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TECHNICAL REPORT

Measuring drag without a force transducer: a terminal velocity assay

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Summary

1. Organisms live surrounded by air or water, which exert drag on an organism when moving. These forces are significant ecologically because they can affect organisms' distribution, behaviour or dispersal.

2. Appropriate techniques for measuring or estimating these forces vary greatly depending on the magnitude of the forces and the flow pattern of the moving fluid (air or water; both gases and liquids are fluids). A simple method for estimating the drag in uniform steady flow is described. This technique is particularly well suited for forces of small magnitude (micronewtons) and slow flows ($1 < Re < 100$), and provides very uniform and well-characterized ambient flow relative to the object.

3. This method capitalizes on the force balance that exists when a falling object reaches terminal velocity. At terminal velocity, all of the forces sum to zero, and therefore the drag may be estimated from the other (known) forces (buoyancy and gravitational force). Orientation during falling may be controlled if necessary.

Key-words: Biophysics, physical model

Functional Ecology (2002) **16**, 268–272

Introduction

The forces that act on an organism due to the relative movement of the surrounding fluid medium are of general interest in understanding the functional design of organisms (Denny 1993; Vogel 1994). These forces are significant ecologically because they can affect distribution, behaviour or dispersal. In addition, these forces cause deformation or deflection of body parts, which can cause change or loss of function. Drag is defined as the force that opposes the movement of a body relative to the surrounding fluid. Measurement of the drag acting on small objects in a slowly moving fluid is complicated by the difficulties not only of measuring such small forces but also of supplying sufficiently slow and steady flow relative to the object.

One way to generate steady flow relative to an object is to move the object at a constant speed through a stationary fluid. In the laboratory this steady movement is commonly generated by a motor-driven translating table. However, steady movement through a stationary fluid also occurs when a falling object has reached terminal velocity; the object is no longer accelerating under the influence of gravity and its velocity remains constant with the further passage of time. The phenomenon of terminal velocity may be exploited to provide steady flow relative to an object for the purpose of estimating the drag within a range of velocities. This method is appropriate for applications other than falling under the influence of gravity, has been used to estimate drag on fish and frogs (Webb 1975; Blake 1981; Gal & Blake 1987), and is well suited for drag measurements on slower or smaller objects. We provide suggestions for applying this method and sample data.

PHYSICAL BACKGROUND OF METHOD: FORCES ACTING ON FALLING BODIES

When an object falls straight down, three main forces act on the body: a gravitational force (F_{grav}), buoyancy (F_{buoy}) and drag (F_{drag}):

$$F_{net} = F_{grav} + F_{buoy} + F_{drag}. \qquad \text{eqn 1}$$

The gravitational force (F_{grav}), will not change in magnitude as the object falls unless there is a change in the mass of the object:

$$F_{grav} = -mg, \qquad \text{eqn 2}$$

where m is the mass of the object, g is the acceleration of gravity ($9{\cdot}8\ m\ s^{-2}$) and upwards is defined as the positive direction. Buoyancy (F_{buoy}) acts upwards, is equivalent in magnitude to the weight of the fluid displaced by the object, and will not change in magnitude while

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the object falls (in the absence of a change of volume of the object or density of the fluid):

$$F_{buoy} = V_{object}\rho_{fluid}g = (m/\rho_{object})\rho_{fluid}g, \qquad \text{eqn 3}$$

where V_{object} is the volume of the object and ρ_{object} is the density of the object. Drag (F_{drag}) acts upwards (for an object falling downward) and will increase in magnitude as the speed of the falling object increases. The speed of the falling object will increase until the combined effects of drag and buoyancy acting upwards cancel the downward gravitational force; after the forces balance the object will no longer accelerate but will continue to fall at this terminal velocity. Cases of gliding, oscillating, rotating and tumbling objects will not be considered here, although this method could be modified appropriately for those applications. The drag on a falling object that has reached terminal velocity is simply (combining equations 1, 2 and 3 with $F_{net} = 0$)

$$F_{drag} = -F_{grav} - F_{buoy} = mg[1 - (\rho_{fluid}/\rho_{object})]. \qquad \text{eqn 4}$$

Hence, we know the drag on an object at its terminal velocity from its mass and density and the density of the fluid through which it is falling (i.e. even before the object falls). We do not know the terminal velocity, and in practice this is what will be measured.

