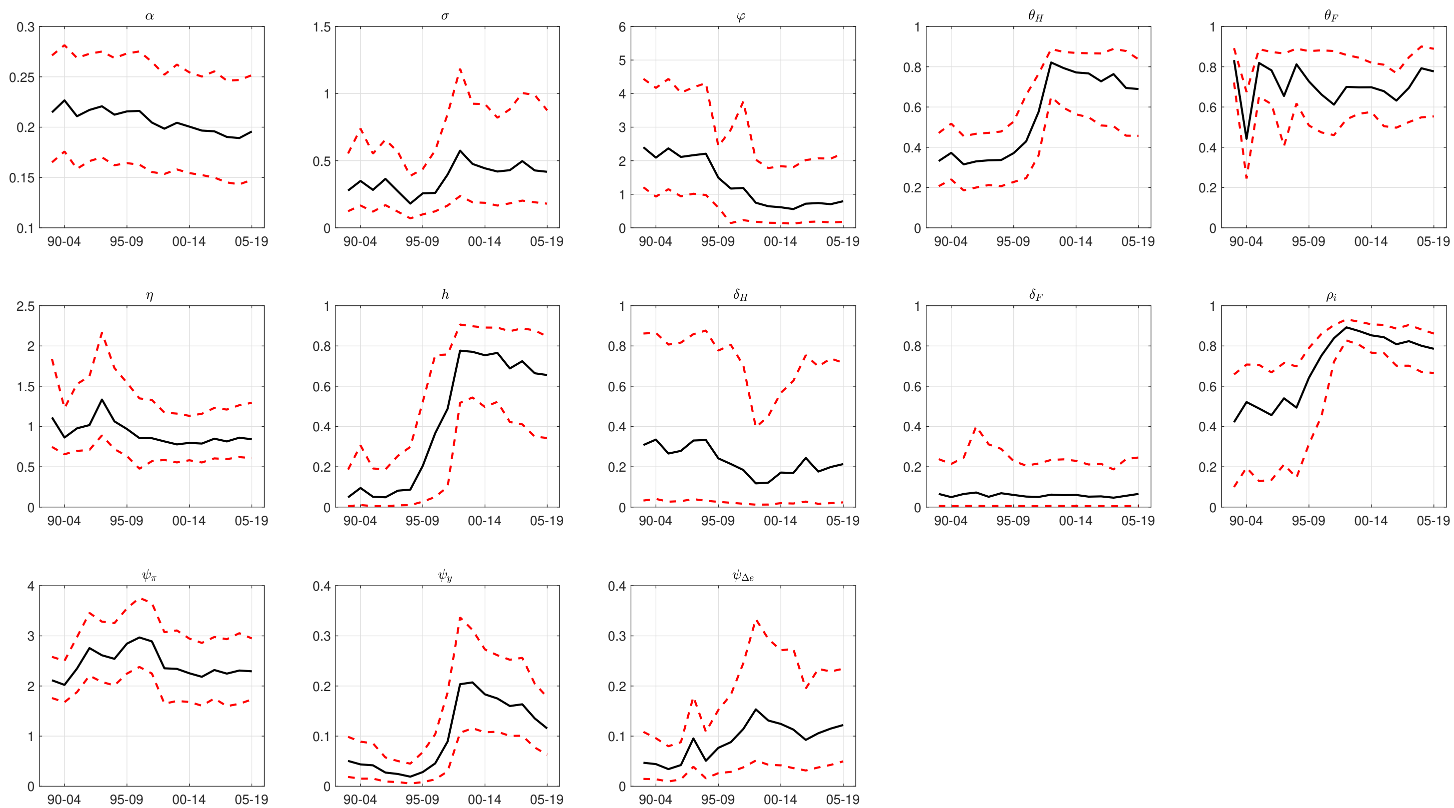
inverse elasticity of labor supply also reveals an evident shift across the rolling windows, drifting from a posterior median of 2.4 in the initial windows to 0.79 in the last one.

Optimal price setting in home goods and imported goods render two contrasting results. The Calvo parameter of home good prices displays a progression from lower to higher posterior medians. For the lower estimates, the results suggest that firms reoptimize prices approximately every 1.5 quarters, while this happens every 3-5 quarters for the higher estimates. On the other hand, I find no evidence of a time-varying behavior by the imported-prices Calvo parameter. Instead, the posterior medians move back and forth between estimations, with rather similar credible bands, and suggest price re-optimization every 2-5 quarters.

Posterior medians for the elasticity of substitution between home and foreign goods are close to unity in all windows, with slightly higher values in the initial subsamples that move below one in the latter ones. These findings consonate with related literature, although the estimates are still somewhat low compared to studies that use micro data.[17] The results for the habit formation parameter are interesting to note. The posterior medians seem to transition from estimates in the vicinity of 0.08 to ones in the neighborhood of 0.65-0.77. In comparison, Justiniano and Preston (2010b) and Liu and Mumtaz (2011) also find habits in consumption to play a smaller role than in other studies. They argue that these differences are likely due to the set of autoregressive shocks included in the model, particularly the fact that the persistence of home goods inflation is largely explained by the technology and preference shocks. Indeed, the results reveal similar patterns in the estimates for the standard deviations of these shocks and those encountered for the habit formation parameter. Moreover, it is remarking to note that the observable drift in all of these parameters starts when the windows include the data that concern the Zero Lower Bound period, when interest rates are low and inflation is (relatively) closer to the target.

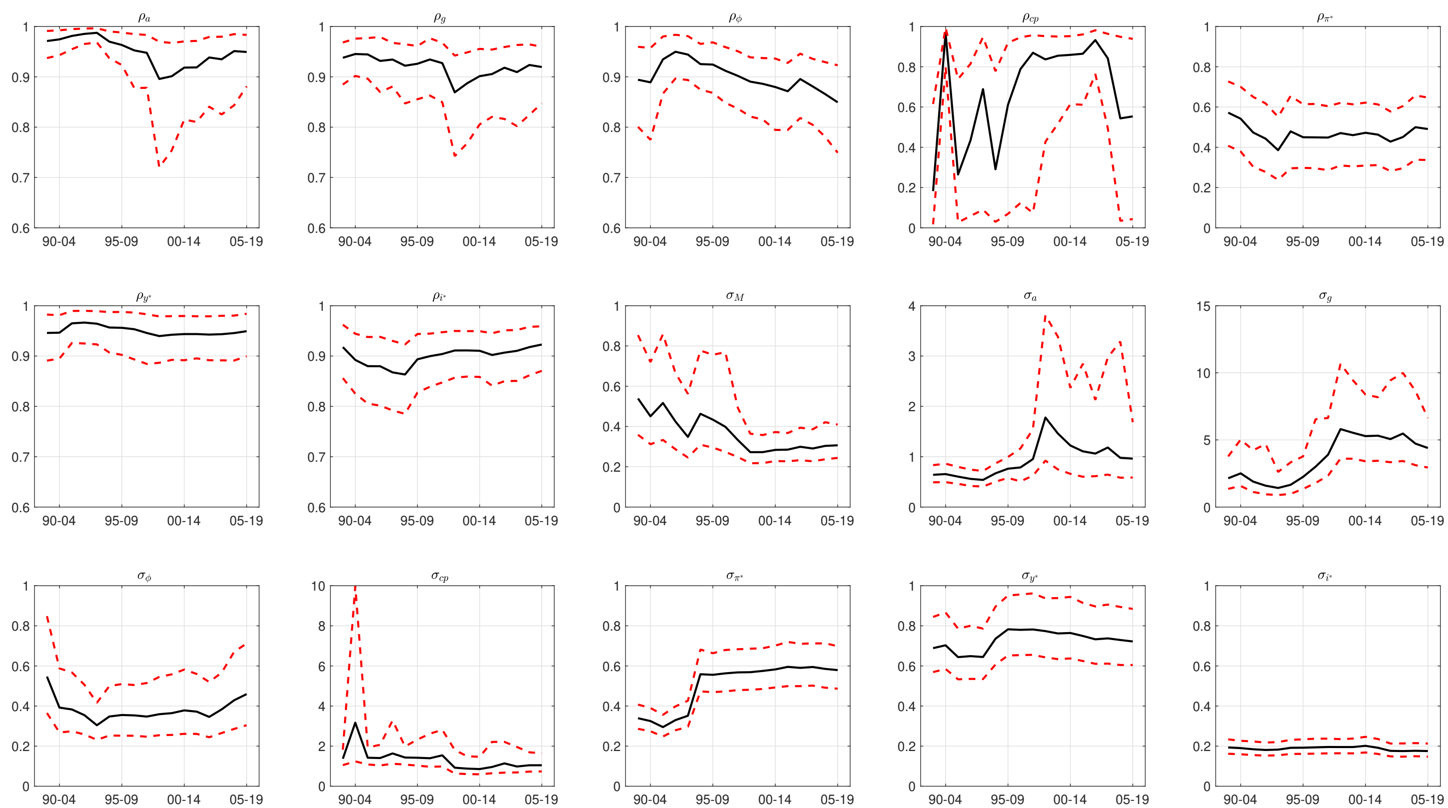
[17] See for example Obstfeld and Rogoff (2000). Refer also to Bajzik et al. (2020) for a survey of estimated values in individual studies.

Figure 3.2: Rolling-window posterior estimates: structural parameters



Note: The figure shows the posterior median (solid line) of each sub-sample across the Metropolis-Hastings draws, along with 95% Bayesian credible interval bands (dashed lines).

Figure 3.2: (cont.) Rolling- window posterior estimates: autoregressive and standard deviations



Note: The figure shows the posterior median (solid line) of each sub-sample across the Metropolis-Hastings draws, along with 95% Bayesian credible interval bands (dashed lines).

As for the price indexation parameters, I do not find substantial evidence of parameter variations. The densities for the price indexation of domestic goods do exhibit a transition to lower posterior medians, but the credible bands are too wide across all the estimations to sustain that inference. The imported goods indexation parameter displays a constant behavior over all windows.

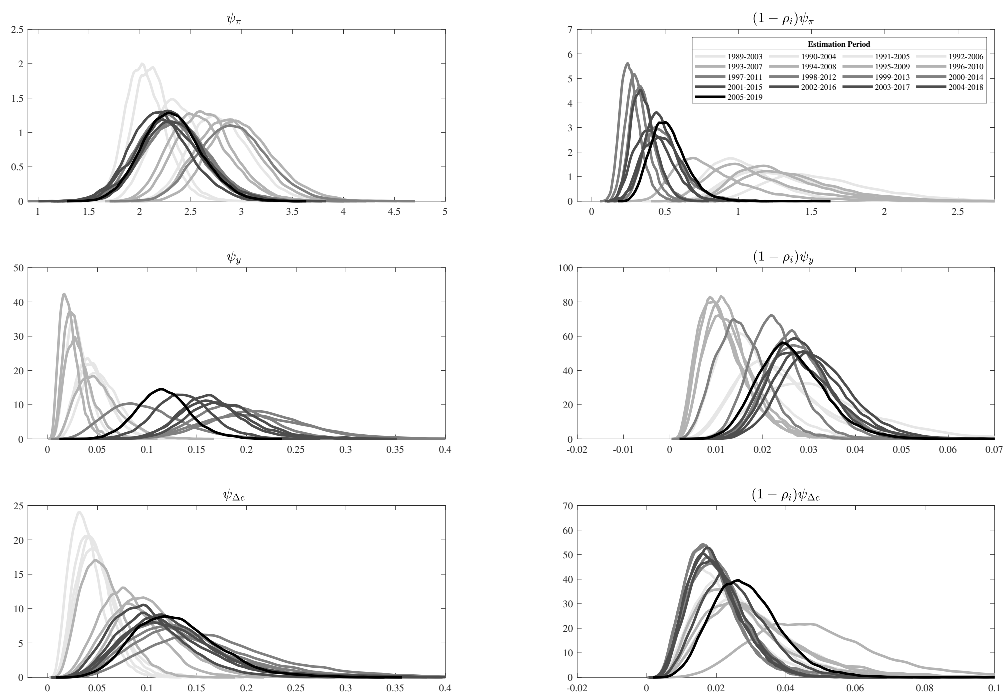
For the parameters associated with the exogenous shocks, I find the following results. First, there is a high degree of persistence in almost all shocks. Except for the cost-push and the foreign inflation autoregressive parameters, most posterior medians stay above 0.8 for all subsamples. Second, only the persistence of the risk-premium shock exhibit an evident shift to slightly lower posterior medians in latter windows. Furthermore, the densities found for the autoregressive coefficient of the cost-push shock show visible instabilities across the estimations, with erratic behavior and wide credible bands. Third, the majority of the standard deviation parameters manifest a time-varying behavior. The standard deviation of the monetary policy innovation decreases in the first half of the windows and then stabilizes in the rest, with wider credible bands first and narrower later. The corresponding estimates for the technology and preference coefficients depict a similar narrative, moving from lower to higher values. Posterior distributions for the risk-premium standard deviation register a U-shape pattern across the estimations. Similar to the persistence coefficient, densities for the standard deviation of the cost-push shock are the most volatile but become relatively more stable in the latter windows while shifting to lower values.[18] Concerning the foreign block, only the standard deviation for foreign inflation drifts to larger posterior medians.[19]

