Title
NONLINEAR DYNAMICS IN SPINWAVES

Authors
Jeffries, C.D.
Bryant, P.H.
Nakamura, K.

Publication Date
1988-06-01

Lawrence Berkeley Laboratory

UNIVERSITY OF CALIFORNIA

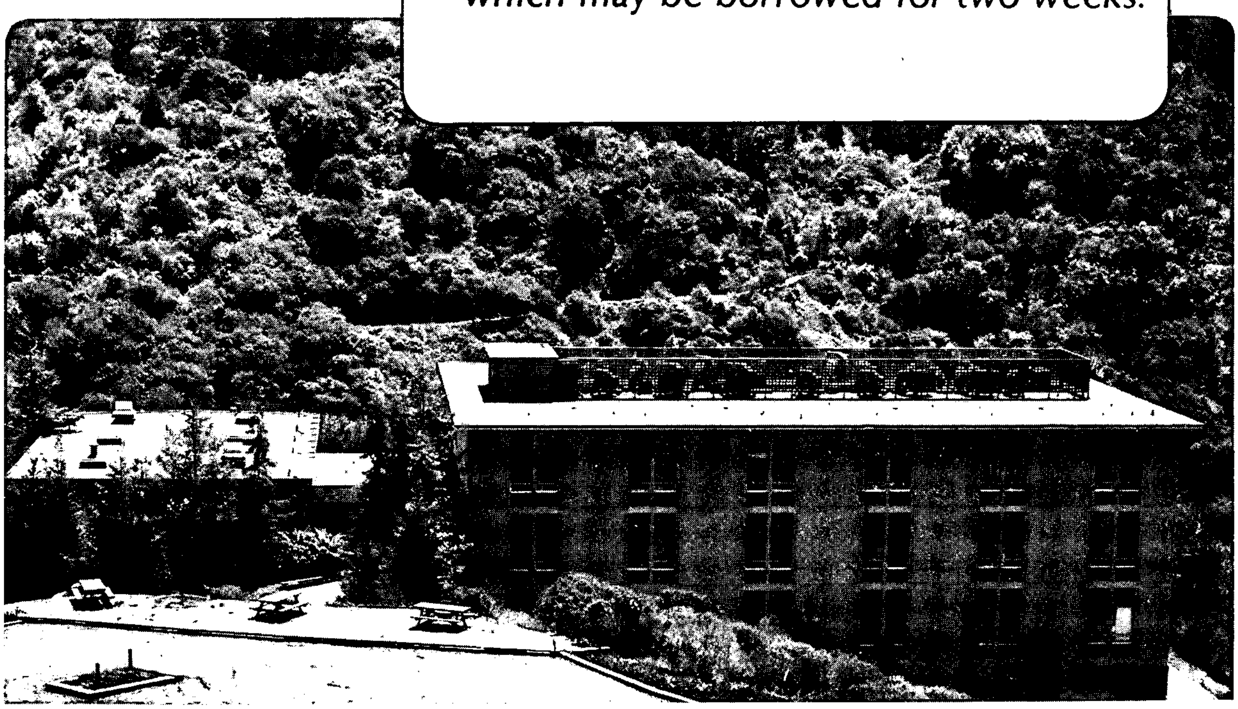
Materials & Chemical Sciences Division

Invited talk to be presented at the 4th Joint MMM-Intermag Conference, Vancouver, British Columbia, July 12–15, 1988, and to be published in Journal of Applied Physics

Nonlinear Dynamics in Spinwaves

C.D. Jeffries, P.H. Bryant, and K. Nakamura

June 1988



Prepared for the U.S. Department of Energy under Contract Number DE-AC03-76SF00098.

DISCLAIMER

Invited talk to be presented at the
4th Joint MMM-Intermag Conference
July 12-15, 1988, Vancouver, B.C.
To be published in J. Applied Phys. (Nov. 1988)

LBL-25409

NONLINEAR DYNAMICS IN SPINWAVES*

Carson D. Jeffries and Paul H. Bryant

Physics Department, University of California, Berkeley, CA 94720

and

Materials and Chemical Sciences Division,
Lawrence Berkeley Laboratory, Berkeley, CA 94720

and

Katsuhiro Nakamura

Fukuoka Institute of Technology, Higashi-ku, Fukuoka 811-02, Japan

June 1988

*Work supported in part by the Director, Office of Energy Research, Office of Basic Energy Sciences, Materials Sciences Division of the U.S. Department of Energy under Contract No. DE-AC03-76SF00098.

NONLINEAR DYNAMICS OF SPINWAVES

Carson D. Jeffries and Paul H. Bryant[a]

Physics Department, University of California, Berkeley, CA 94720

and

Materials and Chemical Sciences Division,
Lawrence Berkeley Laboratory, Berkeley, CA 94720

and

Katsuhiro Nakamura[b]

Fukuoka Institute of Technology, Higashi-ku, Fukuoka 811-02, Japan

ABSTRACT

For a yttrium iron garnet sphere at room temperature, an experimental study is made of the first order Suhl spinwave instability using perpendicular pumping at 9.2 GHz with the dc field parallel to the [111] crystal axis. The dynamical behavior of the magnetization is observed under high resolution by varying two control parameters, dc field ($580 < H_0 < 2100$ G) and microwave pump power ($1 < P_{in} < 200$ mW). Within this parameter space quite varied behavior is found: (i) onset of the Suhl instability by excitation of a single spinwave mode with very narrow linewidth (<0.5 G); (ii) when two or more modes are excited, interactions lead to collective oscillations ("auto-oscillations") with a systematic dependence of frequency (10^4 to 10^6 Hz) on pump power, these oscillations displaying period-doubling to chaos; (iii) quasiperiodicity, locking, and chaos occur when three or more modes are excited; (iv) abrupt transition to wide band power spectra (i.e., turbulence), with hysteresis; (v) irregular relaxation oscillations and aperiodic spiking behavior. A theoretical model of coupled modes is numerically evaluated and found to exhibit a behavior pattern similar to those observed experimentally.

