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Academic literacy in mathematics for English Learners



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ABSTRACT

This paper uses a sociocultural conceptual framework to provide an integrated view of academic literacy in mathematics for English Learners. The proposed definition of academic literacy in mathematics includes three integrated components: mathematical proficiency, mathematical practices, and mathematical discourse. The paper uses an analysis of a classroom discussion to illustrate how the three components of academic literacy in mathematics are intertwined, how academic literacy in mathematics is situated, and how participants engaged in academic literacy in mathematics use hybrid resources. The paper closes by describing the implications of this integrated view of academic literacy in mathematics for mathematics instruction for English Learners, arguing that it is important that the three components not be separated when designing instruction in general, and it is essential that mathematics instruction for English Learners address these three components simultaneously.

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This paper uses a sociocultural conceptual framework to provide an integrated view of academic literacy in mathematics for English Learners.¹ The proposed definition of academic literacy in mathematics includes three integrated components: mathematical proficiency, mathematical practices, and mathematical discourse. The paper uses an analysis of a classroom discussion and questions adapted from Gee's (1999) questions for Discourse analysis, to illustrate how the three components of academic literacy in mathematics are intertwined, how academic literacy in mathematics is situated, and how participants engaged in academic literacy in mathematics use hybrid resources. The paper closes by describing the implications of this integrated view of academic literacy in mathematics for mathematics instruction, arguing that although these three components are important for all mathematics learners, it is essential that mathematics instruction for ELs include and maintain a simultaneous focus on all three components.

The view of academic literacy in mathematics presented here is different than previous approaches to academic language in several ways. First, the definition includes not only cognitive aspects of mathematical activity—such as mathematical reasoning, thinking, concepts, and metacognition—but also sociocultural aspects—participation in mathematical practices—and discursive aspects—participation in mathematical discourse. A sociocultural perspective of academic literacy in mathematics provides a complex view of mathematical proficiency as participation in discipline-based practices that involve conceptual understanding and mathematical discourse.² Most importantly for ELs, this integrated view, rather than separating

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¹ Although there are many labels used to refer to students who are learning English, I will use the term English Learners, abbreviated as ELs. Research on ELs may also be relevant to bilingual or multilingual mathematics learners, and to students learning a language of instruction other than English.

² The sociocultural perspective of academic literacy in mathematics described here builds on previous work that appeared in several publications where I described a sociocultural view of mathematics learners who are bilingual and/or learning English (Moschkovich, 2002a, 2007b), of mathematical discourse (Moschkovich, 2007c), and of mathematical practices (Moschkovich, 2013a, 2013b). In other publications (Moschkovich, 2008, 2009) I described

academic language from mathematical proficiency or practices, views the three components as working in unison. Separating language from mathematical thinking and practices can have dire consequences for English Learners. First, such a separation can make ELs seem more deficient than they might actually be, since they may not be able to express their mathematical ideas through language, but may still be engaged in correct mathematical thinking and participate in mathematical practices that are less language intensive, for example using objects or drawings to show a result, finding regularity in data, or using gestures to illustrate a mathematical concept.

The sociocultural perspective used here expands academic literacy in mathematics beyond simplified views of language as words. Simplified views of academic language focus on words, assume that meanings are static and given by definitions, separate language from mathematical knowledge and practices, and limit mathematical discourse to formal language. In contrast, the view of academic literacy in mathematics proposed here sees meanings for academic mathematical language as socioculturally situated in mathematical practices and the classroom setting. A complex view of mathematical discourse also means that mathematical discourse draws on hybrid resources and involves not only oral and written text, but also multiple modes, representations (gestures, objects, drawings, tables, graphs, symbols, etc.), and registers (school mathematical language, home languages and the everyday register).

One might assume that English Learners cannot participate in academic literacy in mathematics as defined above because they do not know mathematical vocabulary or they need to learn English first. However, research has documented that ELs can, in fact, participate in mathematical discussions as they are learning English. Research shows that English Learners, even as they are learning English, can participate in discussions where they grapple with important mathematical content³ and participate in mathematical practices. Instruction for this population should not emphasize low-level language skills over opportunities to actively communicate about mathematical ideas. One of the goals of mathematics instruction for students who are learning English should be to support all students, regardless of their proficiency in English, in participating in discussions that focus on important mathematical concepts and engage students in mathematical practices, rather than on low-level linguistic skills. By learning to recognize how English Learners actively use academic literacy in mathematics, teachers can provide opportunities for ELs to participate in all three components of academic literacy in mathematics in integrated ways.

If we want students who are learning English to participate in academic literacy in mathematics as defined here, then we first need to use mathematical tasks that will provide opportunities for students to engage in the full spectrum of mathematical proficiency, in mathematical practices, and in mathematical discourse. For students to participate in academic literacy in mathematics, we need to select tasks that require more than using numbers, computation, or symbol manipulation and organize classroom instruction so that students actively use mathematical concepts and show their conceptual understanding through explaining and justifying.

If students are participating in academic literacy in mathematics as defined here, then we see or hear them engaged in the full spectrum of mathematical proficiency, as they participate in mathematical practices, many of which are discursive. If students are participating in academic literacy in mathematics, we see or hear them actively using concepts and showing their conceptual understanding through explaining and justifying. Since mathematical discourse is multimodal and multi-semiotic (O'Halloran, 1999), opportunities for academic literacy in mathematics include multiple modes of communication, sign systems, and types of inscriptions.

The sociocultural theoretical framework draws on situated perspectives of learning mathematics (Brown, Collins, & Duguid, 1989; Greeno, 1998) as a discursive activity (Forman, 1996) that involves participating in a community of practice (Forman, 1996; Lave & Wenger, 1991; Nasir, 2002), developing classroom socio-mathematical norms (Cobb, Wood, & Yackel, 1993), and using multiple material, linguistic, and social resources (Greeno, 1998). Mathematical activity thus involves not only mathematical knowledge, but also mathematical practices and discourse.

Beyond the assumption that mathematical activity is simultaneously cognitive, social, and cultural, a sociocultural perspective brings two other assumptions to a definition of academic literacy in mathematics. First, the focus is on the potential for progress in what learners say and do, not on learner deficiencies or misconceptions. Second, participants bring multiple perspectives to a situation, representations and utterances have multiple meanings for participants, meanings for words are situated and constructed while participating in practices, and multiple meanings are negotiated through interaction.

Shifting from a simplified view of academic language as words to a view of academic literacy in mathematics that integrates mathematical proficiency and practices is crucial for the education of ELs. Research and policy have repeatedly, clearly, and strongly called for mathematics instruction for this student population to maintain high standards (American Educational Research Association, 2004) and high-cognitive demand (AERA, 2006). In order to accomplish these goals, mathematics instruction for ELs needs to move beyond defining academic literacy in mathematics as low-level language skills (i.e. vocabulary) or mathematical skills (i.e. arithmetic computation) and use an expanded definition of academic literacy in mathematics to describe and prescribe instruction that supports academic literacy in mathematics.

how mathematical discourse is situated, involves coordinated utterances and focus of attention, and combines everyday and academic registers (also in Moschkovich, 2011). The definition of academic literacy in mathematics used here brings together and builds on different aspects of those analyses.

³ For examples of lessons where English Learners participate in a mathematical discussions see Moschkovich (1999, 2008) and Khisty (1995).

Such instruction (a) includes the full spectrum of mathematical proficiency, balancing computational fluency with high-cognitive-demand tasks that require conceptual understanding and reasoning; (b) provides opportunities for students to participate in mathematical practices; (c) allows students to use multiple modes of communication, symbol systems, registers, and languages as resources for mathematical reasoning, and (d) supports students in negotiating situated meanings for mathematical language that is grounded in mathematical activity, instead of giving students definitions divorced from mathematical activity. The first step in making these shifts in instruction is to develop the expanded view of academic literacy in mathematics as proposed here.

The paper is organized into three main sections. The first section provides summary descriptions of each component of academic literacy in mathematics. The second section provides an analysis of a transcript of a classroom discussion, using questions adapted from Gee's (1999) questions for Discourse analysis. The analysis shows how the three components of academic literacy in mathematics cannot be separated when analyzing tasks, how the three components function together during student activity, and how academic literacy in mathematics is both situated and hybrid. The analysis of the transcript illustrates the situated nature of academic literacy in mathematics, showing that learners negotiate situated meanings for words and phrases that are grounded in the local sociocultural setting and coordinated with ways of viewing inscriptions. The analysis also illustrates how academic literacy in mathematics is hybrid (Gutiérrez, Baquedano-López, & Tejada, 1999; Gutiérrez, Baquedano-Lopez, & Alvarez, 2001), in the sense that learners draw on multiple resources—modes of communication, symbol systems, as well as both everyday and academic registers. In the third and final section I consider the implications of the proposed definition of academic literacy in mathematics for instruction for ELs.

1. Academic literacy in mathematics

I chose to use the phrase “academic literacy in mathematics” instead of “academic language in mathematics,” to shift from a focus on language as words to a broader sense of *literacy* as participation in practices and discourses. These discursive practices involve multiple aspects of mathematical proficiency, multiple symbol systems (written text, numbers, graphs, tables, etc.), and multiple modes of communication (oral, written, receptive, productive). The motivation for using a new term, academic literacy in mathematics, is to distinguish this proposed approach from two other conceptualizations of academic English, generic approaches that do not take into account the specificity of literacy *in mathematics* and narrow views of academic English that reduce *literacy* in mathematics to its components (lexical, syntactic, or semantic features). A recent review of the literature on teaching academic English (AE) to English Learners (DiCerbo, Anstrom, Baker, & Rivera, 2014) provides support for the need to make such a distinction. The review concluded “there are differences in AE across content areas that must be accounted for in instruction” and that “specific language demands are unique depending on the task and discipline.” The new term makes it clear that the focus is mathematics. That review also concluded “Instruction focused only on an individual language feature, such as academic vocabulary, does not fully address the challenges of AE acquisition.” Using the term literacy, instead of language, is an attempt to move the focus away from any individual language feature and shift to broader literacy practices. The proposed new term, however, does not define literacy in conventional ways.

Conventional interpretations of literacy refer to proficiency in reading and writing. We could imagine that when solving a word problem, *literacy* is necessary for reading and understanding the words and *mathematical proficiency* is necessary for extracting the numbers and relating them through arithmetic operations. In this article, however, I use a different definition of literacy informed by work in sociolinguistics (Gee, 1996, 1999) and New Literacy Studies (Lea & Street, 2006; Street, 2005a, 2005b). New Literacy Studies distinguish between reductionist views of language and a view of language as social practice. This broader view of literacy practices assumes that these practices are processes, not fixed entities, and a set of resources or repertoires, not a set of rules (Street, 2005a, 2005b). This perspective adds two important dimensions to the meaning of *academic literacy* so that (1) it includes using the vernacular even when engaging in academic literacy practices and (2) draws on a full communicative and multimodal repertoire—not only written text but also other inscriptions, oral communication, gestures, and objects. Using the phrase academic literacy, rather than academic language, also draws on Gee's view of Discourses as involving not only written and oral language, but also other modalities such as images, equations, symbols, sounds, gestures, graphs, and artifacts (Gee, 1996, 1999).