PHYSICAL BACKGROUND OF METHOD: DIMENSIONLESS NUMBERS

In the engineering technique of dimensional analysis, dimensionless numbers are substituted for the physical variables to characterize a general physical relationship (McMahon & Bonner 1983; Pennycuick 1992; Vogel 1994). For the case of a steadily falling object there are two dimensionless numbers that completely characterize the falling behaviour: the Reynolds number and the drag coefficient. The Reynolds number *Re* is

$$Re = Lu\rho_{fluid}/\mu, \qquad \text{eqn 5}$$

where L is the characteristic length, u is the ambient velocity of the fluid relative to the object (either can be moving relative to the observer), ρ_{fluid} is the density of the fluid and μ is the dynamic viscosity of the fluid (Vogel 1994). The convention for defining the characteristic length (L) varies somewhat with context (e.g. L may be measured on the solid or within the fluid) and therefore will be identified as needed.

The drag coefficient, C_d, is related to drag by:

$$F_{drag} = 0{\cdot}5C_dA\rho_{fluid}u^2, \qquad \text{eqn 6}$$

where A is the frontal area (area projected onto a plane perpendicular to the direction of fluid flow or object motion). Note that C_d is not a constant, and therefore the drag can depend on the velocity in a complicated way through the dependence of C_d on velocity. At terminal velocity the drag coefficient may be estimated by (combining equations 4 and 6):

$$C_d = \frac{2mg(\rho_{object} - \rho_{fluid})}{A\rho_{fluid}u^2\rho_{object}}. \qquad \text{eqn 7}$$

The value of establishing the relationship between C_d and *Re* for a given shape is that this relationship is expected in any fluid medium as long as the geometry does not change. Dimensionless representation facilitates comparison with other measurements and allows predictions of drag with change in size or speed, or in different fluids. This predictive ability also means that the relationship between drag and velocity may be determined in a different fluid if convenient, such as measurement in water when the biologically relevant fluid is air.

Method

Measurement in the original fluid medium is preferable if the terminal velocity matches the speed of interest and is a convenient magnitude. Slower terminal velocities may be obtained by choosing more viscous fluids or fluids of density more similar to that of the object, while faster terminal velocities may be obtained by choosing less viscous or less dense fluids. The inconvenience of measuring the viscosity and density of an arbitrary fluid may be avoided by using a physically well-characterized fluid (e.g. listed in Weast *et al*. 1988).

If measurements are made in a fluid different from the original, the drag expected in the original medium may be calculated by equating the dimensionless numbers *Re* and C_d in the experimental and original media. For the example of calculating the forces in air from measurements made in water, equating dimensionless drag in the two media results in

$$C_{d,air} = C_{d,water} = \frac{2F_{drag,air}}{\rho_{air}u_{air}^2A} = \frac{2F_{drag,water}}{\rho_{water}u_{water}^2A}. \qquad \text{eqn 8}$$

Solving for the drag in air leads to

$$F_{drag,air} = \frac{F_{drag,water}\rho_{air}u_{air}^2}{\rho_{water}u_{water}^2}. \qquad \text{eqn 9}$$

The only unknown in equation 9 is the velocity in air, u_{air}. We can find this from equating the Reynolds numbers in the two media:

$$Re_{air} = Re_{water} = \frac{\rho_{air}u_{air}L}{\mu_{air}} = \frac{\rho_{water}u_{water}L}{\mu_{water}}. \qquad \text{eqn 10}$$

Solving for u_{air} (equation 10) and substituting into equation 9 results in the drag that would be expected in air:

$$F_{drag,air} = \frac{F_{drag,water}\rho_{water}\mu_{air}^2}{\rho_{air}\mu_{water}^2}. \qquad \text{eqn 11}$$

Note that identical drag coefficients in different media do not imply identical drag; only certain (rare) pairs of

Functional Ecology, **16**, 268–272

fluids result in identical drag for identical *Re* and C_d (such as air at 20 °C and water at 52 °C, Vogel 1983).