PACS numbers: 76.50.+g, 05.45.+b, 47.20.Tg, 75.30.Ds

In ferromagnetic resonance experiments, as the microwave pumping power is increased, instabilities may be observed,[1,2] characterized by abrupt onset of anomalous noisy absorption. Suhl[3] gave a theory for this behavior, remarking (1957) that it resembled turbulence in fluid dynamics. From the viewpoint of nonlinear dynamical systems theory one may view spinwaves in a ferrite sphere as a set of coupled nonlinear standing wave modes with the dynamics controlled by a low dimensional attractor.[4] Accordingly, certain universal aspects of the behavior are expected on quite general grounds. At the Suhl threshold power a single microwave spinwave mode becomes exponentially excited, being critically driven by the uniform precession mode. When the next mode is excited, these two microwave modes (frequency $f \approx 10^{10}$ Hz) can interact by a four-magnon scattering process to produce a low frequency collective oscillation at a frequency $f_{co} \approx 10^4$ to 10^6 Hz. It was predicted[5] and observed[6] that this collective oscillation would show a period-doubling cascade to chaos. When a third microwave mode is excited, it was found from more detailed experimental and theoretical studies[7,8,9] that a second collective oscillation frequency f'_{co} may arise, with the system displaying quasiperiodicity, frequency entrainment, and chaos. Although most spinwave systems studied have a very large number of accessible modes, they often display a rich low-dimensional behavior and are the subject of renewed experimental and theoretical interest.[10-20]

In this paper we describe some codimension-two experiments on a yttrium iron garnet (YIG) sphere in which we excite the Suhl first-order perpendicular pumped instability, the so-called "subsidiary absorption." The data are then compared to the numerical iteration of a theoretical model of coupled modes. Consider spins S_j on the crystal lattice of the ferrite sphere in an external magnetic field H_0, with Zeeman, exchange, and dipolar terms in the hamiltonian

$$\mathcal{H} = -\hbar\gamma\sum_j \vec{S}_j\cdot\vec{H}_o - 2J\sum_{j,j'}\vec{S}_j\cdot\vec{S}_{j'} + \Sigma H_d$$

where γ is the gyromagnetic ratio and J (J>0) the Heisenberg nearest neighbor exchange energy. The Zeeman interaction leads to a uniform precession of the crystal magnetization M about H_o at frequency $\omega_o \equiv \gamma H_o$, and to a narrow ferromagnetic resonance absorption at $\omega_p \approx \omega_o$ when driven by a small ac field $H_1 \sin(\omega_p t)$, perpendicular to H_o. The exchange term gives rise to spinwaves with the dispersion relation[21]

$$\omega_k^2 = (\gamma H_o - \frac{1}{3}\omega_m + \gamma Dk^2)(\gamma H_o - \frac{1}{3}\omega_m + \gamma Dk^2 + \omega_m \sin^2\theta_k)$$

for spinwaves of frequency ω_k and wave vector k in the direction θ_k relative to H_o; for our sample, $\gamma = 1.77\times10^7$ sec^{-1} G^{-1}; $D = 5.4\times10^{-9}$ Gcm^2; $\omega_m = 3.0\times10^{10}$ sec^{-1}. The instability described in this paper occurs when the dc field is set near $H_o \approx (\omega_p/2\gamma)$. At this field a pump photon $\omega_p \approx 2\omega_k$ can excite a uniform magnon which scatters by a three-magnon process into a pair of magnons (ω_k, k) and $(\omega_k, -k)$, i.e. a spinwave "mode," with maximum wave vector $k \approx 3\times10^5$ cm^{-1} at $\theta_k \approx 0$, as shown in Fig. 1(a). When the driving field H_1 exceeds the Suhl threshold value, the amplitude increases exponentially for the first mode to go unstable. The experiment is performed at room temperature with a sphere of pure single crystal YIG (diameter d = 0.066 cm, spherical to $\Delta d/d = 6\times10^{-5}$, smooth to within 0.15 μm), mounted in a resonator with the crystal axis [111] $\parallel H_o \perp H_1$, with incident microwave power P_{in} from a klystron oscillator ($f_p = \omega_p/2\pi = 9.2$ GHz) coupled via a waveguide and precision attenuator to the sphere; see Fig. 1(b). Power not absorbed is reflected to a diode detector, giving a dc signal S_o and also a video frequency signal S(t), 10 to 10^6 Hz, from which is obtained the power spectrum P(f).