In the example analyzed in this article, two students and a teacher are involved in an oral discussion of graphs. This interaction could be seen as an example of literacy in the conventional reading/writing sense because the discussion was based on how these two students had read and understood two kinds of inscriptions, the English text setting up the situation for the mathematics problem (typically called the word problem) and the graph that each student had produced. Thus the example is about literacy (even in the conventional sense) because it involves reading in two modalities, alphabetic reading and reading a graph. Since the example is an oral discussion, it could be seen as primarily involving the oral mode and thus not literacy, in the conventional sense of reading or writing. However, although the discussion is oral, the interaction is not conversational, and the participants use discourse patterns typical of mathematical explanations and arguments. Therefore, this interaction is an example of a literacy practice in school mathematics (as defined by the New Literacy Studies perspective) and of participation in mathematical Discourse (as defined by Gee).

In the next section, I provide a summary of each of the three components of academic literacy in mathematics: mathematical proficiency, mathematical practices, and mathematical discourse. These descriptions will ground the subsequent analysis of academic literacy in mathematics in a classroom discussion.

1.1. Mathematical proficiency

A current description of mathematical proficiency (Kilpatrick et al., 2001a, 2001b) shows five intertwined strands:

1. Conceptual understanding, or comprehension of mathematical concepts, operations, and relations;
2. Procedural fluency, or skill in carrying out procedures flexibly, accurately, efficiently, and appropriately;
3. Strategic competence, or competence in formulating, representing, and solving mathematical problems (novel problems, not routine exercises);
4. Adaptive reasoning, or logical thought, reflection, explanation, and justification; and
5. Productive disposition, a habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy.

Although mathematical proficiency is often reduced to procedural fluency in arithmetic, this is only one component of mathematical proficiency. It is crucial that we define academic literacy in mathematics for ELs as more than computation or symbol manipulation. Conceptual understanding, strategic competence, and reasoning are as, if not more, important than fluent arithmetic computation, for example for applying mathematics and knowing what computation to use when (Hiebert & Carpenter, 1992).

Conceptual understanding is fundamentally about the meanings that learners construct for mathematical solutions: knowing the meaning of a result (what the number, solution, or result represents), knowing why a procedure works, and/or explaining why a particular result is the right answer. Other aspects of conceptual understanding are connecting procedures to concepts and connecting procedures to multiple representations such as words, drawings, symbols, diagrams, tables, graphs, or equations (Hiebert & Carpenter, 1992). Reasoning, logical thought, explanation, and justification are closely related to conceptual understanding. Student reasoning is evidence of conceptual understanding when a student explains why a particular result is the right answer or justifies a conclusion. For example, if students understand addition and multiplication, they have made connections between these two procedures, they can represent them in multiple ways, and they can explain how multiplication and addition are related, for example, saying that whole number multiplication can be described as repeated addition.

ELs need to develop conceptual understanding along with procedural skills, in part, because conceptual understanding and procedural fluency are closely related. Research in cognitive science (Bransford, Brown, & Cocking, 1999) has shown that people remember procedures better, longer, and in more detail if they understand, actively organize, elaborate, and connect new knowledge to prior knowledge. In mathematics this means that in order to remember how to carry out computations, ELs will need to understand, elaborate, and organize procedures.

An emphasis on conceptual understanding, reasoning, and high-cognitive demand is motivated not only by equity considerations for ELs, but also by empirical research on effective mathematics teaching. Mathematics instruction for ELs should provide these students access to high-quality and effective mathematics instruction. According to a review of the research (Hiebert & Grouws, 2007), mathematics teaching that impacts student achievement and conceptual development provides students opportunities to attend explicitly to concepts and time to wrestle with important mathematics. Mathematics instruction for ELs should follow these recommendations for effective mathematics teaching by focusing on conceptual understanding (Hiebert & Grouws, 2007) and maintaining high-cognitive-demand for mathematical tasks (AERA, 2006; Stein, Grover, & Henningsen, 1996), for example, by encouraging students to explain, represent, and discuss their reasoning. By extension, instruction that supports academic literacy in mathematics should emphasize these aspects of mathematical proficiency.

1.2. Mathematical practices

The five strands of mathematical proficiency provide a cognitive account of mathematical activity focused on knowledge, metacognition, and beliefs. From a sociocultural perspective, mathematics students are not only acquiring mathematical knowledge, they are also learning to participate in valued mathematical practices (Moschkovich, 2004, 2007c, 2013a, 2013b). Some of these practices include problem solving, sense-making, reasoning, modeling, and looking for patterns, structure, or regularity.

For this discussion of mathematical practices I draw principally on three sources: my own work on mathematical practices (Moschkovich, 2004, 2007c, 2013a, 2013b), the NCTM Standards, and recent standards for mathematical practices produced by the Common Core State Standards (CCSS, 2010a, 2010b). Although many researchers have used the concept of mathematical practices, it is not my goal here to review different uses of the construct of mathematical practices. Cobb, Stephan, McClain, and Gravemeijer (2001) define mathematical practices as the “taken-as-shared ways of reasoning, arguing, and symbolizing established while discussing particular mathematical ideas” (Cobb et al., p. 126). For Cobb et al. mathematical practices, in contrast to social norms and socio-mathematical norms, are specific to particular mathematical ideas. Here I

will assume that some mathematical practices are general (see CCSS Standards for examples of general practices) and others are specific to a particular topic or concept.

Using the term *practice*⁴ shifts from purely cognitive accounts of mathematical activity to accounts that assume its social, cultural, and discursive nature. I use the terms *practice* and *practices* in the sense used by Scribner (1984) for a practice account of literacy to “. . . highlight the culturally organized nature of significant literacy activities and their conceptual kinship to other culturally organized activities involving different technologies and symbol systems . . . (p. 13).” This definition when applied to mathematical practices implies that these are culturally organized, involve symbols systems, and are related conceptually to other mathematical practices. From this perspective, mathematical practices are not only cognitive—i.e. involve mathematical thinking and reasoning as described in the four strands of mathematical proficiency—but also social and cultural—they arise from communities and mark membership in communities—and semiotic—they involve semiotic systems (signs, tools, and their meanings).

Academic mathematical practices can be understood in general as using language and other symbols systems to think, talk, and participate in the practices that are “the objective of school learning.” There is no single set of mathematical practices or one mathematical community (For a discussion of multiple mathematical practices, see Moschkovich, 2002b). Mathematical activity can involve different communities (mathematicians, teachers, or students) and different genres (explanations, proofs, or presentations). Practices vary across communities of research mathematicians, traditional classrooms, and reformed classrooms. However, across these various communities and genres, there are common practices that can be labeled as academic mathematical practices.

Two sources for descriptions of mathematical practices are the National Council of Teachers of Mathematics (NCTM) Standards (NCTM, 1989, 1991, 1995) and the Common Core State Standards (CCSS) for Mathematical Practices (CCSS, 2010a, 2010b). The CCSS standards for mathematical practice overlap in important ways with the NCTM standards and the National Research Council definition of mathematical proficiency. Although the NCTM Standards did not explicitly label these as practices, they focused on problem solving, sense-making, reasoning, modeling, and looking for patterns, structure, or regularity (Koestler, Felton, Bieda, & Otten, 2013). The Common Core State Standards recently introduced the following list of eight “Standards for Mathematical Practice” (for more details see <http://www.ccsstoolbox.com/>):

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

The Common Core documents state that these standards⁵ “describe varieties of expertise that mathematic educators at all levels should develop in their students” and connect these standards to other “processes and proficiencies” such as problem solving, reasoning and proof, communication, and connections (NCTM Standards, 1989, 1991), as well as to the five strands of mathematical proficiency (conceptual understanding, procedural fluency, adaptive reasoning, strategic competence, and productive disposition).

A sociocultural framing of mathematical practices has implications for connecting practices to discourse. In particular, discourse is central to participation in many mathematical practices, and the meanings for words are situated and constructed while participating in mathematical practices.⁶

1.3. Mathematical discourse

I use the phrase mathematical discourse, rather than mathematical language, to refer to the *communicative competence* (Hymes, 1972) necessary and sufficient for competent participation in mathematical practices. The phrases

⁴ In using the terms *practice* and *practices* in the sense used by Scribner (1984), I make a distinction between the concept of *practices* and other common uses, for example practice as repetition or rehearsal, or practice as in “my teaching practice.”

⁵ This is a list of standards, not practices (I thank my colleague Patricio Herbst for bringing my attention to this distinction) that provide a start in the direction of describing standards for mathematical practices in classrooms. However, these standards should not be interpreted as providing a definition, an exhaustive list, or a description of all the important mathematical practices that students need to develop mathematical proficiency. These standards (like any set of standards) are open to multiple interpretations, depending on theoretical perspectives used to frame mathematical activity. For example, although the word practices can signal a sociocultural framing, without such a perspective it is still possible to interpret these practices as purely cognitive.

⁶ For the sake of brevity, I am putting aside the relationship between mathematical practices and mathematical discourse, including questions regarding whether all mathematical practices are discursive, whether some are more discursive than others, and so on. These complex issues are discussed elsewhere (Moschkovich, 2013a, 2013b).

mathematical language or academic language can have multiple meanings. Many interpretations of the phrase *academic language* in recommendations for teaching mathematics reduce its meaning to vocabulary or grammar (for example, see Cavanagh, 2005). Work on the language of disciplines (e.g., Pimm, 1987; Schleppegrell, 2007, 2010) provides a more complex view of mathematical language as not only specialized vocabulary—new words and new meanings for familiar words—but also as extended discourse that includes other symbolic systems as well as artifacts (Moschkovich, 2002a, 2002b, 2013b), syntax and organization (Crowhurst, 1994), the mathematics register (Halliday, 1978), and discourse practices (Moschkovich, 2007c). (For a sociocultural approach and description of key features of mathematical discourse, see Moschkovich, 2007c).

Mathematical discourse is more than language (Moschkovich, 2007c)—it involves other symbolic systems as well as artifacts, discourse is embedded in mathematical practices, and meanings are situated and develop through participation in mathematical practices. Academic mathematical discourse has been described as having some general characteristics. In general, particular modes of argument, such as precision, brevity, and logical coherence, are valued (Forman, 1996). Abstracting, generalizing, and searching for certainty are also highly valued. Generalizing is reflected in common mathematical statements, such as “The angles of any triangle add up to 180 degrees,” “Parallel lines never meet,” or “ $a + b$ (always) equals $b + a$.” What makes a claim mathematical is, in part, the attention paid to describing in detail when the claim applies and when it does not. Mathematical claims apply only to a precisely and explicitly defined set of situations. Mathematical claims are often tied to mathematical representations (symbols, graphs, tables, or diagrams).