Given the central role of monetary policy in this paper, and to facilitate a visual narrative of the evolution of these parameters, I display the BoE's policy coefficients (and the effective responses) separately in Figure 3.3. The figure portrays the same information as Figure

[18] In contrast, Justiniano and Preston (2010b) also find the cost-push shock to be the most volatile across different small open economies.

[19] Arguably, this is also true for the standard deviation of foreign output.

Figure 3.3: Posterior distributions of policy coefficients and (effective) policy responses



3.2 but overlaps the posterior distributions instead. I employ different shades of gray that periodically become darker to illustrate the transition of the rolling window densities over the different samples. The left-hand panels plot the masses for ψ_π, ψ_y, and $\psi_{\Delta e}$, while the right-hand panels show the distribution of the overall impact in the policy rule (i.e., accounting for the degree of interest rate smoothing. See Eq. (3.11)).

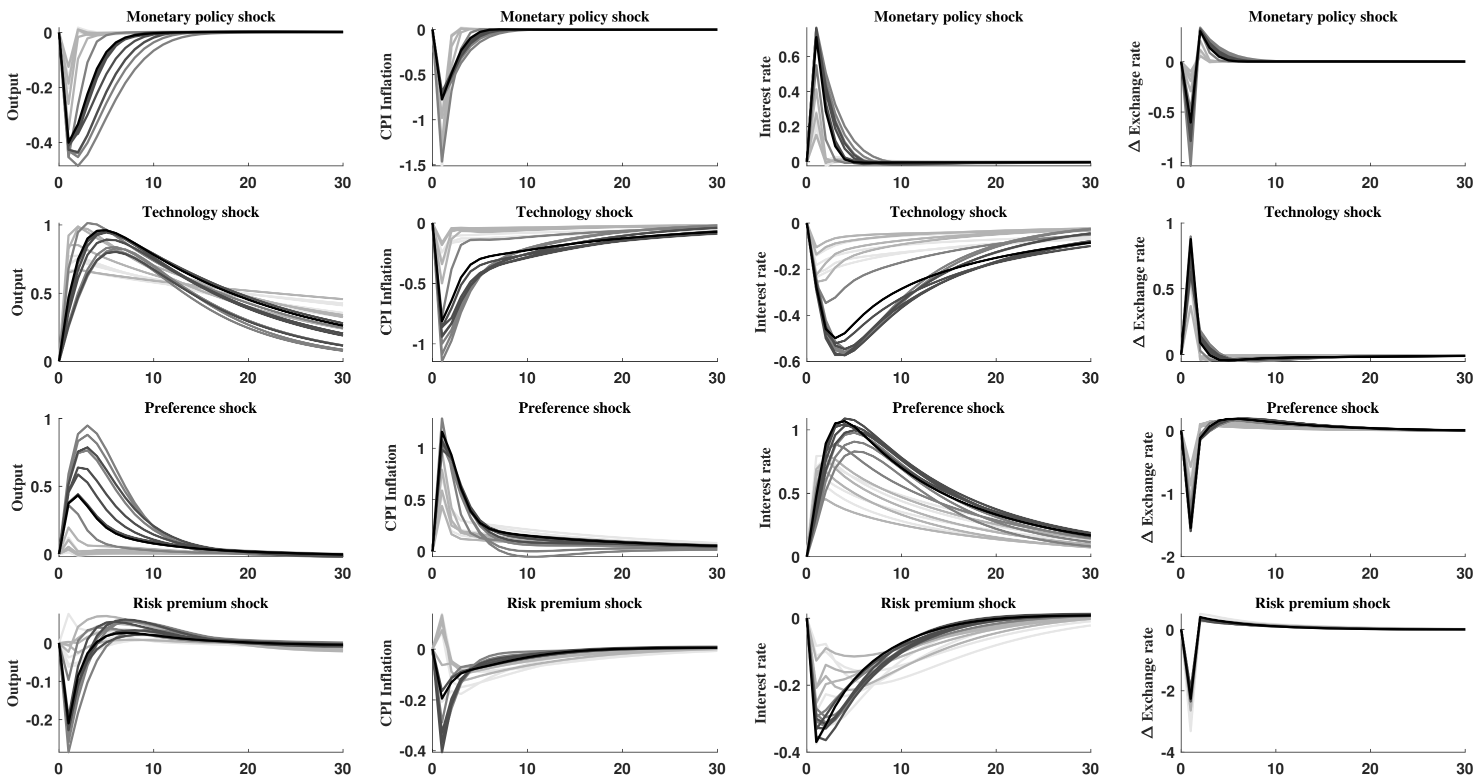
The policy parameters convey an interesting narrative in terms of drifts and uncertainty. The policy coefficient for inflation initially becomes more reactive but shifts back to more passive estimates in latter samples. In contrast, monetary policy appears to become more assertive for output and exchange rate fluctuations, although associated with a higher degree of uncertainty. What is particularly remarking in the results is that, once the smoothing parameter is taken into account, a sharper description of the evolution of the BoE's monetary policy develops. In particular, the *effective* (or *contemporaneous*) response to inflation steadily becomes more passive over the different windows and there is a significant reduction in the degree of uncertainty associated with the overall impact on interest rates. This result conforms with existing evidence that central banks adjust their response to macroeconomic variables when the economy progresses towards the inflation target. On the other hand, the fact that the posterior distributions are wider for the initial windows and narrower for the latter ones seems to capture the different institutional and structural changes that occurred to UK monetary policy during the decade of the 1990s[20]. For output, the effective response steadily becomes more reactive. As seen in Figure 3.1, as the inflation rate starts to decrease and the output gap becomes relatively large, monetary authorities seem to adjust their strategy by responding more aggressively to output. As for the exchange rate changes, the results reveal an oscillating behavior in the posterior distributions across the rolling windows and no significant changes in the contemporaneous response to the policy rate. However, notice that the current methodology does not fully consider whether monetary authorities updated their response to this variable instead by completely removing it from their reaction function. Subsection 3.4.3 formally addresses this issue by performing posterior odds tests with an alternative model specification that features no exchange rate feedback in the policy rule.

3.4.2 Evolving macroeconomic dynamics

In this section, I examine the associated implications to the UK macroeconomic dynamics. First, I start by computing rolling window impulse response functions, which are reported in Figure 3.4. The figure presents the response of the domestic variables (columns) to the various exogenous shocks (rows). For the sake of clarity, I focus the discussion on the time-varying differences depicted in the responses while abstracting to mention overlapping probability intervals, though these are available upon request. As before, I use different shades of gray to facilitate a visual narrative of the evolving dynamics across the subsamples.

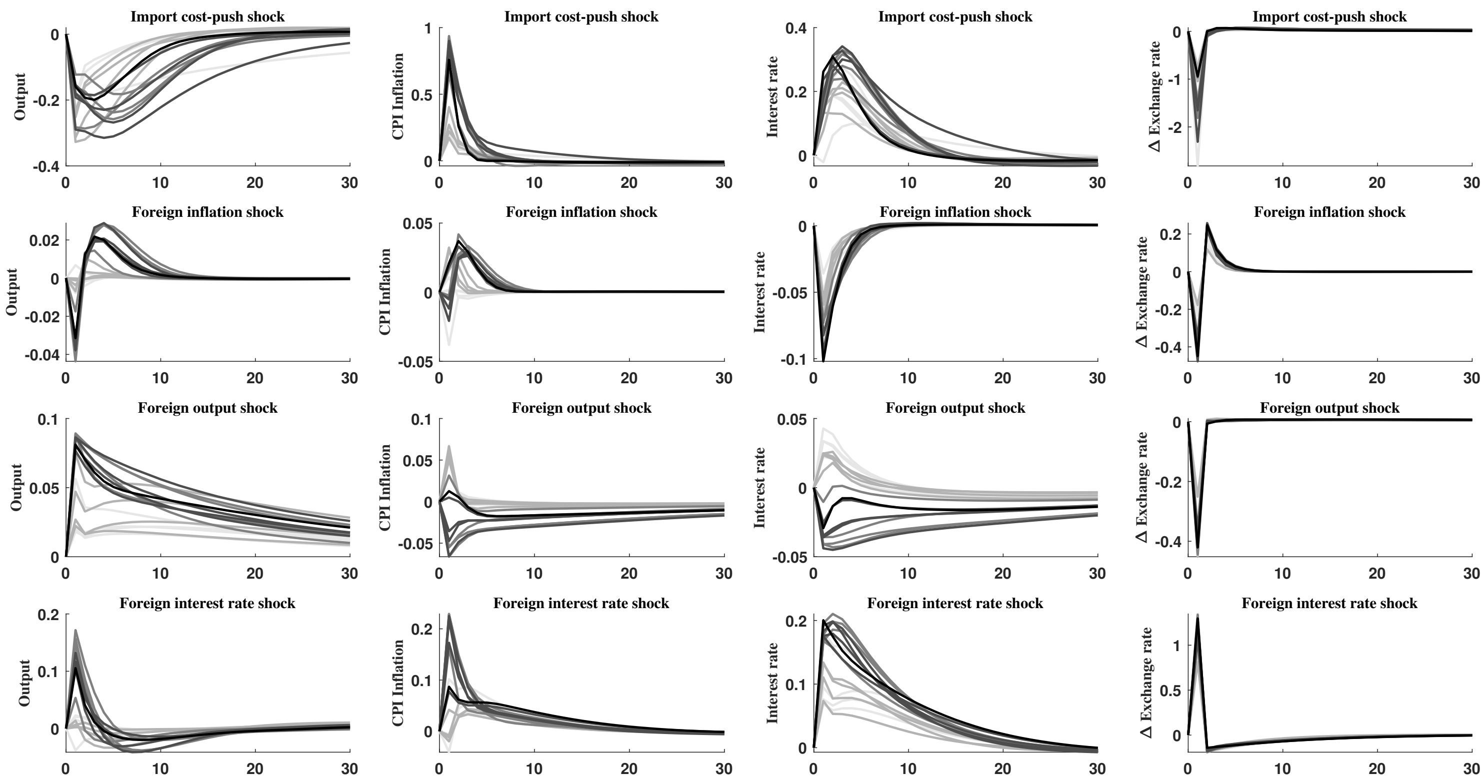
[20]See De Lipsis (2021).