Experimental results. The regions and boundaries of some of the observed behavior are shown in Fig. 2 in the parameter space of the dc field H_0 and the pumping power $P_{in} \propto H_1^2$. The Suhl threshold, shown as the solid line, is experimentally determined by an abrupt change in S_0, which is reversible except in the shaded area where it is abrupt and hysteretic, accompanied by a large increase (50 dB) in the video signal S(t), with no resolved spectral features in P(f). This wide-band "noise" is deterministic; the hysteresis can be understood from the model. In another region of parameter space ($1600 < H_0 < 2000$ G) the dc signal S_0 shows a series of sharp dips as H_0 is increased above the Suhl threshold [Fig. 3(a)], believed to be high-order spatial resonances of single spinwave modes within the sphere.[22] For a small change in wave vector $\Delta k = \pi/d$, the field change computed from the dispersion relation for $k \approx 3\times10^5$ cm^{-1} is $\Delta H_0 = 0.152$ G, compared to the observed spacing 0.157 G in Fig. 3(a). The first few dips in S_0 are not accompanied by a video signal S(t), an indication that only single microwave spinwave modes are excited (these are not detected by the video detector). However, as H_0 is further increased, a sinusoidal signal S(t) [e.g. Fig. 4(a)] may arise owing to the overlap of two microwave modes. This collective oscillation, at frequency $f_{co} = 10^4$ to 10^6 Hz, typically, is found to display period doubling [Fig. 4(b)] for very small changes in H_0, or in P_{in}, or in crystal orientation, or in microwave frequency f_p. A period-doubling cascade to chaos [Fig. 4(e)] can be observed. There is a marked dependence of the frequency f_{co} on pump power [e.g. Fig. 3(b)] owing to the dynamic interaction between modes. The data are fit by the expression $f^2_{co} = k[(P_{in}/P_c)-1]$ where P_c is the threshold value of the pump power P_{in} and k is a constant if all other parameters are held fixed. This frequency dependence is numerically predicted by our model, Fig. 3(c).

The phase diagram (Fig. 2) in the broad regions $1600 \lesssim H_0 \lesssim 2100$ G, $5 \lesssim P_{in} \lesssim 10$ dB, is filled with many small regions of size $\Delta H_0/H_0 \approx 10^{-5}$, $\Delta H_1/H_1 \approx 10^{-3}$ displaying single frequency collective oscillations, f_{co}, many of which do not display a period-doubling cascade to chaos before interruption by the appearance of a second incommensurate frequency f_{co}' which we associate with the excitation and overlap of a third microwave mode; close examination of Fig. 3(a) shows several series of weak peaks with different spacings. Figure 4(c) shows the resulting time series for two-frequency quasiperiodicity, resembling the amplitude of two coupled pendula. The power spectrum P(f) (not shown) shows sharp peaks at $nf_{co} + mf_{co}'$ with n,m integers. For very small changes in any parameter, these two collective oscillations will display frequency locking [e.g. Fig. 4(d)] and a quasiperiodic route to chaos. Two-frequency quasiperiodicity and chaos are abundantly found in this region of the phase diagram. We have no reproducible experimental evidence for three-frequency quasiperiodicity.

In other regions of the phase diagram [Fig. 2] we find additional types of behavior: (i) "relaxation oscillations" [Fig. 4(f)] characterized by a fast rise and slow decay, with power spectra having no resolved spectral peaks; these are sometimes seen as irregularly spaced spikes. (ii) This spiking behavior has a reasonably well defined onset to the region labelled "very noisy collective oscillations," characterized by a high level featureless wide-band power spectrum. (iii) "High amplitude oscillations" are observed at still higher pump power in a region near $H_0 \approx 1900$ G; these oscillations show periodic, quasiperiodic, and chaotic behavior, all superimposed on a high level broad-band background signal.

Theoretical model. Figure 1(b) shows schematically the elements of our theoretical model for the experimental system: a collection of coupled oscillators or modes.[7,8,9,23] The Hamiltonian includes the photon resonator mode (A), uniform magnon mode (B), and spinwave modes b_k with energies ω_p, ω_o, and ω_k, respectively. These oscillators are mutually coupled with coupling constants G between A and B, g_k between B and b_k, and four-magnon interactions $T_{kk'}$, $S_{kk'}$ among $\{b_k\}$. The driving field $(P_{in})^{1/2} \times \exp(-i\omega_p t)$ couples with A. From the Hamiltonian, we obtain the equations of motion for A, B and b_k, and add phenomenological damping terms with their constants Γ, γ_o, and γ_k, respectively. We transform to slow variables C_k via $b_k = C_k \exp(-i\omega_p t/2)$ and adiabatically eliminate A,B, assuming $\Gamma >> \gamma_k$. We assume $C_k = C_{-k}$ and arrive at a set of coupled equations for C_k:

$$\dot{C}_k = -(\gamma_k+i\Delta\Omega_k)C_k-iQg_k^*P_{in}{}^{1/2}C_k^*-i\sum_{k'}\{2T_{kk'}|C_{k'}|^2C_k+(S_{kk'}+Eg_{k'}g_k^*)C_{k'}^2C_k^*\} \tag{1}$$

where $\Delta\Omega_k \equiv (\omega_k-\omega_p/2) = 2\pi\Delta f_k$ is the detuning parameter, and parameters Q and E are functions of G, ω_o, ω_p, Γ, and γ_o.

The fixed points of Eq. (1) may be determined exactly if only one mode is excited. The equation $\dot{C}_k = 0$ may be put in the form of a point on a unit circle, $M + N|C_k|^2 = (C_k^*)^2/|C_k|^2$, where $M = i(\gamma_k + i\Delta\Omega_k)/QP_{in}{}^{1/2}g_k^*$, and $N = -(2T_{kk} + S_{kk} + E|g_k|^2)/QP_{in}{}^{1/2}g_k^*$. The Suhl threshold occurs at $|M| = 1$. For $P_{in} > P_t$ (P_t = threshold power) the stability of the trivial fixed point (C_k = 0) is lost in a symmetry-breaking bifurcation. This occurs in two forms: (1) If Re(M/N) > 0, one obtains a supercritical bifurcation in which stable nonzero fixed points emerge from the origin as P_{in} crosses P_t. (2) For Re(M/N) < 0 a subcritical bifurcation occurs in which stable nonzero fixed points appear below P_t, and the system will jump to these at $P_{in} = P_t$, resulting in hysteretic behavior. The experimentally observed hysteresis is probably a related effect involving the cooperation of neighboring modes.