Academic mathematical discourse is not principally about formal or technical vocabulary (Moschkovich, 2007c). The mathematics register is a complex construct that includes styles of meaning, modes of argument, and mathematical practices and has several dimensions such as the concepts involved, how mathematical discourse positions students, and how mathematics texts are organized. When describing mathematical discourse we should not confuse “mathematical” with “formal” or “textbook.” Textbook definitions and formal ways of talking are only one aspect of school mathematical discourse. In classrooms students use multiple resources, including everyday registers and experiences, to make sense of mathematics. It also important to avoid construing everyday and academic registers as opposites (Moschkovich, 2010).

A shift to a more complex view of mathematical discourse is important for all mathematics students, but it is essential for defining academic literacy in mathematics for ELs. A simplified view of mathematical discourse can lead to the assumption that precision lies primarily in individual word meaning. This (misguided) assumption can have dire consequences for ELs, as they are likely to use imperfect language to describe their mathematical thinking (for analyses of accurate mathematical thinking expressed through imperfect utterances see Moschkovich (2002a, 2012)). The mathematical practice (Number 6 in the CCSS list) “Attending to precision” should not be interpreted as using the perfect word. Attending to precision can also refer to deciding when and what kind of precision is necessary during a computation, including when an exact answer is or is not necessary, a mathematical practice that does not require a precise word. Attending to precision is also involved in making precise claims, a practice that occurs not at the word level but at the discourse level.

For example, we can contrast the claim “Multiplication makes bigger,” which is not precise, with the claim “Multiplication makes the result bigger, only when you multiply by a positive number greater than 1.” When contrasting the two claims, notice that (a) precision does not lie in the individual words used and (b) the words used in the second claim are not more formal mathematical words. Instead, the precision of the second claim lies in specifying when the claim is true. In a classroom, if a teachers’ response to the first claim focused on precision at the word level, a follow up question might be to ask a student to use a more formal word for “bigger.” In contrast, if a teacher was focusing on precision at the discourse level, a follow up question would be “When does multiplication make a result bigger?”

Mathematics instruction for ELs needs to shift from simplified views of language as words. Such views separate language from mathematical proficiency and practices. These views also severely limit the linguistic resources teachers can use to teach mathematics and students can use to learn mathematics. Focusing instruction on words limits students’ access to the five strands of mathematical proficiency and curtails students’ opportunities to participate in mathematical practices (for examples of instruction for ELs focusing on word activities see de Araujo, 2012a, 2012b). In contrast, the view of academic literacy in mathematics proposed provides a complex view of mathematical discourse that is connected to mathematical proficiency and practices.

1.4. Summary

In sum, academic literacy in mathematics involves three intertwined components: mathematical proficiency, mathematical practices, and mathematical discourse. A definition of academic literacy in mathematics is more than a theoretical exercise. A clear definition of academic literacy in mathematics prepares teachers to choose (or design) tasks that support academic literacy in mathematics, provide opportunities for ELs to participate in academic literacy in mathematics, and recognize academic literacy in mathematics in student activity. When designing instruction we can consider how each component of academic literacy in mathematics might appear and how instruction could provide students opportunities to participate in the three aspects of academic literacy in mathematics. The example analyzed in the next section illustrates how these three components cannot be separated when considering mathematical tasks, analyzing student mathematical activity, or designing mathematics instruction.

2. Academic literacy in mathematics during student activity

The example analyzed here is a classroom discussion between two students and a teacher as they compared two graphs using multiple and situated meanings for words and phrases.⁷ The analysis is presented in two parts. In the first I examine the academic literacy in mathematics involved in working on the problem and generating the graph that then became the topic of the classroom discussion. I describe the mathematical proficiency, practices, and symbol systems that students used to generate the graph they brought to class.

In the second part of the analysis I examine the academic literacy in mathematics during the discussion comparing the scales on the two graphs. To clarify what the participants were talking about, I first describe the multiple meanings and competing claims. I also show how students coordinated meanings for utterances with views of the scales. I then describe the mathematical concepts and practices, and examine how the discussion involved a hybrid mix of everyday and academic language.

2.1. Methods

The transcript (see [Appendix A](#)) comes from a larger set of data collected in an eighth grade bilingual mathematics classroom. Classroom observations and videotaping were conducted during a unit from *Connected Mathematics Project* titled *Moving Straight Ahead* ([Lappan, Fey, Fitzgerald, Friel, & Phillips, 1998](#)). I selected this segment because it illustrates how the three components of academic literacy can appear during student activity in the classroom. The purpose of the excerpt was to illustrate theory, not theory confirmation. Therefore, the selection of this particular discussion was not based on how well it represents the data corpus, the frequency of such discussions in this classroom, or other criteria that would be relevant selection of samples in theory confirmation ([Moschkovich & Brenner, 2000](#)). The use of the phrase “I went by. . .” was documented as being used by students in this classroom at several points in time and has also been documented in other classrooms ([Moschkovich, 2008](#)).

The example illustrates how mathematical discourse, talk and text, is embedded in mathematical practices. These mathematical practices involve concepts, ground mathematical meanings, and focus attention on mathematical inscriptions. The example also shows how mathematical meanings are grounded in coordinated text, talk, and inscriptions. The example shows how participants engaged in academic literacy in mathematics used hybrid resources—multiple modes of communication, multiple sign systems, and multiple registers (everyday and academic). The example exemplifies the hybrid and situated nature of academic literacy in mathematics.

In the analysis, I use the following questions⁸:

- How are students displaying mathematical proficiency? How are students participating in mathematical practices and discourse? These focus questions show the complexity of academic literacy in mathematics.
- What resources do students use to communicate mathematically? What sign systems are relevant to the discussion? These focus questions illuminate how academic literacy in mathematics is multimodal and multi-semiotic.
- What are the situated meanings of words and phrases? This focus question illuminates how mathematical meanings are not given or static but instead situated in the socio-cultural context of a discussion.

2.2. Classroom setting

This eighth grade bilingual class was located in an urban area in the United States. In this school, there was a ‘two-way’ or ‘dual immersion’ bilingual program for Grades K-6. In grades K-6, students spend half their instructional time in English and half in Spanish. In grades 7 and 8, classes were no longer two-way bilingual, instead, teachers and students use both languages depending on the setting and participants. Most of the students in this class have been in the program for several years, many since elementary school. Some of the students are recent immigrants, several students are Spanish dominant, and most students are proficient in both Spanish and English. In this classroom some students spoke mainly English, some used both languages, and some spoke mainly Spanish. Instruction was conducted principally in English, with some discussions and explanations in Spanish among students when working in small groups as well as between the teacher and some students. The teacher was bilingual and used Spanish mostly when addressing students who were more recent immigrants and Spanish dominant.

Carlos and David,⁹ the students in this excerpt, are both native Spanish speakers who are bilingual.¹⁰ They are not recent immigrants or Spanish dominant. They arrived in the U.S. from Central America as young children and have both been in the bilingual program since the early grades in elementary school. They report that they sometimes speak Spanish at home. In the classroom they switch easily and fluidly between monolingual and bilingual modes ([Grosjean, 1999](#)). When discussing

⁷ A different analysis of this discussion appeared in [Moschkovich \(2008\)](#).

⁸ These were designed selectively and loosely following [Gee's \(1999\)](#) questions for Discourse analysis.

⁹ All names are pseudonyms.

¹⁰ They were not officially labeled as Limited English Proficiency nor did they receive special services.

a mathematics problem together, they used words, phrases, or extended talk in Spanish. When talking to the teacher, they tended to use mostly English.

The teacher usually started the 90-min class with a brief whole-class discussion about a mathematics problem (the problem for that day, a problem from a previous lesson, or a homework problem). Students then worked in groups of two to four, discussing the problem at their tables. The teacher moved from small group to small group, asking and answering questions in each group. Toward the end of the class period there were usually reports or presentations by each group as well as whole-class discussions led by the teacher.

This is a classroom where students expect to make sense of their work, discuss their work with peers, and also use the teacher as a resource in their discussions. Students took on some of the responsibility for explaining and understanding solutions. Students engaged in serious and extended discussion of their solutions. The group discussions seemed to be important to the students. Nevertheless, while the students shared responsibility for explaining solutions, they sometimes also tended to rely on the teacher as the authority for evaluating a solution.

This discussion occurred toward the beginning of a classroom period. After this discussion, students expected each group to go to the front of the classroom to explain their graphs or charts, describe how and why they solved a problem as they had, and be prepared to answer questions from other students and the teacher. This discussion arose when Carlos and David reviewed their graphs for a problem they had completed for homework. The class had been working on a unit framed by a story about a five-day bicycle tour, and this problem was part of that unit. In this story, while some riders rode bicycles, others rode in a van and recorded the total distance from the starting point for the van and the riders every half-hour. The problem below refers to the second day of the bicycle tour (see Fig. 1). For homework, students had responded to several questions about the problem shown in Fig. 1. Since the ensuing discussion focused on comparing the two graphs that the students had generated in response to the first question “Make a coordinate graph of the (time, distance) data given in the table,” the analysis of the task will focus only on the first question (in bold in Fig. 1).

3. Analysis of academic literacy in mathematics

3.1. Opportunities for academic literacy in mathematics in this task

What academic literacy in mathematics might be involved in generating the graph for this problem? What opportunities for each of the three components of academic literacy in mathematics—mathematical proficiency, practices, and discourse—might this task provide? First, students needed to read and understand the text that describes the situation. The genre is not a traditional word problem, but rather a situation to be mathematized. The purpose of the text, in contrast to text in, for example, language arts or social studies class, is not to tell a story, make an argument, or persuade the reader but to provide a situation to be modeled using mathematics. The structure for this genre in mathematics texts is that there is some information given that describes a real world situation and sets the stage, then there are questions for the reader.

Students need to read and understand not only the text but also a mathematical representation, the table. They need to read the table, extract, and use the information provided in the table. This is not as simple as it may seem. There are two typical interpretations of the second column that often arise in classrooms. One interpretation is that number in the second column refers to *interval distance*, i.e. that after 0.5 h the bikers had traveled 8 miles, and after 1 h the bikers had traveled an additional interval of 15 miles. Thus, after 1 h, the bikers were not 15 miles away from their starting point, but $8 + 15$, or 23 miles from their starting point. The other interpretation is the second column refers to *cumulative distance*, i.e. that after 0.5 h the bikers had traveled 8 miles, and after 1 h the bikers had traveled a *cumulative distance* of 15 miles. Thus, after 1 h, the bikers were 15 miles away from their starting point, not 23 miles. Students reading this table would need to sort out which of these interpretations fits the situation (in fact, these two students were involved in a later discussion not included in Appendix A where they sorted out these two interpretations). And lastly, students need to connect two representations by using the data in the table to construct a graph.