Figure 3.4: Rolling-window Impulse Response Functions



Note: The impulse responses represent the median of each window across draws. Bayesian probability intervals omitted for clarity purposes. Legend is the same as in Figure 3.3 (lines get periodically darker for the more recent windows).

Figure 3.4: (cont.)



Note: The impulse responses represent the median of each window across draws. Bayesian probability intervals are omitted for clarity purposes. Legend is the same as in Figure 3.3 (lines get periodically darker for the more recent windows).

The contractionary monetary policy shock has the expected sign-effect on domestic variables; it appreciates the domestic currency (initially) and lowers output and inflation. Across windows, the impulse responses shift outward (i.e., a larger impact effect in absolute value) and exhibit longer adjustments for output, interest rates, and exchange rates. For inflation, the results show lower and more persistent responses. Although it is difficult to isolate how individual parameters are associated with impulse response drifts, these results seem to match the evolution of monetary policy depicted in Figure 3.3, particularly the behavior of the effective policy responses (right panels).

The responses to the technology and preference shocks become larger on impact and display longer adjustments, except for output, which narrative is not as evident through the rolling windows. In particular, for inflation, the interest rate, and exchange rates, the outward shift seems to occur in the first half of subsamples. For output, the responses reverse to zero at different paces for the technology shock and uniformly for the preference shock.

Impulse responses for the risk premium and cost-push shocks are among the most irregular across the estimation samples. Output and interest rate responses to the risk premium shock appear to become larger (in absolute value) and faster to adjust as the windows progress. Inflation initially responds positively to the risk premium shock, but the impact becomes negative for the rest of the samples. Still, the responses take similar periods to reverse back to zero. Concerning the cost-push shocks, inflation and interest rate impulse responses increase over time in their impact effects. However, for output, the lines instead oscillate around each other with no clear pattern that points toward a drifting effect. Exchange rate responses uniformly display a lower impact effect and virtually the same adjustment periods for both structural innovations.

For the foreign block innovations, the results are as follows. The foreign inflation shock seems to escalate the impacts on output, interest rate, and the exchange rate. For output particularly, the adjustment also becomes slower. The time-varying behavior for the inflation responses, however, is unclear. The effects of the foreign output shock are somewhat similar for output and the exchange rate, increasing the impact effect over the samples. Nonetheless, for the output responses, it now renders different degrees of persistence. For the inflation and interest rate, the responses become smaller in magnitude while fluctuating between positive and negative impact values. Lastly, foreign interest rate shocks display greater impacts for all variables except inflation, which its response behavior is similar to the previous case.

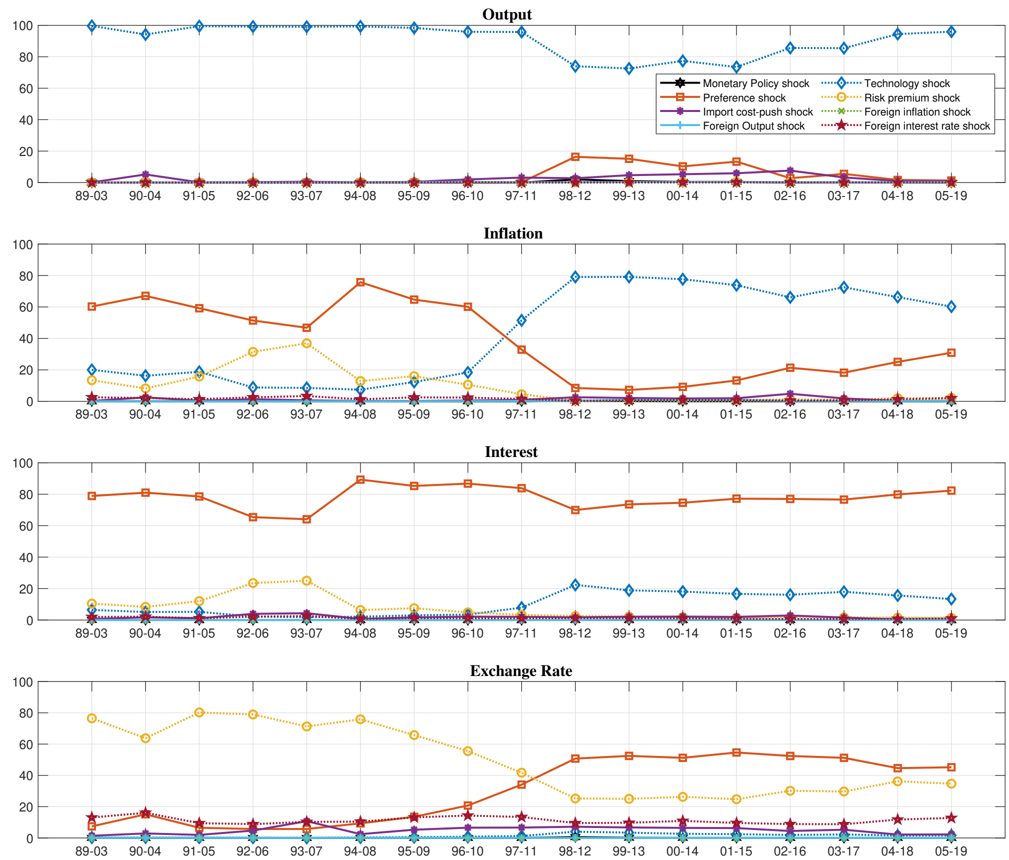
I then investigate how the role of the structural shocks driving the UK macroeconomic performance has evolved by computing forecast error variance decomposition for each rolling window. Figure 3.5 illustrates the results. The figure represents the median forecast error variance share at the 8-period ahead horizon, but similar figures for the 4 and 24 horizons are also available in Appendix.

Output fluctuations are mainly explained by the technology shock, accounting for more than 75% of the variance across samples. The only evolving transition in the dynamics is presented by the contributions of the preference and the cost-push shocks, which seem to transition to larger shares after the 98-12 window and then return to their previous contributions in the last estimation period.

In contrast, inflation fluctuations exhibit various changes in the forecast error shares. The preference and risk premium shocks, which initially are among the main drivers of inflation's variability, decrease their role significantly in the second half of the samples. These shares, on the other hand, are symmetrically gained by the technology shock, which accounts for 60-80% of inflation fluctuations in the latter samples.

The interest rate is also largely driven by the preference shock, which is consistently the major contributor to its variability, with forecast error variance shares of more than 60% in all windows. The visible transition here occurs between the risk premium and the technology shocks, the former being an important driver in the first half of windows and the latter in

Figure 3.5: Evolution of variance decomposition



Note: The figure shows the window-specific median forecast-error variance shares across MH draws at the 8-quarters ahead horizon.

the rest.

The empirical findings for the exchange rate variations depict several time-varying shifts. The most apparent ones are presented by the preference and risk premium shocks. Initially, the risk premium shock accounts for more than 60% of the variance, while the preference shock oscillates between 10-20%. After the 95-09 sample, risk premium and preference shares transition to values of 30-40% and 45-50%, respectively. At a smaller scale, the import cost-push shock becomes more important for the middle samples and the technology shock after the 98-12 window. Lastly, while being one of the main contributors explaining exchange rate fluctuations, the foreign interest shock did not display apparent shifts across the rolling samples.

The results are fairly similar at the 24-period ahead horizon, though a few differences are worth mentioning. The risk premium shock no longer displays a significant contribution explaining inflation and interest rate variability in the initial samples. In fact, in both scenarios, their shares seem to be absorbed by the technology shock. For the exchange rate fluctuations, the import cost-push shock plays a dominant role across all windows, while the contribution of the risk premium shock is more dormant in general.

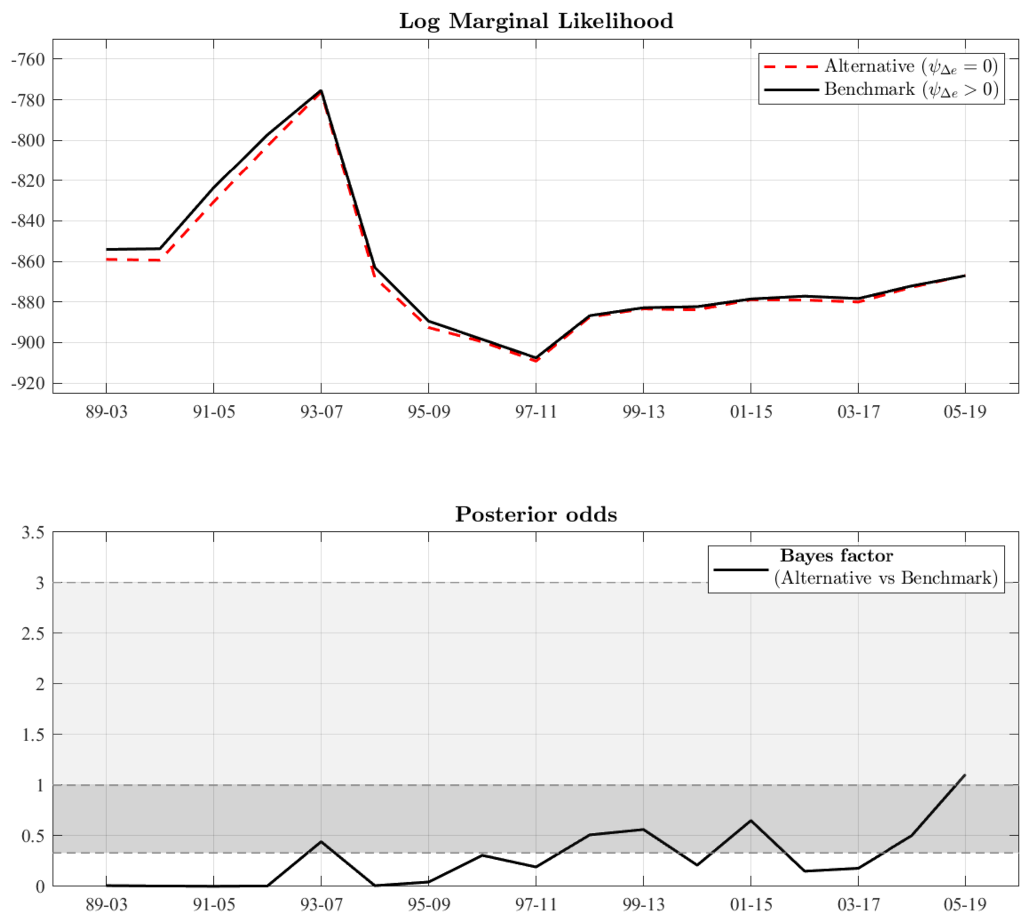
3.4.3 Does the Bank of England respond to exchange rate fluctuations?

As discussed before, the estimation procedure does not consider whether monetary authorities include the exchange rate in their reaction function across the subsamples. In this section, I evaluate this possibility by estimating the model under the restriction $\psi_{\Delta e} = 0$ and comparing the marginal likelihoods of the two model specifications. The analysis is performed as in Lubik and Shorfheide (2004) but extended to contemplate the evolution of marginal likelihood across the rolling samples. The posterior estimates of this alternative specification are available in the Appendix.

Figure 3.6 summarizes the results. The top panel shows the (log) marginal likelihood of the benchmark specification, where the nominal exchange rate is embedded in the policy rule ($\psi_{\Delta e} > 0$), and an alternative specification that assumes monetary policy is not responsive to this variable ($\psi_{\Delta e} = 0$).[21] The bottom panel presents the corresponding posterior odds (which assumes that the prior odds are one) for each rolling window. To facilitate the inter-

[21]For conciseness, I refer to them in the Appendix as $\mathcal{M}_1$ and $\mathcal{M}_2$, respectively.