The orientation of the falling object may not result in oncoming flow striking the object at the appropriate angle. However, orientation (and terminal velocity) may be modified by adding weights, whether external (such as the wires described below) or internal (such as adding lead shot to the body cavities of frogs, Gal & Blake 1987). The size and location of any attached external weights should be chosen keeping in mind that the lower the *Re*, the further the influence of any attachment or nearby surface extends into space. In the Reynolds number range $Re < 100$, falling objects of shapes from flat to elongate (plates to columns) tend to fall in a stable orientation without the oscillating, rotating or tumbling seen at higher values of *Re* (Ward-Smith 1984; *L* is the diameter of the plate or column), making this *Re* range particularly suitable for this method.

The size of the enclosure within which the object is dropped reflects compromises between two potential sources of systematic errors: a container that is too small relative to the object will tend to retard the motion of the falling object ('wall effects'), while a container that is too large is more likely to exhibit natural convection (see Vogel 1994, pp. 338–340, for discussion). Wall effects are more likely to be significant for $Re < 1$ (*L* is the diameter of the object, not the container) (Happel & Brenner 1965; Loudon, Best, & Koehl 1994), and Vogel (1994) provides formulae for their evaluation. The fluid should be as homogeneous as possible to lessen the thermal or concentration gradients that may lead to natural convection within the container. Falling objects tend to interact in complex ways and therefore only one object should be dropped at a time unless their interaction is of interest.

Velocity may be estimated by videotaping the falling object from the side, and dividing the displacement of the object on different video frames by the corresponding time difference. A useful and thorough discussion of sources and magnitudes of error resulting from the calculation of velocities from successive locations of an object may be found in Walker (1998). When successive estimates of the velocity of a falling object no longer increase in time, the object has reached terminal velocity under the experimental conditions. Errors in early published measurements on drag on fish using this method are probably due to the terminal velocity not having been reached (described in Webb 1975).

Sample data

SPHERES

The extensive literature on drag of spheres made falling spheres a logical and convenient choice for evaluation of this method. However, it should be noted that the relationship between drag and velocity is sufficiently complex that even the drag on smooth, uniform and rigid spheres moving at constant velocity relative to the surrounding medium has generated many empirical treatments (e.g. 12 different equations in Clift *et al*. 1978 all within $0 < Re < 3 \times 10^5$). For many purposes the following empirically based formula supplied by White (1991, equation 3-225; also supplied in Vogel 1994 as equation 15·2) for drag on a single isolated sphere is adequate (stated accuracy within 10% for the given *Re* range):

$$C_{d,sphere} = \frac{24}{Re} + \frac{6}{1 + \sqrt{Re}} + 0{\cdot}4, \quad 0 < Re \leq 2 \times 10^5. \qquad \text{eqn 12}$$

In the present study, the Reynolds numbers for the falling spheres ranged from 44 to 306 (*L* is the diameter of the sphere). Individual spheres (3·175-mm diameter plastic spheres of 1138 kg m^{-3} density and 6·35-mm diameter glass spheres of 2530 kg m^{-3} density, Small Parts, Inc., Miami Lakes, FL) were allowed to fall inside a Perspex box (0·15 m × 0·11 m × 0·20 m high) filled with aqueous ethanol or glycerol solutions or water at temperatures ranging from 18·2 to 26 °C (depth of the fluid was 0·17 m). The fluid never varied by more than 1 °C within any set of drops. After each set of drops, the density and viscosity of the fluid was measured (by weighing a known volume of fluid and by using a Gilmont falling ball viscometer (Barnant Co., Barrington, IL, USA) respectively). In the range of conditions used, terminal velocity was always reached within 0·4 s and before falling 40 mm.