The mathematical proficiency required for this task certainly involves conceptual understanding, since connecting and making sense of three symbol systems (text, table, and graph) is a typical way for a task to involve conceptual understanding (Leinhardt, Zaslavsky, & Stein, 1990; Moschkovich, Schoenfeld, & Arcavi, 1993). At the very least this task involves the mathematical practice of modeling with mathematics. However, opportunities for other mathematical practices or mathematical discourse depend on the activity structure framing the task and the norms in the classroom. Because the activity structure provided by the classroom norms required that students discuss their responses in small groups, arrive at joint group solutions, and present a group solution to the whole class, this task and its activity structure provided opportunities for students to engage in other valued mathematical practices, such as constructing arguments and critiquing the reasoning of others. The typical routine in this classroom of solving, sharing, and presenting solutions and the norms for how to participate in those activities set the goals for the ensuing discussion between Carlos and David that involved not only individual sense making and reasoning, but also collectively negotiating meanings through discussion.

3.2. Academic literacy in mathematics when comparing the two graphs

What academic literacy in mathematics was involved in comparing the graphs? What opportunities for each of the three components of academic literacy in mathematics—mathematical proficiency, practices, and discourse—did this discussion

On the second day of their bicycle trip, the group left Atlantic City and rode five hours South to Cape May, New Jersey. This time, Sidney and Sarah rode in the van. From Cape May, they took a ferry across the Delaware Bay to Lewes, Delaware. Sarah recorded the following data about the distance traveled until they reached the ferry.

Time (hours)	Distance (miles)
0.0	0
0.5	8
1.0	15
1.5	19
2.0	25
2.5	27
3.0	34
3.5	40
4.0	40
4.5	40
5.0	45

1. Make a coordinate graph of the (time, distance) data given in the table.
2. Sidney wants to write a report describing Day 2 of the tour. Using information from the table and the graph, what would she write about the day's travel? Be sure to consider the following questions:
 - A. How far did the group travel in the day? How much time did it take them?
 - B. During which interval(s) did the riders make the most progress? The least progress?
 - C. Did the riders go further in the first half or the second half of the day's ride?
3. By analyzing the table, how can you find the time intervals when the riders made the most progress? The least progress? How can you find these intervals by analyzing the graph?

From Lappan, G., Fey, J. T., Fitzgerald, W. M., Friel, S. N., & Phillips, E. D. (1998). *Connected Mathematics*. White Plains, NY: Dale Seymour Publications.

Fig. 1. Problem: From Atlantic City to Lewes.

provide? Before analyzing the academic literacy involved, we need to understand what the participants were doing, first by providing the background and setting the stage for the discussion and, second, by describing the situated and multiple meanings for the phrase "I went by..."

3.2.1. Background for the discussion comparing two graphs

We join Carlos and David as they were discussing their answers to the questions for the problem in Fig. 1. Each student first read his written answer out loud. Carlos and David agreed in two ways in their written answers. First, they both wrote that the bikers traveled a total of 45 miles in 5 h (Question 2a). Second, they both wrote that the half-hour time interval in which the bikers made the least progress was the one between 2.0 and 2.5 h (Question 2b). However, they had written different answers for which half of the trip riders had made the most progress (Question 2c) and for the half-hour interval during which they made the most progress (Question 3). As the students read their answers, Carlos remarked that the written answers were different. David then took his graph (Fig. 2) and compared it to Carlos' graph (Fig. 3). As they looked at the

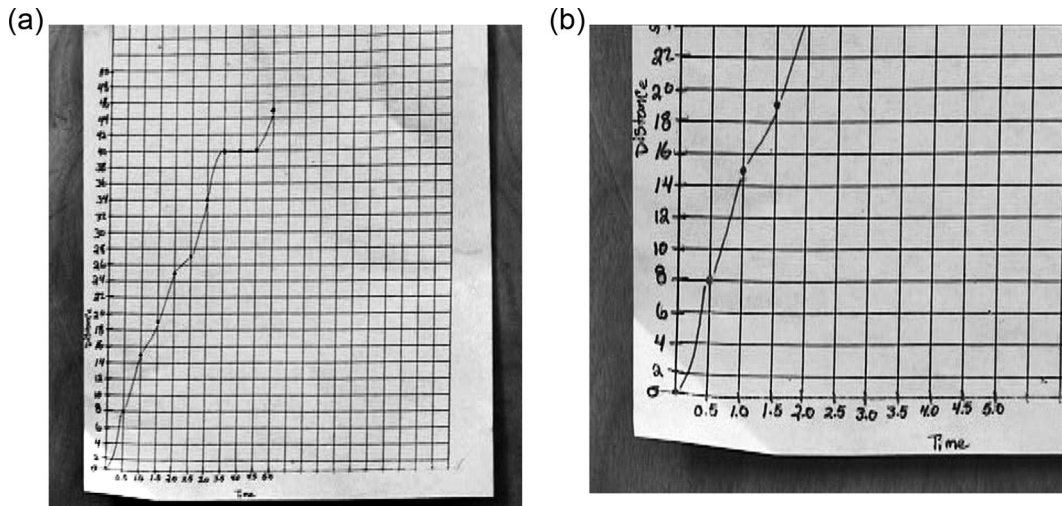


Fig. 2. David's graph: (a) whole view and (b) in close up.

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graphs, they noticed that the two graphs looked different. The discussion originated in their questioning why their graphs looked different (see Appendix A for the transcript).

When the teacher joined them, she focused the conversation on the meaning of the scales on the axes. The students started out with the goal of evaluating the shape of the graphs. The teacher joined the discussion at the students' request (lines 30–33). She initially set the goal of finding similarities and differences between the two graphs and later set the goal of considering the impact of the scale on the shape of the graph. She next asked the students to compare how they had labeled their axes and consider what was different about the two graphs. She suggested that David and Carlos look at their numbers and the way they had placed their numbers (line 40) and concluded that they both had put time on the x-axis. It is at this point in the discussion that the students started using multiple situated meanings for phrases of the form "I went by."

3.2.2. Situated meanings for "I went by..."

Descriptions of the scales reveal three different situated meanings for phrases of the form "I went by..." One meaning refers to the value of the interval between tick marks on a scale, a second to the number of segments between tick marks, and a third to the value of each segment between tick marks. The same utterance was used with different meanings depending on the view of the inscriptions. Participants coordinated multiple meanings with different views of the scale, grid segments, tick marks, and number labels on the scale.

During this discussion, Carlos, David, and the teacher were using the phrase "went by" with three different meanings. David seemed to use "I went by 2" in line 56 to mean that the value of each tick mark on the y-axis of his own graph increased by 2 (see Fig. 4). In David's graph tick marks corresponded to segments so "two" is also the value of each segment. In Carlos'

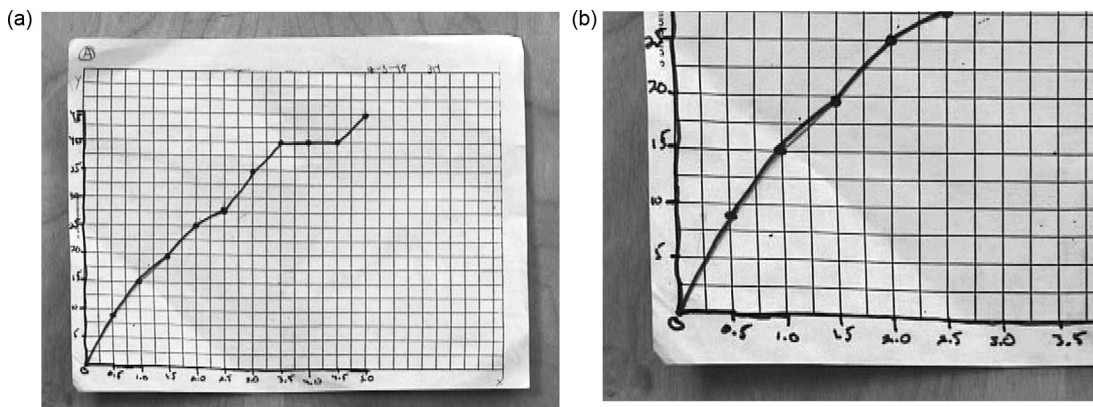


Fig. 3. Carlos's graph (a) whole view and (b) close up.

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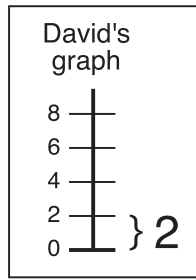


Fig. 4. David describes his scale as “I went by twos.”

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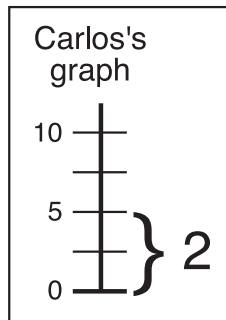


Fig. 5. Carlos describes his scale as “I went by twos.”

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graph, tick marks did not correspond to segments since Carlos had labeled only every other segment with a tick mark. Carlos seemed to use “I went by—” (line 42, 63, 66, and 74) in several ways that are different than David’s or the teacher’s. On the one hand, Carlos used the phrase in line 63 to refer to how many segments there were between tick marks on his graph, in this case 2 segments (see Fig. 5). On the other hand, in lines 66 and 72, Carlos seemed to be using the phrase to refer to how much the value increased for each tick mark on his graph, in this case by 5 (see Fig. 6). There are several ways to interpret Carlos’ utterance in line 69 “Then he (David) only went by ones.” One is that Carlos was referring to how many segments correspond to a tick mark in David’s graph (see Fig. 7). The other is that Carlos was taking the value between tick marks, 2,

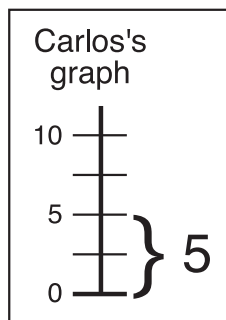


Fig. 6. Carlos describes his scale as “I went by fives”.

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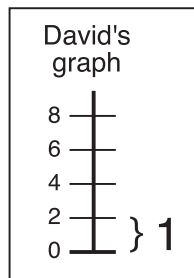


Fig. 7. Carlos describes David's scale as "He went by one".

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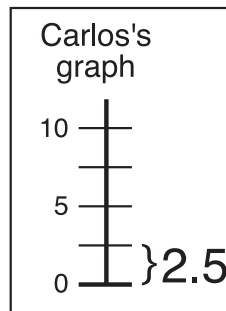


Fig. 8. Teacher describes Carlos' scale as "You went by two and a halves".