Figure 3.6: Evolution of Marginal Likelihood and window-specific Bayes factor



Note: The figure summarizes the posterior odds ratio of each rolling window of the hypothesis $\psi_e = 0$ versus $\psi_e > 0$. The light (dark) gray area denotes anecdotal evidence in favor of the alternative (benchmark). The upper (lower) white area suggests substantial or strong evidence for the alternative (benchmark). Prior odds are assumed to be equal to one.

pretation of the results, I use different shades of gray to illustrate anecdotal or weak evidence in favor of the specifications: light for the alternative and dark for the benchmark. White areas, in turn, suggest substantial or strong evidence: the upper area for the alternative and the lower area for the benchmark.[22]

The results render an interesting narrative. For the initial windows, the posterior odds tests

[22]See Jeffreys (1998).

favor the benchmark specification. This result is consistent with previous literature. For example, Lubik and Schorfheide (2007), whose sample roughly corresponds to the first window, find strong evidence that the Bank of England responds to exchange rate movements. However, the figure shows that this result is not robust across subsamples. Moreover, the gap between the marginal likelihoods shrinks as the windows progress. After the 93-07 window, the evidence in support of the benchmark becomes weak for some estimations and ultimately favors (slightly) the alternative in the last subsample. This result becomes more apparent when repeating the estimation exercise using 20-year windows, which I present in the robustness section.

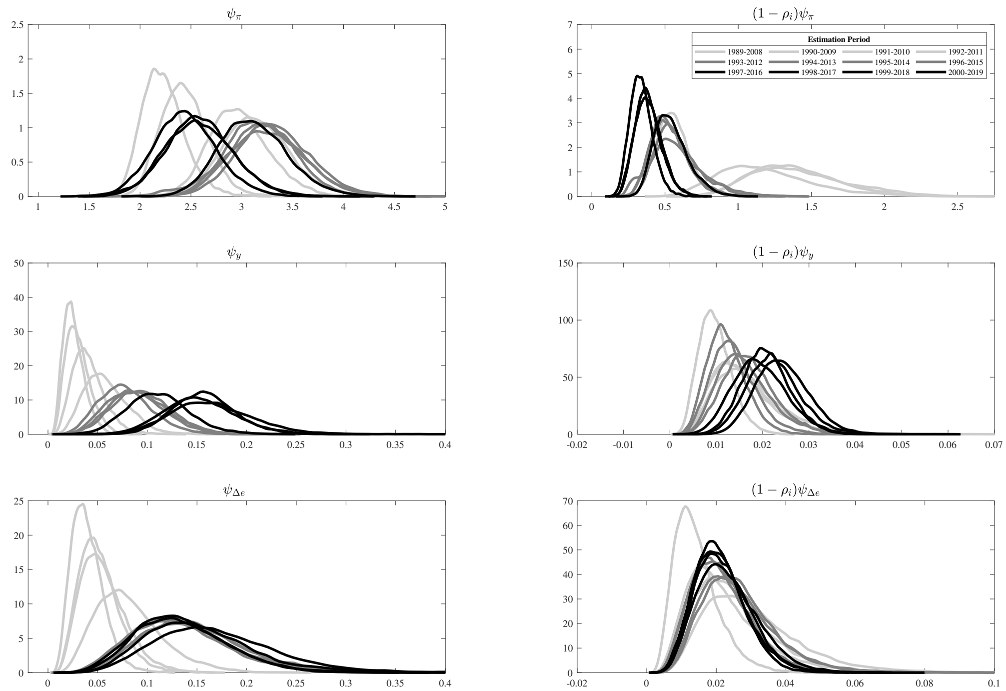
As an additional exercise, I also re-estimate the model for both specifications using the full sample to compare the implications for the policy responses and the marginal likelihoods. The policy responses under this setup are available in the Appendix. In short, the posterior distributions for the effective policy responses are wider than most of the rolling window counterparts and tend to compute similar estimates to the mid subsample periods. In terms of the marginal likelihoods, I find a Bayes factor of 0.3679, slightly favoring the premise of a response to exchange rate movements by the BoE. This result contrasts the importance of considering parameter instabilities in the estimation of DSGE models.

3.4.4 Robustness analysis

3.4.4.1 20-year windows

In this section, I check the robustness of the results by repeating the estimation procedure using 20-year rolling windows. The objective of this exercise is to verify that the width of the subsamples does not drive the main results. For brevity, here I report only the central findings concerning the evolving behavior of monetary authorities, though I provide the full ensemble of results in the Appendix.

Figure 3.7: Posterior distributions of policy coefficients and (effective) policy responses (20-year windows)



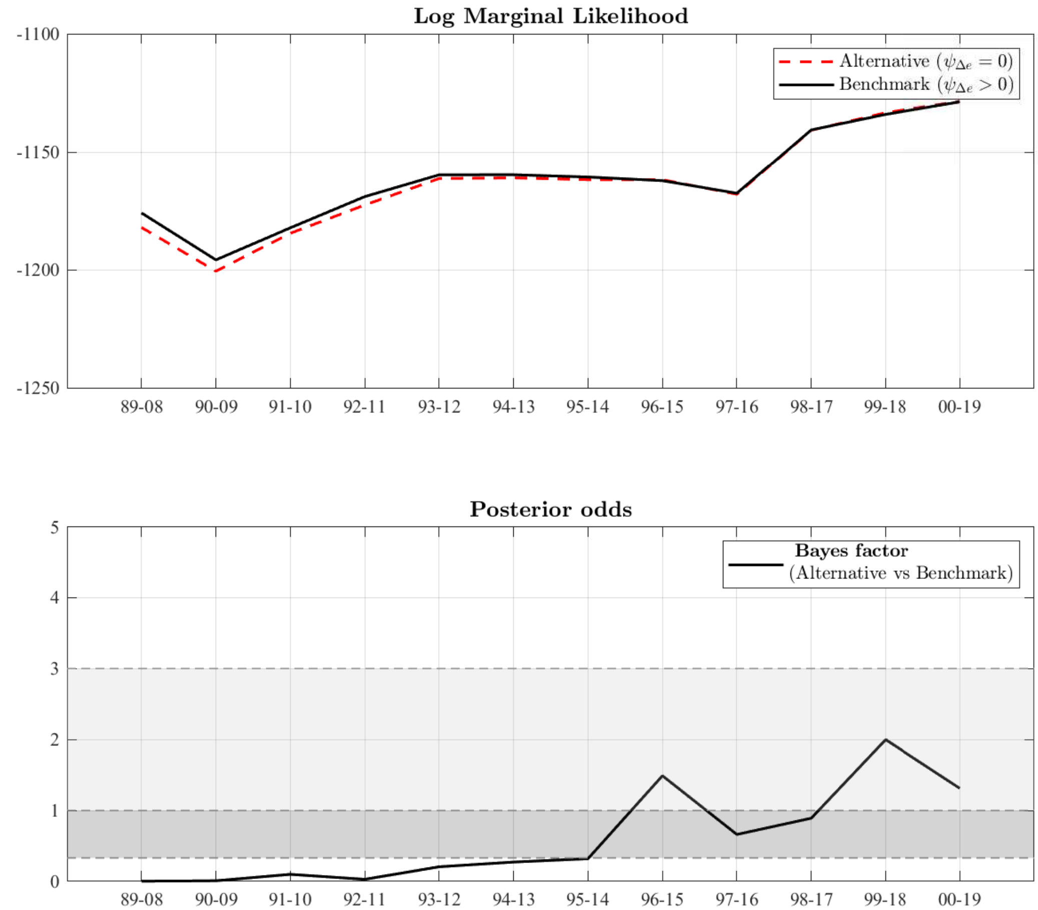
Note: The horizontal axis of the right panels is maintained consistent with Figure 3.3 for comparability reasons.

Figure 3.7 presents the posterior distributions and effective policy responses for the 20-year windows. The results are overall alike and display a more linear transition across subsamples. Monetary policy becomes more passive for inflation and more active for output. As in Zamarripa (2021), this finding conforms with the notion that once central banks approach their inflation targets, they react relatively less to inflation and more to output.[23] For the exchange rate, the figure shows fairly steady effective responses.

Nonetheless, the striking difference is displayed by the posterior odds tests in Figure 3.8. The evolution of the marginal likelihoods and the window-specific Bayes factors becomes more apparent and signals an evident transition in the BoE reaction function. Previously,

[23]To provide some context, the UK adopted (retail) inflation targeting in 1992 with a band of 1-4%. In 2003, changed from RPIX inflation of 2.5% to the current CPI target of 2% (with a band of ± 1%).

Figure 3.8: Evolution of Marginal Likelihood and window-specific Bayes factor (20-year windows)



Note: The figure summarizes the posterior odds ratio of each rolling window of the hypothesis $\psi_e = 0$ versus $\psi_e > 0$. The light (dark) gray area denotes anecdotal evidence in favor of the alternative (benchmark). The upper (lower) white area suggests substantial or strong evidence for the alternative (benchmark). Prior odds are assumed to be equal to one.

posterior odds ratios oscillated between strong and weak evidence supporting the benchmark specification. Now, after the 95-14 subsample, the preferred specification starts transitioning to the model with no exchange rates in the policy function. Although the evidence is still anecdotal when the alternative model is preferred, it is important to remark that the data no longer favors (substantially) one specification against the other one. Indeed, the results

Table 3.2:
Alternative priors

Parameter	Density	Benchmark		Alternative priors	
		P(1)	P(2)	P(1)	P(2)
ρ_i	Beta/Uniform	0.50	0.25	0.00	1.00
ψ_π	Gamma	1.50	0.30	1.50	0.60
ψ_y	Gamma	0.25	0.13	0.75	0.30
ψ_e	Gamma	0.25	0.13	0.75	0.30

Note: P(1) and P(2) refers the mean and standard deviation for the Beta and Gamma distributions, and lower and upper bounds for the Uniform distribution.

confirm a time-varying behavior by UK monetary authorities on the conception of exchange rate targeting.

Concerning the parameter instabilities and evolving macroeconomic dynamics, the overall results hold. In some instances, the rolling window estimates drift smoother across subsamples, but the transitions are comparable. The impulse responses also show more uniform shifts. There are slight differences in the median shares of the exogenous shocks for the forecast error variance decomposition but without disrupting the underlying results.

3.4.4.2 Alternative policy priors

Being the monetary policy parameters a focal element in the analysis, I also re-estimate the model relaxing the priors for the parameters of the BoE reaction function and assess the robustness of the parameter instabilities found earlier and the transition of the effective policy responses. I increase the standard deviation for the three policy parameters and impose a uniform prior for the interest rate smoothing parameters.[24] Table 3.2 summarizes the new set of priors adopted.

[24] I also increase the prior mean of the response to output and the interest rate to account for the non-negative restrictions imposed by the distribution.

The results, which are available in the Appendix, show that the overall characteristics of the posterior densities hold. There is evidence of parameter instabilities across the model parameters and clear transitions in the policy coefficients. Not surprisingly, the response parameters are larger than before due to the influence of the prior means adopted for output and the exchange rate responses. Nonetheless, the higher posterior estimates for the interest smoothing term seem to balance this difference and ultimately render similar effective policy responses.

3.5 Conclusions

This paper revisited evidence on how central banks respond to the exchange rate by investigating the possibility of parameter instabilities estimating a small open economy DSGE model fitted to UK data over rolling windows.

The empirical results provide ample evidence of drifts on several model parameters. I find that monetary policy has become progressively more passive for inflation and more active for output. Concerning the exchange rate, I reconsidered earlier results on the response to this variable by the BoE by comparing posterior odds against an alternative with no exchange rate in the policy rule. The analysis reveals interesting results. For the initial windows, the marginal likelihood of the specification with exchange rate turned to be significantly superior. However, for latter samples, the evidence becomes weaker and, in some cases, even favors the model with no exchange rate policy feedback. This brings a new dimension to consider on the debate of whether central banks respond to exchange rate fluctuations. Indeed, as found in the literature, the results vary across countries or model specifications. In this paper, I show that the results may also be susceptible to the sample period analyzed.