Equation (1) corresponds to a set of k = 1 to n coupled damped driven nonlinear oscillators, each characterized by a detuning parameter $2\pi\Delta f_k$ and by other physical parameters that may be reasonably well chosen from knowledge of the YIG ferrite material and its resonance and relaxation properties. To explore the behavior of Eq. (1) we perform a numerical iteration[24] for n = 1, then n = 2, etc. For n = 1, the system is always attracted to a fixed point, but a hysteresis may be displayed as noted above. For n = 2, periodic oscillations are found [Figs. 5(a) and 5(b)]: Mode 2 exhibits an asymmetric orbit while mode 1 exhibits a symmetrical orbit at twice the period. We simulate the spatial modes of Fig. 3(b) by choosing detuning parameters $\Delta f_1 = f_s - 500$ kHz [corresponding to the 0.15 G splitting of Fig. 3(a)] and $\Delta f_2 = f_s$, and shift f_s to simulate the dc field shift. The computed behavior [Figs. 5(c) and 5(d)] shows period doubling and symmetry breaking, respectively, and eventually chaotic behavior [Figs. 5(e) and 5(f)] for both modes.

For n = 3 modes, new behavior arises: Fig. 5(g) shows quasiperiodic behavior with a smooth Poincaré section of a torus; at higher excitation the section [Fig. 5(h)] is a chaotic attractor. For some other parameter values the behavior shows chaotic bursts [Fig. 5(i)] and other forms of aperiodic behavior similar to that observed [Fig. 4(f)].

In summary, the model is found to moderately explain the experimentally observed phenomena, in particular: The hysteresis at the Suhl threshold; onset of collective oscillations, period doubling and chaos; quasiperiodicity, locking and chaos; aperiodic spiking; and dependence of collective oscillation frequency on pump power. Taken together, the high resolution experimental data and the numerical iteration calculations give a reasonably good picture of spinwave dynamics in YIG spheres in the chaotic regime.

This work was supported in part by the U.S. Department of Energy under Contract No. DE-AC03-76SF00098; and by the Office of Naval Research under

Contract No. N00014-86-K-0154.

References

(a) Present address: Institute for Pure and Applied Physical Sciences, University of California, San Diego, La Jolla, CA 92093.

(b) Permanent address. Present address: James Franck Institute, The University of Chicago, Chicago, IL 60637.

1. R. W. Damon, Rev. Mod. Phys. 25, 239 (1953).
2. N. Bloembergen and S. Wang, Phys. Rev. 93, 72 (1954).
3. H. Suhl, J. Phys. Chem. Solids 1, 209 (1957).
4. D. Ruelle and F. Takens, Commun. Math. Phys. 20, 167 (1971); 23, 343 (1971).
5. K. Nakamura, S. Ohta, and K. Kawasaki, J. Phys. C 15, L143 (1982); K. Nakamura, S. Ohta, and K. Kawasaki, J. Phys. Soc. Japan 52, 147 (1983).
6. G. Gibson and C. Jeffries, Phys. Rev. A 29, 811 (1984).
7. P. Bryant, C. Jeffries, and K. Nakamura, Nucl. Phys. B (Proc. Suppl.) 2, 25 (1987).
8. P. Bryant, C. Jeffries, and K. Nakamura, Phys. Rev. Lett. 60, 1185 (1988).
9. P. Bryant, C. Jeffries, and K. Nakamura, submitted to Phys. Rev. A (1988).
10. S. Ohta and K. Nakamura, J. Phys. C 16, L605 (1983).
11. X. Y. Zhang and H. Suhl, Phys. Rev. A 32, 2530 (1985); H. Suhl and X. Y. Zhang, Phys. Rev. Lett. 57, 1480 (1986).
12. S. M. Rezende, O. F. de Alcontara Bonfin, and F. M. de Aguiar, Phys. Rev. B 33, 5153 (1986).
13. X. Y. Zhang and H. Suhl, submitted to Phys. Rev. B.
14. F. Waldner, D. R. Barberis, and H. Yamazaki, Phys. Rev. A 31, 420 (1985).
15. F. M. de Aguiar and S. M. Rezende, Phys. Rev. Lett. 50, 1070 (1986).

16. M. Mino and H. Yamazaki, J. Phys. Soc. Japan 55, 4168 (1986); H. Yamazaki and M. Warden, J. Phys. Soc. Japan 55, 4477 (1986).

17. T. L. Carrol, L. M. Pecora, and F. J. Ratchford, Phys. Rev. Lett. 59, 2891 (1987).

18. F. Waldner, R. Badii, D. R. Barberis, G. Broggi, W. Floeder, P. F. Meier, R. Stoop, M. Warden, and H. Yamazaki, J. Magn. Magn. Mat. 54-57, 1135 (1986).

19. F. M. de Aguiar, A. Azevedo, and S. M. Rezende, submitted to Phys. Rev. B.

20. S. P. Lim and D. L. Huber, Phys. Rev. B 37, 5426 (1988).

21. A. M. Clogston, H. Suhl, L. R. Walker, and P. W. Anderson, J. Phys. Chem. Solids 1, 129 (1956).

22. W. Jantz and J. Schneider, Phys. Stat. Solidi (a) 31, 595 (1975).

23. Paul Henry Bryant, "Studies of Nonlinear and Chaotic Phenomena in Solid State Systems" (Ph.D. thesis), University Microfilms International, Ann Arbor, Michigan (1987).

24. Typical parameter values used: P_{in} = 0.015 W; γ_k = 1×10^{6} s^{-1}; iQg_k = 1.414×10^{7} $W^{-1/2}$ s^{-1}; $S_{kk'}$ = S_{kk} = 4.3×10^{-8} s^{-1}; $T_{kk'}$ = -2×10^{-8} s^{-1}; T_{kk} = 0; E = 0.