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and dividing by 2 because that was what the teacher had done for Carlos's graph (dividing 5 by 2 to obtain 2.5).¹¹ In contrast, the teacher was referring to the value of one segment in Carlos's graph (line 67, see Fig. 8).

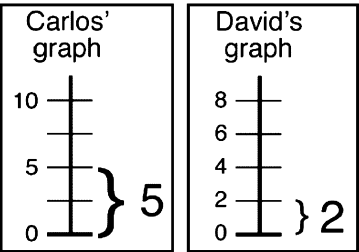
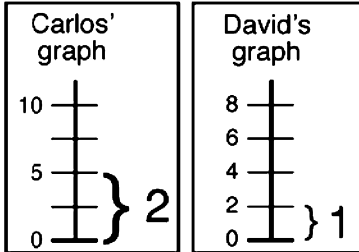
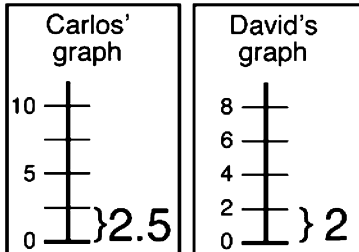
The phrase "I went by twos" could refer to the action one used to construct a scale by making tick marks every two segments and thus mean "I made tick marks every two segments." This description names how many segments are marked by the tick mark that has a number label, but need not refer to the units or the quantity represented by each segment marked. The same utterance, "I went by twos," could also refer to the number of units represented by the chunk created between two tick marks, as in "I made tick marks at every segment *and* each segment represents two units." This meaning refers not only to the marks and the number of segments marked but also to the number of units represented in any marked segment. Table 1 lists three ways of using "I (or you) went by" to describe the scales on these two graphs. One meaning refers to the value of the interval between tick marks, the second to the number of segments between tick marks, and the third to the value of each segment between tick marks. Carlos used the first and second meaning, David used the first and third meaning, and the teacher used the third meaning.

This discussion exemplifies how academic literacy in mathematics involves situated meanings for utterances, rather than static meanings for words, and how mathematical discourse practices involve not just utterances but, instead, utterances coordinated with views of inscriptions. The discussion involved ambiguous meanings for utterances and changing views of the inscriptions, as the utterances referred to imagined segments and units on the scales. Mathematical discourse is thus not simply about using a particular meaning for an utterance, but rather coordinating the meaning of an utterance with a particular focus of attention (Moschkovich, 2008).

In analyzing the situated meanings of phrases the participants used to describe the scales, I used "situated" in several senses. The multiple meanings of the phrase "I went by..." were situated locally in the ecology of this classroom and in the history and interactions that preceded this discussion, and, thus, they may or may not arise in other classrooms. In this particular classroom, these meanings were also situated in time, in the situation, and with respect to artifacts; meanings may shift among participants and for an individual participant at different times, in different situations, and with respect to different artifacts. The multiple meanings for this phrase reflected how each participant "highlighted" (Goodwin, 1994) the

¹¹ Even after the teacher's explanation that each segment in his graph had a value of 2.5, we can see that Carlos used yet another way of describing the graph. Carlos's utterance "three" in line 80 is difficult to interpret. If we assume that Carlos knows how to divide 5 by 2 to obtain the correct answer, then his answer that the point on the graph halfway between 0 and 5 is 3, is perhaps evidence that Carlos does not interpret the segments as corresponding to lengths of equal value.

Table 1
Multiple meanings for “went by”.

Utterance	Focus of attention	Coordinated utterance and focus of attention
Carlos: “I went by fives” David: “I went by twos”	The value of the interval between labeled tick marks	“I went by fives” “I went by twos” 
Carlos: “I went by twos” Carlos: “He (David) went by one”	The number of segments between labeled tick marks	“I went by twos” “He went by one” 
David: “I went by twos” Teacher: “You went by twos...”	The number of units in the interval between tick marks	“You went by two and a halves” “I went by twos” 
Teacher: No, actually you didn't go by fives, you actually went by two and a halves, because you did every two spaces as five.		

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inscription from different perspectives. In general, the multiple meanings for words, phrases, utterances, or written text are situated locally in the ecology of each classroom, in the history and interactions that precede a discussion of a mathematics problem. In each classroom, meanings are also situated in time and may shift among participants and for an individual participant at different times. Meanings are also situated with respect to artifacts used in the classroom such as objects (manipulatives, geometric shapes, etc.), drawing, graphs, and so on. Most importantly, meanings for utterances are situated in practices, in particular where participants focus joint attention (Rogoff, 1990) and how they view inscriptions (Goodwin, 1994; Stevens & Hall, 1998).

3.3. Mathematical proficiency, practices, and discourse

A discussion centered on how two students labeled the axes on their graphs might seem to focus on procedural fluency. However, the deceptively simple actions of labeling, describing, and comparing labels on axes are not so simple for students and, in fact, involved important conceptual understanding. A seemingly procedural question provided opportunities for a conceptual discussion. The first step in constructing a graph is to take the axes and partition each axis into equal segments that will serve as markers for graphing an ordered pair. Labeling an axis with tick marks and numbers divides the number line into segments of equal length and involves an important mathematical concept, unitizing (Lamon, 1994, 1996). As one makes

tick marks or writes number labels on the axis, each segment labeled or marked is assigned a unit. The teacher highlighted (Goodwin, 1994) a “unitized” view of the scales, anchoring her descriptions of the scales on an important concept, unitizing.

Lamon (1996) defined *unitizing* as “the cognitive assignment of a unit of measurement to a given quantity; it refers to the size chunk one constructs in terms of which to think about a given commodity” (1996, p. 170). The example used to illustrate unitizing is how a six-pack of cans of a beverage can be viewed and described as 6 cans (6 one-units) or 1 six-pack (1 six-unit). Similarly, when looking at a case of cans, the case can be viewed as 24 cans (24 one-units), 2 twelve-packs (2 twelve-units), or 4 six-packs (4 six-units). Labeling or making tick marks on a number line or axis is another example of unitizing. As I make a tick mark or write a number label, I am assigning a unit to the segment I label or tick and deciding the size of the chunks I am marking. This example shows how a mathematical concept, unitizing, is not purely a cognitive aspect of mathematical thinking. This concept was involved not only in an individual’s thinking about a situation in a particular way, but also in how the participants used particular meanings for utterances and particular views of the scales.

This discussion represents important aspects of mathematical proficiency that involve more than procedural fluency. Interpreting the different scales and the impact of the scales on the shape of the graphed curves involves substantial conceptual understanding. This is evident in how the teacher used the concept of unitizing (Lamon, 1994, 1996) in her explanation. The teacher’s descriptions focused on making comparisons among quantities and she made a distinction among labels, quantities, and measures. The teacher explicitly distinguished between the labels that go “by fives” and the value of the grid segments as a unit, saying “You actually went by two and a half” (line 67). In the second case, the phrase “you went by” refers to the unit value of one grid segment and is thus an instance of unitizing. The teacher also compared the values of the grid segments on the two scales (line 71), again using the concept of unitizing. Lastly, the teacher provided the students an opportunity to use a *unitized* view of the marks on the scales. She set a new problem, determining the y -coordinate of a point on the two graphs, given the same x -coordinate (three). As she and Carlos jointly estimated the y -coordinates for the same x -coordinate on the two graphs, she engaged him in talking about and viewing the scales from a unitized point of view, thus actively engaging him in unitizing.

Mathematical practices evident in this discussion include constructing an argument, tying claims to mathematical representations, and attending to precision. What makes a claim mathematical is, in part, the attention paid to describing in detail when the claim applies and when it does not. Whether one went by twos, or by fives, or by ones, depends on what “went by” means, and what “went by” means depends on one’s view of the scales. The participants were also talking about an imagined object—a unit that is used to measure but need not be marked on the graph and visualizing an imaginary relationship between marks and units.

This example illustrates that mathematical discourse with conceptual content need not involve formal vocabulary to be mathematical. The phrase “I went by” does not seem, at first glance, particularly mathematical and can be labeled as everyday or vernacular, rather than formal. The example illustrates how, although the phrase is informal or vernacular, the teacher and the students used this phrase repeatedly during this discussion (and at other times in this classroom), this phrase grounded the discussion on the inscriptions, and the students used this phrase to communicate their mathematical ideas and compare competing claims in an argument about the graphs. The teacher connected the meaning of this phrase to a mathematical concept, unitizing, to describe a mathematical situation more precisely. The phrase “I went by . . .” is not in itself mathematical, but rather how the phrase was used in this classroom. The ways participants used this phrase is an example of an informal phrase used by the participants as a resource to make mathematical claims and support an argument. What makes a discussion mathematical is not the use of formal mathematical words, but mathematical concepts, which can sometimes be expressed using informal words and phrases, and mathematical practices, such as justifying a claim, which are not at the word level.

The trajectory for learning vocabulary is a continuum (Nagy & Scott, 2000). However, it is difficult to categorize this phrase along a continuum of academic vocabulary (August, Carlo, Dressler, & Snow, 2005). On one hand, although it is specific to a mathematical situation, it is different than specialized Tier 3 words (August et al., 2005) such as tangent or slope that can be taught through explicit instruction or using definitions. On the other hand, unlike Tier 1 words, it is not easy to demonstrate the meaning of this phrase, as seen in the complexity of this discussion. When learning to describe the scales on a graph students can start with informal words and, at another point, with repeated instruction and use (McKeown, Beck, Omanson, & Pople, 1985), can later develop a description that includes formal words such as scale, ratio, and unit. By starting with a discussion using informal words, these students had an opportunity to participate in what McKeown et al. (1985) label rich classroom instruction “characterized by elaboration and discussion about words, their meanings, and their uses” (p. 524). If the goal is higher order processing that involves integrating words and context, then such elaboration and discussion are preferred to the more traditional instruction that only requires associations between words and their definitions or synonyms.

In summary, the example illustrates the situated nature of the mathematical discourse component of academic literacy in mathematics. The multiple meanings for “I went by . . .” evident in this discussion were situated in this classroom’s history and in the coordinated use of artifacts in the discussion. The example shows that mathematical discourse is not about using static meanings for words. During this discussion participants negotiated meanings that were grounded in utterances *coordinated* with focus of attention on inscriptions. This example also highlights the hybrid nature of the mathematical discourse component of academic literacy in mathematics. Participants were not negotiating the meaning of formal vocabulary but using an everyday phrase with mathematical meaning.