By comparing the estimated impulse response functions, the paper shows evident differences in how the model responds to the exogenous shocks. This turns to be a significant finding that provides more context on the importance of accounting for parameter drifts in DSGE models and analysis of the macroeconomic dynamics. In the paper, I also document how various shocks become more important to explain the variance of the variables as the windows progress to latter samples.

Further, the paper aimed to provide a first look at how parameters drift in small open economies and the corresponding implications to the aggregate dynamics. In this sense, the results are to be interpreted concerning the UK's case. However, taking this inquiry together with similar findings in the literature, the evidence is consistent with the premise that central banks update the relative weights to these variables as the macroeconomy unfolds. In particular, these findings correlate with Dong (2013) and Zamarripa (2021), on that central banks in small open economies seem to have adjusted the weights in their policy rules around the periods when inflation targetting was adopted. Future research could reconcile these results and continue investigating recurrent patterns in the transition of structural parameters in the open economy context, such as during inflation episodes or when systematic changes in monetary policy occurred.

In the paper, I largely focused on showing the role of monetary policy and the evolution of the parameters that govern the BoE's reaction function. In light of the evidence derived from the posterior odds test, future studies could expand the analysis by considering optimal policy design within the rolling samples. It could also be worthwhile to extend the present study and explore how alternative assumptions about the expectation formation process may affect the path of parameter instabilities in small open economy DSGE models, such as with learning mechanisms or heterogeneous expectations.

From a policy standpoint, the results are of particular interest to policymakers as they show that 'structural' parameters in DSGE models may contain a time-varying component. As such, this paper shows that monetary policy is not an invariant process. Disregarding parameter instabilities could lead monetary authorities to assign incorrect weights to the variables in their policy rules when attempting to achieve economic objectives and unfold undesired macroeconomic effects. Likewise, ignoring the possibility of parameter drifts could lead to incorrect analysis of the propagation of shocks or produce relatively poor forecasts.

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Appendix A

Appendix for Chapter 3

A.1 Small open economy model

This section sketches the microfoundations of the model employed in Section 3.2. The content is taken and summarized from Justiniano and Preston (2010b).

A.1.1 Households

Households are assume to maximize the following intertemporal problem:

$$E_0 \sum_{t=0}^{\infty} \beta^t \tilde{\varepsilon}_{g,t} \left[\frac{(C_t - H_t)^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} \right]$$

where N_t is the labor input; $H_t \equiv hC_{t-1}$ refers to an external habit taken as exogenous by the household; $\sigma, \varphi > 0$ are inverse elasticities of intertemporal substitution and labor suply, respectively; and $\tilde{\varepsilon}_{g,t}$ is a preference shock. C_t is a composite consumption index:

$$C_t = \left[(1-\alpha)^{\frac{1}{\eta}} C_{H,t}^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} C_{F,t}^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}}$$

where $C_{H,t}$ and $C_{F,t}$ are Dixit–Stiglitz aggregates of the domestic and foreign produced goods equal to

$$C_{H,t} = \left[\int_0^1 C_{H,t}(i)^{\frac{\varepsilon-1}{\varepsilon}} di\right]^{\frac{\varepsilon}{\varepsilon-1}} \qquad \text{and} \qquad C_{F,t} = \left[\int_0^1 C_{F,t}(i)^{\frac{\varepsilon-1}{\varepsilon}} di\right]^{\frac{\varepsilon}{\varepsilon-1}}$$

where α corresponds to the share of foreign goods in the domestic consumption bundle; $\eta > 0$ is the elasticity of substitution between domestic and foreign goods; and $\varepsilon > 1$ refers to the elasticity of substitution between types differentiated domestic and foreign goods.

The only available assets are one-period domestic and foreign bonds. Hence, the flow budget constraint is given by

$$P_t C_t + D_t + \tilde{e}_t B_t = D_{t-1}(1+\tilde{i}_{t-1}) + \tilde{e}_t B_{t-1}(1+\tilde{i}^*_{t-1})\phi_t(A_t) + W_t N_t + \Pi_{H,t} + \Pi_{F,t} + T_t$$

for all $t > 0$, where D_t denotes the household's holding of one-period domestic bonds, and B_t holdings of one-period foreign bonds with corresponding interest rates $\tilde{i}_t$ and $\tilde{i}^*_t$. The nominal exchange rate is $\tilde{e}_t$. P_t, $P_{H,t}$, $P_{F,t}$ and P^*_t refer to the domestic CPI, domestic goods prices, the domestic currency price of imported goods and the foreign price, respectively, and are formally defined below. Wages W_t are earned on labor supplied and $\Pi_{H,t}$ and $\Pi_{F,t}$ denote profits from holding shares in domestic and imported goods firms. T_t denotes lump-sum taxes and transfers. Debt elastic interest rate premium is given by the function $\phi_t(\cdot)$, such that

$$\phi_t = \exp\big[-\chi(A_t + \tilde{\phi}_t)\big]$$

where

$$A_t \equiv \frac{\tilde{e}_{t-1}B_{t-1}}{\bar{Y}P_{t-1}}$$

is the real quantity of outstanding foreign debt expressed in terms of domestic currency as a fraction of steady-state output and $\tilde{\phi}_t$ a risk premium shock.

The budget constraint implicitly assumes that all households in the domestic economy receive an equal fraction of both domestic and retail firm. Thus, nominal income in each period is $W_tN_t+\Pi_{H,t}+\Pi_{F,t}$, which in equilibrium equals $P_{H,t}Y_{H,t}+(P_{F,t}-\tilde{e}_tP_t^*)C_{F,t}$ for all households.

The household's optimization problem requires allocation of expenditures across all types of domestic and foreign goods, both intratemporally and intertemporally. This yields the following set of optimality conditions.

$$C_{H,t}(i) = \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\theta} C_{H,t} \qquad \text{and} \qquad C_{F,t}(i) = \left(\frac{P_{F,t}(i)}{P_{F,t}}\right)^{-\theta} C_{F,t}$$

for all i with associated aggregate price indexes for the domestic and foreign consumption bundles given by $P_{H,t}$ and $P_{F,t}$. Optimal allocation of expenditure across domestic and foreign goods imply the demand functions

$$C_{H,t} = (1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t \qquad \text{and} \qquad C_{F,t} = \alpha\left(\frac{P_{F,t}}{P_t}\right)^{-\eta} C_t$$

where

$$P_t = \left[(1-\alpha)P_{H,t}^{1-\eta} + \alpha P_{F,t}^{1-\eta}\right]^{\frac{1}{1-\eta}}$$

is the consumer price index.

The allocation of expenditures on the aggregate consumption bundle and optimal labor supply satisfy

$$\lambda_t = \tilde{\varepsilon}_{g,t}\big(C_t - H_t\big)^{-\frac{1}{\sigma}}$$
$$\lambda_t = \tilde{\varepsilon}_{g,t}\frac{P_t N_t^{\varphi}}{W_t}$$

and portfolio allocation is determined by the optimality conditions

$$\lambda_t \tilde{e}_t P_t = E_t\big[(1+\tilde{i}_t^*)\beta\phi_{t+1}\lambda_{t+1}\tilde{e}_{t+1}P_{t+1}\big]$$
$$\lambda_t P_t = E_t\big[(1+\tilde{i}_t^*)\beta\lambda_{t+1}P_{t+1}\big]$$

for the Lagrange multiplier λ_t.

A.1.2 Domestic producers

There is a continuum of monopolistically competitive domestic firms producing differentiated goods. Calvo-style price setting is assumed, allowing for indexation to past domestic goods price inflation. Hence, in any period t, a fraction $1-\theta_H$ of firms set prices optimally, while a fraction $0 < \theta_H < 1$ of goods prices are adjusted according to the indexation rule

$$\log P_{H,t}(i) = \log P_{H,t-1}(i) + \delta_H \pi_{H,t-1}$$

where $0 \le \delta_H \le 1$ measures the degree of indexation to the previous period's inflation rate and $\pi_{H,t} = \log(P_{H,t}/P_{H,t-1})$.

Since all firms having the opportunity to reset their price in period t face the same decision problem they set a common price $P_{H,t}'$. The Dixit–Stiglitz aggregate price index therefore evolves according to the relation

$$P_{H,t} = \left[(1-\theta_H)P_{H,t}^{'(1-\varepsilon)} + \theta_H\left(P_{H,t-1}\Big(\frac{P_{H,t-1}}{P_{H,t-2}}\Big)^{\delta_H}\right)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}$$

Firms setting prices in period t face a demand curve

$$y_{H,T}(i) = \left(\frac{P_{H,t}(i)}{P_{H,T}} \cdot \Big(\frac{P_{H,T-1}}{P_{H,t-1}}\Big)^{\delta_H}\right)^{-\varepsilon}(C_{H,T} + C_{H,T}^*)$$

for all t and take aggregate prices and consumption bundles as parametric. Good i is produced using a single labor input $N_t(i)$ according to the relation $y_{H,t}(i) = \varepsilon_{a,t}N_t(i)$, where $\varepsilon_{a,t}$ is an exogenous technology shock.

The firm's price-setting problem in period t is to maximize the expected present discounted value of profits:

$$E_t\sum_{T=t}^{\infty}\theta_H^{T-t}Q_{t,T}y_{H,T}(i)\left[P_{H,t}(i)\Big(\frac{P_{H,T-1}}{P_{H,t-1}}\Big)^{\delta_H} - P_{H,t}MC_t\right]$$

where $MC_T = W_T/P_{H,T}\varepsilon_{a,T}$ is the real marginal cost function for each firm, assuming homogeneous factor markets. The factor θ_H^{T-t} in the firm's objective function is the probability that the firm will not be able to adjust its price in the next $(T-t)$ periods.

A.1.3 Retail firms

Retail firms import foreign differentiated goods for which the law of one price holds at the docks. In determining the domestic currency price of the imported good, firms are assumed to be monopolistically competitive. This small degree of pricing power leads to a violation of the law of one price in the short run.

In any period t, a fraction $1-\theta_F$ of firms set prices optimally, while a fraction $0 < \theta_F < 1$

of goods prices are adjusted given $\log P_{F,t}(i) = \log P_{F,t-1}(i) + \delta_F \pi_{F,t-1}$. The Dixit–Stiglitz aggregate price index consequently evolves according to the relation

$$P_{F,t} = \left[(1-\theta_F) P_{F,t}^{\prime(1-\varepsilon)} + \theta_F \left(P_{F,t-1} \Big(\frac{P_{F,t-1}}{P_{F,t-2}} \Big)^{\delta_F} \right)^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}}$$

and firms setting prices in period t face a demand curve

$$C_{F_T}(i) = \left(\frac{P_{F,t}(i)}{P_{F,T}} \cdot \Big(\frac{P_{F,T-1}}{P_{F,t-1}} \Big)^{\delta_F} C_{F,T} \right)^{-\varepsilon}$$

for all t and take aggregate prices and consumption bundles as parametric.

The firm's price-setting problem in period t is to maximize the expected present discounted value of profits:

$$E_t \sum_{T=t}^{\infty} \theta_F^{T-t} Q_{t,T} C_{F,T}(i) \left[P_{F,t}(i) \Big(\frac{P_{F,T-1}}{P_{F,t-1}} \Big)^{\delta_F} - \tilde{e}_T P_{F,t}^*(i) \right]$$

A.1.4 International risk sharing

From the asset-pricing conditions that determine domestic and foreign bond holdings, the uncovered interest rate parity condition

$$E_t \lambda_{t+1} P_{t+1} \left[(1+\tilde{i}_t) - (1+\tilde{i}_t^*) \Big(\frac{\tilde{e}_{t+1}}{\tilde{e}_t} \Big) \phi_{t+1} \right] = 0$$

The real exchange rate is defined as $\tilde{q} \equiv \tilde{e}_t P_t^* / P_{F,t}$. Since $P_t^* = P_{F,t}^*$, when the law of one price fails to hold, we have $\tilde{\Psi}_{F,t} \equiv \tilde{e}_t P_t^* / P_{F,t} \neq 1$ (the law of one price gap).