FIGURE CAPTIONS

Fig. 1 - (a) Spinwave dispersion diagram, ω_k vs. k, computed for YIG sphere with H_0 = 1700 G. The uniform mode excited by microwave perpendicular pumping at ω_p can excite a spinwave pair (k,-k) travelling in the azimuthal direction θ. The first-order Suhl instability occurs when the population of this spinwave mode exponentially increases. (b) Schematic diagram of the experimental system, together with the elements of the corresponding theoretical model.

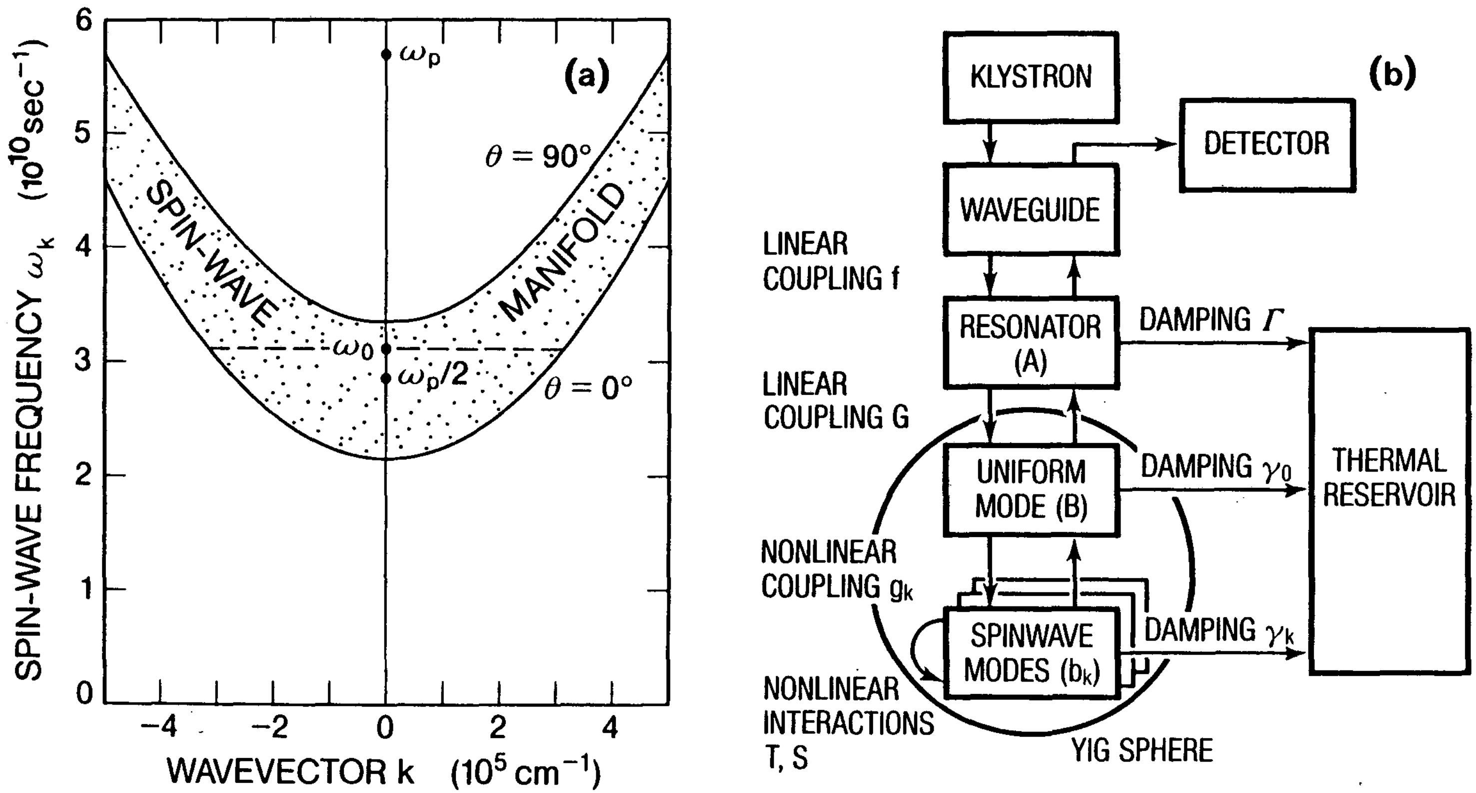
Fig. 2 - Regions and boundaries of types of experimentally observed behavior in the perpendicular pumped spinwave instability in a YIG sphere; dc field H_0 vs. microwave pump power $P_i \propto H_1^2$. P_{in} = 20 dB corresponds to H_1 = 5 G.

Fig. 3 - (a) Single spinwave modes from the region indicated in Fig. 2 in a YIG sphere for microwave pumping at f_p = 9.2 GHz. The modes are spaced by $\Delta H_0 \approx 0.157$ G. (b) Square of observed collective oscillation frequency f_{co} vs. microwave pump power P_{in} relative to threshold value P_c. The solid line is a fit to the data. (c) 10 × f_{co}^2 vs. P_{in}/P_c computed from model; solid line is a fit to the computed points.

Fig. 4 - Observed ac signals S(t) in spinwave instability showing (a) periodic oscillation at 16 kHz; (b) period doubled; (c) quasiperiodic; (d) frequency locking; (e) chaotic; (f) aperiodic relaxation oscillation.