4. Conclusions

The definition, descriptions, and analysis provided here show how a socio-cultural conceptual framework broadens notions of academic literacy in mathematics, expanding it beyond competence with words. The proposed definition of academic literacy in mathematics integrates three components of mathematical activity: mathematical proficiency, mathematical practices, and mathematical discourse. The example shows how the three components of academic literacy in mathematics cannot be separated when analyzing tasks or student activity. The analysis illustrates how academic literacy in mathematics is situated, in that learners engaged in academic literacy in mathematics negotiated situated meanings for utterances that were grounded in the socio-cultural setting and coordinated with ways of viewing inscriptions. The example also illustrates how academic literacy in mathematics is hybrid, in that participants drew on and coordinated multiple resources—modes of communication, symbol systems, as well as both everyday and academic registers.

The view of academic literacy in mathematics presented here has important implications for mathematics instruction. First, instruction needs to consider not only cognitive aspects of mathematical activity—mathematical reasoning, thinking, concepts, and metacognition—but also sociocultural aspects—participation in mathematical practices—and discursive aspects—participation in mathematical discourse. Most importantly, because these three components of academic literacy in mathematics function together during mathematical activity, instruction intended to support academic literacy in mathematics for ELs should not address academic language as an isolated goal, but integrate mathematical proficiency, practices, and discourse whenever possible. Instruction should also include the full spectrum mathematical proficiency, balancing computation fluency, conceptual understanding, and reasoning. And lastly, instruction should implement a complex view of mathematical discourse that goes beyond competence with words. In the concluding section I expand on each of these three recommendations for instruction.

4.1. Provide opportunities for integrating components of academic literacy in mathematics

A socio-cultural framing of academic literacy in mathematics implies that instruction should provide opportunities for joint problem solving, appropriation of mathematical practices, participation in mathematical discourse, and negotiation of situated meanings for mathematical language. Opportunities for ELs to focus on language should be connected to mathematical activity, instead of giving students definitions in a vacuum or divorcing language work from mathematical work.

Classroom activities need to provide opportunities for students to participate in integrated ways in mathematical proficiency, practices, and discourse. Since the three components of academic literacy in mathematics function together in intertwined ways during mathematical problem solving, isolating one component during instruction can have unintended consequences. Isolating mathematical discourse from mathematical proficiency or practices removes the context, concepts, and practices that ground the meanings for mathematical language. Isolating mathematical proficiency from mathematical practices or discourse limits access to an impoverished version of mathematical activity that does not parallel mathematical expertise.

One of the goals of mathematics instruction for ELs should be to support all students, regardless of their proficiency in English, in participating in discussions that focus on understanding and reasoning, rather than on low-level computational skills. Another goal for mathematics instruction for ELs should be to support all students, regardless of their proficiency in English, in participating in discussions that focus on high cognitive demands tasks, conceptual understanding, and reasoning, rather than on pronunciation, vocabulary, or low-level linguistic skills. One way to accomplish these two goals is by integrating the three components of academic literacy in mathematics described here.

ELs learning mathematics need opportunities to engage in mathematical practices and discourse because these are both important for conceptual understanding. Mathematical activity occurs in the context of mathematical practices that provide the goals and purposes for doing mathematics. Student participation in mathematical discourse is also important in supporting conceptual understanding. The more opportunities a learner has to make connections among multiple representations, or to explain why something is the case, or to describe their reasoning, the more opportunities that learner has to develop conceptual understanding. But not all kinds of communication will support conceptual understanding. Communication needs to be focused on important mathematical concepts and be embedded within mathematical practices. Communication that provides opportunities for students to participate in mathematical practices (explaining, constructing arguments, justifying, and proving) supports conceptual understanding. Communication that includes multiple modes (talking, listening, writing, drawing, etc.) also supports learners in making connections among multiple ways of representing mathematical concepts and thus supports conceptual understanding. Conceptual understanding, mathematical practices, and mathematical discourse should thus be closely connected when designing instruction.

4.2. Maintain high cognitive demand and focus on conceptual understanding

Mathematics instruction for ELs needs to provide this student population equitable access to mathematics instruction that goes beyond low-level computational skills. Therefore, mathematics instruction for ELs should begin with tasks

that require high cognitive demand, conceptual understanding, and reasoning. Instruction should balance conceptual and procedural knowledge, connect the two types of knowledge, use high-cognitive-demand math tasks, maintain the rigor of mathematical tasks throughout lessons and units, and provide opportunities for students to engage in mathematical practices.

A focus on conceptual understanding does not mean that students should be taught to recite definitions of concepts or draw pictures by following a set of steps modeled by the teacher. Classroom discussions, in pairs, small groups, and whole class, which focus on concepts, mathematical practices, and student explanations of their solutions, have been shown to support both procedural fluency and conceptual understanding (Hiebert & Grouws, 2007). Students should be focusing on making connections, understanding multiple representations of mathematical concepts, communicating their thinking, and justifying their reasoning. In order to engage students in mathematical practices, instruction needs to include time and support for mathematical discussions and use a variety of participation structures (teacher-led, small group, pairs, student presentations, etc.) that support students in learning to participate in such discussions.

This view of mathematical proficiency has important implications for instruction for ELs that supports academic literacy in mathematics. If EL students need support in procedural skills for whole number multiplication, instruction should balance a focus on procedural fluency or drill approaches to multiplication facts with other aspects of mathematical proficiency that help students understand, represent, apply, and connect multiplication to other important mathematical ideas. This can be accomplished, for example, by representing multiplication using arrays and area models, solving multi-digit multiplication exercises by grouping and regrouping and making a connection to the distributive property, and solving multiplication word problems. Instruction can also support an integrated approach to academic literacy in mathematics by designing multiplication lessons that focus not only on mathematical proficiency but also provide opportunities for students to participate in mathematical practices—solving problems, looking for regularity, etc.—and discourse—reading word problems, explaining solutions orally and in writing, providing mathematical justification, etc.

4.3. Implement complex view of mathematical discourse

Previous research provides several guidelines for mathematics instruction for ELs that addresses language: (1) treat language as a resource, not a deficit (Gándara & Contreras, 2009; Moschkovich, 2000, 2002a); (2) address much more than vocabulary and support ELs' participation in mathematical discussions as they learn English (Moschkovich, 1999, 2002a, 2007a, 2007b, 2007c); and (3) draw on multiple resources available in classrooms—such as objects, drawings, graphs, and gestures—as well as home languages and experiences outside of school. Overall, instruction for this population should not emphasize low-level language skills but, instead, provide students opportunities to actively communicate about their mathematical solutions, ideas, and reasoning.

Mathematics instruction for ELs needs to shift from simplified views of language as words, vocabulary, or definitions. Such views severely limit the linguistic resources teachers can use to teach mathematics and students can use to learn mathematics, and separate language from mathematical proficiency and mathematical practices. Focusing instruction on words, vocabulary, or definitions, limits students' access to the five strands of mathematical proficiency and curtails students' opportunities to participate in mathematical practices (for examples see de Araujo, 2012a, 2012b). In contrast, the view of academic literacy in mathematics proposed here provides a complex view of mathematical discourse.

Assessment should also be informed by a complex view of academic literacy in mathematics. The following word problem illustrates how the academic literacy in mathematics for word problems cannot be reduced to any single aspect of mathematical text:

A boat in a river with a current of 3 mph can travel 16 miles downstream in the same amount of time it can go 10 miles upstream. Find the speed of the boat in still water.

The language complexity in this word problem is not at the word level. First, the complexity lies in the background knowledge (Martiniello, 2008; Martiniello & Wolf, 2012) to imagine the situation. In this case, the reader needs to imagine and understand that there is a boat traveling up and down a river, that the speed was measured in still water (presumably a lake), and that the speed of the boat increases (by the speed of the current) when going downstream, and decreases (by the speed of the current) when going upstream. Supporting students in learning non-math words (upstream, downstream, and the phrase “in still water”) would certainly help. However, much of the language complexity is not at the word level, but at the sentence & paragraph level, in the use of the passive voice without an agent and in the multiple subordinate clauses and nested constructions (Cook & MacDonald, 2012).

According to Abedi (2002), linguistic complexity of test items unrelated to the content being assessed may at least be partly responsible for the performance gap between ELs and non-ELs, and linguistic complexity of test items may invalidate achievement on tests. The language features of text that have been documented as problematic for ELs are at three levels: background, syntactic, and lexical (Moschkovich & Scott, 2013). The syntactic level involves complex sentences, multiple subordinate clauses, nested constructions, long noun phrases, and the use of the passive voice without an

agent. The lexical level involves unfamiliar words, unfamiliar phrases, and unfamiliar connotations of words with multiple meanings. Since the language complexity of mathematics word problems and the language complexity issues for ELs are not all necessarily at the word level, instruction should focus on the background knowledge in word problems, sentence and paragraph level, and word phrases. Instruction should not reduce the language complexity of word problems since that will not support students in learning to work with this mathematical genre. Instead, instruction should include examples of word problems with passive voice, sentences with unclear subject, nested constructions, and subordinate clauses.

The definition of academic literacy in mathematics provided here reflects a view of mathematical discourse as the complex interaction of three semiotic systems—natural language, mathematics symbol systems, and visual displays (O'Halloran, 1999). The complexity of mathematical discourse in the classroom involves:

- Multiple modes (oral, written, receptive, expressive, etc.)
- Multiple representations (including objects, pictures, words, symbols, tables, graphs, etc.)
- Different types of written texts (textbooks, word problems, student explanations, teacher explanations, etc.)
- Different types of talk (exploratory and expository), and
- Different audiences (presentations to the teacher or by the teacher, presentations to peers or by peers, exploration with peers, etc.).

Instruction that supports academic literacy in mathematics for ELs and, in particular competence in mathematical discourse, should not be interpreted as helping students acquire static meanings for words provided by the teacher or a textbook. As the analysis here shows, mathematical meanings are situated, negotiated, and grounded in activity. Instruction should provide opportunities for students to negotiate and refine meanings that are grounded in mathematical activity. The design of instruction that provides opportunities for mathematical discourse should consider tasks not in isolation but as they are framed by the routines used in the classroom, the activity structure for each task, the inscriptions used, and other relevant contextual aspects of mathematical activity as it occurs in each classroom setting.

A shift to a complex view of mathematical discourse is particularly important for defining academic literacy in mathematics for ELs. A simplified view of mathematical discourse can lead to the assumption that precision lies primarily in individual word meaning, and assumption that could have dire consequences for ELs, as they are likely to use imperfect language to describe their mathematical thinking. It is important to consider what we mean by *precision* for all students learning mathematics, since all students are likely to need time and support for moving from expressing their reasoning in imperfect form. However, it is *essential* for teachers of ELs to consider when and how to focus on precision for ELs. Attending to precision only at the level of individual word meaning can curtail opportunities for students to express their emerging mathematical ideas. Although students' use of imperfect language is likely to interact with teachers' own interpretations of precision, we should not confuse the two, and remember that precise claims can be expressed in imperfect language.

Appendix A. Transcript

(When reading the transcript, it is important to look at the graphs.)