A.1.5 General equilibrium

Goods market clearing in the domestic economy requires

$$Y_{H,t} = C_{H,t} + C^*_{H,t}$$

The model is closed assuming foreign demand for the domestically produced good is specified as

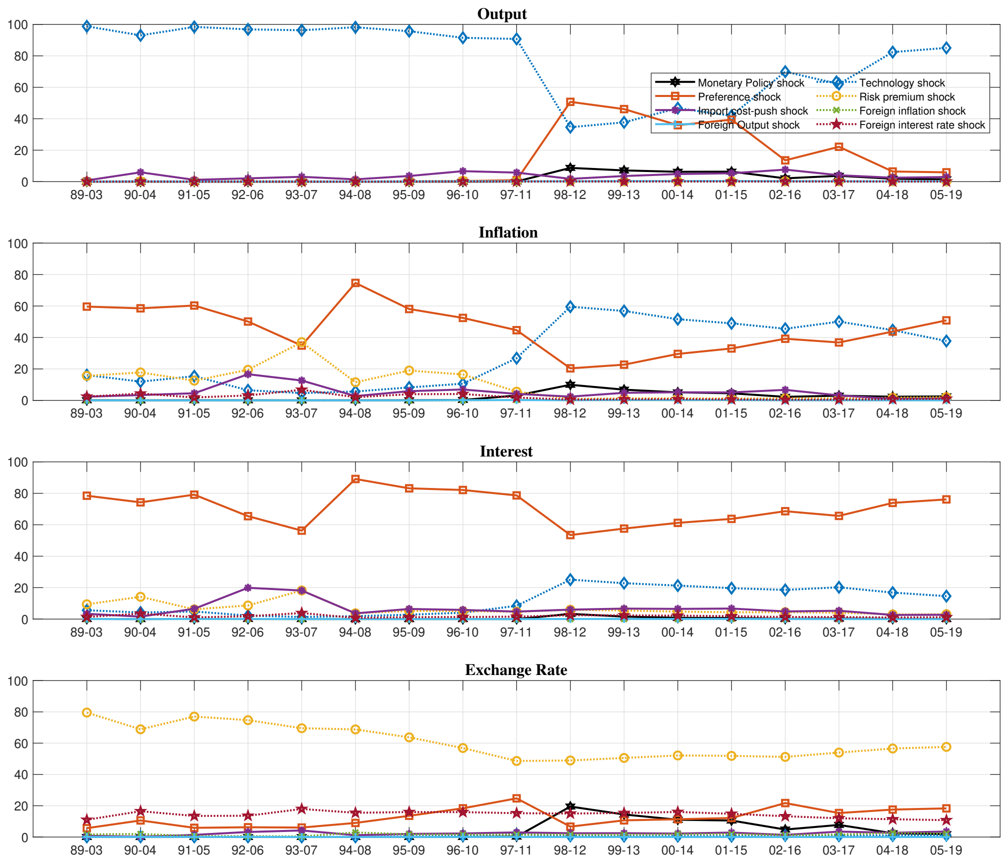
$$C^*_{H,t} = \left(\frac{P^*_{H,t}}{P^*_t}\right)^{-\lambda} Y^*_t$$

where $\lambda > 0$.

Domestic debt is assumed to be in zero net supply so that $D_t = 0$ for all t. The model considers a symmetric equilibrium in which all domestic producers and all retailers setting prices in period t set common prices $P_{H,t}$ and $P_{F,t}$, respectively. Households are assumed to have identical initial wealth, so that each faces the same period budget constraint and therefore makes identical consumption and portfolio decisions. Monetary policy is assumed to be conducted according to a Taylor-type rule. Fiscal policy is specified as a zero debt policy, with taxes equal to the subsidy required to eliminate the steady-state distortion induced by imperfect competition in the domestic and imported goods markets.

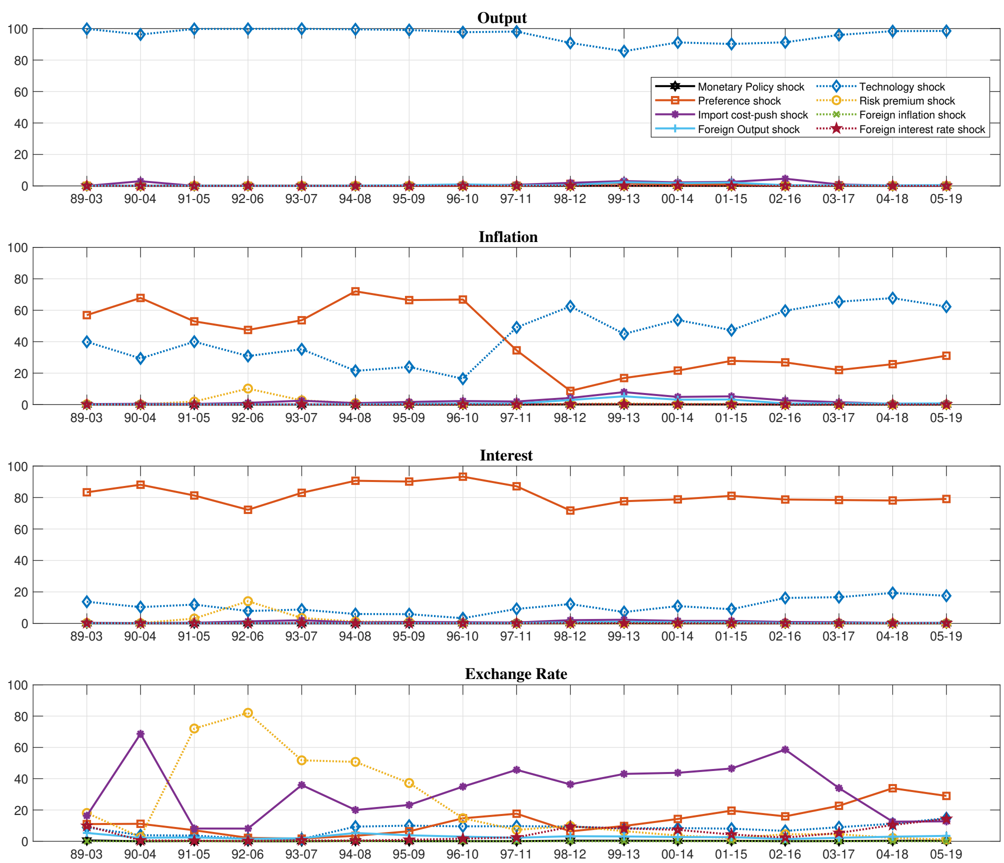
A.2 15-year windows

Figure A.1: Evolution of variance decomposition (4-quarters ahead horizon)



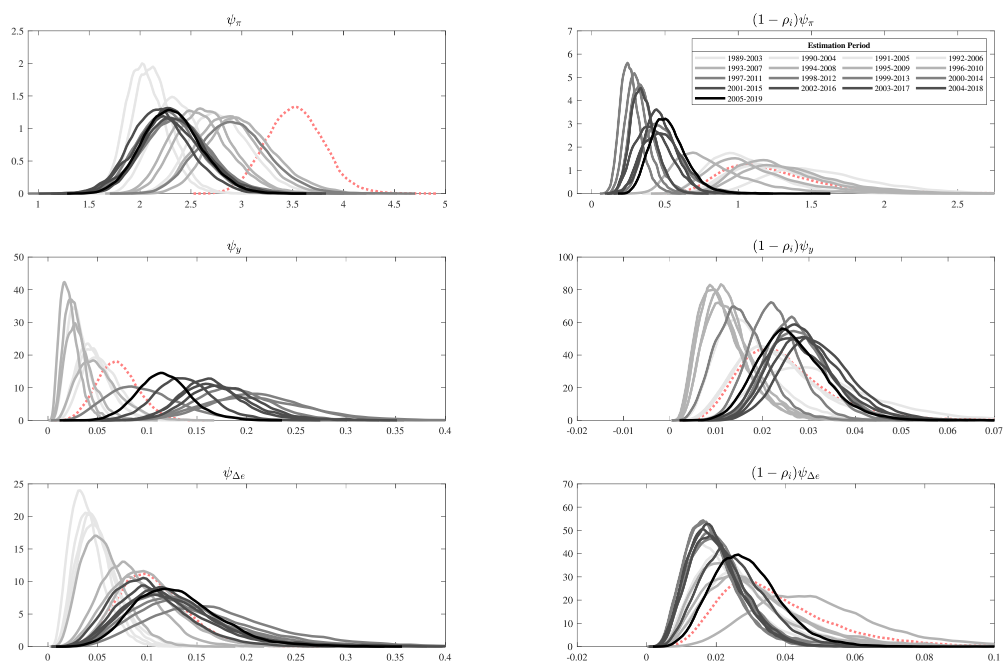
Note: The figure shows the window-specific median forecast-error variance shares across MH draws at the 4-quarters ahead horizon.

Figure A.2: Evolution of variance decomposition (24-quarters ahead horizon)



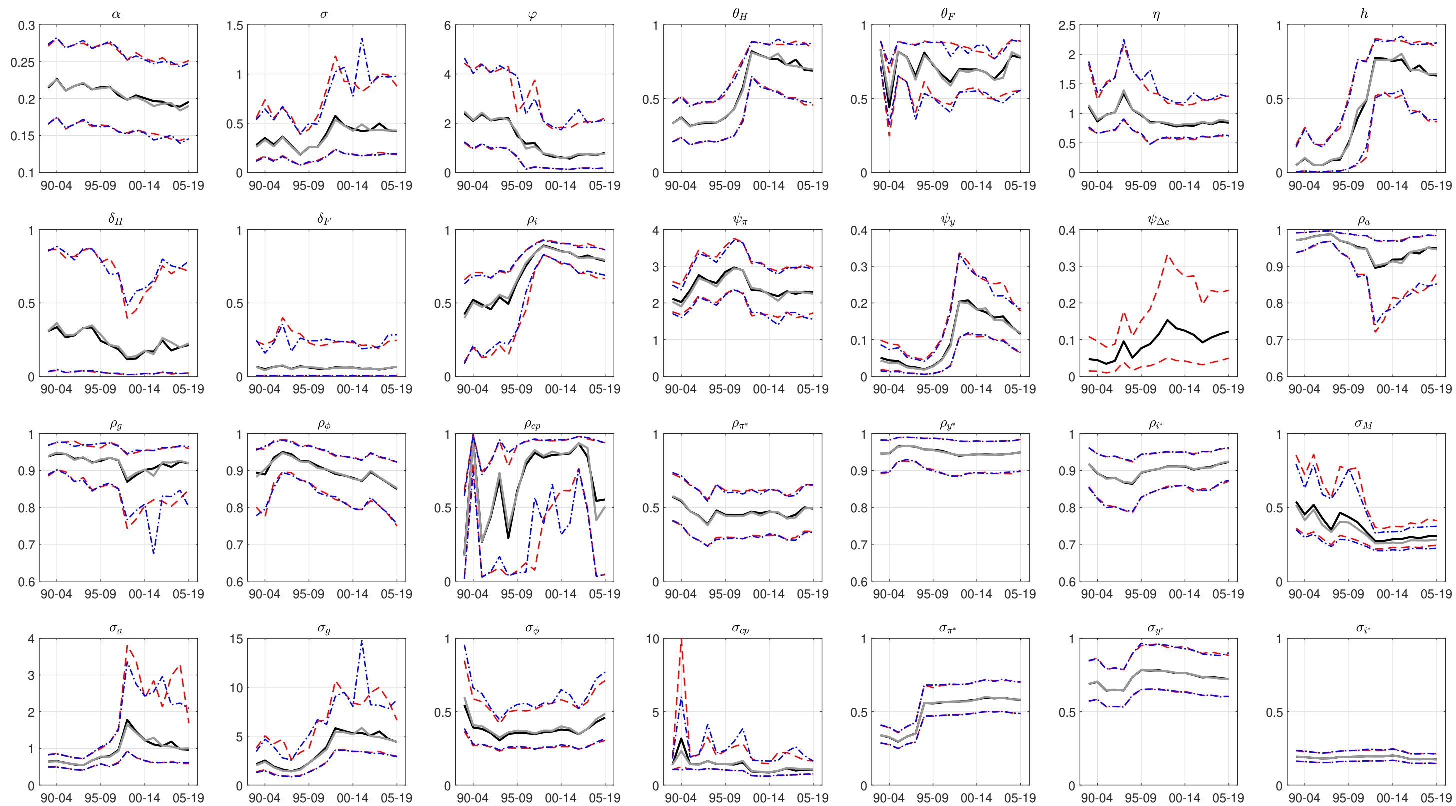
Note: The figure shows the window-specific median forecast-error variance shares across MH draws at the 24-quarters ahead horizon.