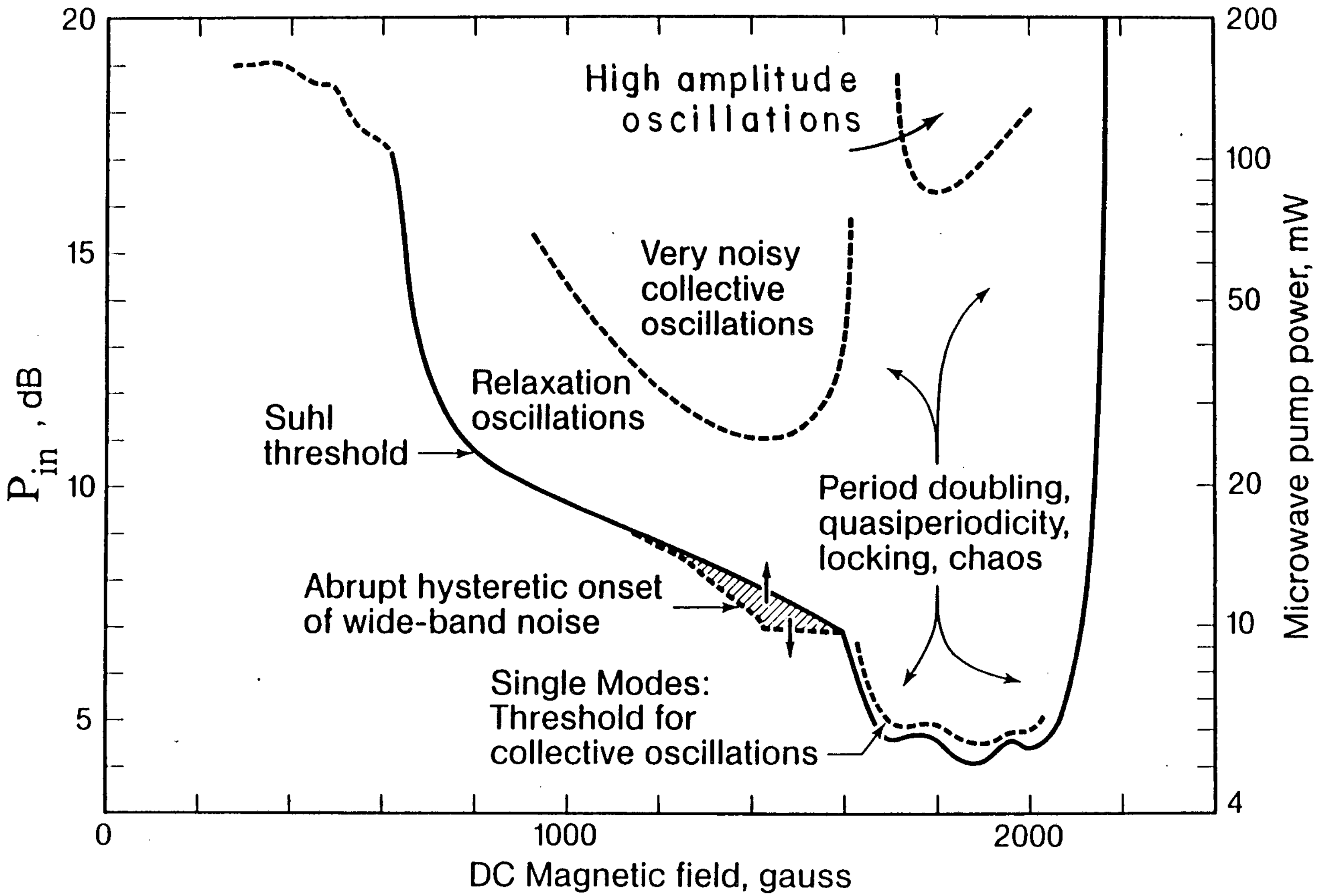
Fig. 5 - (a) Computed behavior for two modes: phase portrait for periodic oscillations, asymmetric mode; Δf_1 = -300 kHz, Δf_2 = 200 kHz. (b) Symmetric mode. (c) Period doubling of asymmetric mode; Δf_1 = -385 kHz, Δf_1 = 115 kHz. (d) Symmetry breaking of symmetric mode. (e) Chaotic orbit following period doubling cascade; Δf_1 = -410 kHz, Δf_2

= 90 kHz. (f) Power spectrum of chaotic orbit, f_{max} = 2.5 MHz. (g) Computed phase portrait for quasiperiodic behavior for three modes, with Poincaré section. (h) Poincaré section of chaotic orbit; proximity to period-5 locking produces the five points. (i) Chaotic bursts.