Falling objects were videotaped from the side using a Panasonic WV CL1200 camera (Panasonic Communications, Secaucus, NJ, USA) at standard speed (60 fields s^{-1}) in a horizontal orientation from a distance of 2 m. The location of the object was digitized on successive video fields (Peak Performance Motus Motion Measurement System, Englewood, CO). The velocity series in time was smoothed using a Butterworth filtering method to decrease the noise generated at the digitizing frequency. The terminal velocity for any individual drop was estimated from the average velocity for the middle third of the 0·17 m fluid height in the tank. Three drops in succession for any object were averaged as one replicate. The results for the drag coefficients of the spheres estimated using equation 7 were within 10% of the values predicted from equation 12 (Fig. 1).

POROUS OBJECTS OF COMPLEX MORPHOLOGY: MOTH ANTENNAE

The terminal velocity technique was used to measure drag on single moth antennae (*Bombyx mori* L., the commercial silkworm moth) using the specific methods described above for the spheres. Drag on antennae is of interest in order to calculate the proportion of approaching air that passes through an antenna and is therefore available for chemical sampling (J. Zhang & C. Loudon, unpublished). Individual antennae were removed from adult moths and dropped in a variety of

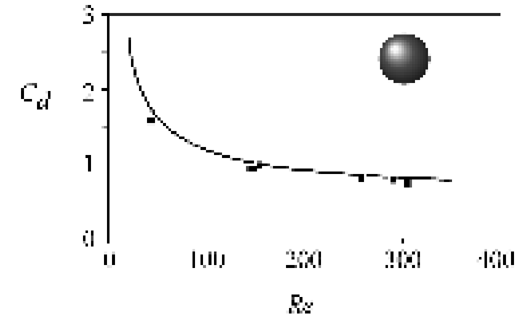

Fig. 1. The drag coefficient (C_d) and Reynolds number (*Re*) for falling spheres were calculated from the terminal velocity method as explained in the text. These data (squares) were within 10% of the C_d predicted from an equation from the engineering literature (solid line; see equation 12).

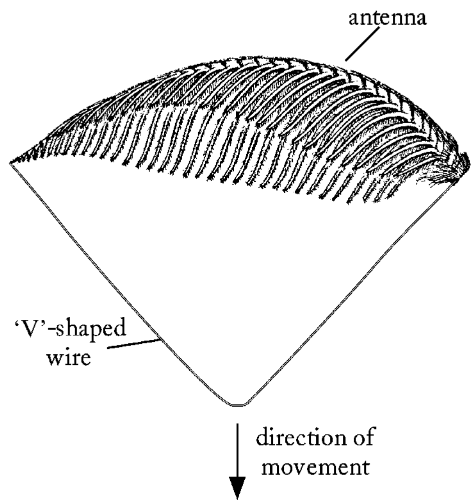

Fig. 2. A single moth antenna (*Bombyx mori*) tethered with a bent wire falls in the orientation shown.

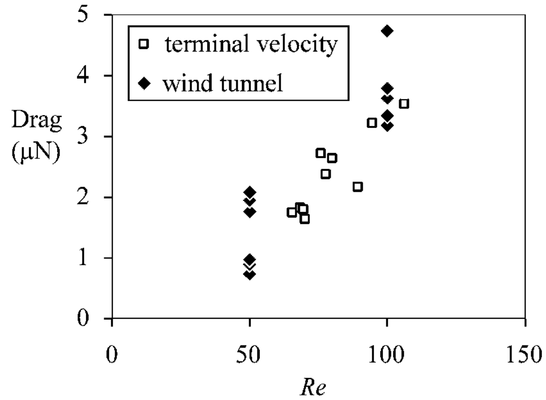

Fig. 3. Drag measured on single antennae using the terminal velocity assay were comparable in magnitude to drag measured on antennae using a different method in a wind tunnel. Each point corresponds to a measurement on a single antenna.