Figure A.3: Posterior distributions of policy coefficients and policy responses (with full-sample estimates)



Note: The red dashed densities refer to full-sample estimates.

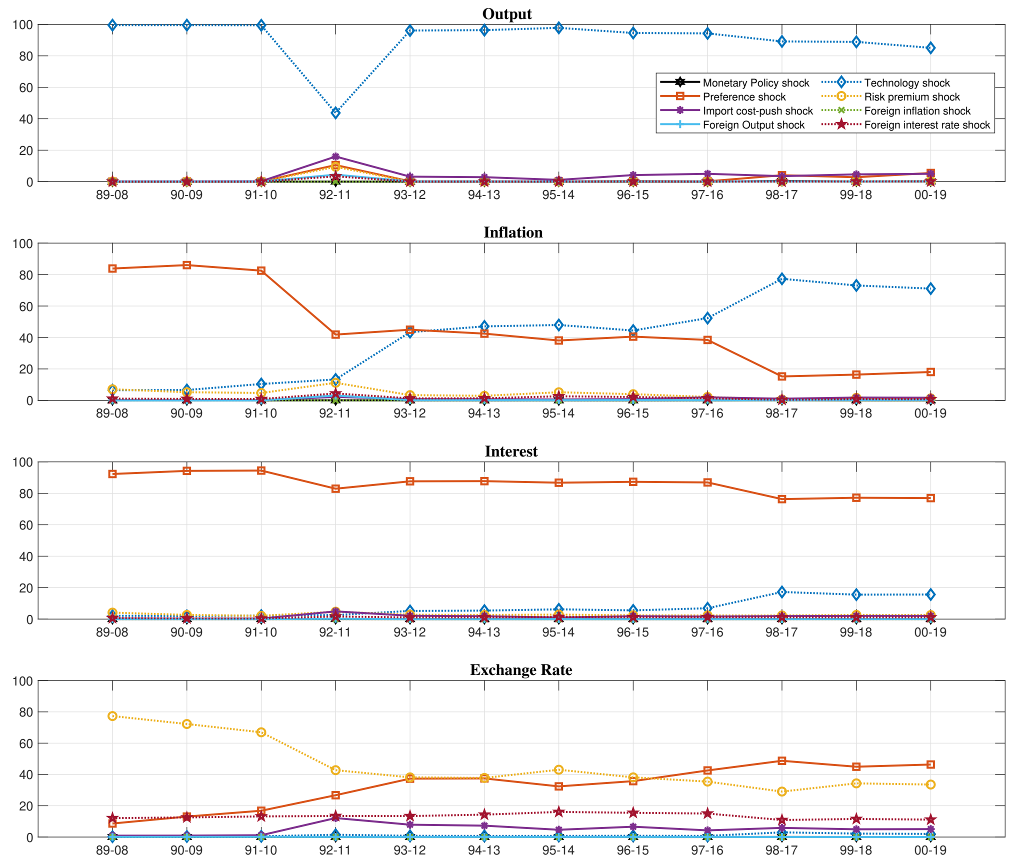
Figure A.4: Rolling-window posterior estimates ($\mathcal{M}_1$ vs $\mathcal{M}_2$ - 15-year windows)



Note: The figure shows the posterior median (solid lines) of each sub-sample across the Metropolis-Hastings draws, along with 95% Bayesian credible interval bands (dashed lines). Black solid lines and red interval bands describe $\mathcal{M}_1$ estimates, gray solid lines and blue interval bands describe $\mathcal{M}_2$ estimates.

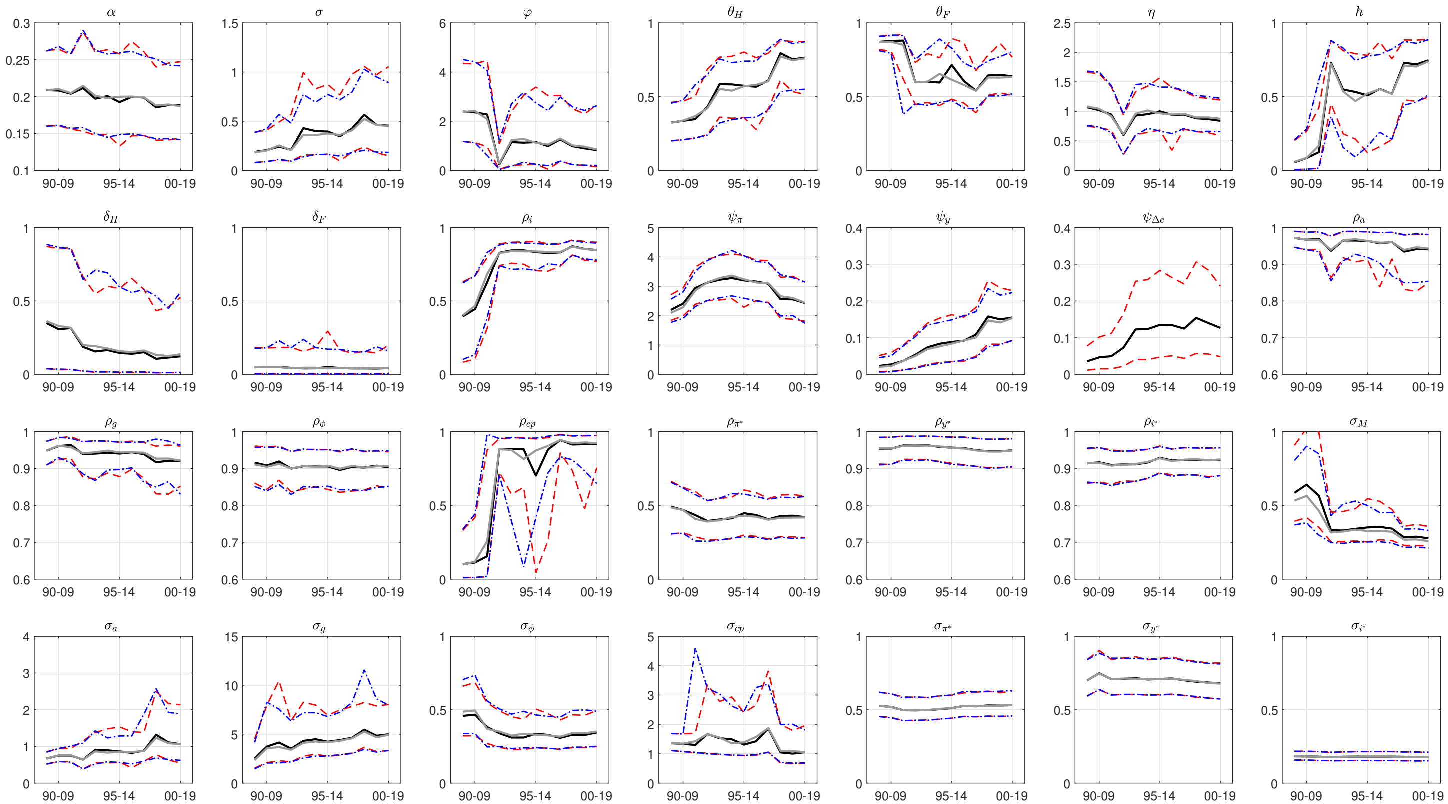
A.3 20-year windows

Figure A.5: Evolution of variance decomposition (8-quarters ahead horizon)



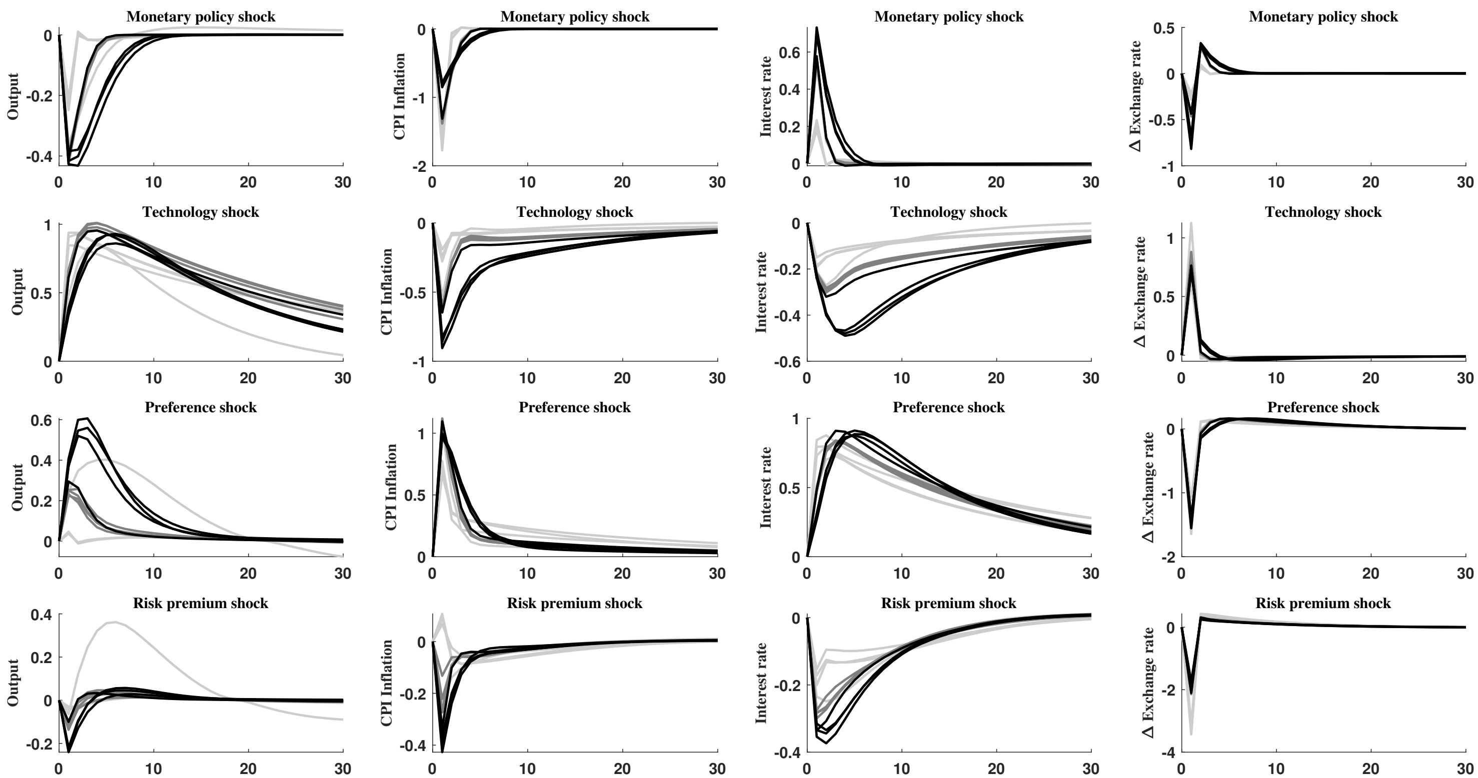
Note: The figure shows the window-specific median forecast-error variance shares across MH draws at the 8-quarters ahead horizon.