22. David: Here's my graph. Did it come out like yours?
23. David's gesture: ((Turning his paper towards Carlos.))
24. Carlos: I don't know.
25. Carlos' gesture: ((He looks at both papers and compares graphs.)) ((0.2 s))
26. David: I'll do mine. =
27. Carlos: =It's because you did it upwards.
28. Carlos' gesture: ((Sweeping his right hand, with his finger pointing up.))
29. David: Oh, you did. ((0.2 s)) ((Teacher approaches the group.))
30. Carlos: Were we supposed to do the graph upwards? Or to-
31. Carlos' gesture: ((Sweeping his hand upwards again and then moving his hand horizontally in the air.))
32. David: Or, or or crooked like this? Whatever.
33. Carlos: Or horizontally? ((0.2 s))
34. Teacher: Doesn't matter. ((shaking her head))
35. Carlos: 'Cause like, like when we look at our graphs his is going up and mine is going toward the side.
36. Carlos' gesture: ((Moving his right hand upwards and then moving his right hand horizontally in the air.)) ((0.2 s))
37. Teacher: Did you have the same, did you put both of the same things on the x -axis and on the y -axis?
38. Teacher's gesture: ((Looking through students' graphs.))

39. Carlos: No, ((0.2 s)) yes, actually.
40. Teacher: You both put time here (referring to the x -axes). OK, so what's different about these? I don't think it's, I don't think it's the positioning of them. Look at your numbers, the way you placed your numbers.
41. Teacher's gesture: ((Points at the x -axis on both papers, back and forth. Then turns David's paper in the same direction as Carlos' paper.))
42. Carlos: Oh, that's true, 'cause I went by twos, I went 1, 2 ((0.2 s)) and then I put that one (—) he went by one.
43. Carlos' gesture: ((Begins counting with his right finger following the numbers on his paper.))
44. Teacher: Aha, You skipped one. So how does that change how it looks?
45. Carlos: 'Cause it doesn't go up as far, it only goes, it's more steeper. It looks more steeper.
46. Carlos' gesture: ((Moving his right hand outward. Then moving his right hand straight up.))
47. Teacher: Remem-. Similar to the difference between this one and ((0.1 s)) and this one here. Right?
48. Teacher's gesture: ((Makes a sign with right thumb and index of her hand to show interval differences on their papers. Then she points to a graph on the blackboard. Next she points to a second graph on the blackboard. The first and second graphs have different scales on the x -axis so that the second graph is compressed along the x -direction.))
49. David: That one.
50. Carlos: Yeah.
51. Teacher: Here the numbers are closer together so it looks looks steeper. Other than that, are they the same graph?
52. Teacher's gesture: ((Makes a sign with thumb and index finger. Then she gestures upwards with her right hand.)) ((0.1 s))
53. Carlos: No, also here in the x -axis.
54. Carlos' gesture: ((Carlos point to the x -axis on his paper.))
55. David: (the distance) ((Points to the axis on his paper.))
56. David: I went by twos. =
57. Carlos: = This is the x -axis. Right?
58. ((Carlos points to the axis.))
59. Teacher: This is the y -axis, ((0.1 s)) this is the x -axis.
60. Teacher's gesture: ((Sweeps her pencil vertically to represent the y -axis and then horizontally to represent the x -axis.))
61. David: I went by twos.
62. Teacher: You went by twos and you went b:y- ((0.2 s))
63. Carlos: I went by twos. You (didn't) you went by ones! What are you talking about.
64. Teacher: No, here on the y -axis.
65. Teacher's gesture: ((Points to the axis.))
66. Carlos: Oh, I went by fives.
67. Teacher: You went by fives. ((0.2 s)) No, actually you didn't go by fives. You actually went by two and a halves because you'd, you did every two spaces was five.
68. Teacher's gesture: ((She points to Carlos' paper while she explains.)) ((0.3 s))
69. Carlos: Then he only went by one.
70. Carlos' gesture: ((Carlos points to David's paper.)) ((0.2 s))
71. Teacher: Every one space was two of his. You see, they're almost the same. If you look at the next two-((Puts down her notebook and points to the graphs.))
72. Carlos: Wait! But I don't get what you're saying.
73. Teacher: OK.
74. Carlos: 'Cause I went by fives. ((David stands up.))
75. Teacher: OK, your numbers, right, the numbers you have are by five ((0.2 s)) OK ((0.1 s)). If you look at one line here, what number is he at?
76. Teacher's gesture: ((Takes David's paper and places it next to Carlos' paper then points to David's graph.))
77. Carlos: Two.
78. Teacher: What number would you be at if you had a number here?
79. Teacher's gesture: ((Points to Carlos' graph.))
80. Carlos: Three.
81. Teacher: Almost, two and a half.
82. Carlos: Yeah.
83. Teacher: Because that'd be half way to five. OK. ((0.1 s)) At this point, after 1, 2, 3, he's got 6. For you after three, 1, 2, 3, you'd be at 7 and a half.
84. Teacher's gesture: ((She counts the squares with her pencil.))
85. Carlos: O:K:.
86. Teacher: See what I mean? So it's actually two and a half. The numbers you wrote are by fives but since you skipped a line in between, each one is two and a half.
87. Teacher's gesture: ((Raises her hand and in the air uses thumb and index to show interval.))

Appendix B. Transcript conventions

Timing		
Equal sign	=	Indicates the end or beginning of two 'latched' utterances that continue without an intervening pause. Where necessary, can be combined with brackets.
Timed pause	(1.8)	Measured in seconds, this symbol represents intervals of silence occurring within and between speakers' utterances.
Delivery		
Period	.	Indicates a falling pitch or intonation at the conclusion of an utterance that may or may not mark the completion of a grammatically constructed unit.
Question mark	?	Rising vocal pitch or intonation at the conclusion of an utterance that may or may not have the grammatical structure of a question.
Exclamation point	!	Marks the conclusion of an utterance delivered with emphatic and animated tone. The utterance itself may or may not be an exclamation.
Comma	,	Indicates a continuing intonation with slight upward or downward contour that may or may not occur at the end of a grammatical phrase.
Other		
Parentheses	()	Talk for which transcriber doubt exists.
Double parentheses	(())	Transcript annotations Translations are italicized

These are selected conventions taken from a fuller list provided in Charlotte Linde (1993), *Life Stories: The Creation of Coherence* (Oxford Studies in Sociolinguistics).

References

- Abedi, J. (2002). *Standardized achievement tests and English language learners: Psychometrics issues*. *Educational Assessment*, 8(3), 231–257.
- American Educational Research Association. (2004). *Closing the gap: High achievement for students of color*. *Research Points*, 2(3)
- American Educational Research Association. (2006). *Do the math: Cognitive demand makes a difference*. *Research Points*, 4(2).
- August, D., Carlo, M., Dressler, C., & Snow, C. (2005). *The critical role of vocabulary development for English language learners*. *Learning Disabilities Research & Practice*, 20(1), 50–57.
- Bransford, J. D., Brown, A. L., & Cocking, R. R. (1999). *How people learn: Brain, mind, experience, and school*. National Academy Press.
- Brown, J. S., Collins, A., & Duguid, P. (1989). *Situated cognition and the culture of learning*. *Educational Researcher*, 18(1), 32–42.
- Cavanagh, S. (2005). *Math: The not-so-universal language*. *Education Week*, (July). Retrieved from: <http://www.barrow.k12.ga.us/esol/Math.The.Not.So.Universal.Language.pdf>
- Cobb, P., Stephan, M., McClain, K., & Gravemeijer, K. (2001). *Participating in classroom mathematical practices*. *Journal of the Learning Sciences*, 10(1–2), 113–164.
- Cobb, P., Wood, T., & Yackel, E. (1993). *Discourse, mathematical thinking and classroom practice*. In E. Forman, N. Minick, & C. A. Stone (Eds.), *Contexts for learning: Sociocultural dynamics in children's development* (pp. 91–119). New York, NY: Oxford University Press.
- Common Core State Standards Initiative. (2010). *Common Core State Standards for Mathematics*. Washington, DC: National Governors Association Center for Best Practices and the Council of Chief State School Officers.
- Common Core State Standards. (2010). *Common core state standards for mathematical practice*. Retrieved from: <http://www.corestandards.org/Math/Practice>
- Cook, G., & MacDonald, R. (2012). *Draft of "Can Do" descriptors for CCSS standards for mathematical practice*. WIDA, University of Wisconsin.
- Crowhurst, M. (1994). *Language and learning across the curriculum*. Scarborough, Ontario: Allyn and Bacon.
- de Araujo, Z. (2012a). *An examination of non-mathematical activities in the mathematics classroom*. In L. R. Van Zoest, J. Lo, & J. L. Kratky (Eds.), *Proceedings of the 34th annual meeting for the North American chapter for the psychology of mathematics education* (pp. 339–342). Kalamazoo, MI: Western Michigan University.
- de Araujo, Z. (2012b). *Diminishing demands: Secondary teachers' modifications to tasks for English language learners*. In L. R. Van Zoest, J. Lo, & J. L. Kratky (Eds.), *Proceedings of the 34th annual meeting for the North American chapter for the psychology of mathematics education* (pp. 76–79). Kalamazoo, MI: Western Michigan University.
- DiCerbo, P. A., Anstrom, K. A., Baker, L. L., & Rivera, C. (2014). *A review of the literature on teaching academic English to English language learners*. *Review of Educational Research*, 84(3), 446–482.
- Forman, E. (1996). *Learning mathematics as participation in classroom practice: Implications of sociocultural theory for educational reform*. In L. Steffe, P. Nesher, P. Cobb, G. Goldin, & B. Greer (Eds.), *Theories of mathematical learning* (pp. 115–130). Mahwah, NJ: Lawrence Erlbaum Associates.
- Gándara, P., & Contreras, F. (2009). *The Latino education crisis: The consequences of failed social policies*. Cambridge, MA: Harvard University Press.
- Gee, J. (1996). *Social linguistics and literacies: Ideology in discourses*. London, UK: The Falmer Press.
- Gee, J. (1999). *An introduction to discourse analysis: Theory and method*. New York, NY: Routledge.
- Goodwin, C. (1994). *Professional vision*. *American Anthropologist*, 96(3), 606–633.
- Greeno, J. G. (1998). *The situativity of knowing, learning, and research*. *American Psychologist*, 53(1), 5.
- Grosjean, F. (1999). *Individual bilingualism*. In B. Spolsky (Ed.), *Concise encyclopedia of educational linguistics* (pp. 284–290). London, UK: Elsevier.
- Gutiérrez, K., Baquedano-Lopez, P., & Alvarez, H. (2001). *Literacy as hybridity: Moving beyond bilingualism in urban classrooms*. In M. de la Luz Reyes, & J. Halcon (Eds.), *The best for our children Critical perspectives on literacy for Latino students* (pp. 122–141). New York, NY: Teachers College Press.
- Gutiérrez, K. D., Baquedano-López, P., & Tejada, C. (1999). *Rethinking diversity: Hybridity and hybrid language practices in the third space*. *Mind, Culture, and Activity*, 6(4), 286–303.
- Halliday, M. A. K. (1978). *Sociolinguistics aspects of mathematical education*. In M. Halliday (Ed.), *The social interpretation of language and meaning* (pp. 194–204). London, UK: University Park Press.
- Hiebert, J., & Carpenter, T. P. (1992). *Learning and teaching with understanding*. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning: A project of the National Council of Teachers of Mathematics* (pp. 65–97). New York, NY: Macmillan Publishing.
- Hiebert, J., & Grouws, D. (2007). *The effects of classroom mathematics teaching on students' learning*. In F. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 371–404). Reston, VA: National Council of Teachers of Mathematics.
- Hymes, D. (1972). *On communicative competence*. In J. B. Pride, & J. Holmes (Eds.), *Sociolinguistics* (pp. 269–293). London, England: Penguin. Reprinted in Duranti, A. (Ed.). (2009). *Linguistic anthropology: A reader* (Vol. 1). John Wiley & Sons.
- Khisty, L. (1995). *Making inequality: Issues of language and meanings in mathematics teaching with Hispanic students*. In W. Secada, E. Fennema, & L. Byrd Adajian (Eds.), *New directions for equity in mathematics education* (pp. 279–297). New York, NY: Cambridge University Press.
- Kilpatrick, J., Swafford, J., & Findell, B. (Eds.). (2001). *Adding it up: Helping children learn mathematics*. Washington, DC: National Academy Press.
- Kilpatrick, J., Swafford, J., & Findell, B. (2001). In J. Kilpatrick, J. Swafford, & B. Findell (Eds.), *Adding it up: Helping children learn mathematics*. Washington, DC: National Research Council, National Academy Press.