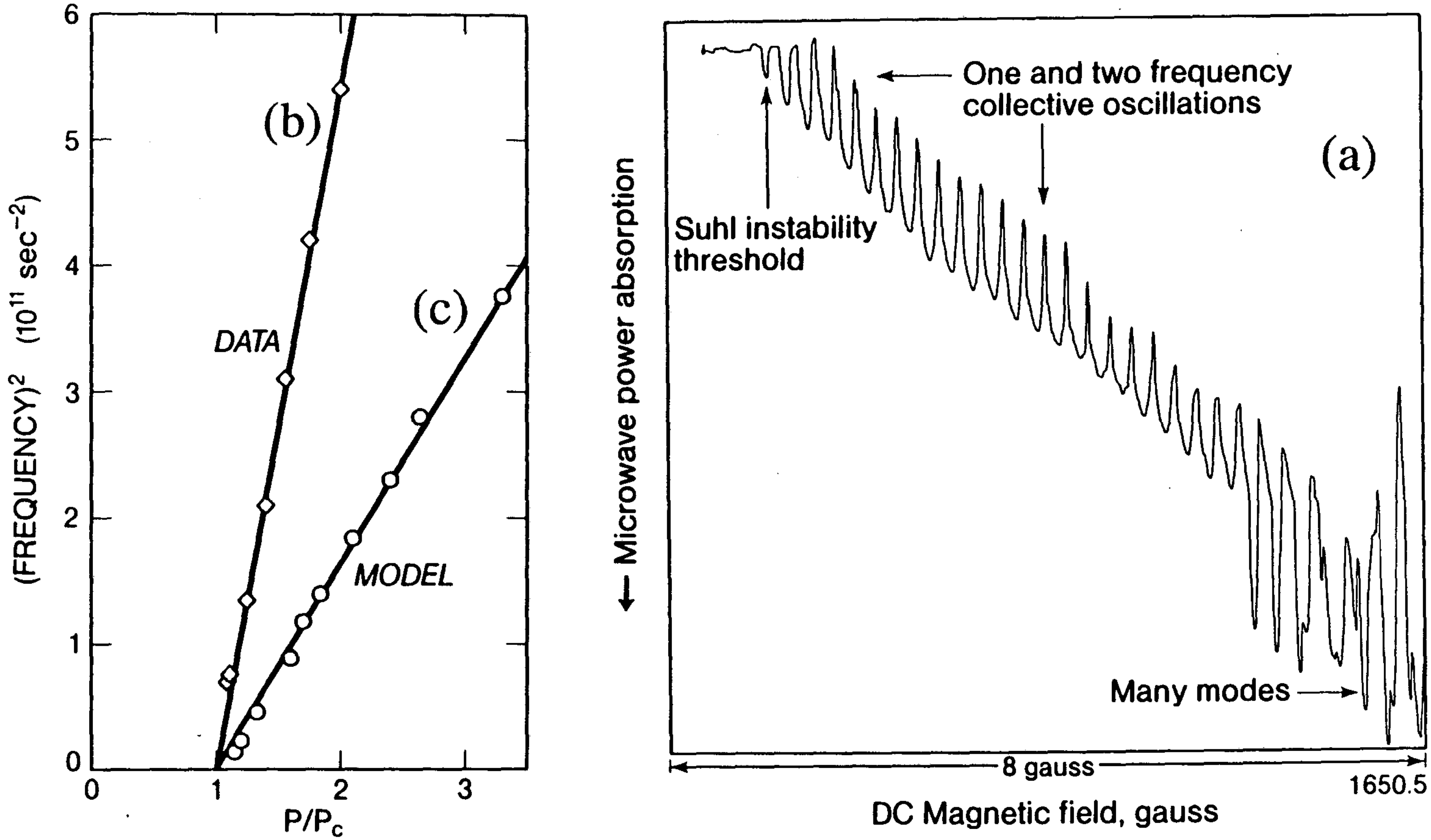
XBL 873-653

Fig. 1



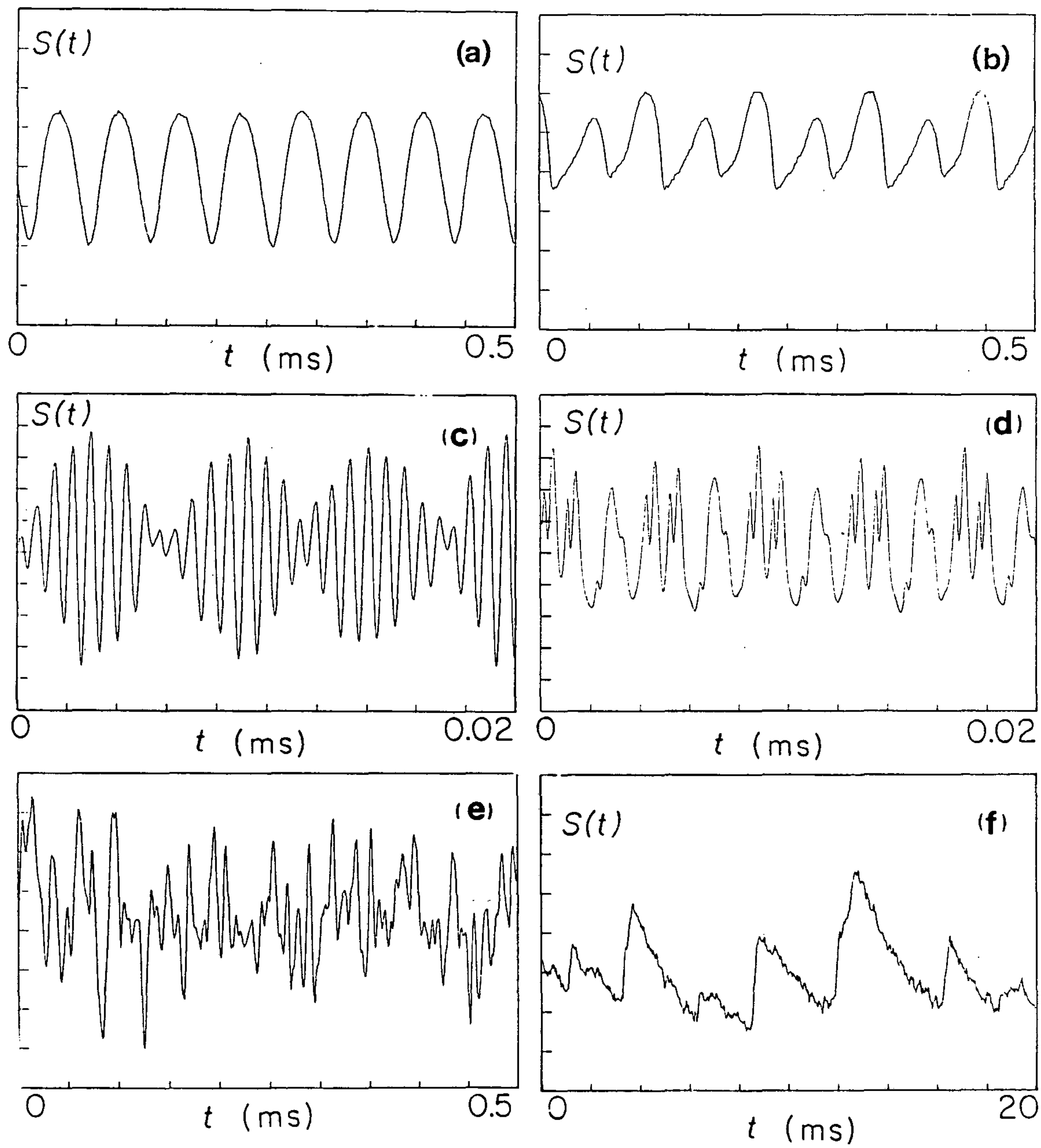
XBL 873-652 A

Fig. 2



XBL 872-649 A

Fig. 3



XBL 881-228

Fig. 4

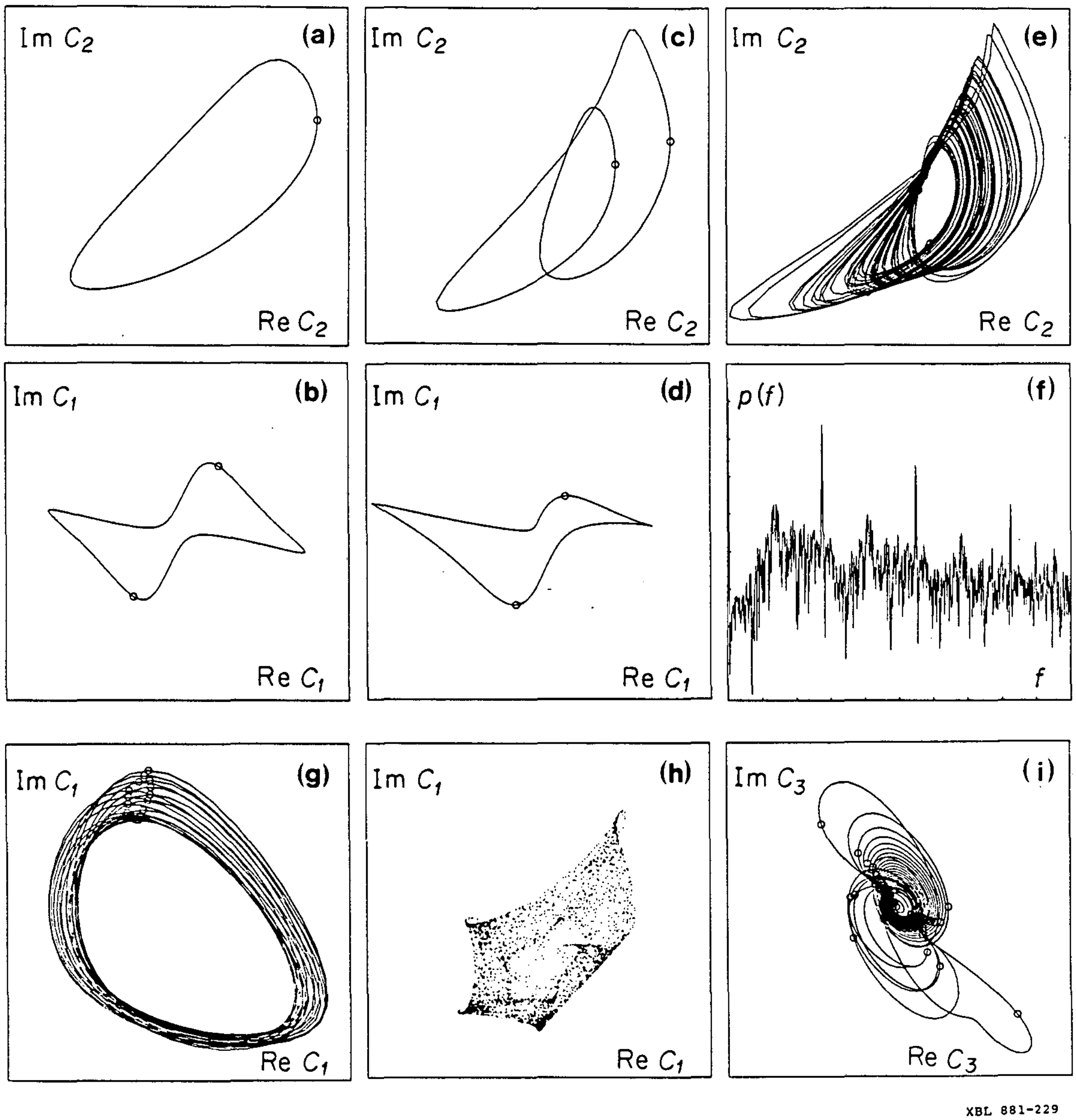

Im C_2
(a)
Re C_2
Im C_2
(c)
Re C_2
Im C_2
(e)
Re C_2
Im C_1
(b)
Re C_1
Im C_1
(d)
Re C_1
$p(f)$
(f)
f
Im C_1
(g)
Re C_1
Im C_1
(h)
Re C_1
Im C_3
(i)
Re C_3
XBL 881-229

Fig. 5

LAWRENCE BERKELEY LABORATORY
TECHNICAL INFORMATION DEPARTMENT
UNIVERSITY OF CALIFORNIA
BERKELEY, CALIFORNIA 94720