fluids (water and aqueous ethanol and glycerin solutions) to generate the C_d–*Re* relationship for this pectinate morphology (J. Zhang & C. Loudon, unpublished). The maximum air flow speed in the vicinity of the antennae for this species is about 0·4 m s^{-1} (Loudon & Koehl 2000). An isolated antenna tended to fall with the base downward, which generated fluid flow in a biologically irrelevant direction from the base to the distal tip of the antenna. In order to generate flow perpendicular to the concave surface of the antennae, a thin wire (0·15 mm diameter) was attached to an antenna to make it fall in the correct orientation (Fig. 2). The drag due to the wire must be taken into account when estimating the drag on the antenna, and therefore the C_d–*Re* relationship for the isolated wires was also determined by the terminal velocity method. The drag of the wire (estimated from the equation fit to the isolated wires) was subtracted from the total drag to estimate the drag due to the antenna. This subtraction method is only valid for the flow range within which these individual components of drag are additive. In order to identify the Reynolds number range within which these drag components are additive, drag estimated for antennae using the terminal velocity method was compared to drag measured in a wind tunnel using a different method without such 'V'-shaped wires upstream. When the drag estimates were the same using the two methods, the drag components were assumed to be approximately additive. The method for drag estimation in the wind tunnel used a vertically suspended antenna pivoting on a low-friction 'sting' attached to the ceiling of the wind tunnel with the concave surface of the antenna facing upwind; the angle of deflection from the vertical of the antenna with its support was greater in faster air (the support was sheltered from moving air and hence did not experience drag itself) and could be used to estimate the drag using the known masses for the antenna and its support and assuming that the forces acting on the antenna and its support summed to zero.

Drag on individual antennae measured by the terminal velocity method (using subtraction for the wire drag) was comparable in magnitude (6% lower on average) to drag measured by the different method in a wind tunnel without the 'V'-shaped wire upstream (Fig. 3). The sources of experimental error differ somewhat in the two methods and are assumed to be largely unrelated; the variation in individual measurements (Fig. 3) indicates the magnitude of the experimental error and the individual variation between antennae in the two methods. The overlap in the drag estimated by these two methods suggests that the drag components are approximately additive for $Re > 50$ (L is the width of the antenna; 2 mm) and therefore that it is valid to subtract the drag of the wire from the total drag to obtain the drag on an antenna within the stated *Re* range. This result may be used as an approximate guide for identifying the minimum *Re* for which the drag may be assumed to be additive for comparable situations. For lower *Re* applications, less obtrusive means for modifying the orientation should be used.

The switch from air (the original medium for these antennae) to aqueous solutions for these measurements meant that the magnitude of the drag was greater under the experimental conditions and therefore any deformation of the antenna had to be assessed. The antennal branches and hairs were not measurably deflected due to flow under the experimental conditions (measured under a dissecting microscope using video-microscopy; deflection of hair tips by 5 µm would have been clearly visible).

Discussion

While there are many different ways to measure the drag on a biological object, the usual method is to supply flow at a speed of interest (using a wind tunnel or flow tank) and use a force transducer to measure the drag directly (e.g. Vogel 1994). The simple method described in this paper is most likely to appeal to functional ecologists who are interested in estimating forces on small (mm scale) biological structures in slow flows but do not have easy access to appropriately sensitive force transducers, flow tanks and wind tunnels with good control in the slow-speed range, or motor-driven translating tables. The phenomenon of terminal velocity is a useful means to generate smooth, reliable flow for drag measurement, even though the biological context may differ from an object falling under the influence of gravity. Biologists studying dispersal who have measured terminal velocities of seeds or small insects may also use this method to estimate the forces acting on these organisms. While these forces are expected to be small in magnitude, they are large enough to generate circulation in a drop of water falling through the air (Clift *et al*. 1978).

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