- Koestler, C., Felton, M., Bieda, K., & Otten, S. (2013). *Connecting the NCTM process standards and the CCSSM practices*. Reston, VA: National Council of Teachers of Mathematics.
- Lamon, S. (1994). Ratio and proportion: Cognitive foundations in unitizing and norming. In G. Harel, & J. Confrey (Eds.), *The development of multiplicative reasoning in the learning of mathematics* (pp. 89–120). New York, NY: SUNY Press.
- Lamon, S. (1996). The development of unitizing: Its role in children's partitioning strategies. *Journal for Research in Mathematics Education*, 27(2), 170–193.
- Lappan, G., Fey, J. T., Fitzgerald, W. M., Friel, S. N., & Phillips, E. D. (1998). *Connected mathematics*. White Plains, NY: Dale Seymour Publications.
- Lave, J., & Wenger, E. (1991). *Situated learning: Legitimate peripheral participation*. New York, NY: Cambridge University Press.
- Lea, M. R., & Street, B. V. (2006). The "academic literacies" model: Theory and applications. *Theory into Practice*, 45(4), 368–377.
- Leinhardt, G., Zaslavsky, O., & Stein, M. K. (1990). Functions, graphs and graphing: Tasks learning and teaching. *Review of Educational Research*, 60(1), 1–64.
- Martiniello, M. (2008). Language and the performance of English language learners in math word problems. *Harvard Educational Review*, 78(2), 333–368.
- Martiniello, M., & Wolf, M. K. (2012). Exploring ELLs' understanding of word problems in mathematics assessments: The role of text complexity and student background knowledge. In S. Celedón-Pattichis, & N. Ramirez (Eds.), *Beyond good teaching: Strategies that are imperative for English language learners in the mathematics classroom*. Reston, VA: NCTM.
- McKeown, M. G., Beck, I. L., Omanson, R. C., & Pople, M. T. (1985). Some effects of the nature and frequency of vocabulary instruction on the knowledge and use of words. *Reading Research Quarterly*, 522–535.
- Moschkovich, J. N. (1999). Supporting the participation of English language learners in mathematical discussions. *For the Learning of Mathematics*, 19(1), 11–19.
- Moschkovich, J. N. (2000). Learning mathematics in two languages: Moving from obstacles to resources. In W. Secada (Ed.), *Changing the faces of mathematics (Vol. 1): Perspectives on multiculturalism and gender equity* (pp. 85–93). Reston, VA: National Council of Teachers of Mathematics.
- Moschkovich, J. N. (2002a). A situated and sociocultural perspective on bilingual mathematics learners. In N. Nasir, & P. Cobb (Eds.), *Mathematical thinking and learning. Special issue on diversity, equity, and mathematical learning* (Vol. 4 (2–3)) (pp. 189–212).
- Moschkovich, J. N. (2002b). An introduction to examining everyday and academic mathematical practices. In M. Brenner, & J. Moschkovich (Eds.), *Everyday and academic mathematics: Implications for the classroom*. *Journal for Research in Mathematics Education*, 1–11. Monograph Number 11.
- Moschkovich, J. N. (2004). Appropriating mathematical practices: A case study of learning to use and explore functions through interaction with a tutor. *Educational Studies in Mathematics*, 5, 49–80.
- Moschkovich, J. N. (2007a). Beyond words to mathematical content: Assessing English Learners in the mathematics classroom. In A. Schoenfeld (Ed.), *Assessing Mathematical Proficiency* (pp. 345–352). New York, NY: Cambridge University Press.
- Moschkovich, J. N. (2007b). Bilingual mathematics learners: How views of language, bilingual learners and mathematical communication impact instruction. In N. Nasir, & P. Cobb (Eds.), *Diversity, Equity, and Access to Mathematical Ideas* (pp. 89–104). New York, NY: Teachers College Press.
- Moschkovich, J. N. (2007c). Examining mathematical discourse practices. *For The Learning of Mathematics*, 27(1), 24–30.
- Moschkovich, J. N. (2008). I went by twos, he went by one: "Multiple interpretations of inscriptions as resources for mathematical discussions. *Journal of the Learning Sciences*, 17(4), 551–587.
- Moschkovich, J. N. (2009). How language and graphs support conversation in a bilingual mathematics classroom. In R. Barwell (Ed.), *Multilingualism in mathematics classrooms: Global perspectives* (pp. 78–96). Bristol, UK: Multilingual Matters Press.
- Moschkovich, J. N. (2010). Language(s) and learning mathematics: Resources, challenges and issues for research. In J. N. Moschkovich (Ed.), *Language and mathematics education: Multiple perspectives and directions for research* (pp. 1–28). Charlotte, NC: Information Age Publishing.
- Moschkovich, J. N. (2011). Supporting mathematical reasoning and sense making for English Learners. In M. Strutchens, & J. Quander (Eds.), *Focus in high school mathematics: Fostering reasoning and sense making for all students* (pp. 17–36). Reston, VA: National Council of Teachers of Mathematics.
- Moschkovich, J. N. (2012). Mathematics, the common core standards, and language: Mathematics instruction for ELs aligned with the common core. In *Proceedings of the annual conference of the North American chapter of the international group for the psychology of mathematics education*. Kalamazoo, MI: Western Michigan University.
- Moschkovich, J. N. (2013a). Issues regarding the concept of mathematical practices. In Y. Li, & J. N. Moschkovich (Eds.), *Proficiency and beliefs in learning and teaching mathematics: Learning from Alan Schoenfeld and Günter Toerner* (pp. 257–275). Rotterdam, Netherlands: Sense Publishers.
- Moschkovich, J. N. (2013b). Principles and guidelines for equitable mathematics teaching practices and materials for English language learners. *Journal of Urban Mathematics Education*, 6(July (1)), 45–57.
- Moschkovich, J. N., & Brenner, M. (2000). Integrating a naturalistic paradigm into research on mathematics and science cognition and learning. In R. Lesh, & A. Kelly (Eds.), *Handbook of research design in mathematics & science education* (pp. 457–486). New Jersey: Lawrence Erlbaum Associates, Inc.
- Moschkovich, J. N., Schoenfeld, A., & Arcavi, A. (1993). Aspects of understanding: On multiple perspectives and representations of linear relations and connections among them. In T. A. Romberg, E. Fennema, & T. P. Carpenter (Eds.), *Integrating research on the graphical representation of function* (pp. 69–100). Hillsdale, NJ: Erlbaum.
- Moschkovich, J., & Scott, J. (2013). *Linguistic complexity of assessment items*. Unpublished report to Smarter Balanced Assessment Consortium.
- Nagy, W. E., & Scott, J. A. (2000). Vocabulary processes. *Handbook of Reading Research*, 3(269–284).
- Nasir, N. (2002). Identity, goals, and learning: Mathematics in cultural practice. *Mathematical Thinking and Learning*, 4(2–3), 213–247.
- National Council of Teachers of Mathematics. (1989). *Curriculum and evaluation standards for school mathematics*. Reston, VA: NCTM.
- National Council of Teachers of Mathematics. (1991). *Professional standards for teaching mathematics*. Reston, VA: NCTM.
- National Council of Teachers of Mathematics. (1995). *Assessment standards for school mathematics*. Reston, VA: NCTM.
- O'Halloran, K. (1999). Towards a systemic functional analysis of multimedial mathematics texts. *Semiotica*, 124(1/2), 1–29.
- Pimm, D. (1987). *Speaking mathematically: Communication in mathematics classrooms*. London, UK: Routledge.
- Rogoff, B. (1990). *Apprenticeship in thinking: Cognitive development in social context*. New York, NY: Oxford University Press.
- Schleppegrell, M. (2007). The linguistic challenges of mathematics teaching and learning: A research review. *Reading & Writing Quarterly*, 23, 139–159.
- Schleppegrell, M. (2010). Language in mathematics teaching and learning: A research review. In J. N. Moschkovich (Ed.), *Language and mathematics education: Multiple perspectives and directions for research*. Charlotte, NC: Information Age Publishing.
- Scribner, S. (1984). Studying working intelligence. In B. Rogoff, & J. Lave (Eds.), *Everyday cognition: Its development in social context* (pp. 9–40). Cambridge, MA: Harvard University Press.
- Stein, M. K., Grover, B., & Henningsen, M. (1996). Building student capacity for mathematical thinking and reasoning: An analysis of mathematical tasks used in reform classrooms. *American Educational Research Journal*, 33(2), 455–488.
- Stevens, R., & Hall, R. (1998). Disciplined perception: Learning to see in technoscience. In M. Lampert, & M. L. Blunk (Eds.), *Talking mathematics in school: Studies of teaching and learning* (pp. 107–149). New York, NY: Cambridge University Press.
- Street, B. V. (2005). At last: Recent applications of new literacy studies in educational contexts. *Research in the Teaching of English*, 417–423.
- Street, B. (2005). The hidden dimensions of mathematical language and literacy. *Language and Education*, 19(2), 135–140.