Figure A.6: Rolling-window posterior estimates ($\mathcal{M}_1$ vs $\mathcal{M}_2$ - 20-year windows)



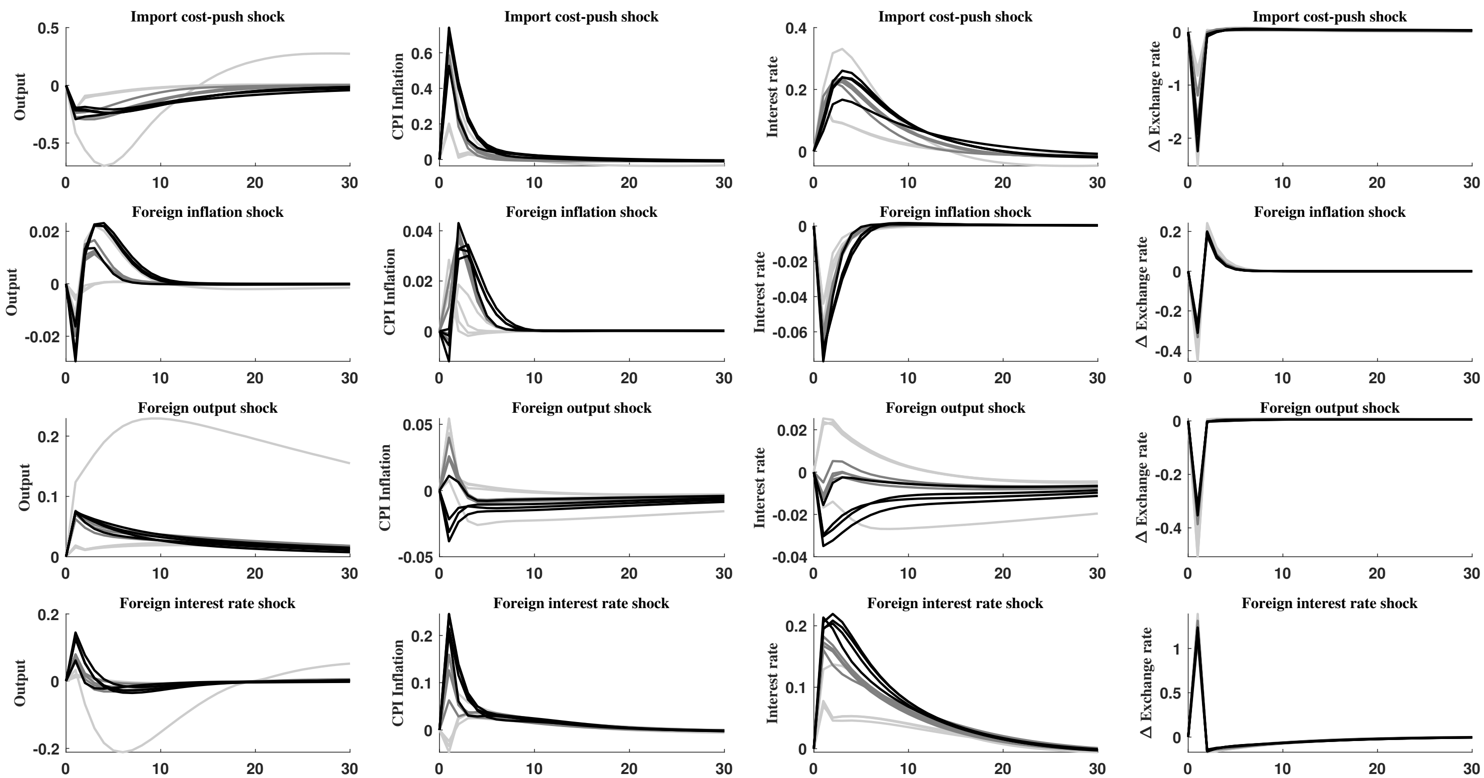
Note: The figure shows the posterior median (solid lines) of each 20-year rolling window across the Metropolis-Hastings draws, along with 95% Bayesian credible interval bands (dashed lines). Black solid lines and red interval bands describe $\mathcal{M}_1$ estimates, gray solid lines and blue interval bands describe $\mathcal{M}_2$ estimates.

Figure A.7: Rolling-window Impulse Response Functions (20-year windows)



Note: The impulse responses represent the median of each window across draws. Bayesian probability intervals omitted for clarity purposes. Legend is the same as in Figure 3.7 (lines get periodically darker for the more recent windows).

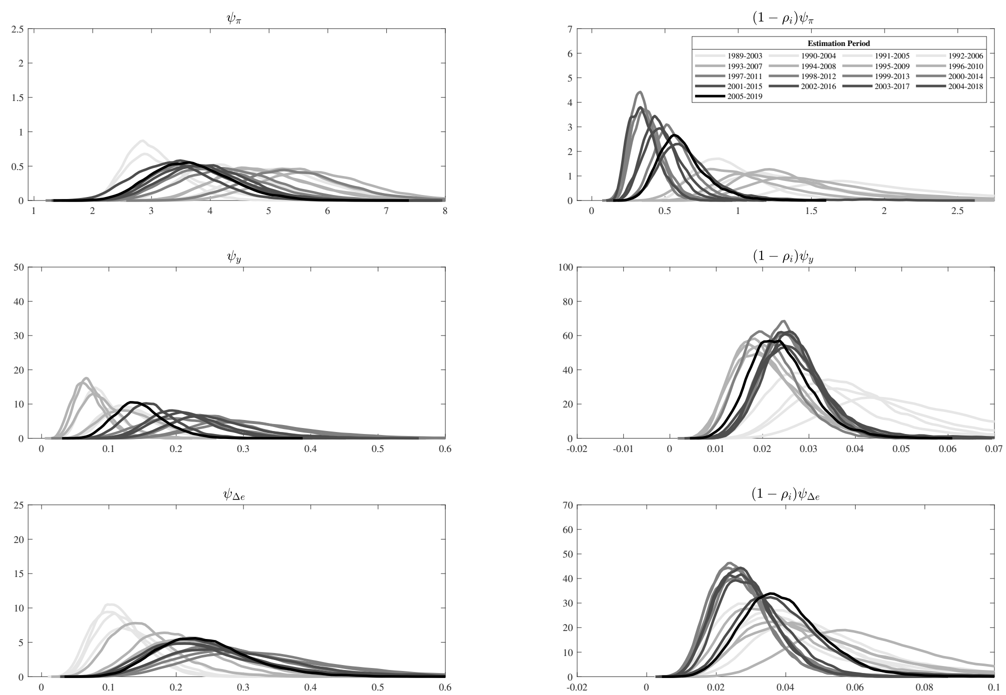
Figure A.7: (cont.)



Note: The impulse responses represent the median of each window across draws. Bayesian probability intervals are omitted for clarity purposes. Legend is the same as in Figure 3.7 (lines get periodically darker for the more recent windows).

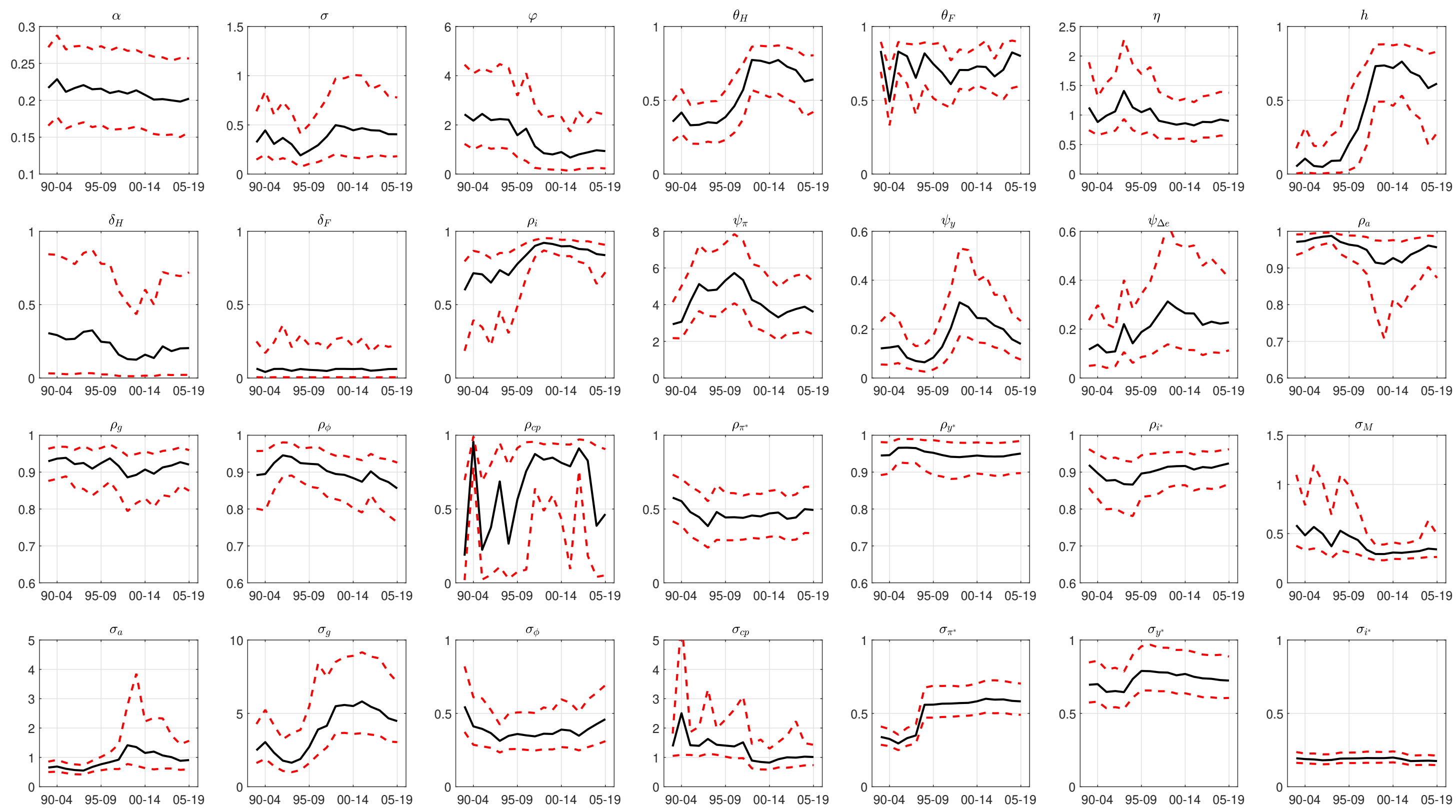
A.4 Alternative priors

Figure A.8: Posterior distributions of policy coefficients and policy responses (Alternative priors)



Note: The horizontal axis of the right panels is maintained consistent with Figure 3.3 for comparability reasons.

Figure A.9: Posterior distributions of policy coefficients (Alternative priors)



Note: The figure shows the posterior median (solid lines) of each sub-sample across the Metropolis-Hastings draws, along with 95% Bayesian credible interval bands (dashed lines).